GLASGOW COLLEGE UESTC

Exam

Dynamics and Control (UESTC3001)

Date: 20th June 2022

Time: 09:30-11:30

Attempt All Questions. Total 100 marks

All questions bear equal marks [25 marks]
Use one answer sheet for each of the questions in this exam.
Show all work on the answer sheet.
Make sure that your University of Glasgow and UESTC Student Identification Numbers are on all answer sheets.
An electronic calculator may be used provided that it does not allow text storage or display, or graphical display.
All graphs should be clearly labelled and sufficiently large so that all elements are easy to read.
The numbers in square brackets in the right-hand margin indicate the marks allotted to the part of the question against which the mark is shown. These marks are for guidance only.

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Q1

a) Explain the logical sequence for the design of a feedback control system.

[6 marks]

b) The block diagram of a control system is shown in Figure Q1.

i) Using block diagram reduction techniques, you can represent multiple subsystems as a single transfer function. Briefly explain the importance of it.

[3 marks]

ii) Reduce the block diagram shown in Figure Q1 to a single block using block diagram reduction techniques showing the intermediate steps of your simplification. [10 marks]

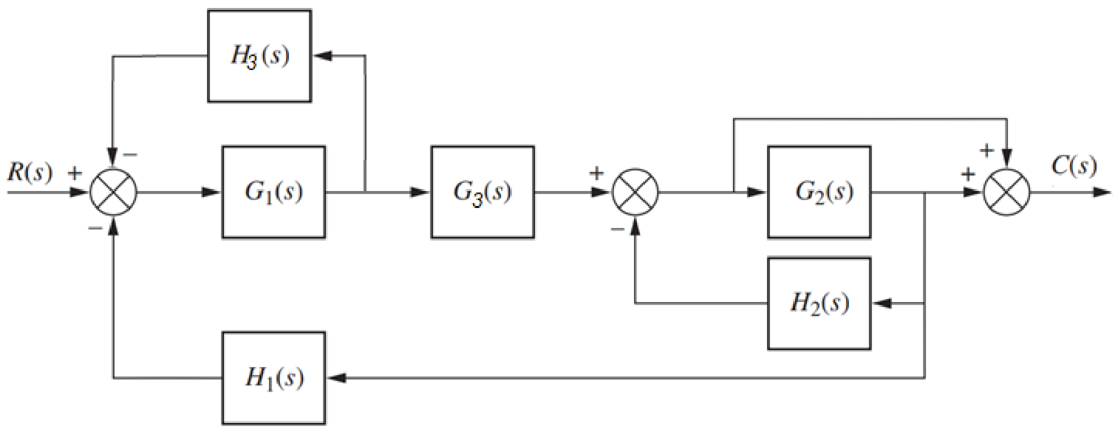

Figure Q1

c) Given the characteristic equation below, determine the stability of the system.

$$s^4 + 2s^3 + 2s^2 + 3s + 1 = 0$$

[6 marks]

Q2

a) Derivative control is almost never used by itself; it is usually augmented by proportional control. Explain why. [5 marks]

b) A first-order system with proportional feedback control can be modelled as shown in Figure Q2.

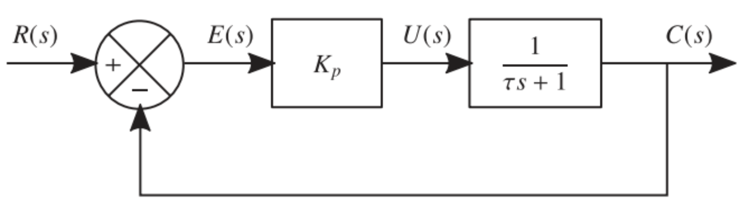

Figure Q2

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i) Obtain expressions for the closed-loop transfer function and for the error transfer function. [6 marks]

ii) For a unit step input, obtain the expressions for final response and for the steady-state error of the system. [6 marks]

iii) If the time constant, τ, is 1 second, find the range for the proportional gain, K_p, which is required to ensure a steady-state error of 5% or below. [3 marks]

iv) In Figure Q2, if you replace proportional gain, K_p, with an integral controller, comment on the steady-state error of the system for a unit step input with appropriate mathematical proof. [5 marks]

Q3

A unity feedback control system has an open loop transfer function $KG(s) = \frac{K}{s(s^2+4s+13)}$ your task is to sketch the root locus plot by determining the following

a. The value of the poles and zeros and the order of the system [3 marks]
b. The centroid and angle of asymptotes [4 marks]
c. Breakaway points from the real axis if any [3 marks]
d. Now sketch the bode plot showing the direction of travel of the poles [6 marks]
e. The value of k and the frequency at which the loci cross the jw-axis [6 marks]
f. Write a reflection on the change in characteristics as K increases. (Hint: discuss the regions where the system is stable, marginally stable, and unstable) [3 marks]

Q4

4. You have been asked to show the frequency response of a damper system with the transfer function $G(s) = \frac{10K}{s(1+0.5s)(1+0.01s)}$.

a) For system gain K =1, determine the four components of the transfer function and their corner frequencies [7 marks]
b) evaluate the slopes and phase angles of the main components in the transfer function [6 marks]
c) sketch the Bode plots showing the magnitude in decibels and phase angle in degrees as a function of log frequency [6 marks]
d) determine the value of the system gain K for the gain cross-over frequency ω_c to be 2 rad/sec (hint: at gain cross over frequency, $|G(j\omega)| = 1$) [6 marks]

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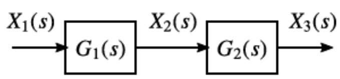
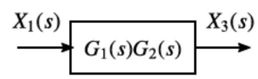
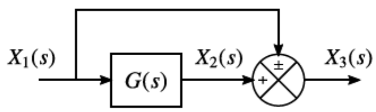
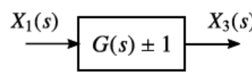
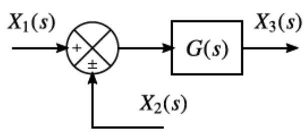
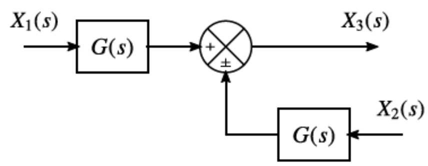
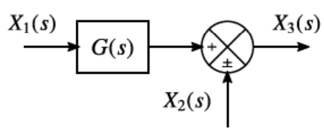
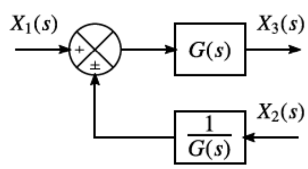
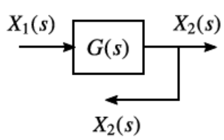
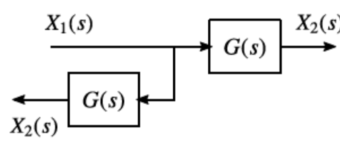
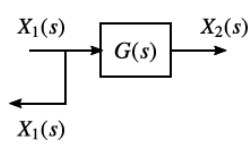
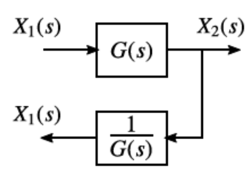
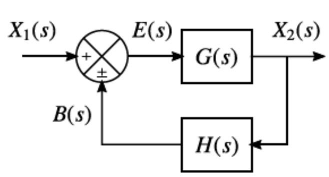
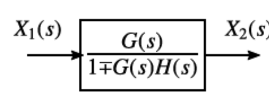
Appendix 1

Rules for block diagram reduction

Rule	Original	Equivalent
1. Cascaded blocks		
2. Summing two signals		
3. Moving a summing point behind a block		
4. Moving a summing point ahead of a block		
5. Moving a branch point ahead of a block		
6. Moving a branch point behind a block		
7. Eliminating a feedback loop		

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Appendix 2

Root Locus Analysis

Properties of the Open Loop Transfer Function

The characteristic equation of a system may be written in the form: $1 + F(s) = 0$ where $F(s)$ is the open loop transfer function. This gives the magnitude condition that

$$|F(s)| = 1$$

.

and given that he open loop transfer function takes the form:

$$F(s) = \frac{K(s - z_1)(s - z_2)\cdots(s - z_v)}{s^n(s - p_1)(s - p_2)\cdots(s - p_u)}$$

we can write

$$K = \frac{\prod_{j=1}^{n+u} |s - p_j|}{\prod_{i=1}^{v} |s - z_i|}$$

Sketching a Root Locus

Angle of Asymptotes: $\phi = \dfrac{(2m+1)\times 180}{P - Z}$ degrees for m = 0, 1, 2, ... (P - Z - 1)

The intersection point: $\sigma_A = \dfrac{\sum_{j=1}^{P} \mathrm{Re}(p_j) - \sum_{i=1}^{Z} \mathrm{Re}(z_i)}{P - Z}$

Breakaway Point: $\dfrac{dK}{ds} = 0$

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Appendix 3: Frequency Response Analysis

General theory

For a system with closed loop transfer function: $G(s) = \dfrac{X(s)}{Y(s)}$

a periodic input $y(t) = Y \sin \omega t$

yields a steady state response $x(t)_{ss} = Y|G(i\omega)|\sin(\omega t + \phi)$

where $|G(i\omega)| =$ the gain of the system,

and $\phi = \angle G(i\omega)$ = the phase of the system.

Elements of the Bode plot

Factor	**G(s)**	**G(iω)**	$\lvert G(i\omega)\rvert_{dB}$	$\angle G(i\omega)$ **rad**
Gain	K	K	$20\log_{10} K$	0
Poles at Origin	$\dfrac{1}{s^n}$	$\dfrac{1}{i^n \omega^n}$	$-20n\log_{10}\omega$	$-n\dfrac{\pi}{2}$
Zeros at Origin	s^n	$i^n \omega^n$	$20n\log_{10}\omega$	$n\dfrac{\pi}{2}$
Pole	$\dfrac{1}{1+\tau s}$	$\dfrac{1}{1+i\omega\tau}$	$\omega \to 0:\ 0$ $\omega \to \infty:$ $-20\log_{10}(\tau\omega)$	$\omega \to 0:\ 0$ $\omega \to \infty:\ -\dfrac{\pi}{2}$
Zero	$1 + \tau s$	$1 + i\omega\tau$	$\omega \to 0:\ 0$ $\omega \to \infty:$ $20\log_{10}(\tau\omega)$	$\omega \to 0:\ 0$ $\omega \to \infty:\ \dfrac{\pi}{2}$
Quadratic Poles	$\dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$	$\dfrac{\omega_n^2}{(\omega_n^2 - \omega^2) + 2\zeta\omega_n \omega i}$	$\omega \to 0:\ 0$ $\omega \to \infty:$ $-40\log_{10}(\tau\omega)$	$\omega \to 0:\ 0$ $\omega \to \infty:\ -\pi$
Quadratic Zeros	$s^2 + 2\zeta\omega_n s + \omega_n^2$	$(\omega_n^2 - \omega^2) + 2\zeta\omega_n \omega i$	$\omega \to 0:\ 0$ $\omega \to \infty:$ $40\log_{10}(\tau\omega)$	$\omega \to 0:\ 0$ $\omega \to \infty:\ \pi$

End of question paper