Dynamics and Control (UESTC 3001)

Notes prepared by: Dr Ola R. Popoola

Syllabus Delivery

- Lecture 1: Root Locus Analysis
- Lecture 2: Root Locus II and Nyquist Plots
- Lecture 3: Bode Plots
- Lecture 4: Bode Plots II
- Lecture 5: Stability in Frequency Domain
- Lecture 6: Stability Examples
- Lecture 7: Compensators
- Lecture 8: Tutorials and Test Exercises

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Intended Learning Objectives

At the end of this lecture, you will be able to:

- Explain frequency response analysis and discuss how it relates to stability of a control system.
- Draw and analyze Nyquist plots of control systems and infer the stability of the system.

Additional Reading Materials

- "Modern Control Systems" - R. C. Dorf, R.H. Bishop Addison Wesley
- "Modern Control Engineering" - K. Ogata. Prentice Hall International
- "Feedback Control of Dynamic Systems" - Franklin, Powell, Emami-Naeini. Addison Wesley.
- "Feedback Systems: An Introduction for Scientists and Engineers" – K. J. Åström, R. M. Murray. Princeton University Press, Princeton.
  Available at
  https://www.cds.caltech.edu/~murray/amwiki/index.php/Main_Page
- "Control Engineering: An introduction with the use of Matlab", D. P. Atherton.
  Available at http://bookboon.com/en/control-engineering-matlab-ebook

These are recommended only and are not required to pass the course.

Additional Notes

- All required notes/tutorials/exam answers may be found on Moodle.
- You are encouraged to do your own reading to complement the lecture material.
- You can also use the forum on Moodle to ask questions about course material, rather than using email.

Revision –RLC Summary

1. Identify number of poles (n), zeros (m), and determine the order (n − m).
2. Plot the location of the poles (x) and zeros (o) in the complex plane
3. Determine which parts of the real axis are on the locus
4. Identify the number and directions of the asymptotes
5. Find where the asymptotes meet and draw the asymptotes on the graph.
6. Identify any poles uniquely connected to zeros along the x-axis — these form one branch of the locus.
7. If there are complex poles or zeros the angles of arrival or departure need to be determined (Chapter 5 of reference book – rule 4)
8. If two poles on the axis need to become complex to go to asymptotes, they must pass through a double point found by the solutions of $\frac{\mathrm{d}G(s)}{\mathrm{d}s} = 0$.

In many cases, steps 1–6 allow a reasonable estimation of the approximate shape of the root-locus.

Guided Classwork

Sketch Root Locus for a system with open loop transfer function:

$$G(s) = \left(\frac{(s+3)}{s(s+1)(s+2)(s+4)}\right)$$

Steps:

1. Identify number of poles (n), zeros (m), and determine the order (n − m).

   n_poles = 4; n_zeros=1, order = 4-1=3

2. Plot the location of the poles (x) and zeros (o) in the complex plane
3. Determine which parts of the real axis are on the locus
4. Identify the number and directions of the asymptotes

   n_asymptote=n_p-n_z=3, directions = 180, 60, -60

5. Find where the asymptotes meet and draw the asymptotes on the graph.

Guided Classwork

Sketch Root Locus for a system with open loop transfer function:

$$G(s) = \left(\frac{(s+3)}{s(s+1)(s+2)(s+4)}\right)$$

Steps:

$$\text{CoG} = \frac{(\Sigma_p - \Sigma_z)}{\mathrm{n_p} - \mathrm{n_z}} = \frac{0-1-2-4-(-3)}{4-1} = -\frac{4}{3}$$

. Identify number of poles (n), zeros (m), and determine the order (n − m).

6. Identify any poles uniquely connected to zeros along the x-axis — these form one branch of the locus.

7. Identify breakaway points if any

Guided Classwork

Breakaway points

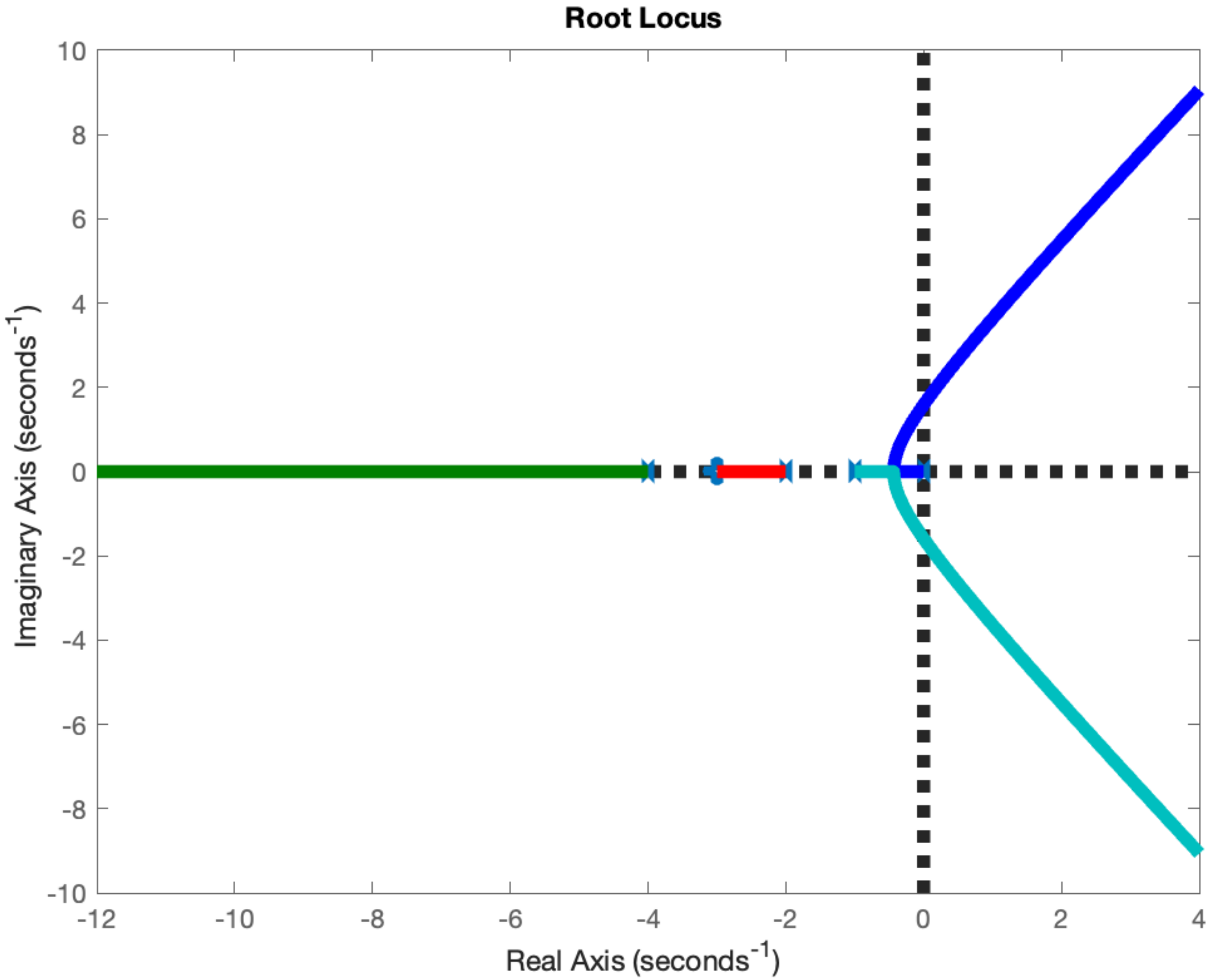
$$\frac{dG(s)}{ds} = \frac{d}{ds}\left(\frac{(s+3)}{s(s+1)(s+2)(s+4)}\right) = 0$$

$$\frac{3s^4 + 26s^3 + 77s^2 + 84s + 24}{???} = 0$$

$$s = -1.61, -0.43, -3.3 \pm 0.68$$

Only valid breakaway point is -0.43.

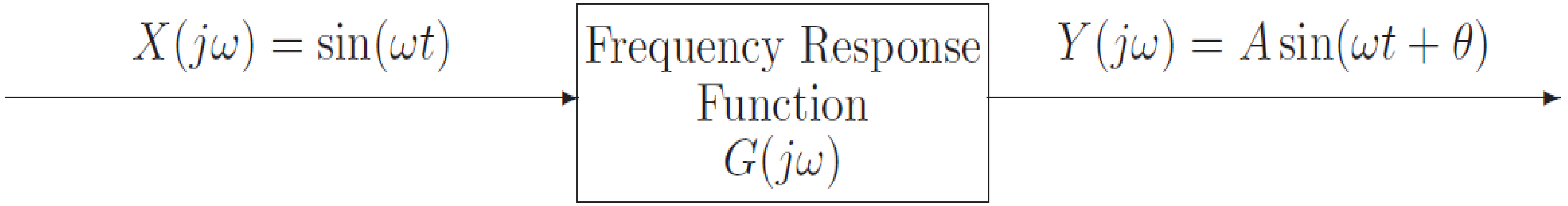
Guided Classwork

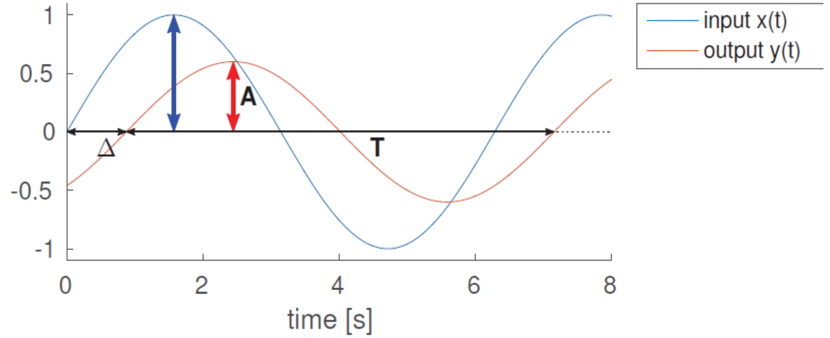


Background Discussion –Frequency Response

Frequency Response

- The Frequency Response of a system is defined as the steady-state response of the system to a sinusoidal input signal.
- Frequency response analysis is a method used to understand how a system responds to different frequencies of input signals.
- It provides insights into how a system behaves across a range of frequencies, which is crucial for designing stable and robust control systems.
- For linear time invariant systems, the frequency response is independent of the amplitude and phase of the input signal.





Time vs Frequency Response

- Revision of time domain parameters
  - Rise time t_r
  - Settling time t_s
  - Overshoot M_p
  - Steady-state error e_{ss}
  - Natural frequency ω_n
  - Damping factor ζ

Time vs Frequency Response

- Consider a second order system: $\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$, where ζ is the damping factor and ω_n is the undamped natural frequency.
- Given the open loop transfer function in time $G_t(s) = \frac{K_v}{s(\tau s+1)}$ and in frequency $G_f(s) = \frac{\omega_n^2}{s^2+2\zeta\omega_n s}$ and $G_t(s) \equiv G_f(s)$.
- For $s = j\omega$, $\frac{C(j\omega)}{R(j\omega)} = T(j\omega) = \frac{\omega_n^2}{(j\omega)^2+2\zeta\omega_n(j\omega)+\omega_n^2} = \frac{1}{1-u^2+j2\zeta u}$, where $u = \frac{\omega}{\omega_n}$ is the normalised driving signal frequency (1).
- We can write $|T(j\omega)| = M = \frac{1}{\sqrt{(1-u^2)^2+(2\zeta u)^2}}$ and $\angle T(j\omega) = \phi = -\tan^{-1}\left[\frac{2\zeta u}{1-u^2}\right]$, M: magnitude and ϕ: phase (2,3).

Time vs Frequency Response

- The maximum magnitude, known as resonant peak, can be computed at the resonant frequency $\omega = \omega_r$ at $\frac{dM}{du}|_{u=u_r} = 0$

- $4u_r^3 - 4u_r + 8\zeta^2 u_r = 0, u_r = \sqrt{1 - 2\zeta^2}, \omega_r = \omega_n\sqrt{1 - 2\zeta^2}$ (4)

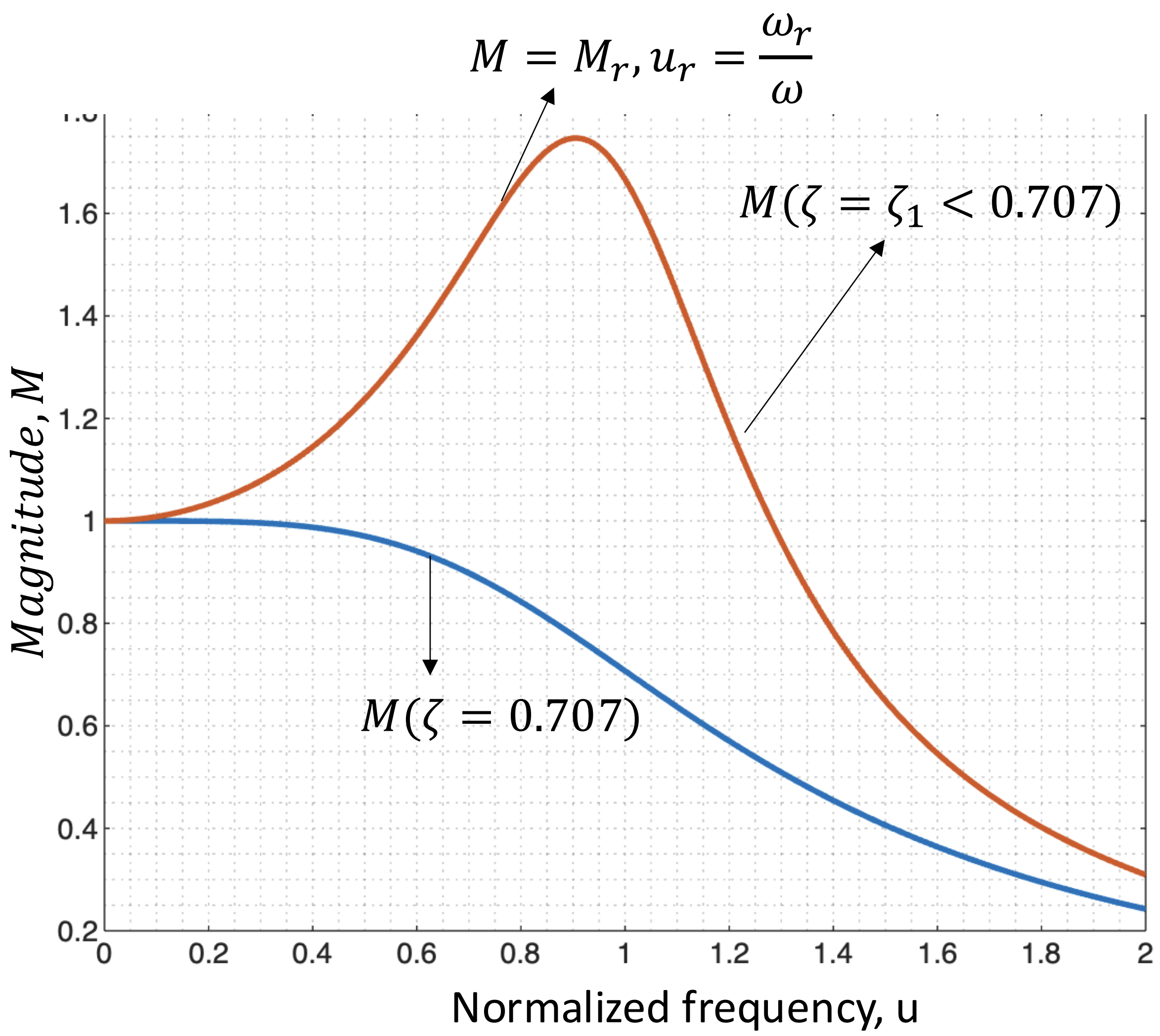
- Resonant peak $M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$ (5)

- Phase at resonant frequency $\phi_r = -\tan^{-1}\frac{\sqrt{1-2\zeta^2}}{\zeta}$

Time vs Frequency Response

From (4) and (5)

- As ζ approaches zero:
  - ω_r approaches ω_n
  - M_r approaches ∞
- For $0 < \zeta \leq \frac{1}{\sqrt{2}}$
  - $\omega_r < \omega_n$
  - $M_r > 1$



Time vs Frequency Response

Study the response for various damping factor with respect to (4) and (5)

- For $\zeta > \frac{1}{\sqrt{2}}$

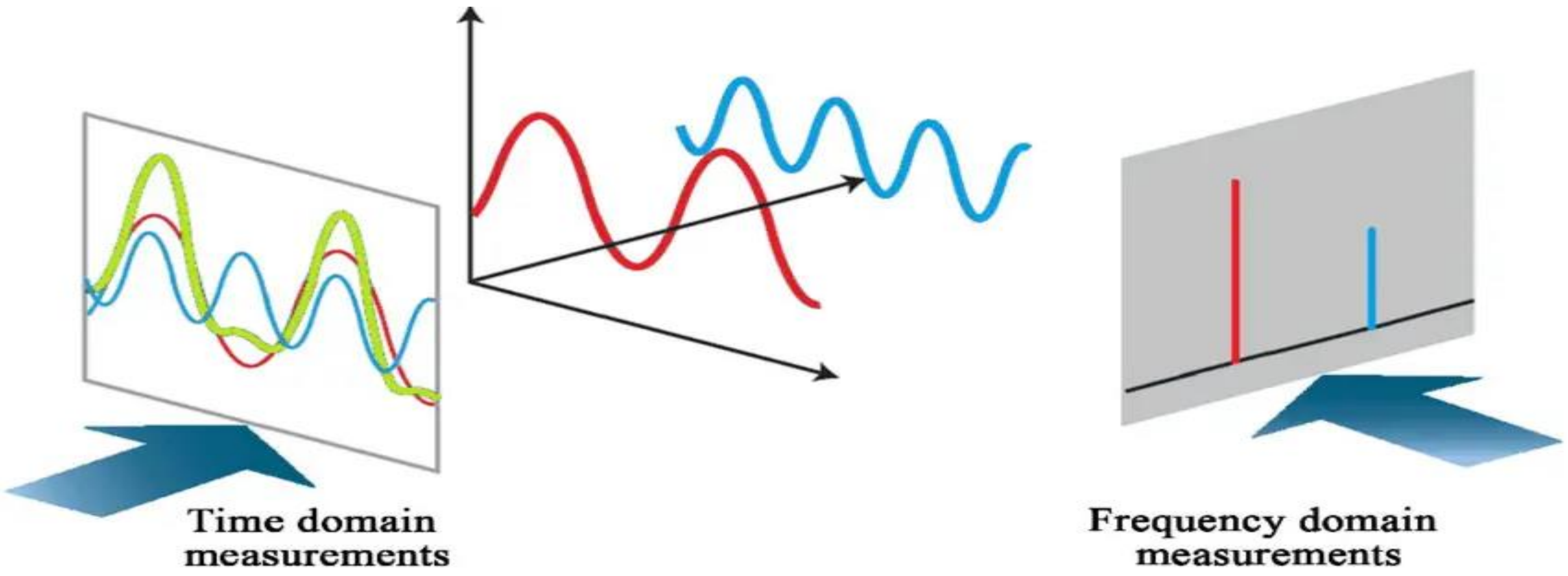
- For $0 < \zeta \leq \frac{1}{\sqrt{2}}$

- $\omega_r = \omega_n\sqrt{1 - 2\zeta^2}$ (4)

- Resonant peak $M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$ (5)

Concepts in Frequency Domain

- **Frequency Domain Representation:** Signals and systems are represented in frequency domain instead of time domain.
- **Gain Margin:** Very important concept that looks at how much the system's gain can be increased before the system is unstable.
- **Phase Margin:** How much phase lag can be added to the system before instability occurs



Source: www.studyelectrical.com

Frequency Response Analysis

Evaluating the frequency response function G(jω) for a frequency ω generally results in a complex value which can be represented using:

- its magnitude |G(jω)|

- its phase argG(jω),

or by its real and imaginary components,

$$G(j\omega) = |G(j\omega)|e^{j\arg G(j\omega)}$$
$$= \Re\big(G(j\omega)\big) + \Im\big(G(j\omega)\big)$$

Source: www.studyelectrical.com

Frequency Response Plots

There are 2 ways of plotting the frequency response:

- The polar locus (or **Nyquist** plot) displays the values of $G(j\omega)$ for a range of frequencies in the complex plane

- The **Bode** Diagram displays the magnitude and phase in separate plots as a function of ω.

Source: www.studyelectrical.com

Nyquist Plot

Polar Locus/Nyquist Plot

The polar locus is a graph showing how the phase and magnitude of the frequency response varies with frequency ω.

There are a few steps to sketch the polar locus of a system:

1. Determine the behaviour of the magnitude and phase as $\omega \rightarrow 0$. This is determined by the number of pure integrators (if any) and the gain.
2. Determine the behaviour of the magnitude and phase as $\omega \rightarrow \infty$. The phase is determined by the relative order of the system. For all real systems $|G(j\omega)| \rightarrow 0$ as $\omega \rightarrow \infty$.
3. Determine the corner frequencies of any LEAD or LAG terms, and evaluate the magnitude and phase at these values.
4. The frequency and the point at which the locus crosses the negative real axis is important.

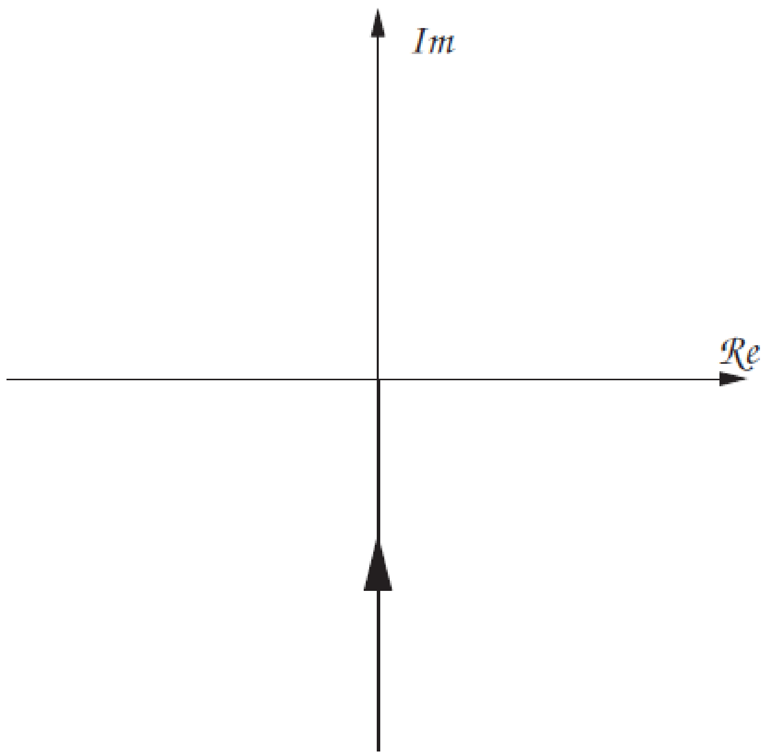
Nyquist Plot –Single Integrator

Transfer function: $\boldsymbol{G(s) = \frac{1}{s}}$

Frequency response function: $G(j\omega) = \frac{1}{j\omega}$

This can be seen to have:

- a phase of -90^0
- and a magnitude given by: $|G(j\omega)| = \frac{1}{\omega}$



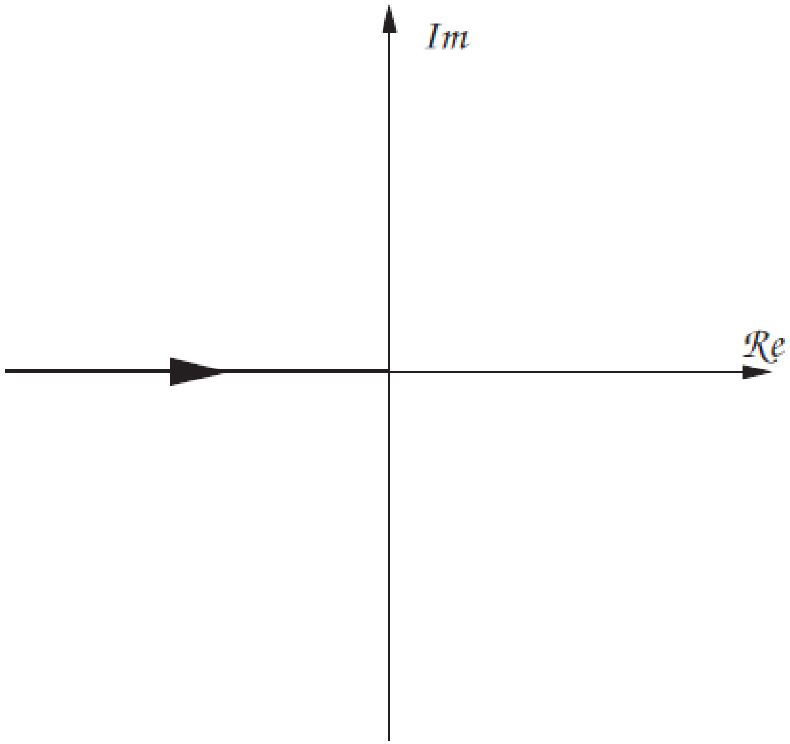
Nyquist Plot –Double Integrator

Transfer function: $\boldsymbol{G(s) = \frac{1}{s^2}}$

Frequency response function: $G(j\omega) = \frac{1}{(j\omega)^2}$

This can be seen to have:

- a phase of -180^0
- and a magnitude given by: $|G(j\omega)| = \frac{1}{\omega^2}$



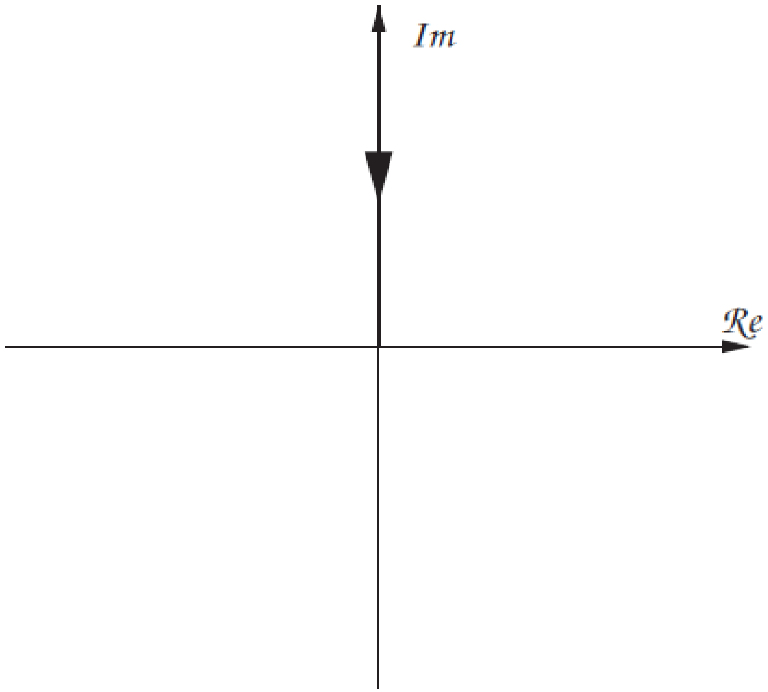
Nyquist Plot –Tripple Integrator

Transfer function: $\boldsymbol{G(s) = \frac{1}{s^3}}$

Frequency response function: $G(j\omega) = \frac{1}{(j\omega)^3}$

This can be seen to have

- a phase of -270^0
- and a magnitude given by: $|G(j\omega)| = \frac{1}{\omega^3}$



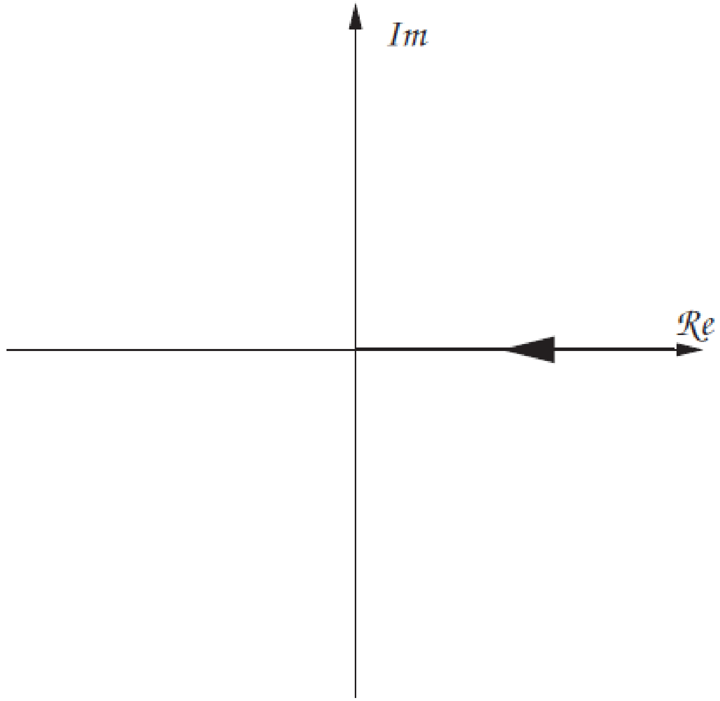
Nyquist Plot –Quadruple Integrator

Transfer function: $\boldsymbol{G(s)} = \frac{\mathbf{1}}{\boldsymbol{s}^{\mathbf{4}}}$

Frequency response function: $G(j\omega) = \frac{1}{(j\omega)^4}$

This can be seen to have

- a phase of -360^0
- and a magnitude given by: $|G(j\omega)| = \frac{1}{\omega^4}$



Nyquist Plot –Single Lag

Transfer function: $\boldsymbol{G(s)} = \frac{\mathbf{1}}{\boldsymbol{s}+\mathbf{1}}$

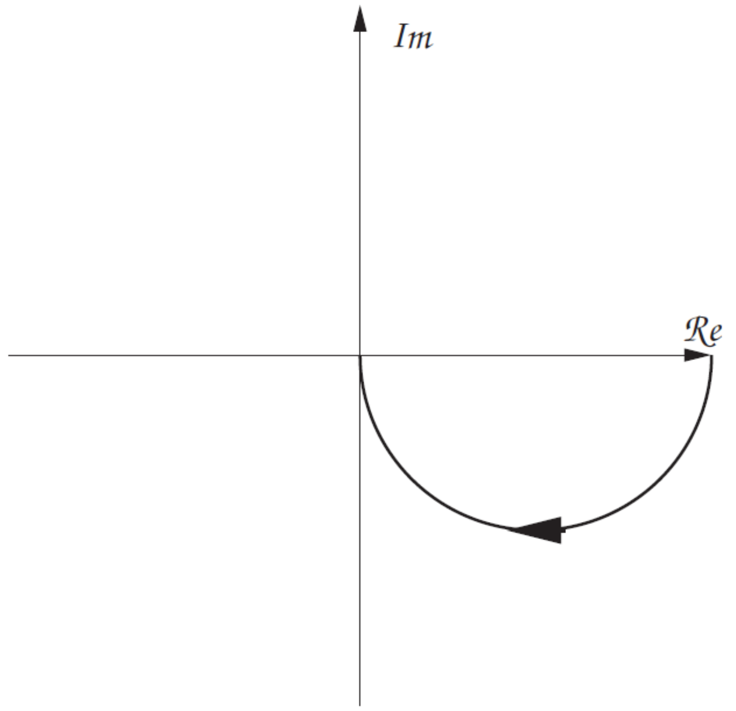
Frequency response function: $G(j\omega) = \frac{1}{1+j\omega}$

This can be seen to have a phase of **$-\tan^{-1}\omega$**. This varies between **0** when ω = 0 and **-90^0** when ω→∞.

The magnitude given by:

$$|G(j\omega)| = \frac{1}{\sqrt{1+\omega^2}}$$

Which varies between 1 when ω = 0 and zero when ω→∞.

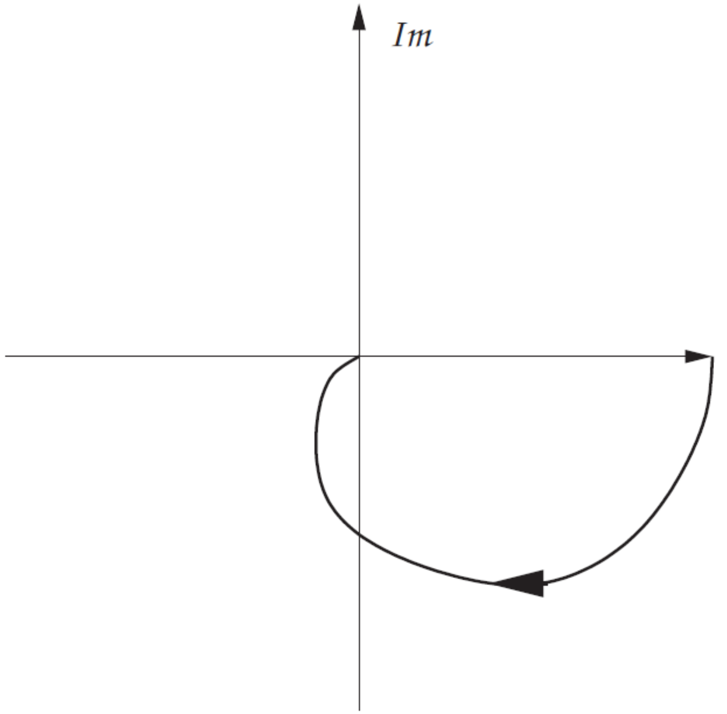


Nyquist Plot –Double Lag

Transfer function: $\boldsymbol{G(s) = \frac{1}{(s+1)^2}}$

Frequency response function: $G(j\omega) = \frac{1}{(1+j\omega)^2}$

This can be seen to have a phase of **$-2\tan^{-1}\omega$**. This varies between **0** when ω = 0 and **-180^0** when ω→∞.



The magnitude given by:

$$|G(j\omega)| = \frac{1}{1+\omega^2}$$

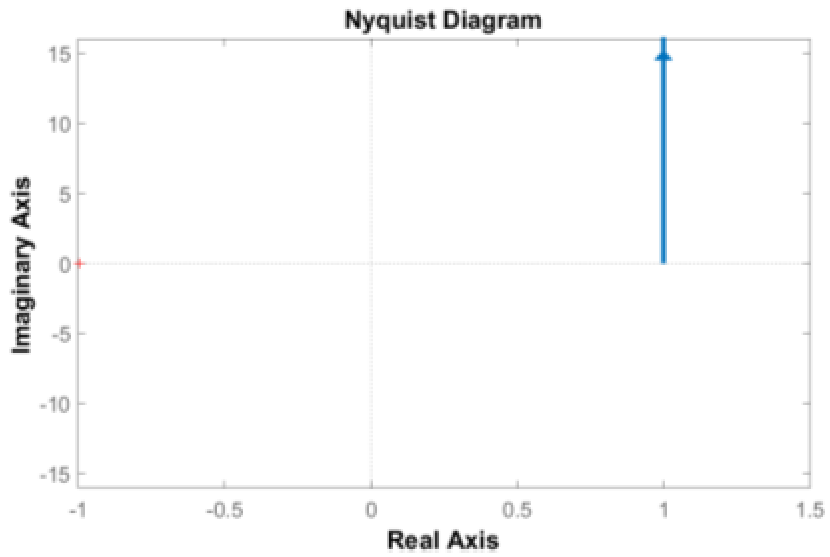
Which varies between **1** when ω = 0 and **0** when ω→∞.

Nyquist Plot –Single Lead

Transfer function: $\boldsymbol{G(s) = s + 1}$

Frequency response function: $G(j\omega) = 1 + j\omega$

This can be seen to have a phase of **$\tan^{-1}\omega$**. This varies between **0** when ω = 0 and **$+90^0$** when ω→∞.



The magnitude given by:

$|G(j\omega)| = \sqrt{1 + \omega^2}$

Which varies between

1 when ω = 0 and

+∞ when ω→∞.

Nyquist Plot –MATLAB Examples

$$G(s) = \frac{10(1+10s)}{s\left(1+\frac{s}{10}\right)^2} = \frac{100s+10}{\frac{1}{100}s^3+\frac{1}{5}s^2+s}$$

In Matlab, it can be represented as a transfer function object:

```
sys = tf([100 10],[1/100 1/5 1 0]);
```

- Root locus: rlocus(sys)
- Nyquist plot: nyquist(sys)

  N.B. : don’t forget to switch off the display of negative frequencies

Nyquist Plot –Examples

Draw the Nyquist plot of the transfer functions:

$$(a) H(s) = \frac{10}{s(s+2)}$$

$$(b) H(s) = \frac{1}{(s-3)(s-4)}$$