GLASGOW COLLEGE UESTC

Exam

Dynamics and Control (UESTC3001)

Date: 27th June 2021
Time: 09:30-11:30AM

Time: 2h

Attempt All Questions. Total 100 marks

- **All questions bear equal marks [25 marks]**
- **Use one answer sheet for each of the questions in this exam.**
- **Show all work on the answer sheet.**
- **Make sure that your University of Glasgow and UESTC Student Identification Numbers are on all answer sheets.**
- **An electronic calculator may be used provided that it does not allow text storage or display, or graphical display.**
- **All graphs should be clearly labelled and sufficiently large so that all elements are easy to read.**
- **The numbers in square brackets in the right-hand margin indicate the marks allotted to the part of the question against which the mark is shown. These marks are for guidance only.**

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Q1.

a) Explain the primary reasons for using control systems in engineering applications. [4]

b) The block diagram of a control system is shown in Figure Q1. Using block diagram reduction techniques, determine the closed-loop transfer function of the output C(s) over the command R(s).

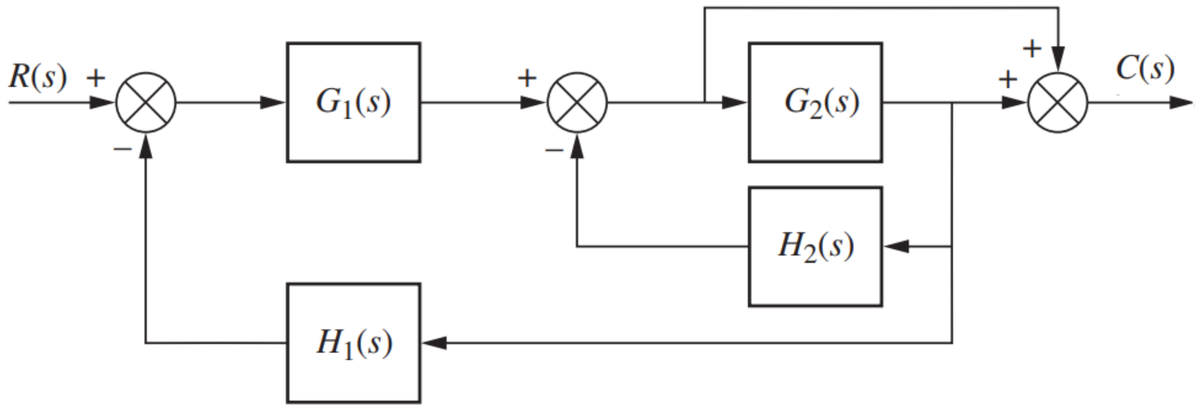

Figure Q1

i) Redraw the simplified block diagram. [5 marks]

ii) Reduce the block diagram to a single block using block reduction techniques. [8 marks]

c) Given the transfer function below, use the Routh's criteria to determine the range of values of *K* for which the system remains stable.

$$\text{Transfer function} = \frac{s^3+3s^2+s+6}{s^4+12s^3+s^2+4s+K}$$

[8 marks]

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Q2.

In order to regulate the speed of a rotor ω, a control system has been designed. The equation linking the torque $T(t)$ and the angular speed $\omega(t)$ is given below:

$$T(t) = 0.1\dot{\omega}(t) + 5\omega(t)$$

The controller has a transfer function $G_c(s)$ which operates on the error between the measured voltage $v_m(t)$ representing the actual speed of the rotor $\omega(t)$ and the voltage $v_r(t)$ representing the required speed $\omega_r(t)$. The voltage $v_m(t)$ supplied by the sensor is proportional to the actual speed of the rotor $\omega(t)$.

$$v_m(t) = \frac{1}{2\alpha}\omega(t)$$

The cable linking the output of the sensor to the comparator is not shielded, thus a noise $N(t)$ is added to the measured voltage $v_m(t)$. The required voltage $v_r(t)$ is computed mathematically from the required speed $\omega_r(t)$ using the same relationship as the sensor. The controller (via an electric servo motor) supplies the driving torque $T(t)$.

Consider the steps described in parts (a) to (d) below, then draw the block diagram in (e):

(a) Identify all the variables as a function of the complex variable s that are a part of the system. [2]

(b) Translate the transfer function for torque and output angular speed from time domain to the s-domain. [2]

(c) Translate the transfer function for the measured speed and the output angular speed from the time domain to the s-domain. [2]

(d) Draw the relations linking the variables to each other based on the description in the text. [6]

(e) Draw the block diagram of the system described above. [7]

(f) A proportional + integral + derivative controller is used with constant proportional and integral gain, K_p and K_i. What is the effect of increasing the derivative gain on the system response (natural frequency, frequency, oscillation and damping) to a unit step input for a first-order system? [6]

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Q3. Consider the control system shown in Figure Q3.

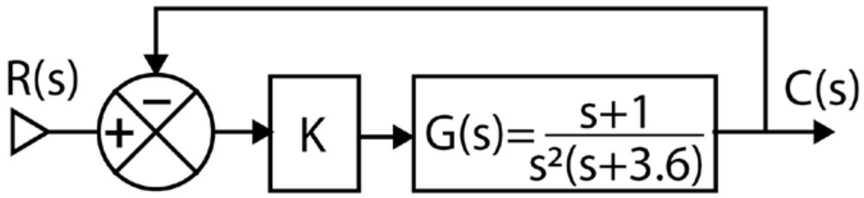

Figure Q3

(a) Using the plant transfer function $G(s) = \frac{s+1}{s^2(s+3.6)}$ in a closed-loop system with unit negative feedback with a controller gain of K, explain what is meant by root-locus analysis of a system KG(s)? Explain the relevance of the Characteristic Equation of the system when considering such analysis. [6]

(b) Before sketching the root-locus diagram for the open loop feedback system with the plant transfer function $G(s)$ given above in Figure Q3, considering a unit negative feedback loop with a controller gain of $K > 0$.

Show all calculations, including the following:

i) The values of the poles and zeros and the order of the open loop system [3]

ii) The intersection point and angle of the asymptotes [4]

iii) The breakaway point [5]

iv) Investigate if there are any points where root locus branches cross the imaginary axis. [3]

v) Sketch of the root loci for this system. [4]

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Q4.

a) Consider the open-loop transfer function of a process given by:

$$G(s) = \frac{K}{s(s-5)(s+100)}$$

Answer the following questions for $K = 300$ (you are expected to fully justify your answers).

(i) Sketch the Nyquist plot for *G*. As part of this question show clearly the calculations for both the gain and phase of the system at the key frequencies. [10]

(Hint to draw the Nyquist plot: You may need to consider the real part of the transfer function and its limit when ω tends to 0 to trace the sketch accurately.)

(ii) Calculate the gain and phase margins. You must show your calculations and reasoning. [8]

Hint: The following factorisation may be useful

$$\omega^6 + 10025\omega^4 + 2.5.10^5\omega^2 - 9.10^4$$
$$= (\omega + 0.5958)(\omega - 0.5958)(\omega^2 + 10^4)(\omega^2 + 25.356)$$

(iii) Based on your work in (i) and (ii), conclude on the stability of the closed loop system. [2]

b) A closed-loop system has the following characteristic equation:

$$s^4 + 8s^3 + 8s^2 + (K+1)s + Ka = 0$$

where K is a gain term.

Using Routh-Hurwitz, find the conditions that *K* and *a* must meet for the closed-loop to be stable. [5]

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Data Sheet: Important Laplace Transform Pairs

Function $f(t)$	**Laplace Transform $F(s)$**
Unit step function $u(t)$	$\frac{1}{s}$
t^n	$\frac{n!}{s^{n+1}}$
e^{-at}	$\frac{1}{s+a}$
te^{-at}	$\frac{1}{(s+a)^2}$
$\frac{1}{a}\left[1-e^{-at}\right]$	$\frac{1}{s(s+a)}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
Unit impulse function $\delta(t)$	1

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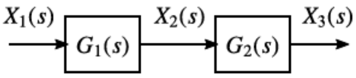
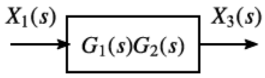
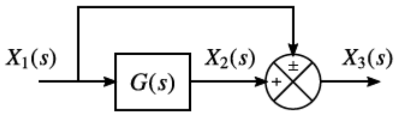
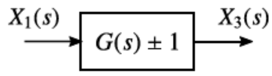
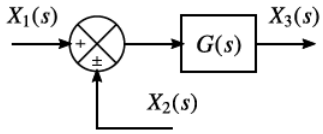
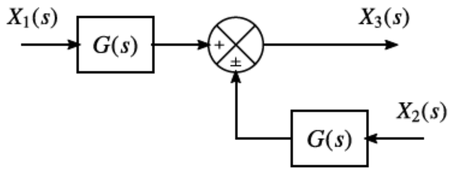
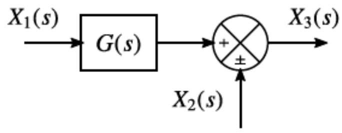
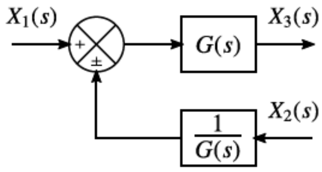
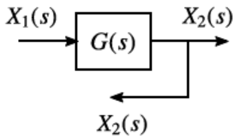
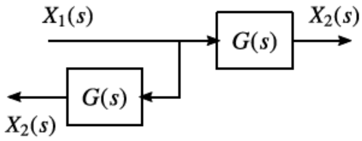
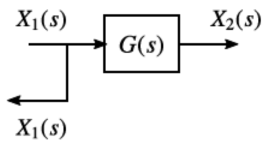
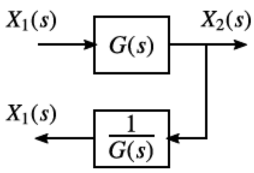
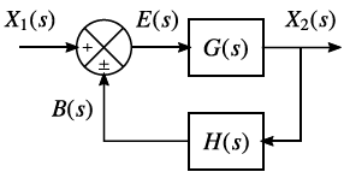
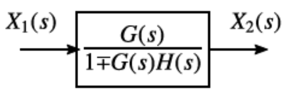
Data sheet 2: Rules for block diagram reduction

Rule	Original	Equivalent
1. Cascaded blocks		
2. Summing two signals		
3. Moving a summing point behind a block		
4. Moving a summing point ahead of a block		
5. Moving a branch point ahead of a block		
6. Moving a branch point behind a block		
7. Eliminating a feedback loop		

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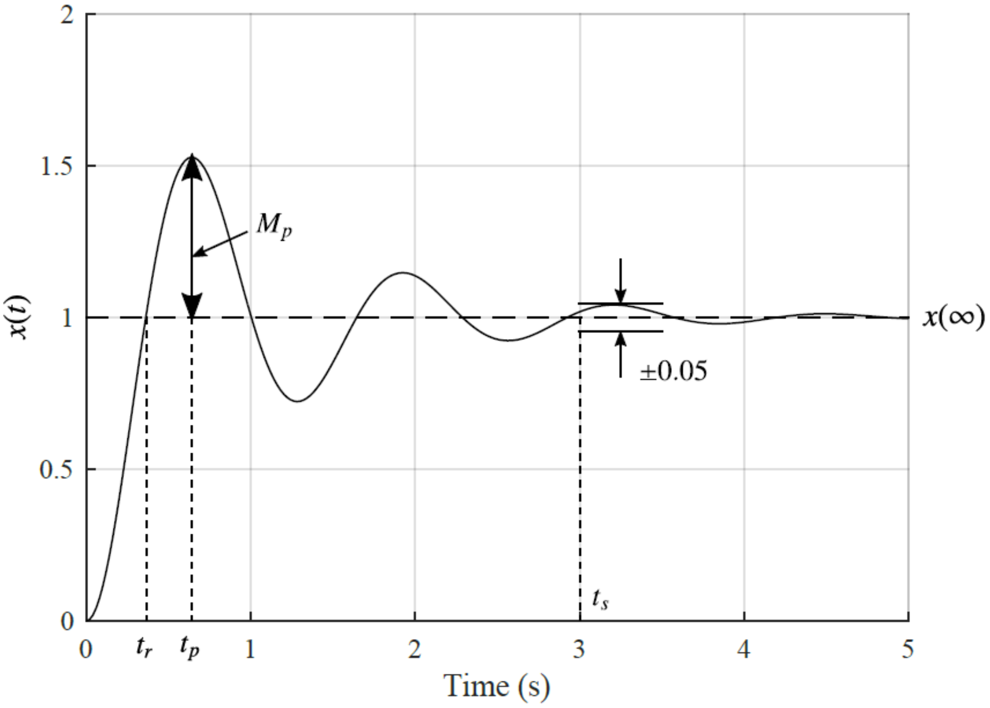
Datasheet 3: Second order systems

The general form of the second-order system is

$$\ddot{x}(t) + 2\zeta\omega_n\dot{x}(t) + \omega_n^2 x(t) = Ku(t), \qquad \frac{X(s)}{U(s)} = \frac{K}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

where ζ is the **damping ratio** and ω_n is the **natural frequency**, as stated previously.



Characteristics of an underdamped second-order system in response to a step input.

<u>Characteristics of the Response of an Underdamped 2nd Order System to a Step Input</u>

Rise Time	$t_r = \dfrac{1}{\omega_d}\tan^{-1}\left(\dfrac{\sqrt{1-\zeta^2}}{\zeta}\right)$
Peak Time	$t_p = \dfrac{\pi}{\omega_d}$
Maximum Overshoot	$M_p = x(\infty)e^{-\left(\pi\zeta/\sqrt{1-\zeta^2}\right)}$ or $M_p = 100e^{-\zeta\omega_n t_p}$ %
Settling Time	$t_s = \dfrac{3}{\zeta\omega_n}$
Damped natural Frequency	$\omega_d = \omega_n\sqrt{1-\zeta^2}$

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Datasheet 4: Root Locus

Root Locus Analysis

Properties of the Open Loop Transfer Function

The characteristic equation of a system may be written in the form: $1 + F(s) = 0$ where $F(s)$ is the open loop transfer function. This gives the magnitude condition that

$$|F(s)| = 1$$

.

and given that he open loop transfer function takes the form:

$$F(s) = \frac{K(s-z_1)(s-z_2)\cdots(s-z_v)}{s^n(s-p_1)(s-p_2)\cdots(s-p_u)}$$

we can write

$$K = \frac{\prod_{j=1}^{n+u} |s-p_j|}{\prod_{i=1}^{v} |s-z_i|}$$

Sketching a Root Locus

Angle of Asymptotes: $\phi = \dfrac{(2m+1)\times 180}{P-Z}$ degrees for m = 0, 1, 2, ... (P - Z - 1)

The intersection point: $\sigma_A = \dfrac{\sum_{j=1}^{P} \mathrm{Re}(p_j) - \sum_{i=1}^{Z} \mathrm{Re}(z_i)}{P-Z}$

Breakaway Point: $\dfrac{dK}{ds} = 0$

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Departure Angle from a Complex Pole, ϕ_d, is given by:

$\phi_d = 180^\circ -$ Σ angles of vectors to the complex pole from all other poles
$+ \Sigma$ angles of vectors to the complex pole from all zeros.

Arrival Angle to a Complex Zero, ϕ_a, is given by:

$\phi_a = 180 -$ Σ angles of vectors to the complex zero from all other zeros
$+ \Sigma$ angles of vectors to the complex zero from all poles

Newton-Raphson Method: For a function A(s) = 0, successive estimates of a root may be obtained from:

$$s_{i+1} = s_i - \frac{A(s_i)}{A'(s_i)}$$

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Datasheet 5: Frequency Response Analysis

General theory

For a system with closed loop transfer function: $G(s) = \dfrac{X(s)}{Y(s)}$

a periodic input $y(t) = Y \sin \omega t$

yields a steady state response $x(t)_{ss} = Y\left|G(i\omega)\right|\sin(\omega t + \phi)$

where $\left|G(i\omega)\right| =$ the gain of the system,

and $\phi = \angle G(i\omega)$ = the phase of the system.

Elements of the Bode plot

Factor	**G(s)**	**G(iω)**	$\left\|G(i\omega)\right\|_{dB}$	$\angle G(i\omega)$ **rad**
Gain	K	K	$20\log_{10} K$	0
Poles at Origin	$\dfrac{1}{s^n}$	$\dfrac{1}{i^n \omega^n}$	$-20n\log_{10}\omega$	$-n\dfrac{\pi}{2}$
Zeros at Origin	s^n	$i^n \omega^n$	$20n\log_{10}\omega$	$n\dfrac{\pi}{2}$
Pole	$\dfrac{1}{1+\tau s}$	$\dfrac{1}{1+i\omega\tau}$	$\omega \to 0$: 0 $\omega \to \infty$: $-20\log_{10}(\tau\omega)$	$\omega \to 0$: 0 $\omega \to \infty$: $-\dfrac{\pi}{2}$
Zero	$1 + \tau s$	$1 + i\omega\tau$	$\omega \to 0$: 0 $\omega \to \infty$: $20\log_{10}(\tau\omega)$	$\omega \to 0$: 0 $\omega \to \infty$: $\dfrac{\pi}{2}$
Quadratic Poles	$\dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$	$\dfrac{\omega_n^2}{(\omega_n^2 - \omega^2) + 2\zeta\omega_n\omega i}$	$\omega \to 0$: 0 $\omega \to \infty$: $-40\log_{10}(\tau\omega)$	$\omega \to 0$: 0 $\omega \to \infty$: $-\pi$
Quadratic Zeros	$s^2 + 2\zeta\omega_n s + \omega_n^2$	$(\omega_n^2 - \omega^2) + 2\zeta\omega_n\omega i$	$\omega \to 0$: 0 $\omega \to \infty$: $40\log_{10}(\tau\omega)$	$\omega \to 0$: 0 $\omega \to \infty$: π

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Properties of the Bode Plot

For quadratic poles: Resonant Frequency, ω_r: $\omega_r = \omega_n \sqrt{1-2\zeta^2}$

Resonant Peak, M_r $M_r = \dfrac{1}{2\zeta\sqrt{1-\zeta^2}}$

End of question paper