Linear Regression: Example Simulation

Example

x1 (length)	x2 (weight)		y (price)
2	11		70
5	9		85
3	7		80
7	5		100

Assumptions

x1 (length)	x2 (weight)		y (price)
2	11		70
5	9		85
3	7		80
7	5		100

Regression Equation: $y = \omega_0 + \omega_1 x_1 + \omega_2 x_2$

Error Function: $e = \frac{1}{2} \sum_{i=1}^{m} \left((\omega_0 + \omega_1 x_1(i) + \omega_2 x_2(i)) - y(i) \right)^2$

Weight Update Equations:

$$\omega_0(new) = \omega_0(old) - \alpha \frac{\delta e}{\delta \omega_0}$$

$$\omega_1(new) = \omega_1(old) - \alpha \frac{\delta e}{\delta \omega_1}$$

$$\omega_2(new) = \omega_2(old) - \alpha \frac{\delta e}{\delta \omega_2}$$

Simulation

x1 (length)	x2 (weight)		y (price)
2	11		70
5	9		85
3	7		80
7	5		100

Initial Values: Assume,

$$\omega_0 = 0.5$$
$$\omega_1 = 0.5$$
$$\omega_2 = 0.5$$
$$\alpha = 0.25$$

Slope Equations:

$$\frac{\delta e}{\delta \omega_0} = \sum_{i=1}^{4}((\omega_0 + \omega_1 x_1(i) + \omega_2 x_2(i)) - y(i))$$

$$\frac{\delta e}{\delta \omega_1} = \sum_{i=1}^{4}((\omega_0 + \omega_1 x_1(i) + \omega_2 x_2(i)) - y(i)).\, x_1(i)$$

$$\frac{\delta e}{\delta \omega_2} = \sum_{i=1}^{4}((\omega_0 + \omega_1 x_1(i) + \omega_2 x_2(i)) - y(i)).\, x_2(i)$$

Simulation: Iteration 1

Data Sample	Predicted Value (p)	Expected Value (y)	(p-y)	(p-y)*x1	(p-y)*x2
1	7	70	-63	-126	-693
2	7.5	85	-77.5	-387.5	-697.5
3	5.5	80	-74.5	-223.5	-521.5
4	6.5	100	-93.5	-654.5	-467.5
Total Sum			-308.5	-1391.5	-2379.5

x1 (length)	x2 (weight)		y (price)
2	11		70
5	9		85
3	7		80
7	5		100

$$\omega_0 = 0.5$$
$$\omega_1 = 0.5$$
$$\omega_2 = 0.5$$
$$\alpha = 0.25$$

$$\omega_0(new) = 0.5 - 0.25 \times (-308.5) = 77.625$$
$$\omega_1(new) = 0.5 - 0.25 \times (-1391.5) = 348.375$$
$$\omega_2(new) = 0.5 - 0.25 \times (-2379.5) = 595.375$$

Simulation: Iteration 2

Data Sample	Predicted Value (p)	Expected Value (y)	(p-y)	(p-y)*x1	(p-y)*x2
1	7323.5	70	7253.5	14507	79788.5
2	7177.875	85	7092.875	35464.375	63835.875
3	5290.375	80	5210.375	15631.125	36472.625
4	5493.125	100	5393.125	37751.875	26965.625
Total Sum			24949.875	103354.375	207062.625

x1 (length)	x2 (weight)		y (price)
2	11		70
5	9		85
3	7		80
7	5		100

$$\omega_0 = 77.625$$
$$\omega_1 = 348.375$$
$$\omega_2 = 595.375$$
$$\alpha = 0.25$$

$$\omega_0(new) = 77.625 - 0.25 \times (24949.875) = -6159.844$$
$$\omega_1(new) = 348.375 - 0.25 \times (103354.375) = -25490.219$$
$$\omega_2(new) = 595.375 - 0.25 \times (207062.625) = -51170.281$$

Simulation: Iteration 3

Data Sample	Predicted Value (p)	Expected Value (y)	(p-y)	(p-y)*x1	(p-y)*x2
1	-620013.373	70	-620083.373	-1240166.746	-6820917.103
2	-594143.468	85	-594228.468	-2971142.34	-5348056.212
3	-440822.468	80	-440902.468	-1322707.404	-3086317.276
4	-440442.782	100	-440542.782	-3083799.474	-2202713.91
Total Sum			-2095757.091	-8617815.964	-17458004.5

x1 (length)	x2 (weight)		y (price)
2	11		70
5	9		85
3	7		80
7	5		100

$$\omega_0 = -6159.844$$
$$\omega_1 = -25490.219$$
$$\omega_2 = -51170.281$$
$$\alpha = 0.25$$

$$\omega_0(new) = -6159.844 - 0.25 \times (-2095757.091) = 517779.429$$
$$\omega_1(new) = -25490.219 - 0.25 \times (-8617815.964) = 2128963.772$$
$$\omega_2(new) = -51170.281 - 0.25 \times (-174580004.5) = 4313330.844$$

Simulation: Iteration …….

Data Sample	Predicted Value (p)	Expected Value (y)	(p-y)	(p-y)*x1	(p-y)*x2
1					
2					
3					
4					
Total Sum					

x1 (length)	x2 (weight)		y (price)
2	11		70
5	9		85
3	7		80
7	5		100

$$\omega_0 = 517779.429$$
$$\omega_1 = 2128963.772$$
$$\omega_2 = 4313330.844$$
$$\alpha = 0.25$$

Continue to get your expected result !!!

$$\omega_0(new) = \quad ???$$
$$\omega_1(new) = \quad ???$$
$$\omega_2(new) = \quad ???$$

Linear Regression:
Example Simulation Using Linear Algebra

Simulation

x1 (length)	x2 (weight)		y (price)
2	11		70
5	9		85
3	7		80
7	5		100

$$X = \begin{bmatrix} 1 & 2 & 11 \\ 1 & 5 & 9 \\ 1 & 3 & 7 \\ 1 & 7 & 5 \end{bmatrix} \qquad Y = \begin{bmatrix} 70 \\ 85 \\ 80 \\ 100 \end{bmatrix} \qquad X^T = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 5 & 3 & 7 \\ 11 & 9 & 7 & 5 \end{bmatrix}$$

$$W = \left(X^T X\right)^{-1} X^T Y$$

Simulation

$$
\begin{aligned}
W &= \left(\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 5 & 3 & 7 \\ 11 & 9 & 7 & 5 \end{bmatrix} \begin{bmatrix} 1 & 2 & 11 \\ 1 & 5 & 9 \\ 1 & 3 & 7 \\ 1 & 7 & 5 \end{bmatrix} \right)^{-1} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 5 & 3 & 7 \\ 11 & 9 & 7 & 5 \end{bmatrix} \begin{bmatrix} 70 \\ 85 \\ 80 \\ 100 \end{bmatrix} \\
&= \left(\begin{bmatrix} 4 & 17 & 32 \\ 17 & 87 & 123 \\ 32 & 123 & 276 \end{bmatrix} \right)^{-1} \begin{bmatrix} 335 \\ 1505 \\ 2595 \end{bmatrix} \\
&= \begin{bmatrix} 17.625 & -1.5 & -1.375 \\ -1.5 & 0.15873 & 0.10317 \\ -1.375 & 0.10317 & 0.11706 \end{bmatrix} \begin{bmatrix} 335 \\ 1505 \\ 2595 \end{bmatrix} \\
&= \begin{bmatrix} 78.75 \\ 4.1148 \\ -1.58345 \end{bmatrix}
\end{aligned}
$$

Thank You