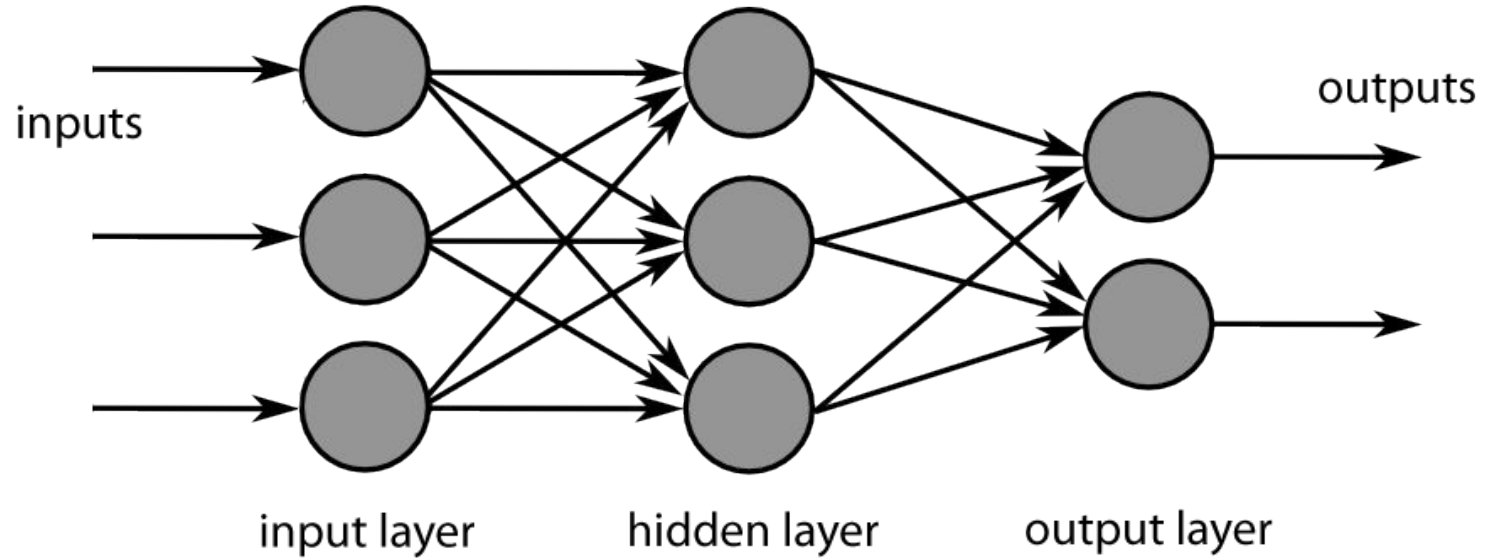


# Neural Network: Multi-layer Perceptron

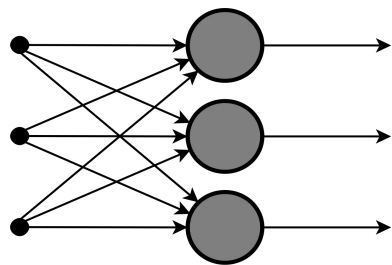
# Introduction

- Robust approach to predict real, discrete or vector value
- Inspired from Biological learning system
- Solve both biological and non-biological problems
- Representation Element
  - Nodes
  - Edges
  - Edge Weights
  - Bias
- Instances are represented by many attribute-value pairs
- The training examples may contain errors
- Long training times are acceptable / *required*
- Fast evaluation of the learned target function may be required
- The ability of humans to understand the learned target function is not important

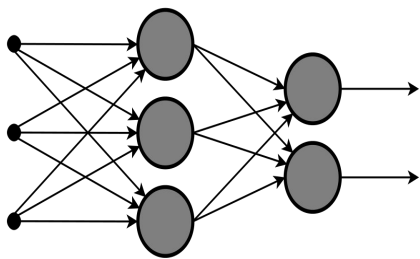
# Introduction



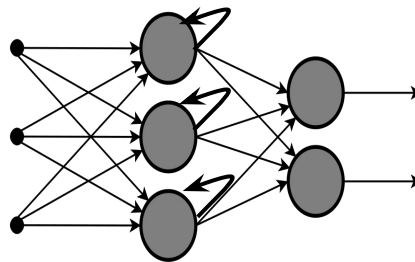
# Types of Neural Networks



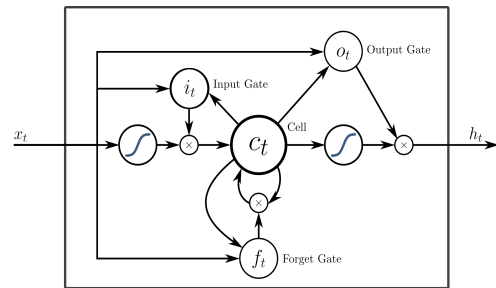
Perceptron or Single Layer NN



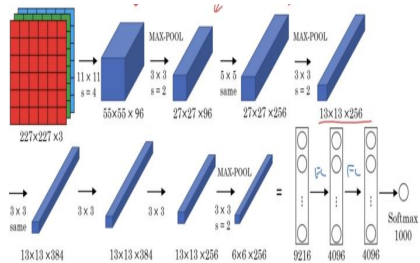
Feed Forward (MLP) NN



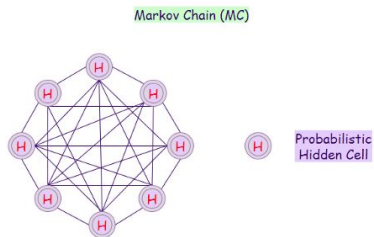
Recurrent (RNN) NN



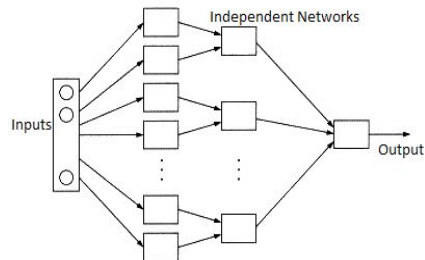
Long Short Term Memory (LSTM) NN



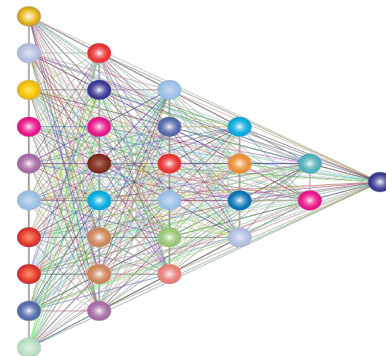
Convolutional (CNN) NN



Markov Chain (MCNN) NN

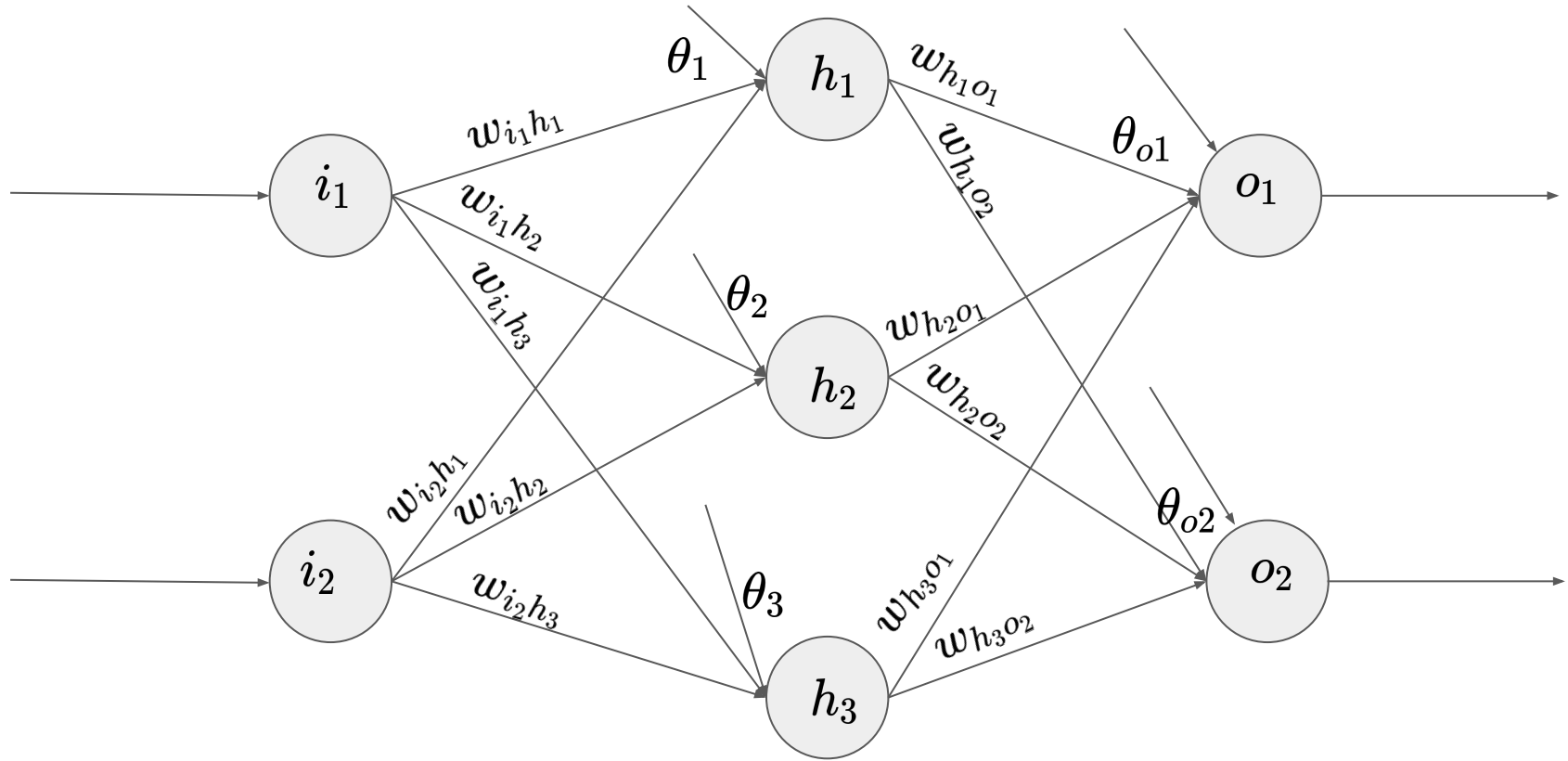


Modular (MNN) NN





Deep Neural Network

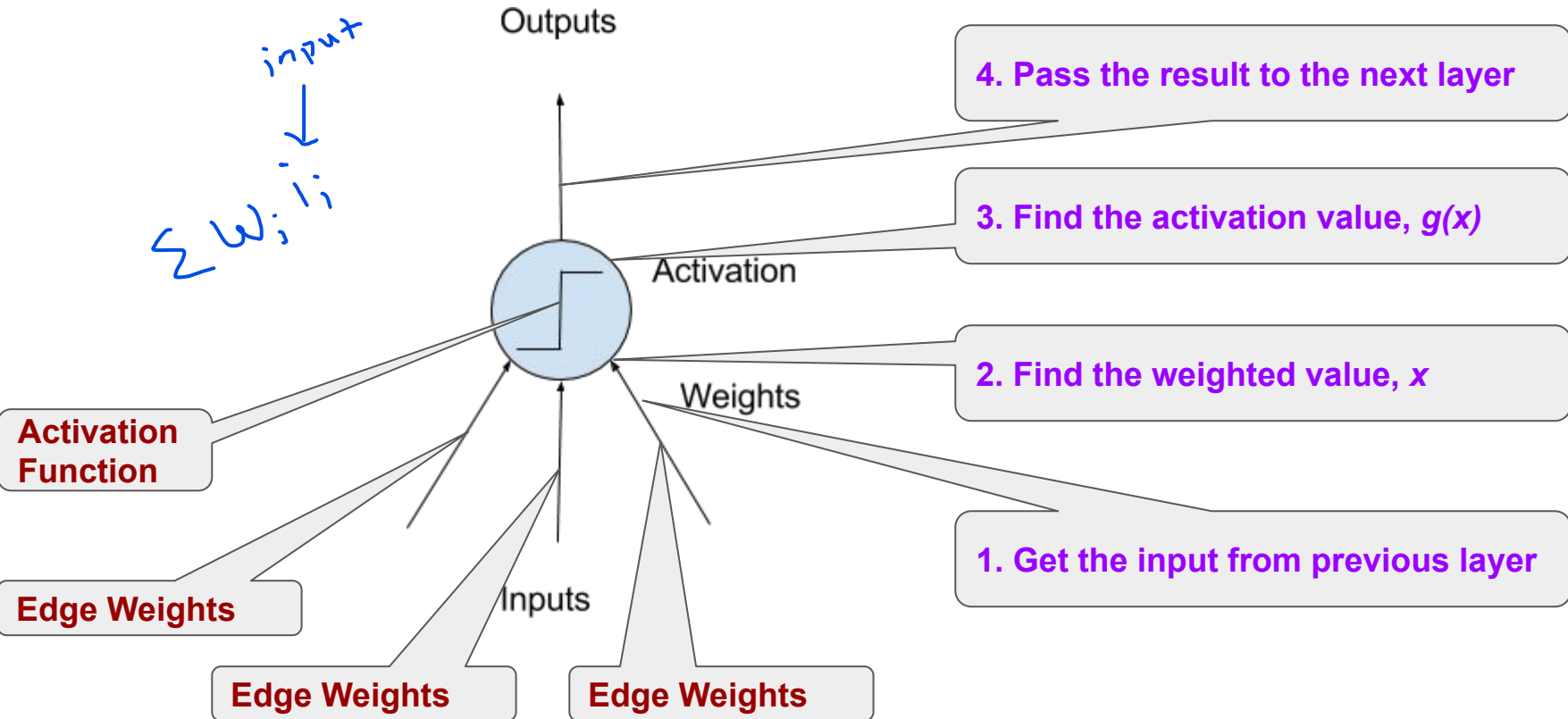
# Multi-layer Perceptron



# Multi-layer Perceptron NN

- The most useful type of neural network
- A perceptron is a single neuron model
- Investigates how simply biological brains can be modeled to solve difficult computational tasks
- Develop robust algorithms and data structures
- Power of the ability to learn the representation in training data
- Neurons are the building block
- Neurons are arranged into network of neurons
- Two layer learning and predicting methodologies:
  -  Feed Forward
  -  Backpropagation
- Activation Function needed on neuron

# Neuron



# Feedforward Algorithm

$$h_1 = \sum_{k=1}^2 i_k * w_{i_k h_1} + \theta_1$$

$$= i_1 * w_{i_1 h_1} + i_2 * w_{i_2 h_1} + \theta_1$$

$x_{h_1} \rightarrow$   $ho_1 = g(h_1) = \frac{1}{1 + e^{-h_1}}$  ← sigmoid f

$$h_2 = \sum_{k=1}^2 i_k * w_{i_k h_2} + \theta_2$$

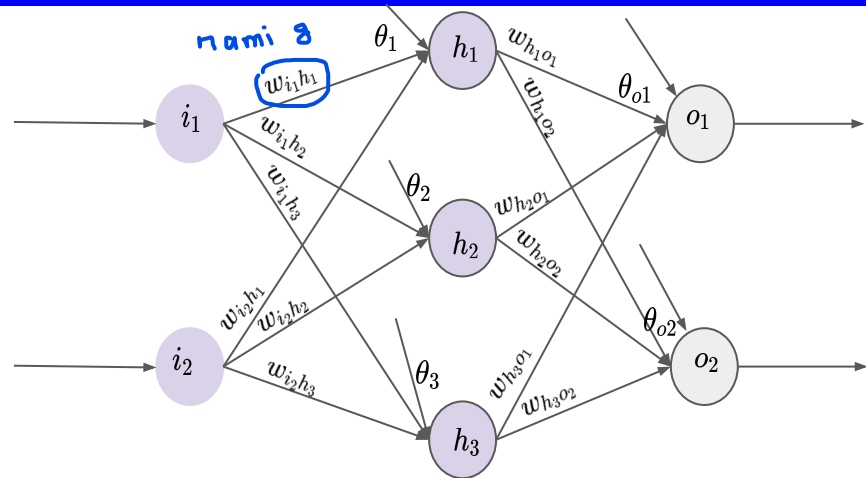
$$= i_1 * w_{i_1 h_2} + i_2 * w_{i_2 h_2} + \theta_2$$

$$ho_2 = g(h_2) = \frac{1}{1 + e^{-h_2}}$$

$$h_3 = \sum_{k=1}^2 i_k * w_{i_k h_3} + \theta_3$$

$$= i_1 * w_{i_1 h_3} + i_2 * w_{i_2 h_3} + \theta_3$$

$$ho_3 = g(h_3) = \frac{1}{1 + e^{-h_3}}$$



$$h_j = \sum_{k=1}^{|i|} i_k * w_{i_k h_j} + \theta_j$$

$$ho_j = g(h_j) = \frac{1}{1 + e^{-h_j}}$$



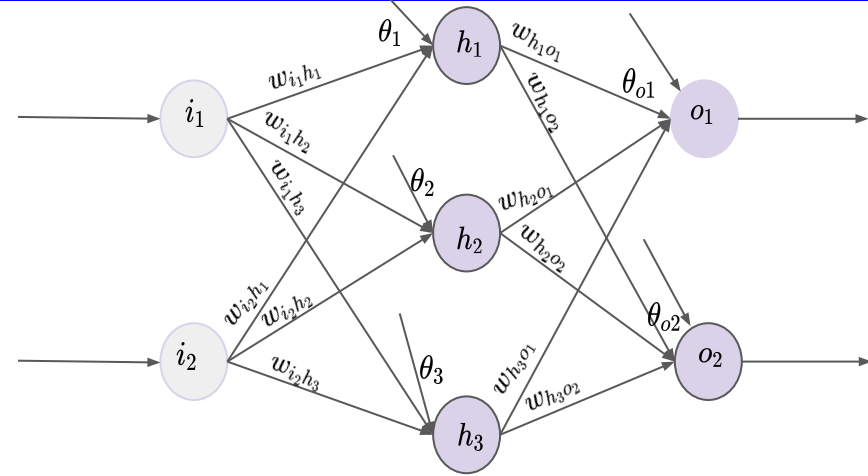
# Feedforward Algorithm

01

$$\begin{aligned}\bar{y}_1 &= \sum_{k=1}^3 h_{o_k} * w_{h_k o_1} + \theta_{o_1} \\ &= h_{o_1} * w_{h_1 o_1} + h_{o_2} * w_{h_2 o_1} + h_{o_3} * w_{h_3 o_1} + \theta_{o_1} \\ y_1 &= g(\bar{y}_1) = \frac{1}{1 + e^{-\bar{y}_1}}\end{aligned}$$

101

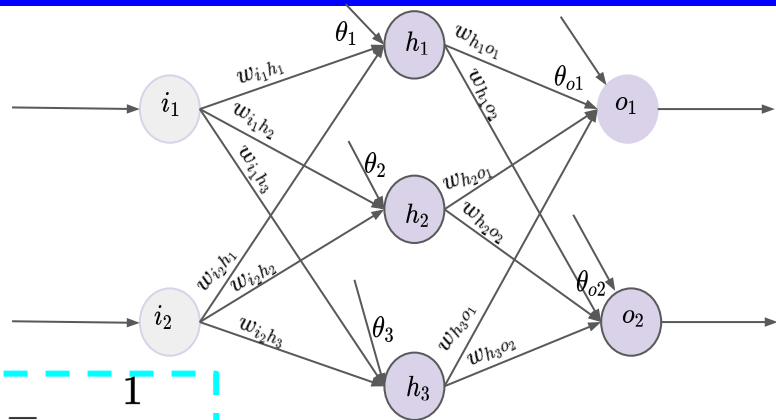
$$\begin{aligned}\bar{y}_2 &= \sum_{k=1}^3 h_{o_k} * w_{h_k o_2} + \theta_{o_2} \\ &= h_{o_1} * w_{h_1 o_2} + h_{o_2} * w_{h_2 o_2} + h_{o_3} * w_{h_3 o_2} + \theta_{o_2} \\ y_2 &= g(\bar{y}_2) = \frac{1}{1 + e^{-\bar{y}_2}}\end{aligned}$$



$$\begin{aligned}\bar{y}_j &= \sum_{k=1}^{|h|} h_k * w_{h_k o_j} + \theta_{o_j} \\ y_j &= g(\bar{y}_j) = \frac{1}{1 + e^{-\bar{y}_j}}\end{aligned}$$

# Backpropagation Algorithm

$$\begin{aligned}
 h_1 &= i_1 * w_{i_1 h_1} + i_2 * w_{i_2 h_1} + \theta_1, \quad \underline{h_{o1} = g(h_1) = \frac{1}{1 + e^{-h_1}}} \\
 h_2 &= i_1 * w_{i_1 h_2} + i_2 * w_{i_2 h_2} + \theta_2, \quad h_{o2} = g(h_2) = \frac{1}{1 + e^{-h_2}} \\
 h_3 &= i_1 * w_{i_1 h_3} + i_2 * w_{i_2 h_3} + \theta_3, \quad h_{o3} = g(h_3) = \frac{1}{1 + e^{-h_3}}
 \end{aligned}$$



$$\begin{aligned}
 \bar{y}_1 &= h_{o1} * w_{h_1 o_1} + h_{o2} * w_{h_2 o_1} + h_{o3} * w_{h_3 o_1} + \theta_{o1}, \quad \underline{y_1 = g(\bar{y}_1) = \frac{1}{1 + e^{-\bar{y}_1}}} \\
 \bar{y}_2 &= h_{o1} * w_{h_1 o_2} + h_{o2} * w_{h_2 o_2} + h_{o3} * w_{h_3 o_2} + \theta_{o2}, \quad y_2 = g(\bar{y}_2) = \frac{1}{1 + e^{-\bar{y}_2}}
 \end{aligned}$$

$$E = \frac{1}{2} \sum_{k=1}^{|Output|} (y_k - y_{ex_k})^2$$

$$\forall w_k, w_{k(new)} = w_{k(old)} - \Delta w_k \quad \Delta w_k = \frac{\partial E}{\partial w_k}$$

gradient descent

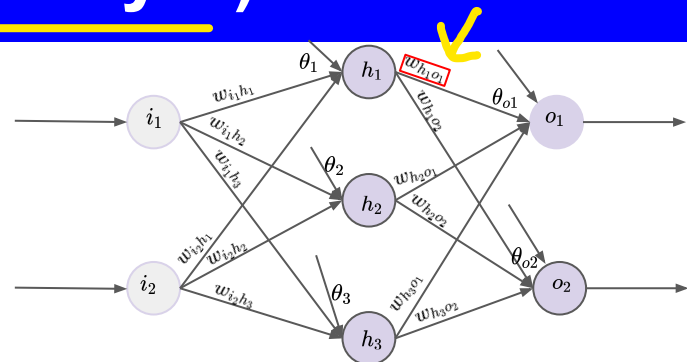
# Backpropagation (Last Layer)

$$\Delta w_{h_1 o_1} = \frac{\partial E}{\partial w_{h_1 o_1}}$$

$$\Delta w_{h_1 o_1} = \frac{\partial E}{\partial y_1} \frac{\partial y_1}{\partial \bar{y}_1} \frac{\partial \bar{y}_1}{\partial w_{h_1 o_1}}$$

✓

$$\begin{aligned} \frac{\partial E}{\partial y_1} &= \frac{\partial}{\partial y_1} \left( \frac{1}{2} \left( (y_1 - y_{ex_1})^2 + (y_2 - y_{ex_2})^2 \right) \right) \\ &= \frac{1}{2} \cdot \frac{\partial}{\partial y_1} \left( (y_1 - y_{ex_1})^2 \right) \\ &= \frac{1}{2} \cdot 2 \cdot (y_1 - y_{ex_1}) \cdot \frac{\partial}{\partial y_1} ((y_1 - y_{ex_1})) \\ &= \frac{1}{2} \cdot 2 \cdot (y_1 - y_{ex_1}) \cdot 1 \\ &= (y_1 - y_{ex_1}) \end{aligned}$$



$$\begin{aligned} h_1 &= i_1 * w_{i_1 h_1} + i_2 * w_{i_2 h_1} + \theta_1, h_{o1} = g(h_1) = \frac{1}{1 + e^{-h_1}} \\ h_2 &= i_1 * w_{i_1 h_2} + i_2 * w_{i_2 h_2} + \theta_2, h_{o2} = g(h_2) = \frac{1}{1 + e^{-h_2}} \\ h_3 &= i_1 * w_{i_1 h_3} + i_2 * w_{i_2 h_3} + \theta_3, h_{o3} = g(h_3) = \frac{1}{1 + e^{-h_3}} \end{aligned}$$

$$\begin{aligned} \bar{y}_1 &= h_{o1} * w_{h_1 o_1} + h_{o2} * w_{h_2 o_1} + h_{o3} * w_{h_3 o_1} + \theta_{o1}, y_1 = g(\bar{y}_1) = \frac{1}{1 + e^{-\bar{y}_1}} \\ \bar{y}_2 &= h_{o1} * w_{h_1 o_2} + h_{o2} * w_{h_2 o_2} + h_{o3} * w_{h_3 o_2} + \theta_{o2}, y_2 = g(\bar{y}_2) = \frac{1}{1 + e^{-\bar{y}_2}} \end{aligned}$$

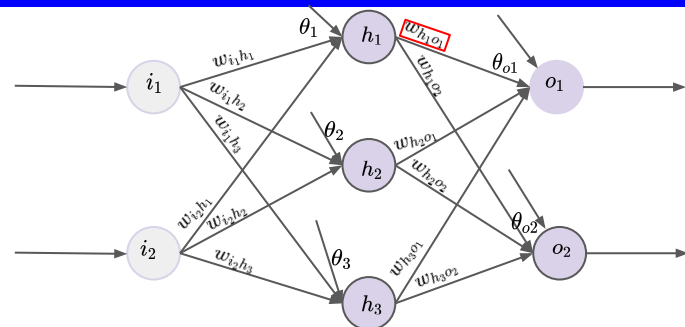
$$E = \frac{1}{2} \sum_{k=1}^{|Output|} (y_k - y_{ex_k})^2$$

# Backpropagation (Last Layer)

$$\Delta w_{h_1 o_1} = \frac{\partial E}{\partial w_{h_1 o_1}}$$

$$\Delta w_{h_1 o_1} = \frac{\partial E}{\partial y_1} \frac{\partial y_1}{\partial \bar{y}_1} \frac{\partial \bar{y}_1}{\partial w_{h_1 o_1}}$$

$$\begin{aligned} \frac{\partial y_1}{\partial \bar{y}_1} &= \frac{\partial}{\partial \bar{y}_1} (g(\bar{y}_1)) \\ &= g(\bar{y}_1)(1 - g(\bar{y}_1)) \\ &= y_1(1 - y_1) \end{aligned}$$



$$\begin{aligned} h_1 &= i_1 * w_{i_1 h_1} + i_2 * w_{i_2 h_1} + \theta_1, h_{o1} = g(h_1) = \frac{1}{1 + e^{-h_1}} \\ h_2 &= i_1 * w_{i_1 h_2} + i_2 * w_{i_2 h_2} + \theta_2, h_{o2} = g(h_2) = \frac{1}{1 + e^{-h_2}} \\ h_3 &= i_1 * w_{i_1 h_3} + i_2 * w_{i_2 h_3} + \theta_3, h_{o3} = g(h_3) = \frac{1}{1 + e^{-h_3}} \end{aligned}$$

$$\bar{y}_1 = h_{o1} * w_{h_1 o_1} + h_{o2} * w_{h_2 o_1} + h_{o3} * w_{h_3 o_1} + \theta_{o1}, y_1 = g(\bar{y}_1) = \frac{1}{1 + e^{-\bar{y}_1}}$$

$$\bar{y}_2 = h_{o1} * w_{h_1 o_2} + h_{o2} * w_{h_2 o_2} + h_{o3} * w_{h_3 o_2} + \theta_{o2}, y_2 = g(\bar{y}_2) = \frac{1}{1 + e^{-\bar{y}_2}}$$

$$E = \frac{1}{2} \sum_{k=1}^{|Output|} (y_k - y_{ex_k})^2$$

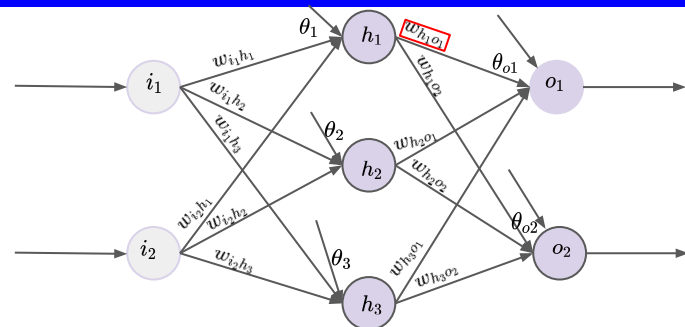
# Backpropagation (Last Layer)

$$\Delta w_{h_1 o_1} = \frac{\partial E}{\partial w_{h_1 o_1}}$$

$$\Delta w_{h_1 o_1} = \frac{\partial E}{\partial y_1} \frac{\partial y_1}{\partial \bar{y}_1} \frac{\partial \bar{y}_1}{\partial w_{h_1 o_1}}$$



$$\begin{aligned} \frac{\partial \bar{y}_1}{\partial w_{h_1 o_1}} &= \frac{\partial}{\partial w_{h_1 o_1}} (h_{o_1} * w_{h_1 o_1} + h_{o_2} * w_{h_2 o_1} + h_{o_3} * w_{h_3 o_1} + \theta_{o_1}) \\ &= h_{o_1} \end{aligned}$$



$$h_1 = i_1 * w_{i_1 h_1} + i_2 * w_{i_2 h_1} + \theta_1, h_{o1} = g(h_1) = \frac{1}{1 + e^{-h_1}}$$

$$h_2 = i_1 * w_{i_1 h_2} + i_2 * w_{i_2 h_2} + \theta_2, h_{o2} = g(h_2) = \frac{1}{1 + e^{-h_2}}$$

$$h_3 = i_1 * w_{i_1 h_3} + i_2 * w_{i_2 h_3} + \theta_3, h_{o3} = g(h_3) = \frac{1}{1 + e^{-h_3}}$$

$$\bar{y}_1 = h_{o1} * w_{h_1 o_1} + h_{o2} * w_{h_2 o_1} + h_{o3} * w_{h_3 o_1} + \theta_{o1}, y_1 = g(\bar{y}_1) = \frac{1}{1 + e^{-\bar{y}_1}}$$

$$\bar{y}_2 = h_{o1} * w_{h_1 o_2} + h_{o2} * w_{h_2 o_2} + h_{o3} * w_{h_3 o_2} + \theta_{o2}, y_2 = g(\bar{y}_2) = \frac{1}{1 + e^{-\bar{y}_2}}$$

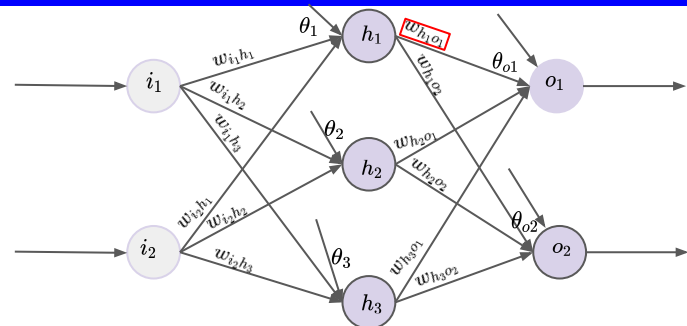
$$E = \frac{1}{2} \sum_{k=1}^{|Output|} (y_k - y_{ex_k})^2$$

# Backpropagation (Last Layer)

$$\Delta w_{h_1 o_1} = \frac{\partial E}{\partial w_{h_1 o_1}}$$

$$\Delta w_{h_1 o_1} = \frac{\partial E}{\partial y_1} \frac{\partial y_1}{\partial \bar{y}_1} \frac{\partial \bar{y}_1}{\partial w_{h_1 o_1}}$$

$$\begin{aligned} \Delta w_{h_1 o_1} &= \frac{\partial E}{\partial y_1} \frac{\partial y_1}{\partial \bar{y}_1} \frac{\partial \bar{y}_1}{\partial w_{h_1 o_1}} \\ &= (y_1 - y_{ex_1}) \cdot y_1 \cdot (1 - y_1) \cdot h_{o1} \end{aligned}$$



$$\begin{aligned} h_1 &= i_1 * w_{i_1 h_1} + i_2 * w_{i_2 h_1} + \theta_1, h_{o1} = g(h_1) = \frac{1}{1 + e^{-h_1}} \\ h_2 &= i_1 * w_{i_1 h_2} + i_2 * w_{i_2 h_2} + \theta_2, h_{o2} = g(h_2) = \frac{1}{1 + e^{-h_2}} \\ h_3 &= i_1 * w_{i_1 h_3} + i_2 * w_{i_2 h_3} + \theta_3, h_{o3} = g(h_3) = \frac{1}{1 + e^{-h_3}} \end{aligned}$$

$$\bar{y}_1 = h_{o1} * w_{h_1 o_1} + h_{o2} * w_{h_2 o_1} + h_{o3} * w_{h_3 o_1} + \theta_{o1}, y_1 = g(\bar{y}_1) = \frac{1}{1 + e^{-\bar{y}_1}}$$

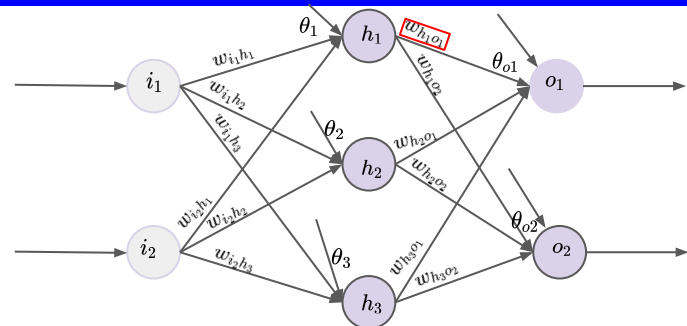
$$\bar{y}_2 = h_{o1} * w_{h_1 o_2} + h_{o2} * w_{h_2 o_2} + h_{o3} * w_{h_3 o_2} + \theta_{o2}, y_2 = g(\bar{y}_2) = \frac{1}{1 + e^{-\bar{y}_2}}$$

$$E = \frac{1}{2} \sum_{k=1}^{|Output|} (y_k - y_{ex_k})^2$$

# Backpropagation (Last Layer)

$$\begin{aligned}\Delta w_{h_1 o_1} &= (y_1 - y_{ex_1}) \cdot y_1 \cdot (1 - y_1) \cdot h_{o1} \\ \Delta w_{h_2 o_1} &= (y_1 - y_{ex_1}) \cdot y_1 \cdot (1 - y_1) \cdot h_{o2} \\ \Delta w_{h_3 o_1} &= (y_1 - y_{ex_1}) \cdot y_1 \cdot (1 - y_1) \cdot h_{o3} \\ \Delta w_{h_1 o_2} &= (y_2 - y_{ex_2}) \cdot y_2 \cdot (1 - y_2) \cdot h_{o1} \\ \Delta w_{h_2 o_2} &= (y_2 - y_{ex_2}) \cdot y_2 \cdot (1 - y_2) \cdot h_{o2} \\ \Delta w_{h_3 o_2} &= (y_2 - y_{ex_2}) \cdot y_2 \cdot (1 - y_2) \cdot h_{o3}\end{aligned}$$

Do Yourself ...



$$h_1 = i_1 * w_{i_1 h_1} + i_2 * w_{i_2 h_1} + \theta_1, h_{o1} = g(h_1) = \frac{1}{1 + e^{-h_1}}$$

$$h_2 = i_1 * w_{i_1 h_2} + i_2 * w_{i_2 h_2} + \theta_2, h_{o2} = g(h_2) = \frac{1}{1 + e^{-h_2}}$$

$$h_3 = i_1 * w_{i_1 h_3} + i_2 * w_{i_2 h_3} + \theta_3, h_{o3} = g(h_3) = \frac{1}{1 + e^{-h_3}}$$

$$\bar{y}_1 = h_{o1} * w_{h_1 o_1} + h_{o2} * w_{h_2 o_1} + h_{o3} * w_{h_3 o_1} + \theta_{o1}, y_1 = g(\bar{y}_1) = \frac{1}{1 + e^{-\bar{y}_1}}$$

$$\bar{y}_2 = h_{o1} * w_{h_1 o_2} + h_{o2} * w_{h_2 o_2} + h_{o3} * w_{h_3 o_2} + \theta_{o2}, y_2 = g(\bar{y}_2) = \frac{1}{1 + e^{-\bar{y}_2}}$$

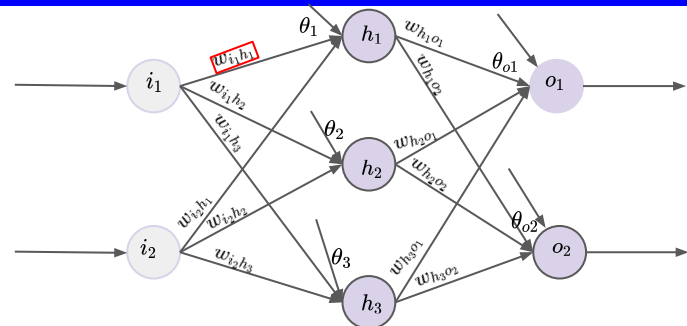
$$E = \frac{1}{2} \sum_{k=1}^{|Output|} (y_k - y_{ex_k})^2$$

# Backpropagation (Internal Layer)

$$\Delta w_{i_1 h_1} = \frac{\partial E}{\partial w_{i_1 h_1}}$$

$$\Delta w_{i_1 h_1} = \frac{\partial E}{\partial h_{o1}} \frac{\partial h_{o1}}{\partial h_1} \frac{\partial h_1}{\partial w_{i_1 h_1}}$$

$$\begin{aligned} \frac{\partial h_1}{\partial w_{i_1 h_1}} &= \frac{\partial}{\partial w_{i_1 h_1}} (i_1 * w_{i_1 h_1} + i_2 * w_{i_2 h_1} + \theta_1) \\ &= i_1 \end{aligned}$$



$$h_1 = i_1 * w_{i_1 h_1} + i_2 * w_{i_2 h_1} + \theta_1, h_{o1} = g(h_1) = \frac{1}{1 + e^{-h_1}}$$

$$h_2 = i_1 * w_{i_1 h_2} + i_2 * w_{i_2 h_2} + \theta_2, h_{o2} = g(h_2) = \frac{1}{1 + e^{-h_2}}$$

$$h_3 = i_1 * w_{i_1 h_3} + i_2 * w_{i_2 h_3} + \theta_3, h_{o3} = g(h_3) = \frac{1}{1 + e^{-h_3}}$$

$$\bar{y}_1 = h_{o1} * w_{h_1 o_1} + h_{o2} * w_{h_2 o_1} + h_{o3} * w_{h_3 o_1} + \theta_{o1}, y_1 = g(\bar{y}_1) = \frac{1}{1 + e^{-\bar{y}_1}}$$

$$\bar{y}_2 = h_{o1} * w_{h_1 o_2} + h_{o2} * w_{h_2 o_2} + h_{o3} * w_{h_3 o_2} + \theta_{o2}, y_2 = g(\bar{y}_2) = \frac{1}{1 + e^{-\bar{y}_2}}$$

$$E = \frac{1}{2} \sum_{k=1}^{|Output|} (y_k - y_{ex_k})^2$$

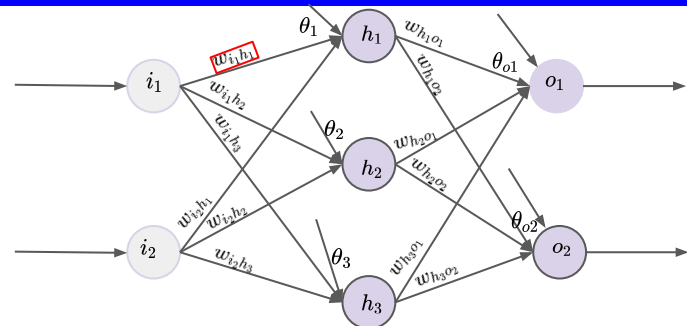


# Backpropagation (Internal Layer)

$$\Delta w_{i_1 h_1} = \frac{\partial E}{\partial w_{i_1 h_1}}$$

$$\Delta w_{i_1 h_1} = \frac{\partial E}{\partial h_{o1}} \frac{\partial h_{o1}}{\partial h_1} \frac{\partial h_1}{\partial w_{i_1 h_1}}$$

$$\begin{aligned} \frac{\partial h_{o1}}{\partial h_1} &= \frac{\partial}{\partial h_1} g(h_1) \\ &= g(h_1) \cdot (1 - g(h_1)) \\ &= h_{o1} \cdot (1 - h_{o1}) \end{aligned}$$



$$\begin{aligned} h_1 &= i_1 * w_{i_1 h_1} + i_2 * w_{i_2 h_1} + \theta_1, h_{o1} = g(h_1) = \frac{1}{1 + e^{-h_1}} \\ h_2 &= i_1 * w_{i_1 h_2} + i_2 * w_{i_2 h_2} + \theta_2, h_{o2} = g(h_2) = \frac{1}{1 + e^{-h_2}} \\ h_3 &= i_1 * w_{i_1 h_3} + i_2 * w_{i_2 h_3} + \theta_3, h_{o3} = g(h_3) = \frac{1}{1 + e^{-h_3}} \end{aligned}$$

$$\begin{aligned} \bar{y}_1 &= h_{o1} * w_{h_1 o_1} + h_{o2} * w_{h_2 o_1} + h_{o3} * w_{h_3 o_1} + \theta_{o1}, y_1 = g(\bar{y}_1) = \frac{1}{1 + e^{-\bar{y}_1}} \\ \bar{y}_2 &= h_{o1} * w_{h_1 o_2} + h_{o2} * w_{h_2 o_2} + h_{o3} * w_{h_3 o_2} + \theta_{o2}, y_2 = g(\bar{y}_2) = \frac{1}{1 + e^{-\bar{y}_2}} \end{aligned}$$

$$E = \frac{1}{2} \sum_{k=1}^{|Output|} (y_k - y_{ex_k})^2$$

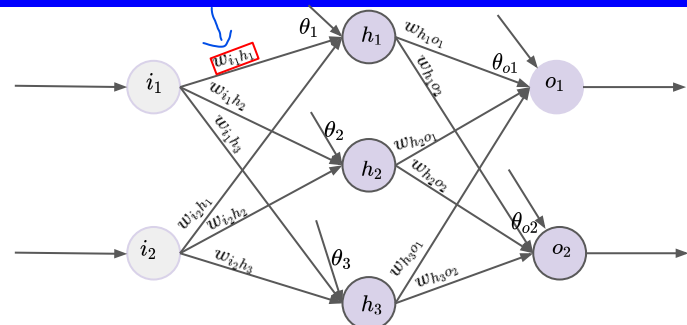
# Backpropagation (Internal Layer)

$$\Delta w_{i_1 h_1} = \frac{\partial E}{\partial w_{i_1 h_1}}$$

$$\Delta w_{i_1 h_1} = \frac{\partial E}{\partial h_{o_1}} \frac{\partial h_{o_1}}{\partial h_1} \frac{\partial h_1}{\partial w_{i_1 h_1}}$$

$$\frac{\partial E}{\partial h_{o_1}} = \frac{\partial}{\partial h_{o_1}} \left( \frac{1}{2} (y_1 - y_{ex_1})^2 \right) + \frac{\partial}{\partial h_{o_1}} \left( \frac{1}{2} (y_2 - y_{ex_2})^2 \right)$$

$$\begin{aligned} \frac{\partial}{\partial h_{o_1}} \left( \frac{1}{2} (y_1 - y_{ex_1})^2 \right) &= \frac{1}{2} \cdot 2 \cdot (y_1 - y_{ex_1}) \cdot \frac{\partial}{\partial h_{o_1}} (y_1 - y_{ex_1}) \\ &= (y_1 - y_{ex_1}) \cdot \frac{\partial}{\partial h_{o_1}} (y_1) \\ &= (y_1 - y_{ex_1}) \cdot w_{h_1 o_1} \end{aligned}$$



$$h_1 = i_1 * w_{i_1 h_1} + i_2 * w_{i_2 h_1} + \theta_1, h_{o1} = g(h_1) = \frac{1}{1 + e^{-h_1}}$$

$$h_2 = i_1 * w_{i_1 h_2} + i_2 * w_{i_2 h_2} + \theta_2, h_{o2} = g(h_2) = \frac{1}{1 + e^{-h_2}}$$

$$h_3 = i_1 * w_{i_1 h_3} + i_2 * w_{i_2 h_3} + \theta_3, h_{o3} = g(h_3) = \frac{1}{1 + e^{-h_3}}$$

$$\bar{y}_1 = h_{o1} * w_{h_1 o_1} + h_{o2} * w_{h_2 o_1} + h_{o3} * w_{h_3 o_1} + \theta_{o1}, y_1 = g(\bar{y}_1) = \frac{1}{1 + e^{-\bar{y}_1}}$$

$$\bar{y}_2 = h_{o1} * w_{h_1 o_2} + h_{o2} * w_{h_2 o_2} + h_{o3} * w_{h_3 o_2} + \theta_{o2}, y_2 = g(\bar{y}_2) = \frac{1}{1 + e^{-\bar{y}_2}}$$

$$E = \frac{1}{2} \sum_{k=1}^{|Output|} (y_k - y_{ex_k})^2$$

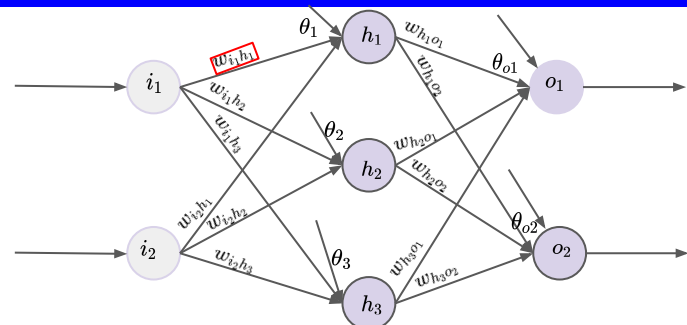
# Backpropagation (Internal Layer)

$$\Delta w_{i_1 h_1} = \frac{\partial E}{\partial w_{i_1 h_1}}$$

$$\Delta w_{i_1 h_1} = \frac{\partial E}{\partial h_{o_1}} \frac{\partial h_{o_1}}{\partial h_1} \frac{\partial h_1}{\partial w_{i_1 h_1}}$$

$$\frac{\partial E}{\partial h_{o_1}} = \frac{\partial}{\partial h_{o_1}} \left( \frac{1}{2} (y_1 - y_{ex_1})^2 \right) + \frac{\partial}{\partial h_{o_1}} \left( \frac{1}{2} (y_2 - y_{ex_2})^2 \right)$$

$$\begin{aligned} \frac{\partial}{\partial h_{o_1}} \left( \frac{1}{2} (y_2 - y_{ex_2})^2 \right) &= \frac{1}{2} \cdot 2 \cdot (y_2 - y_{ex_2}) \cdot \frac{\partial}{\partial h_{o_1}} (y_2 - y_{ex_2}) \\ &= (y_2 - y_{ex_2}) \cdot \frac{\partial}{\partial h_{o_1}} (y_2) \\ &= (y_2 - y_{ex_2}) \cdot w_{h_1 o_2} \end{aligned}$$



$$h_1 = i_1 * w_{i_1 h_1} + i_2 * w_{i_2 h_1} + \theta_1, h_{o1} = g(h_1) = \frac{1}{1 + e^{-h_1}}$$

$$h_2 = i_1 * w_{i_1 h_2} + i_2 * w_{i_2 h_2} + \theta_2, h_{o2} = g(h_2) = \frac{1}{1 + e^{-h_2}}$$

$$h_3 = i_1 * w_{i_1 h_3} + i_2 * w_{i_2 h_3} + \theta_3, h_{o3} = g(h_3) = \frac{1}{1 + e^{-h_3}}$$

$$\bar{y}_1 = h_{o1} * w_{h_1 o_1} + h_{o2} * w_{h_2 o_1} + h_{o3} * w_{h_3 o_1} + \theta_{o1}, y_1 = g(\bar{y}_1) = \frac{1}{1 + e^{-\bar{y}_1}}$$

$$\bar{y}_2 = h_{o1} * w_{h_1 o_2} + h_{o2} * w_{h_2 o_2} + h_{o3} * w_{h_3 o_2} + \theta_{o2}, y_2 = g(\bar{y}_2) = \frac{1}{1 + e^{-\bar{y}_2}}$$

$$E = \frac{1}{2} \sum_{k=1}^{|Output|} (y_k - y_{ex_k})^2$$

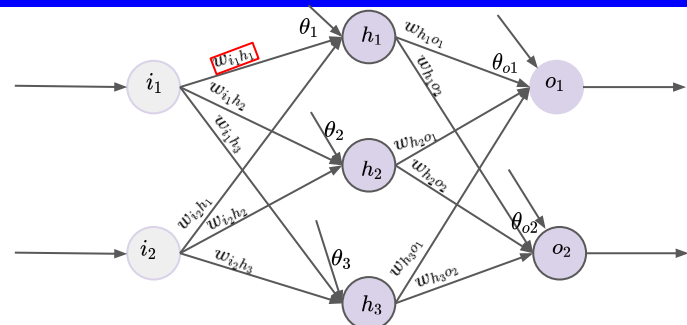
# Backpropagation (Internal Layer)

$$\Delta w_{i_1 h_1} = \frac{\partial E}{\partial w_{i_1 h_1}}$$

$$\Delta w_{i_1 h_1} = \frac{\partial E}{\partial h_{o_1}} \frac{\partial h_{o_1}}{\partial h_1} \frac{\partial h_1}{\partial w_{i_1 h_1}}$$

$$\frac{\partial E}{\partial h_{o_1}} = \frac{\partial}{\partial h_{o_1}} \left( \frac{1}{2} (y_1 - y_{ex_1})^2 \right) + \frac{\partial}{\partial h_{o_1}} \left( \frac{1}{2} (y_2 - y_{ex_2})^2 \right)$$

$$\begin{aligned} \frac{\partial E}{\partial h_{o_1}} &= \frac{\partial}{\partial h_{o_1}} \left( \frac{1}{2} (y_1 - y_{ex_1})^2 \right) + \frac{\partial}{\partial h_{o_1}} \left( \frac{1}{2} (y_2 - y_{ex_2})^2 \right) \\ &= (y_1 - y_{ex_1}) \cdot w_{h_1 o_1} + (y_2 - y_{ex_2}) \cdot w_{h_1 o_2} \\ &= \sum_{k=1}^{|Output|} w_{h_1 o_k} \cdot (y_k - y_{ex_k}) \end{aligned}$$



$$h_1 = i_1 * w_{i_1 h_1} + i_2 * w_{i_2 h_1} + \theta_1, h_{o1} = g(h_1) = \frac{1}{1 + e^{-h_1}}$$

$$h_2 = i_1 * w_{i_1 h_2} + i_2 * w_{i_2 h_2} + \theta_2, h_{o2} = g(h_2) = \frac{1}{1 + e^{-h_2}}$$

$$h_3 = i_1 * w_{i_1 h_3} + i_2 * w_{i_2 h_3} + \theta_3, h_{o3} = g(h_3) = \frac{1}{1 + e^{-h_3}}$$

$$\bar{y}_1 = h_{o1} * w_{h_1 o_1} + h_{o2} * w_{h_2 o_1} + h_{o3} * w_{h_3 o_1} + \theta_{o1}, y_1 = g(\bar{y}_1) = \frac{1}{1 + e^{-\bar{y}_1}}$$

$$\bar{y}_2 = h_{o1} * w_{h_1 o_2} + h_{o2} * w_{h_2 o_2} + h_{o3} * w_{h_3 o_2} + \theta_{o2}, y_2 = g(\bar{y}_2) = \frac{1}{1 + e^{-\bar{y}_2}}$$

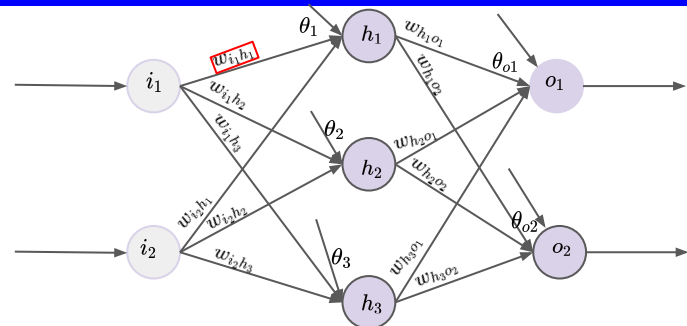
$$E = \frac{1}{2} \sum_{k=1}^{|Output|} (y_k - y_{ex_k})^2$$

# Backpropagation (Internal Layer)

$$\Delta w_{i_1 h_1} = \frac{\partial E}{\partial w_{i_1 h_1}}$$

$$\Delta w_{i_1 h_1} = \frac{\partial E}{\partial h_{o_1}} \frac{\partial h_{o_1}}{\partial h_1} \frac{\partial h_1}{\partial w_{i_1 h_1}}$$

$$\begin{aligned} \Delta w_{i_1 h_1} &= \frac{\partial E}{\partial h_{o_1}} \frac{\partial h_{o_1}}{\partial h_1} \frac{\partial h_1}{\partial w_{i_1 h_1}} \\ &= \left( \sum_{k=1}^{|Output|} w_{h_1 o_k} \cdot (y_k - y_{ex_k}) \right) \cdot h_{o_1} \cdot (1 - h_{o_1}) \cdot i_1 \\ &= i_1 \cdot h_{o_1} \cdot (1 - h_{o_1}) \cdot \left( \sum_{k=1}^{|Output|} w_{h_1 o_k} \cdot (y_k - y_{ex_k}) \right) \end{aligned}$$



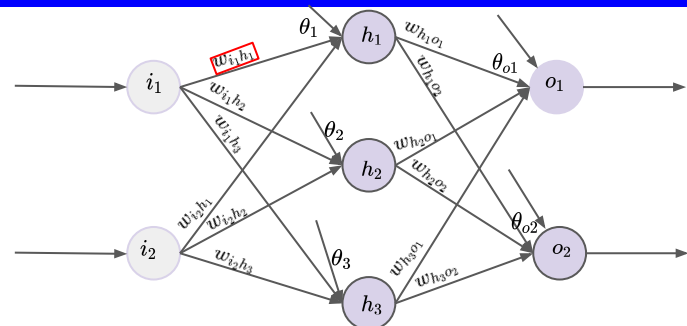
$$\begin{aligned} h_1 &= i_1 * w_{i_1 h_1} + i_2 * w_{i_2 h_1} + \theta_1, h_{o1} = g(h_1) = \frac{1}{1 + e^{-h_1}} \\ h_2 &= i_1 * w_{i_1 h_2} + i_2 * w_{i_2 h_2} + \theta_2, h_{o2} = g(h_2) = \frac{1}{1 + e^{-h_2}} \\ h_3 &= i_1 * w_{i_1 h_3} + i_2 * w_{i_2 h_3} + \theta_3, h_{o3} = g(h_3) = \frac{1}{1 + e^{-h_3}} \\ \bar{y}_1 &= h_{o1} * w_{h_1 o_1} + h_{o2} * w_{h_2 o_1} + h_{o3} * w_{h_3 o_1} + \theta_{o1}, y_1 = g(\bar{y}_1) = \frac{1}{1 + e^{-\bar{y}_1}} \\ \bar{y}_2 &= h_{o1} * w_{h_1 o_2} + h_{o2} * w_{h_2 o_2} + h_{o3} * w_{h_3 o_2} + \theta_{o2}, y_2 = g(\bar{y}_2) = \frac{1}{1 + e^{-\bar{y}_2}} \end{aligned}$$

$$E = \frac{1}{2} \sum_{k=1}^{|Output|} (y_k - y_{ex_k})^2$$

# Backpropagation (Internal Layer)

$$\begin{aligned}\Delta w_{i_1 h_1} &= i_1 \cdot h_{o1} \cdot (1 - h_{o1}) \cdot \left( \sum_{k=1}^{|Output|} w_{h_1 o_k} \cdot (y_k - y_{ex_k}) \right) \\ \Delta w_{i_1 h_2} &= i_1 \cdot h_{o2} \cdot (1 - h_{o2}) \cdot \left( \sum_{k=1}^{|Output|} w_{h_2 o_k} \cdot (y_k - y_{ex_k}) \right) \\ \Delta w_{i_1 h_3} &= i_1 \cdot h_{o3} \cdot (1 - h_{o3}) \cdot \left( \sum_{k=1}^{|Output|} w_{h_3 o_k} \cdot (y_k - y_{ex_k}) \right) \\ \Delta w_{i_2 h_1} &= i_2 \cdot h_{o1} \cdot (1 - h_{o1}) \cdot \left( \sum_{k=1}^{|Output|} w_{h_1 o_k} \cdot (y_k - y_{ex_k}) \right) \\ \Delta w_{i_2 h_2} &= i_2 \cdot h_{o2} \cdot (1 - h_{o2}) \cdot \left( \sum_{k=1}^{|Output|} w_{h_2 o_k} \cdot (y_k - y_{ex_k}) \right) \\ \Delta w_{i_2 h_3} &= i_2 \cdot h_{o3} \cdot (1 - h_{o3}) \cdot \left( \sum_{k=1}^{|Output|} w_{h_3 o_k} \cdot (y_k - y_{ex_k}) \right)\end{aligned}$$

Do Yourself ...



$$\begin{aligned}h_1 &= i_1 * w_{i_1 h_1} + i_2 * w_{i_2 h_1} + \theta_1, h_{o1} = g(h_1) = \frac{1}{1 + e^{-h_1}} \\ h_2 &= i_1 * w_{i_1 h_2} + i_2 * w_{i_2 h_2} + \theta_2, h_{o2} = g(h_2) = \frac{1}{1 + e^{-h_2}} \\ h_3 &= i_1 * w_{i_1 h_3} + i_2 * w_{i_2 h_3} + \theta_3, h_{o3} = g(h_3) = \frac{1}{1 + e^{-h_3}} \\ \bar{y}_1 &= h_{o1} * w_{h_1 o_1} + h_{o2} * w_{h_2 o_1} + h_{o3} * w_{h_3 o_1} + \theta_{o1}, y_1 = g(\bar{y}_1) = \frac{1}{1 + e^{-\bar{y}_1}} \\ \bar{y}_2 &= h_{o1} * w_{h_1 o_2} + h_{o2} * w_{h_2 o_2} + h_{o3} * w_{h_3 o_2} + \theta_{o2}, y_2 = g(\bar{y}_2) = \frac{1}{1 + e^{-\bar{y}_2}}\end{aligned}$$

$$E = \frac{1}{2} \sum_{k=1}^{|Output|} (y_k - y_{ex_k})^2$$

*Thank You*

# References

- <http://deeplearning.stanford.edu/tutorial/supervised/MultiLayerNeuralNetworks/>
- <https://www.investopedia.com/terms/n/neuralnetwork.asp#:~:text=A%20neural%20network%20is%20a,organic%20or%20artificial%20in%20nature.>
- <https://wiki.pathmind.com/neural-network>
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- [https://scikit-learn.org/stable/modules/neural\\_networks\\_supervised.html](https://scikit-learn.org/stable/modules/neural_networks_supervised.html)
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