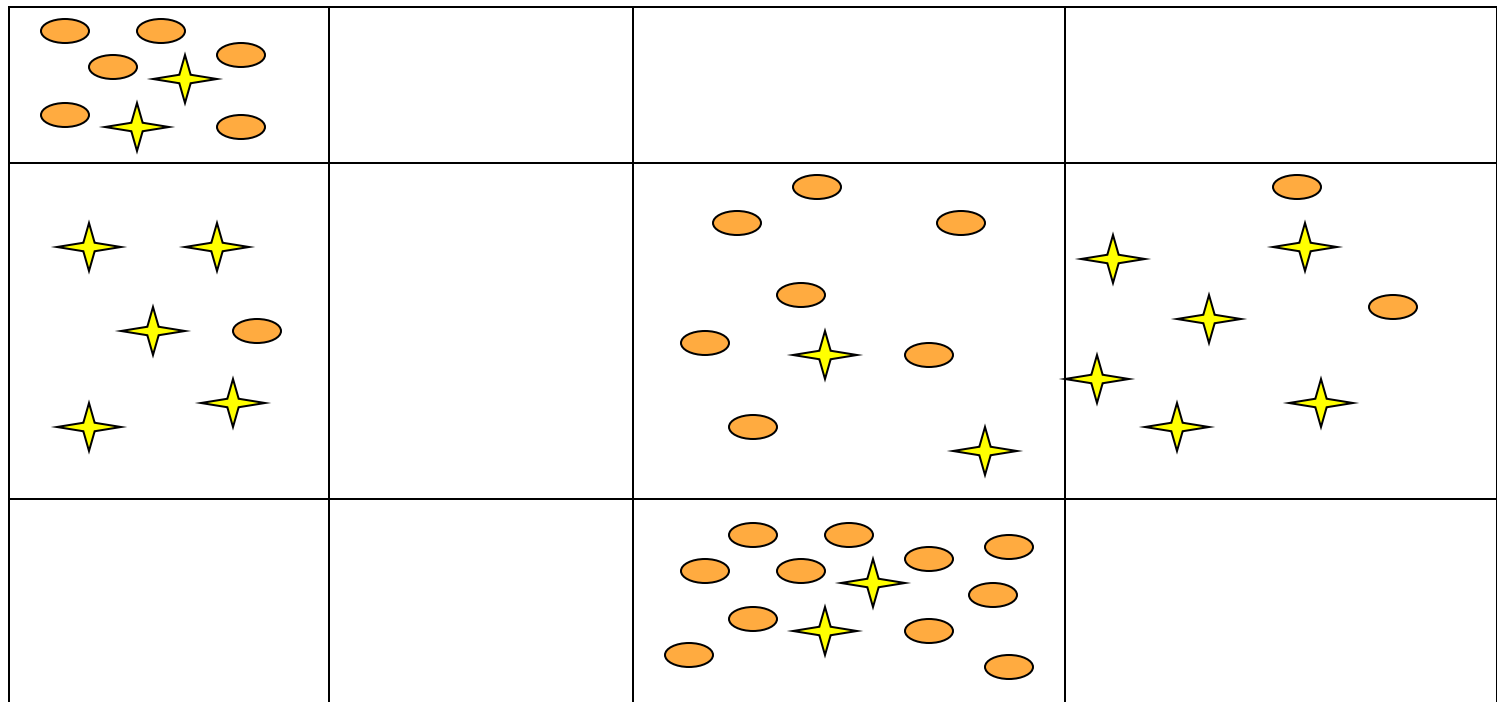


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# Basic Concepts of Classifications and Decision Trees

# An Example Problem



# Classification: Definition

- Given a collection of records (*training set*)
  - Each record contains a set of *attributes*, one of the attributes is the *class*.
- Find a *model* for class attribute as a function of the values of other attributes.
- **Goal:** previously unseen records should be assigned a class as accurately as possible.
  - A *test set* is used to determine the accuracy of the model. Usually, the given data set is divided into training and test sets, with training set used to build the model and test set used to validate it.

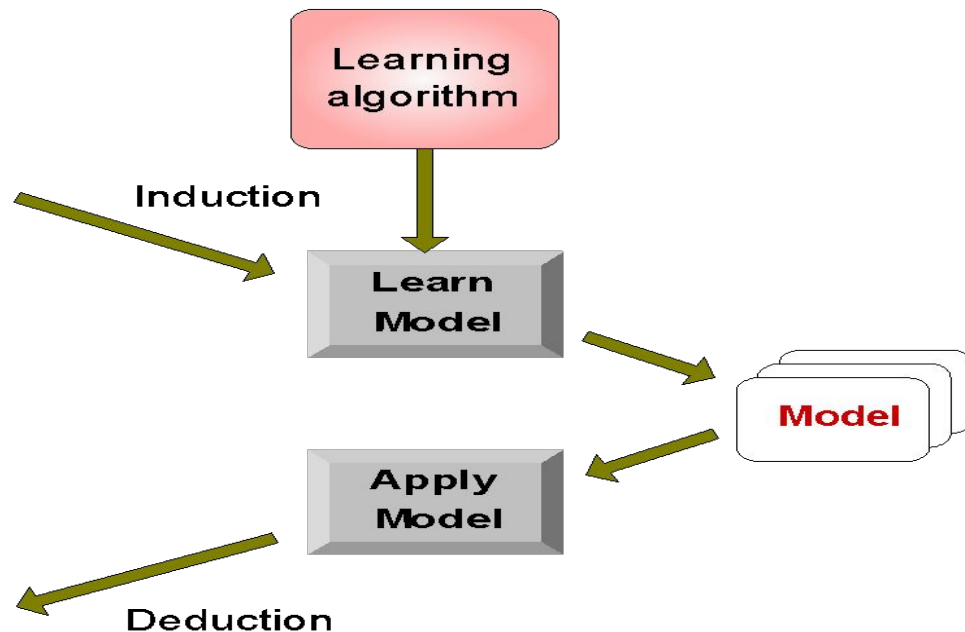
# Illustration of Classification Task

Tid	Attrib1	Attrib2	Attrib3	Class
1	Yes	Large	125K	No
2	No	Medium	100K	No
3	No	Small	70K	No
4	Yes	Medium	120K	No
5	No	Large	95K	Yes
6	No	Medium	60K	No
7	Yes	Large	220K	No
8	No	Small	85K	Yes
9	No	Medium	75K	No
10	No	Small	90K	Yes

Training Set

Tid	Attrib1	Attrib2	Attrib3	Class
11	No	Small	55K	?
12	Yes	Medium	80K	?
13	Yes	Large	110K	?
14	No	Small	95K	?
15	No	Large	67K	?

Test Set



# Examples of Classification Problems

- Predicting **tumor cells** as benign or malignant
- Classifying credit **card transactions** as legitimate or fraudulent
- Classifying **secondary structures** of protein as alpha-helix, beta-sheet, or random coil
- Categorizing **news stories** as finance, weather, entertainment, sports, etc

# Classification Using Distance

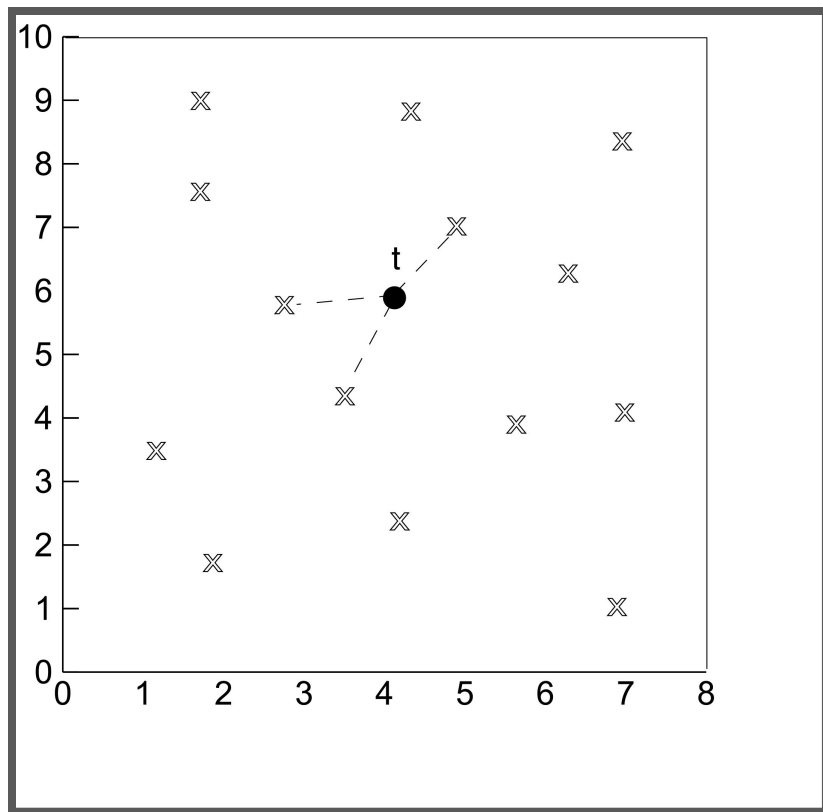
- Place items in class to which they are “closest”.
- Must determine distance between an item and a class.
- Classes represented by
  - ***Centroid***: Central value.
  - ***Medoid***: Representative point.
  - Individual points
- Algorithm: KNN

# *K Nearest Neighbor (KNN):*

- Training set includes classes.
- Examine  $K$  items near item to be classified.
- New item placed in class with the most number of close items.
- $O(q)$  for each tuple to be classified.

(Here  $q$  is the size of the training set.)

# KNN





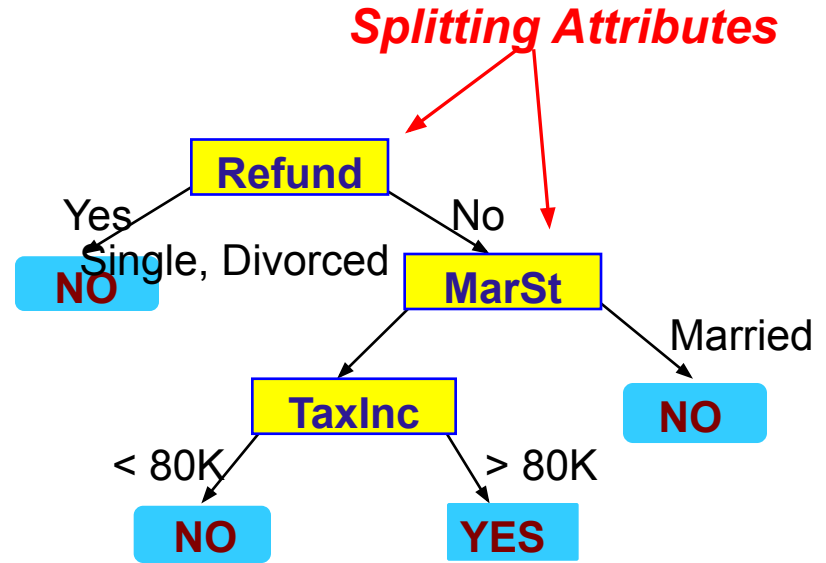
# Classification Techniques

- Decision Tree based Methods
- Rule-based Methods
- Memory based reasoning
- Neural Networks
- Naïve Bayes and Bayesian Belief Networks
- Support Vector Machines

# Example of a Decision Tree

<i>categorical</i>	<i>categorical</i>	<i>continuous</i>	<i>class</i>	
<i>Tid</i>	<i>Refund</i>	<i>Marital Status</i>	<i>Taxable Income</i>	<i>Cheat</i>
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

Training Data

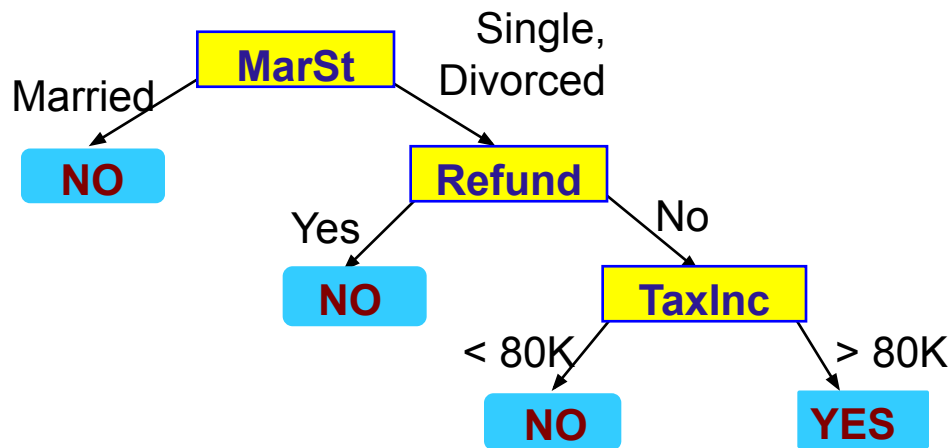


Model: Decision Tree

# Another Example of Decision Tree

*categorical*  
*categorical*  
*continuous*  
*class*

Tid	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes



**There could be more than one tree that fits the same data!**

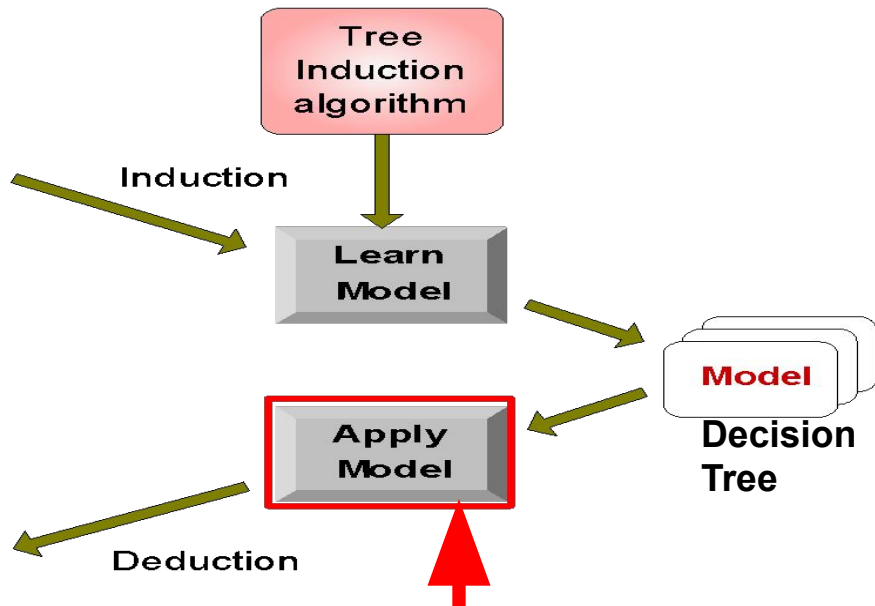
# Decision Tree Classification Task

Tid	Attrib1	Attrib2	Attrib3	Class
1	Yes	Large	125K	No
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5	No	Large	95K	Yes
6	No	Medium	60K	No
7	Yes	Large	220K	No
8	No	Small	85K	Yes
9	No	Medium	75K	No
10	No	Small	90K	Yes

Training Set

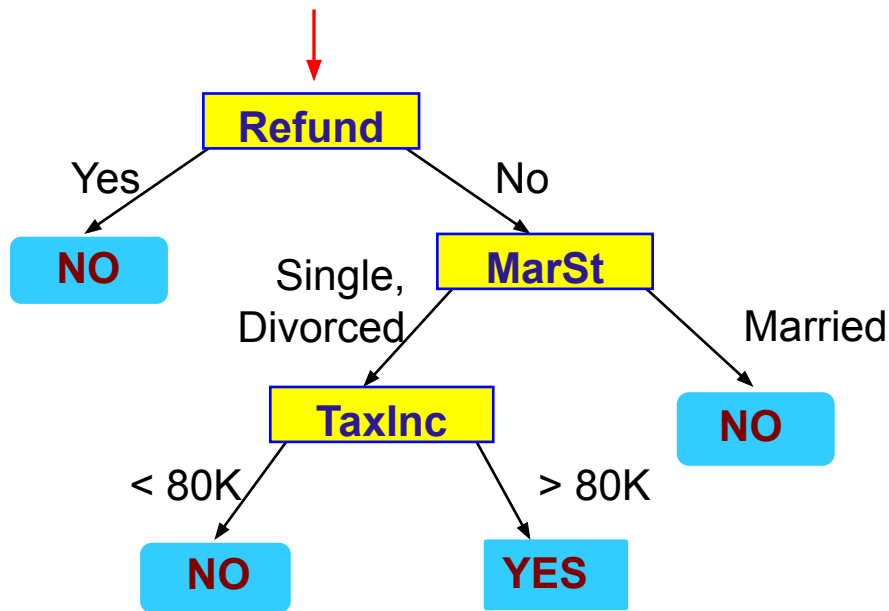
Tid	Attrib1	Attrib2	Attrib3	Class
11	No	Small	55K	?
12	Yes	Medium	80K	?
13	Yes	Large	110K	?
14	No	Small	95K	?
15	No	Large	67K	?

Test Set



# Apply Model to Test Data

Start from the root of tree.



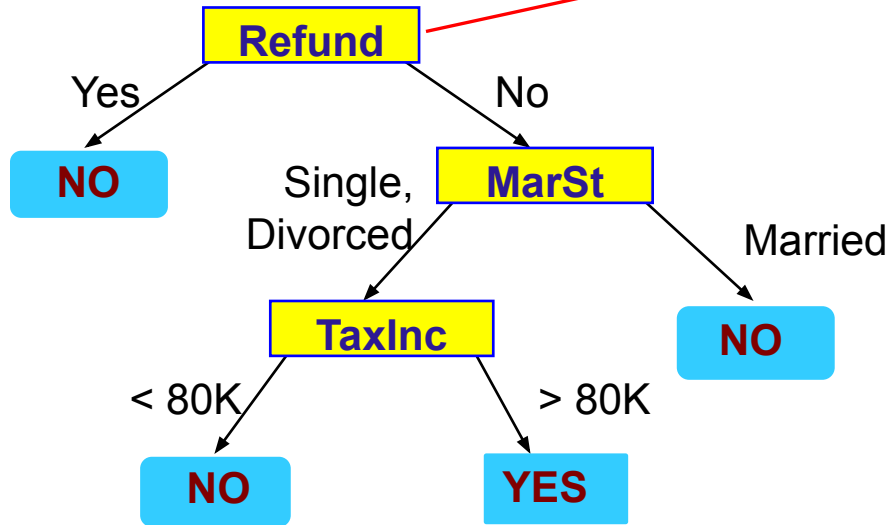
## Test Data

Refund	Marital Status	Taxable Income	Cheat
No	Married	80K	?

# Apply Model to Test Data

## Test Data

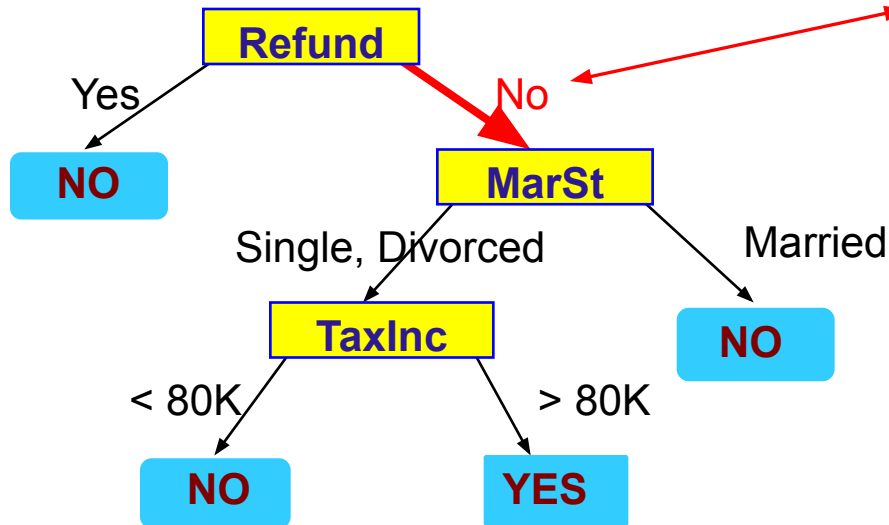
Refund	Marital Status	Taxable Income	Cheat
No	Married	80K	?



# Apply Model to Test Data

## Test Data

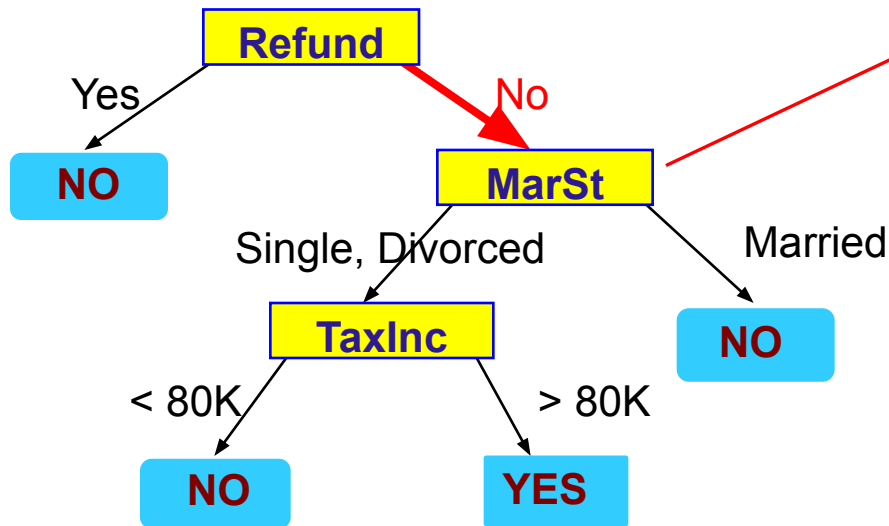
Refund	Marital Status	Taxable Income	Cheat
No	Married	80K	?



# Apply Model to Test Data

## Test Data

Refund	Marital Status	Taxable Income	Cheat
No	Married	80K	?

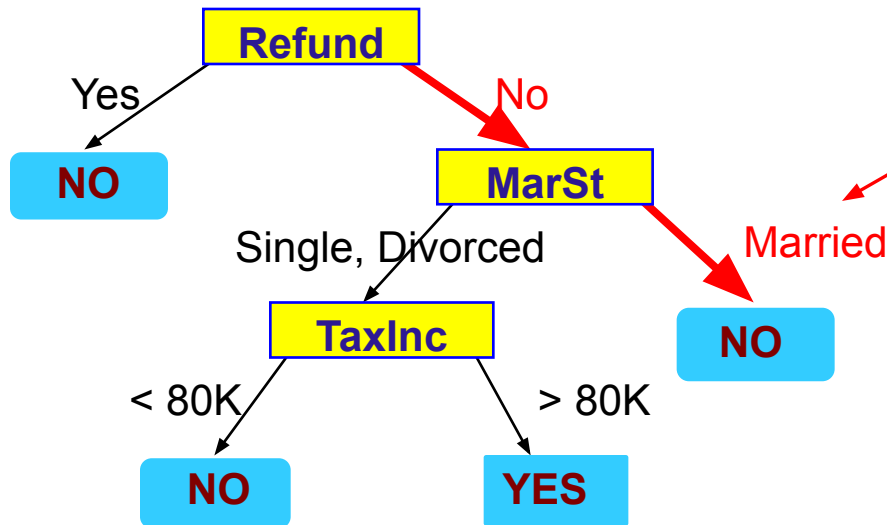




# Apply Model to Test Data

## Test Data

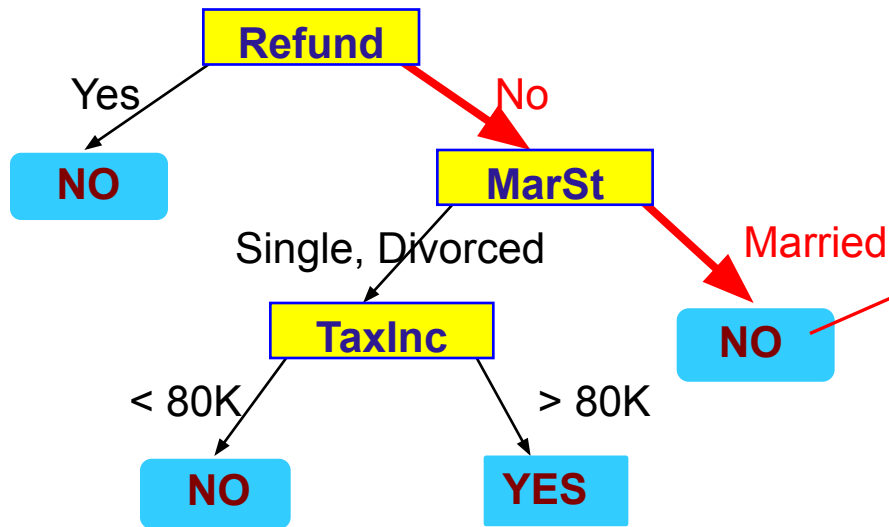
Refund	Marital Status	Taxable Income	Cheat
No	Married	80K	?



# Apply Model to Test Data

## Test Data

Refund	Marital Status	Taxable Income	Cheat
No	Married	80K	?



Assign Cheat to "No"

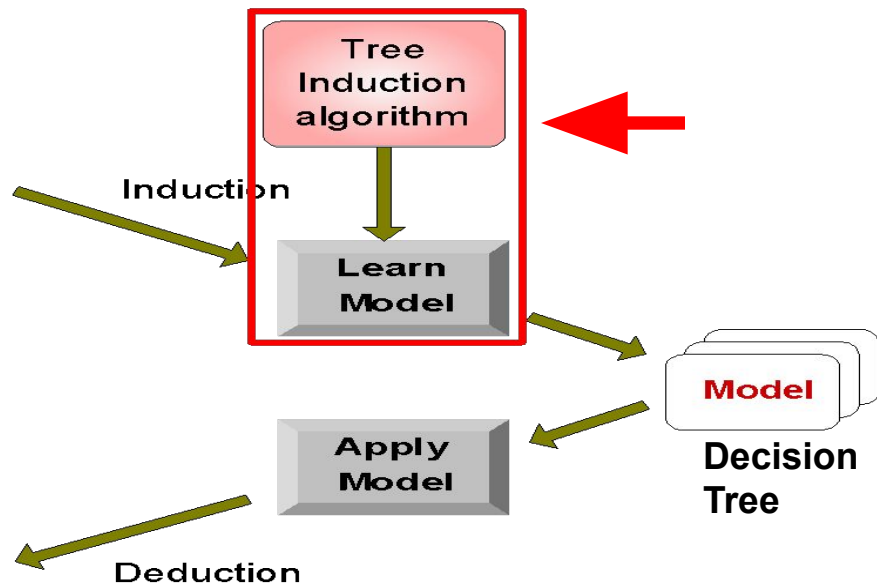
# Decision Tree Classification Task

Tid	Attrib1	Attrib2	Attrib3	Class
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4	Yes	Medium	120K	No
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6	No	Medium	60K	No
7	Yes	Large	220K	No
8	No	Small	85K	Yes
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Training Set

Tid	Attrib1	Attrib2	Attrib3	Class
11	No	Small	55K	?
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13	Yes	Large	110K	?
14	No	Small	95K	?
15	No	Large	67K	?

Test Set



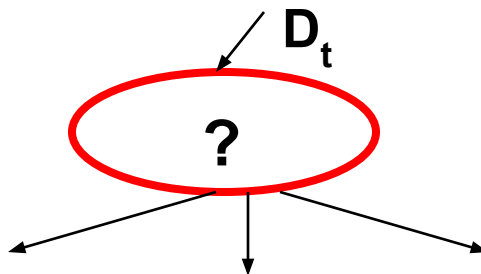
# Decision Tree Induction

- Many Algorithms:
  - Hunt's Algorithm (one of the earliest)
  - CART
  - ID3, C4.5
  - SLIQ, SPRINT

# General Structure of Hunt's Algorithm

- Let  $D_t$  be the set of training records that reach a node  $t$
- General Procedure:
  - If  $D_t$  contains records that belong the same class  $y_t$ , then  $t$  is a leaf node labeled as  $y_t$
  - If  $D_t$  is an empty set, then  $t$  is a leaf node labeled by the default class,  $y_d$
  - If  $D_t$  contains records that belong to more than one class, use an attribute test to split the data into smaller subsets. Recursively apply the procedure to each subset.

<i>Tid</i>	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes



# Tree Induction

- Greedy strategy.
  - Split the records based on an attribute test that optimizes certain criterion.
- Issues
  - Determine how to split the records
    - How to specify the attribute test condition?
    - How to determine the best split?
  - Determine when to stop splitting

# Tree Induction

- Greedy strategy.
  - Split the records based on an attribute test that optimizes certain criterion.
- Issues
  - Determine how to split the records
    - How to specify the attribute test condition?
    - How to determine the best split?
  - Determine when to stop splitting

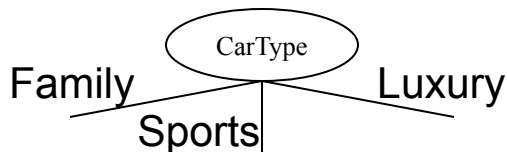
# How to Specify Test Condition?

- Depends on attribute types
  - Nominal
  - Ordinal
  - Continuous
- Depends on number of ways to split
  - 2-way split
  - Multi-way split

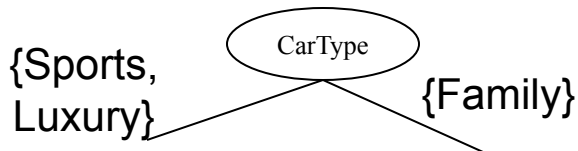


# Splitting Based on Nominal Attributes

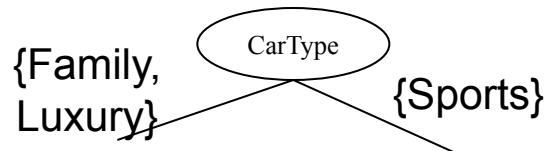
- **Multi-way split:** Use as many partitions as distinct values.



- **Binary split:** Divides values into two subsets.  
Need to find optimal partitioning.

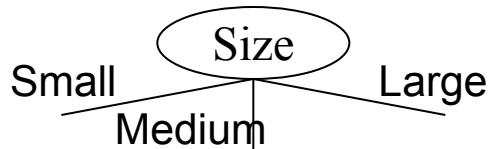


OR

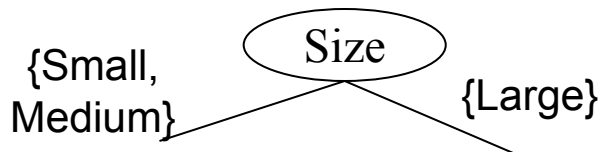


# Splitting Based on Ordinal Attributes

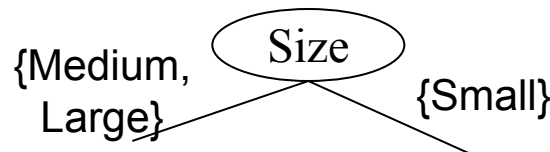
- **Multi-way split:** Use as many partitions as distinct values.



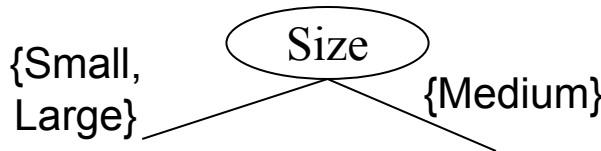
- **Binary split:** Divides values into two subsets.  
Need to find optimal partitioning.



OR



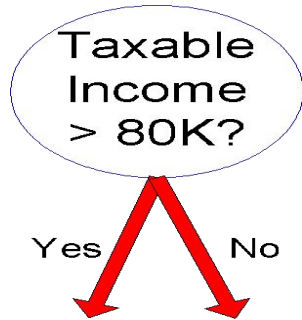
- What about this split?



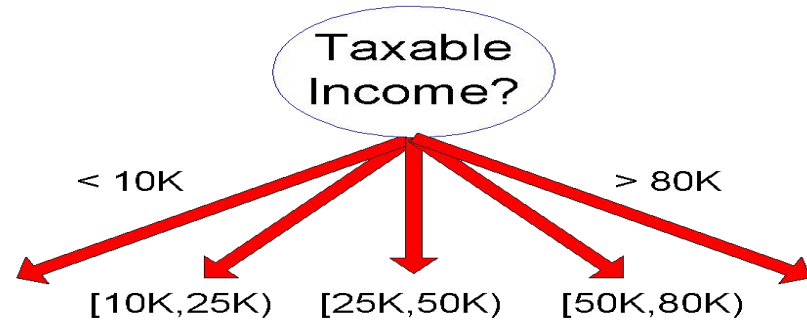
# Splitting Based on Continuous Attributes

- Different ways of handling
  - **Discretization** to form an ordinal categorical attribute
    - Static – discretize once at the beginning
    - Dynamic – ranges can be found by
      - equal interval bucketing,
      - equal frequency bucketing (percentiles)
      - clustering.
  - **Binary Decision**:  $(A < v)$  or  $(A \geq v)$ 
    - consider all possible splits and finds the best cut
    - can be more compute intensive

# Splitting Based on Continuous Attributes



(i) Binary split



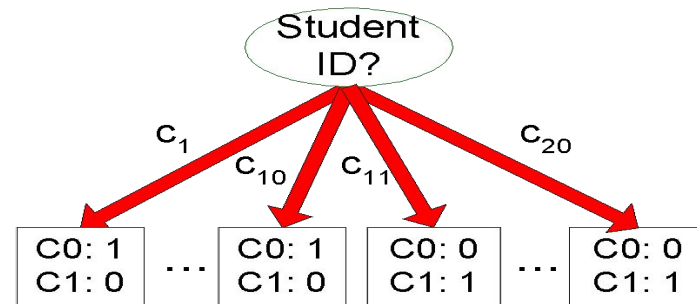
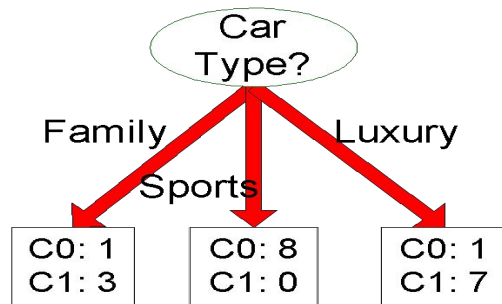
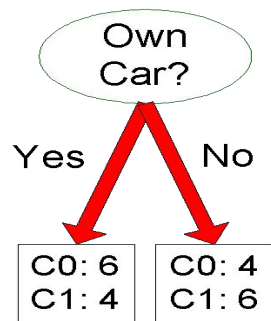
(ii) Multi-way split

# Tree Induction

- Greedy strategy.
  - Split the records based on an attribute test that optimizes certain criterion.
- 
- Issues
  - Determine how to split the records
    - How to specify the attribute test condition?
    - How to determine the best split?
  - Determine when to stop splitting

# How to determine the Best Split

**Before Splitting: 10 records of class 0,  
10 records of class 1**



**Which test condition is the best?**

# How to determine the Best Split

- Greedy approach:
  - Nodes with **homogeneous** class distribution are preferred
- Need a measure of node impurity:

C0: 5
C1: 5

**Non-homogeneous,  
High degree of impurity**

C0: 9
C1: 1

**Homogeneous,  
Low degree of impurity**

# Measures of Node Impurity

- Gini Index
- Entropy
- Misclassification error



# Splitting Criteria based on INFO

- Entropy at a given node t:

$$\text{Entropy}(t) = -\sum_j p(j | t) \log p(j | t)$$

- (NOTE:  $p(j | t)$  is the relative frequency of class j at node t).
- Measures homogeneity of a node.
  - Maximum ( $\log n_c$ ) when records are equally distributed among all classes implying least information
  - Minimum (0.0) when all records belong to one class, implying most information
- Entropy based computations are similar to the GINI index computations

# Examples for computing Entropy

$$\text{Entropy}(t) = -\sum_j p(j | t) \log_2 p(j | t)$$

C1	<b>0</b>
C2	<b>6</b>

$$P(C1) = 0/6 = 0 \quad P(C2) = 6/6 = 1$$

$$\text{Entropy} = -0 \log 0 - 1 \log 1 = -0 - 0 = 0$$

C1	<b>1</b>
C2	<b>5</b>

$$P(C1) = 1/6 \quad P(C2) = 5/6$$

$$\text{Entropy} = - (1/6) \log_2 (1/6) - (5/6) \log_2 (1/6) = 0.65$$

C1	<b>2</b>
C2	<b>4</b>

$$P(C1) = 2/6 \quad P(C2) = 4/6$$

$$\text{Entropy} = - (2/6) \log_2 (2/6) - (4/6) \log_2 (4/6) = 0.92$$

# Splitting Based on INFO...

- Information Gain:

$$GAIN_{split} = Entropy(p) - \left( \sum_{i=1}^k \frac{n_i}{n} Entropy(i) \right)$$

- Parent Node,  $p$  is split into  $k$  partitions;
- $n_i$  is number of records in partition  $i$
- Measures Reduction in Entropy achieved because of the split. Choose the split that achieves most reduction (maximizes GAIN)
- Used in ID3 and C4.5
- Disadvantage: Tends to prefer splits that result in large number of partitions, each being small but pure.

# Splitting Based on INFO...

- Gain Ratio:

$$GainRatio_{split} = \frac{GAIN_{Split}}{SplitINFO}$$

$$SplitINFO = -\sum_{i=1}^k \frac{n_i}{n} \log \frac{n_i}{n}$$

- Parent Node, p is split into k partitions
- $n_i$  is the number of records in partition i
- Adjusts Information Gain by the entropy of the partitioning (SplitINFO). Higher entropy partitioning (large number of small partitions) is penalized!
- Used in C4.5
- Designed to overcome the disadvantage of Information Gain

# Splitting Criteria based on Classification Error

- Classification error at a node  $t$  :

$$Error(t) = 1 - \max_i P(i | t)$$

# Examples for Computing Error

$$\text{Error}(t) = 1 - \max_i P(i | t)$$

C1	<b>0</b>
C2	<b>6</b>

$$P(C1) = 0/6 = 0 \quad P(C2) = 6/6 = 1$$

$$\text{Error} = 1 - \max(0, 1) = 1 - 1 = 0$$

C1	<b>1</b>
C2	<b>5</b>

$$P(C1) = 1/6 \quad P(C2) = 5/6$$

$$\text{Error} = 1 - \max(1/6, 5/6) = 1 - 5/6 = 1/6$$

C1	<b>2</b>
C2	<b>4</b>

$$P(C1) = 2/6 \quad P(C2) = 4/6$$

$$\text{Error} = 1 - \max(2/6, 4/6) = 1 - 4/6 = 1/3$$

# Measure of Impurity: GINI

- Gini Index for a given node  $t$  :

$$GINI(t) = 1 - \sum_j [p(j | t)]^2$$

(NOTE:  $p(j | t)$  is the relative frequency of class  $j$  at node  $t$ ).

C1	<b>0</b>
C2	<b>6</b>
<b>Gini=0.000</b>	

C1	<b>1</b>
C2	<b>5</b>
<b>Gini=0.278</b>	

C1	<b>2</b>
C2	<b>4</b>
<b>Gini=0.444</b>	

C1	<b>3</b>
C2	<b>3</b>
<b>Gini=0.500</b>	

# Examples for computing GINI

$$GINI(t) = 1 - \sum_j [p(j|t)]^2$$

C1	<b>0</b>
C2	<b>6</b>

$$P(C1) = 0/6 = 0 \quad P(C2) = 6/6 = 1$$

$$Gini = 1 - P(C1)^2 - P(C2)^2 = 1 - 0 - 1 = 0$$

C1	<b>1</b>
C2	<b>5</b>

$$P(C1) = 1/6 \quad P(C2) = 5/6$$

$$Gini = 1 - (1/6)^2 - (5/6)^2 = 0.278$$

C1	<b>2</b>
C2	<b>4</b>

$$P(C1) = 2/6 \quad P(C2) = 4/6$$

$$Gini = 1 - (2/6)^2 - (4/6)^2 = 0.444$$



# Splitting Based on GINI

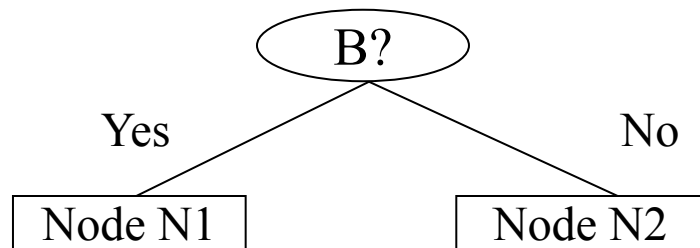
- Used in CART, SLIQ, SPRINT.
- When a node  $p$  is split into  $k$  partitions (children), the quality of split is computed as,

$$GINI_{split} = \sum_{i=1}^k \frac{n_i}{n} GINI(i)$$

where,  $n_i$  = number of records at child  $i$ ,  
 $n$  = number of records at node  $p$ .

# Binary Attributes: Computing GINI Index

- Splits into two partitions
- Effect of Weighing partitions:
  - Larger and Purer Partitions are sought for.



	N1	N2
C1	5	1
C2	2	4
Gini=0.333		

	Parent
C1	6
C2	6
Gini = 0.500	

# Categorical Attributes: Computing Gini Index

- For each distinct value, gather counts for each class in the dataset
- Use the count matrix to make decisions

Multi-way split

	CarType		
	Family	Sports	Luxury
C1	1	2	1
C2	4	1	1
Gini	0.393		

Two-way split

(find best partition of values)

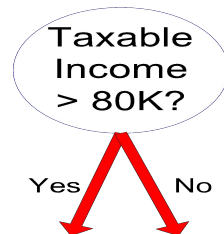
	CarType	
	{Sports, Luxury}	{Family}
C1	3	1
C2	2	4
Gini	0.400	

	CarType	
	{Sports}	{Family, Luxury}
C1	2	2
C2	1	5
Gini	0.419	

# Continuous Attributes: Computing Gini Index

- Use Binary Decisions based on one value
- Several Choices for the splitting value
  - Number of possible splitting values  
= Number of distinct values
- Each splitting value has a count matrix associated with it
  - Class counts in each of the partitions,  $A < v$  and  $A \geq v$
- Simple method to choose best  $v$ 
  - For each  $v$ , scan the database to gather count matrix and compute its Gini index
  - Computationally Inefficient! Repetition of work.

<i>Tid</i>	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes



# Continuous Attributes: Computing Gini Index...

- For efficient computation: for each attribute,
  - Sort the attribute on values
  - Linearly scan these values, each time updating the count matrix and computing gini index
  - Choose the split position that has the least gini index

		Cheat	No		No		No		Yes		Yes		Yes		No		No		No		No			
			Taxable Income																					
			60		70		75		85		90		95		100		120		125		220			
Sorted Values	Split Positions		55		65		72		80		87		92		97		110		122		172		230	
		<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	
		Yes	0	3	0	3	0	3	0	3	1	2	2	1	3	0	3	0	3	0	3	0	3	0
		No	0	7	1	6	2	5	3	4	3	4	3	4	3	4	4	3	5	2	6	1	7	0
		Gini	0.420		0.400		0.375		0.343		0.417		0.400		<u>0.300</u>		0.343		0.375		0.400		0.420	

# Tree Induction

- Greedy strategy.
  - Split the records based on an attribute test that optimizes certain criterion.
- Issues
  - Determine how to split the records
    - How to specify the attribute test condition?
    - How to determine the best split?
  - Determine when to stop splitting

# Stopping Criteria for Tree Induction

- Stop expanding a node when all the records belong to the same class
- Stop expanding a node when all the records have similar attribute values
- Early termination (to be discussed later)

# Decision Tree Based Classification

- **Advantages:**

- Inexpensive to construct
- Extremely fast at classifying unknown records
- Easy to interpret for small-sized trees
- Accuracy is comparable to other classification techniques for many simple data sets



# C4.5 Algorithm

- Simple depth-first construction.
- Uses Information Gain
- Sorts Continuous Attributes at each node.
- Needs entire data to fit in memory.
- Unsuitable for Large Datasets.
  - Needs out-of-core sorting.

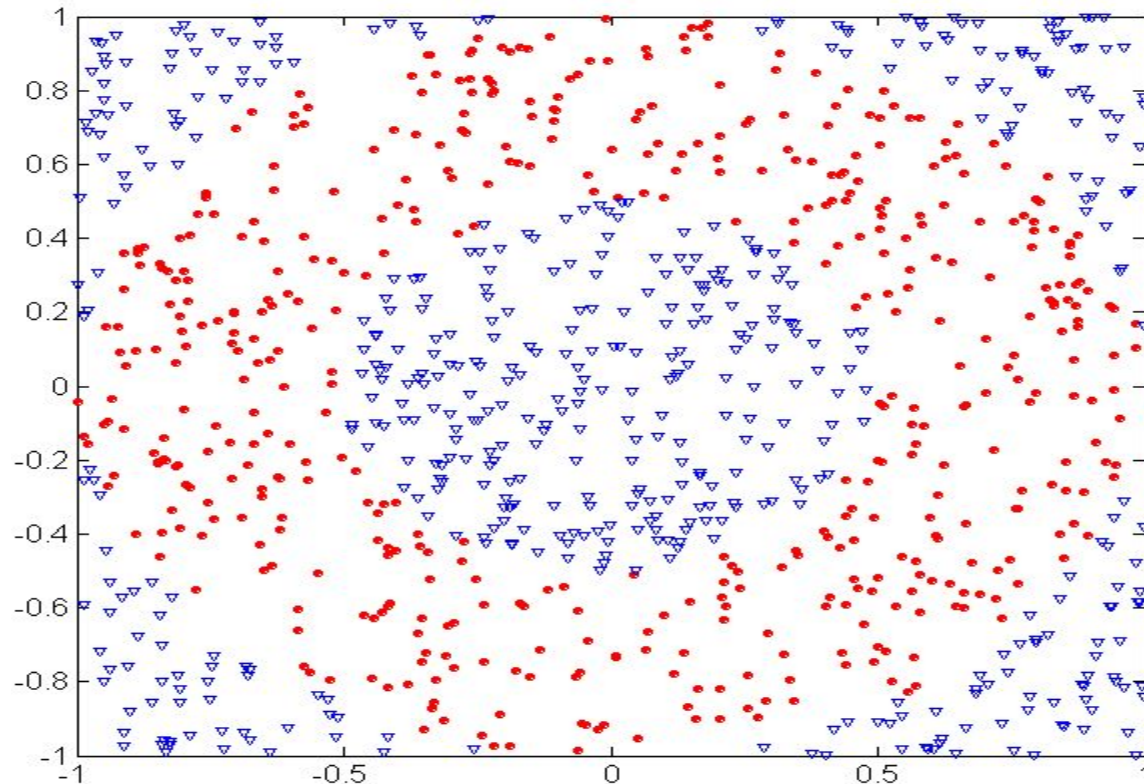
# Practical Issues of Classification

Underfitting and Overfitting

Missing Values

Costs of Classification

# Underfitting and Overfitting (Example)



**500 circular and 500  
triangular data points.**

**Circular points:**

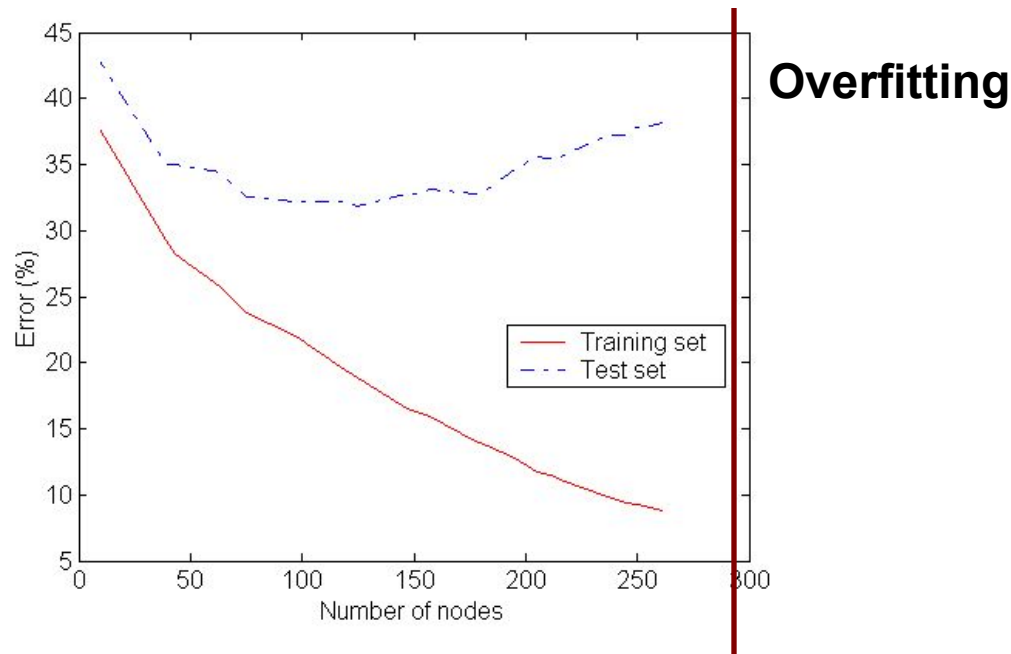
$$0.5 \leq \sqrt{x_1^2 + x_2^2} \leq 1$$

**Triangular points:**

$$\sqrt{x_1^2 + x_2^2} > 0.5 \text{ or}$$

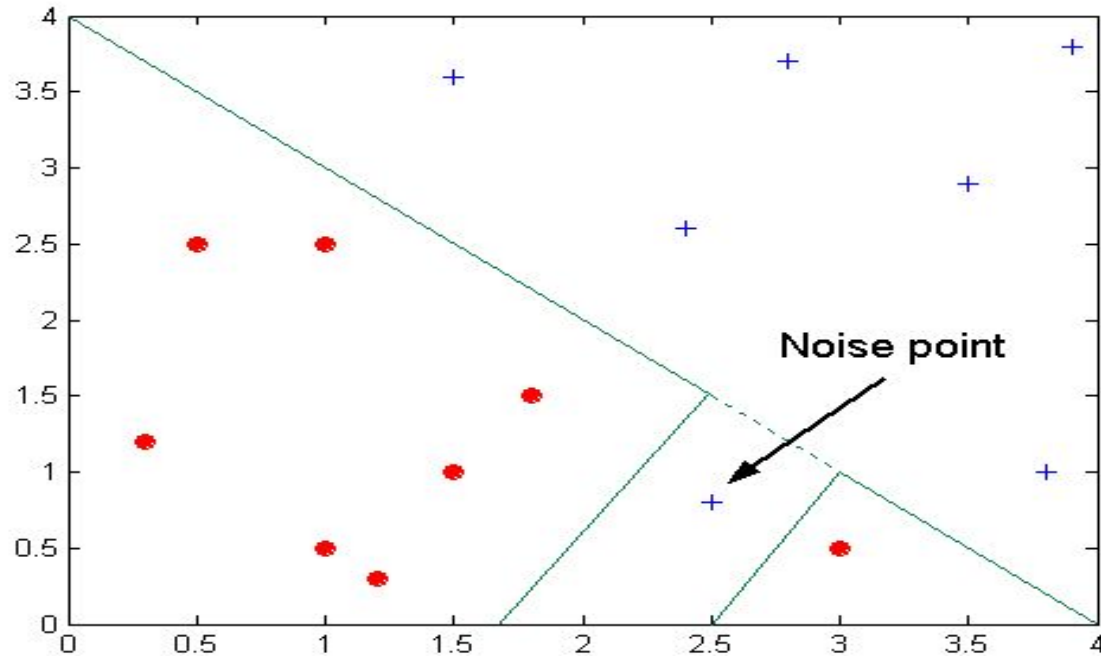
$$\sqrt{x_1^2 + x_2^2} < 1$$

# Underfitting and Overfitting



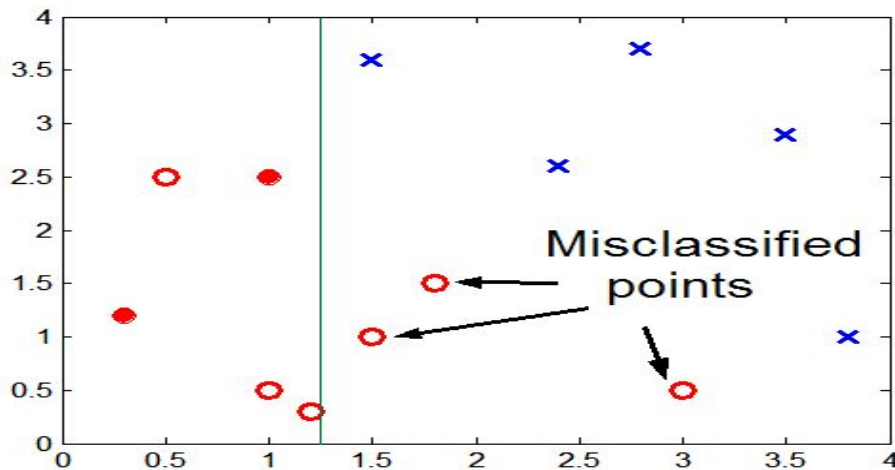
**Underfitting:** when model is too simple, both training and test errors are large

# Overfitting due to Noise



**Decision boundary is distorted by noise point**

# Overfitting due to Insufficient Examples



**Lack of data points in the lower half of the diagram makes it difficult to predict correctly the class labels of that region**

**- Insufficient number of training records in the region causes the decision tree to predict the test examples using other training records that are irrelevant to the classification task**

# Notes on Overfitting

- Overfitting results in decision trees that are more complex than necessary
- Training error no longer provides a good estimate of how well the tree will perform on previously unseen records
- Need new ways for estimating errors

# How to Address Overfitting

- **Pre-Pruning (Early Stopping Rule)**

- Stop the algorithm before it becomes a fully-grown tree
- Typical stopping conditions for a node:
  - Stop if all instances belong to the same class
  - Stop if all the attribute values are the same
- More restrictive conditions:
  - Stop if number of instances is less than some user-specified threshold
  - Stop if class distribution of instances are independent of the available features (e.g., using  $\chi^2$  test)
  - Stop if expanding the current node does not improve impurity measures (e.g., Gini or information gain).



# How to Address Overfitting...

- **Post-pruning**

- Grow decision tree to its entirety
- Trim the nodes of the decision tree in a bottom-up fashion
- If generalization error improves after trimming, replace sub-tree by a leaf node.
- Class label of leaf node is determined from majority class of instances in the sub-tree
- Can use MDL for post-pruning

# Handling Missing Attribute Values

- Missing values affect decision tree construction in three different ways:
  - Affects how impurity measures are computed
  - Affects how to distribute instance with missing value to child nodes
  - Affects how a test instance with missing value is classified

# Scalable Decision Tree Induction Methods

- **SLIQ** (EDBT'96 — Mehta et al.)
  - Builds an index for each attribute and only class list and the current attribute list reside in memory
- **SPRINT** (VLDB'96 — J. Shafer et al.)
  - Constructs an attribute list data structure
- **PUBLIC** (VLDB'98 — Rastogi & Shim)
  - Integrates tree splitting and tree pruning: stop growing the tree earlier
- **RainForest** (VLDB'98 — Gehrke, Ramakrishnan & Ganti)
  - Builds an AVC-list (attribute, value, class label)
- **BOAT** (PODS'99 — Gehrke, Ganti, Ramakrishnan & Loh)
  - Uses bootstrapping to create several small samples

*Thank You*