UESTC3001 Dynamics & Control
Lecture 5

Characteristics and Performance of Feedback Control Systems - I

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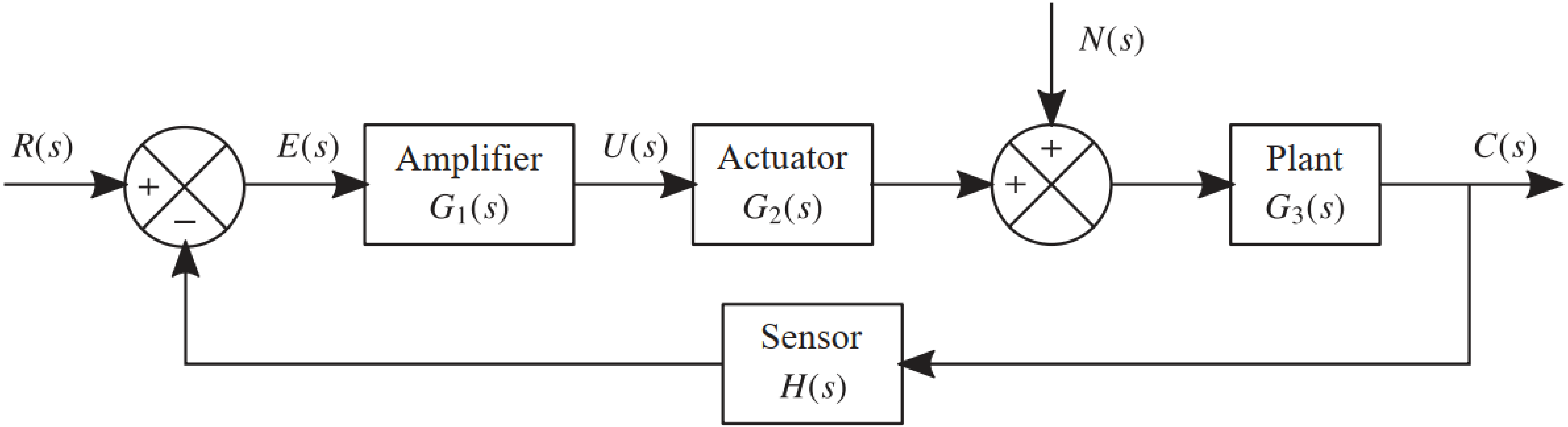
Outline

- Proportional Control, Derivative Control, Integral Control
- Proportional plus Integral Control, Proportional + Derivative Control, Proportional + Integral + Derivative Control
- Proportional Control of a First-Order/Second-order System and Effect on a First-Order/Second-order System
- Proportional + Derivative Control of a First-Order System and Effect on a First-Order System

Basic Control Actions

- Controller compares actual O/P with desired O/P
- Produce a control signal to reduce the deviation
- Different forms for the actuator, plant and sensor transfer functions



Proportional Control (P)

- Control signal is linearly proportional to the system error

$$u(t) = K_p e(t)$$

Derivative Control (D)

- Improve C/L system stability, speed up the transient response etc.
- Control signal is proportional to the derivative of the system error

$$u(t) = K_d \frac{de(t)}{dt}$$

- Usually augmented by proportional control
- Tends to amplify noise
- Introduced into the feedback path to eliminate response to I/P

Integral Control (I)

- Control signal is proportional to the integral of the system error

$$u(t) = K_i \int_0^t e(t)\,dt$$

- Minimize steady-state error; output response to disturbances
- Superior performance in the steady state
- Constant disturbances can be cancelled with zero error

Proportional plus Derivative Control (PD Control)

- Derivative action may be added to control action

$$u(t) = K_p e(t) + K_d \frac{de(t)}{dt}$$

- Derivative action speed the effect of the proportional action

Proportional plus Integral Control (PI Control)

- Proportional action adds a steady offset to a system's response. This may be reduced by adding integral action.

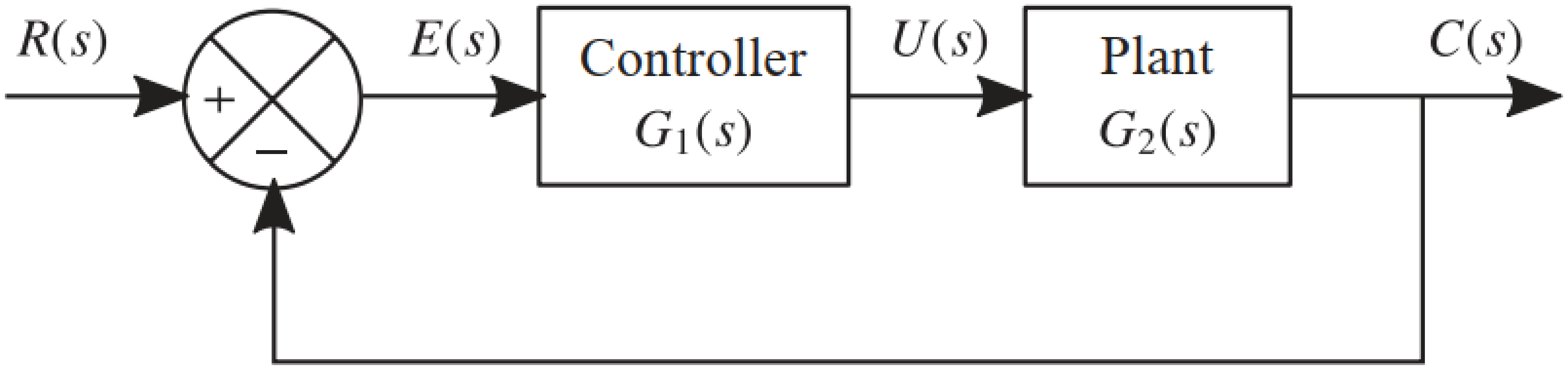
$$u(t) = K_p e(t) + K_i \int_0^t e(t)\,dt$$

Proportional plus Integral plus Derivative Control (PID Control)

- Putting all the three terms together results in PID

$$u(t) = K_p e(t) + K_i \int_0^t e(t)\,dt + K_d \frac{de(t)}{dt}$$

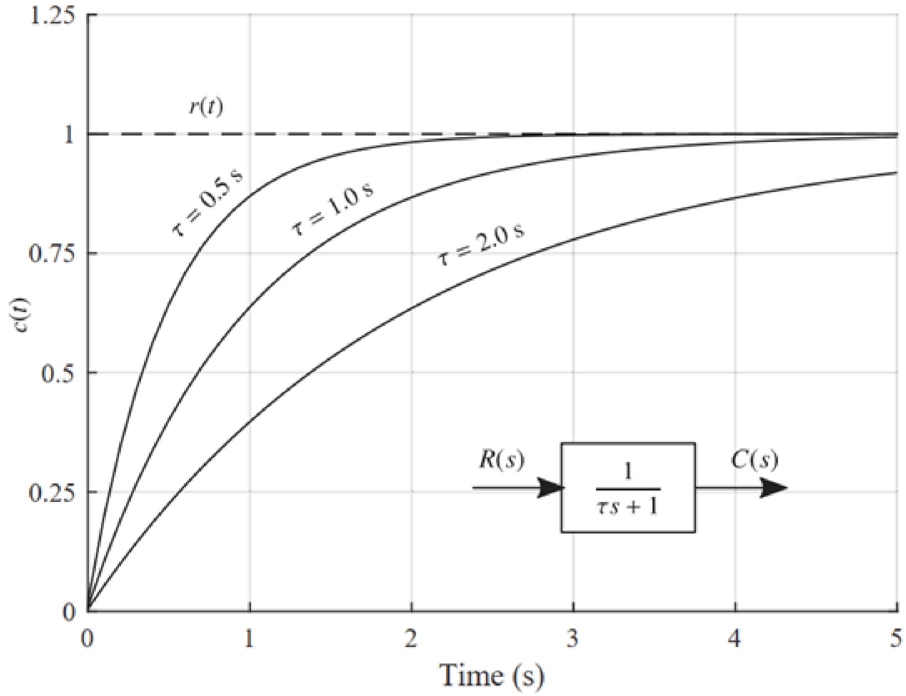
Effect of Control Actions



Uncontrolled Open-Loop Response of a First-Order System

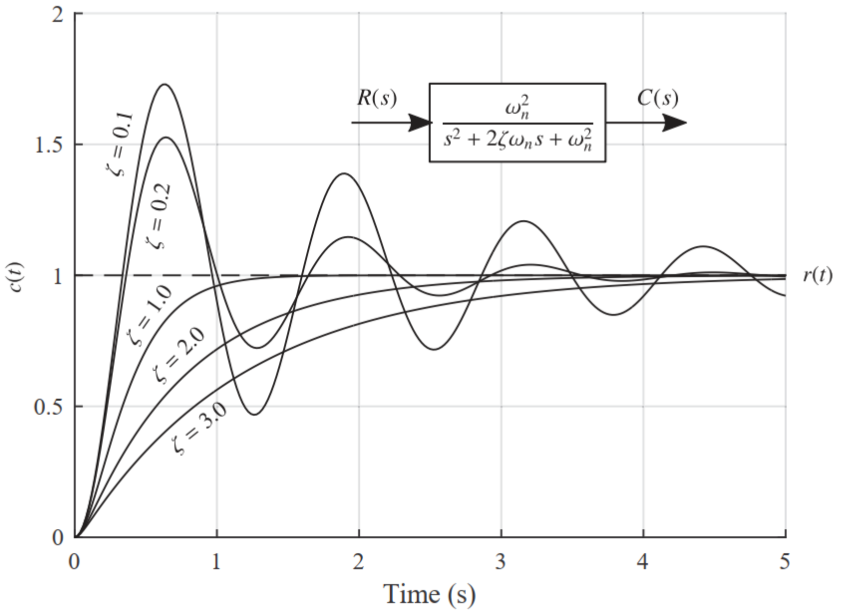
$$\frac{C(s)}{R(s)} = \frac{1}{\tau s + 1}$$

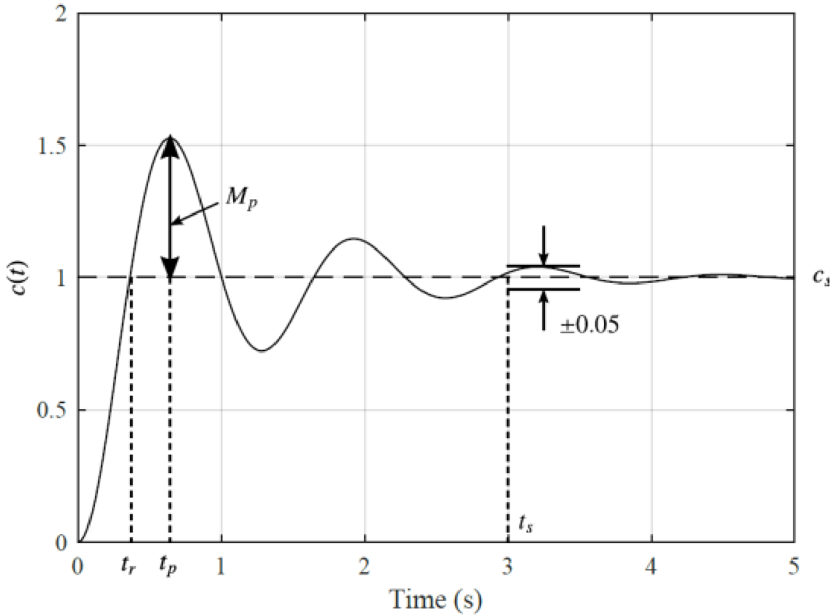
- E.g. find response for a unit step input



Uncontrolled Open-Loop Response of a Second-Order System

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$





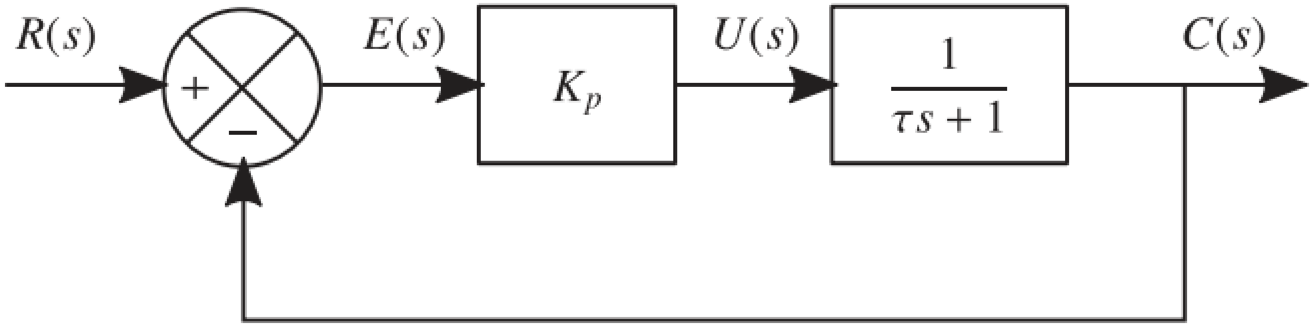
$$t_r = \frac{1}{\omega_d}\tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$$

$$t_p = \frac{\pi}{\omega_d}$$

$$M_p = 100e^{-\zeta\omega_n t_p}\%$$

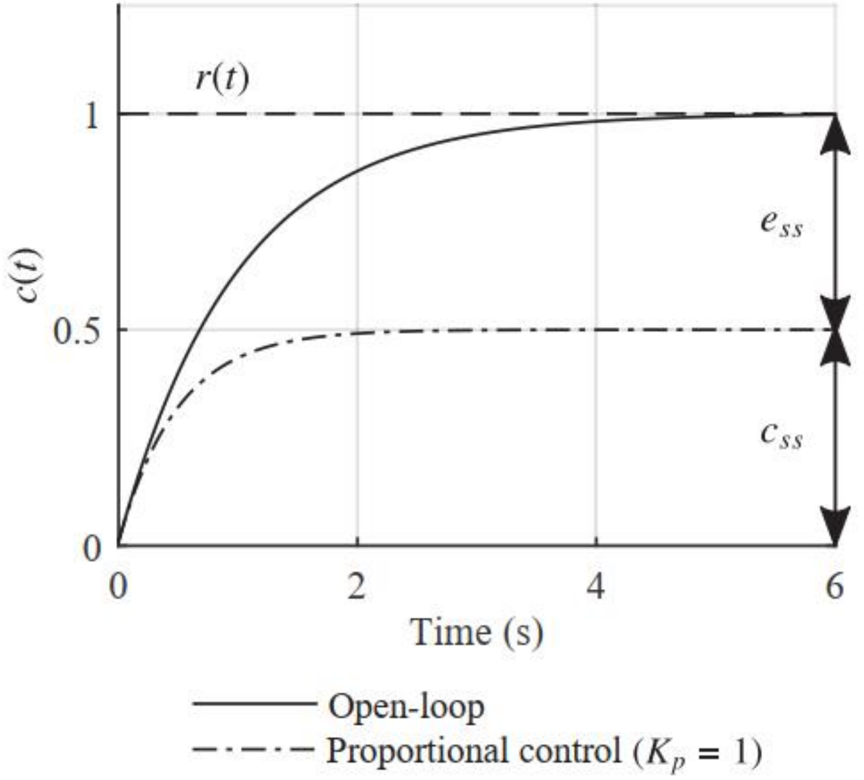
$$t_s = \frac{3}{\zeta\omega_n}$$

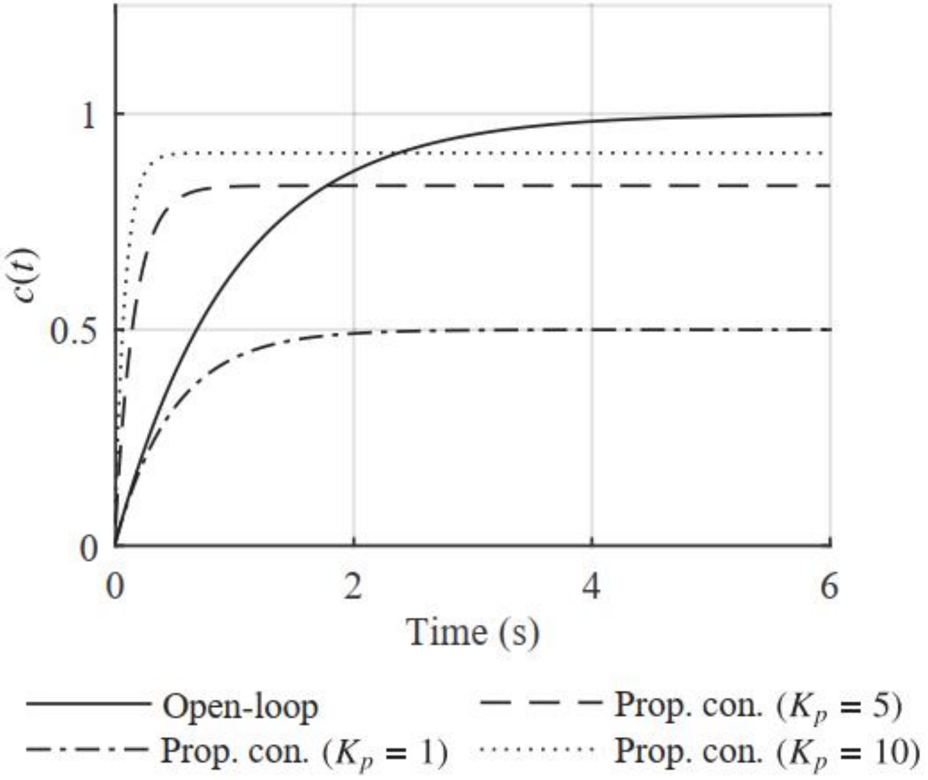
Proportional Control of a First-Order System



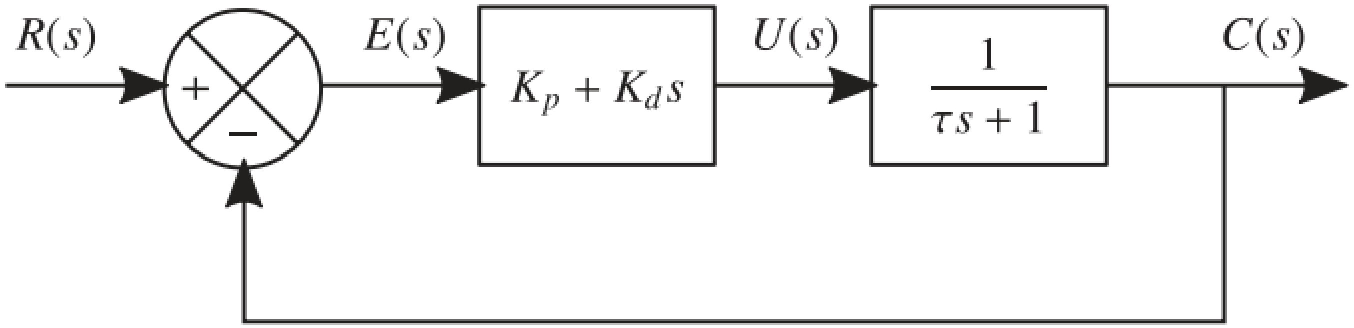
Effect of Proportional Control on a First-Order System

- Closed-loop response: $$c(t) = \frac{K_p}{K_p + 1}\left(1 - e^{-\frac{t}{\tau_{cl}}}\right) \qquad \tau_{cl} = \frac{\tau}{K_p + 1}$$



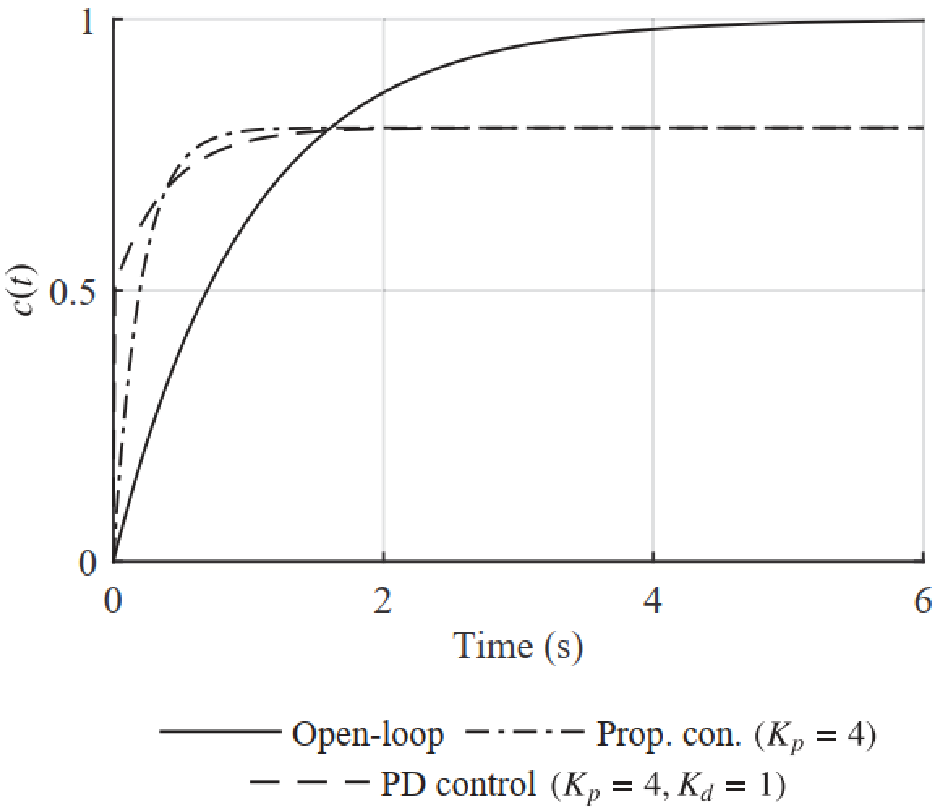


Proportional Plus Derivative Control of a First-Order System



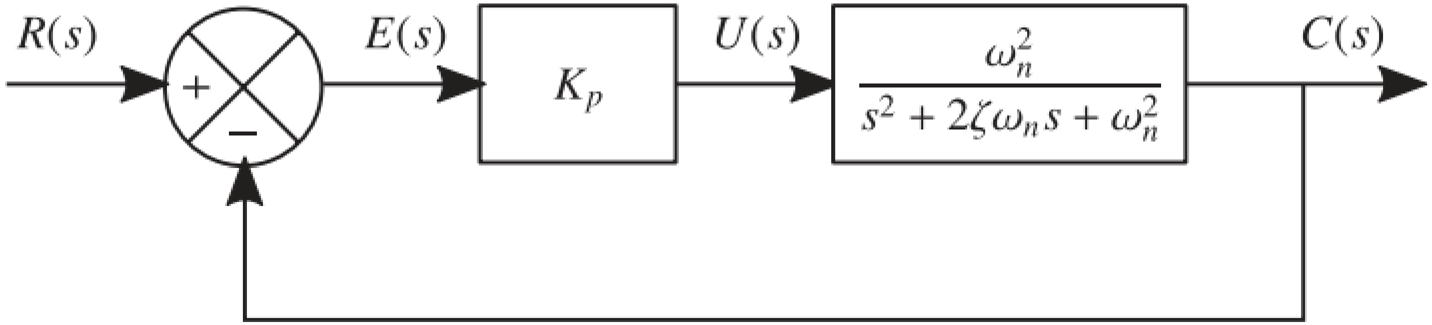
Effect of Proportional Plus Derivative Control on a First-Order System

- Closed-loop response: $$c(t) = \frac{K_p}{K_p + 1}\left[1 - \frac{K_p\tau - K_d}{K_p(\tau + K_d)}e^{-\frac{t}{\tau_{cl}}}\right]$$



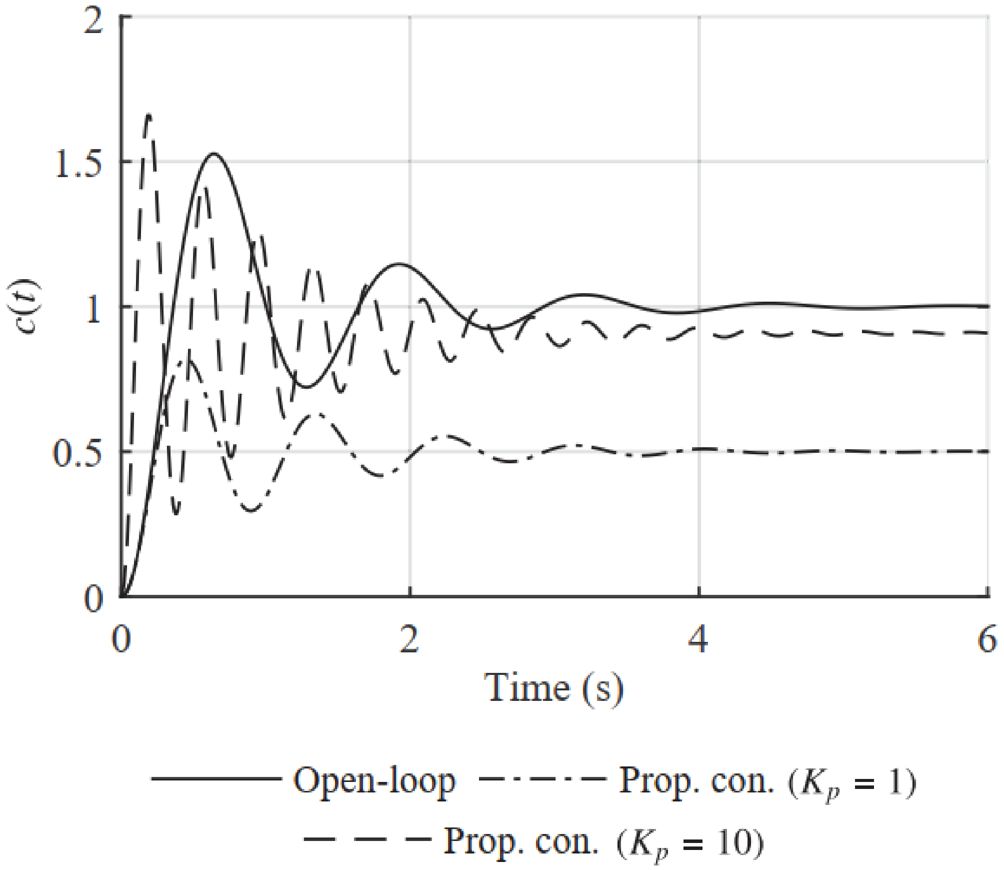
$$\tau_{cl} = \frac{\tau + K_d}{K_p + 1}$$

Proportional Control of a Second-Order System



Effect of Proportional Control on a Second-Order System

- Closed-loop response: $$c(t) = \frac{K_p}{K_p+1}\left[1 - e^{-\zeta\omega_n t}\left(\cos\omega_{d,cl}t + \frac{\zeta}{\sqrt{K_p+1-\zeta^2}}\sin\omega_{d,cl}\right)\right]$$



$$\omega_{d,cl} = \omega_n\sqrt{K_p+1-\zeta^2}$$

Exercise

A plant with transfer function *G(s)* is controlled by a controller of variable proportional gain K_p and unity negative feedback. Given

$$G(s) = \frac{1}{s\left(s^2 + 1.2s + 1\right)}$$

Show that the value of the proportional gain K_p has no influence on the steady state value of the response of the plant to a unit step input. What is the effect on the stability of the system of a negative value of K_p?

Investigate the stability of the system for the values: $K_p = 1; 1.5$

Summary

- Proportional Control, Derivative Control, Integral Control
- PI Control, PD Control, PID Control
- Proportional Control of a First-Order/Second-order System and Effect on a First-Order/Second-order System
- PD Control of a First-Order System and Effect on a First-Order System

Reference:
-Control Systems Engineering, 7th Edition, N.S. Nise
-UESTC3001 2019/20 Notes, J. Le Kernec