Learning from Delayed Outcomes with Intermediate Observations

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Abstract

Optimizing for long term value is desirable in many practical applications, e.g. recommender systems. The most common approach for long term value optimization is supervised learning using long term value as the target. Unfortunately, long term metrics take a long time to measure (e.g., will customers finish reading an ebook?), and vanilla forecasters cannot learn from examples until the outcome is observed. In practical systems where new items arrive frequently, such delay can increase the training-serving skew, thereby negatively affecting the model's predictions for new products. We argue that intermediate observations (e.g., if customers read a third of the book in 24 hours) can improve a model's predictions. We formalize the problem as a semi-stochastic model, where instances are selected by an adversary but, given an instance, the intermediate observation and the outcome are sampled from a factored joint distribution. We propose an algorithm that exploits intermediate observations and theoretically quantify how much it can outperform any prediction method that ignores the intermediate observations. Motivated by the theoretical analysis, we propose two neural network architectures: Factored Forecaster (FF) which is ideal if our assumptions are satisfied, and Residual Factored Forecaster (RFF) that is more robust to model mis-specification. Experiments on two real world datasets, a dataset derived from GitHub repositories and another dataset from a popular marketplace, show that RFF outperforms both FF as well as an algorithm that ignores intermediate observations.

1 Introduction

Long term utility of recommendations is an important indicator of recommendation quality (cf. [Wu et al., 2017]); hence optimizing for long term value is desirable in many practical applications. For instance, an online book seller may want to maximize the sale of ebooks that customers finish reading (assuming that customers who engage with purchased content are likely to return to the store). Similarly, an online video subscription service may want to maximize the number of videos a customer finishes watching (to avoid recommending 'click-bait' content). Unfortunately, long term value is only revealed after a significant delay, which is problematic when training a model to predict long term value based on historical data. The delay causes *training-serving skew* [Zinkevich, 2017], and hence the model's predictions for new products may be inaccurate, particularly in large-scale systems where new items arrive frequently. While generalization can help to mitigate this problem, per product memorization is important for large scale recommender systems [Cheng et al., 2016].

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Motivating Example: Consider an online marketplace for ebooks. We define an engagement event to occur when a customer finishes an ebook within 90 days of conversion. Similar to YouTube video recommendations [Davidson et al., 2010], when a customer visits, the marketplace (1) generates a small set of ebook candidates, (2) scores each of the candidates, and (3) presents them in descending order of predicted probability of engagement. If an ebook is purchased, whether or not a successful engagement will occur is unknown. The approach taken by much of the prior work on learning with delay [Weinberger and Ordentlich, 2002, Mesterharm, 2005, Joulani et al., 2013, Quanrud and Khashabi, 2015] would imply waiting the whole 90 days to see the outcome. However, new books are added to the marketplace every day. If we waited 90 days before we can accurately predict the engagement probability of a new book, the marketplace will miss out on many potential sales. An alternative approach could make use of intermediate observations, which we denote by ISym (for intermediate symbol) throughout this paper. For example, one day after a purchase we can define ISyms based on the furthest page reached in the ebook. Clearly, these ISyms provide some information about the eventual outcome.

Intermediate Observations and Outcomes: To exploit ISyms, we need to formalize their relationship to outcomes. For example, ISyms based on independent, random noise obviously do not provide useful information about outcomes. The key assumption we use is that the outcome distribution can be factored into two models: (1) one that generates ISyms given an instance, and (2) one that generates outcomes given an ISym. An intuitive algorithm could try to learn both models. The first model can be updated as soon as an ISym is available, while the second model is slower to learn but generalizes across instances.

The Problem: In the ebook marketplace, ebook candidates are not generated by a fixed distribution, since new books are being added over time. Furthermore, a useful scoring algorithm should be agnostic to the candidate retrieval process. For these reasons, we model the problem as a semistochastic online learning problem with ISyms, where instances may be selected by an adversary but, given an instance, the outcome and the ISyms are sampled from a joint distribution. In our setting, if ISyms are ignored, a lower bound on the cumulative error is $\Omega(DN+\sqrt{NT})$ where $D\geq 0$ is the maximum outcome delay, N is the number of instances (e.g., total corpus size), and T is the performance horizon. Note that in a recommender system where we make 10,000 predictions per day and only update the forecaster each evening, a 90 day delay would translate into a lower bound on cumulative error proportional to $10,000\times 90\times N+\sqrt{NT}$. We show that a much improved $\widetilde{O}(D+N+Z\sqrt{NT})$ cumulative error can be achieved by a predictor with a proper factored structure, where Z is the number of possible ISyms and the tilde indicates that we have suppressed logarithmic factors. Examining these two bounds, we can see that ISyms help when the delay D and number of instances N are large and the number of ISyms Z is small.

Practical Deep Implementations: Using this intuition we propose a neural network-based online learner (FF) with two modules learning the above two models separately. However, we found that the factorization assumption is violated in both of our experimental domains. To mitigate this problem, we introduce a second neural network-based learner (RFF) that introduces a residual correction term. We compare both of these factored algorithms to an algorithm that ignores ISyms (DF) on two experimental domains: (1) predicting commit activity for GitHub repositories, and (2) predicting engagement with items acquired from a popular marketplace. In both of these domains, RFF outperforms both DF and FF.

Contribution: This paper offers three main contributions:

- 1. We formalize the problem of learning from delayed outcomes with ISyms as a semi-stochastic online learning problem. Many prior works have analyzed learning with delayed outcomes [e.g., Weinberger and Ordentlich, 2002, Mesterharm, 2005, Joulani et al., 2013], however, we believe this is the first work to consider a setting where ISyms are exploited to mitigate the impact of delay.
- 2. We quantify the potential gain for using ISyms under an assumption on the relationship between ISyms and outcomes. In particular, exploiting ISyms helps most when the outcome delay is large and the number of ISyms is small.
- 3. Finally, we introduce a practical neural network implementation, RFF, for exploiting ISyms. Our experiments provide evidence that RFF outperforms a delayed learner even when the assumptions required by our analysis are violated.

2 Formal Problem Description & Approach

Let $\mathcal{X}, \mathcal{Y}, \mathcal{Z}$ be finite, nonempty sets representing the instances, outcomes, and ISyms, respectively. We denote the number of instances $|\mathcal{X}|$ by N, the number of labels $|\mathcal{Y}|$ by C, the number of different ISyms $|\mathcal{Z}|$ by Z, and the d-dimensional simplex by Δ^d for any $d \geq 2$. The outcome distribution function is $p^*: \mathcal{X} \to \Delta^C$. Let $T, D \geq 0$ where T is the number of prediction rounds and D is the maximum number of steps that an outcome can be delayed (the delay is defined to be 0 if the label is received at the end of the round). At each round, the environment generates $(x_t, y_t, z_t) \in \mathcal{X} \times \mathcal{Y} \times \mathcal{Z}$ such that x_t is chosen by an adversary and y_t is sampled according to $p^*(\cdot|x_t)$, independently in every round t. We will discuss how ISyms are generated below. Using the instance x_t , the forecaster makes a prediction $p_t \in \Delta^C$ and incurs instantaneous loss $\ell(p_t, x_t, y_t) = |p_t(y_t|x_t) - p^*(y_t|x_t)|$. At the end of round $t \in \{1, 2, \ldots, T\}$, the forecaster receives a possibly empty set $\mathcal{D}_t \subset [t] \times \mathcal{Y}$ containing pairs (s, y_s) where s indicates the round that the label y_s was generated on. The goal of the forecaster is to minimize cumulative error $\xi(T) = \sum_{t=1}^T |p_t(y_t|x_t) - p^*(y_t|x_t)|$ over T rounds.

Joulani et al. [2013] introduce a strategy for converting a base online learner into a delayed online learner, both for the adversarial and stochastic setting (but not for the hybrid setting we consider). We refer to this approach or any other approach that learns a direct relationship between instances and outcomes as a *direct forecaster* for delayed outcomes. As mentioned in the introduction, a lower bound on the cumulative error in our setting where ISyms are ignored is $\Omega(DN + \sqrt{NT})$. Thus, if we want to do better, we need to make an assumption about the relationship between how the outcome and the ISyms are generated.

Assumption 1. At round t the environment generates $(x_t, y_t, z_t) \in \mathcal{X} \times \mathcal{Y} \times \mathcal{Z}$, x_t is revealed at the beginning of the round, z_t is revealed at the end of the round, 2 and y_t is revealed after at most D additional rounds. Given x_t , $z_t \sim h(\cdot|x_t)$ and given z_t , $y_t \sim g(\cdot|z_t)$, independently for all rounds.

Note that Assumption 1 implies that for all $x, y \in \mathcal{X} \times \mathcal{Y}$, the probability of y is

$$p^*(y|x) = \sum_{z \in \mathcal{Z}} g(y|z)h(z|x) . \tag{1}$$

Assumption 1 says that the outcome is generated by a stochastic process that can be factorized into two conditional probability distributions g and h. Since g does not depend on an instance it can be estimated regardless of the sequence of instances chosen by the adversary. On the other hand, h does depend on an instance, but the ISyms are revealed by the environment much sooner than the label. So h can also be updated more quickly than trying to estimate p^* directly from labels.

The formal setting in this paper is motivated by online marketplaces where we want to predict consumer engagement with purchased products. An instance $x \in \mathcal{X}$ represents features about the users and the content being considered for recommendation, while a label $y \in \mathcal{Y}$ indicates what kind of engagement occurred, with the simplest being $\mathcal{Y} = \{0,1\}$ where y=0 represents "no engagement" and y=1 represents "successful engagement". While it is tempting to analyze this as a bandit problem, real online market places often rank content based on multiple factors. Since multiple factors are used to determine which items are recommended, there is rarely a predictable distribution over future instances available. Thus, we allow instances to be chosen adversarially.

Proposed Approach: The basic idea is to exploit the factored model for the outcome distribution introduced in Assumption 1. Algorithm 1 learns empirical estimates of $g(\cdot|z)$ and $h(\cdot|x)$ and then uses the factorization (1) to make predictions. Since g does not depend on an instance $x \in \mathcal{X}$, the algorithm can improve the estimate of g no matter what sequence of instances the adversary selects. Although h does depend on an instance, the ISym is revealed on the same round. So the agent can update h quickly when the adversary selects an instance that has not been observed in the past.

Before we analyze Algorithm 1, we need one additional assumption.

Assumption 2. Let $\zeta > 0$ and $\mathbb{A} \subset \Delta^N$ be a subset of distributions over documents such that $\forall_{\alpha \in \mathbb{A}} \forall_{z \in \mathbb{Z}} \sum_{x \in \mathcal{X}} \alpha(x) h(z|x) \geq \zeta$. At each round $t \in [T]$, the adversary first chooses $\alpha_t \in \mathbb{A}$ and then selects the instance $x_t \sim \alpha_t(\cdot)$, independently at all rounds.

¹This deviates from the typical adversarial setting where both instances and labels are generated adversarially.

²Our experiments consider the case where intermediate observations are also delayed.

Assumption 2 is needed to ensure that we can estimate g quickly. If $\zeta=0$, the adversary might select instances where some ISym $z\in\mathcal{Z}$ does not occur and then towards the end of the episode start introducing instances where the observation z is very probable causing any algorithm to have to wait for the label rather than approximating (1).

Theorem 1. Given Assumptions 1 and 2, for any delay $D \geq 0$, the expected cumulative error of Algorithm 1 satisfies $\mathbb{E}\left[\xi(T)\right] \leq \widetilde{O}\left(D+N+Z\sqrt{TN}+\sqrt{\frac{T}{\zeta}}+\frac{1}{\zeta}\right)$.

While the analysis is straightforward, Theorem 1 provides valuable insight about when learning from intermediate observations is helpful. The proof can be found in Section A of the supplementary material.

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Algorithm 1 Factored Forecaster for Delayed Labels
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Require: \mathcal{A}^{(g)} and \mathcal{A}^{(h)} {Forecasters for g and h.} history \leftarrow {Stores observation history.} for t=1,2,\ldots,T do

Environment generates (x_t,y_t,z_t) and reveals x_t. \hat{h}(\cdot|x_t) \leftarrow \mathcal{A}^{(h)}: predict(x_t) \hat{p} \leftarrow \sum_{z \in \mathcal{Z}} \hat{h}(z|x_t) \mathcal{A}^{(g)}: predict(z). Environment reveals z_t. history[t] \leftarrow z_t. \mathcal{A}^{(h)}: update(x_t,z_t). \mathcal{D}_t is revealed by the environment for (s,y_s) \in \mathcal{D}_t do

z \leftarrow \text{history}[s]. \mathcal{A}^{(g)}: update(z,y_s). end for end for
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Before discussing Theorem 1, note that if intermediate observations are ignored, a lower bound

$$\mathbb{E}\left[\xi(T)\right] \ge \Omega(DN + \sqrt{TN}) \quad , \tag{2}$$

can be derived. It is well-known that estimating the mean of a Bernoulli random variable from T samples results in an $\Omega(\sqrt{T})$ error (see, e.g., Devroye et al. [1996]). If the outcomes are delayed by D rounds, at least a constant positive loss is suffered on average in the first D+1 rounds (before any observation is received). This implies that in the delayed case, a lower bound on the error is $\Omega(D+\sqrt{T})$ for a single Bernoulli distribution. Now consider the case that we have N different instances, and each is repeated T/N times. In each of these segments the minimum loss suffered is $\Omega(D+\sqrt{T/N})$, and so the cumulative loss is at least $\Omega(DN+\sqrt{TN})$.

Compared to the lower bound that ignores intermediate observations, the bound of Theorem 1 scales more favorably when the delay and number of instances are large, since it depends on D+N rather than DN. However, we pay an additional $\widetilde{O}\left(Z\sqrt{TN}+\sqrt{\frac{T}{\zeta}}+\frac{1}{\zeta}\right)$ price for learning with ISyms. If ζ is very small (meaning that there is at least one ISym that is unlikely to be observed), it can take many rounds to learn a good approximation of $g(\cdot|z)$ for each $z\in\mathcal{Z}$. Furthermore, we also pay a price for introducing a large number of ISyms.

2.1 Neural Network Architectures for Learning from Delayed Outcomes

Based on our analysis, we propose three neural network architectures for learning from delayed outcomes with intermediate observations.

Direct forecaster (DF): We use a single neural network to predict the distribution over outcomes given an instance (Figure 1(a)). This approach ignores ISyms.

Factored forecaster (FF): This approach learns two neural networks (Figure 1(b) and 1(c)). The first (Figure 1(b)) predicts a distribution over ISyms given an instance, while the second (Figure 1(c)) predicts an outcome distribution given an ISym. This approach is similar to Algorithm 1.

Factored with Residual forecaster (RFF): Similarly to the factored approach, this approach learns two neural networks (Figure 1(b) and 1(d)). However, the neural network that predicts an outcome distribution has two towers. The first tower only uses the ISym to predict the logits, while the second tower is an instance-dependent residual correction that can help to correct predictions when Assumption 1 does not hold exactly. Furthermore, we train the residual tower with a separate loss and stop backpropogation from that loss to the first tower. This ensures that the second tower is treated as a residual correction helping to preserve generalization across instances.

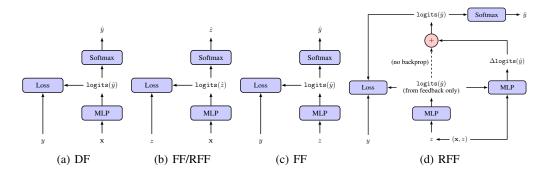


Figure 1: Neural network architectures for delayed prediction, where x denotes an instance, y denotes a label, \hat{y} denotes a predicted outcome distribution, z denotes an intermediate observation, and \hat{z} denotes a predicted distribution over intermediate observations. The direct model (a) directly predicts \hat{y} an outcome distribution from an instance x. The factored model is defined by two neural networks (b) and (c), where (b) predicts \hat{z} the intermediate observation distribution given an instance x and (c) predicts \hat{y} an outcome distribution from an intermediate observation z. Finally, the factored with residual model is defined by two networks (b) and (d), where (d) uses both an instance x and intermediate observation z to predict \hat{y} an outcome distribution.

For all experiments and unless stated otherwise, we update network weights using Stochastic Gradient Descent³ with a learning rate of 0.1 minimizing the log-loss. Except for the networks predicting the outcome distribution from ISyms (Figure 1(c) and left tower in Figure 1(d)), all network towers have two hidden layers. Their output layer is sized appropriately (to output the correct number of class logits) and use a softmax activation. We apply L_2 regularization on the weights with a scale parameter of 0.01. The networks predicting the outcome distribution from ISyms do not contain hidden layers, they do not use regularization or bias in their output layer. Training samples are stored in a fixed-sized FIFO replay buffer from which we sample uniformly.

For the experiment explained in Section 3.1, the networks predicting the outcome distribution from ISyms use a learning rate of 1. Network towers have two hidden layers with 40 and 20 units. The training buffer has a size of 1,000. We start training once we have 128 examples in the buffer and perform one gradient step with a batch size of 128 every four rounds.

For the experiment explained in Section 3.2, the networks predicting the outcome distribution from ISyms use a learning rate of 0.1. Network towers have two hidden layers with 20 and 10 units. The training buffer has a size of 3,000. We start training once we have 500 examples in the buffer and perform 20 gradient steps with a batch size of 128 every 1,000 rounds.⁴ The parameters were tuned using grid search.

3 Experiments & Results

We compare cumulative error of DF, FF, and RFF in two domains: (1) predicting the commit activity of GitHub repositories, and (2) predicting engagement with items acquired from a marketplace.

3.1 GitHub Commit Activity

The goal is to predict the number of commits made to repositories from GitHub⁵ - a popular website hosting open source software projects. Given a repository, the question we want the online learner to answer is "will there be at least three commits in the next three weeks?". This information could be used to predict churn rate. For example, GitHub could potentially intervene by sending a reminder email. We obtained historical information about commits to GitHub repositories from the BigQuery GitHub database. We started with 100,000 repositories and filtered out repositories with fewer than

³We tried to use other optimizers but found that most were not sensitive enough to changes in the distribution over instances as they keep track of an historical average over gradients.

⁴To simulate less frequent updates to the networks.

⁵http://www.github.com

⁶https://cloud.google.com/bigquery/public-data/github

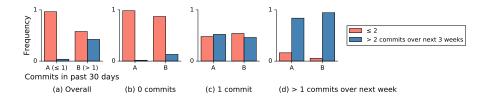


Figure 2: Comparison of outcome distributions for each ISym under different adversarial modes (A and B). The fact that these distributions are not identical under A and B violates Assumption 1. (a) shows the change in overall outcome distributions between the two adversarial distributions, while (b-d) show the marginal outcome distribution conditioned on each ISym.

five unique days with commits between May 1, 2017 and January 8, 2018. This resulted in about 8,300 repositories. In our experiments, an adversary selects both a repository and a timestamp. The outcome is one if there were at least three commits over the 21 days following the chosen timestamp and zero otherwise. The ISym is based on the number of commits over the first seven days following the chosen timestamp. These were mapped to three ISyms: (1) no commits, (2) one commit, and (3) more than one commits. The ISym is delayed by one week and the outcomes are delayed by three weeks. The forecaster receives the equivalent of one sample every 10 minutes.

The adversary initially selects repositories from a subset with a low number of commits - one or fewer commits over the past month. After four and a half weeks, the adversary switches to a distribution that samples from repositories with two or more commits within the past week. Figure 2 shows the outcome probabilities for each ISym under the two distributions used by the adversary. Since the outcome probabilities given an ISym are not equal, this violates Assumption 1. Thus, this makes the task more difficult for the factored architectures but does not matter for the direct architecture. In this experiment, the historical information about a repository defines an instance. We used used binary features to represent the programming languages present in a repository as well as time bucketized counts of historical commit activity for that repository.

To generate the cumulative error, we subtract the loss of an optimal forecaster. Since we do not have access to an optimal forecaster, we trained two models with the same architecture as DF on both modes used by the adversary. We trained these models using the Adam optimizer for 10,000 steps and using an initial learning rate of 0.0005.

Figure 3(a) compares the loss⁷ of the direct and factored architectures averaged over 200 independent trials. The vertical dashed line indicates the time at which the adversary switches from its initial distribution to a distribution over high commit repositories. Due to the outcome delay, all algorithms suffer the same initial loss for roughly 3 weeks. Then all three algorithms quickly achieve a low loss until the adversary changes the distribution over repositories. Finally, the factored architectures (FF and RFF) recover roughly 2.5 weeks more quickly than DF (as can be seen in Figure 3(a) from week 5.5–8). Figure 3(b) shows the cumulative error for the same experiment. FF and RFF achieve smaller cumulative error as they are better able to adapt to changing distributions. We can also clearly observe that FF is not able to maintain a low loss (when compared to DF and RFF) as it is unable to cope with the slightly incorrect factorization assumption.

3.2 Engagement with Marketplace Items

For a popular marketplace with personalized recommendations, we want to predict the probability that after acquiring an item a user will (1) engage with that item more than once, and (2) not delete that item within 7 days after acquiring it (e.g., a user that downloads a new ebook will read at least two chapters and not delete this ebook from their device). The ISyms are measured two days after an item is acquired and the possible outcomes are: (1) *Deleted:* The item was deleted. (2) *Zero Engagements:* The user engaged with the item exactly once and did not delete it. (3) *One Engagements:* The user engaged with the item two or more times and did not delete it. (4) *Many Engagements:* The user engaged with the item two or more times and did not delete it. The outcomes are the same as the ISyms but measured seven days after conversion.

⁷For all experiments, we show the excess log-loss with respect an optimal forecaster (since we train on that loss). However, results are similar when using the L1-loss instead.

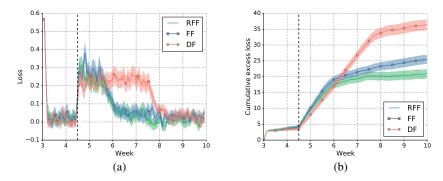


Figure 3: Comparisons of (a) average excess loss and (b) cumulative excess loss (averaged over 200 independent trials with (a) further averaged over five consecutive datapoints) in a prediction task where the adversary shifts the distribution of instances partway through the episode (indicated by the dashed vertical line). We measure the excess loss with respect to an optimally trained model. The shaded area represents 95% confidence intervals.

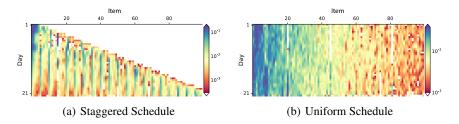


Figure 4: Two different item distribution schedules derived from an actual marketplace. The X-axis corresponds to 100 item indices sorted in decreasing order of total conversions, while the Y-axis corresponds to days. (a) Empirical distribution (logscale) with new items being added each day. (b) More stable empirical distribution of uniformly sampled items.

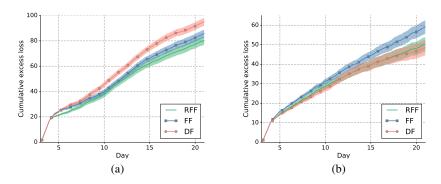


Figure 5: Cumulative excess loss (and 95% confidence intervals) of DF, FF, and RFF averaged over 200 independent trials on items sampled from different schedules. (a) In the staggered schedule, the factored approaches outperform DF, which is crippled by delay. (b) The uniform schedule demonstrates how RFF can achieve similar performance to DF, when the factorization assumption does not help. Note the differences in Y-axis.

We collected 21 days of data between the 10th and 31st of January 2018. For each item with at least 100 conversions, we stored the empirical probability of each ISym and the probability of each outcome conditioned on ISyms. In addition, we stored the empirical probability that an item was acquired on each of the 21 days. We consider two distribution schedules (Figure 4). The first schedule, which we refer to as Staggered (Figure 4(a)), subsampled 100 items for each day (at noon) that appear for the first time on that day in the marketplace (i.e., the candidate items with higher indices only appear on increasingly later days). This schedule simulates items continually being added to the marketplace. This is a very hard setting, which we expect DF to perform poorly in due the need to constantly make predictions about new instances. The second schedule, which we refer to as Uniform, is derived by subsampling 100 items uniformly (Figure 4(b)) and creating a 21 day schedule with half-day intervals by interpolating between the empirical distribution for each item based on the frequencies from the 21 logged days. This schedule is more favorable for DF because instances are sampled according to a slowly shifting distribution. Overall, the ISyms are delayed by two days and the outcomes are delayed by seven days. The forecaster receives the equivalent of one sample every 40 seconds.

In this experiment, we encoded an item/instance using a 100-dimensional one-hot encoding specifying items by index. Similarly to the GitHub experiment, to generate the cumulative error, we subtract the loss of an optimal forecaster. We trained 42 models (one for each half day interval) with the same architecture as DF on all modes used by the adversary (one model for each half-day interval). We train these models using the Adam optimizer and trained for 40,000 steps using an initial learning rate of 0.0005.

Figure 5 compares the average cumulative error of all three forecasters over 200 independent trials for both adversarial schedules. For the Staggered schedule (Figure 5(a)), FF and RFF outperform DF, as expected. DF must wait until the outcomes become available, but FF and RFF are able to generalize to new instances. RFF performs slightly better than FF, indicating that RFF can mitigate the incorrect factorization assumption as delayed outcomes become available.

The Uniform schedule (Figure 5(b)) is easier since the distribution over instances is shifting slowly. DF, FF, and RFF all achieve smaller cumulative error compared to the Staggered schedule. In the Uniform schedule, DF outperforms FF because the factorization assumption is violated by the data. However, RFF achieves similar results to DF because its residual tower allows it to mitigate the error introduced by the incorrect factorization assumption.

4 Related Work

Chapelle [2014] proposes a model for learning from delayed conversions. However, this approach does not take advantage of potential intermediate observations. Learning from delayed labels is also related to survival analysis [Yu et al., 2011, Fernández et al., 2016], where the goal is to model the time until a delayed event. A significant difference of our work is the use of intermediate observations.

A large body of literature exists in online learning for both the adversarial and stochastic partial monitoring settings with delayed intermediate observation, analyzing the regret (see, e.g., Weinberger and Ordentlich [2002], Mesterharm [2005], Agarwal and Duchi [2011], Joulani et al. [2013, 2016]). However, none of the settings takes intermediate observations into consideration for each prediction. Either the intermediate observation for a prediction at round t has been revealed or it has not. In our setting, each prediction round is associated with an intermediate observation symbol revealed sooner than the label.

In a stochastic online learning setting, where the instances and labels are sampled from the same distribution at each round, the regret [Joulani et al., 2013] or mistake bounds [Mesterharm, 2005] scale with D+N (i.e., delay plus number of instances; the exact dependence on the time horizon T depends on the loss function). However, the stochastic assumption is not realistic for online marketplaces because new products are being added on a regular basis. Thus, the distribution over instances is not independent and identically distributed from day to day.

5 Discussion

We present a way to leverage intermediate observations to learn faster in scenarios where the long term labels are delayed. Our theoretical analysis shows that the cumulative error of the factored approach which exploits intermediate observations scales as D+N unlike a naive approach that scales as $D\times N$. We present experimental results on a dataset from GitHub as well as a dataset from a real marketplace, and show that our algorithms can learn faster when intermediate observations are helpful, and can gracefully recover the baseline performance when intermediate observations are unhelpful. We believe that the proposed approach can be beneficial in many real-world applications where the goal is to optimize for long term value. It would be interesting to extend our theoretical analysis to ranking measures.

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Learning from Delayed Outcomes with Intermediate Observations: Supplementary Material

A Proof of Theorem 1

Proof. Let $T_{t-1,zy}$ denote the number of times we observe a feedback-label pair (z,y) by the end of round t-1, and $T_{t-1,z}$ the number of times we observe an ISyms z. Note that $T_{t-1-D,z} = \sum_{y \in \mathcal{Y}} T_{t-1,zy}$. Furthermore, let $T_{t-1,xz}$ and $T_{t-1,x}$ denote the number of times we observe a side information-feedback pair (x,z) and, respectively, a feature vector x by the end of round t-1 (note that $T_{t-1,x} = \sum_{z \in \mathcal{Z}} T_{t-1,xz}$).

Define the estimators $g_t(y|z) = T_{t-1,yz}/T_{t-1-D,z}$ and $h_t(z|x) = T_{t-1,xz}/T_{t-1,x}$; note that $T_{t-1-D,z}$ is the number of times we observe labels for the feedback z before the tth prediction is made. Let $\delta_t = \frac{3\delta}{\pi^2 t^2}$ (so that $\sum_{t=1}^{\infty} \delta_t = \delta/2$).

By the union bound and the Hoeffding-Azuma inequality [Devroye et al., 1996], we can obtain concentration bounds for g_t and h_t . Let E_1 denote the event that

$$|g_t(y|z) - g^*(y|z)| \le \sqrt{\frac{1}{2T_{t-1-D,z}} \log \frac{2ZC}{\delta_t}}$$

 $|h_t(z|x) - h^*(z|x)| \le \sqrt{\frac{1}{2T_{t-1,x}} \log \frac{2ZN}{\delta_t}}$

hold simultaneously for all $1 \le t \le T, x \in \mathcal{X}, y \in \mathcal{Y}, z \in \mathcal{Z}$, where the right hand side is defined to be infinity when the corresponding counts in the denominator $(T_{t-1-D,z})$ and, resp., $T_{t-1,x}$ are zero. Then E_1 holds with probability at least $1 - \delta/2$.

The error of the estimate at time t for any $x \in \mathcal{X}$ and $y \in \mathcal{Y}$ can be bounded as

$$|p_{t}(y|x) - p^{*}(y|x)| = \left| \sum_{z \in \mathcal{Z}} g_{t}(y|z) h_{t}(z|x) - \sum_{z \in \mathcal{Z}} g^{*}(y|z) h^{*}(z|x) \right|$$

$$\leq \left| \sum_{z \in \mathcal{Z}} g_{t}(y|z) \left(h_{t}(z|x) - h^{*}(z|x) \right) \right| + \left| \sum_{z \in \mathcal{Z}} h^{*}(z|x) \left(g_{t}(y|z) - g^{*}(y|z) \right) \right|$$
(3)

Given E_1 , if $T_{t-1,x} \geq 1$, the first term can be bounded by $Z\sqrt{\frac{1}{2T_{t-1,x}}\log\frac{2ZN}{\delta_T}}$. Furthermore, taking into account that $\sum_{z\in\mathcal{Z}}h^*(z|x)=1$, the second term is bounded by $\sqrt{\frac{1}{2\min_{z\in\mathcal{Z}}T_{t-1-D,z}}\log\frac{2ZC}{\delta_t}}$ as long as $\min_{z\in\mathcal{Z}}T_{t-1-D,z}\geq 1$.

Bounding the first expression for all t is simple, as we get the ISyms z_t immediately after making a prediction. For any fixed side information x, we can use the concentration bounds for our estimates from the second time x is observed. Thus, given E_1 ,

$$\sum_{t \in \mathcal{T}_x} \left| \sum_{z \in \mathcal{Z}} g_t(y_t|z) \left(h_t(z|x) - h^*(z|x) \right) \right| \leq 1 + \sum_{t \in \mathcal{T}_x} Z \sqrt{\frac{1}{2T_{t-1,x}} \log \frac{2ZN}{\delta_T}} \\
\leq 1 + Z \sqrt{\log \frac{2ZN}{\delta_T}} \cdot \sum_{t=1}^{T_{T-1,x}} \frac{1}{\sqrt{2t}} \\
\leq 1 + Z \sqrt{2T_{T-1,x} \log \frac{2ZN}{\delta_T}} .$$

Summing up for all $x \in \mathcal{X}$ and using Jensen's inequality with the concavity of the square root function and $\sum_{x \in \mathcal{X}} T_{T-1,x} = T-1$, we get

$$\sum_{t=1}^{n} \left| \sum_{z \in \mathcal{Z}} g_t(y_t|z) \left(h_t(z|x_t) - h^*(z|x_t) \right) \right| \le N + Z \sqrt{2TN \log \frac{2ZN}{\delta_T}}. \tag{4}$$

To handle the second term, we are going to use Assumption 2. Fix $z \in \mathcal{Z}$. By the assumption, there exists a sequence of independent and identically distributed random variables z'_t coupled to z_t such that $z_t = z$ if $z'_t = z$, and $\Pr(z'_t = z) = \zeta$. Assuming we observe z'_t exactly when we observe z_t , let $T'_{t,z}$ denote the observations $z'_s = z$ up to the end of round t. Then clearly $T'_{t-1,z} \geq T_{t-1,z}$. Furthermore, since $\mathbb{E}[T'_{t,z}] = \zeta t$, by the multiplicative Chernoff bound [Angluin and Valiant, 1979], for any $\delta'_t \in (0,1)$, $T'_{t,z} \geq \zeta t/2$ if $t \geq (8/\zeta) \log(1/\delta'_t)$ with probability at least $1 - \delta'_t$. Defining $\delta'_t = \frac{3\delta}{Z\pi^2t^2}$, by Lemma 1 below, $t \geq (8/\zeta) \log(1/\delta'_t)$ is satisfied for all $t \geq C = \frac{32}{\zeta} \log \frac{16\pi\sqrt{Z}}{\zeta\sqrt{3\delta}}$. Therefore, since $Z\sum_{t=1}^\infty \delta_t = \delta/2$, with probability at least $1 - \delta/2$, for all $t \geq C$ with probability at least $1 - \delta/2$, giving $\min_{z \in \mathcal{Z}} T_{t-1-D,z} \geq \zeta(t-1-D)/2$ for $t \geq D + C + 1$. Also assuming E_1 holds, we see that with probability at least $1 - \delta$,

$$\sum_{t=1}^{T} \left| \sum_{z \in \mathcal{Z}} h^*(z|x_t) \left(g_t(y_t|z) - g^*(y_t|z) \right) \right| \le D + C + \sum_{t=1}^{T} \sqrt{\frac{4}{\zeta t} \log \frac{ZC}{\delta_T}} \\
\le D + C + 4\sqrt{\frac{T}{\zeta} \log \frac{ZC}{\delta_T}} \tag{5}$$

Putting together (3), (4), and (5) and substituting the constants, we obtain that with probability at least $1 - \delta$

$$\sum_{t=1}^{T} |p_t(y_t|x_t) - p^*(y_t|x_t)| \leq D + N + \frac{32}{\zeta} \log \frac{16\pi\sqrt{Z}}{\zeta\sqrt{3\delta}} + Z\sqrt{2TN\log \frac{2ZNT^2\pi^2}{3\delta}} + 4\sqrt{\frac{T}{\zeta}\log \frac{ZCT^2\pi^2}{3\delta}}$$

Setting $\delta = 1/T$, the expected loss can be bounded as

$$\sum_{t=1}^{T} \mathbb{E}\left[|p_t(y_t|x_t) - p^*(y_t|x_t)|\right] \le D + N + \frac{32}{\zeta} \log \frac{16\pi\sqrt{Z}}{\zeta\sqrt{3\delta}} + Z\sqrt{2TN\log \frac{2ZNT^3\pi^2}{3}} + 4\sqrt{\frac{T}{\zeta}\log \frac{ZCT^3\pi^2}{3}}$$

Lemma 1. Let a > 0 and b real. Then $t \ge a \log t + b$ if $t \ge 2a \log a + 2b$.

Proof. By the convexity of the logarithm, using a first order Taylor expansion at 2a, for any t > 0,

$$\log t \le \log(2a) + \frac{t - 2a}{2a} \le \log a + \frac{t}{2a}.$$

Now the statement follows by solving

$$a\log a + \frac{t}{2} + b \le t.$$