# Lab Report Glasgow College, UESTC Dynamic and Control (UESTC 3001)

## 1. UESTC ID: 2022190504031

## 2. GUID: 2839915H

## 3. Introduction

Dynamic and control is the course that focuses on analyzing and designing dynamic system. A dynamic system is one which is in motion. Electronic circuits, motors and even airplanes are dynamic systems. These lab sessions take some typical dynamic systems that we have learnt on the lectures, aiming to improve our abilities to analyze and implement theses dynamic systems.

## 4. Lab 1

### 4.1. Baisc principles

This lab session's topic is the RLC circuit which are commonly used as bandpass and notch filters. The response of the RLC circuit can be classified as **overdamped, critically damped and underdamped**. The transition between initial and final conditions for component voltages and currents is fastest in a critically damped circuit which is difficult to achieve.

## 4.2. Principles

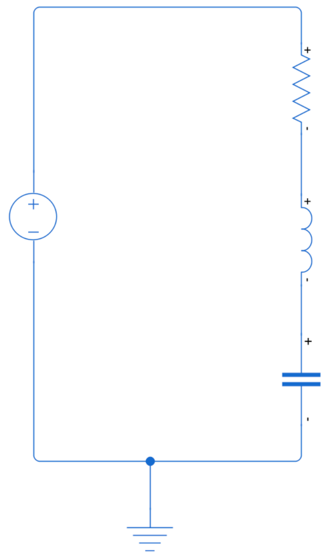

Figure 1: Lab1 setup

A basic RLC circuit is shown in Figure 1. An essential factor that can describe this factor is the damping ratio.

$$\zeta = \frac{R}{2}\sqrt{\frac{C}{L}}$$

- $\zeta > 1$, overdamped

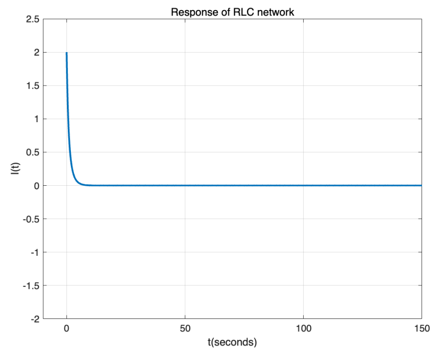


Figure 2: overdamped

- $\zeta = 1$, critically damped

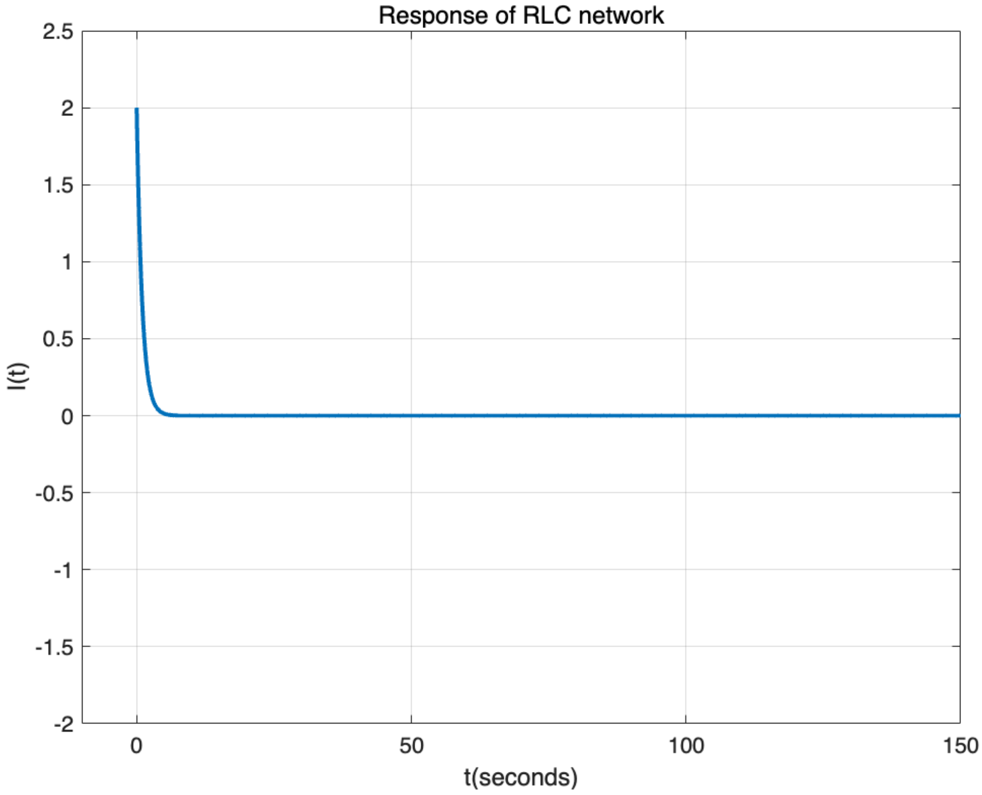


Figure 3: critically damped

- $\zeta < 1$, underdamped

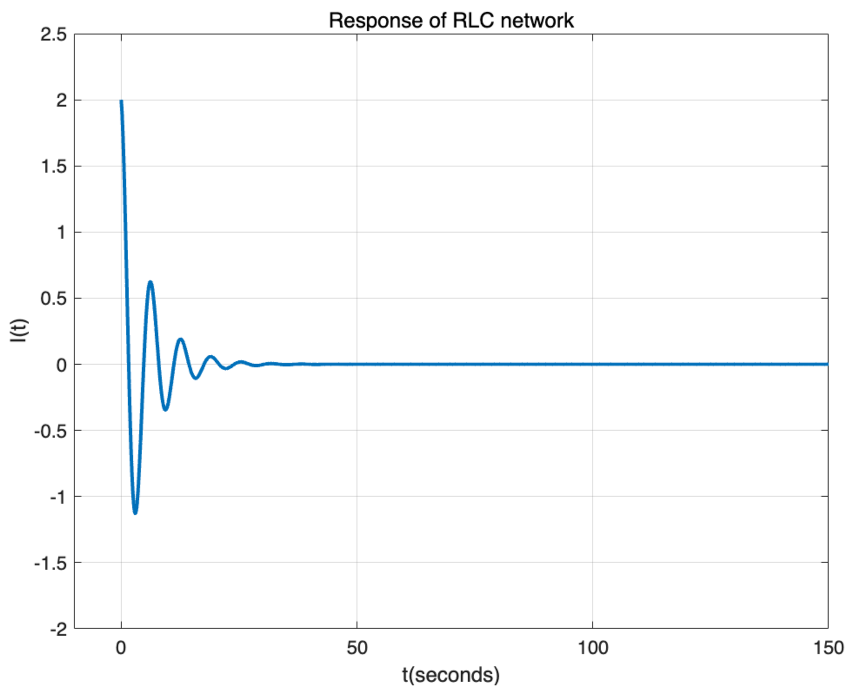


Figure 4: underdamped

## 4.3. Q2 (A)

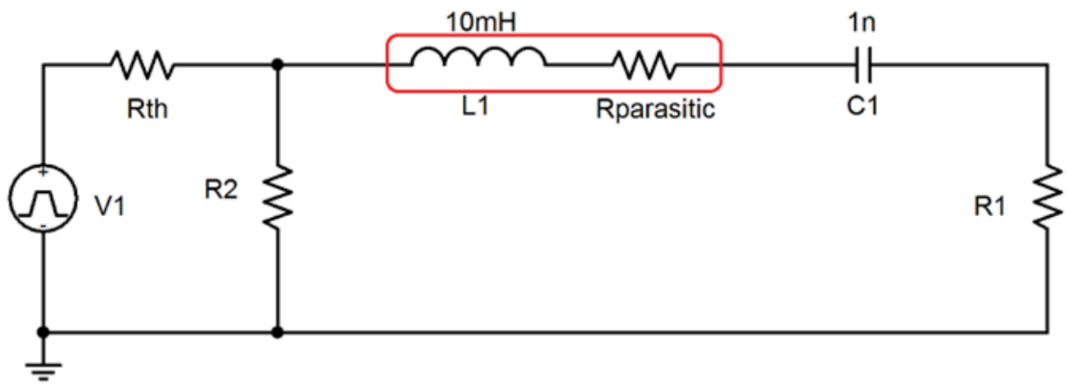


Figure 5: RLC circuit with series and non-series components for experiment

Analysis of the response:

1. Convert $V_1$ and $R_{\text{th}}$ into its equivalent current source $I_x$ and parallel resistor

$$I_x = \frac{V_1}{R_{\text{th}}}$$

$$R_p = R_{\text{th}}$$

2. Combine the parallel resistor $R_2$ with giving $R_{\text{eq}}$

$$R_{\text{eq}} = \frac{R_{\text{th}} R_2}{R_{\text{th}} + R_2}$$

3. Convert $I_x$ and $R_{\text{eq}}$ into $V_x$ and $R_{\text{eq2}}$

$$V_x = \frac{V_1 R_2}{R_{\text{th}} + R_2}$$

$$R_{\text{eq2}} = R_{\text{eq}} = \frac{R_{\text{th}} R_2}{R_{\text{th}} + R_2}$$

4. Rearrange the components so as to get a RLC series circuit

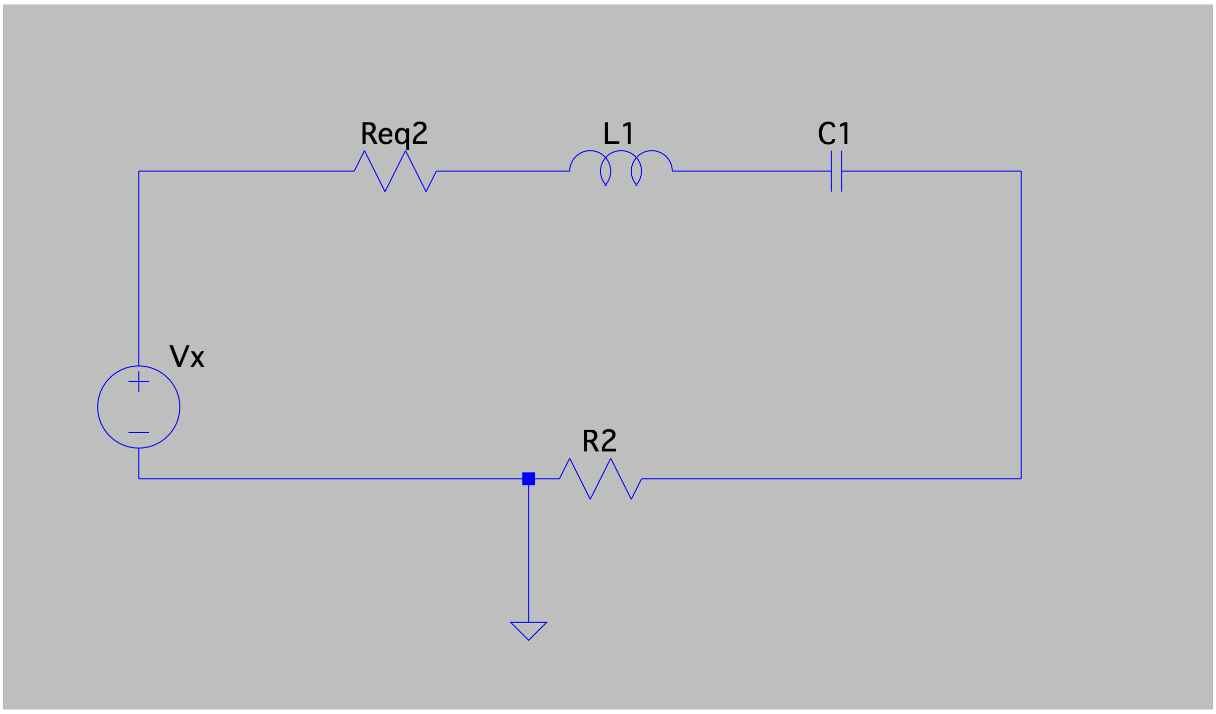

Figure 6: Lab1 Q2

The equivalent impedance of $C_1$ and $R_2$ is

$$Z_p = \frac{1}{\frac{1}{R_2} + sC_1} = \frac{R_2}{1 + sR_2C_1}$$

The total impedance is

$$Z_{\text{total}} = R_{\text{eq2}} + sL_1 + Z_p = R_{\text{eq2}} + sL_1 + \frac{R_2}{1 + sR_2C_1}$$

Apply the voltage divider rule

$$H(s) = \frac{V_{\text{C1}(S)}}{V_{x(s)}} = \frac{\frac{R_2}{1+sR_2C_1}}{R_{\text{eq2}} + sL_1 + \frac{R_2}{1+sR_2C_1}}$$

$$H_0(s) = \frac{V_{\text{C1}(S)}}{V_1} = \frac{V_{\text{C1}(S)}}{V_{x(s)}} = \frac{\frac{R_2}{1+sR_2C_1}}{R_{\text{eq2}} + sL_1 + \frac{R_2}{1+sR_2C_1}} \times \frac{R_2}{R_2 + R_{\text{th}}}$$

## 4.4. Q2 (B)

- Natural frequency

$$H(s) = \frac{K}{as^2 + bs + c}$$

$$as^2 + bs + c = a\left[s^2 + \frac{b}{a}s + \frac{c}{a}\right]$$

$$\omega_0^2 = \frac{c}{a}$$

$$\omega_0 = \sqrt{\frac{R_{\text{eq2}} + R_2}{L_1 R_2 C_1}}$$

- Attenuation ($\alpha$)

$$2\alpha = \frac{b}{a}$$

$$\alpha = \frac{R_{\text{eq2}} R_2 C_1 + L_1}{2 L_1 R_2 C_1}$$

- Damping factor ($\zeta$)

$$2\zeta\omega_0 = \frac{b}{a}$$

$$\zeta = \frac{R_{\text{eq2}} R_2 C_1 + L_1}{2\sqrt{(L_1 R_2 C_1)\left(R_{\text{eq2}} + R_2\right)}}$$

- Damping frequency ($\omega_d$)

$$\omega_d = \omega_0 \sqrt{1 - \zeta^2}$$

If $R_{\text{ps}}$ is considered:
- $a = L_1 R_2 C_1$
- $b = R_s R_2 C_1 + L_1$
- $c = R_s + R_2$
- $R_s = R_{\text{eq2}} + R_{\text{ps}}$

$$\omega_0 = \sqrt{\frac{R_s + R_2}{L_1 R_2 C_1}}$$

$$\alpha = \frac{1}{2}\left(\frac{R_s}{L_1} + \frac{1}{R_2 C_1}\right)$$

$$\zeta = \frac{\frac{R_s}{L_1} + \frac{1}{R_2 C_1}}{2\sqrt{\frac{R_s + R_2}{L_1 R_2 C_1}}}$$

$$\omega_d = \sqrt{\omega_0^2 - \zeta^2 \omega_0^2} \zeta < 1$$

### 4.5. Q2 (C)

```
function [f_n, alpha, zeta, f_d] = system_charateristics(R_th,
R_2, R_ps, R_1)
    Req=(R_th*R_2)/(R_th+R_2);
    R_2=R_1;
    Rs=Req+R_ps;
    L1=10e-3;
    C1=1e-9;
    omega_0=sqrt((Rs+R_2)/(L1*R_2*C1));
    f_n=(2*pi)/omega_0;
    alpha=0.5*(Rs/L1+1/(R_2*C1));
    zeta=(Rs/L1+1/(R_2*C1))/(2*sqrt((Rs+R_2)/(L1*R_2*C1)));
    omega_d=sqrt(omega_0^2-zeta^2*omega_0^2);
    f_d=(2*pi)/(omega_d);
    disp(['the natural frequency is: ', num2str(f_n), ' Hz']);
    disp(['the attenuation is: ', num2str(alpha)]);
    disp(['the damping factor is: ', num2str(zeta)]);
    disp(['the damping frequency is: ', num2str(f_d), ' Hz']);
end
```

## 5. Lab 2

### 5.1. Problem IV

$$g = 9.81, m = 5, l = 0.4, c = 0.7$$

- moment of inertia: $I = m\left(\frac{l}{2}\right)^2 = 0.2$

- undamped natural frequency:$\omega_n = \sqrt{\frac{mgl}{2I}} = 7.0036$
- damping coefficient:$\xi = \frac{c}{2 * I * \omega_n}$
- damped natural frequency:$\omega_d = \omega_n \sqrt{1 - \xi^2} = 6.7814$
- rise time:10% to 90%:$t_r = \frac{1}{\omega_d} \arctan\left(\frac{\sqrt{1-\xi^2}}{x_i}\right) = 0.1944$
- peaking time from application of step:$t_p = \frac{\pi}{\omega_d} = 0.4633$
- settling time based on $\pm 1\%$ tolerance:$t_s = \frac{3}{\xi \omega_n} = 1.7143$
- overshoot in %:$M_p = 100 \times e^{-\xi \omega_n t_p} = 44.4540$
- CLTF(Closed-Loop Transfer Function):

$$\frac{500}{s^2 + 3.5s + 549}$$

- sys(transfer function):

$$\frac{5}{s^2 + 3.5s + 54.05}$$

- Difference: Different from the open-loop transfer function that only contains forward loops, the closed-loop transfer function also contains the feedback, the output has an impact on the input. The closed-loop transfer function has lower gain and is more stable than the open-loop transfer function.

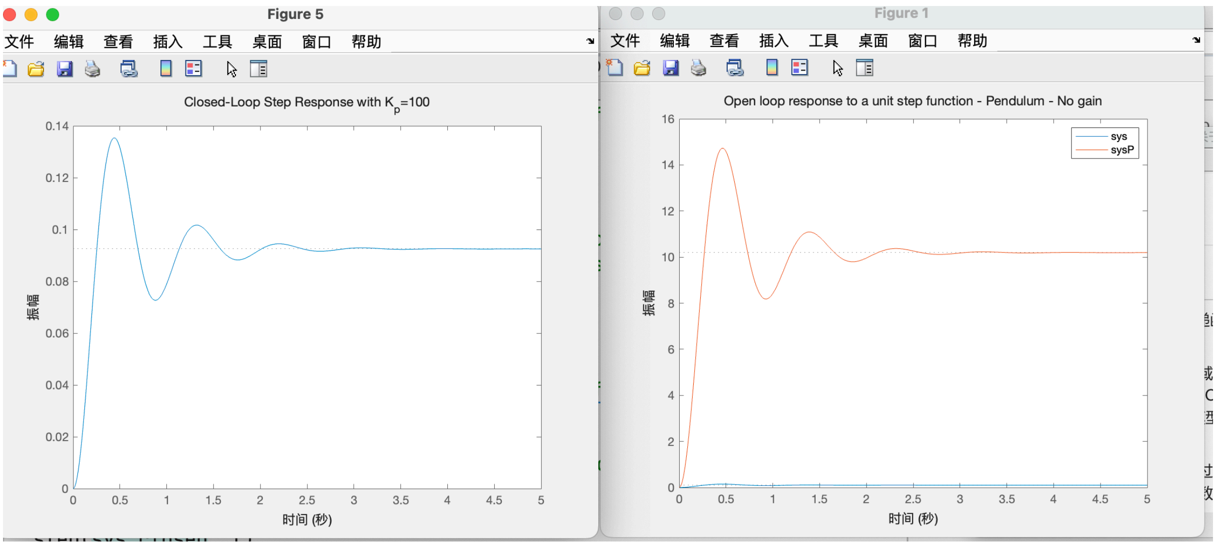


Figure 7: The open-loop and closed-loop transfer function

## 5.2. Calculation of the open-loop transfer function

$$\text{sys} = \text{tf}\left(\frac{1}{I}, 2\xi\omega_n\omega_n^2\right)$$

$$I = 0.2$$

$$2\xi\omega_n = 2 \cdot 0.25 \cdot 7.004 \approx 3.502$$

$$\omega_n^2 = (7.004)^2 \approx 49.056$$

$$\text{sys} = \frac{5}{s^2 + 3.502s + 49.056}$$

By transforming these frequency-domain equation to the time-domain equation:

$$\text{sys(t)} = \frac{5}{\omega_n^2}\left[1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \cdot \left(\cos(\omega_d t) + \frac{\xi}{\sqrt{1-\xi^2}}\sin(\omega_d t)\right)\right]$$

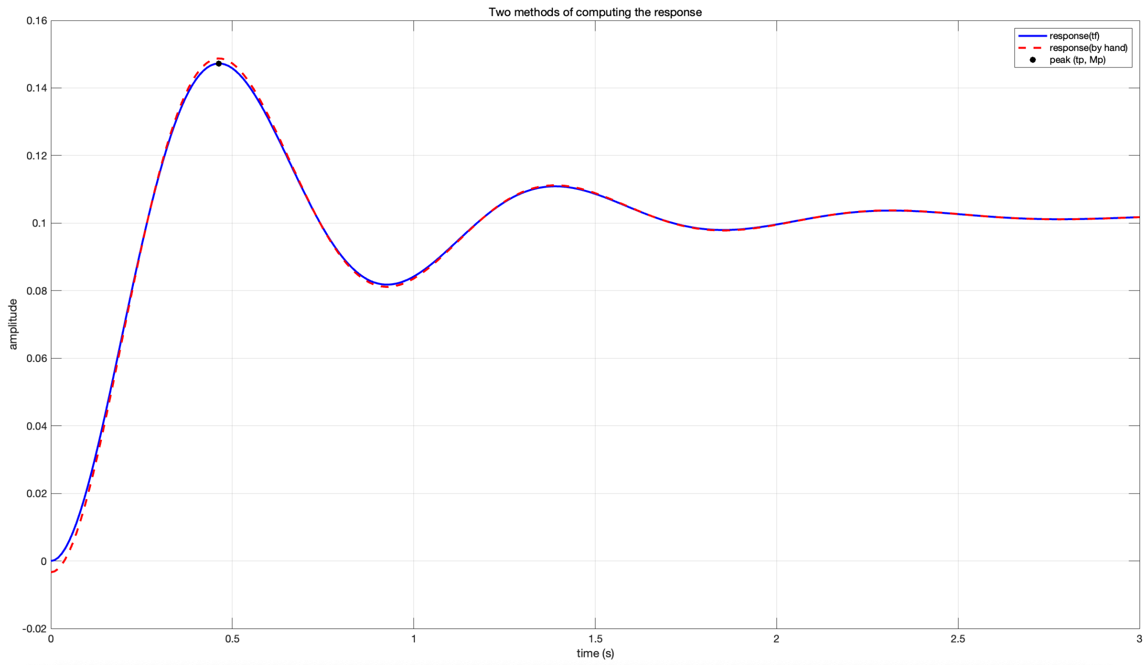


Figure 8: Response derived by two methods

The matlab's result has lower peak value.

## 5.3. Poles and zeros

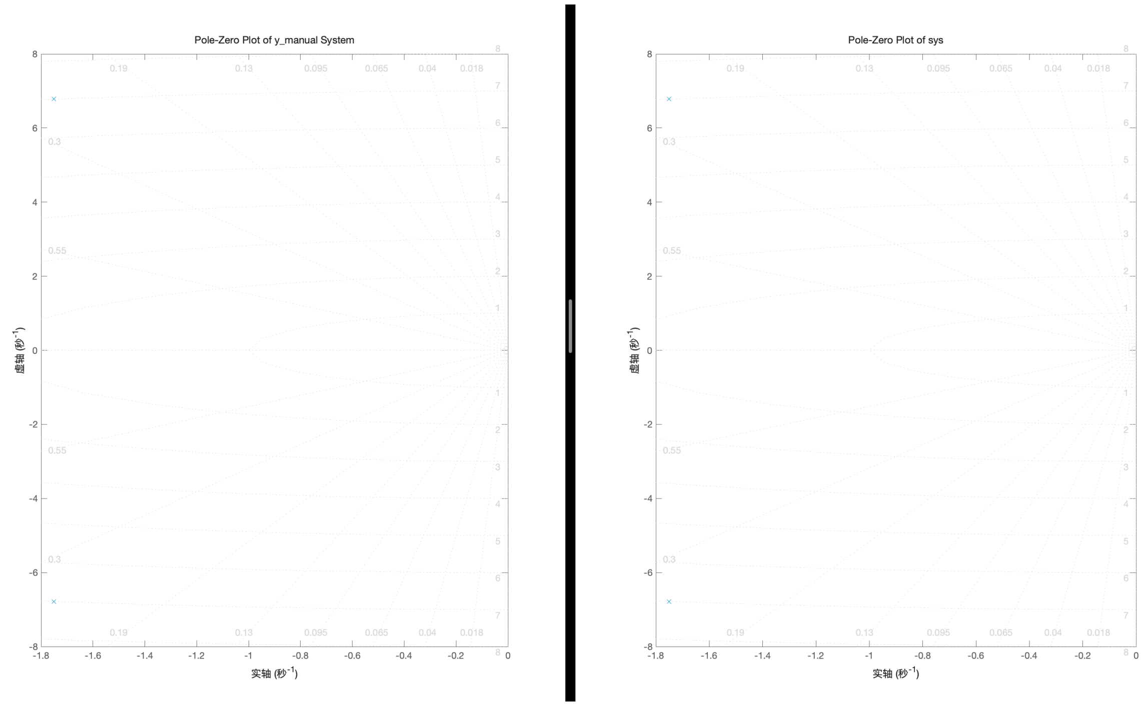


Figure 9:

## 5.4. Analysis of polse and zeros of two methods' results

The poles and zeros from the two methods are the same

## 5.5. 12

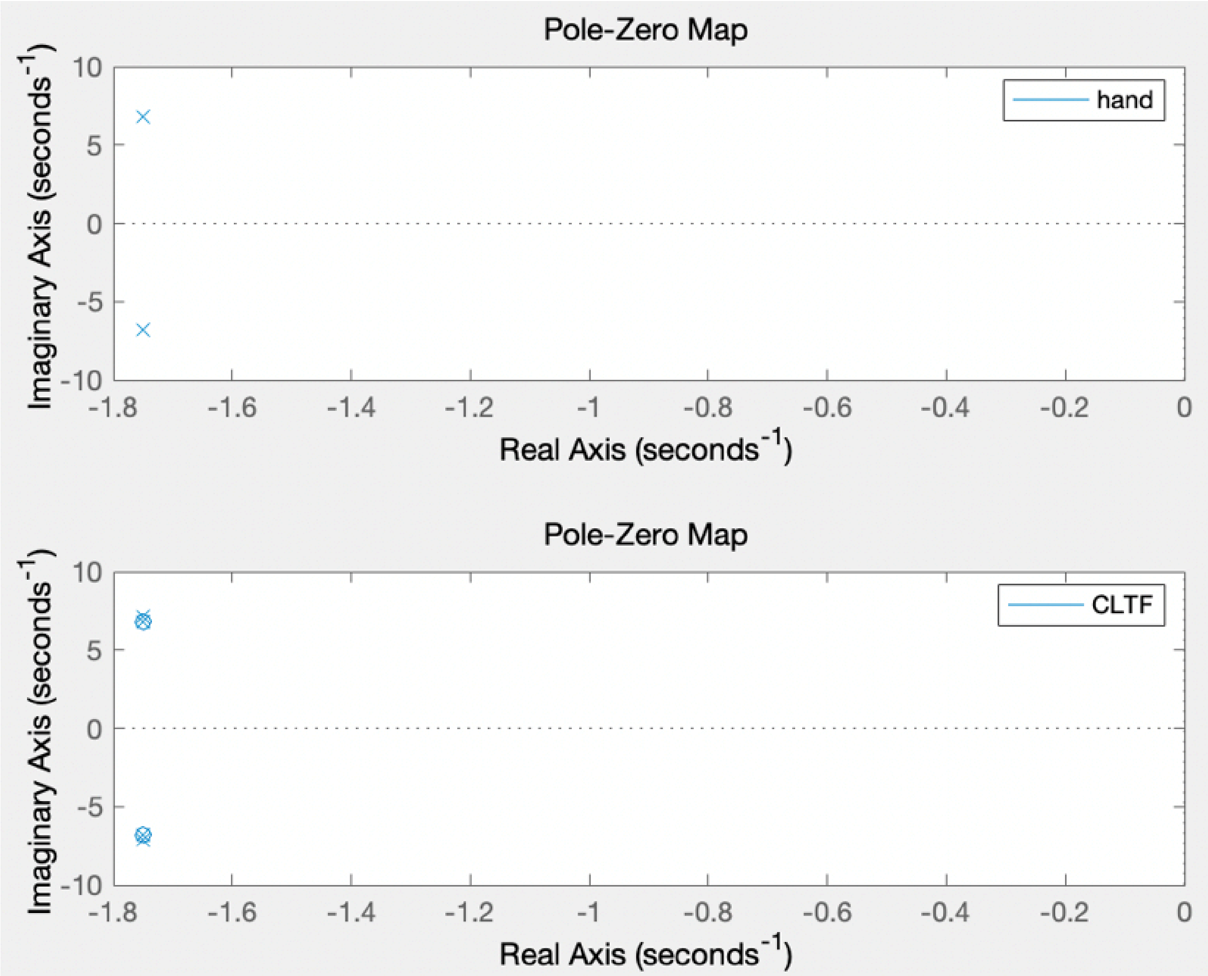


Figure 10: PZ plot

## 5.6. 13

## 5.7. 14

# 6. Lab 3

## 6.1. Problem X

- Question 4

As a first experiment, assume the controller transfer function above. By setting Kd and Ki to zero and influencing Kp, note what happens to the output response. Why does the output behave like this?

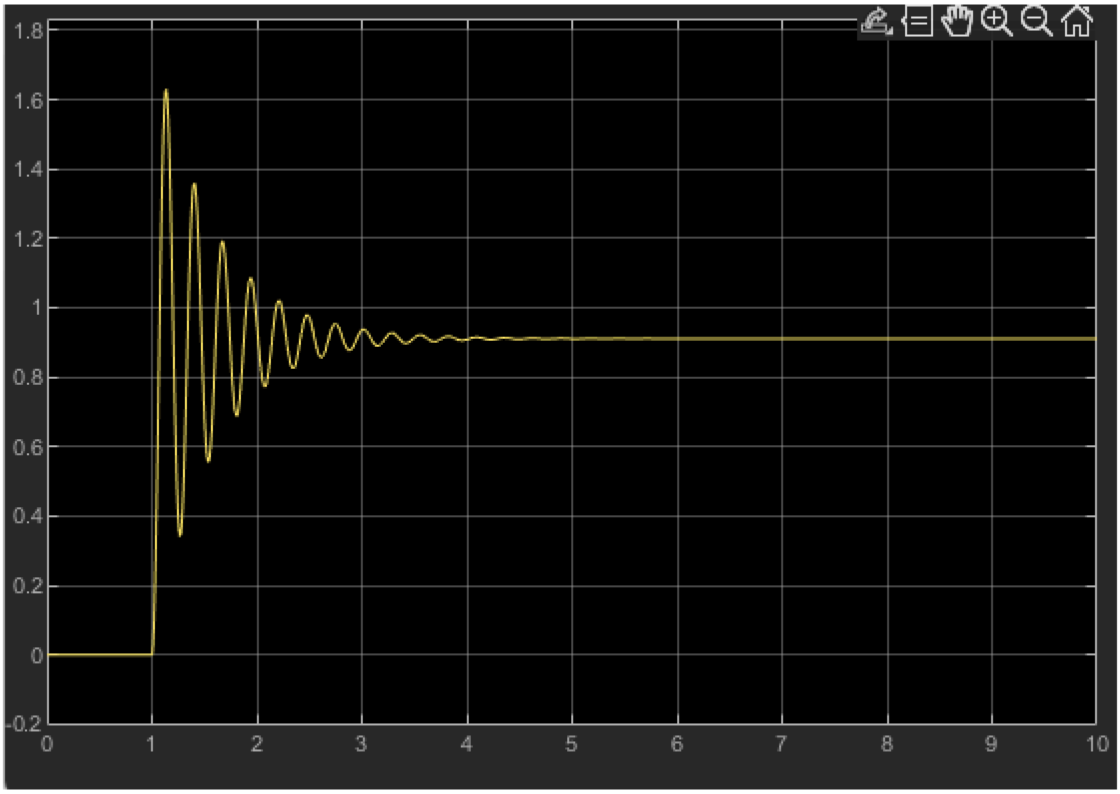


Figure 11: $K_p$=100

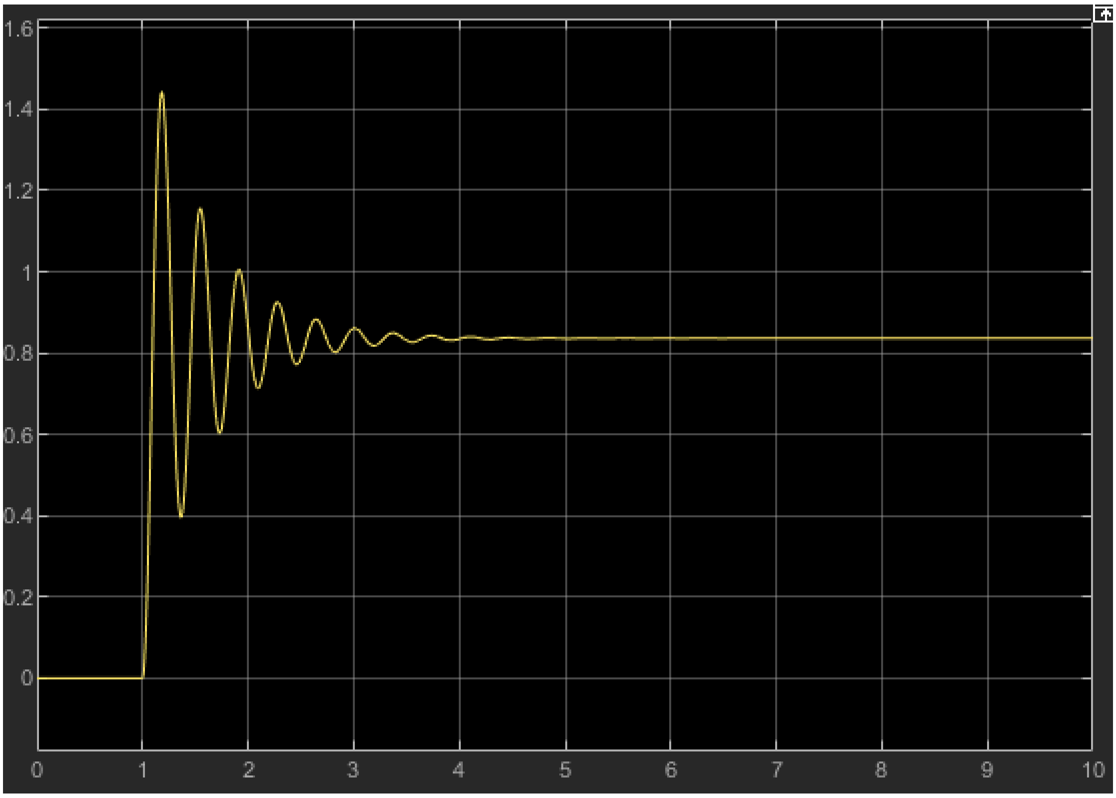


Figure 12: $K_p$=50

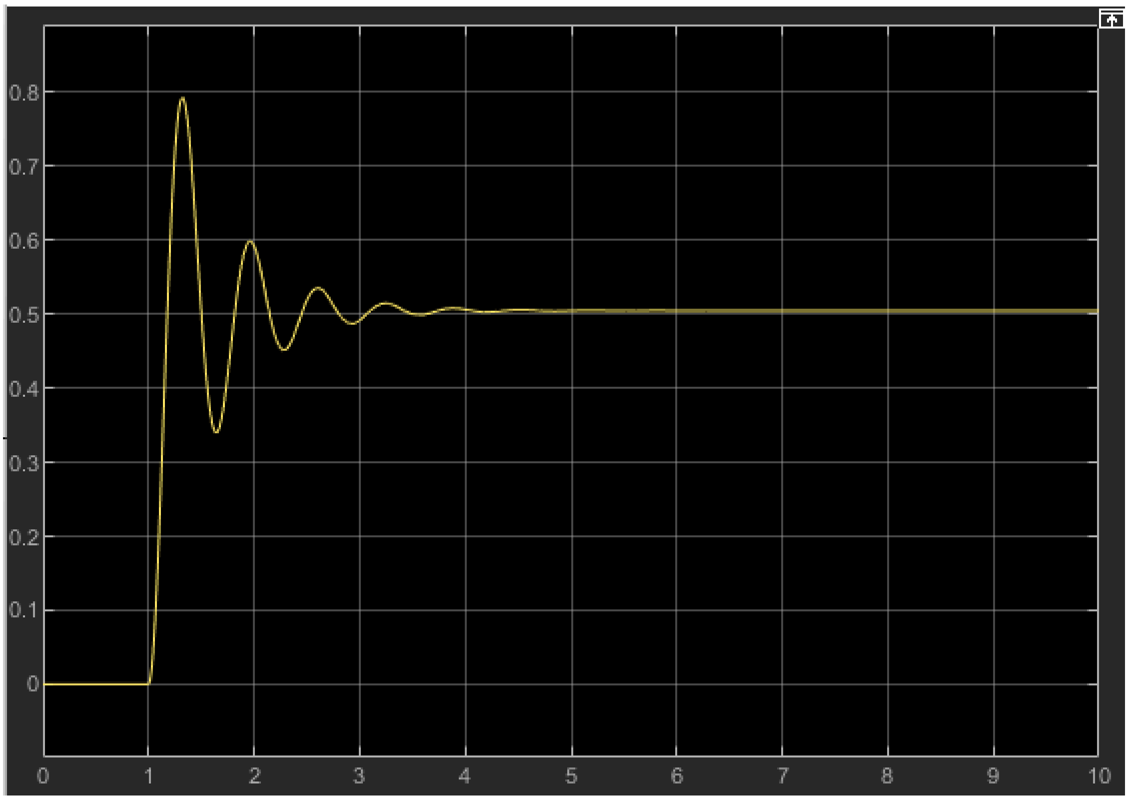


Figure 13: $K_p$=10

As $K_p$ increases, the setting time decreases, the steady state error increases, and it may oscillate.

- Question 5

You can create a script that will plot the poles to allow you to visualise easily where the poles move to depending on the system gain configuration. So from this what do you conclude is the reason for the output behaving as it does with Kp=1, Ki and Kd=0?

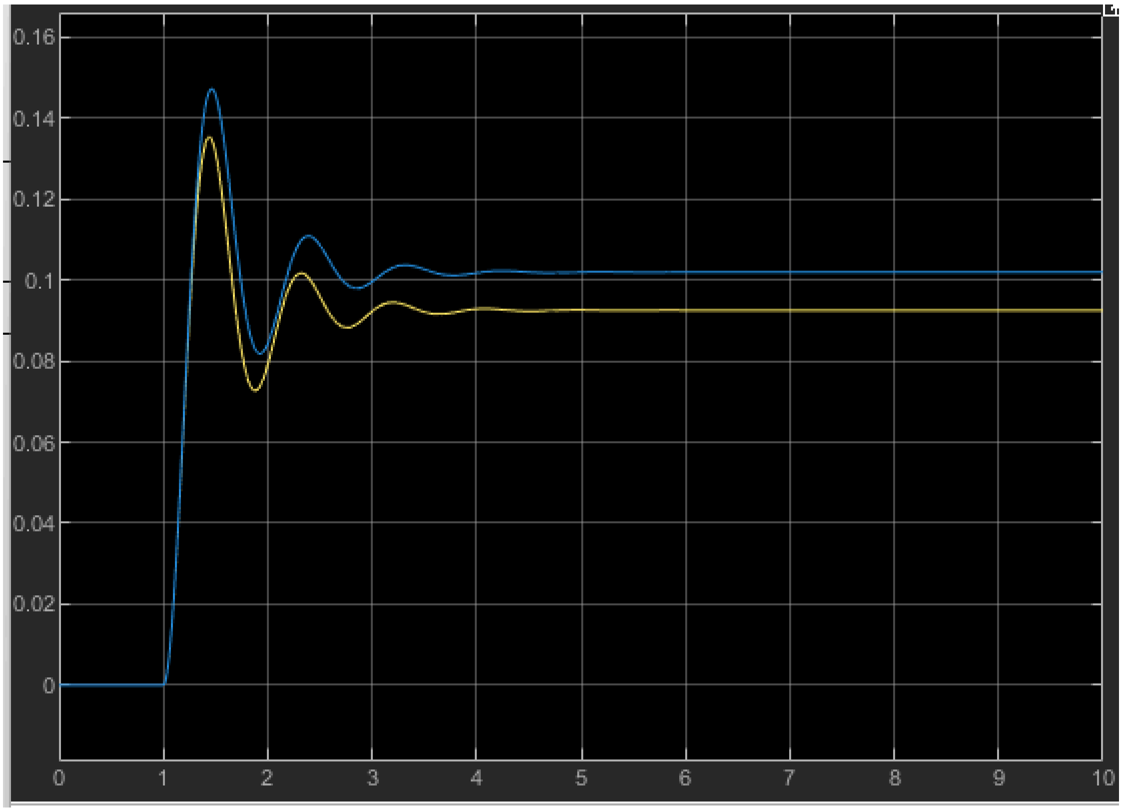

Figure 14: $K_p = 1, K_i = K_d = 0$

$$e_{\text{ss}} = \frac{1}{1 + K_p}$$

$K_p = 1$, the steady error is 0.5.

- Question 6

Now in addition to Kp=1, add derivative gain, Kd=1 and note the change in behaviour. Explain this by again looking at the poles and zeros in the system

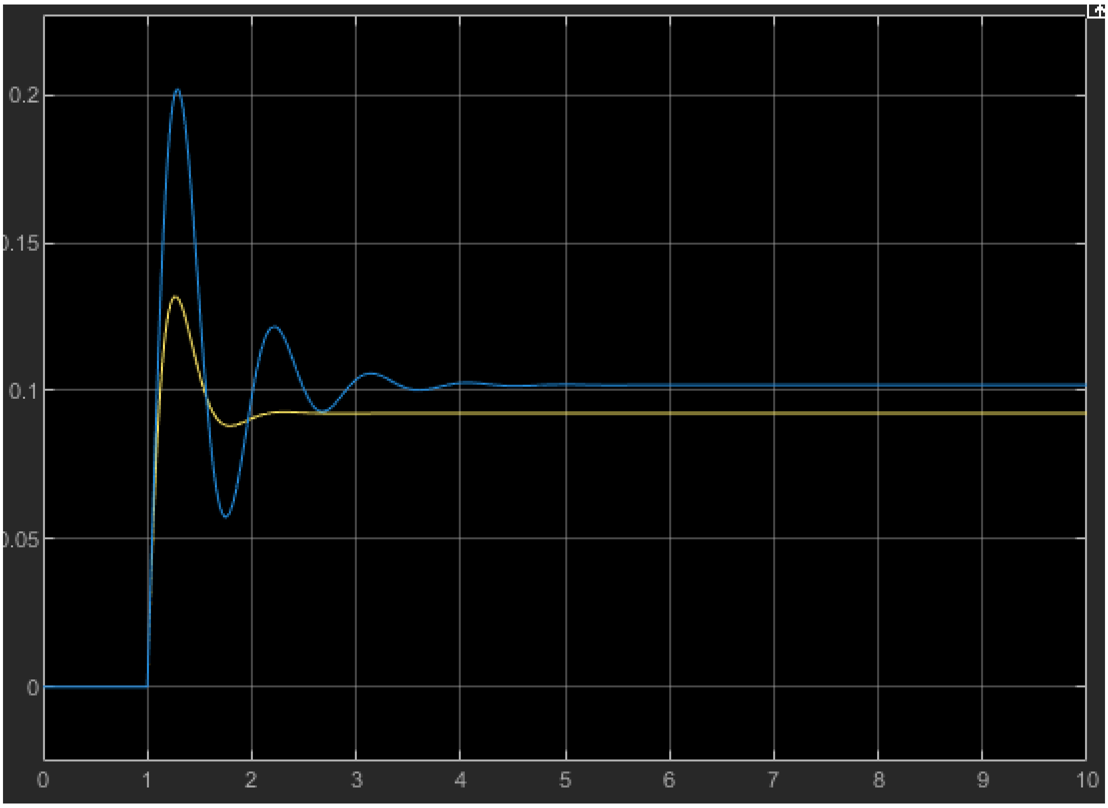


Figure 15: $K_d = 1$

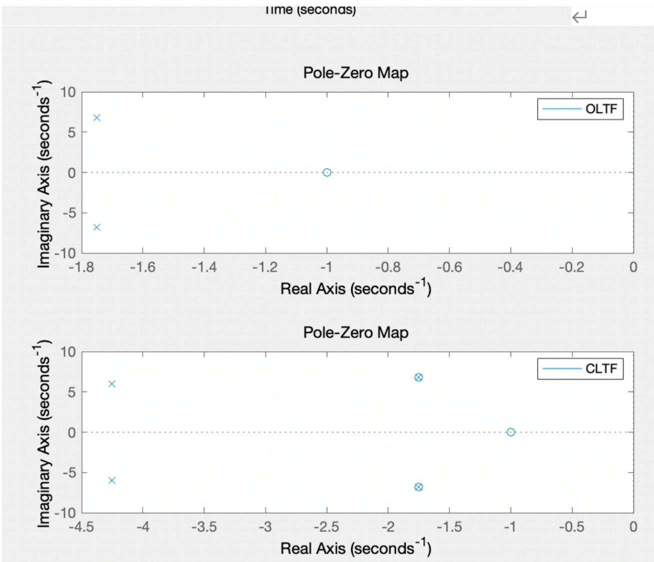


Figure 16: Zeors-poles graph

$$\text{CLTF} = \frac{5s^3 + 22.5s^2 + 262.7s + 245.2}{s^4 + 12s^3 + 132.8s^2 + 606.1s + 2651}$$

- Problem 7

$$\text{CLTF} = \frac{5s^5 + 22.5s^4 + 267.7s^3 + 262.7s^2 + 245.2s}{s^6 + 12s^5 + 132.8s^4 + 611.1s^3 + 2669s^2 + 245.2s}$$

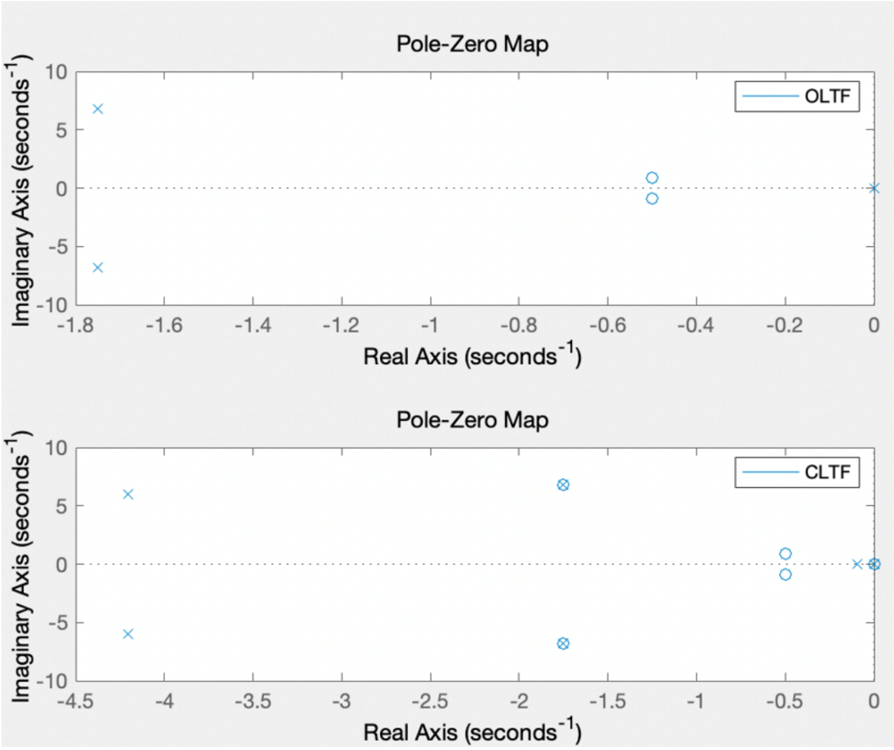


Figure 17: ZP graph of Problem 7

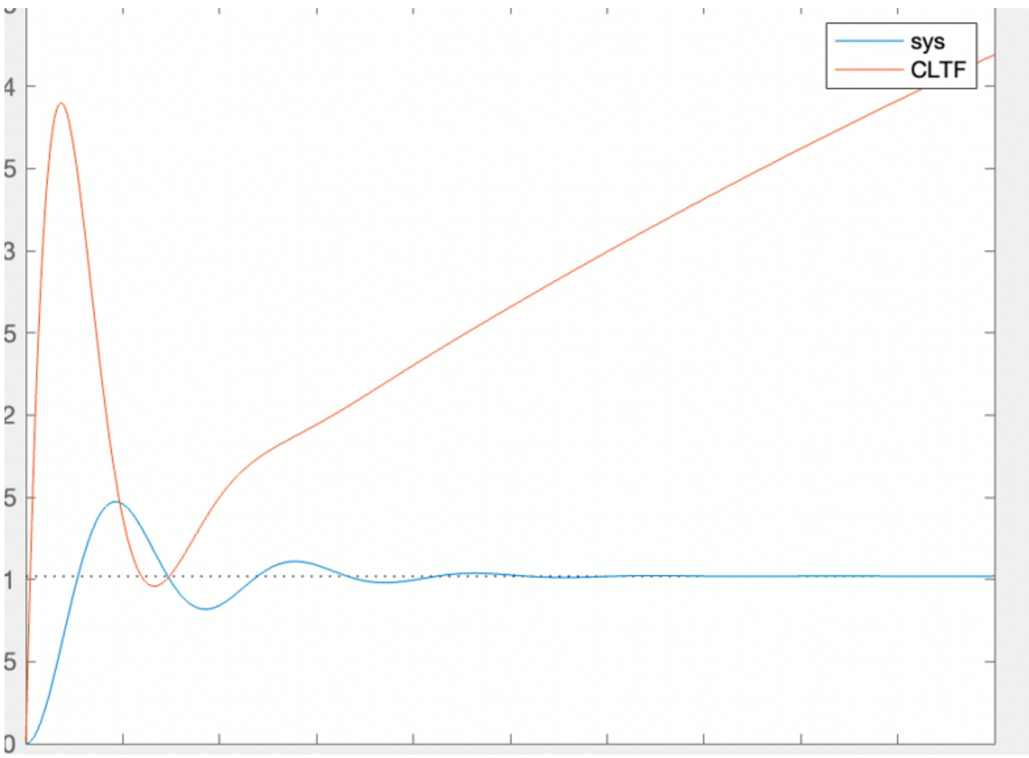


Figure 18: Response of Problem 7

- Problem 8

The proportional gain, Kp, is always set to a non-zero value and influences both the rise time and steady-state error. However, it cannot eliminate steady-state error entirely on its own.

The derivative gain, Kd, responds to input signal changes, helping the system reach steady state faster.

The integral gain, Ki, removes steady-state error by introducing a pole at DC.

- Problem 9 and Problem 10

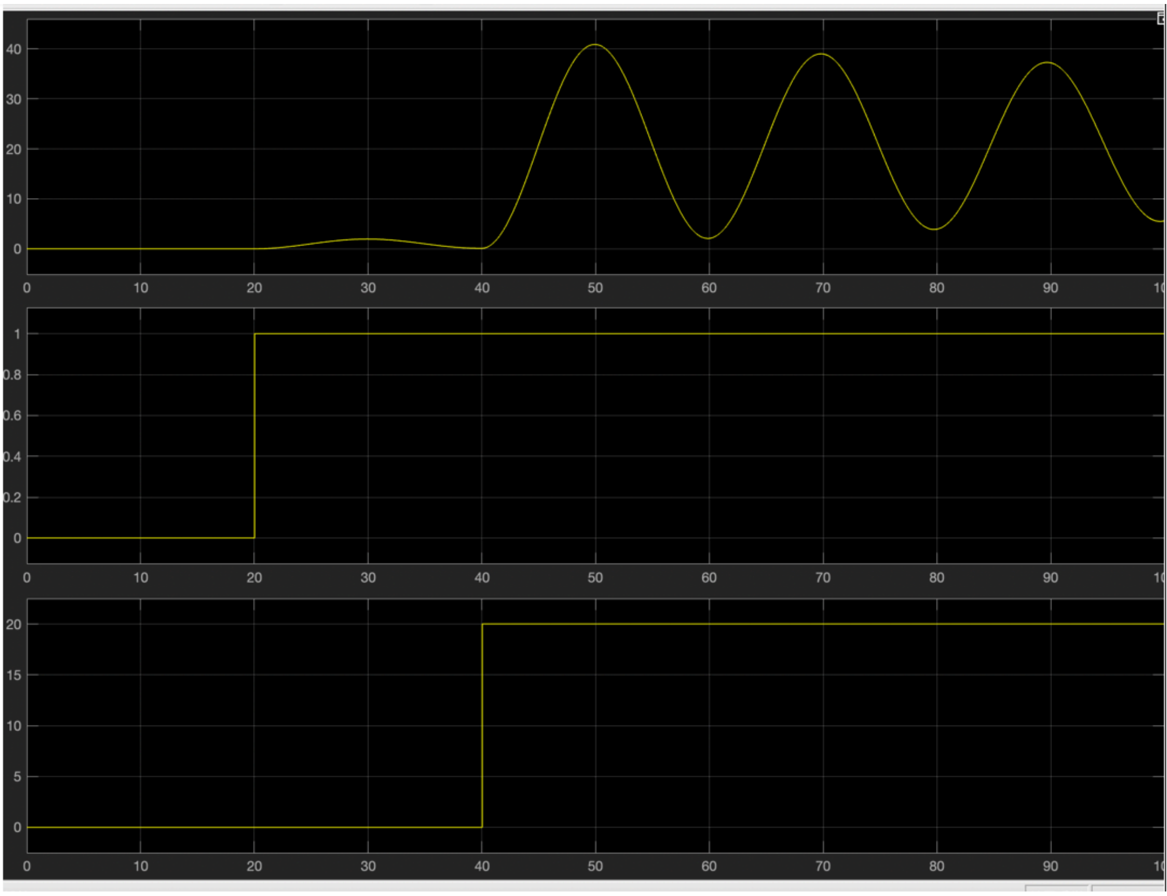

Figure 19: $K_p = 1, K_d = K_i = 0$

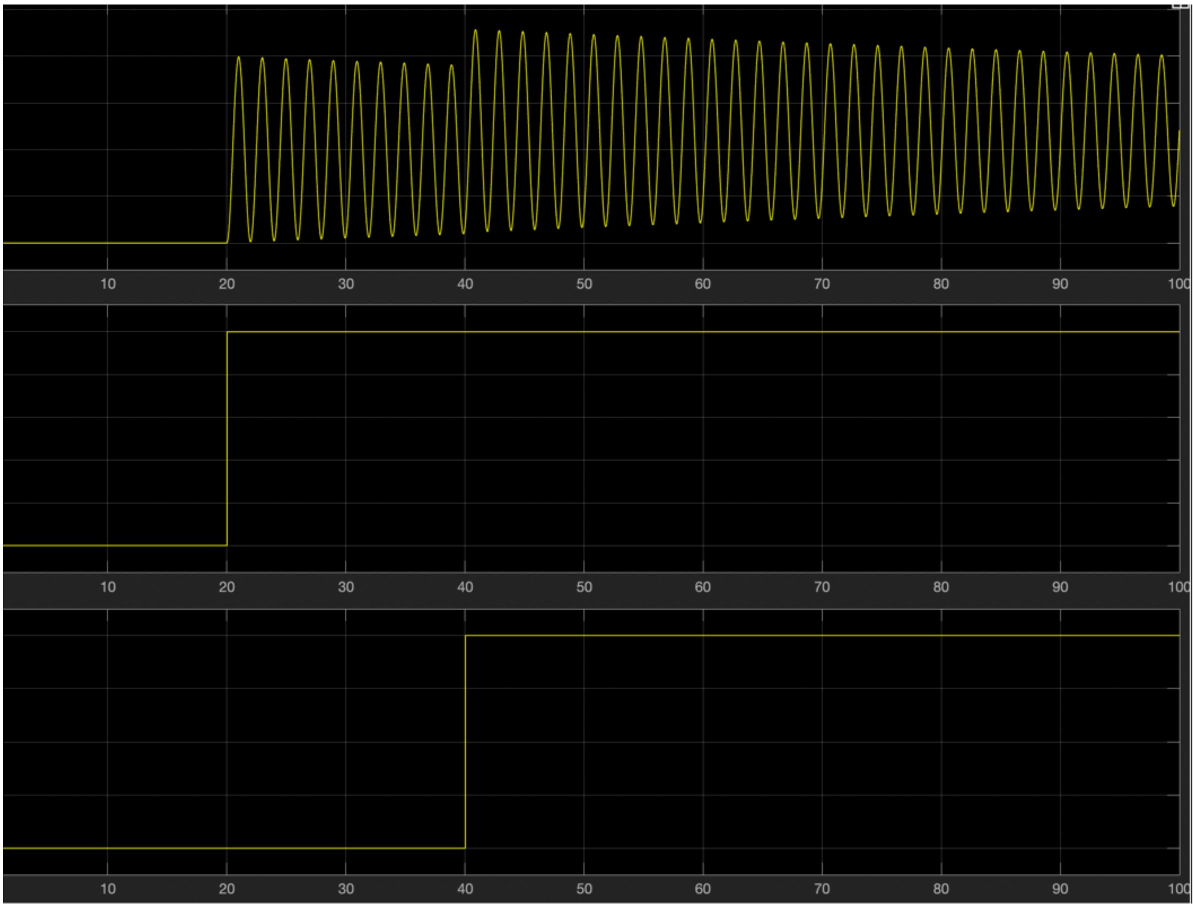


Figure 20: $K_p = 100$

Plant transfer function:

$$\frac{K_p}{10s^2 + 0.1s + K_p}$$

- Problem 11
  - Part I

$$\frac{C(s)}{R(s)} = \frac{K_p + sK_d}{10s^2 + (0.1 + K_d)s + K_p}$$

$$\omega_n = \sqrt{\frac{K_p}{10}}, \xi = \frac{0.1 + K_d}{20} \times \sqrt{\frac{10}{K_p}}$$

$$c(\infty) = \lim_{s \to 0} s \times \frac{1}{s} \times \frac{K_p + sK_d}{10s^2 + (0.1 + K_d)s + K_p} = 1$$

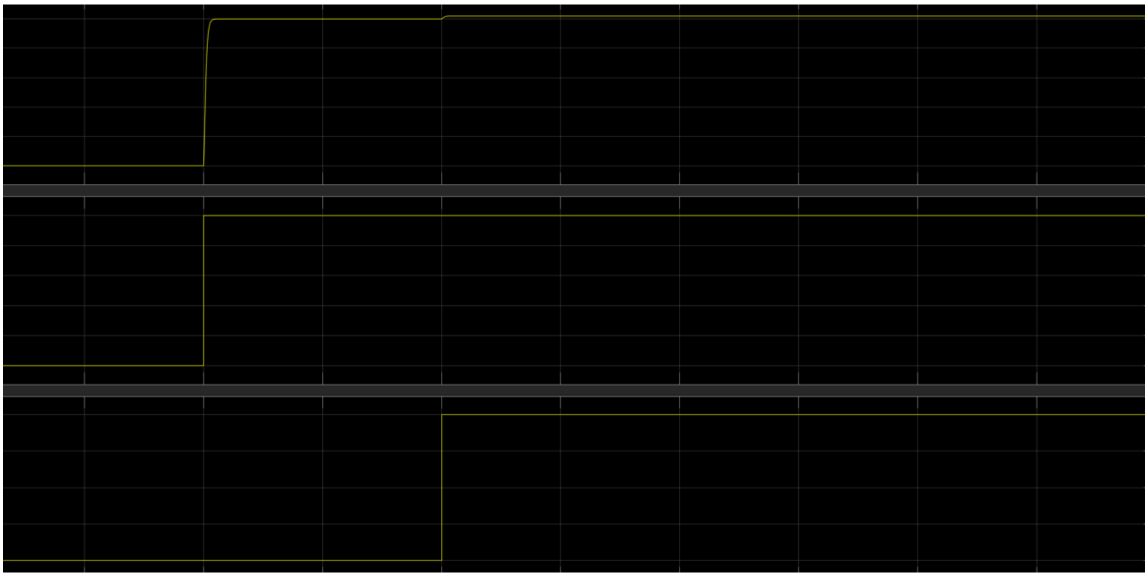

Figure 21: Part I

- Part II
  Characteristic equation:$s^3 + 20s^2 + 100s + \frac{K_i}{10} = 0$, by using Routh's stability criterion:$K_i < 20000$, For $K_i = 19050$:

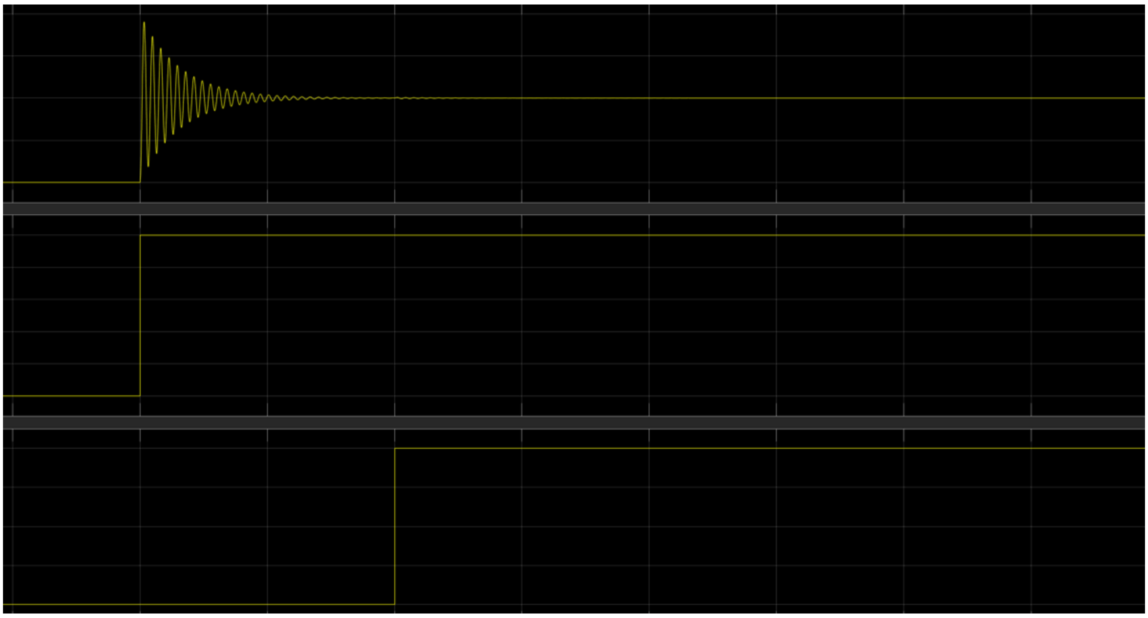

Figure 22: Part II

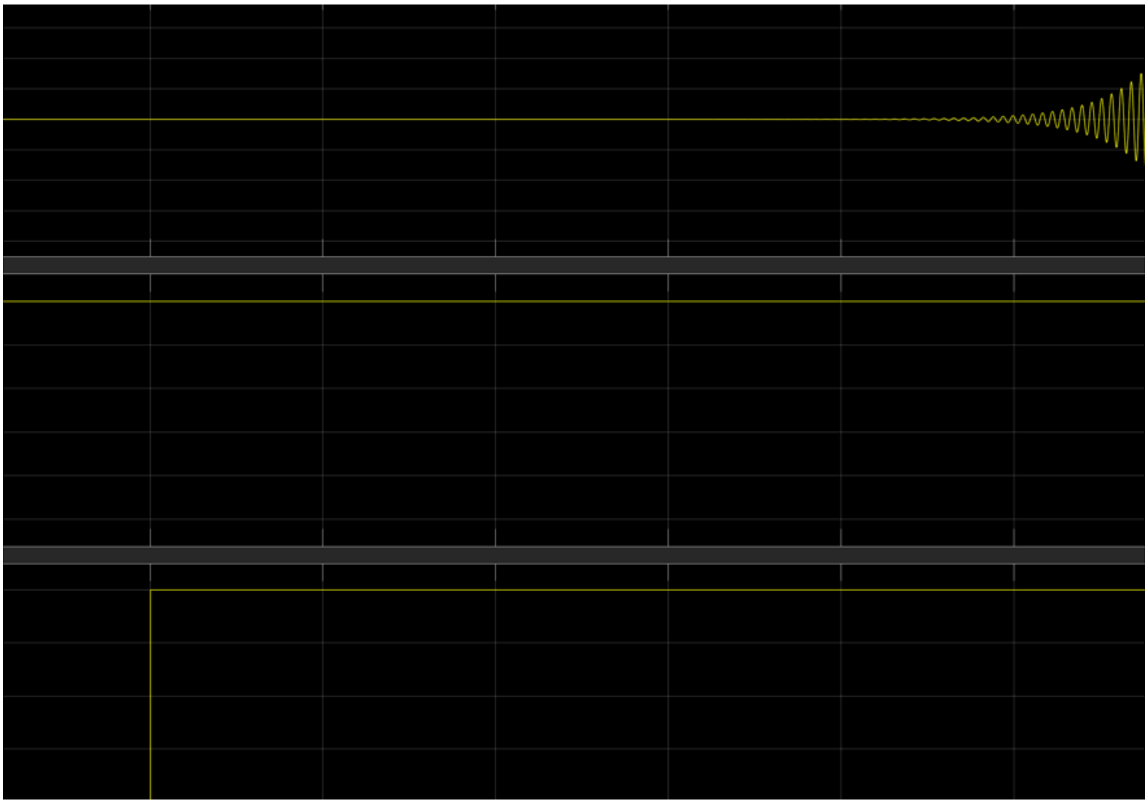

Figure 23: Part II, $K_i = 26000$

- Does the output respond faster or slower as you increase $\omega_n$? $K_p$ increases $\Rightarrow \omega_n$ increases $\Rightarrow$ faster response.

# 7. Lab 4

A PID controller (Proportional-Integral-Derivative controller) is a widely used feedback control mechanism that continuously calculates an error value as the difference between a desired setpoint and a measured process variable. It applies a correction based on three terms:

1. Proportional (P)-Responds to the current error
2. Integral (I)-Eliminate steady-state offset by addressing accumulated past error
3. Derivate (D)-Improve stability by prediciting the future error

## 7.1. Exercise I

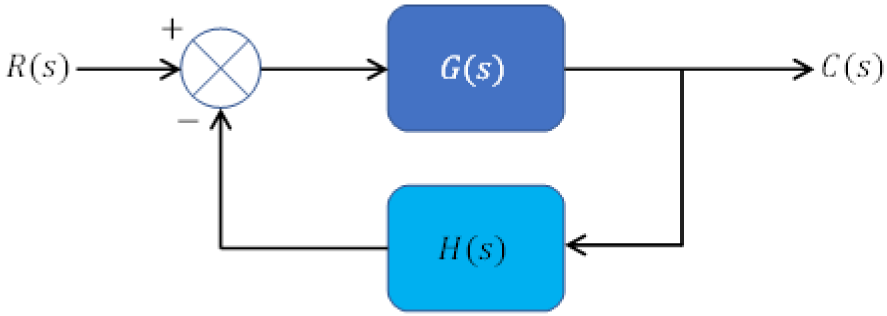


Figure 24: System for exercise I

$$G(s) = \frac{(s+1)(s+2)}{s^2(s+3)(s+4)(s+10)}$$

$$H(s) = 1$$

- OLTF

$$\text{OLTF} = G(s)H(s) = \frac{(s+1)(s+2)}{s^2(s+3)(s+4)(s+10)}$$

- CLTF

$$\text{CLTF} = \frac{G(s)}{1+G(s)H(s)}$$

$$\text{CLTF} = \frac{(s+1)(s+2)}{s^2(s+3)(s+4)(s+10)+(s+1)(s+2)}$$

• Root locus

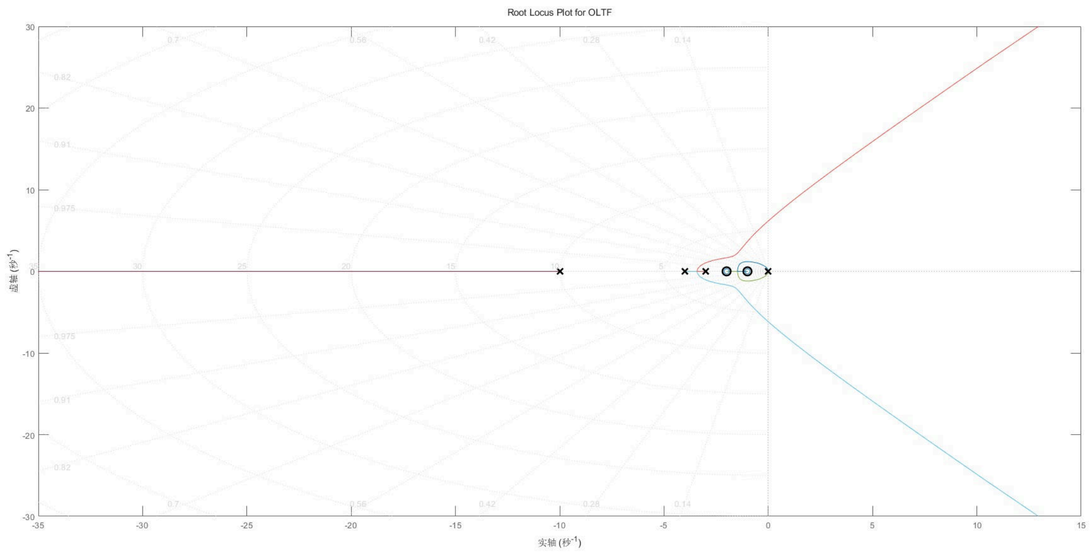


Figure 25: Root locus

• Bode magnitude and phase plots

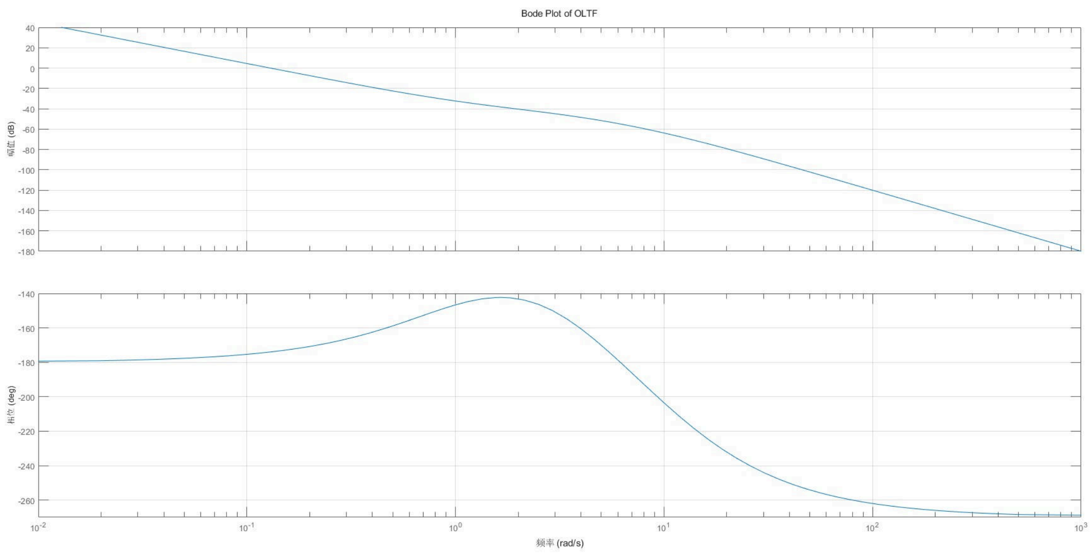


Figure 26: Bode Plot of OLTF

- Nyquist plot of OLTF

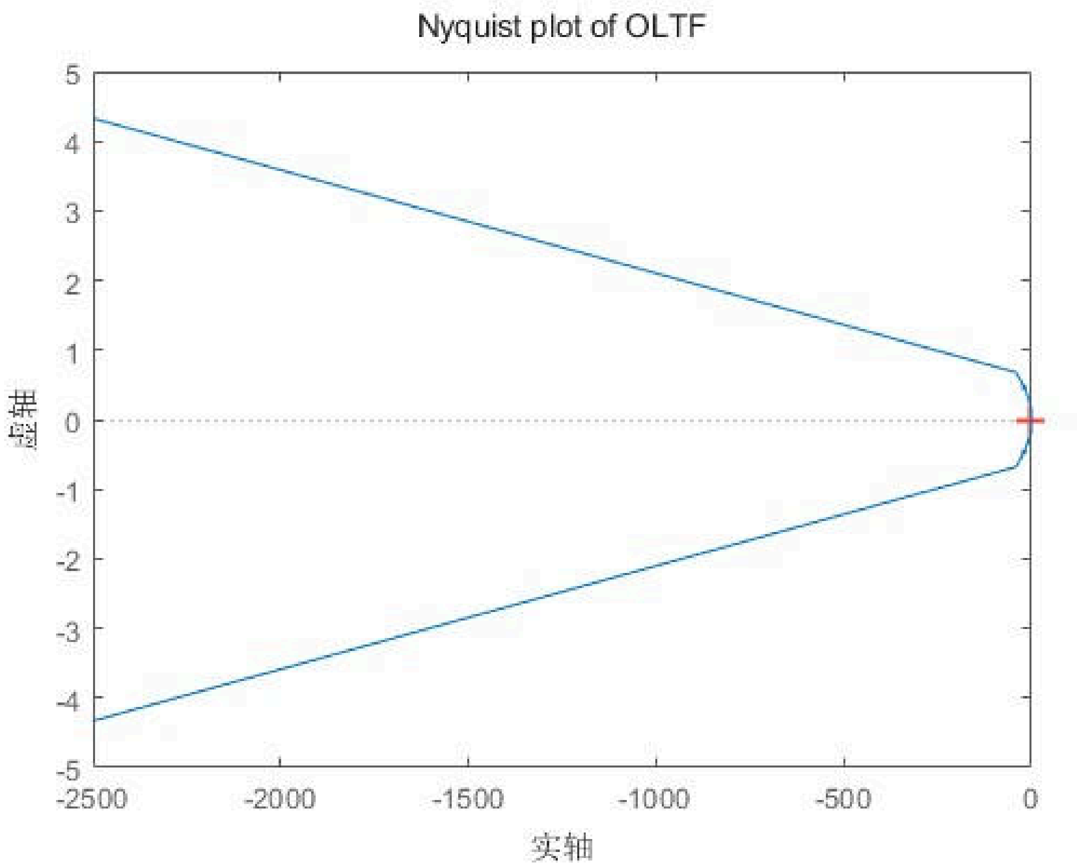


Figure 27: Nyquist plot of OLTF

- Some parameters

```
num = [1, 3, 2];
den = poly([0, 0, -3, -4, -10]);
G = tf(num, den);
[GM, PM, GMFrequency, PMFrequency] = margin(G);
DM = PM / (180 * pi * PMFrequency);
fprintf('Gain Margin (GM): %.4f (linear), %.4f dB\n', GM,
20*log10(GM));
fprintf('GM Frequency: %.4f rad/s\n', GMFrequency);
fprintf('Phase Margin (PM): %.4f degrees\n', PM);
fprintf('PM Frequency: %.4f rad/s\n', PMFrequency);
fprintf('Delay Margin (DM): %.4f seconds\n', DM);
fprintf('DM Frequency: %.4f rad/s\n', PMFrequency);
if PM > 0 && GM > 1
    fprintf('The system is stable.\n');
else
```

```
    fprintf('The system is unstable or marginally stable.\n');
end
```

Gain Margin (GM): 557.5962 (linear), 54.9264 dB

GM Frequency: 6.1756 rad/s

Phase Margin (PM): 6.0247 degrees

PM Frequency: 0.1297 rad/s

Delay Margin (DM): 0.0821 seconds

DM Frequency: 0.1297 rad/s

The system is stable.

# 8. Conclusion