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Distinguishing duration from convexity

Vanguard research

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Executive summary. For equity investors, the perception of risk is generally straightforward: Market risk—the possibility that prices may move in an unprofitable manner—tends to be paramount. However, for bond investors, defining risk is more complicated. Many important risks that are specific to an individual bond—such as default risk and prepayment risk—require consideration, but only interest rate risk is systematic. Like market risk for stocks, interest rate risk is a universal risk for bond investors, and the principal measure of interest rate risk is modified duration (hereafter referred to simply as duration).

Because the topic of duration and a related concept—convexity—can be very complex, we offer this brief paper as a fundamental, rather than a comprehensive, overview of the topic. Our goal is to help investors understand why it's important to consider both duration and convexity when evaluating the interest rate risk of bonds.

Authors
Donald G. Bennyhoff, CFA
Yan Zilbering

Understanding duration

Duration is a measure of the approximate sensitivity of a non-callable bond's price to changes in interest rates. For bonds with embedded options—such as callable bonds, mortgage-backed securities (MBS), and collateralized mortgage obligations (CMOs)—effective duration is generally the preferred measure. Effective duration incorporates the terms for early retirement for a bond. In other cases, such as with Treasury Inflation-Protected Securities (TIPS), duration reflects the sensitivity to a change in *real* interest rates rather than nominal interest rates, as with other bonds.¹

Specifically, duration is a way to estimate the approximate percentage change in the price of a bond for a 100-basis-point (1 percentage point) change in interest rates. A bond with a duration value of 4 would be expected to lose 4% of its market value if interest rates rose by 100 basis points. So why is duration measured in years? The answer: Because duration is technically a weighted average of the time to maturity of a bond's cash flows. As such, it can help an investor assess how soon an initial investment will be repaid—taking into consideration not only when the principal should be returned but also any interest payments that are due along the way.

Duration: An example

A bond is trading at \$10,000, the price at which it was sold. If this bond has a duration of 5 years, and if interest rates rise one full percentage point, then the price of the bond would be expected to fall by about 5% to \$9,500—if everything else remains the same.

A bond is a loan from the bond holder or investor to the bond issuer (for example, a corporation), so it is only natural for investors' concern to center on the timely payment of interest and principal. The bond's indenture, which is an agreement between the issuer and investors, sets forth the specific terms of the bond, including:

- When will the bond mature and principal be repaid?
- Can the principal be repaid prior to the maturity date and, if so, at what price?
- What interest rate (coupon) will be paid to the bond owners?
- How often will coupons be paid and is this rate fixed or variable?

Two bonds with the same coupon rate and the same stated maturity date can have different cash flow patterns and hence different durations. For example, one bond may pay interest more frequently than the other. Duration can help an investor quantify how these cash-flow differences may affect the value of both bonds. For any given maturity date, higher cash flows from a higher coupon rate result in a faster payback period for the investor, hence a shorter duration. A zero-coupon bond has a duration equal to its remaining time to maturity. The duration of a coupon bond, on the other hand, is *shorter* than its time to maturity—because the investor receives periodic interest payments.

In addition to the timing of cash flows, inflation matters because changes in interest rates are frequently associated with changing trends for economic growth and inflation. Higher-than-expected inflation during the loan repayment period reduces the purchasing power of both the principal at maturity

¹ For more information on TIPS, see "Investing in Treasury Inflation Protected Securities."

and the periodic interest payments, reducing the real return. As a result, investors with longer-duration loans—that is, holding longer-duration bonds—are more exposed to changes in interest rates than those with shorter-duration loans.

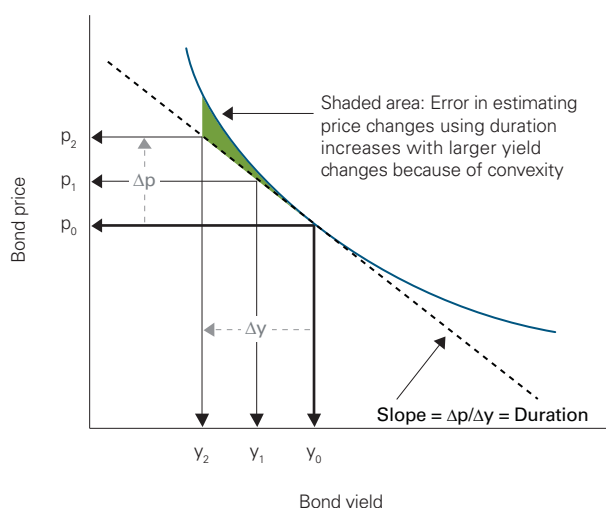
Understanding convexity

Duration, however, captures only one aspect of the relationship between bond prices and interest rate changes. For larger interest rate moves, the relationship between the change in rates and the change in bond prices is asymmetric. The bond price *decrease* resulting from a large interest rate increase will generally be smaller than the price *increase* resulting from an interest rate decline of the same magnitude. This asymmetry arises from the convex payoff pattern shown by the solid curved line in **Figure 1**. This convex shape also means that a portion of the interest rate move remains uncaptured by duration.

Figure 1 plots the relationship between the price of a bond and its yield. The dashed line approximates the effect of a change in yield on the price of the bond. Duration, a measure of interest rate risk, is mathematically derived from the slope of this dashed line. The curved line represents convexity.

Many bonds illustrate positive convexity and behave in the manner illustrated in Figure 1. Convexity captures the degree to which the actual bond price, the solid curved line, deviates from the estimated bond price, the dashed line, when the interest rate changes. However, as shown by the dotted line in **Figure 2**, on page 4, some bonds, such as callable

Figure 1. Duration versus convexity:
An illustration of the difference

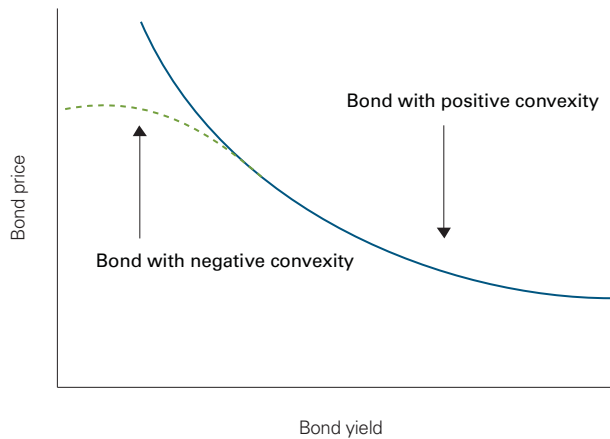


Notes: This figure is an illustration only and is not intended to represent a specific mathematical relationship. Y_0 is the initial yield; Y_1 and Y_2 are subsequent yields. P_0 is the initial price; P_1 is the price after the first yield change and P_2 is the price after the second yield change.

Source: Vanguard.

bonds and mortgage-backed securities, have *negative* convexity at lower interest rate levels. This negative convexity arises from the embedded options discussed earlier: As interest rates fall, the investor is more likely to receive his principal back early, i.e. before maturity, shortening the life of the security and making it less valuable from the standpoint of total cash flow received. Thus, for a bond with negative convexity, falling interest rates do not significantly drive up the bond's price.

Figure 2. Illustration of positive and negative convexity



Note: This figure is an illustration only and is not intended to represent a specific mathematical relationship.
Source: Vanguard.

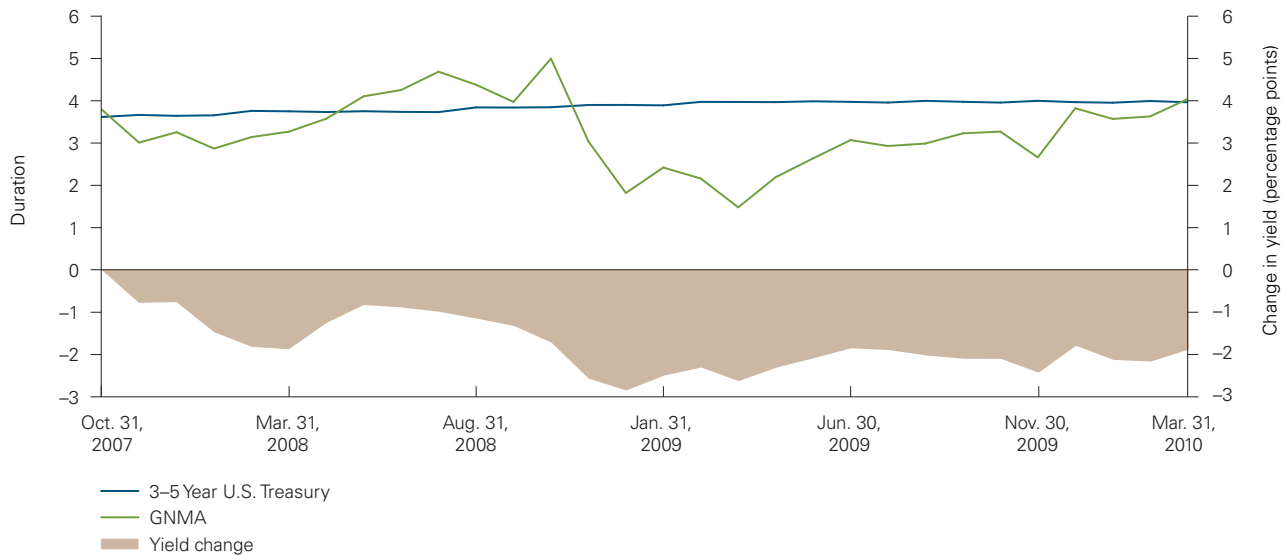
As Figure 2 shows, duration is only an approximate measure of interest rate risk because small and large interest rate changes do not result in the same percentage change in bond price. Because of its linearity, in the absence of negative convexity, duration will underestimate the actual price of a bond for a given rate change. To obtain a more accurate picture of the sensitivity of a bond's price to changes in interest rates, it's important to consider both duration and convexity.

A real world example

Figures 3 and 4 illustrate the importance of considering not only a bond's duration but also its convexity. We show two proxies for a short-to intermediate-term duration portfolio—the Barclays Capital U.S. GNMA Bond Index and the Barclays

Figure 3. Duration relative to yield change for two benchmarks: October 2007 through March 2010

Barclays Capital U.S. GNMA and U.S. 3–5 Year Treasury Bond Indexes

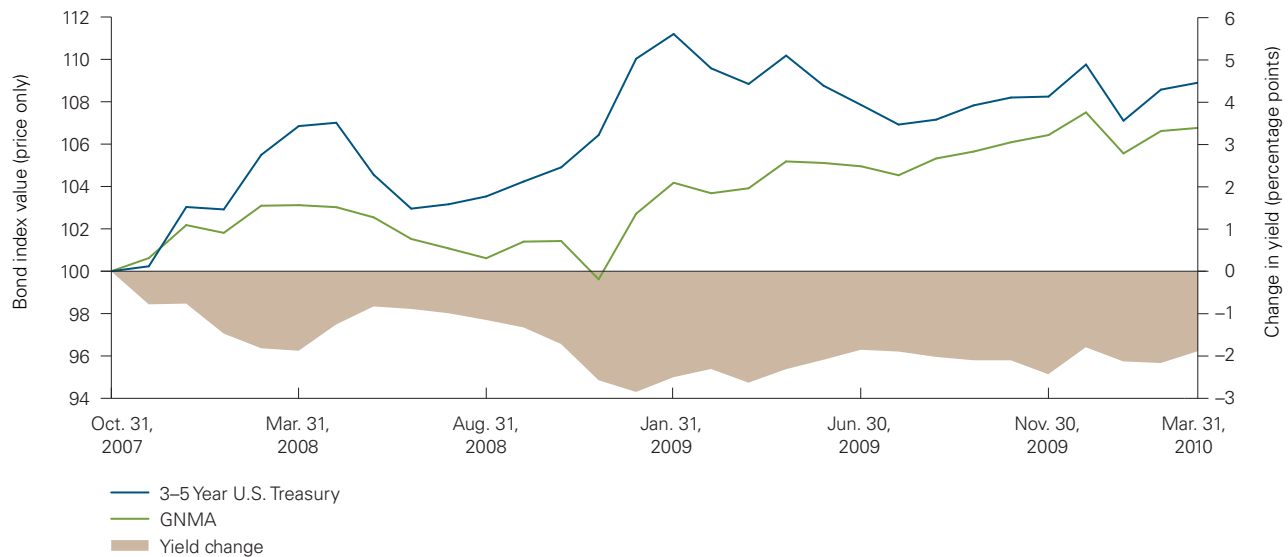


Notes: GNMA's are represented by the Barclays Capital U.S. GNMA Bond Index, and Treasuries by the Barclays Capital U.S. 3–5 Year Treasury Bond Index. Changes in yield, represented on the right vertical axis, are based on monthly data for the Barclays Capital U.S. 3–5 Year Treasury Bond Index. Over the entire period, the cumulative change in yield for this index was a decrease of approximately 2 percentage points.
Sources: Barclays Capital and Vanguard.

Past performance is no guarantee of future results. The performance of an index is not an exact representation of any particular investment, as you cannot invest directly in an index.

Figure 4. Cumulative return relative to yield change for two benchmarks: October 2007 through March 2010

Barclays Capital U.S. GNMA and U.S. 3–5 Year Treasury Bond Indexes



Notes: GNMA's are represented by the Barclays Capital U.S. GNMA Bond Index, and Treasuries by the Barclays Capital U.S. 3–5 Year Treasury Bond Index. Changes in yield, represented on the right vertical axis, are based on monthly data for the Barclays Capital U.S. 3–5 Year Treasury Bond Index. For the entire period, the cumulative change in yield for this index was a decrease of approximately 2 percentage points. The bond index values, represented on the left vertical axis, are based on bond prices only (not total return including income).

Sources: Barclays Capital and Vanguard.

Capital U.S. 3–5 Year Treasury Bond Index. Both indexes include government securities and therefore have similar credit ratings, and had similar durations on the starting date. During the period from October 31, 2007, through March 31, 2010, the duration of the GNMA portfolio—as shown by the green line—was significantly more volatile than that of the Treasury portfolio even though the initial durations were approximately the same. This difference in duration is due in part to the negative convexity of GNMA bonds:

- As interest rates fell, from October 2007 through late 2008, the price of the GNMA portfolio declined with rising expectations for mortgage prepayments and lower mortgage interest payments to the portfolio.

- As interest rates increased from the intra-period lows in December 2008, the duration of the GNMA portfolio increased (Figure 3), as did the prices of the bonds in its portfolio (Figure 4).
- In contrast, the duration of the 3–5 Year Treasury Index remained more stable, increasing slightly as one would expect after rates fell (Figure 3).

When building bond portfolios—whether using bond funds, exchange-traded funds (ETFs), or individual bonds—to help moderate interest rate risk, investors need to be cognizant of both duration and convexity. It's also important to understand the practical limitations and nuances associated with these measures.

Reference

Vanguard Investment Counseling & Research, 2006.
Investing in Treasury Inflation Protected Securities.
Valley Forge, Pa.: The Vanguard Group.

Key terms

Convexity—A measure of a bond's actual price change in response to an interest rate change, compared with the estimated price change based on duration alone. Convexity captures the asymmetry in the relationship between bond prices and interest rate changes, and is plotted as a curved line.

Effective duration—A duration measure that takes into consideration any features that could affect the maturity of the bond and the timing of its cash flows. Such features would include an issuer's ability to "call" the bond, or retire it before the maturity date.

Embedded option—A feature that is inseparable from the bond itself. For example, if the bond issuer has the right to call the bond after five years, that is an embedded option in the bond.

Modified duration (duration)—An indicator, often stated in years, of the approximate percentage change in the price of a bond for a 100-basis-point (1 percentage point) change in interest rates. Duration is plotted as a straight line.

Zero-coupon bond—A non-interest-bearing bond that is sold at a deep discount from its face or par value and is redeemed at par. Interest is effectively being paid to the investor in the form of the price appreciation of the bond as it reaches maturity.



P.O. Box 2600
Valley Forge, PA 19482-2600

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