# Noise and the Operational Amplifier.

## 1. Introduction

In this teaching block we will consider noise generated within op-amps. We will not consider external noise which they may pick up (We did that in block 3). In an op-amp there are three noise sources: a voltage noise, which appears differentially across the two inputs and a current noise in each input. These sources can be considered independent of each other. In addition to these three internal noise sources, it is necessary to consider the thermal (Johnson) noise of the external resistors, which are used with the op-amp in the feedback network.

The causes of noise are random in nature and so are computed using root-mean-square (rms) values of noise voltages and noise currents. Op-amp data sheets specify noise voltage and current in the form of spectral densities. We will look at the necessary equations to convert noise spectral densities into rms noise.

An op-amp circuit exhibits random internal noise for two reasons:

1. Due to the random generation and recombination of electron-hole pairs in semi-conductors.
2. The thermal agitation of electrons in resistors. This is termed thermal or Johnson noise. The thermal agitation of an electron inside a resistor constitutes a minuscule current. These currents add up to a net current and, hence, a net voltage. This thermal noise voltage is zero on average but is constantly fluctuating due to the random distribution of the instantaneous magnitudes and directions of the individual electron currents.

## 2. Signal-to-noise ratio

Noise has the effect of degrading the quality of a signal. It poses a limit on the size of the signal that can be successfully detected. A measure of specifying the signal quality in the presence of noise is the signal-to-noise ratio (SNR). The SNR defined as the ratio of the rms signal voltage, to the rms noise voltage in dB:

$$SNR = 20Log_{10}\frac{\text{rms signal voltage } (V_s)}{\text{rms noise voltage } (E_n)} \qquad (1)$$

## 3. Op-amp noise model

Typically, Op-amp noise is modelled by a noiseless op-amp with two equivalent noise sources at the input. The voltage noise is considered as a voltage source with spectral density, $e_n$ and the current noise as a current source with spectral density, $i_n$. This is depicted in Figure 1. It is convention to assign $e_n$ to the non-inverting input.

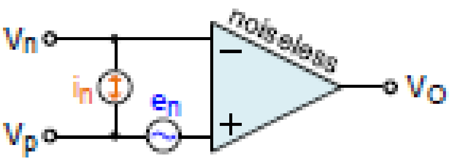


*Figure 1. Noise model of an op-amp: source [1]*

## 4. Thermal noise in resistors

All resistors have a thermal Johnson noise given by:

$$E_r = \sqrt{4RkT\Delta f} \qquad (2)$$

where, k is Boltzmann's Constant ($1.38\times10^{-23}$J/K), T is the absolute temperature in Kelvin, $\Delta f$ is the bandwidth, and R is the resistance value. This is intrinsic to all resistors, so it is not possible to obtain resistors which do not have Johnson noise, unless you operate a 0 Kelvin. **Note** that the unit for thermal noise is $V/\sqrt{Hz}$. (Strictly speaking this is the thermal noise spectral density).

## 5. What do we mean by bandwidth?

First, we need to define something called white noise. White noise is noise that has a flat spectral density. That means that it has the same magnitude across the entire electromagnetic spectrum, as shown in Figure 2.

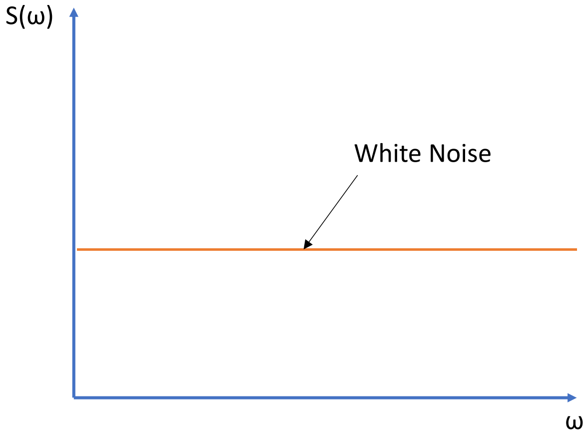


*Figure 2. Spectral density of white noise.*

If we ran white noise through a first order low-pass filter with $f_H$ as its -3dB frequency the noise will behave as if filtered by a "brick-wall filter" with a higher cutoff frequency than the -3 dB frequency. In this case $f_c = 1.57f_H$. The increased bandwidth is termed the white Noise Equivalent Bandwidth (NEB). This is depicted in Figure 3.

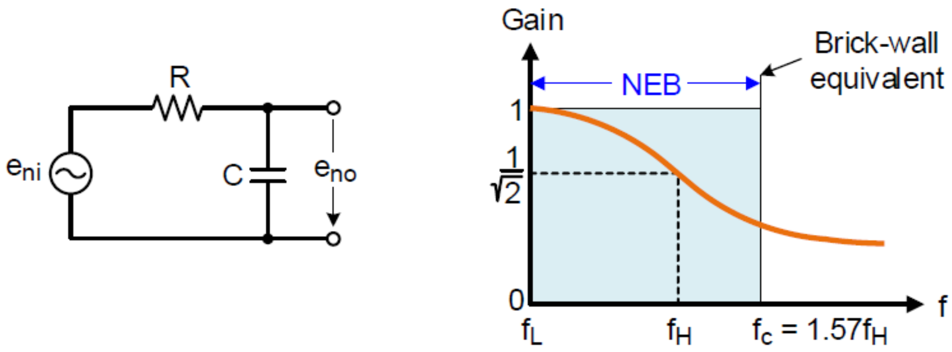


*Figure 3. Noise Equivalent Bandwidth of first order low-pass filter. (Source reference [1])*

If an amplifier with purely resistive feedback is considered then the closed-loop gain represents a first order low-pass with a -3dB bandwidth, $f_H$. This amplifier would then have a NEB with a cutoff frequency, $f_c$, of $1.57f_H$.

### i. In practise how do we find NEB for a given op=amp?

For the ideal op-amp, the gain would be infinite, and it has infinite bandwidth. Since no op-amp is ideal it would have a finite bandwidth and a finite gain. An op-amp would have a flat voltage gain up to a certain frequency after which it would fall at 20 dB per decade, as we would expect from a first order filter. Figure 4 shows the closed loop frequency response of an op-amp. The op-amp considered here has a gain of 100 dB. The frequency at which the gain falls by 3 dB is the cut-off frequency. (Note: Normally it is termed $f_c$ but to be consistent with the previous section we will term it $f_H$.) We then see a first order drop in gain of 20 dB per decade until we reach unity gain, in this case at 1 MHz. The point of unitary gain is termed the Gain-Bandwidth (GBW). If a closed loop is used, as in an inverting or non-inverting amplifier, the gain will be lower but the cut-off frequency will be higher. The purple line shows am amplifier with 40 dB (x100) gain, and an $f_H$ of 10 kHz. If is common for the GBW to be given in data sheets, since it is a constant for all gain, so the Noise Equivalent Bandwidth (NEB) can be found from:

$$NEB = 1.57 \times \frac{GBW}{\text{Noise Gain}} \tag{3}$$

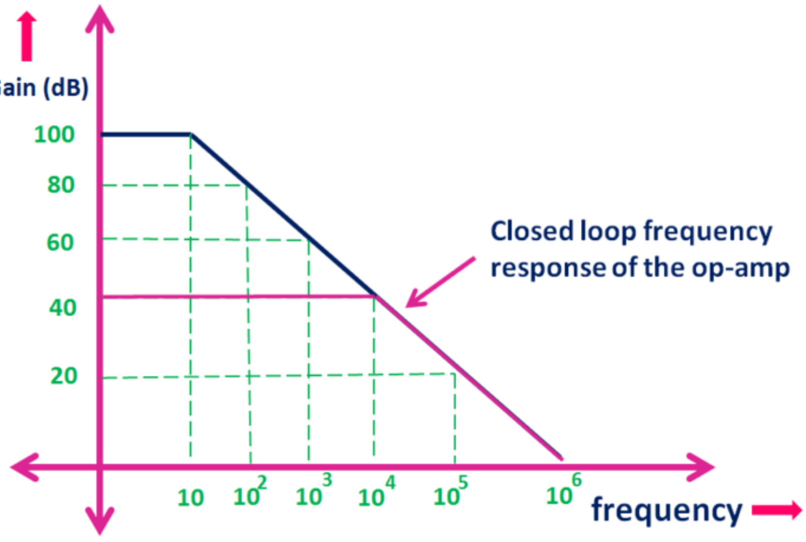


*Figure 4. Closed loop frequency response of an op-amp. (Source: https://www.allaboutelectronics.org/gain-bandwidth-product-of-the-op-amp)[2]*

More accurately we should write equation 3 as:

$$NEB = n \times \frac{GBW}{\text{Noise Gain}} \tag{4}$$

where n is the brick-wall factor for a given filter order. The effective order of the filter will increment every time an amplifier stage is added (so for two cascaded amplifiers the order is 2). Table 1 will give the value of n for filter orders up to 5. In practise, it is unlikely we would want to cascade for than 2 amplifiers.

*Table 1. Values of n for filter order up to 5.*

| Filter order | n |
|---|---|
| 1 | 1.57 |
| 2 | 1.22 |
| 3 | 1.16 |
| 4 | 1.13 |
| 5 | 1.12 |

## 6. Total RMS input noise, $E_{ni}$

In this analysis we will consider an inverting amplifier as shown in Figure 5. We will then add the sources of noise, Figure 6, and look at how they can be calculated. Some approximations are used and these will be justified.

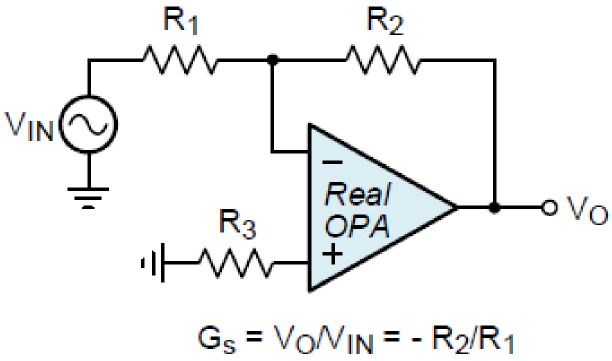


*Figure 5. Inverting amplifier.[1]*

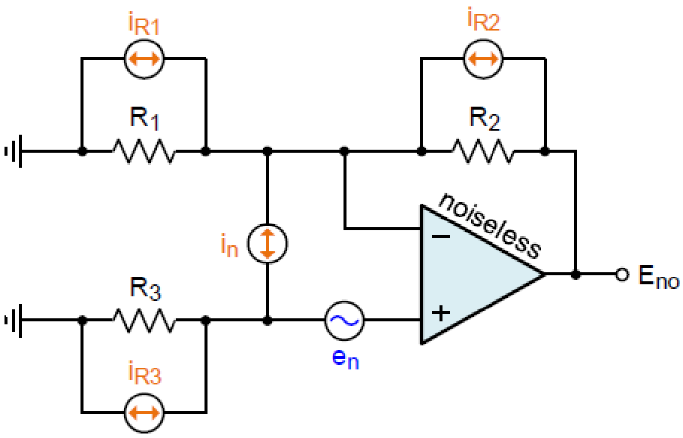


*Figure 6. Inverting op-amp with spectral noise sources*

**i. Thermal noise due to $R_1$ and $R_2$**

$R_1$ and $R_2$ are connected in parallel and so the RMS noise equivalent voltage is:

$$E_{RP} = \sqrt{4KT \times 1.57 f_H \times (R_1 \| R_2)} \quad (5)$$

**ii. Thermal noise due to $R_3$**

It can be assumed here that the value of $R_3$ is the same as $R_1||R_2$. Therefore, the RMS noise equivalent voltage due to $R_3$ will be the same as for $R_1$ and $R_2$.

$$E_3 = E_{RP} = \sqrt{4KT \times 1.57\, f_H \times (R_1 \,||\, R_2)} \tag{6}$$

**iii. RMS noise voltages that are due to op-amp current noise**

At the negative input of the op-amp this noise voltage, $E_{nn}$, is the product of the rms noise current, $I_n$, and the parallel resistance $R_1||R_2$.

$$E_{nn} = R_1 \,||\, R_2 \cdot i_{nw}\sqrt{1.57\, f_H} \tag{7}$$

At the positive input of the op-amp this noise voltage, $E_{pp}$, is the product of the rms noise current, $I_n$, and $R_3$.

$$E_{np} = R_1 \,||\, R_2 \cdot i_{nw}\sqrt{1.57\, f_H} = E_{nn} \tag{8}$$

**iv. RMS input noise voltage due to $e_n$.**

We can find this by:

$$E_n = e_{nw}\sqrt{1.57\, f_h} \tag{9}$$

$E_{ni}$ can now be found by summing all the noise voltages given by equations (5-9).

$$E_{ni} = \sqrt{E_{RP}^2 + E_{R3}^2 + E_{nn}^2 + E_{np}^2 + E_n^2} \tag{10}$$

Equation 10 can be simplified to:

$$E_{ni} = \sqrt{1.57\, f_H} \cdot \sqrt{8kTR_P + 2R_p^2 i_{nm}^2 + e_{nw}^2} \tag{11}$$

Where $R_P = R_1||R_2$.

## 7. Total rms output noise voltage

To find the output noise voltage the $E_{ni}$ has to be multiplied by the noise gain. It is important to realise that $E_{ni}$ is always referred to the non-inverting input, so the noise gain will be the gain of a non-inverting amplifier, so:

$$E_{no} = E_{ni}\left(1 + \frac{R_2}{R_1}\right) \tag{12}$$

## 8. Further example:

Read reference [3] as we will go through this example in class. You will find the reference on Moodle.

## References


1. Noise Calculations of Op-Amp Circuits, Renesas Application Note, R13AN0010EU0100 Rev.1.00, August 2020. (Available on Moodle)
2. Op-Amp Gain Bandwidth Product, https://www.allaboutelectronics.org/gain-bandwidth-product-of-the-op-amp/ (Viewed on 13th November 2023)
3. Noise Calculations in Instrumentation Amplifier Circuits, Renesas Application Note, R13AN0011EU0100, Rev.1.00, August 2020. (Available on Moodle).


## Bibliography


1. Noise Calculations of Op-Amp Circuits, Renesas Application Note, R13AN0010EU0100 Rev.1.00, August 2020.
2. James Karki, Calculating noise figure in op amps, Analog Applications Journal, Texas Instruments Incorporated, 4Q 2003. (Ti White paper available on moodle)