UESTC3001 Dynamics & Control
Lecture 6

Characteristics and Performance of Feedback Control Systems - II

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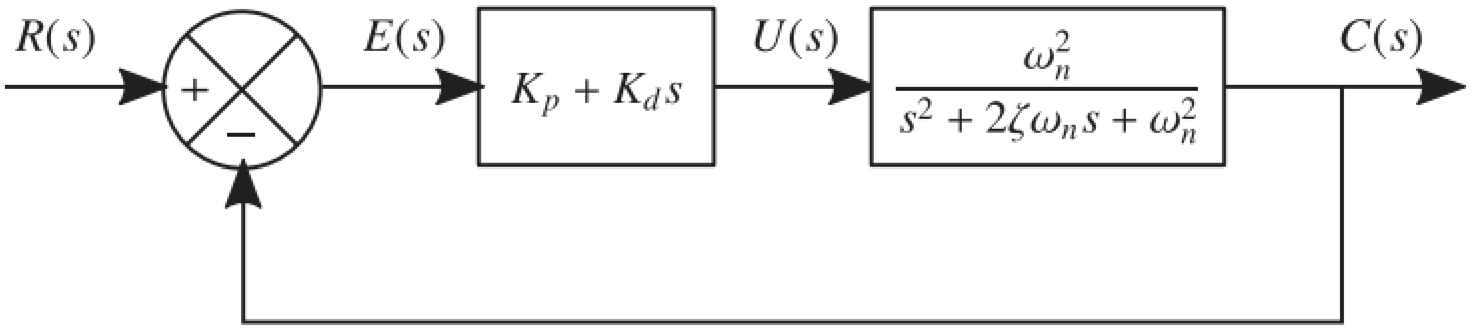
Outline

- Proportional + Derivative Control of a Second-Order System and Effect on a Second-Order System
- Integral Control of a First-Order/Second-Order System and Effect on a First-Order/Second-Order System
- Proportional + Integral Control of a First-Order System and Effect on a First-Order System
- Proportional + Derivative + Integral Control of a First-Order /Second-Order System and Effect on a First-Order System

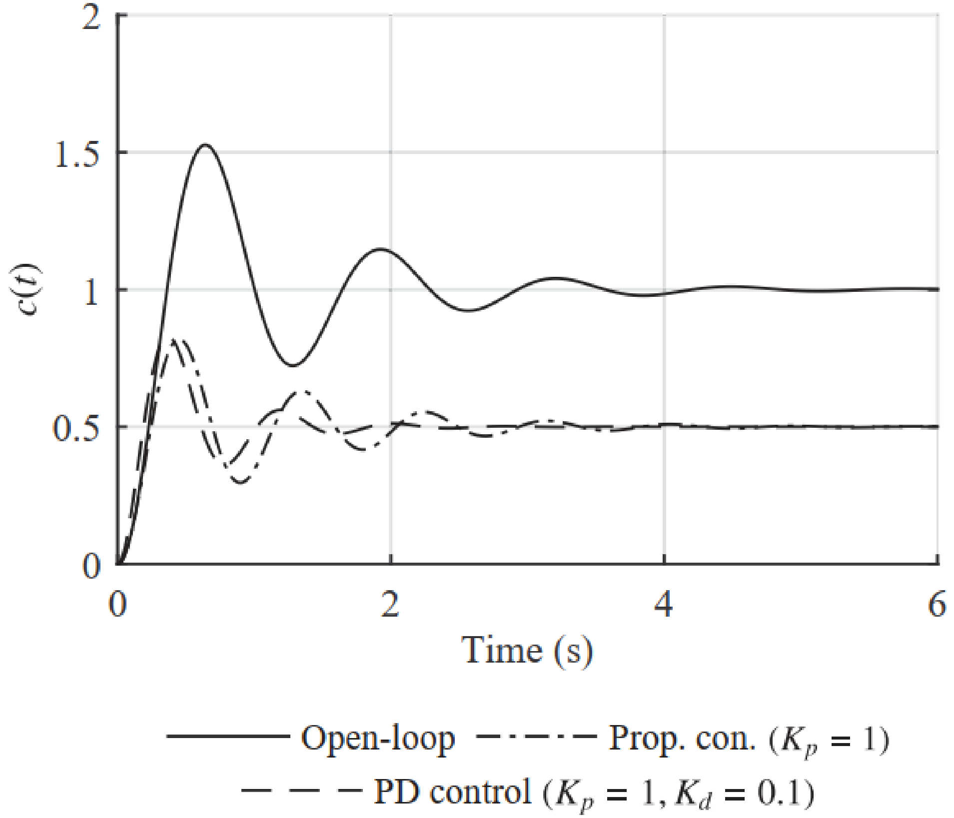
Proportional Plus Derivative (PD) Control of a Second-Order System



Effect of Proportional Plus Derivative (PD) Control on a Second-Order System

$$s^2 + 2\zeta_{cl}\omega_{n,cl}s + \omega_{n,cl}^2 = 0$$

$$c(t) = \frac{K_p}{K_p+1}\left[1 - e^{-\zeta_{cl}\omega_{n,cl}t}\left(\cos\omega_{d,cl}t + \frac{K_p\zeta_{cl} - K_d\omega_{n,cl}}{K_p\sqrt{1-\zeta_{cl}^2}}\sin\omega_{d,cl}\right)\right]$$



$$\omega_{n,cl} = \omega_n\sqrt{K_p+1}$$

$$\zeta_{cl} = \frac{\zeta + \frac{1}{2}K_d\omega_n}{\sqrt{K_p+1}}$$

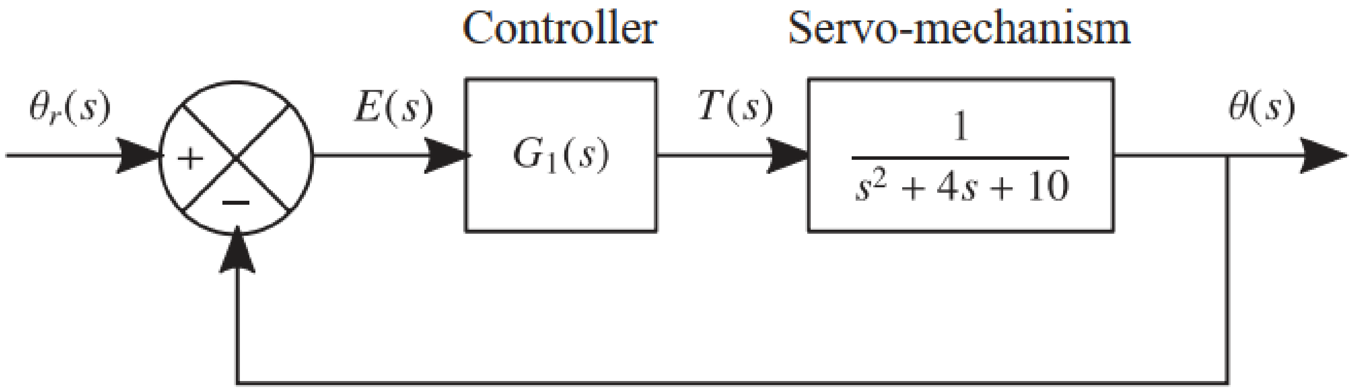
Exercise 1

The angular output of a servo-mechanism θ is controlled as indicated in the figure below where θ_r is the required (reference) displacement. The controller transfer function is given by $G_1(s)$, the torque it applies to the servo-mechanism is denoted by $T(s)$ and the error signal $E(s)$.

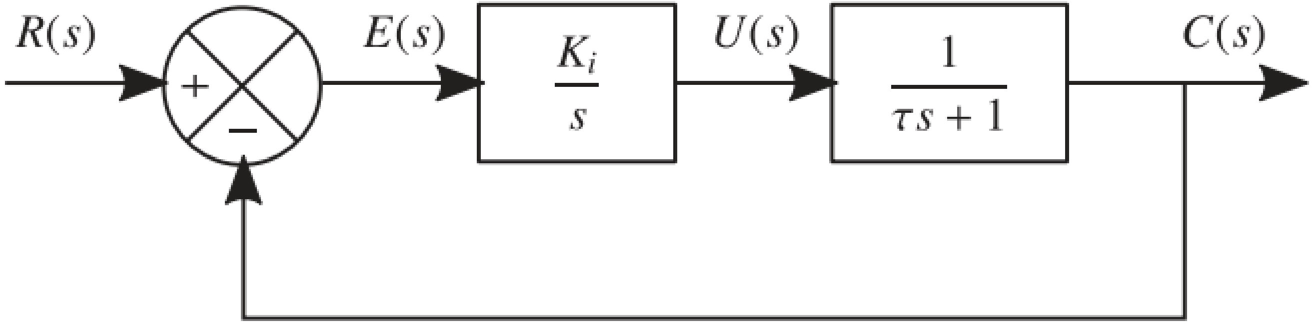
(a) Calculate the damping and natural frequency of the uncontrolled open-loop system. What is the maximum overshoot of the uncontrolled system in response to a unit step change in torque $T(s)$?

(b) Proportional control with gain $K_p = 10$ is applied. Calculate the closed-loop natural frequency and damping. What is the steady state error in response to a unit step input in $\theta_r(s)$? Calculate the resulting maximum overshoot.

(c) Derivative action with gain $K_d = 4$ is now added to the proportional action. Calculate the closed-loop natural frequency and damping. What is the steady state error in response to a unit step input in $\theta_r(s)$? Calculate the resulting maximum overshoot.



Integral Control of a First-Order System

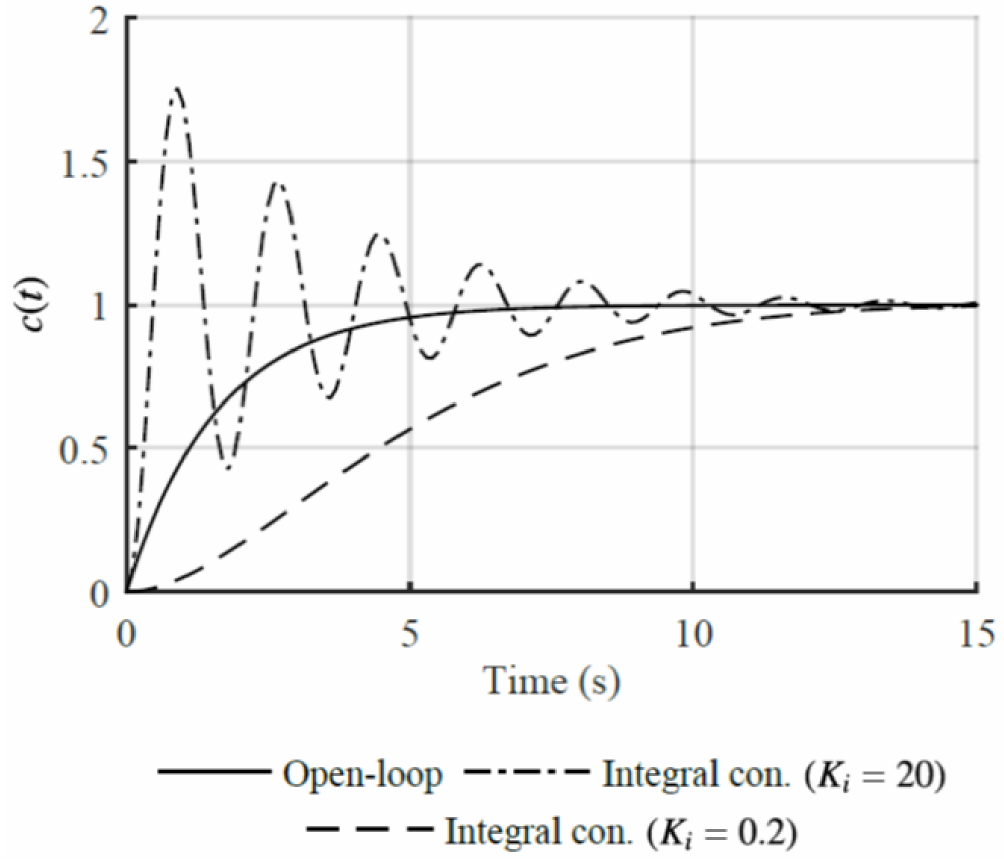


Effect of Integral Control on a First-Order System

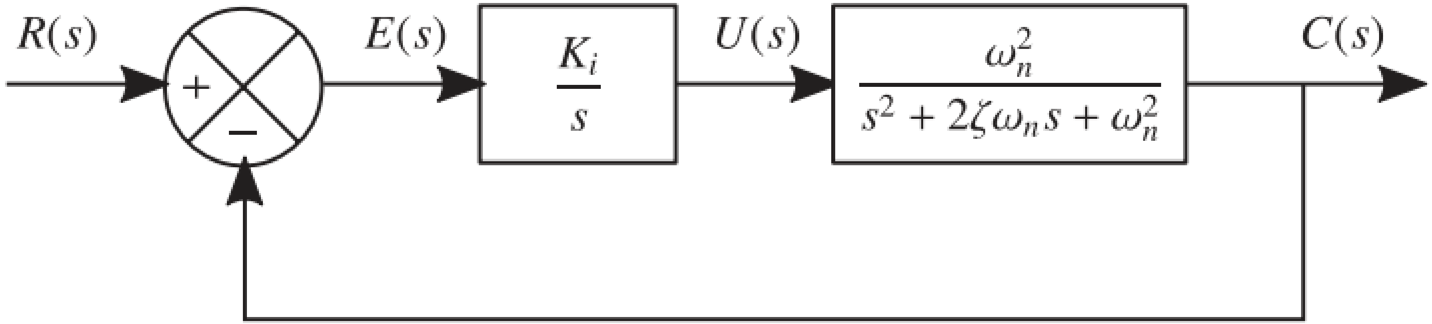
$$s^2 + 2\zeta cl\omega_{n,cl}s + \omega_{n,cl}^2 = 0$$

$$\omega_{n,cl} = \sqrt{\frac{K_i}{\tau}}$$

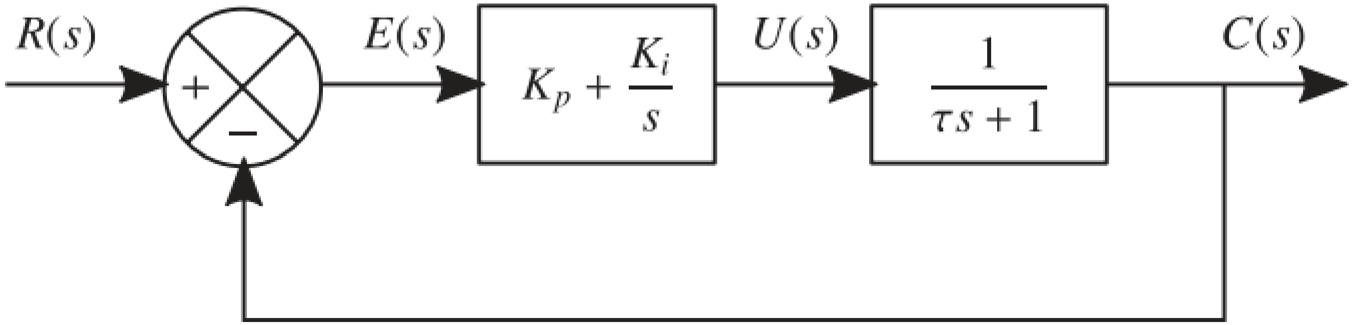
$$\zeta_{cl} = \frac{1}{2\sqrt{\tau K_i}}$$



Integral Control of a Second-Order System



PI Control of a First-Order System

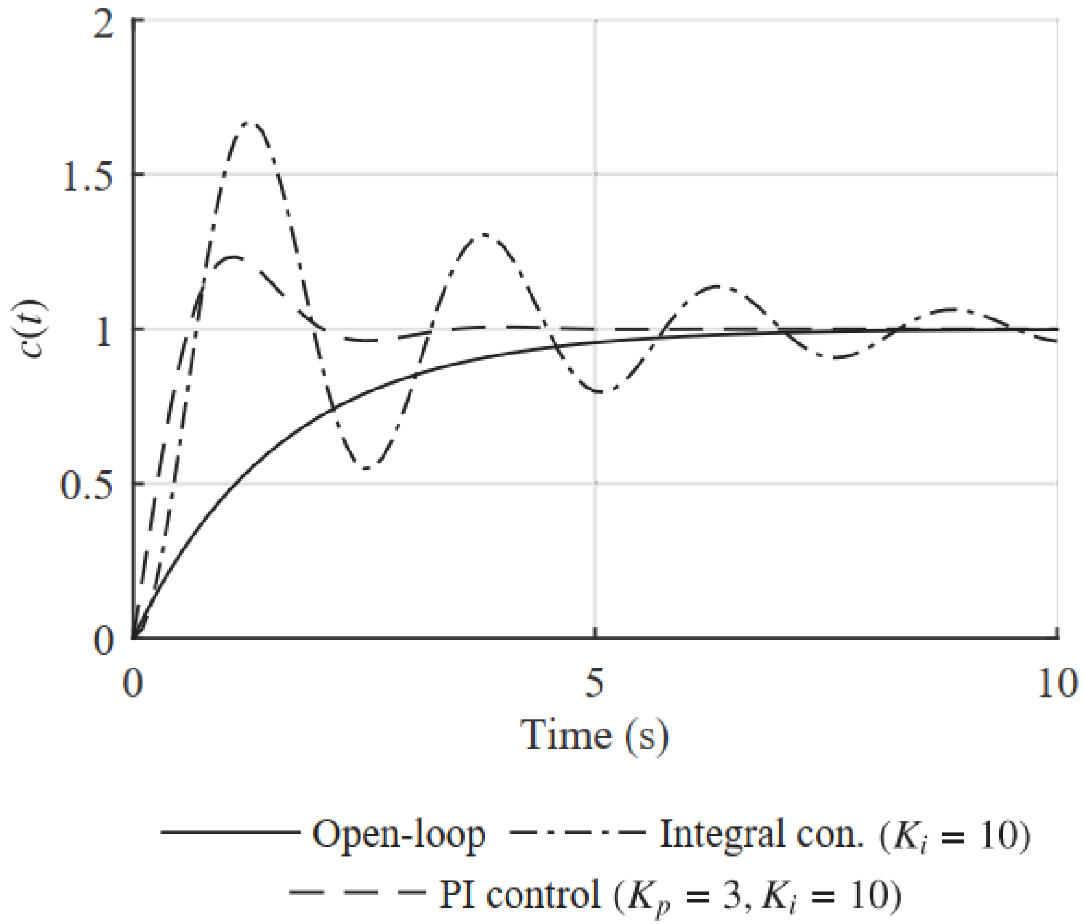


Effect of PI Control on a First-Order System

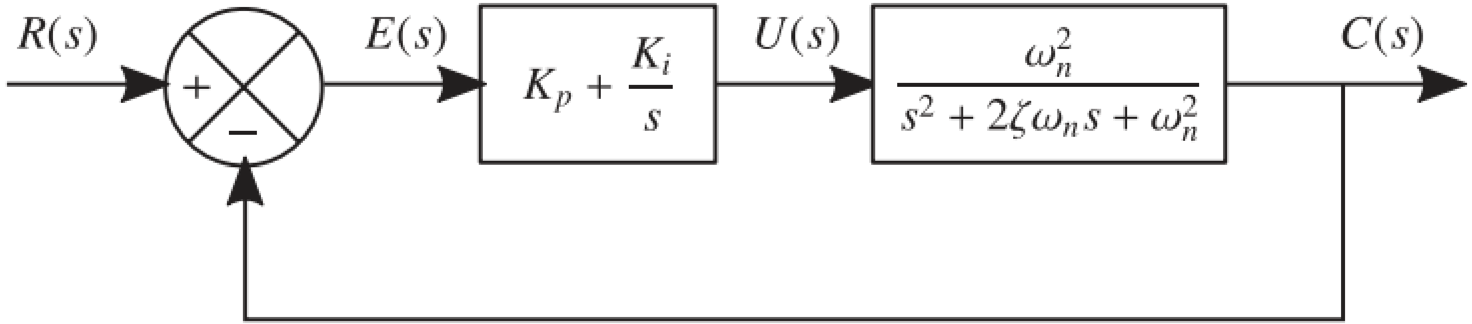
$$s^2 + 2\zeta cl\omega_{n,cl}s + \omega_{n,cl}^2 = 0$$

$$\omega_{n,cl} = \sqrt{\frac{K_i}{\tau}}$$

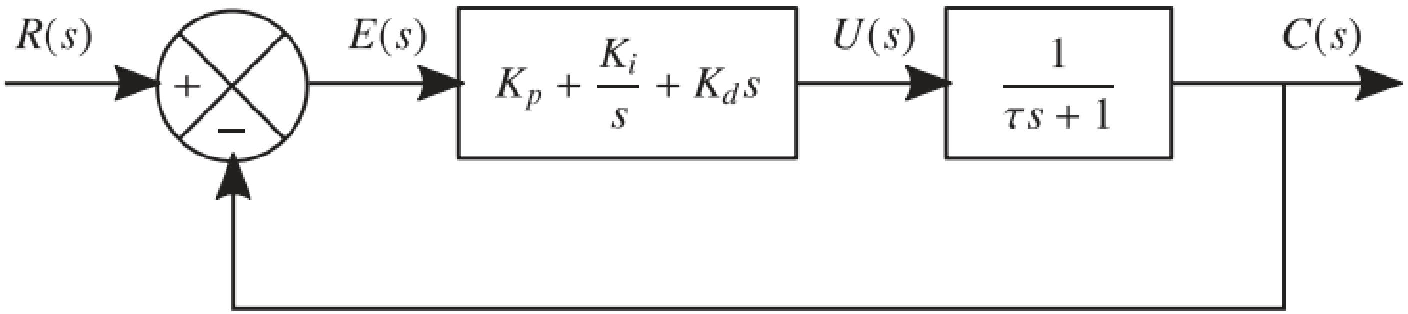
$$\zeta_{cl} = \frac{K_p + 1}{2\sqrt{\tau K_i}}$$



PI Control of a Second-Order System



PID Control of a First-Order System

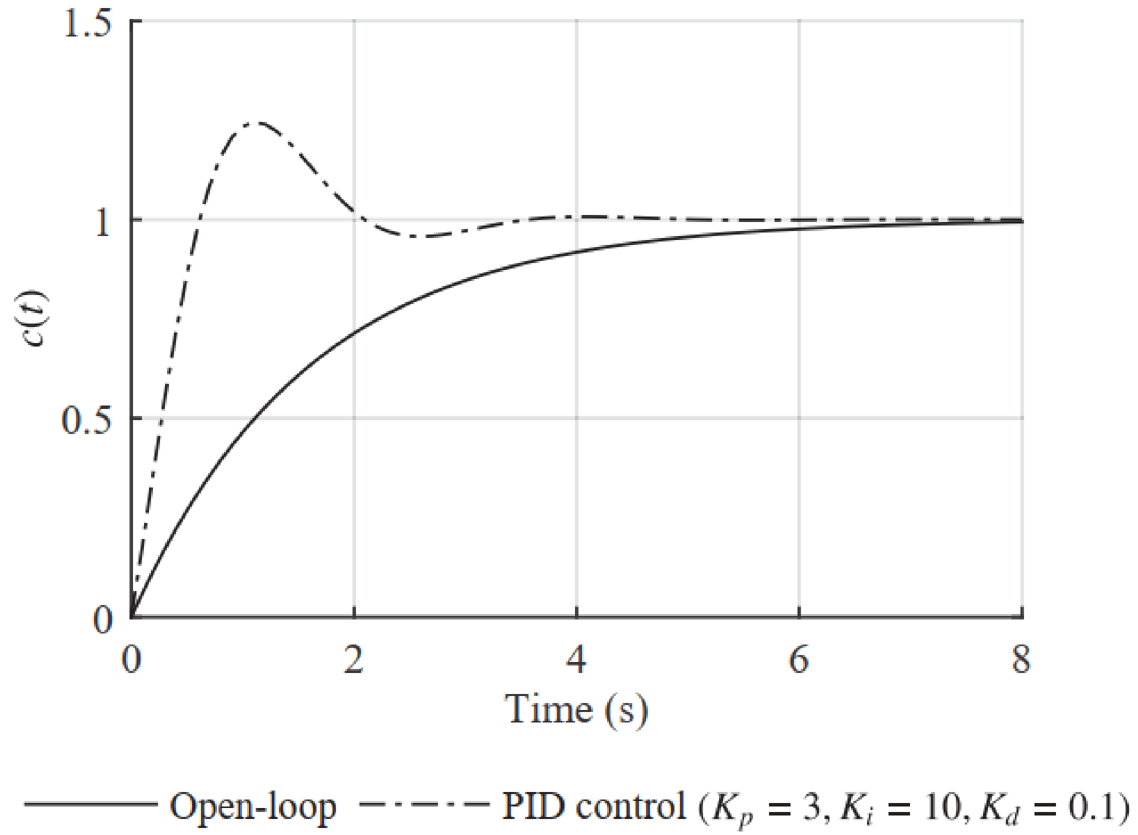


Effect of PID Control on a First-Order System

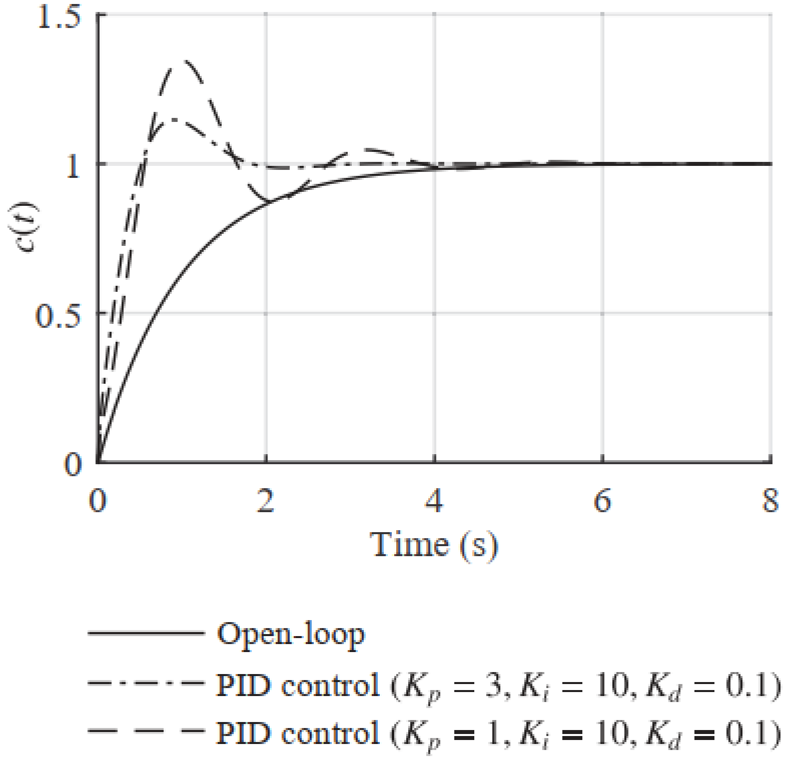
$$s^2 + 2\zeta cl\omega_{n,cl}s + \omega_{n,cl}^2 = 0$$

$$\omega_{n,cl} = \sqrt{\frac{K_i}{\tau + K_d}}$$

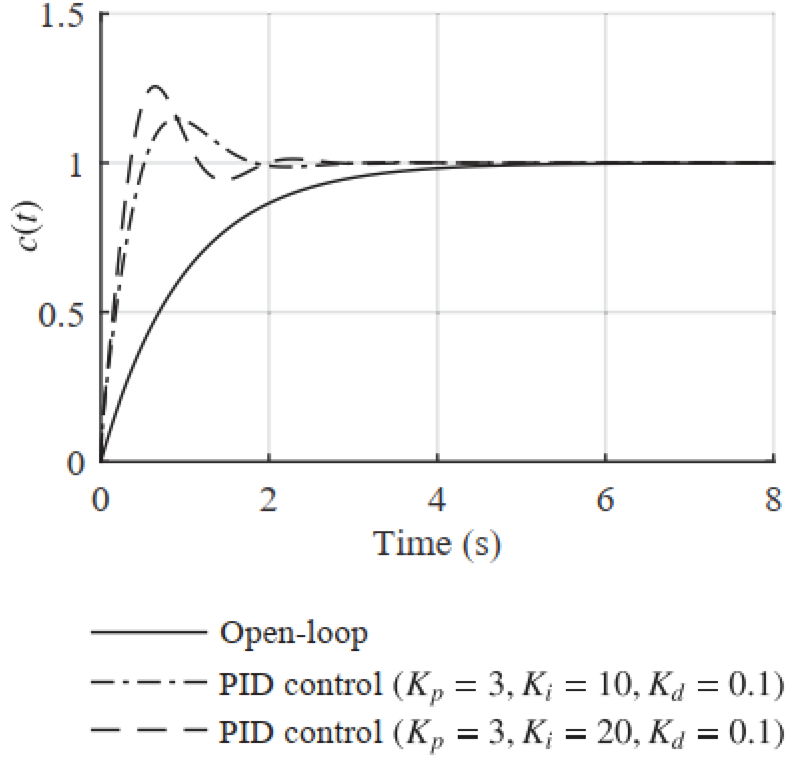
$$\zeta_{cl} = \frac{K_p + 1}{2\sqrt{K_i(\tau + K_d)}}$$



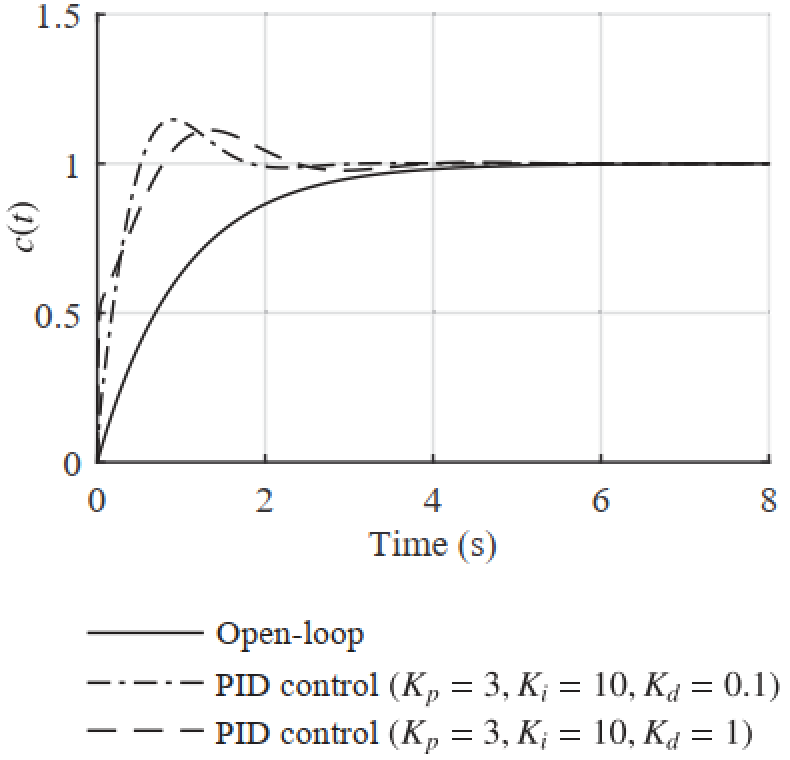
Effect of PID Control on a First-Order System cont.



(a) Proportional action.

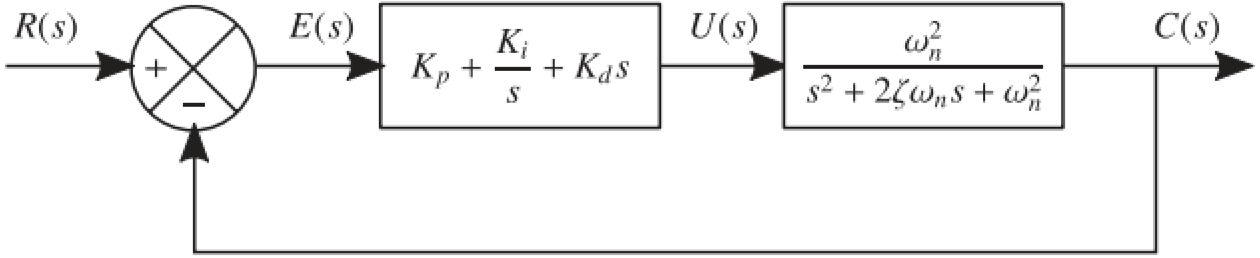


(b) Integral action.



(c) Derivative action.

PID Control of a Second-Order System



Summary

- PD Control of a Second-Order System and Effect on a Second-Order System
- Integral Control of a First-Order/Second-Order System and Effect on a First-Order/Second-Order System
- PI Control of a First-Order System and Effect on a First-Order System
- PID Control of a First-Order /Second-Order System and Effect on a First-Order System

Reference:

-Control Systems Engineering, 7th Edition, N.S. Nise

-UESTC3001 2019/20 Notes, J. Le Kernec