Dynamics and Control (UESTC 3001)

Notes prepared by: Dr Ola R. Popoola

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- Lecture 1: Root Locus Analysis
- Lecture 2: Root Locus II and Nyquist Plots
- Lecture 3: Bode Plots
- Lecture 4: Bode Plots II
- Lecture 5: Stability in Frequency Domain
- Lecture 6: Stability cont. and Stability Examples
- Lecture 7: Compensators
- Lecture 8: Tutorials and Test Exercises

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Intended Learning Objectives

At the end of this lecture, you will be able to:

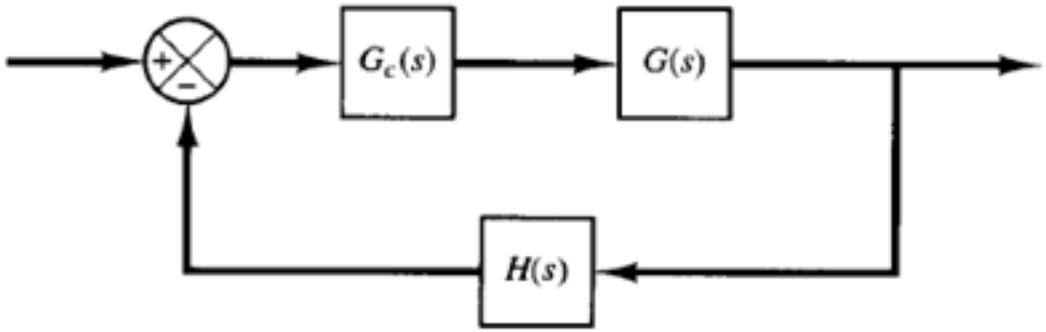
- Explain the concept of design, controllers and controller design
- Design lead compensators, lag compensators, and lead-lag compensators
- Discuss the purposes and benefits of the compensators
- Design compensator systems based on required system performances via worked examples

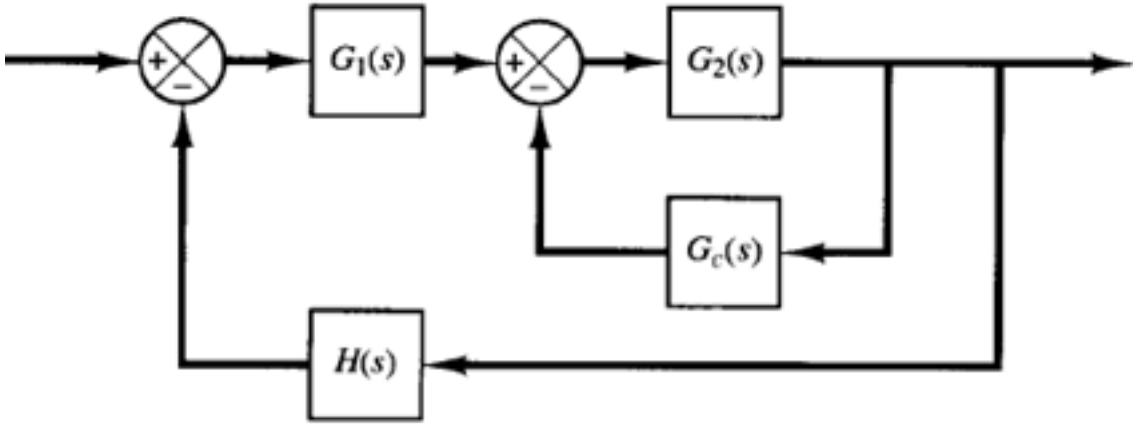
Introduction to Design

- The design of automatic control systems is one of the most important functions you will be required to carry out in your practice
- Design methods could either be by direct analysis or using trial by error methods where analysis are repeatedly applied
- Control system design for specific applications are required to meet certain performance specifications in time domain and/or in frequency domain such as:
  - Peak overshoot
  - Settling time
  - Gain margin
  - Phase margin
  - Bandwidth

Introduction to Design

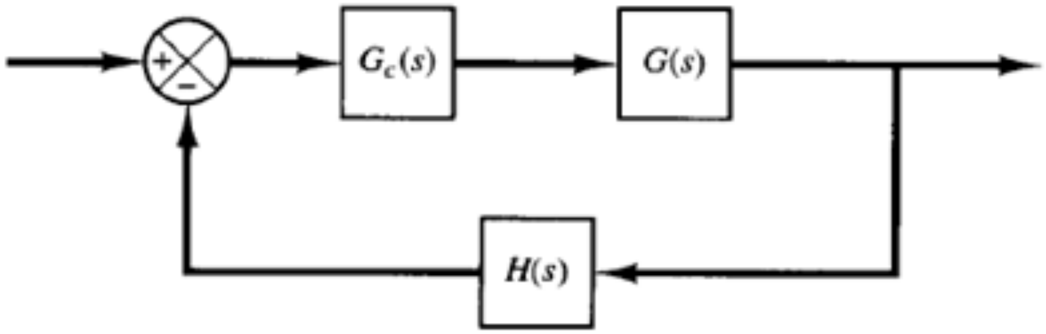
- In frequency domain the measure of relative stability is resonant peak M_r, or phase margin ϕ_{pm}, while the measure of speed of response is resonant frequency ω_r or bandwidth ω_b.
- Once specifications have been selected, you select a configuration for the system
- Compensator configuration could be in cascade or in parallel

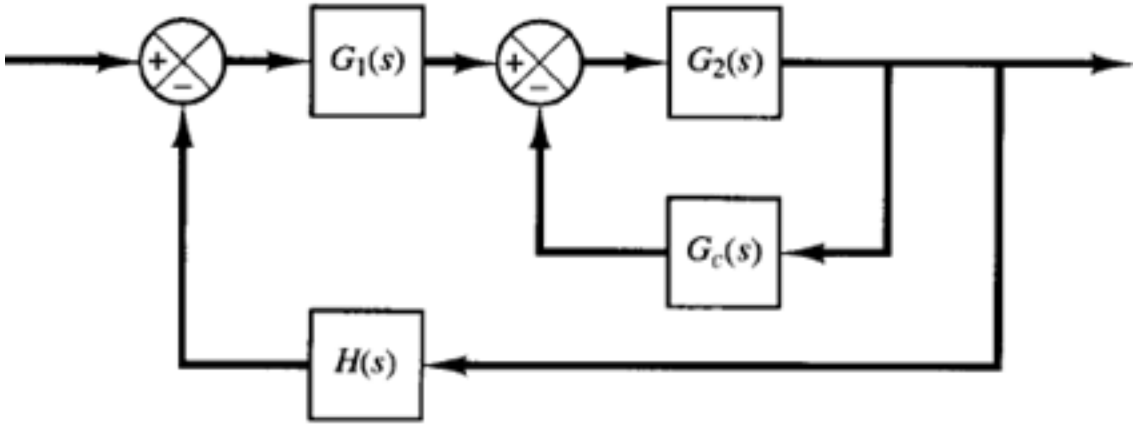




Introduction to Design

- In frequency domain the measure of relative stability is resonant peak M_r, or phase margin ϕ_{pm}, while the measure of speed of response is resonant frequency ω_r or bandwidth ω_b.
- Once specifications have been selected, you select a configuration for the system
- Compensator configuration could be in cascade (or series compensation) or in parallel (or feedback compensation)





Controllers

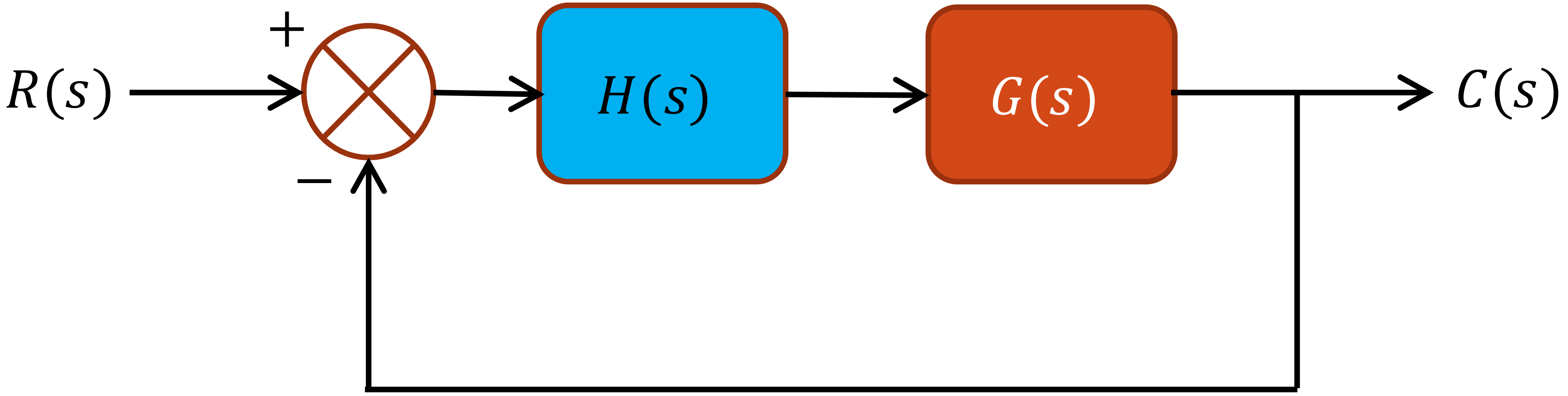
- **For a closed loop system, controller H(s) can be:**

Simple gain: K_p

An integrator: $H(s) = \frac{K_i}{s}$

A differentiator: $H(s) = K_d s$

PID



Compensators

- Compensation is the modification of the system dynamics to satisfy the given specifications.
- A controller manages system behaviour using feedback to maintain desired performance.
- A compensator, often a part of a controller, specifically modifies system dynamics to enhance performance by addressing issues like stability and transient response.
- Controllers ensure overall regulation, while compensators fine-tune specific system characteristics.

Compensator Properties

- Sometimes the job of the control system designer is simply to choose an appropriate value of controller gain, K.
- But often a compensator network is required to reshape the Nyquist locus.
- Compensators are often designed to leave one part of the locus unchanged, while having a significant effect on the important part of the locus where the uncompensated system is unsatisfactory.

Compensator Types

- There are three main types of compensators and they are:

- Lead compensators
  - Improves transient response of the system

- Lag compensators
  - Improves the steady-state performance at the expense of slower settling time

- Lead-Lag compensators
  - Combines the performance of both Lead and Lag compensators

Selection of a compensator

- Two situations that requires compensation are:

- Absolutely unstable system, compensation required to stabilize and achieve a specified performance.
  - Lead compensators are used to increase the margin of stability

- Stable system, compensation required to obtain desired performance
  - Lead, Lag, Lead-lag compensators could be used
  - Particular choice are based on factors subsequently discussed

Lead Compensator

- Transfer function of a Lead compensator

$$G_c(s) = K\frac{\left(1+\frac{s}{\omega_c}\right)}{1+\frac{s}{\alpha\omega_c}} = K\alpha\frac{s+\omega_c}{s+\alpha\omega_c}$$

- $\alpha > 1$, usually > 10
- Zeros of compensator at $s = -\omega_c$
- Poles of compensator at $s = -\alpha\omega_c$

- Some properties
  - Low frequency gain: K
  - High frequency gain: αK

Lead Compensator

- **Phase property:**

$$\angle G_c(j\omega) = \tan^{-1}\frac{\omega}{\omega_C} - \tan^{-1}\frac{\omega}{\alpha\omega_C} = \tan^{-1}\left[\frac{\frac{\omega}{\omega_C}\left(1-\frac{1}{\alpha}\right)}{1+\frac{\omega^2}{\alpha\omega_C^2}}\right]$$

- **Phase angle is always positive**
- **Maximum phase angle occurs when:**

$$\left(1+\frac{\omega^2}{\alpha\omega_C^2}\right).\frac{1}{\omega_C} - \frac{\omega}{\omega_C}.\frac{2\omega}{\alpha\omega_C^2} = 0$$

$$\omega^2 = \alpha\omega_C^2 \implies \omega = \sqrt{\alpha}\omega_C$$

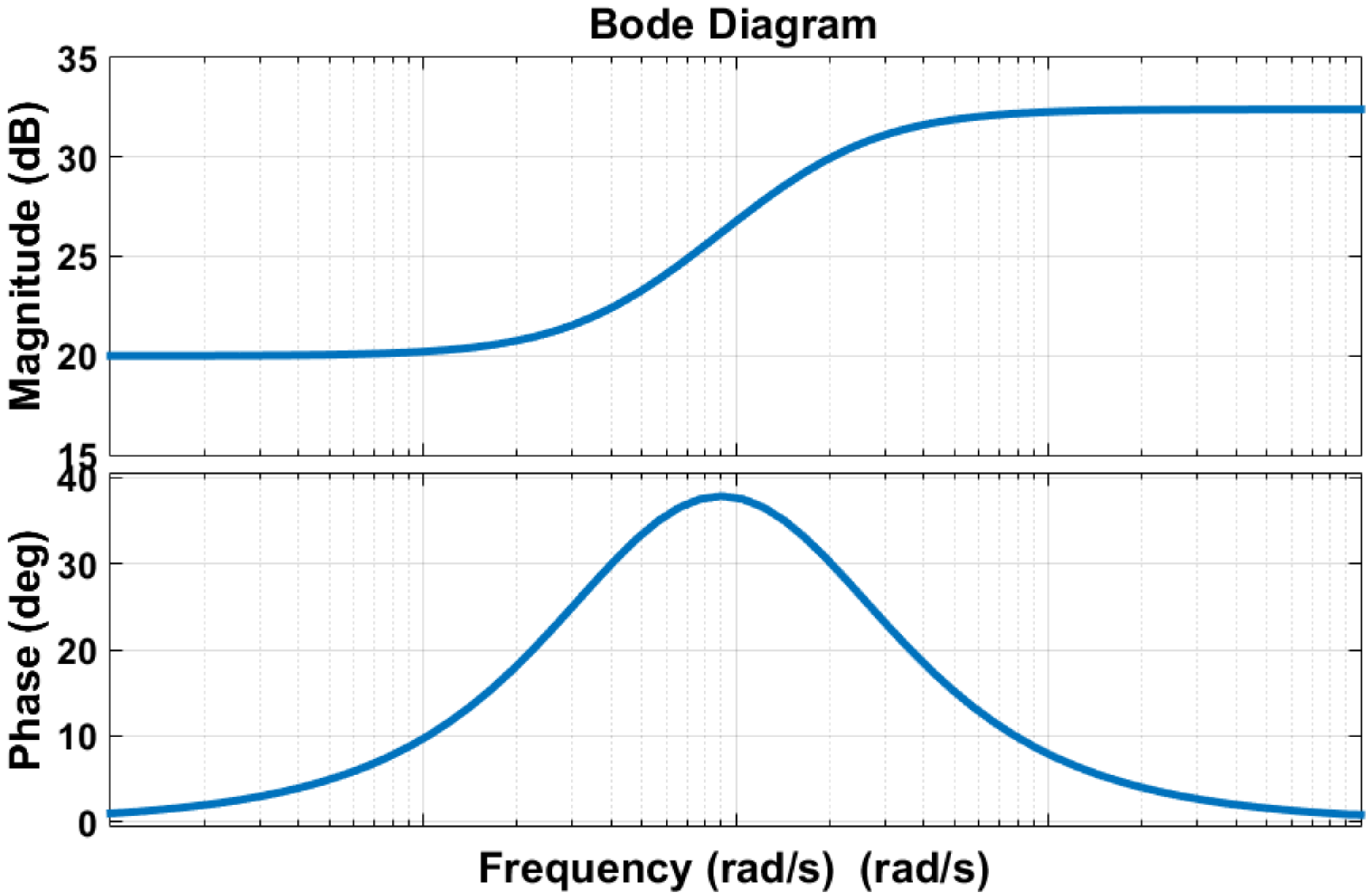
Lead Compensator

- **Corresponds to "half way" point between two breakpoints in the bode diagram**

- **By substitution, we can compute the maximum phase as:**

$$\phi_m = \tan^{-1}\left(\frac{\alpha - 1}{2\sqrt{\alpha}}\right)$$

Bode plot of Lead Compensator



Additional Properties of Lead Compensator

- The polar locus of the compensator is a semi-circle (see example of next slide).

- At any frequency the gain > 1 (or K if an extra gain is added)

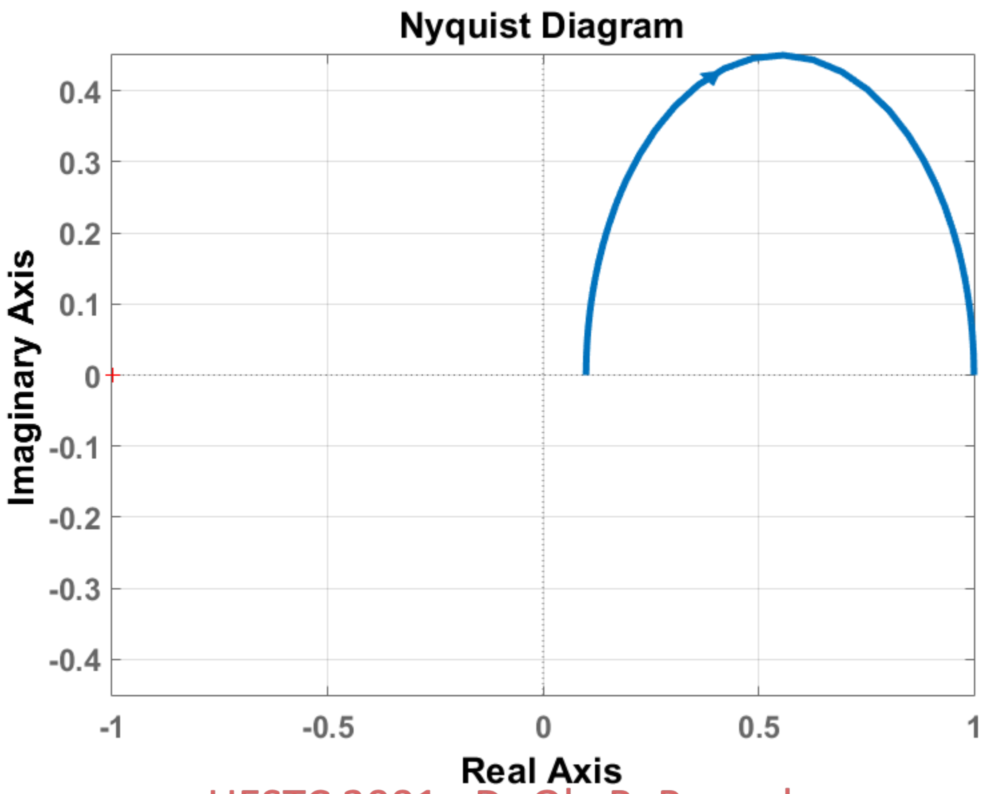
- The purpose of the LEAD compensator is to improve the high-frequency behaviour of the system. It increases bandwidth (faster) and phase margins (less resonant).

- Control system designers have two variables to choose: α and ω_c.

Lead Compensator

- **Sketch the Nyquist plot of a compensator given by:**

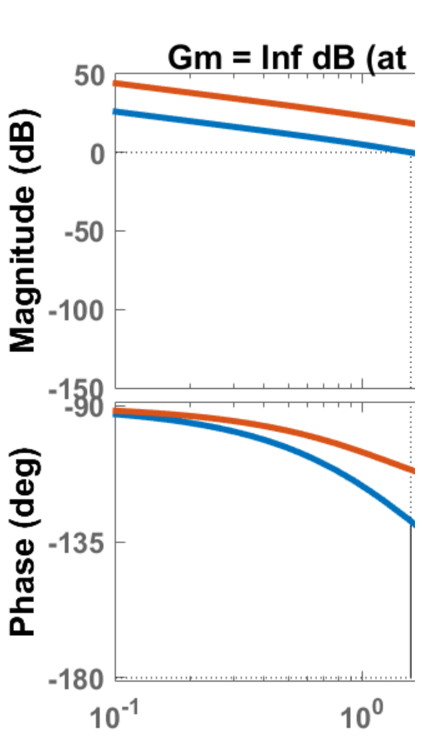
$$C(s) = \frac{10+s}{100+s}$$

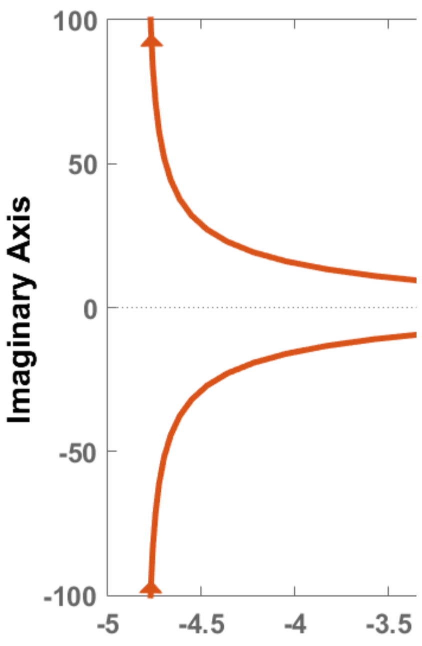


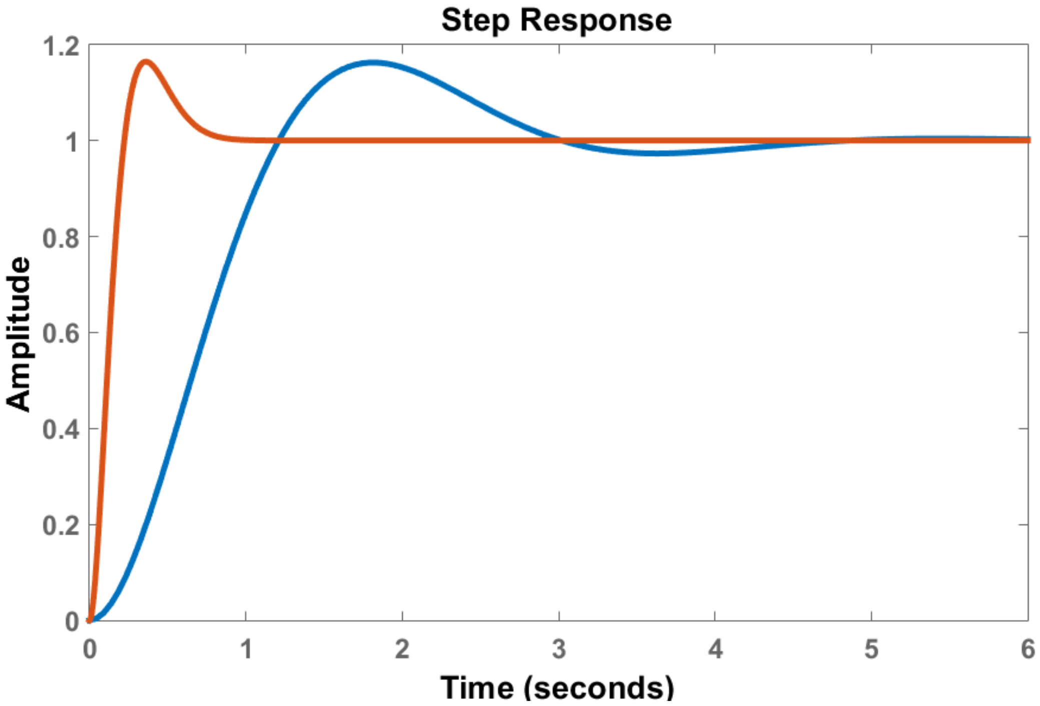
Examples of Impact of Lead Compensator

- **Given a system and compensator transfer function given below:**

$$G(s) = \frac{4}{s(s+2)};\ C_s(s) = 40\,\frac{s+4}{s+20}$$







Lag Compensator

- Transfer function of a Lead compensator

$$G_c(s) = K\alpha \frac{\left(1 + \dfrac{s}{\alpha\omega_c}\right)}{1 + \dfrac{s}{\omega_c}} = K\frac{s + \alpha\omega_c}{s + \omega_c}$$

- $\alpha > 1$, usually < 10
- Zeros of compensator at $s = -\alpha\omega_c$
- Poles of compensator at $s = -\omega_c$

- Some properties
  - Low frequency gain: αK
  - High frequency gain: K

Lag Compensator

- **Phase property:**

$$\angle G_c(j\omega) = \tan^{-1}\frac{\omega}{\alpha\omega_C} - \tan^{-1}\frac{\omega}{\omega_C} = \tan^{-1}\left[\frac{\frac{\omega}{\omega_C}\left(\frac{1}{\alpha} - 1\right)}{1 + \frac{\omega^2}{\alpha\omega_C^2}}\right]$$

- **Phase angle is always negative**
- **Maximum phase angle occurs when:**

$$\left(1 + \frac{\omega^2}{\alpha\omega_C^2}\right) \cdot \frac{1}{\omega_C} - \frac{\omega}{\omega_C} \cdot \frac{2\omega}{\alpha\omega_C^2} = 0$$

$$\omega^2 = \alpha\omega_C^2 \implies \omega = \sqrt{\alpha}\omega_C$$

Lag Compensator

- **Corresponds to "half way" point between two breakpoints in the bode diagram**

- **By substitution, we can compute the maximum phase as:**

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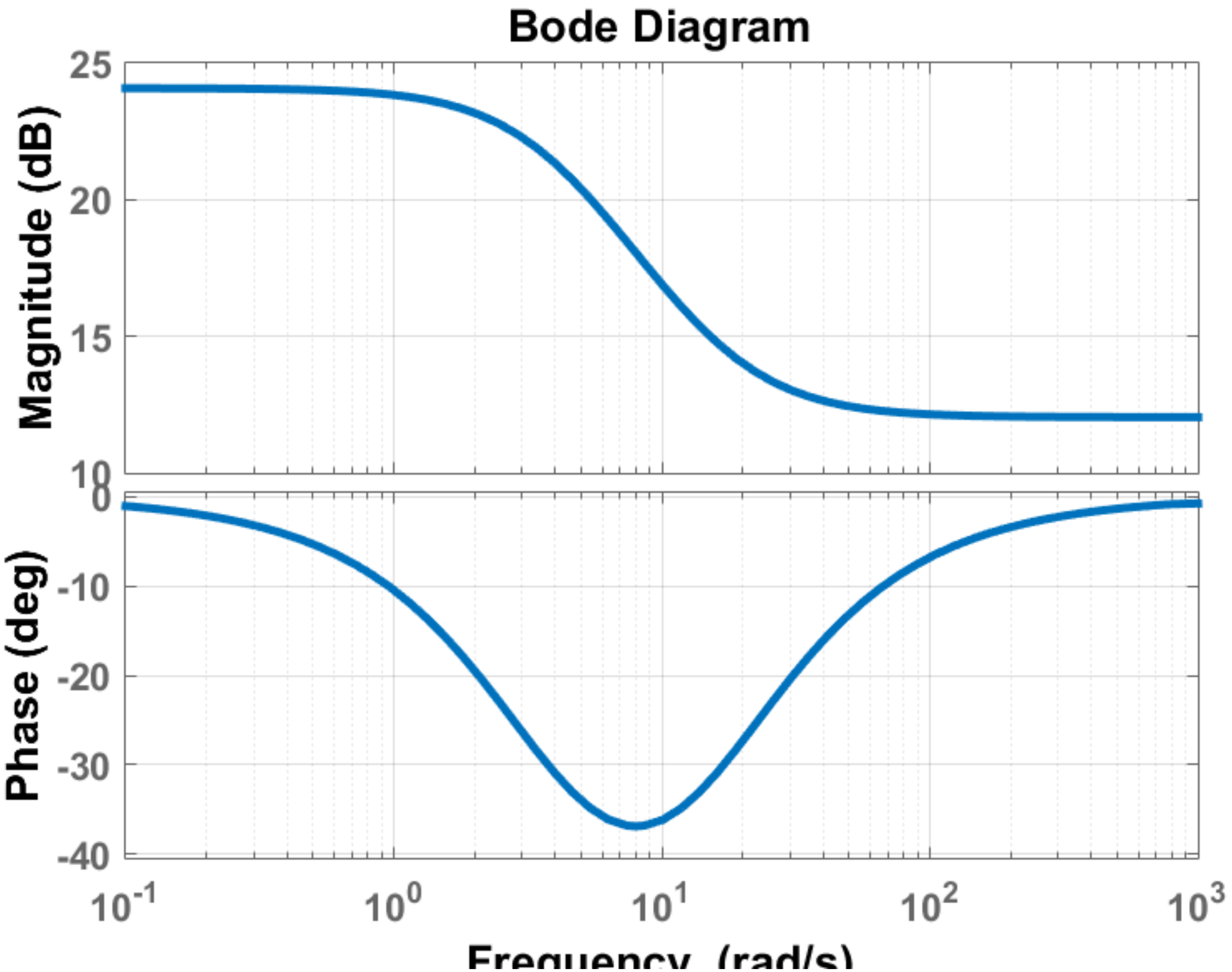
Bode plot of Lag Compensator



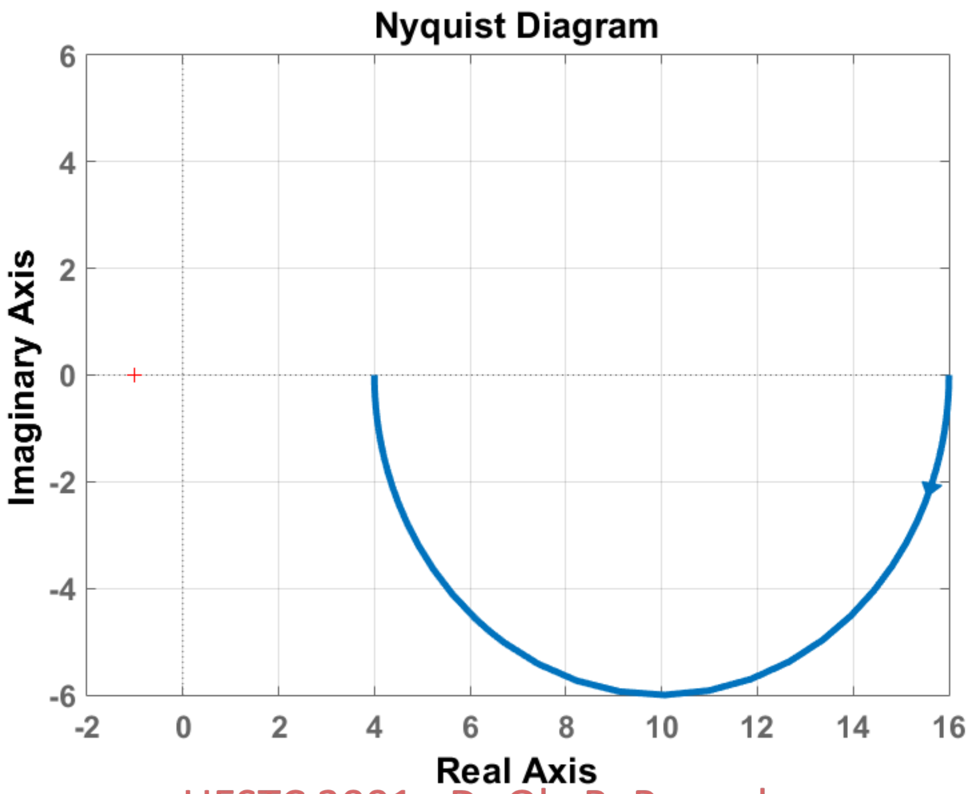
Additional Properties of Lag Compensator

- The polar locus of the compensator is a semi-circle.
- At any frequency the gain > 1
- The purpose of the LAG compensator is to improve the low-frequency behaviour of the system. It increases the gain of the system, while preserving approximately the same phase-margin, hence improving steady-state and low frequency performance.
- Control system designer has two variables to choose: α and ω_c.

Lag Compensator

- **Sketch the Nyquist plot of a compensator given by:**

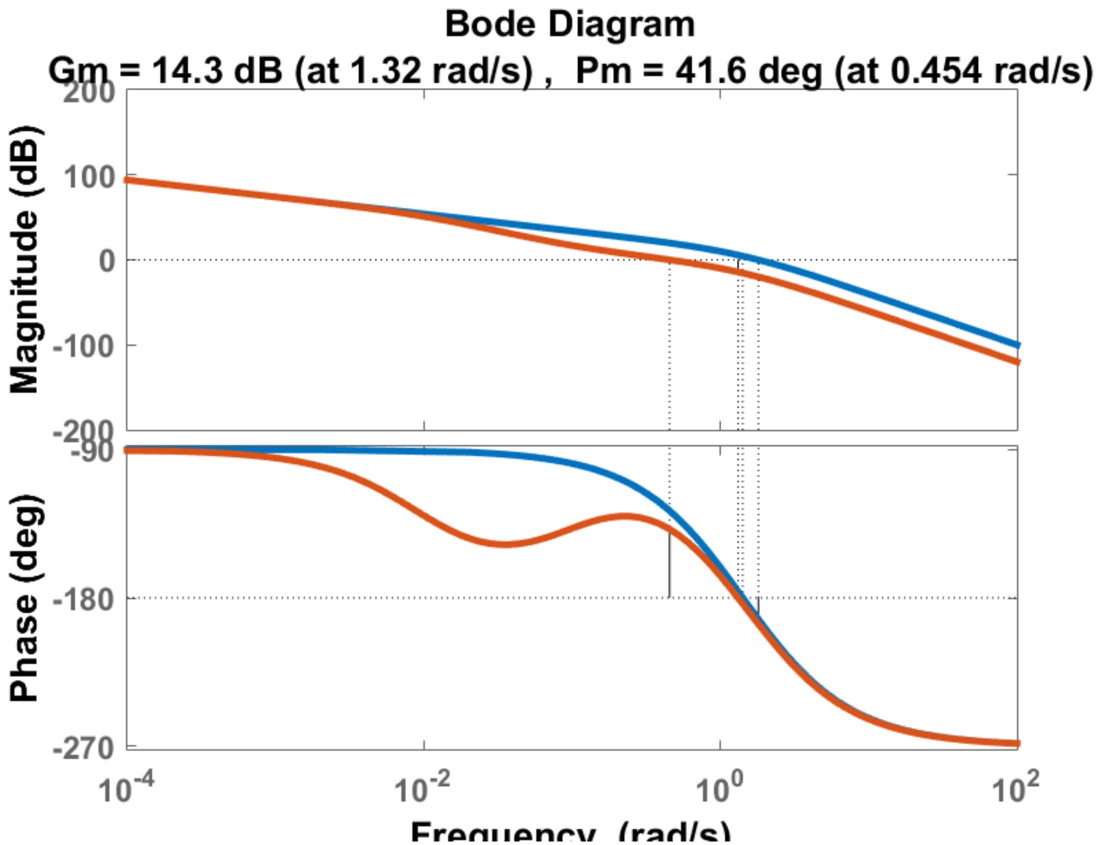
$$C(s) = 4\frac{16+s}{4+s}$$

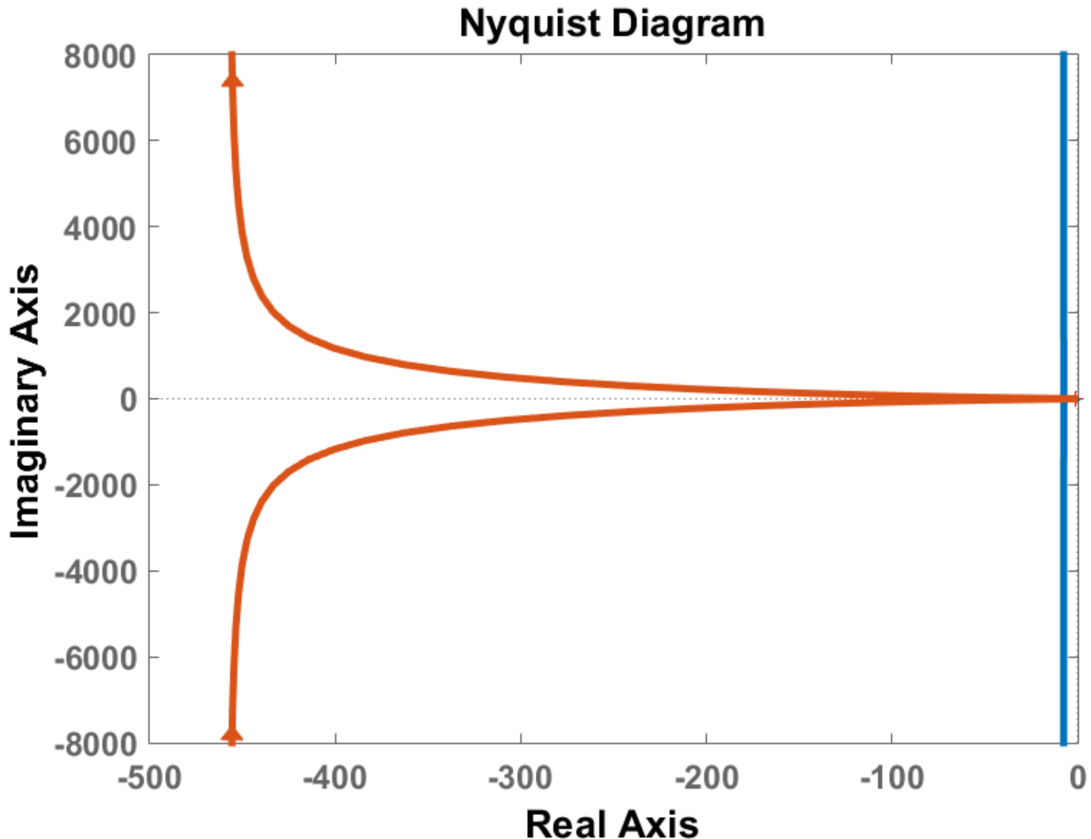


Examples of Impact of Lag Compensator

- **Given a system and compensator transfer function given below:**

$$G(s) = \frac{4}{s(s+1)(0.5s+1)};\ C_s(s) = 0.1\frac{s+0.1}{s+0.05}$$





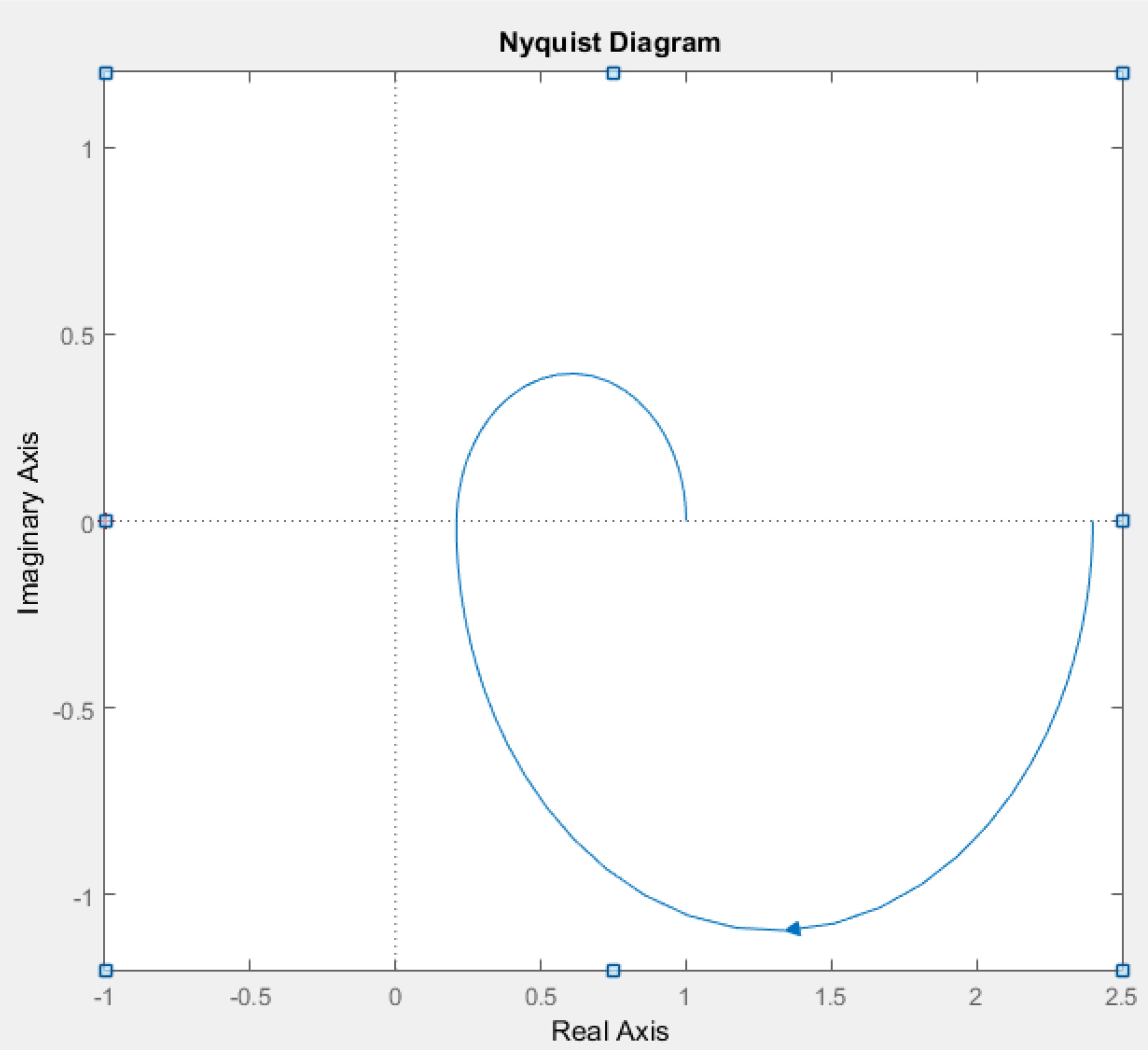
Lead-Lag compensators

- The LAG compensator increases the gain of the open loop for approximately the same phase margin, but at the expense of having a slow mode in the response.

- For some systems both the gain and the natural frequency need to be improved. This can be done using LEAD-LAG compensators.

Lead-Lag Compensators

$$C(s) = \frac{\left(1+\frac{s}{\omega_2}\right)\left(1+\frac{s}{\omega_3}\right)}{\left(1+\frac{s}{\omega_1}\right)\left(1+\frac{s}{\omega_4}\right)}$$



Comparison between lead and lag compensators

LEAD Compensator	LAG Compensator
High Pass	Low Pass
Approximates derivative plus proportional control	Approximates integral plus proportional control
Contributes phase lead	Attenuation at high frequencies
Increases gain crossover frequency	Moves gain crossover frequency lower
Increases bandwidth	Reduces bandwidth

Lead compensator design in frequency domain

1. Determine the loop gain K to satisfy specified error constant

2. Using this value of K, find the phase margin of uncompensated system

3. Determine the phase-lead required by $\phi_l = \phi_s - \phi_1 + \epsilon$, where ϕ_s is specified PM, ϕ_1 is PM of uncompensated system, ϵ is margin of safety required by the fact that the cross-over frequency will increase due to compensation.

Lead compensator design in frequency domain

4. Let $\phi_m = \phi_l$ and determine α-parameter by $\alpha = \frac{1-sin\phi_m}{1+sin\phi_m}$

5. Calculate the db-gain $10\log(\frac{1}{\alpha})$ provided by the network at ω_m. Find the crossover frequency of compensated system $\omega_{c2} = \omega_m$, where uncompensated gain is $-10\log(\frac{1}{\alpha})$

6. Compute corner frequencies as $\omega_1 = \frac{1}{\tau} = \omega_m\alpha; \omega_2 = \frac{1}{\alpha\tau} = \frac{\omega_m}{\sqrt{\alpha}}$

7. Draw magnitude and phase plot of compensated system, examine and repeat above if necessary.

Lead compensator design in frequency domain

- For a unity feedback system with an open loop transfer function given as: $G_f(s) = \frac{K_v}{s(s+1)}$. It is specified that $K_v = 12$ sec^{-1} and $\phi_{pm} = 40°$

- Draw the Bode plot with $K_v = 12$

- The PM of uncompensated system is 15 deg. If a lead compensator is used, phase lead required is: $\phi_l = 40 - 15 + 5 = 30° = \phi_m$,

$$\alpha = \left((1 - \sin 30)/(1 + \sin 30)\right) = 0.333$$

Lead compensator design in frequency domain

- Magnitude contribution at ω_m is 10log(1/0.333) = 4.8 dB

- New crossover frequency, frequency where uncompensated system has magnitude of -4.8dB from Bode plot is: $\omega_{c2} = \omega_m = 4.6$ rad/sec

- Lower corner frequency $\omega_1 = \omega_m\sqrt{\alpha}$ = 4.6x√0.33 = 2.65 rad/sec

- Upper corner frequency $\omega_1 = \frac{\omega_m}{\sqrt{\alpha}}$ = 4.6/√0.333 = 8 rad/sec

Lead compensator design in frequency domain

- Transfer function of the lead network becomes $\frac{1+\frac{s}{2.65}}{1+\frac{s}{8}} = \frac{0.377s+1}{0.125s+1}$
- Amplification necessary to cancel the lead network attenuation is $A = \frac{1}{\alpha} = 3$
- Draw the bode plot of uncompensated system and compare with bode plot of compensated system

Lag compensator design in frequency domain

- Determine the loop gain K to satisfy specified error constant

- Find frequency ω_{c2} where uncompensated system makes a phase margin contribution of $\phi_2 = \phi_s + \epsilon$. ϕ_2 is measured above the -180 deg line, 5< ϵ<15

- Measure gain of uncompensated system at ω_{c2} and equate it to high frequency network attenuation (20 log β), find β

Lag compensator design in frequency domain

- Choose corner frequency of network ω_2 one octave to decade below $\omega_{c2} : \omega_2 = \frac{\omega_{c2}}{2}\ to\ \frac{\omega_{c2}}{10} = \frac{1}{\tau}$, find τ

- Use β and τ to draw the frequency response of the system and check the phase margin

- Check if specifications are met and adjust ϵ and repeat if necessary

- Lag compensator reduces bandwidth but improves signal/noise ratio