Dynamics and Control (UESTC 3001)

Notes prepared by: Dr Ola R. Popoola

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- Lecture 1: Root Locus Analysis
- Lecture 2: Root Locus II and Nyquist Plots
- Lecture 3: Bode Plots
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- Lecture 5: Stability in Frequency Domain
- Lecture 6: Stability Examples
- Lecture 7: Compensators
- Lecture 8: Tutorials and Test Exercises

Lecture delivered by: Dr Ola Popoola (Olaoluwa.Popoola@Glasgow.ac.uk)

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Intended Learning Objectives

At the end of this lecture, you will be able to:

- Sketch bode plots
- Worked examples on Bode plots

Review of Bode Plots

$$G(s) = \frac{K\left(1+\frac{s}{\omega_{Z1}}\right)\left(a+\frac{s}{\omega_{z2}}\right)\ldots}{s^r\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)..\left(1+\frac{2\zeta s}{\omega_n}+\frac{s^2}{\omega_n^2}\right)\ldots}$$

Component	Formular	Gain	Phase
a. Gain	K	$20\log\lvert K\rvert$	0^o
b. Integrator	$\frac{1}{s^r}$	$-20r\log\omega$	$-r.90^o$
c. First Order lead	$1+\frac{s}{\omega_{Z1}}$	0, 20 dB/dec after ω_{z1}	$\tan^{-1}(\frac{\omega}{\omega_{z1}})$
d. First Order lag	$\frac{1}{1+\frac{s}{\omega_{p1}}}$	0, -20 dB/dec after ω_{p1}	$-\tan^{-1}(\frac{\omega}{\omega_{p1}})$
e. Second Order lag	$\frac{1}{\left(1+\frac{2\zeta s}{\omega_n}+\frac{s^2}{\omega_n^2}\right)}$	0, -40 dB/dec after $\omega=\omega_n$	$-\tan^{-1}(\frac{\frac{2\zeta\omega}{\omega_n}}{1-\frac{\omega^2}{\omega_n^2}})$

Constructing Bode Plots for G(jω)

1. Rewrite transfer function in time constant form
2. Find the corresponding corner frequencies for each factor.
3. Now we are required one semi-log graph chooses a frequency range such that the plot should start with the frequency which is lower than the lowest corner frequency. Mark angular frequencies on the x-axis, mark slopes on the left hand side of the y-axis by marking a zero slope in the middle and on the right hand side mark phase angle by taking -180^{o} in the middle.
4. Calculate the gain factor and the type of order of the system.
5. calculate slope corresponding to each factor then add.

Bode Plots –Examples

$$G(s) = \frac{10(1+10s)}{s\left(1+\frac{s}{10}\right)^2}$$

Components:

A simple gain 10

A simple lead term $(1+10s)$

An integrator $\frac{1}{s}$

Two lag terms $\frac{1}{1+\frac{s}{10}}$

Bode Plots –Examples

$$G(s) = \frac{10(1+10s)}{s\left(1+\frac{s}{10}\right)^2}$$

Write out G(jw)

$$\mathrm{G}(\mathrm{j}\omega) = \frac{10(1+10j\omega)}{j\omega\left(1+\frac{j\omega}{10}\right)^2}$$

Bode Plots –Examples

Identify components and corner frequencies:

Component	Magnitude	Phase
10	20 log 10 = 20 dB	0 deg
$\frac{1}{j\omega}$	-20dB/dec	-90 deg
$1 + 10j\omega$	$\omega_c = 0.1;$ 0 for $\omega \ll 0.1;$ 20dB/dec for $\omega \gg 0.1$	0 deg for $\omega \ll \omega_c$ (<~0.02) 45 deg for $\omega = \omega_c$ 90 deg for $\omega \gg \omega_c$ (~>0.5)
$\frac{1}{1+\frac{j\omega}{10}}$	$\omega_c = 10;$ 0 for $\omega \ll 10;$ -20dB/dec for $\omega \gg 10$	0 deg for $\omega \ll \omega_c$ (~<2) -45 deg for $\omega = \omega_c$ -90 deg for $\omega \gg \omega_c$ (~>50)
$\frac{1}{1+\frac{j\omega}{10}}$	$\omega_c = 10;$ 0 for $\omega \ll 10;$ -20dB/dec for $\omega \gg 10$	0 deg for $\omega \ll \omega_c$ -45 deg for $\omega = \omega_c$ -90 deg for $\omega \gg \omega_c$

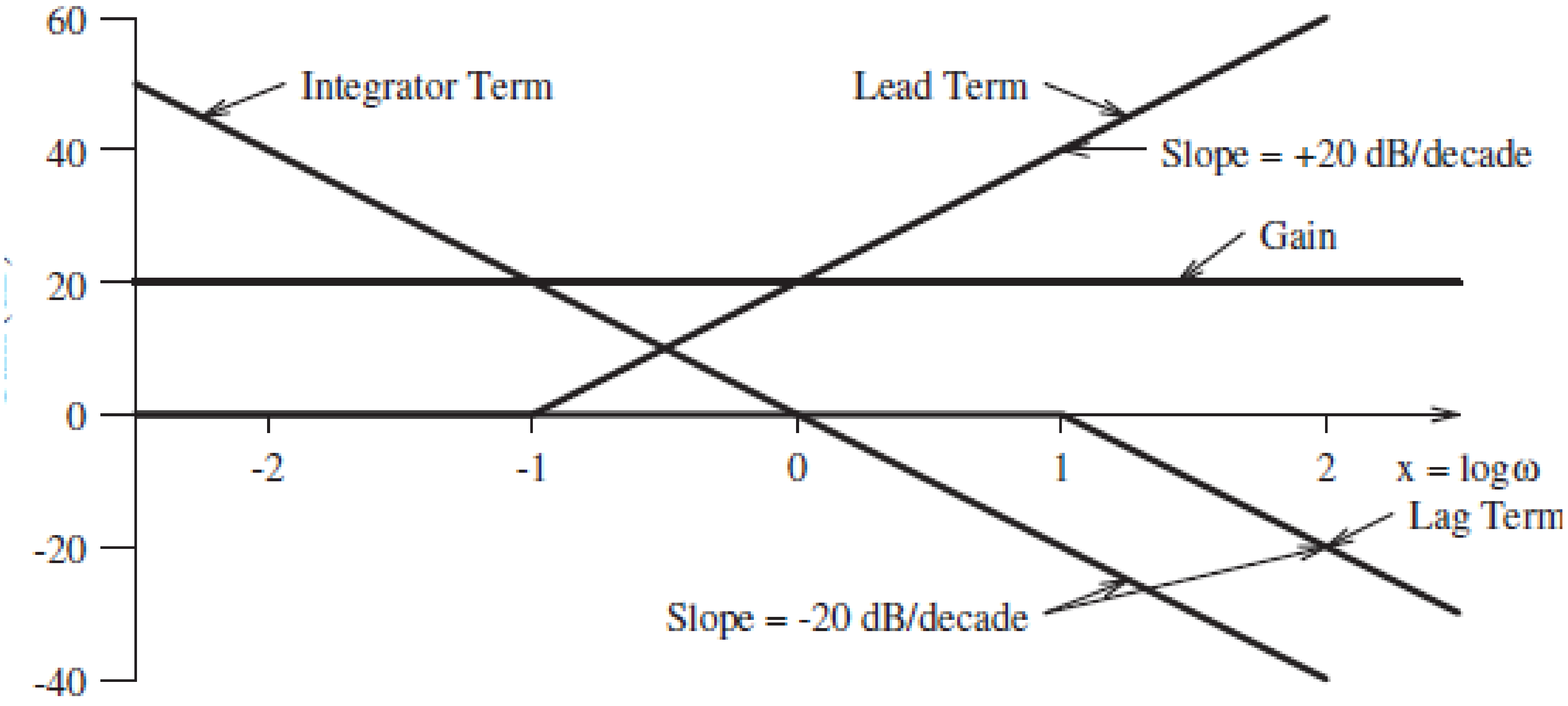
Bode Plots –Examples

Identify components and corner frequencies:

Component	Magnitude	Phase
10	20 log 10 = 20 dB	0 deg
$\frac{1}{j\omega}$	-20dB/dec	-90 deg
$1+10j\omega$	$\omega_c = 0.1;$ 0 for $\omega \ll 0.1;$ 20dB/dec for $\omega \gg 0.1$	0 deg for $\omega \ll \omega_c$ 45 deg for $\omega = \omega_c$ 90 deg for $\omega \gg \omega_c$
$\frac{1}{\left(1+\frac{j\omega}{10}\right)^2}$	$\omega_c = 10;$ 0 for $\omega \ll 10;$ -40dB/dec for $\omega \gg 10$	0 deg for $\omega \ll \omega_c$ -90 deg for $\omega = \omega_c$ -180 deg for $\omega \gg \omega_c$

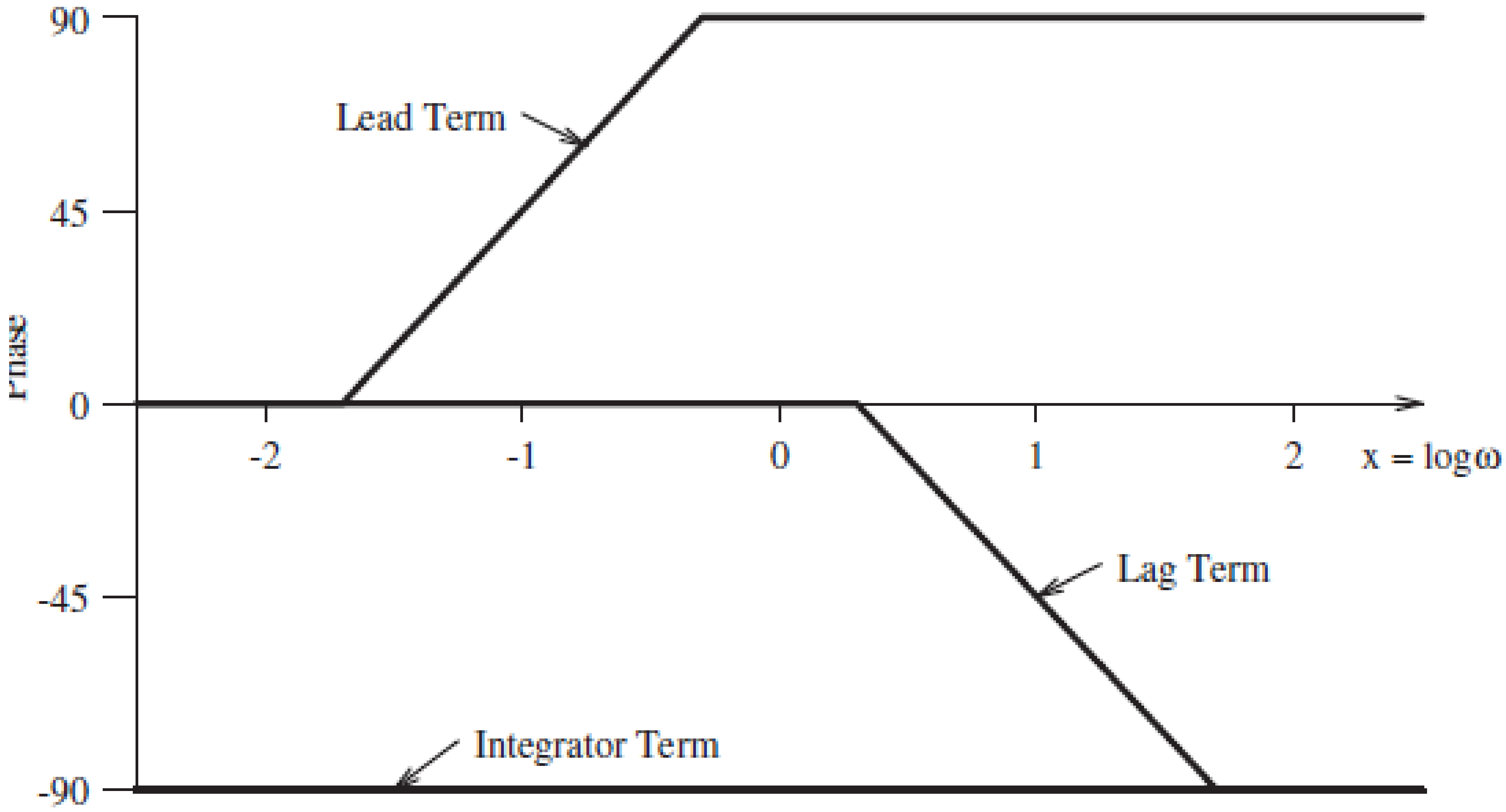
Magnitude plot

$$G(s) = \frac{10(1+10s)}{s\left(1+\frac{s}{10}\right)^2}$$



Phase plot

$$G(s) = \frac{10(1 + 10s)}{s\left(1 + \frac{s}{10}\right)^2}$$

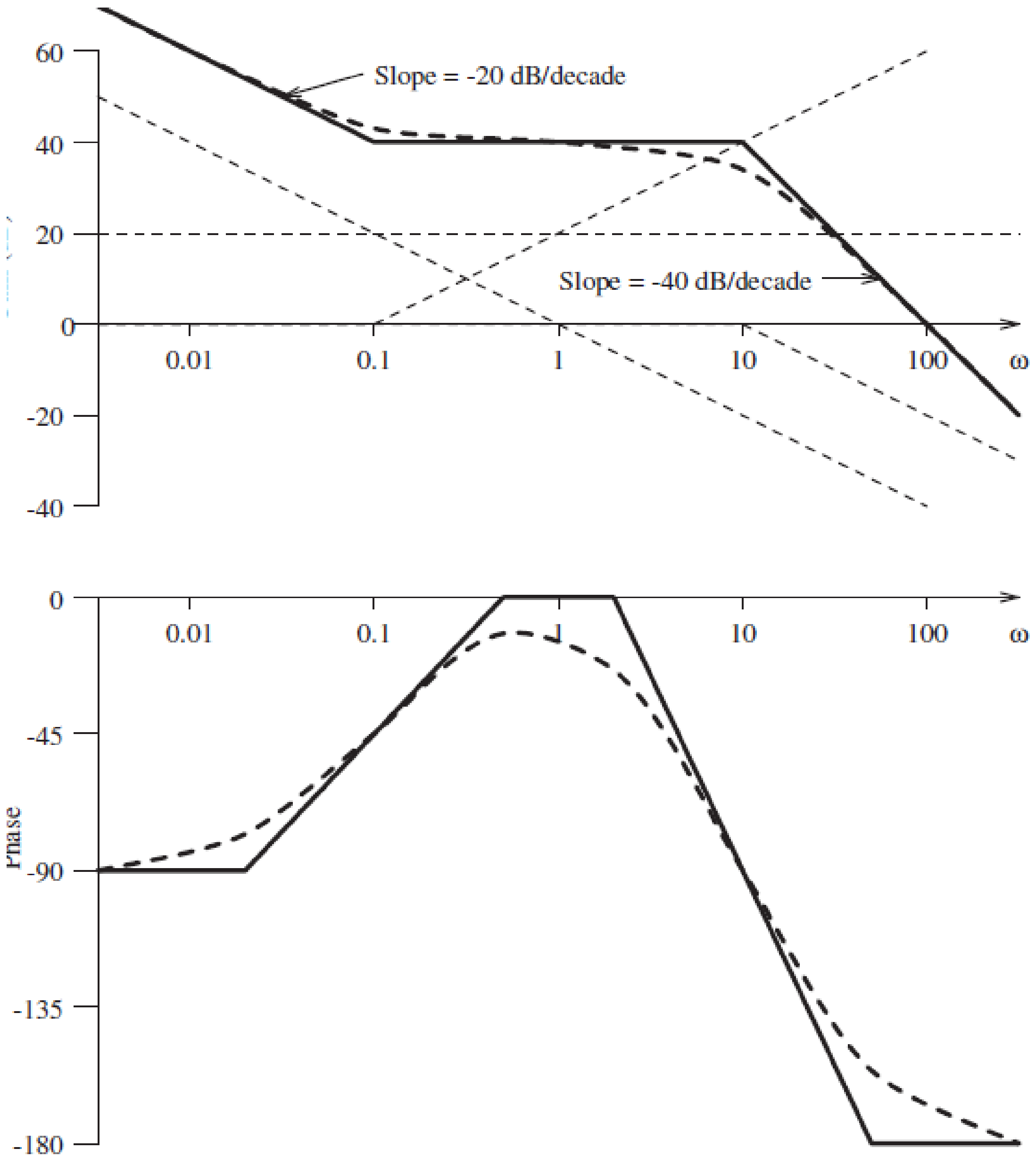


Bode Plots –Examples

$$G(s) = \frac{10(1+10s)}{s\left(1+\frac{s}{10}\right)^2}$$

Phase Plot

$0 < \omega < 0.02$	$\text{Angle} = -90^\circ$
$\omega = 0.1$	$\text{Angle} = -45^\circ$
$0.5 < \omega < 2$	$\text{Angle} = 0^\circ$
$\omega = 10$	$\text{Angle} = -90^\circ$
$50 < \omega < \infty$	$\text{Angle} = -180^\circ$



Bode Plots –Matlab

$$G(s) = \frac{10(1+10s)}{s\left(1+\frac{s}{10}\right)^2} = \frac{100s+10}{\frac{1}{100}s^3+\frac{1}{5}s^2+s}$$

In Matlab, it can be represented as a transfer function object:
sys = tf([100 10],[1/100 1/5 1 0]);

Bode plot: bodeplot(sys)

Exercise

Consider the transfer function:

$$H(s) = \frac{10(s+0.5)}{s(s+10)}$$

For this transfer function, sketch its Bode diagrams

Solution:

Write H(jw) in time constant form

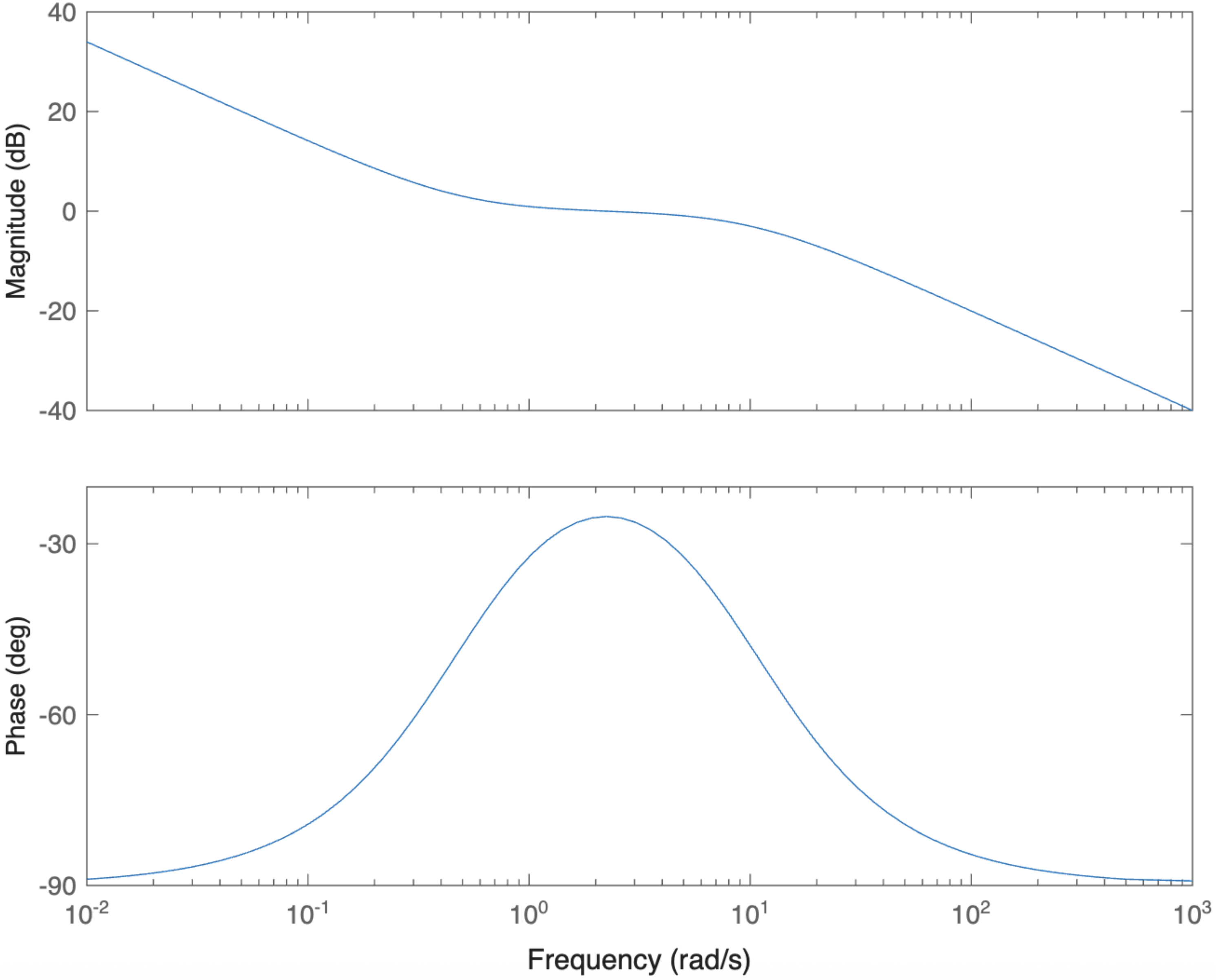
$$H(j\omega) = \frac{10(j\omega+0.5)}{j\omega(j\omega+10)} = 10 \times \frac{0.5\left(1+\frac{j\omega}{0.5}\right)}{10j\omega \times \left(1+\frac{j\omega}{10}\right)}$$

Bode Plots –Examples

Identify components and corner frequencies:

Component	Magnitude	Phase
0.5	20 log 0.5 = -6.02dB	0 deg
$\frac{1}{j\omega}$	-20dB/dec	-90 deg
$1+\frac{j\omega}{0.5}$	$\omega_c = 0.5$; 0 for $\omega \ll 0.5$; 20dB/dec for $\omega \gg 0.5$	0 deg for $\omega \ll \omega_c$ (<~0.05) 45 deg for $\omega = \omega_c$ 90 deg for $\omega \gg \omega_c$ (~>5)
$\frac{1}{1+\frac{j\omega}{10}}$	$\omega_c = 10$; 0 for $\omega \ll 10$; -20dB/dec for $\omega \gg 10$	0 deg for $\omega \ll \omega_c$ (~<1) -45 deg for $\omega = \omega_c$ -90 deg for $\omega \gg \omega_c$ (~>100)

Bode Plots –Examples



Complex conjugate component

$$G(s) = s^2 + 2\zeta\omega_n s + \omega_n^2$$

$$\mathrm{G}(j\omega) = -\omega^2 + j2\zeta\omega_n\omega + \omega_n^2$$

Time constant form can be written as:

$$G(j\omega) = 1 + j\frac{2\zeta\omega}{\omega_n} - \left(\frac{\omega}{\omega_n}\right)^2$$

Corner frequency is given as ω_n

1. Show that the magnitude for complex conjugate is 0 for $\omega \ll \omega_n$ and for $\omega \gg \omega_n$, it rises by 20dB/dec
2. Show that the phase angle varies from 0 to 180 deg and angle at ω_n is 90 deg

Exercise

Consider the transfer function:

$$G(s) = \frac{64(s+2)}{s(s+0.5)(s^2+3.2s+64)}$$

For this transfer function, sketch its Bode diagrams

Solution:

Write G(s) in time constant form

$$G(s) = 64 \times \frac{2\left(1+\frac{s}{2}\right)}{0.5 \times 64 \times s\left(1+\frac{s}{0.5}\right)\left(1+\frac{3.2s}{64}+\frac{s^2}{64}\right)}$$

Exercise

$$G(s)=\frac{4\left(1+\frac{s}{2}\right)}{s\left(1+\frac{s}{0.5}\right)\left(1+\frac{3.2s}{64}+\frac{s^2}{64}\right)}$$

Write out G(jw)

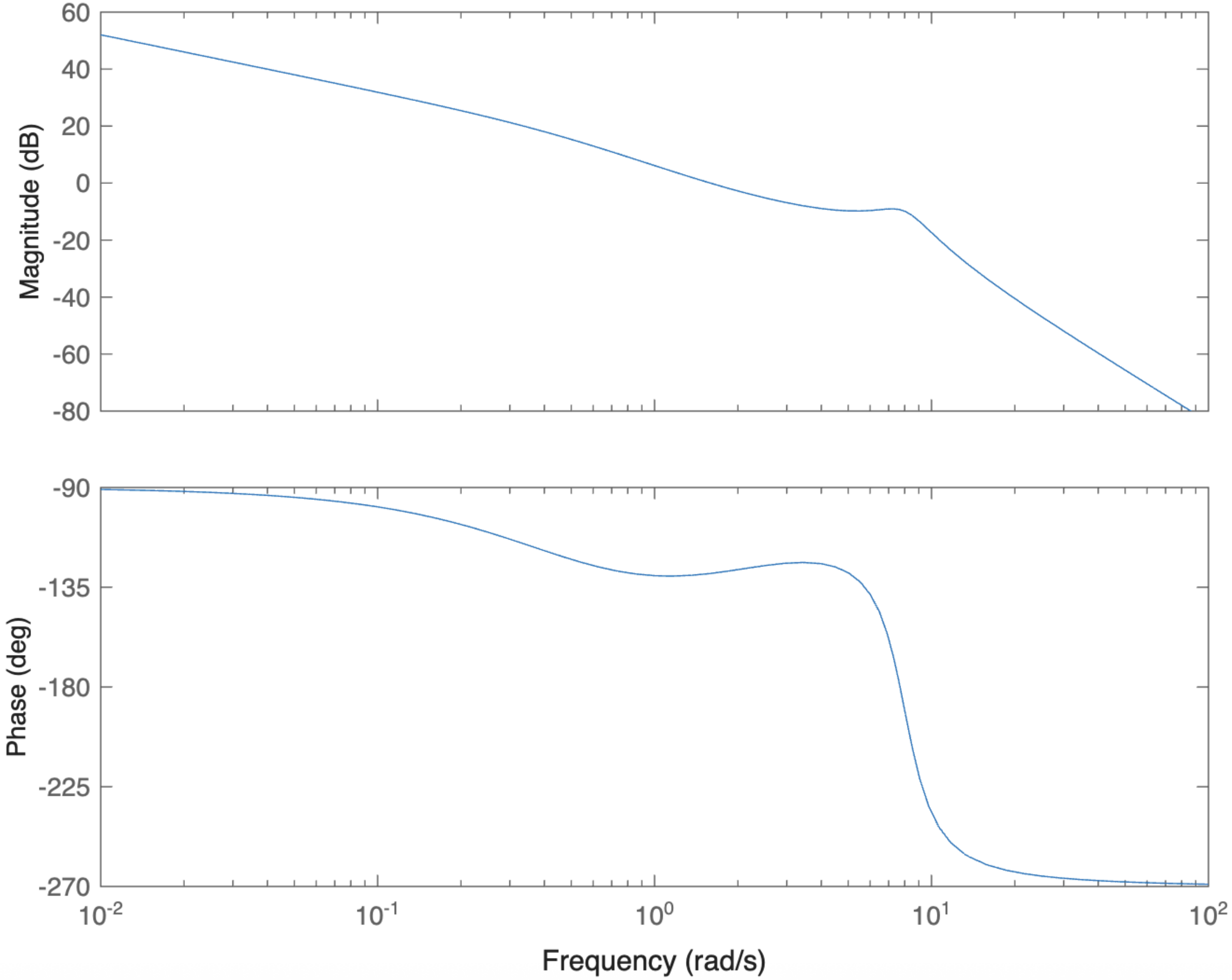
$$G(j\omega)=\frac{4\left(1+\frac{j\omega}{2}\right)}{j\omega\left(1+\frac{j\omega}{0.5}\right)\left(1+j\frac{0.4\omega}{8}-\left(\frac{\omega}{8}\right)^2\right)}$$

Identify components and corner frequencies

Bode Plots –Examples

Component	Magnitude	Phase
4	20 log 4 = 12.04 dB	0 deg
$\dfrac{1}{j\omega}$	-20dB/dec	-90 deg
$1+\dfrac{j\omega}{2}$	$\omega_c = 2;$ 0 for $\omega \ll 2;$ +20dB/dec for $\omega \gg 2$	0 deg for $\omega \ll \omega_c$ (<~0.2) 45 deg for $\omega = \omega_c$ 90 deg for $\omega \gg \omega_c$ (~>20)
$1/(1+\dfrac{j\omega}{0.5})$	$\omega_c = 0.5;$ 0 for $\omega \ll 0.5;$ 20dB/dec for $\omega \gg 0.5$	0 deg for $\omega \ll \omega_c$ (<~0.05) 45 deg for $\omega = \omega_c$ 90 deg for $\omega \gg \omega_c$ (~>5)
$\dfrac{1}{1+j\dfrac{0.4\omega}{8}-\left(\dfrac{\omega}{8}\right)^2}$	$\omega_c = 8, \zeta = 0.2;$ 0 for $\omega \ll 8;$ -40dB/dec for $\omega \gg 8$	0 deg for $\omega \ll \omega_c$ (~<0.8) -90 deg for $\omega = \omega_c$ -180 deg for $\omega \gg \omega_c$ (~>80)

Bode Plots –Examples



Response of a delay

- Laplace transform of a delay of T seconds is:

$$H(s) = e^{-sT}$$

In the frequency domain:

$$s = j\omega$$

The frequency response of a delay is:

$$H(j\omega) = e^{-j\omega T}$$

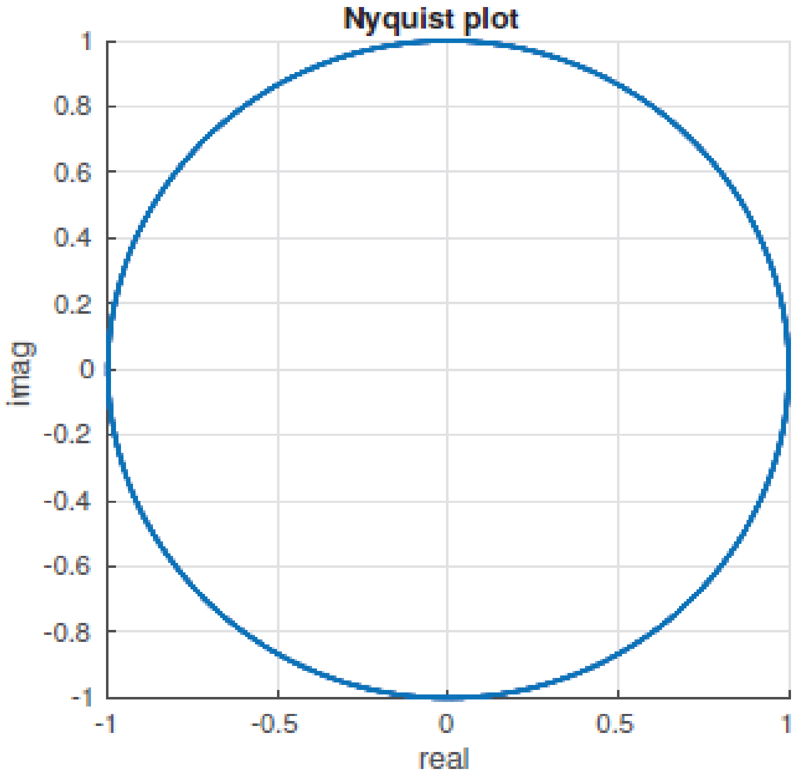
Magnitude:

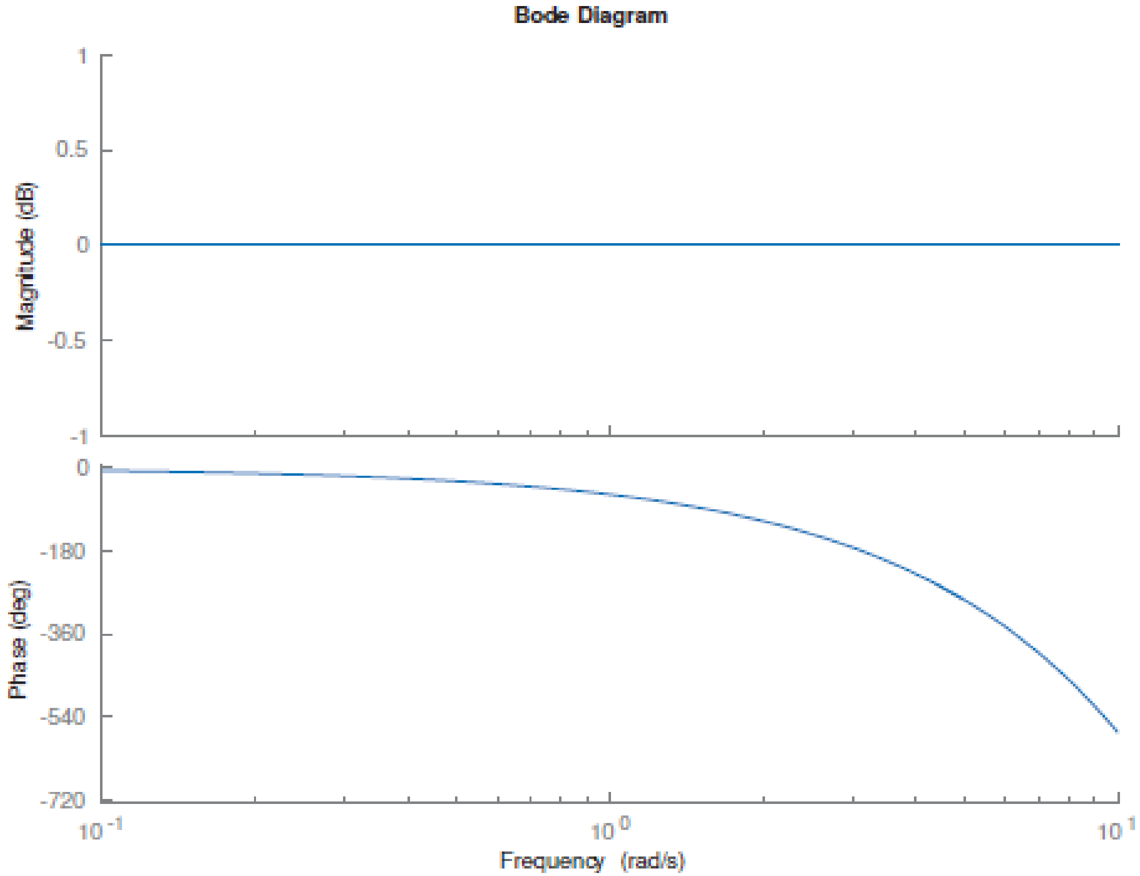
$$|H(j\omega)| = 1$$

Phase:

$$\angle H(j\omega) = -\omega T \; radians$$

Response of a delay





Experimental determination of TF

Bode plots are of great value in situations where transfer function of a system is unknown.

We capture the frequency response data experimentally in the desired frequency range of interest.

Approximate transfer function is gotten by fitting an asymptotic log-magnitude plot to the experimental data

Steps for experimental determination of TF

1. Use experimental data to plot exact log-magnitude and phase angle vs frequency curves on a semilog graph sheet
2. Draw asymptotes in multiples of 20db/dec. Adjust corner frequency so dB value difference captures dB correction.
3. Changes of -20m db/dec at $\omega = \omega_1$ indicates a factor of $\dfrac{1}{\left(1+\frac{j\omega}{\omega_1}\right)^m}$
4. Changes of -40m db/dec at $\omega = \omega_2$ indicates either a double pair or complex conjugate

Steps for experimental determination of TF

5. In low frequency range the plot is determined by $K/(j\omega)^r$ where r usually will be 0,1, or 2

a. If asymptote is horizontal line,20log $K = x, K = \frac{1}{20}\log^{-1}x$

b. slope of -20dB/dec, there is $K/j\omega$, frequency where asymptote intersects 0db represents K. Asymptote has a gain of 20logK at $\omega = 1$

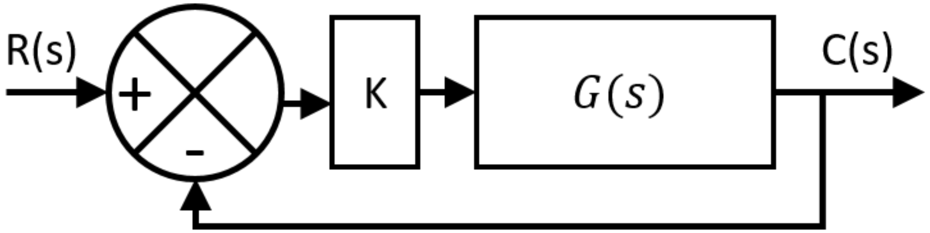
c. slope of -40dB/dec, there is $K/(j\omega)^2$ frequency where asymptote intersects 0db represents √K. Asymptote has a gain of 20logK at $\omega = 1$

Steps for experimental determination of TF

6. After obtaining TF, draw phase plot and compare with experimental

Study question

a. In the system below, if $G(s) = \frac{1}{s(s+a)}$ find the value of K and a, to satisfy the following frequency domain specification $M_r = 1.04$, $\omega_r = 11.55$rad/s



b. For the values of K and a determined in a. above, calculate the settling time and bandwidth of the system.

Solution: K=475, a =26.2, Ts=0.305s, bandwidth = 25.1 rad/s