UESTC3001 Dynamics & Control
Lecture 1

Overview of Dynamic Systems

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Systems

Sets of connected objects or things

They can be living things

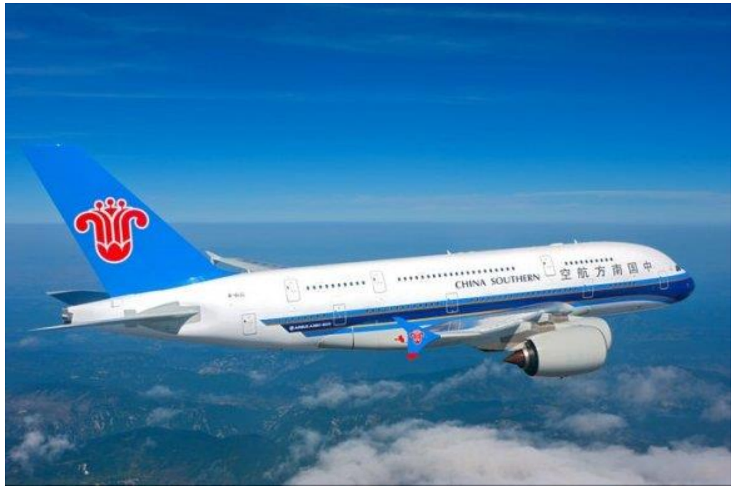
They can be mechanical things

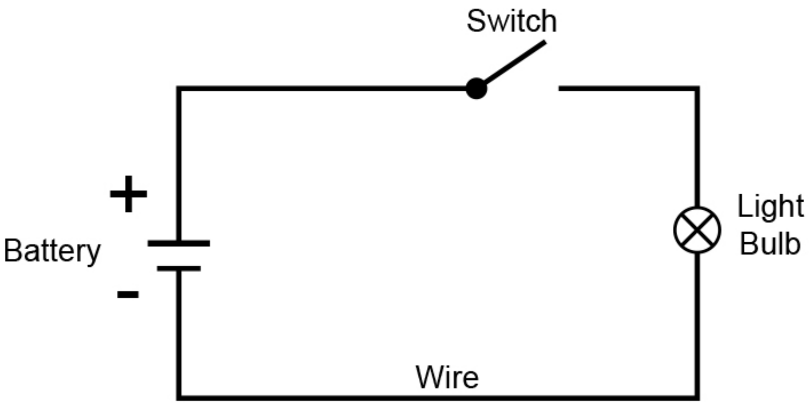
Behaviors of these systems are shaped by many factors

All these systems exhibit some common behavior patterns

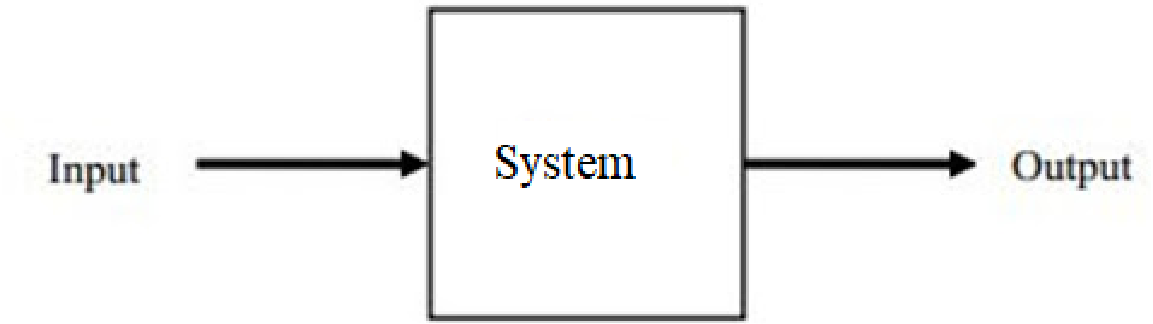
What is a Dynamic System?

A dynamic system is one which is in motion.





The output of the system is dependent on the input and the behaviour of the system itself.

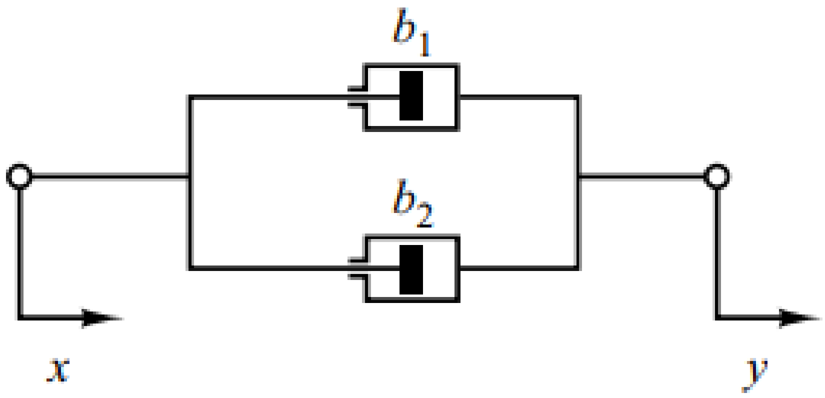


I/P and O/P relationship

- Model dynamic systems in mathematical terms
- A set of equations that represents the dynamics of the system
- System may have many mathematical models
- The dynamics of systems may be described in terms of differential equations
- Use physical laws governing a particular system
- Testing a prototype of the device, or measuring its response to inputs
- Deriving reasonable mathematical models is the most important part

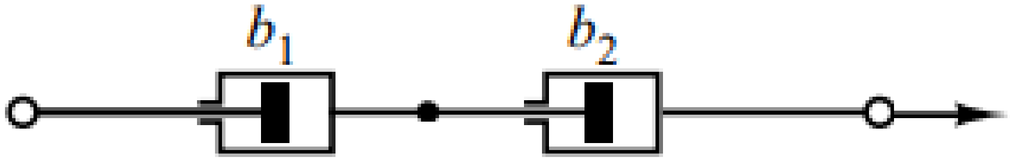
Mechanical System Modelling

System consisting of two dampers connected in parallel



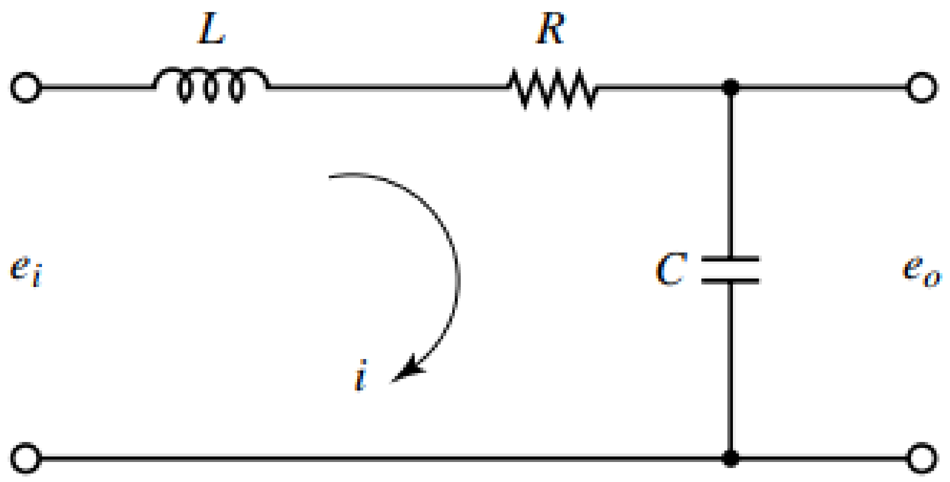
Exercise: Mechanical System Modelling

System consisting of two dampers connected in series



Electrical System Modelling

Consider the electrical circuit shown in the Figure

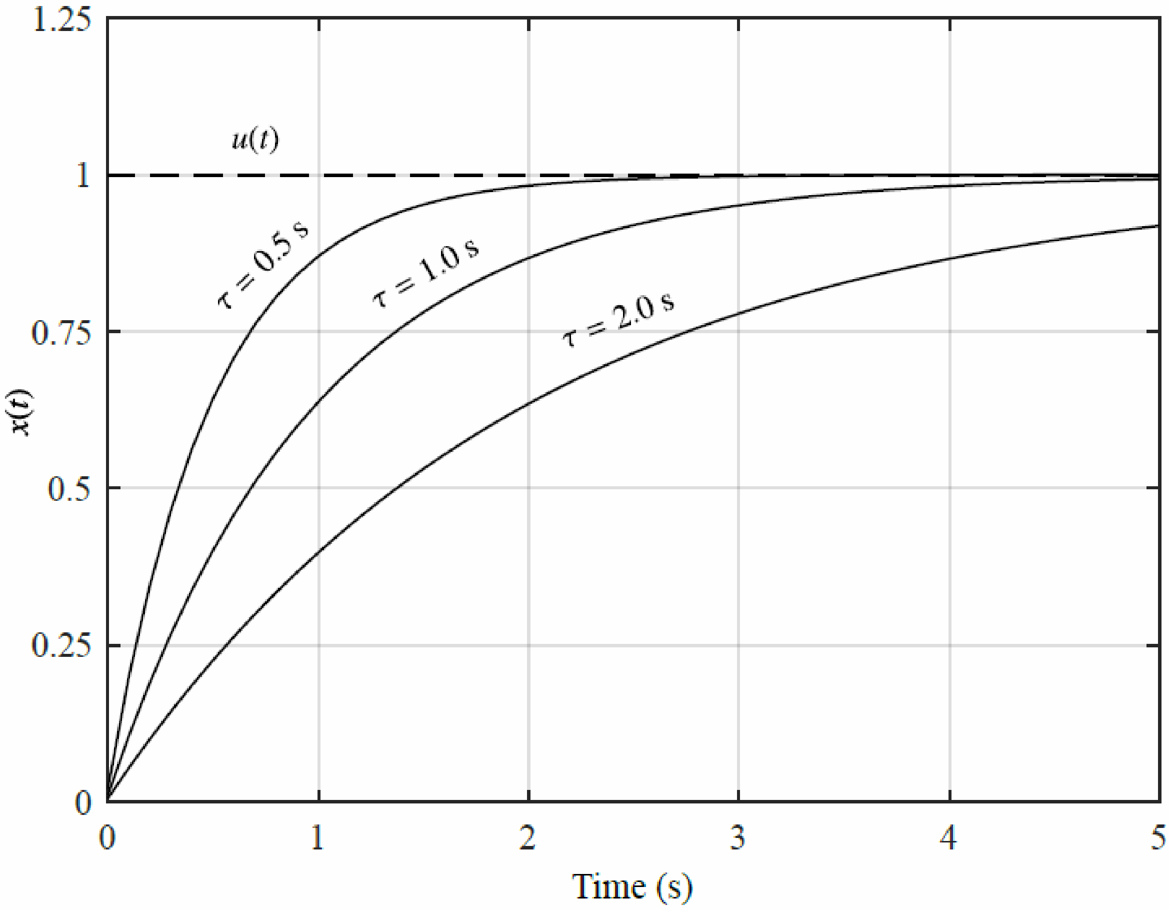


Describing a Dynamic System

- Equations describing the systems known as system’s equation of motion.
- Describes the O/P of the system
- Describes its derivatives as a function of the I/P
- Can have MIMO

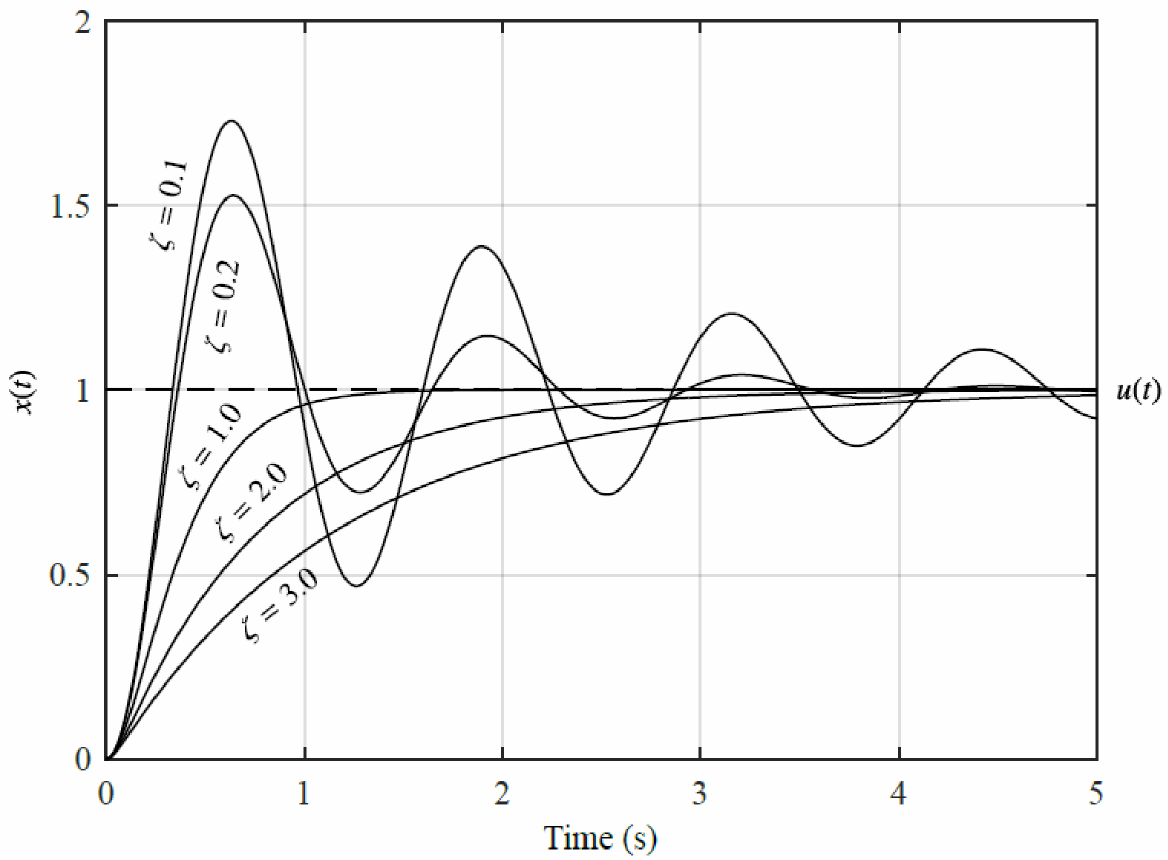
First-Order System

$$\dot{x}(t) + \frac{1}{\tau}x(t) = Ku(t)$$



Second-Order System

$$\ddot{x}(t) + 2\zeta\omega_n\dot{x}(t) + \omega_n^2 x(t) = Ku(t)$$



Higher-Order Systems

An nth-order system has the general equation of motion

$$x^{(n)}(t) + a_1 x^{(n-1)}(t) + a_2 x^{(n-2)}(t) + \cdots + a_n x(t) = Ku(t)$$

The Laplace Transform

- Differential equation is difficult to model as a block diagram
- Represent the input, output, and system as separate entities
- Their interrelationship will be simply algebraic

$$\mathscr{L}[f(t)] = F(s) = \int_{0-}^{\infty} f(t)e^{-st}dt$$

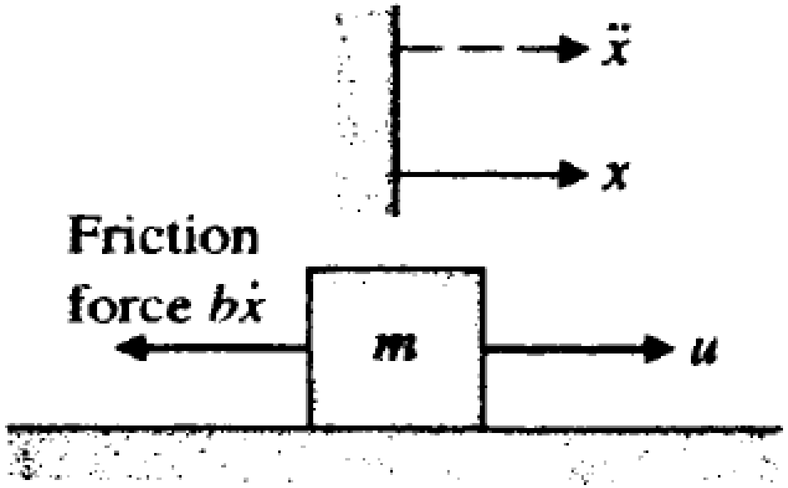
$f(t)$	$F(s)$
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
$tu(t)$	$\frac{1}{s^2}$
$t^n u(t)$	$\frac{n!}{s^{n+1}}$
$e^{-at}u(t)$	$\frac{1}{s+a}$
$\sin \omega t u(t)$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t u(t)$	$\frac{s}{s^2+\omega^2}$

Transfer Functions

- Describe the relationship between O/P and I/P
- Transform it into the Laplace domain

$$\dot{x}(t) + \frac{1}{\tau}x(t) = Ku(t)$$

Exercise: Transfer Functions



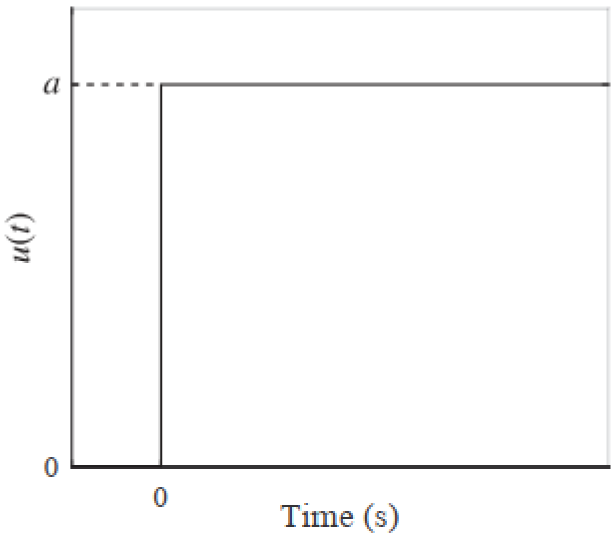
Obtain transfer functions for the below systems

Second-order system: $\ddot{x}(t) + 2\zeta\omega_n\dot{x}(t) + \omega_n^2 x(t) = Ku(t)$

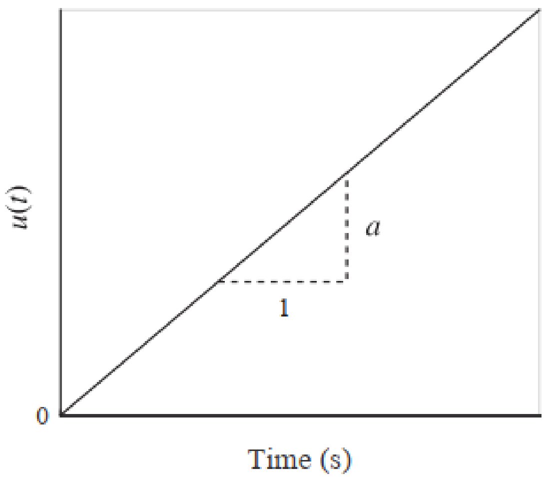
nth-order system: $x^{(n)}(t) + a_1 x^{(n-1)}(t) + a_2 x^{(n-2)}(t) + \cdots + a_n x(t) = Ku(t)$

Types of Input

Step I/P

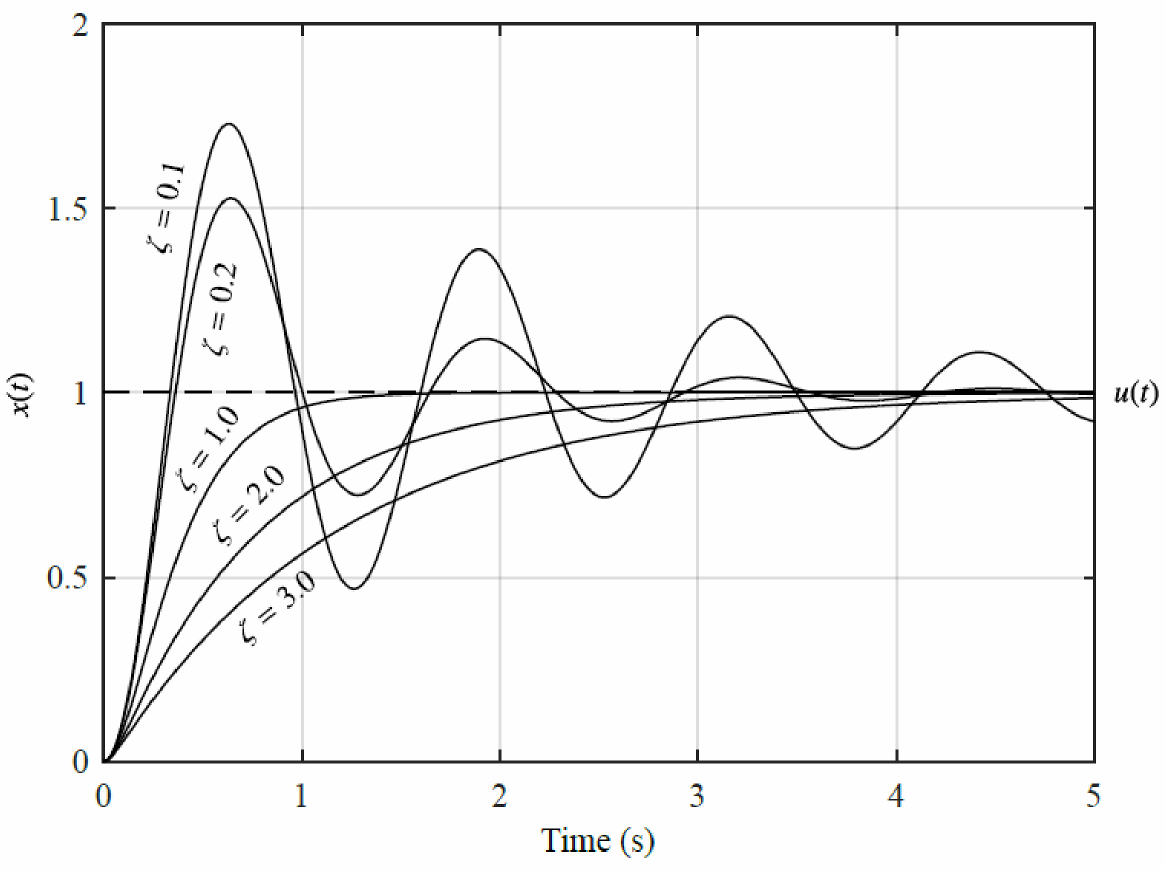


Ramp I/P



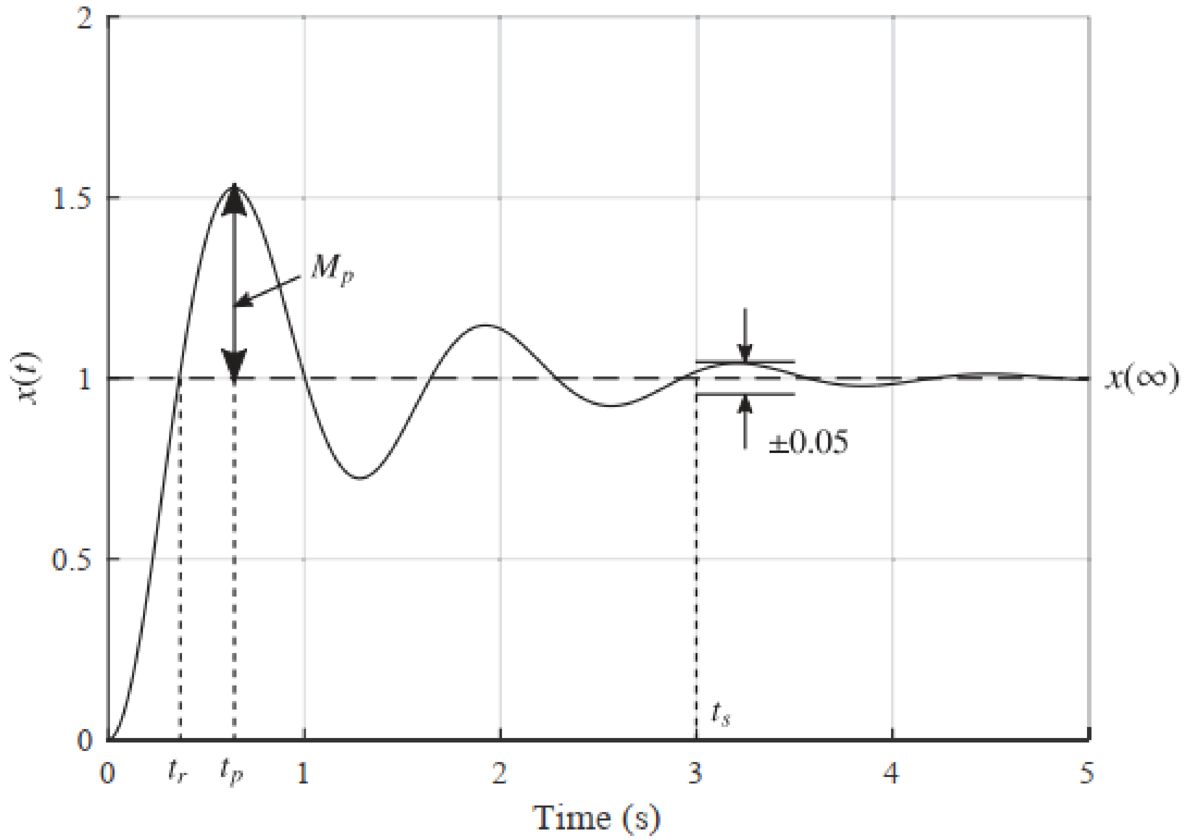
Response Properties

- By applying a step input to a system which is at rest, we cause the system to be in forced vibration.



Response Properties etc.

The response of a system can be described by several properties



Summary

- Basics of Dynamic Systems
- Mathematical Models
- Transfer Function
- Response Characteristics

Reference:

-Modern Control Engineering, 5th Edition, K. Ogata

-UESTC3001 2019/20 Notes, J. Le Kernec etc.