Dynamics and Control (UESTC 3001)

Notes prepared by: Dr Ola R. Popoola

Dynamics and Control (UESTC 3001)

- Lecture 1: Root Locus Analysis
- Lecture 2: Root Locus II and Nyquist Plots
- Lecture 3: Bode Plots
- Lecture 4: Bode Plots II
- Lecture 5: Stability in Frequency Domain
- Lecture 6: Stability Examples
- Lecture 7: Compensators
- Lecture 8: Tutorials and Test Exercises

Lecture delivered by: Dr Ola Popoola (Olaoluwa.Popoola@Glasgow.ac.uk)

Notes prepared by: Dr Ola R. Popoola

Intended Learning Objectives

At the end of this lecture, you will be able to:

- Sketch Bode magnitude plots and Bode phase plots
- Analyze bode plots and compute stability margins

Additional Reading Materials

- “Modern Control Systems” - R. C. Dorf, R.H. Bishop Addison Wesley
- “Modern Control Engineering” - K. Ogata. Prentice Hall International
- “Feedback Control of Dynamic Systems” - Franklin, Powell, Emami-Naeini. Addison Wesley.
- “Feedback Systems: An Introduction for Scientists and Engineers” – K. J. Åström, R. M. Murray. Princeton University Press, Princeton.
  Available at
  https://www.cds.caltech.edu/~murray/amwiki/index.php/Main_Page
- “Control Engineering: An introduction with the use of Matlab”, D. P. Atherton.
  Available at http://bookboon.com/en/control-engineering-matlab-ebook

These are recommended only and are not required to pass the course.

Additional Notes

- All required notes/tutorials/exam answers may be found on Moodle.
- You are encouraged to do your own reading to complement the lecture material.
- You can also use the forum on Moodle to ask questions about course material, rather than using email.

Revision

- Root Locus Plots
- Frequency Response Analysis
- Nyquist Plots

Nyquist Plot –Simple Lead

Transfer function: $\boldsymbol{G(s) = s + 1}$

Frequency response function: $G(j\omega) = 1 + j\omega$

This can be seen to have a phase of **$\tan^{-1}\omega$**. This varies between **0** when ω = 0 and **$+90^0$** when ω→∞.

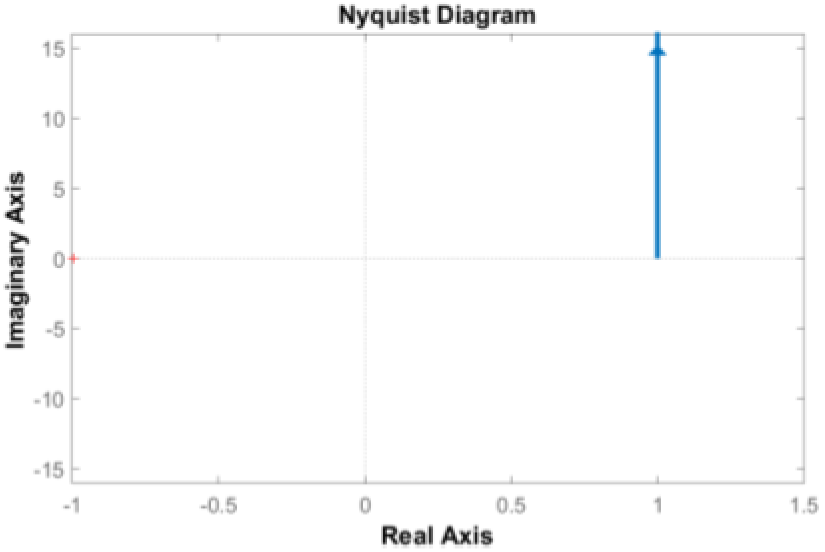
The magnitude given by:

$|G(j\omega)| = \sqrt{1 + \omega^2}$

Which varies between

1 when ω = 0 and

+∞ when ω→∞.



Nyquist Plot –MATLAB Examples

$$G(s) = \frac{10(1+10s)}{s\left(1+\frac{s}{10}\right)^2} = \frac{100s+10}{\frac{1}{100}s^3 + \frac{1}{5}s^2 + s}$$

In Matlab, it can be represented as a transfer function object:

```
sys = tf([100 10],[1/100 1/5 1 0]);
```

- Root locus: rlocus(sys)
- Nyquist plot: nyquist(sys)
  N.B. : don't forget to switch off the display of negative frequencies

Nyquist Plot –Class Examples

Draw the Nyquist plot of the transfer functions:

$$H(s) = \frac{10}{s(s+2)}$$

- Write out $H(j\omega)$
- Write in complex form i.e. $a + jb$
- Evaluate magnitude and phase as $\omega \to 0$, $\omega = 2$, and $\omega \to \infty$
- Sketch Nyquist plot

Nyquist Plot –Class Examples

Draw the Nyquist plot of the transfer functions:

$$H(s) = \frac{10}{s(s+2)}$$

- Write out $H(j\omega)$

$$H(j\omega) = \frac{10}{j\omega(\omega+2)}$$

- Write in complex form i.e. $a + jb$

$$\mathrm{H(j\omega)} = -\frac{10\omega^2}{\omega^4 + 4\omega^2} - \frac{j20\omega}{\omega^4 + 4\omega^2}$$

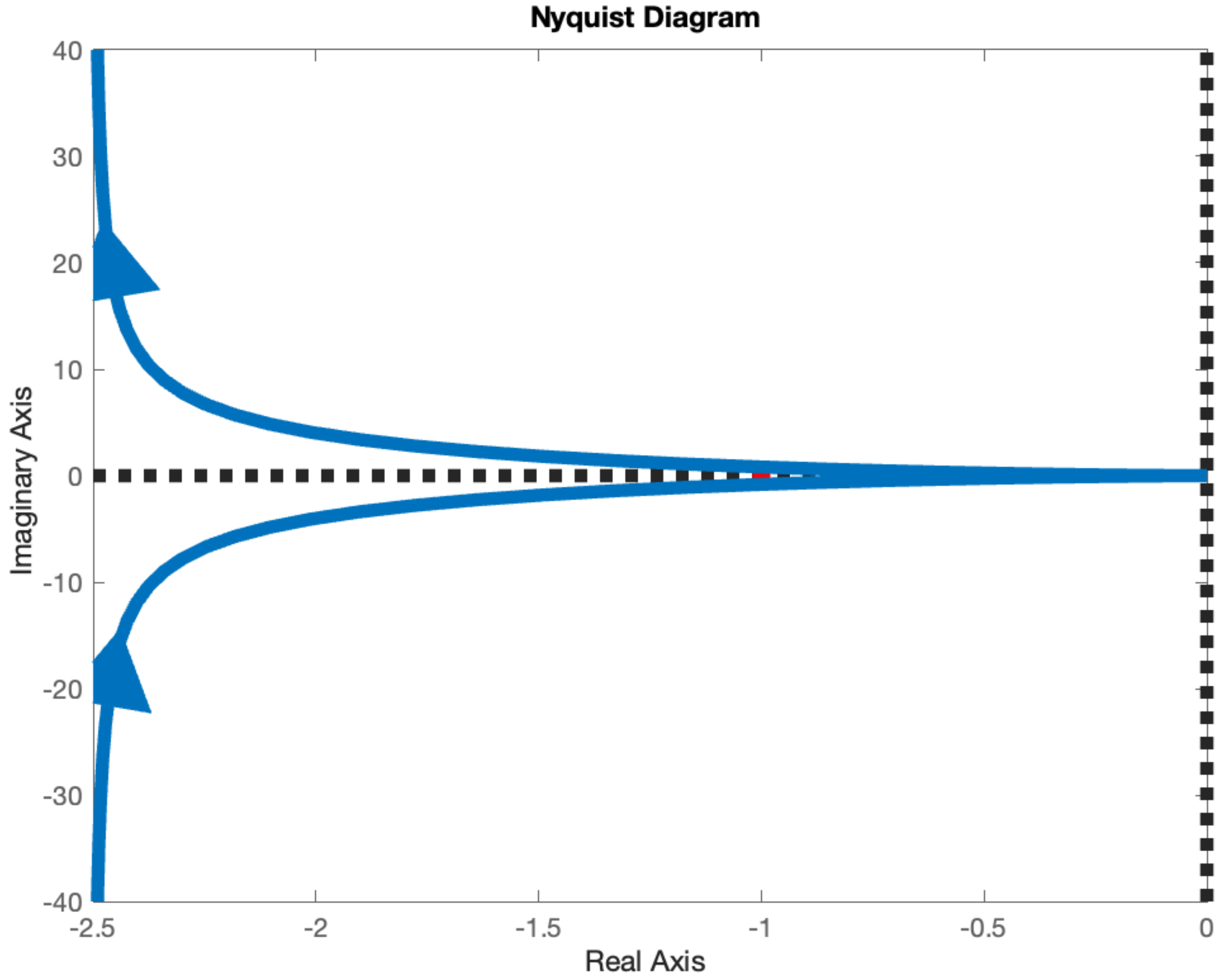
- Evaluate magnitude and phase as $\omega \to 0$, $\omega = 2$, and $\omega \to \infty$
  - Mag: $-2.5 + j\infty, -1.25 - j1.25, -0\ - j0$

- Sketch Nyquist plot

Nyquist Plot –Class Examples

Draw the Nyquist plot of the transfer functions:

$$(a)\, H(s) = \frac{10}{s(s+2)}$$



Nyquist Plot –Classwork

Draw the Nyquist plot of the transfer functions:

$$H(s) = \frac{1}{(s-3)(s-4)}$$

- Write out $H(j\omega)$
- Write in complex form i.e. $a + jb$
- Evaluate magnitude and phase as $\omega \to 0$, $\omega = 2$, and $\omega \to \infty$
- Sketch Nyquist plot

Nyquist Plot –Classwork

Draw the Nyquist plot of the transfer functions:

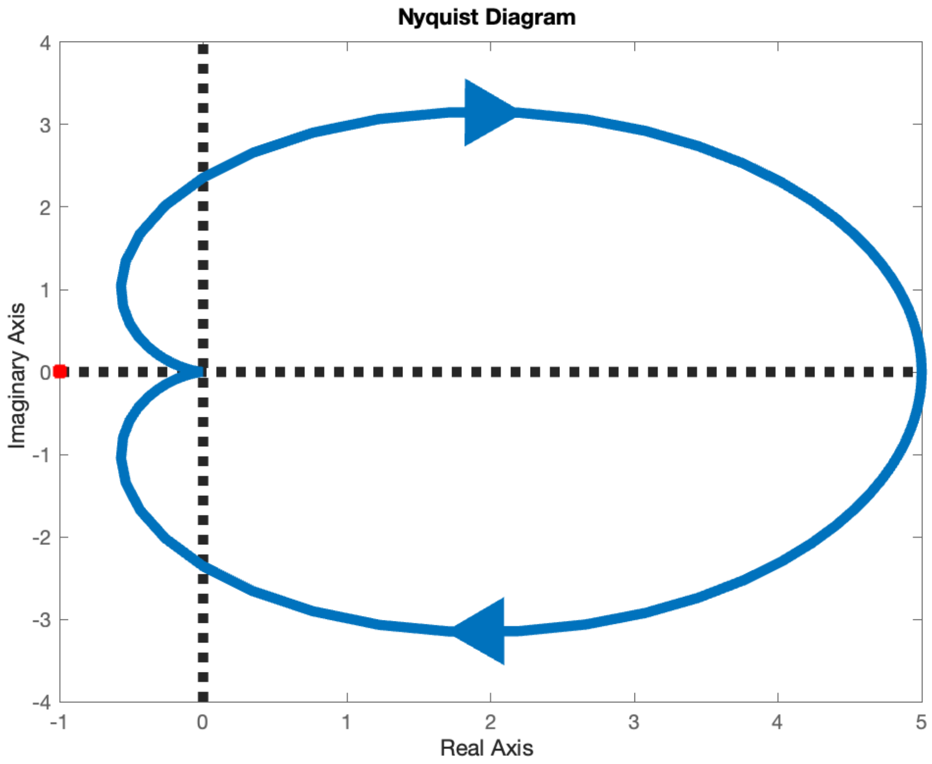
$$H(s) = \frac{1}{(s-3)(s-4)}$$

$$H(j\omega) = \frac{12-\omega^2}{(12-\omega^2)^2+(7\omega)^2} + \frac{j7\omega}{(12-\omega^2)^2+(7\omega^2)}$$

Nyquist Plot –Classwork

Draw the Nyquist plot of the transfer functions:

$$H(s) = \frac{90}{(s+3)(s+6)}$$



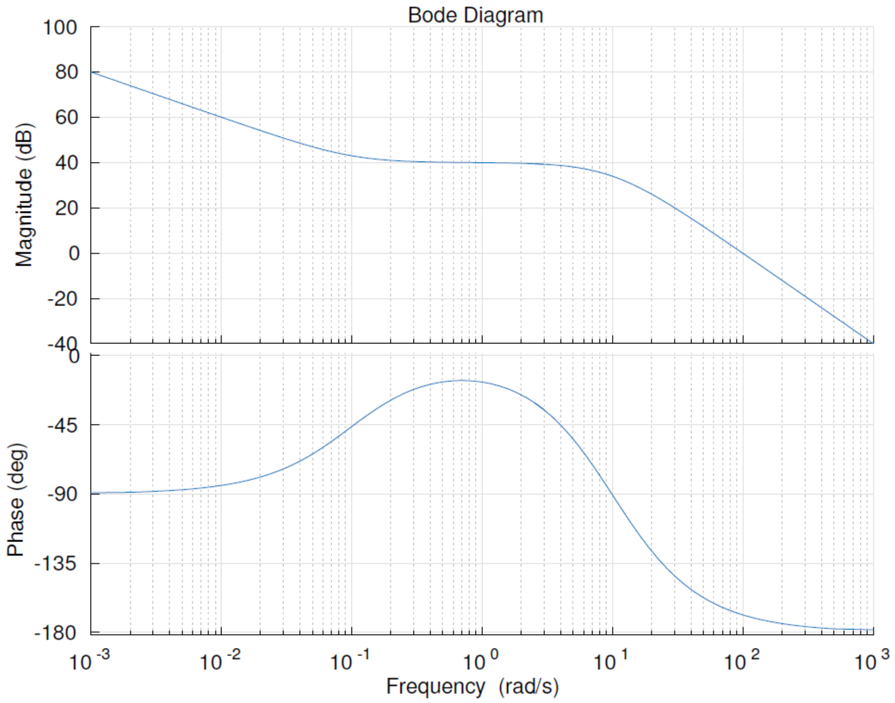
Bode Plots/Diagrams

Bode Plots Introduction

Bode Plots are an alternative method for displaying frequency responses graphically.

The Bode plots use two separate plots

- Magnitude/gain
- Phase



Why Bode Plots?

- Nyquist plots for complex systems are hard to draw accurately without reasonable computations.

- Bode plots are sometimes used instead as accurate sketches are possible.

- To guide us to draw the Bode diagrams of ‘complex’ transfer functions, we can use asymptotic plots.

Bode Components and Time Constant form

- The general form a transfer function can be factorised by writing in the time constant form such as:

$$G(s) = \frac{K\left(1+\frac{s}{\omega_{z1}}\right)\left(1+\frac{s}{\omega_{z2}}\right)\ldots}{s^r\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)\ldots}$$

- The plots are:

gain measured in dB $= 20\log_{10}|G(j\omega)|$

Phase, $arg[G(j\omega)] = \angle G(j\omega)$

Plotted against frequency, ω, on a logarithmic scale.

Writing transfer functions in time constant form

Time constant form –Classwork

Write the transfer functions below in the time constant form:

$$H(s) = \frac{1}{s(s+2)}$$

$$H(s) = \frac{1}{(s+3)(s+4)}$$

$$H(s) = \frac{1}{s^2+3s+3}$$

For quadratic polynomials, comment on transfer function and damping factor.

Bode Plots -Magnitude

Magnitude:

$$G(s) = \frac{K\left(1+\frac{s}{\omega_{z1}}\right)\left(1+\frac{s}{\omega_{z2}}\right)\ldots}{s^r\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)\ldots}$$

$$\begin{aligned}
20\log_{10}|G(j\omega)| = {} & 20\log_{10} K \\
& +20\log_{10}\left|1+j\frac{\omega}{\omega_{z1}}\right| + 20\log_{10}\left|1+j\frac{\omega}{\omega_{z2}}\right| \\
& -20r\log_{10}|j\omega| \\
& -20\log_{10}\left|1+j\frac{\omega}{\omega_{p1}}\right| - 20\log_{10}\left|1+j\frac{\omega}{\omega_{p2}}\right|
\end{aligned}$$

Bode Plots -Phase

$$G(s) = \frac{K\left(1+\frac{s}{\omega_{z1}}\right)\left(1+\frac{s}{\omega_{z2}}\right)\ldots}{s^r\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)\ldots}$$

Phase:

$$\angle G(j\omega) = Arg[G(j\omega)] = \tan^{-1}\left(\frac{\omega}{\omega_{z1}}\right) + \tan^{-1}\left(\frac{\omega}{\omega_{z2}}\right) + \ldots$$

$$-r.90^o$$

$$-\tan^{-1}\left(\frac{\omega}{\omega_{p1}}\right) - \tan^{-1}\left(\frac{\omega}{\omega_{p2}}\right) + \ldots$$

Bode Plots –Simple Gain

$$G(s) = \frac{K\left(1+\frac{s}{\omega_{z1}}\right)\left(1+\frac{s}{\omega_{z2}}\right)\ldots}{s^r\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)\ldots}$$

Simple Gain:

$$G(s) = K$$

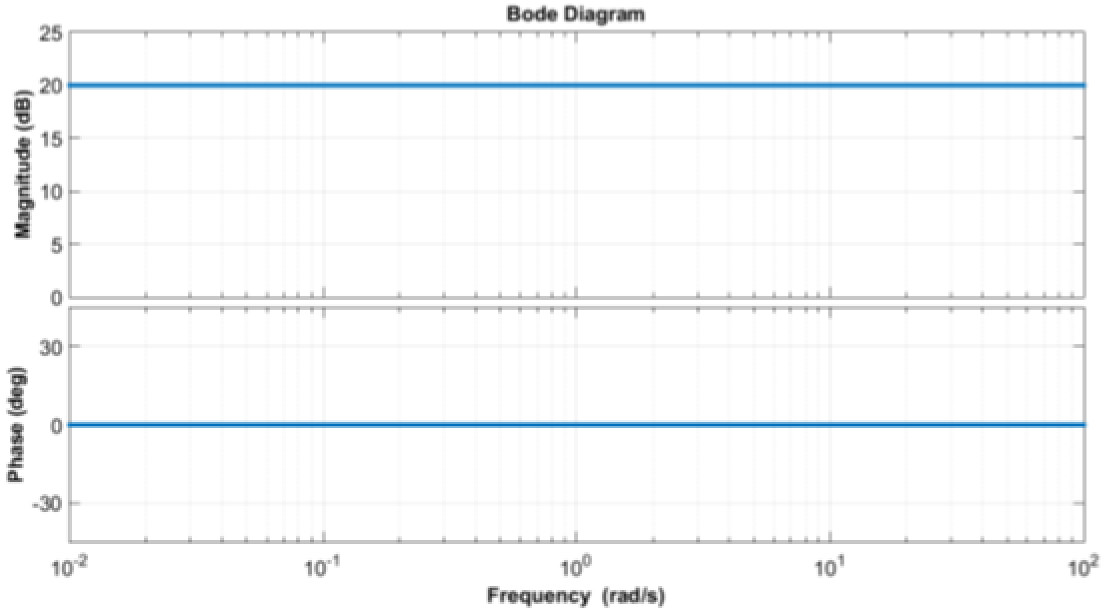
Gain:

$$20\log_{10}|G| = 20\log_{10}|K|$$

Phase:

$$\arg G(j\omega) = 0°$$

Example for K=10



Bode Plots -Integrator

$$G(s) = \frac{K\left(1+\frac{s}{\omega_{z1}}\right)\left(1+\frac{s}{\omega_{z2}}\right)\dots}{s^r\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)\dots}$$

Integrator:

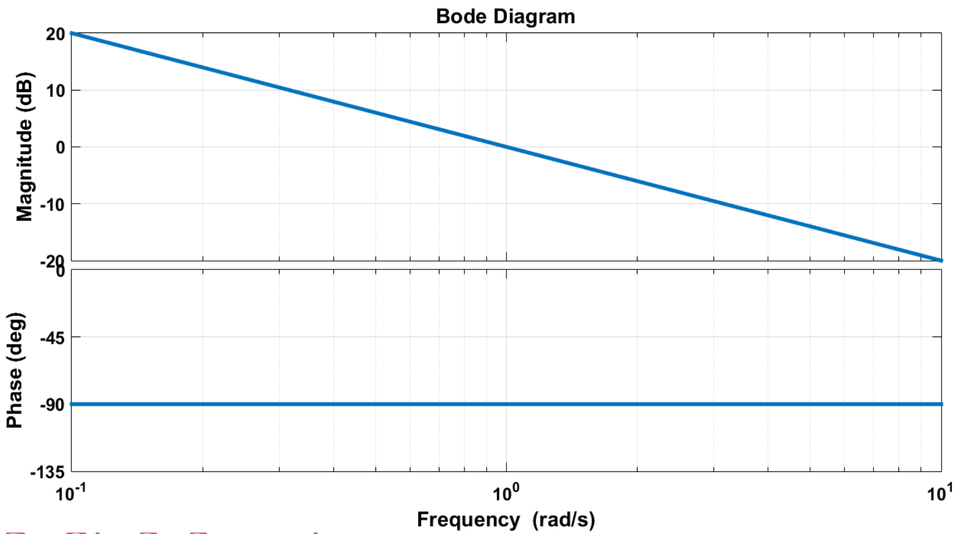
$$G(s) = \frac{1}{s}$$

Gain:

$$20\log_{10}|G| = -20\log_{10}\omega$$

Phase:

$$\arg G(j\omega) = -90°$$



Bode Plots –Simple lag Gain

$$G(s) = \frac{K\left(1+\frac{s}{\omega_{z1}}\right)\left(1+\frac{s}{\omega_{z2}}\right)\ldots}{s^r\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)\ldots}$$

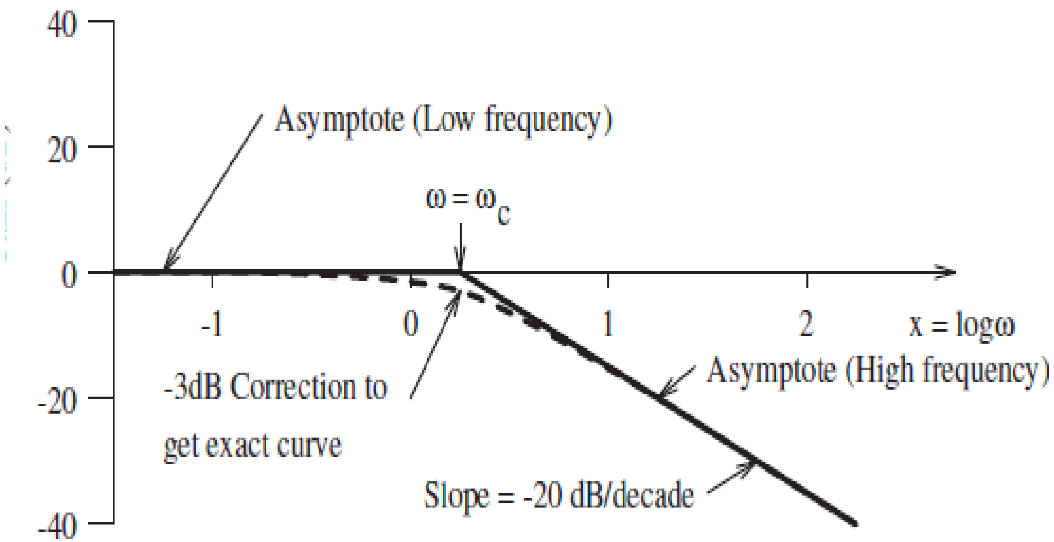
- **Simple lag**: $G(s) = \frac{1}{1+\frac{s}{\omega_C}}$

Gain: $20\log_{10}|G| = -10\log_{10}\left(1+\frac{\omega^2}{\omega_C^2}\right)$

$\omega \ll \omega_C$ ➔ $-10\log_{10}(1) = 0\text{ dB}$

$\omega = \omega_C$ ➔ $-10\log_{10}(2) = -3\text{ dB}$

$\omega \gg \omega_C$ ➔ $-20\log_{10}\left(\frac{\omega}{\omega_C}\right)$ ➔ -20 dB/decade crossing the 0 dB line at $\omega = \omega_C$



Bode Plots –Simple Lag Phase

$$G(s) = \frac{K\left(1+\frac{s}{\omega_{z1}}\right)\left(1+\frac{s}{\omega_{z2}}\right)\dots}{s^r\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)\dots}$$

- **Simple lag**: $G(s) = \frac{1}{1+\frac{s}{\omega_C}}$

Phase: $\tan^{-1} G = -\tan^{-1}\frac{\omega}{\omega_C}$

$\omega \ll \omega_C$ ➔ $-\tan^{-1}\frac{\omega}{\omega_C} \simeq 0^o$

$\omega = \omega_C$ ➔ $-\tan^{-1}\frac{\omega_C}{\omega_C} = -45^o$

$\omega \gg \omega_C$ ➔ $-\tan^{-1}\frac{\omega}{\omega_C} \simeq -90^o$

What is an acceptable limit for $\omega \ll \omega_C$ (resp. $\omega \gg \omega_C$) ?

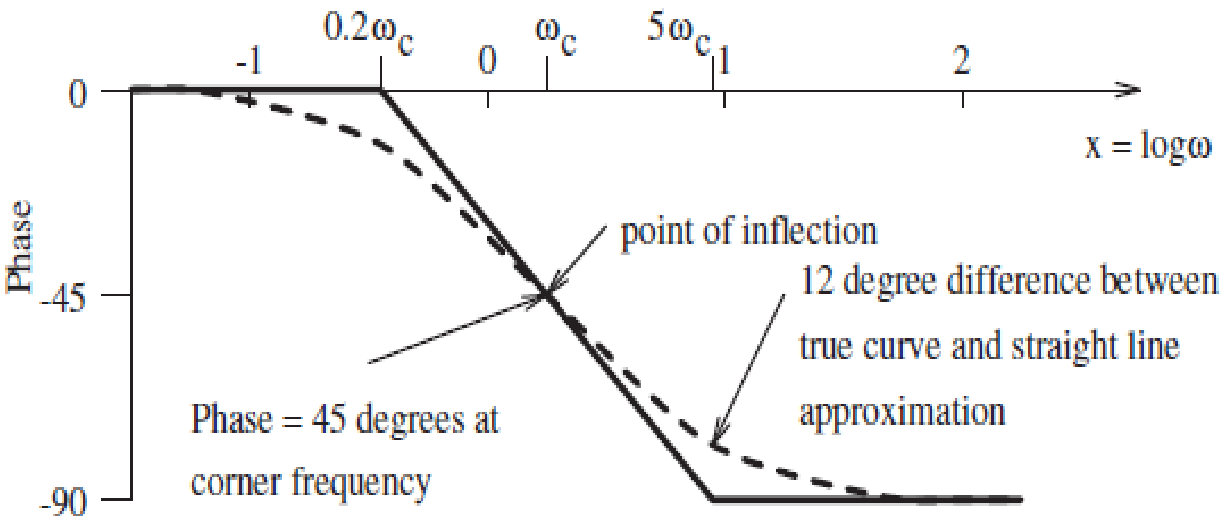
$\frac{\omega_C}{5}$ (resp. $5\omega_C$)

or

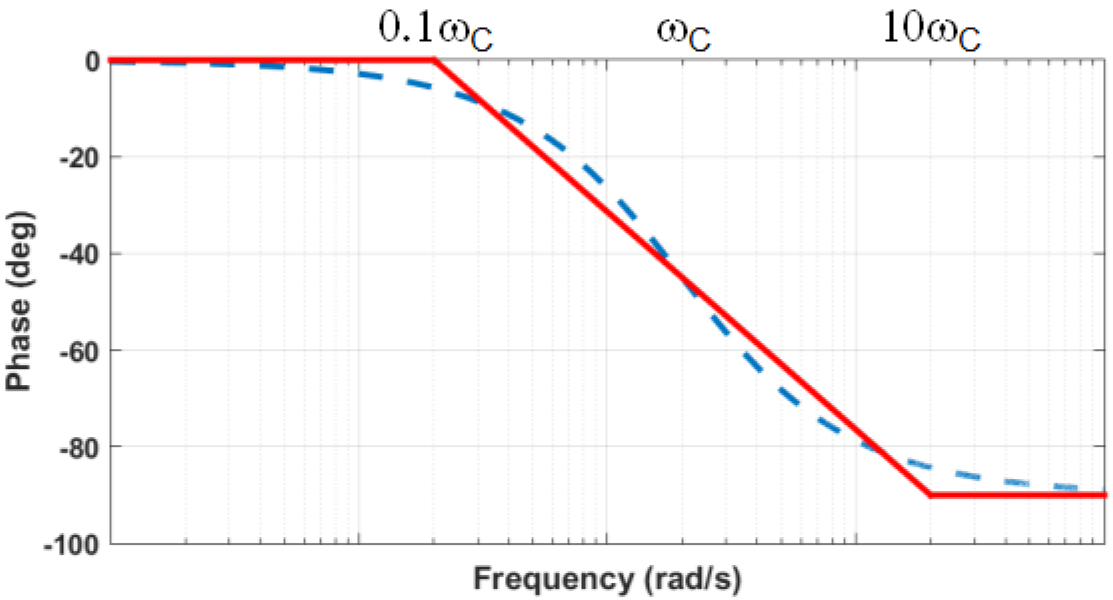
$\frac{\omega_C}{10}$ (resp. $10\omega_C$)

Bode Plots – Simple lag phase plot

Version 1



Version 2



Bode Plots –Simple Lead Gain

$$G(s) = \frac{K\left(1+\frac{s}{\omega_{z1}}\right)\left(1+\frac{s}{\omega_{z2}}\right)\ldots}{s^r\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)\ldots}$$

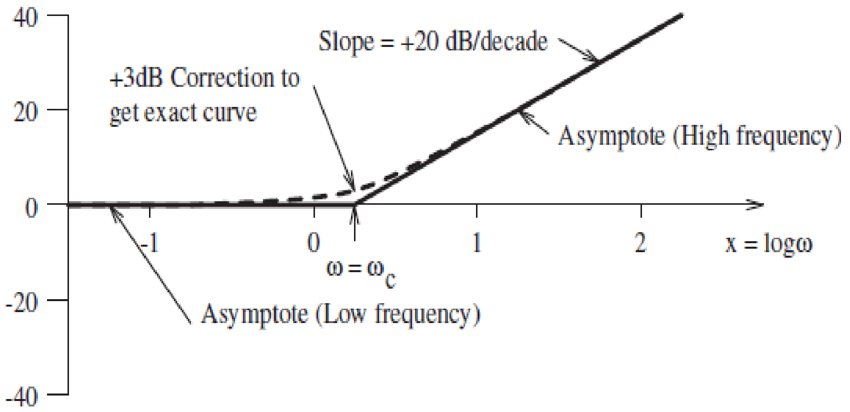
- Simple lead $G(s) = 1 + \frac{s}{\omega_c}$

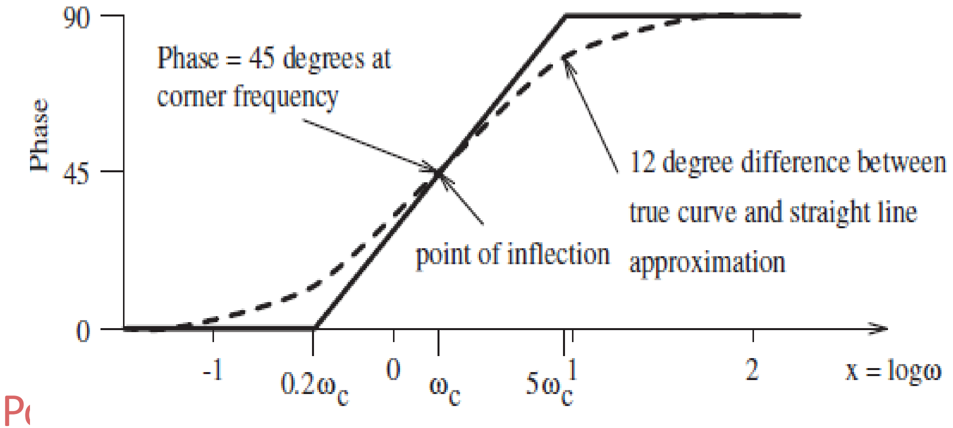
Gain: $20\log_{10}|G| = 10\log_{10}\left(1+\frac{\omega^2}{\omega_C^2}\right)$

$\omega \ll \omega_C$ → $10\log_{10}(1) = 0$ dB

$\omega = \omega_C$ → $10\log_{10}(2) = 3$ dB

$\omega \gg \omega_C$ → $20\log_{10}\left(\frac{\omega}{\omega_c}\right)$ → 20 dB/decade crossing the 0 dB line at $\omega = \omega_C$





Contribution of Components

$$G(s)=\frac{K\left(1+\frac{s}{\omega_{Z1}}\right)\left(a+\frac{s}{\omega_{z2}}\right)\ldots}{s^r\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)\ldots\left(1+\frac{2\zeta s}{\omega_n}+\frac{s^2}{\omega_n^2}\right)\ldots}$$

Component	Formular	Gain	Phase
a. Gain	K	$20\log\lvert K\rvert$	0^o
b. Integrator	$\frac{1}{s^r}$	$-20r\log\omega$	$-r.90^o$
c. First Order lead	$1+\frac{s}{\omega_{Z1}}$	0, 20 dB/dec after ω_{z1}	$\tan^{-1}(\frac{\omega}{\omega_{z1}})$
d. First Order lag	$\frac{1}{1+\frac{s}{\omega_{p1}}}$	0, -20 dB/dec after ω_{p1}	$-\tan^{-1}(\frac{\omega}{\omega_{p1}})$
e. Second Order lag	$\frac{1}{\left(1+\frac{2\zeta s}{\omega_n}+\frac{s^2}{\omega_n^2}\right)}$	0, -40 dB/dec after $\omega=\omega_n$	$-\tan^{-1}(\frac{\frac{2\zeta\omega}{\omega_n}}{1-\frac{\omega^2}{\omega_n^2}})$

Bode Plot Process

Now let us discuss the procedure of drawing a Bode plot:

1. Substitute the $s = j\omega$ in the open loop transfer function $G(s) \times H(s)$.
2. Find the corresponding corner frequencies and tabulate them.
3. Now we are required one semi-log graph chooses a frequency range such that the plot should start with the frequency which is lower than the lowest corner frequency. Mark angular frequencies on the x-axis, mark slopes on the left hand side of the y-axis by marking a zero slope in the middle and on the right hand side mark phase angle by taking -180^o in the middle.
4. Calculate the gain factor and the type of order of the system.
5. calculate slope corresponding to each factor then add.

Bode Plots –Examples

$$G(s) = \frac{10(1+10s)}{s\left(1+\frac{s}{10}\right)^2}$$

Components:

A simple gain 10

A simple lead term $(1+10s)$

An integrator $\frac{1}{s}$

Two lag terms $\frac{1}{1+\frac{s}{10}}$

Bode Plots –Examples

$$G(s) = \frac{10(1+10s)}{s\left(1+\frac{s}{10}\right)^2}$$

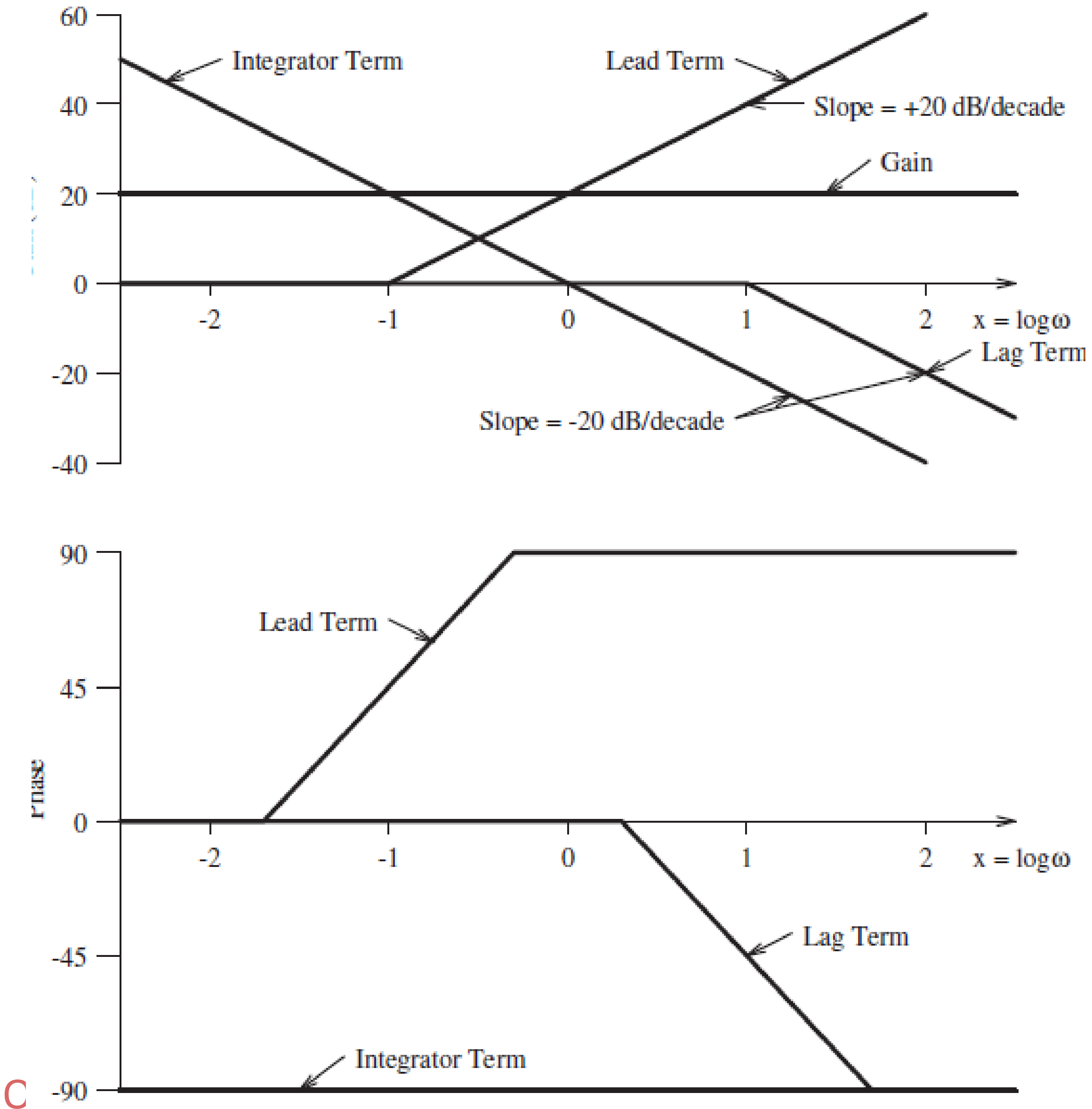
Components:

A simple gain 10

A simple lead term $(1+10s)$

An integrator $\frac{1}{s}$

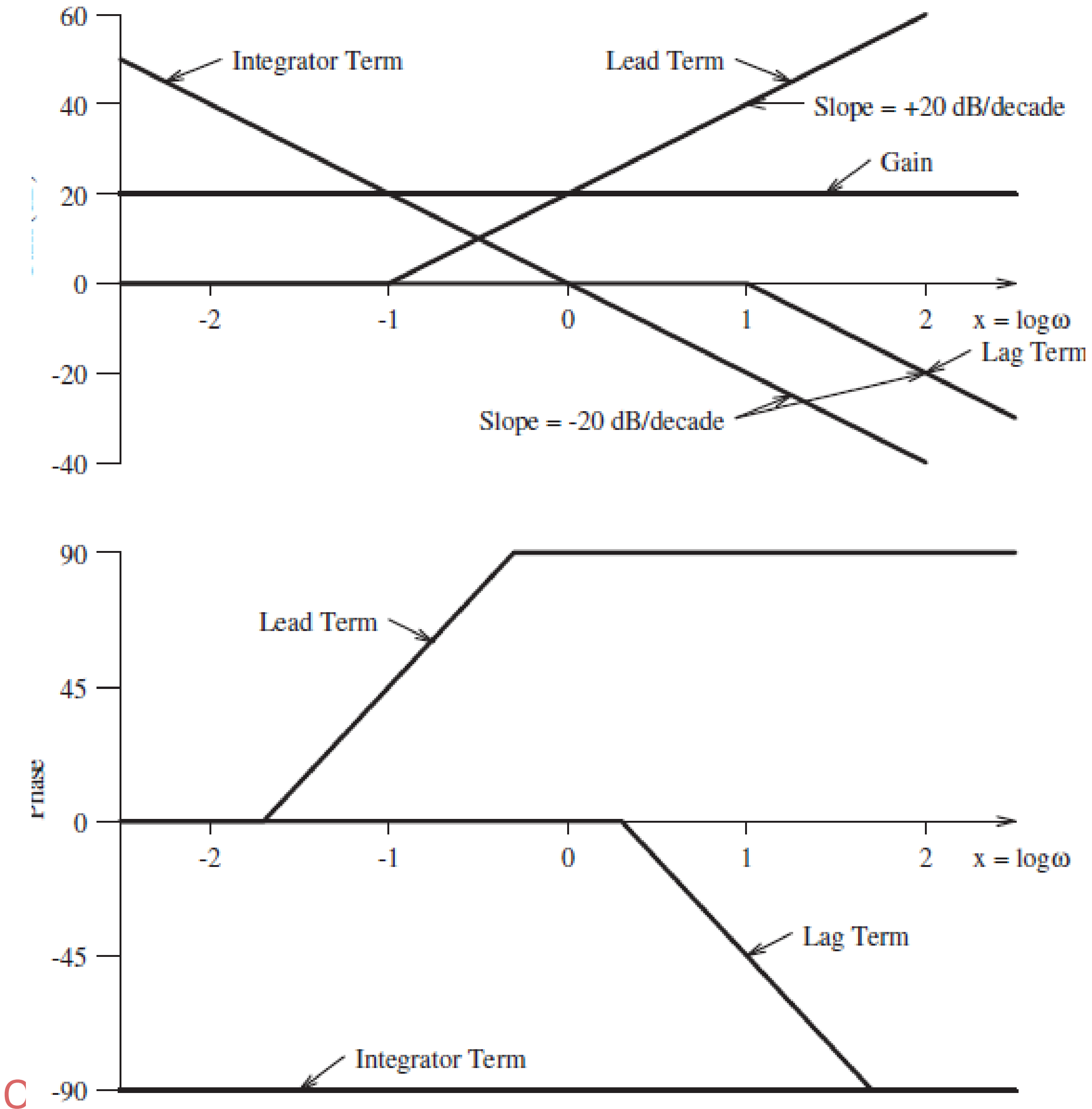
Two lag terms $\frac{1}{1+\frac{s}{10}}$



Bode Plots –Examples

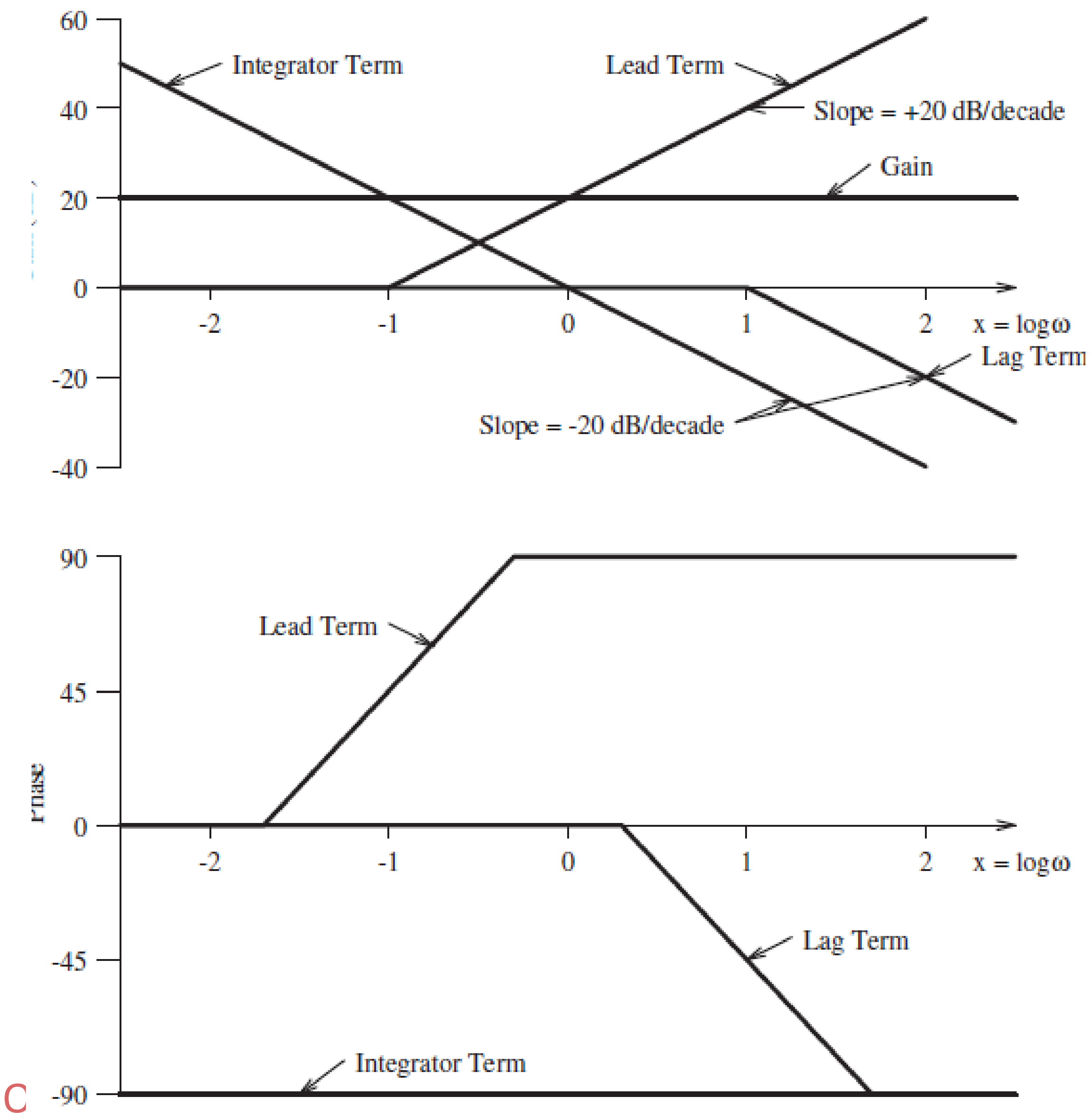
$$G(s) = \frac{10(1+10s)}{s\left(1+\frac{s}{10}\right)^2}$$

Components:
A simple gain 10
A simple lead term $(1+10s)$
An integrator $\frac{1}{s}$
Two lag terms $\frac{1}{1+\frac{s}{10}}$



Bode Plots –Examples

$$G(s) = \frac{10(1+10s)}{s\left(1+\frac{s}{10}\right)^2}$$

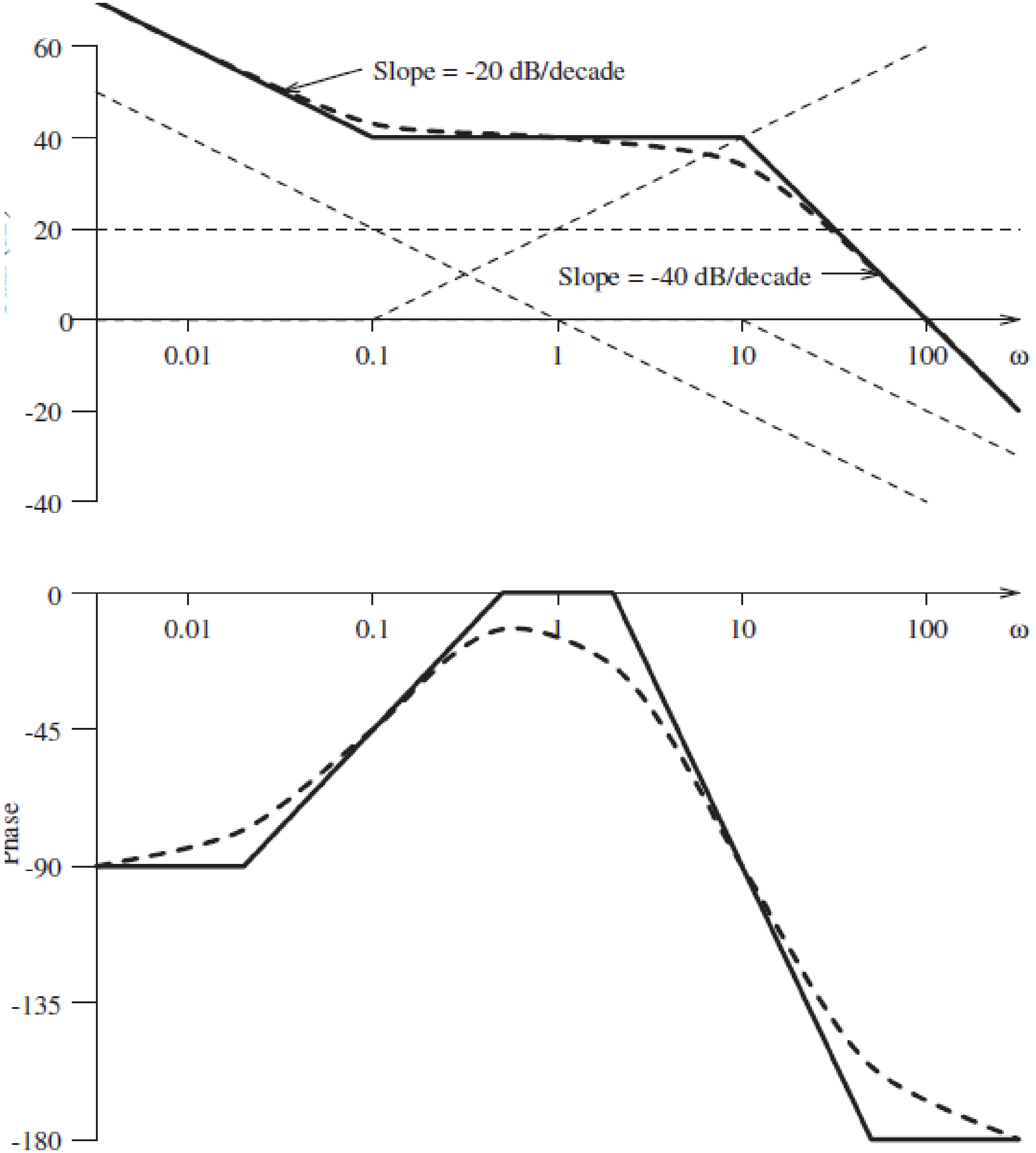


Phase Plot

$0 < \omega < 0.02$	$\text{Angle} = -90^\circ$
$\omega = 0.1$	$\text{Angle} = -45^\circ$
$0.5 < \omega < 2$	$\text{Angle} = 0^\circ$
$\omega = 10$	$\text{Angle} = -90^\circ$
$50 < \omega < \infty$	$\text{Angle} = -180^\circ$

Bode Plots –Examples

$$G(s) = \frac{10(1+10s)}{s\left(1+\frac{s}{10}\right)^2}$$



Bode Plots –Matlab

$$G(s) = \frac{10(1+10s)}{s\left(1+\frac{s}{10}\right)^2} = \frac{100s+10}{\frac{1}{100}s^3+\frac{1}{5}s^2+s}$$

In Matlab, it can be represented as a transfer function object:

sys = tf([100 10],[1/100 1/5 1 0]);

Bode plot: bodeplot(sys)

Nyquist plot: nyquistplot(sys)

Response of a delay

- Laplace transform of a delay of T seconds is:

$$H(s) = e^{-sT}$$

In the frequency domain:

$$s = j\omega$$

The frequency response of a delay is:

$$H(j\omega) = e^{-j\omega T}$$

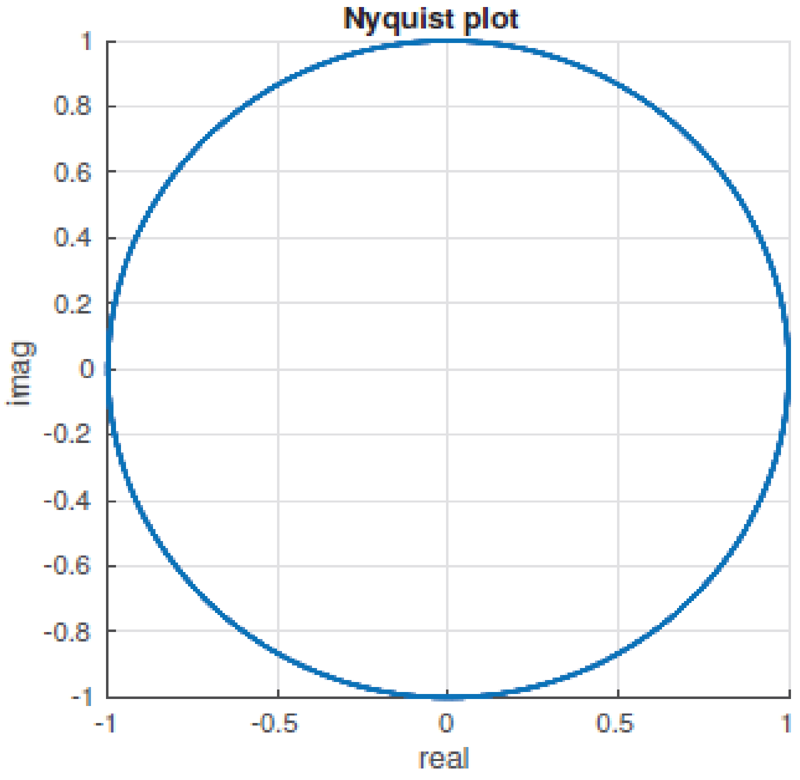
Magnitude:

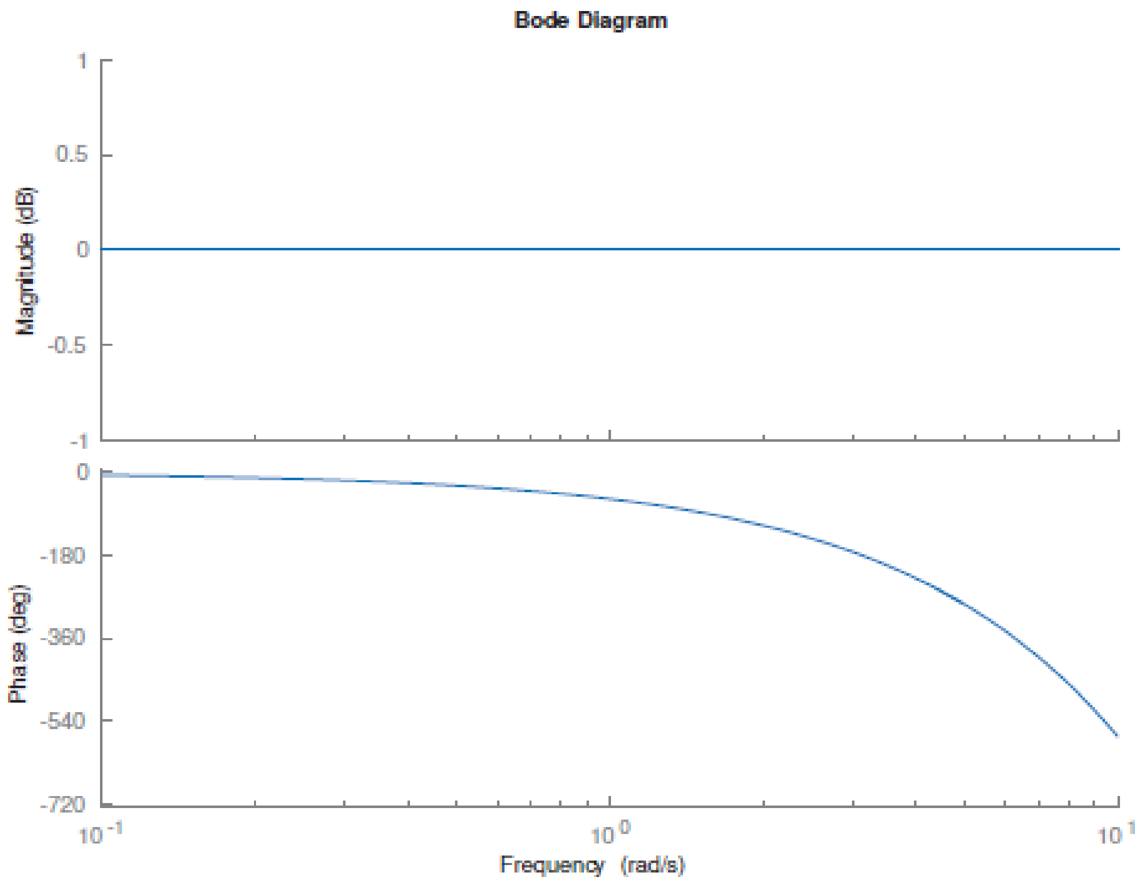
$$|H(j\omega)| = 1$$

Phase:

$$\angle H(j\omega) =- \omega T \; radians$$

Response of a delay





Practice Exercise

Consider the transfer function:

$$H(s) = \frac{10.(s + 0.5)}{(s + 10)}$$

For this transfer function, find its Bode diagrams