UESTC3001 Dynamics & Control
Lecture 4

Control System Stability

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Outline

- Properties of the Transfer Function
- System Stability

Properties of the Transfer Function

- Separate the input, system, and output
- Algebraically combine mathematical representations of subsystems
- G(s) is the ratio of polynomials in s domain

e.g.

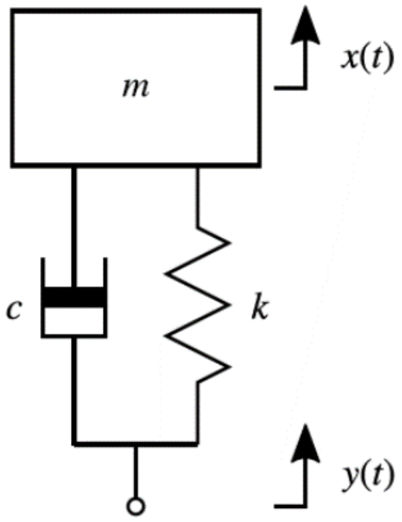
$$\frac{dc(t)}{dt} + 2c(t) = r(t)$$

Poles, Zeros, Characteristic Equation

- Fundamental to the analysis and design of control systems
- Simplifies the evaluation of a system's response
- Poles - roots of the denominator of the transfer function
- Zeros - roots of the numerator of the transfer function
- Characteristic Equation – denominator polynomial set to zero

$$\frac{C(s)}{R(s)} = G(s) = \frac{(b_m s^m + b_{m-1} s^{m-1} + \cdots + b_0)}{(a_n s^n + a_{n-1} s^{n-1} + \cdots + a_0)}$$

Exercise: Poles, Zeros, Characteristic Equation



$$G(s) = \frac{X(s)}{Y(s)} = \frac{2\zeta\omega_n s + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

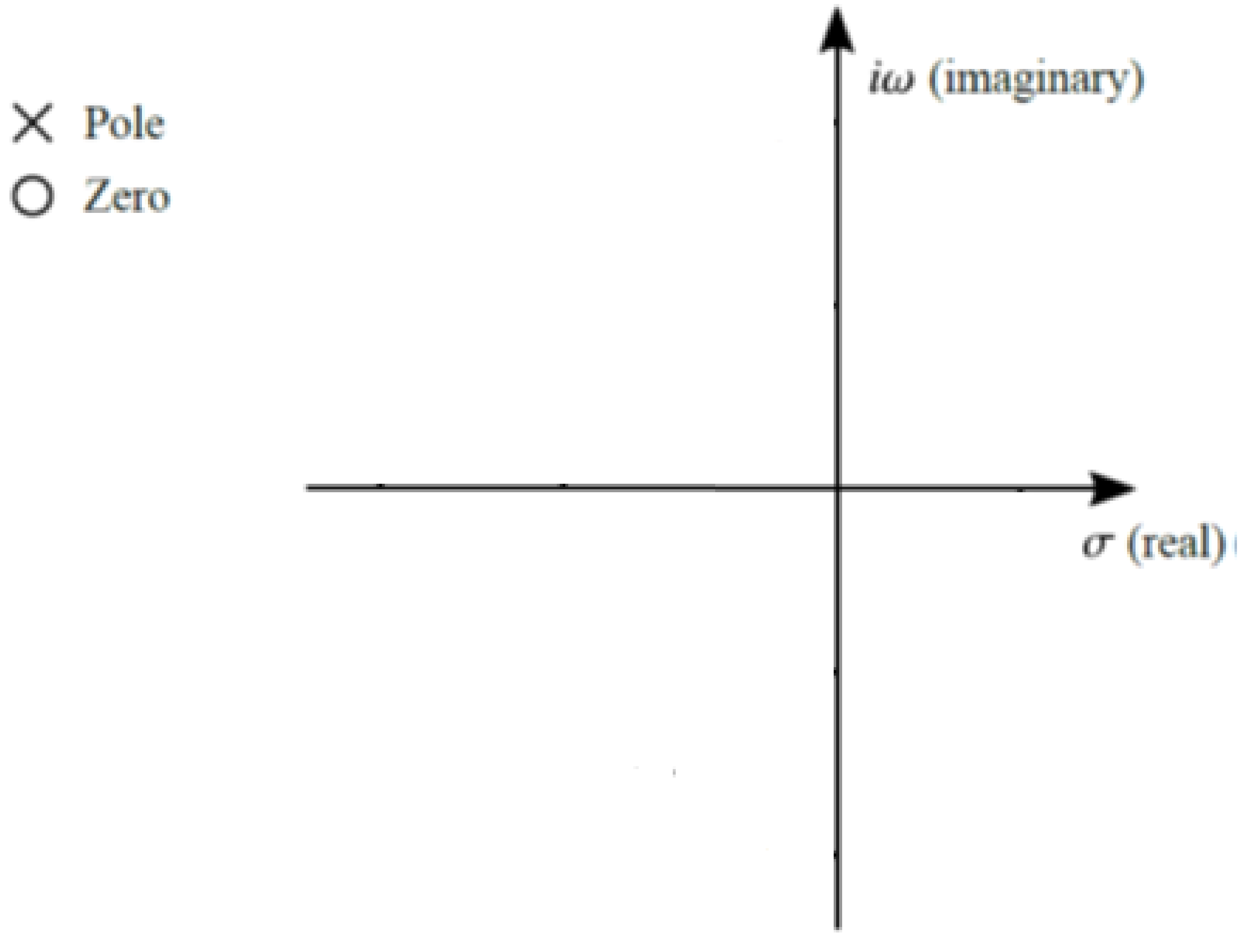
Find poles and zeros:

i) Case a: overdamped ($\zeta = 1.25$)

ii) Case b: underdamped ($\zeta = 0.4$)

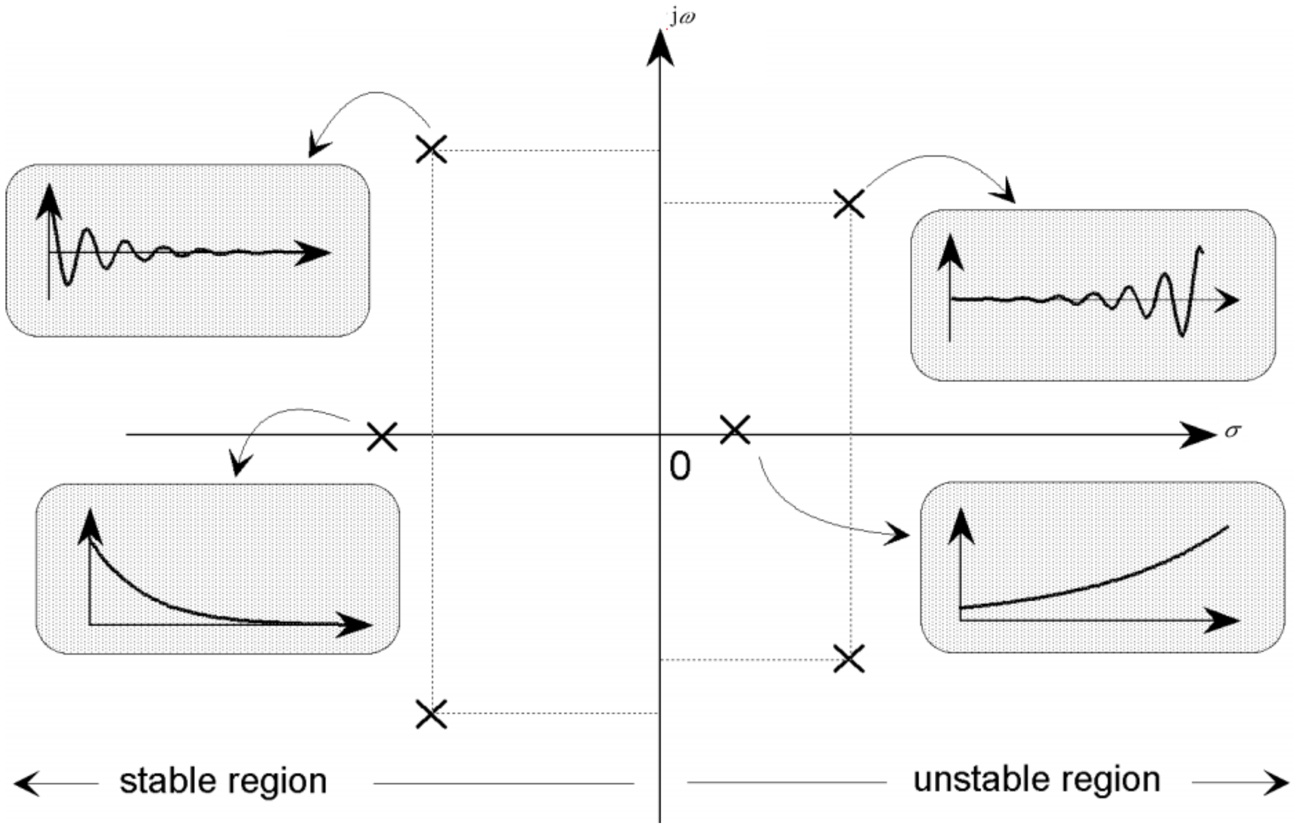
Note, $\omega_n = 4$ rad/s

Complex Plane (s-plane)



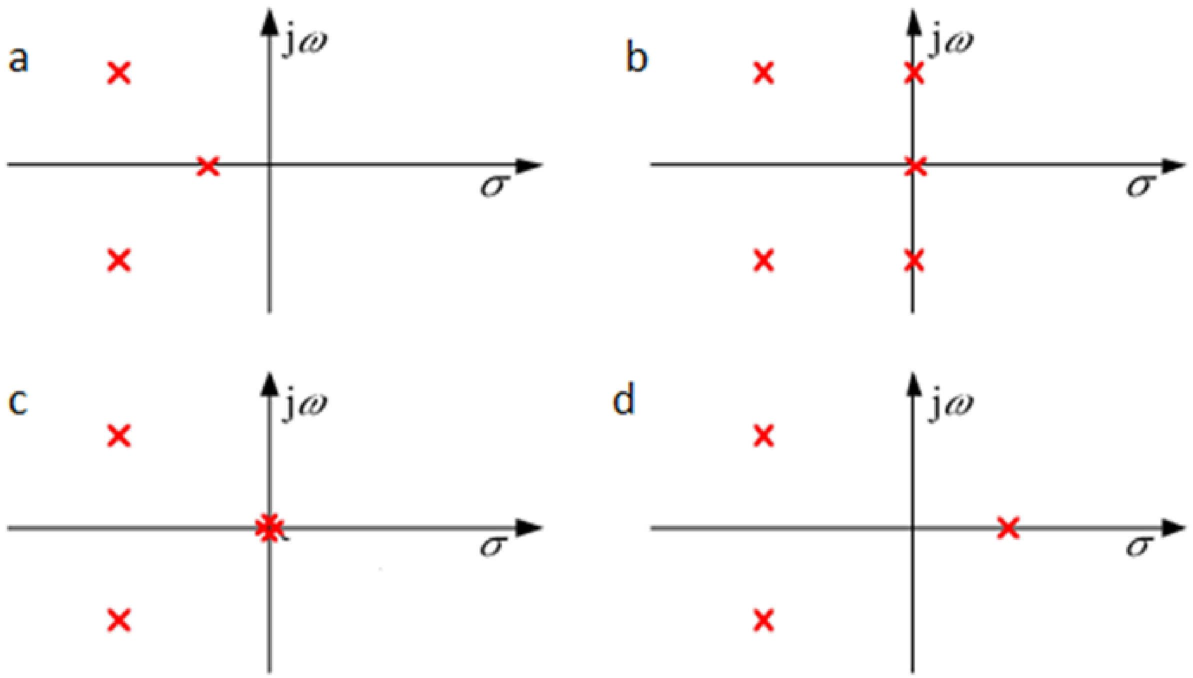
Stability (LTIs)

- System is stable if its transient response decays.
- (σ<0) → stable ; (σ>0) → unstable; (σ=0) → marginally stable



Exercise

Consider the below s plane pole plots and comment on the expected form of stability for each system.



Routh's Stability Criterion

- Allow to find stability without solving the characteristic equation

$$s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_{n-1} s + a_n = 0$$

- A necessary (but not sufficient) condition for stability is that all the coefficients of the characteristic polynomial be positive.
- A system is stable if and only if all the elements in the first column of the Routh array are positive.

Exercise: Routh's Stability Criterion

Consider the below polynomial:

$$a(s) = s^6 + 4s^5 + 3s^4 + 2s^3 + s^2 + 4s + 4$$

Determine the stability of the system.

Summary

- Poles, Zeros, Characteristic Equation
- Characteristics of System Stability
- Routh's Stability Criterion

Reference:

-Control Systems Engineering, 7th Edition, N.S. Nise

-UESTC3001 2019/20 Notes, J. Le Kernec