Introduction to difference and instrumentation amplifiers

Introduction

In this lecture note the Instrumentation amplifier (IA) will be introduced. IA's are circuit elements designed to allow users to extract and amplify the difference between two signals or sources. As we will see we could use an operational amplifier to do this, but this introduces several potentials for error and/or difficulties in design. A major issue from using a single op-amp as a differential amplifier is a lower common-mode rejection ratio (CMRR), usually stemming from a negative feedback loop to increase stability. The feedback loop allows for common-mode voltages to appear on the output of the amplifier, reducing the usable range of the amplifier. An instrumentation amplifier consists of several operational amplifiers (op-amps) and is primarily designed to combat this design flaw. In this section of the course we will look at using a single op-amp differential amplifier and see how we can improve performance using a three op-amp instrumentation amplifier. A number of examples will be covered in the lectures.

Common mode rejection ratio (CMRR) and op-amps

CMRR is a measure of the capability of an op-amp to reject a signal that is common to both inputs. Ideally, the CMRR is infinite since if both inputs fluctuate by the same amount while remaining constant relative to each other, we would like this change to have no bearing on the output (i.e. We only really want to process differential signals).

For a differential input amplifier, common-mode voltage (V_{cm}) is defined as the average of the two input voltages. If we consider a differential amplifier, as shown in Figure 1, it can be seen that:

$$V_{cm} = \frac{V1 + V2}{2} \tag{1}$$

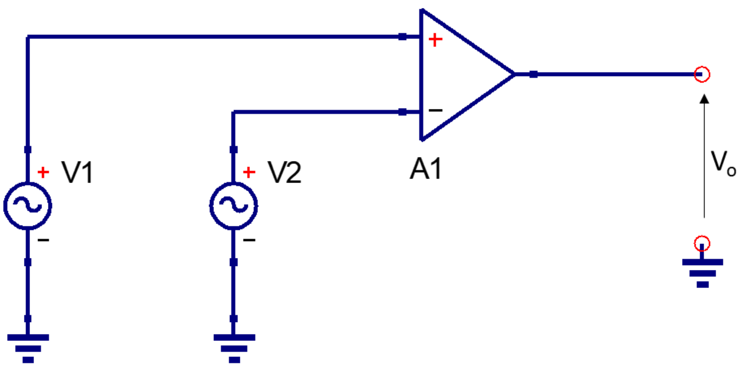

Figure 1. Op-Amp with input voltages to both inputs

If we now define *V(+)* and *V(-)* as the voltages at the non-inverting and inverting inputs to the Op-amp respectively then, considering Figure 2:

$$V_{cm} = \frac{V(+)+V(-)}{2}$$
$$V(+) = V_{cm} + \frac{V_{diff}}{2} \qquad (2)$$
$$V(-) = V_{cm} - \frac{V_{diff}}{2}$$

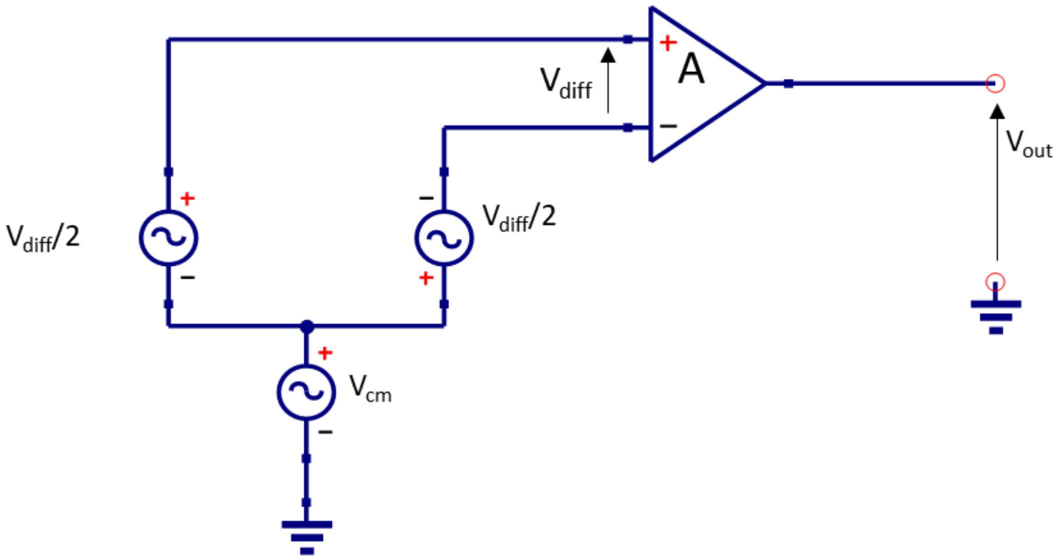

Figure 2. Common mode rejection ratio of op-amp

We can now write:

$$V_{out} = A_{dm}(V_{diff}) + A_{cm}(V_{cm}) \qquad (3)$$

where A_{dm} is the differential mode gain and A_{cm} is the common mode gain. For a perfect amplifier we would only want to amplify the differential signal, but since no amplifier is perfect, we define a Common Mode Rejection Ratio (CMRR), given by:

$$CMRR = \left|\frac{A_{dm}}{A_{cm}}\right| \qquad (4)$$

We also define the Common Mode Rejection:

$$CMR(dB) = 20\log_{10}(CMRR) \qquad (5)$$

Ideally, a differential input amplifier only responds to a differential input voltage, not a common mode voltage, so for an ideal op-amp:

$$CMRR_{ideal-OA} = \frac{A_{dm}}{A_{cm}} = \frac{A_{dm} \to \infty}{A_{cm} \to 0} \to \infty \qquad (6)$$

Now we have some idea what CMRR is we need to consider how it will affect a circuit. To do this we need a model. To produce a useful model the CMRR needs to be referred-to-input (RTI). This could be done by representing it as a voltage source in series with an input to the amplifier. The magnitude RTI is $V_{cm} / CMRR$. Now let's look at a non-inverting buffer, as shown in Figure 3.

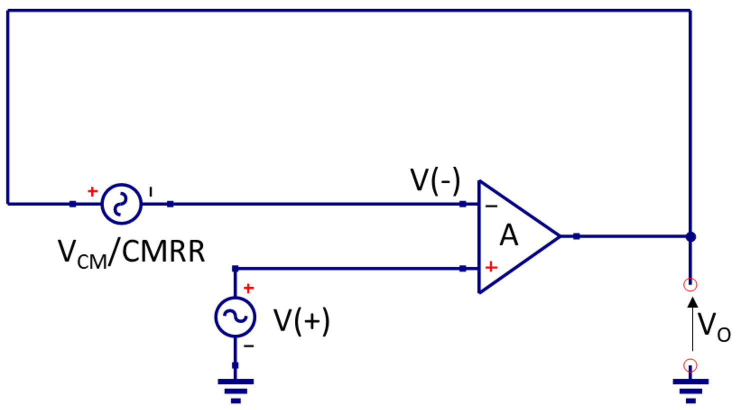

Figure 3. Non-inverting buffer

$$V_O = A(V(+) - V(-))$$
$$V(-) = V_O \pm \frac{V_{CM}}{CMRR} \qquad (7)$$

So:

$$V_O = A\left[V(+) - V_O \pm \frac{V(+)}{CMRR}\right]$$
$$\frac{V_O}{V(+)} = \frac{A\left[1 \pm \frac{1}{CMRR}\right]}{1 + A} \qquad (8)$$

As $A \to \infty$

$$\frac{V_O}{V(+)} \to 1 \pm \frac{1}{CMRR} \qquad (9)$$

What equation (9) shows is that the larger the CMRR the smaller the error.

CMRR of Difference Amplifiers

Let us now consider a difference amplifier made up of an op-amp and a resistor network, as shown in Figure 4. The circuit meets our definition of a differential amplifier since the output is proportional to the difference between the input signals.

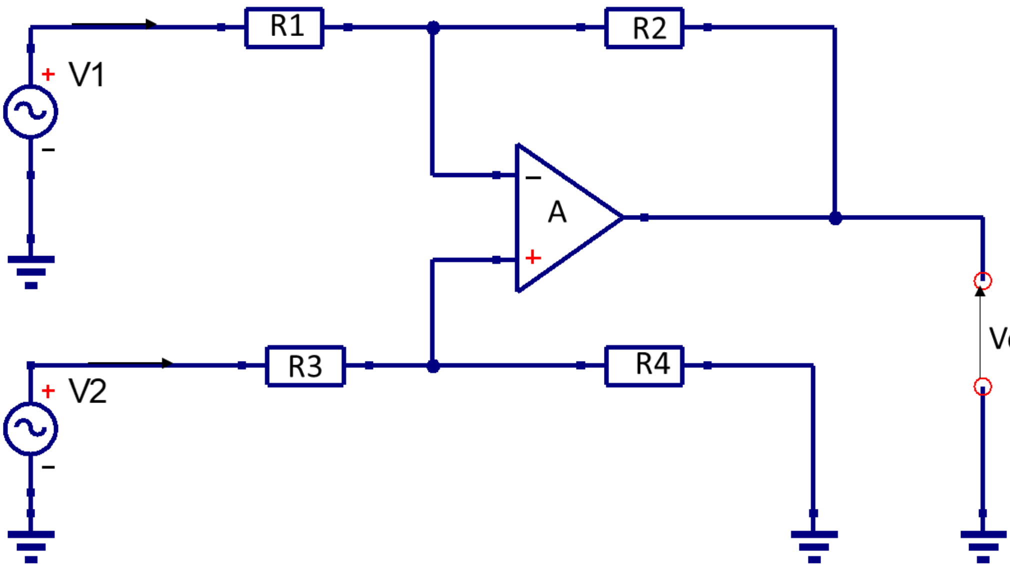

Figure 4. Difference Amplifier

If we now replace V1 and V2 with our alternate definition of the inputs, in terms of the differential and common mode signals (Figure 5), then it becomes apparent that if the amplifier is ideal the output should amplify the differential-mode signal and not the common mode.

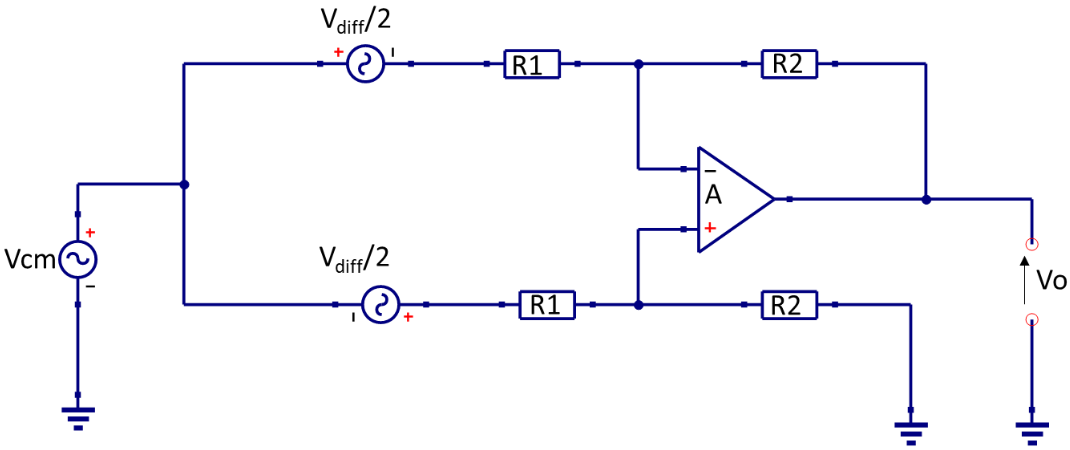

Figure 5. Difference amplifier with differential and common mode signals.

$$V_1 = V_{CM} - \frac{V_{diff}}{2}$$

$$V_2 = V_{CM} + \frac{V_{diff}}{2}$$

$$V_O = \frac{R_2}{R_1}(V_2 - V_1) \tag{10}$$

$$V_O = \frac{R_2}{R_1}\left[\left(V_{CM} + \frac{V_{diff}}{2}\right) - \left(V_{CM} - \frac{V_{diff}}{2}\right)\right]$$

$$V_O = \frac{R_2}{R_1}\left(V_{diff}\right)$$

The equation (10) is based on the premise that the op-amp is ideal and the resistors are balanced. Let us now see what happens if one of the resistors has an imbalance, ε, as depicted in Figure 6.

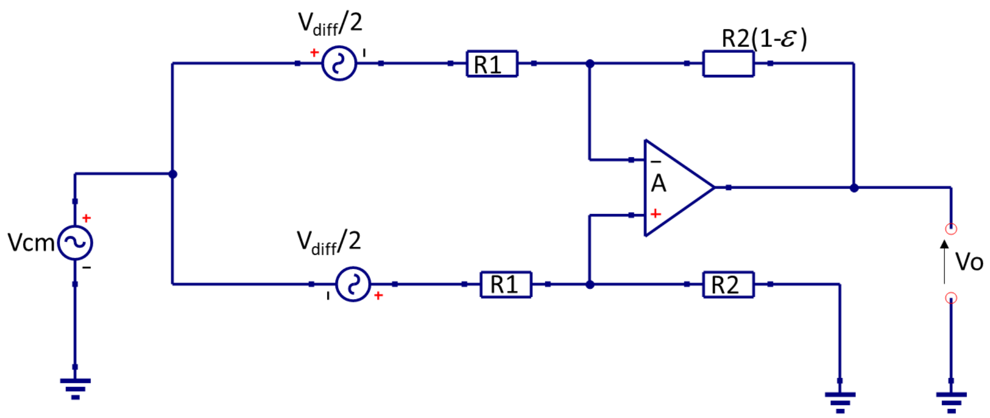

Figure 6. Difference Amplifier with imbalance in resistor.

Using superposition, it can be found that:

$$V_O = \left(V_{CM} - \frac{V_{diff}}{2}\right)\left(-\frac{R_2(1-\varepsilon)}{R_1}\right) + \left(V_{CM} + \frac{V_{diff}}{2}\right)\left(\frac{R_2}{R_1 + R_2}\right)\left(1 + \frac{R_2(1-\varepsilon)}{R_1 + R_2(1-\varepsilon)}\right) \tag{11}$$

Since $V_O = A_{diff}V_{diff} + A_{CM}V_{CM}$, then

$$A_{diff} = \frac{R_2}{R_1}\left(1 - \frac{R_1 + 2R_2}{R_1 + R_2} \times \frac{\varepsilon}{2}\right) \tag{12}$$

and

$$A_{CM} = \frac{R_2}{R_1 + R_2} \times \varepsilon \tag{13}$$

We now have equations for A_{CM} and A_{diff} so we can write the CMR as:

$$CMR(dB) = 20\log_{10}\left(\frac{A_{diff}}{A_{CM}}\right) = 20\log_{10}\left[\frac{\frac{R_2}{R_1}\left(1 - \frac{R_1 + 2R_2}{R_1 + R_2} \times \frac{\varepsilon}{2}\right)}{\frac{R_2}{R_1 + R_2} \times \varepsilon}\right] \tag{14}$$

If the imbalance is sufficiently small then the effect of ε on A_{diff} can be neglected and, with some algebra:

$$CMR(dB) \cong 20\log_{10}\left[\frac{1 + \frac{R_2}{R_1}}{\varepsilon}\right] \tag{15}$$

Equation (15) shows that the CMR will decrease as ε increases. It should be remembered that this shows the effect of the resistor network and assumes an ideal amplifier.

(Note: Another possible source of CMRR degradation is the impedance of the reference pin. If there is an impedance here it will negatively affect the CMR.)

Input Impedance into a Difference Amplifier.

Consider Figure 4 again (re-drawn here as Figure 7). We will start by looking at the impedance (R_{in2}) seen by V2. It can be seen that current (I_2) entering R3 will flow through R4 to ground. No current can flow into the non-inverting input of the op amp, because it has an infinite (or at least very large) input impedance. So:

$$I_2 = \frac{V_2}{R3 + R4} \tag{16}$$

Hence:

$$R_{in2} = R3 + R4 \tag{17}$$

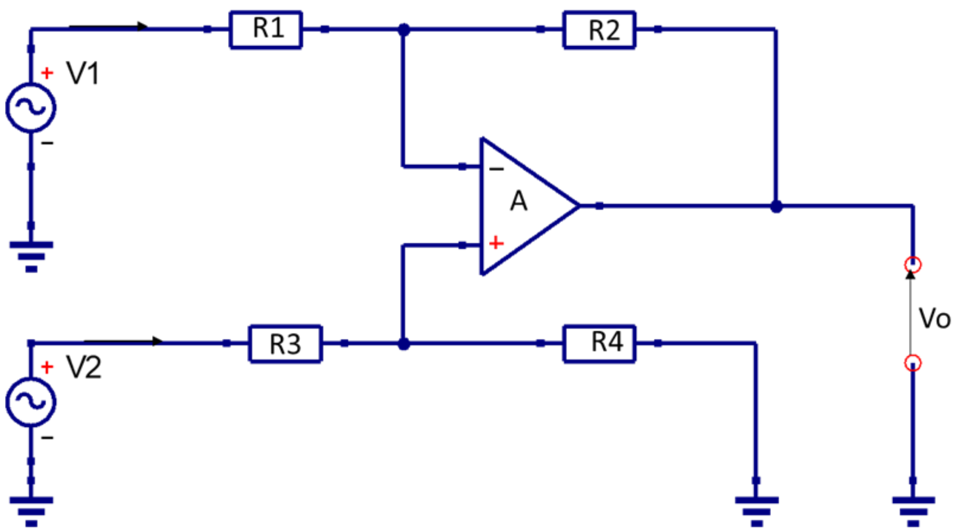

Figure 7. A difference amplifier

The situation at the other input is much more complicated but considering the current flowing through R1 (I_1) we can write:

$$V1 = I_1 R1 - V2 \frac{R4}{R3 + R4} \tag{18}$$

From equation (18) the resistance seen by the source V1 (R1in) will not be the same as R2in, but a function of V2. This is important because the input sources must provide these input currents, which are different. It would be much better if the input impedance seen by both input sources was very large, ideally infinite, so there was no requirement for the input sources to provide current.

In summary then: We have seen that the simple differential amplifier has issues with reduced CMRR, due to resistor mismatch, and low and unmatched input impedances. Despite these problems the difference amplifier is useful as a "building block" within high performance instrumentation amplifiers, as we will see in the following section.

Instrumentation Amplifiers

Instrumentation amplifiers (IA) consist of using more than one op-amp as a difference amplifier and is primarily designed to combat this design flaw. Many IA consist of three op-amps, with two serving as a buffer for the two input circuits, and the third as a differential op-amp. A classical 3 op-amp instrumentation amplifier is shown in Figure 8.

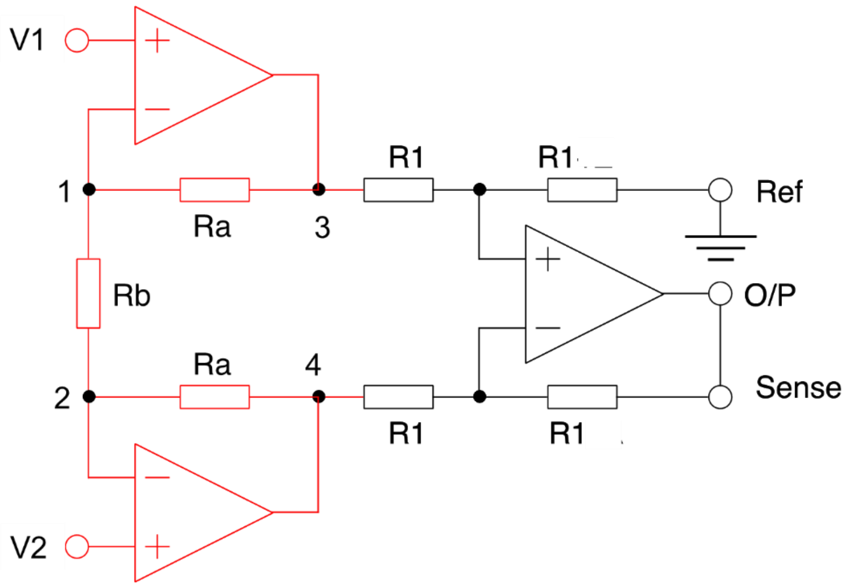

Figure 8. Three op-amp instrumentation amplifier

This IA is in two stages, namely a buffering stage and a differential stage. The signals are applied one each to a non-inverting terminal of the op-amps in the first (buffering) stage. This allows the input sources to see a large resistance and thus reduce the current draw. In an ideal op-amp, the voltage at the non-inverting terminal is equal to that at the inverting terminal. Therefore, at points 1 and 2, the voltage is V1 and V2, respectively.

The second stage of the IA is a unity gain differential op-amp, with the inputs being the voltages at points 3 and 4, V3 and V4. The output of this stage is Vout:

$$V_{OUT} = (V_4 - V_3) \quad (19)$$

Having the feedback resistors on both pins of the differential stage being equal makes this a unity-gain differential op-amp. To get the output in terms of the inputs, the relationship between V4 – V3 and V1 – V2 must be found. Between point 3 and 4, there are three resistors with no connection to ground. This causes the current through all three resistors, I, to be equal:

$$I = \frac{V3 - V4}{2R_a + R_b} \quad (20)$$

Also, considering the voltages at points 1 and 2:

$$I = \frac{V1 - V2}{R_b} \quad (21)$$

Combining and rearranging (20) and (21) yields:

$$V3 - V4 = (V1 - V2)\left(\frac{2R_a + R_b}{R_b}\right) \quad (22)$$

$$V_{OUT} = (V1 - V2)\left(\frac{2R_a + R_b}{R_b}\right) \quad (23)$$

Now we have an expression for the differential gain of the IA, but what about the common mode gain? Referring to the input (buffer) section in Figure 8 (marked in Red) then let V1=V2. If we do this then the current through R_b will be zero since there is no potential difference across that resistor. Also, since for a perfect op-amp the impedance between the inverting and non-inverting inputs is infinite, the current flowing though R_b must flow through the two resistors labelled R_a. This means that V3=V1 and V4=V2. The common mode gain for the input (buffering) stage is then given by:

$$\text{Common-mode gain} = \frac{V3 - V4}{V1 - V2} = 1 \quad (24)$$

If we now consider the scenario depicted in Figure 6, where for the IA, R2=R1, then the differential gain for the IA would be given by:

$$A_{diff} = \frac{2R_a + R_b}{R_b} \quad (25)$$

and:

$$A_{CM} = \frac{\varepsilon}{2} \quad (26)$$

We can see that providing all the gain in the front end of the IA, so unity gain is required from the single op-amp of the output stage, will significantly improve the CMRR, which is now given by:

$$CMRR(IA) = \frac{A_{diff}}{A_{CM}} = \frac{4R_a + 2R_b}{\varepsilon R_b} \quad (27)$$

The CMRR of the IA will be significantly higher than that of a single op-amp differential amplifier, so there will be high attenuation of unwanted common mode signals. The CMR of an IA will typically be around 120 dB. This is very large, which means the output can be considered to be the desired signal.

Bibliography

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