EEF262: Physics for Engineering II

Presented by

Dr DJOB Roger

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Contents

List of Figures

EEF262: Physics for Engineering II 4 Credits (40-20-0)

Course content

Objective

Extend the scope of the **course EEF269** entitled "Physics for Electrical Engineering I"

Content

1. Electromagnetism
   - Magnetism
   - Ampere Theorem
   - Biot and Savart Law
   - Energy and magnetic forces
2. Waves
   - Free Oscillations
   - Progressive waves
   - Electromagnetic Waves
3. Thermal phenomena
   - Heat transmission
   - Thermal dissipation of materials

Outcome

At the end of this course, students will be able to: " Apply knowledge of electromagnetism to explain the operation of many transducers and sensors used in Electrical Engineering " Apply knowledge of electromagnetism to explain the operation of many instruments and systems encountered in Electrical Engineering " Relate the theory of Electromagnetism to the design and operation of Electrical Power Systems

Chapter 1

Magnetism

1.1 Magnetism and Magnetic Field

1.1.1 Definition of magnetism

Magnetism is the force of attraction or repulsion that acts between magnets and other magnetic materials. We know, for example, that magnets attract pieces of iron, deflect compass needles, attract or repel other magnets, and so on.

1.1.2 Definition of magnetic field

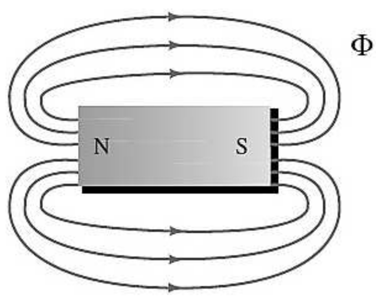


Figure 1.1: *Flux lines*

The region where the force is felt is called the "field of the magnet" or simply, its **magnetic field**. Thus, a magnetic field is a force field. Using Faraday's representation, magnetic fields are shown as lines in space. These lines, called flux lines or lines of force, show the direction and intensity of the field at all points (see Figure 1.1). The field is strongest at the poles of the magnet (where flux lines are most dense), its direction is from north (N) to south (S) external to the magnet, and flux lines never cross. The symbol for magnetic flux as shown is the Greek letter is Φ.

1.1.3 What happens when two magnets are brought close together?

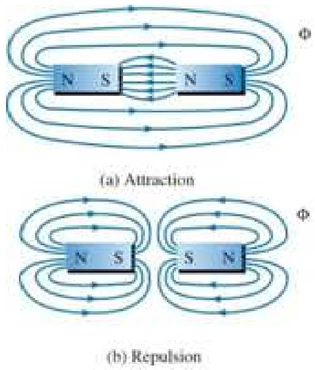


Figure 1.2: *Attraction-repulsion phenomena*

Unlike poles attract, and flux lines pass from one magnet to the other. Like poles repel, and the flux lines are pushed back as indicated by the flattening of the field between the two magnets (see 1.2).

1.1.4 Ferromagnetic Materials

Ferromagnetic Materials are magnetic materials that are attracted by magnets such as **iron, nickel, cobalt, and their alloys**. Ferromagnetic materials provide an easy path for magnetic flux. The flux lines take the longer (but easier) path through the soft iron, rather than the shorter path that they would normally take. Note, however, that nonmagnetic materials (plastic, wood, glass, and so on) have no effect on the field.

1.2 Electromagnetism

Most applications of magnetism involve magnetic effects due to electric currents. The current, I, creates a magnetic field that is concentric about the conductor, uniform along its length, and whose strength is directly proportional to I. Note the direction of the field may be remembered with the aid of the right-hand rule. Imagine placing your right hand around the conductor with your thumb pointing in the direction of current. Your fingers then point in the direction of the field. If you reverse the direction of the current, the direction of the field reverses.

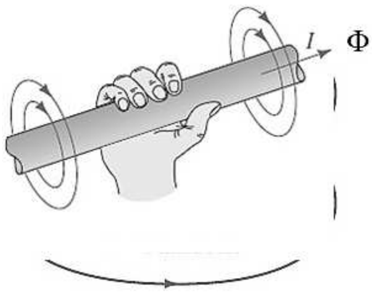


Figure 1.3: *Right hand rule*

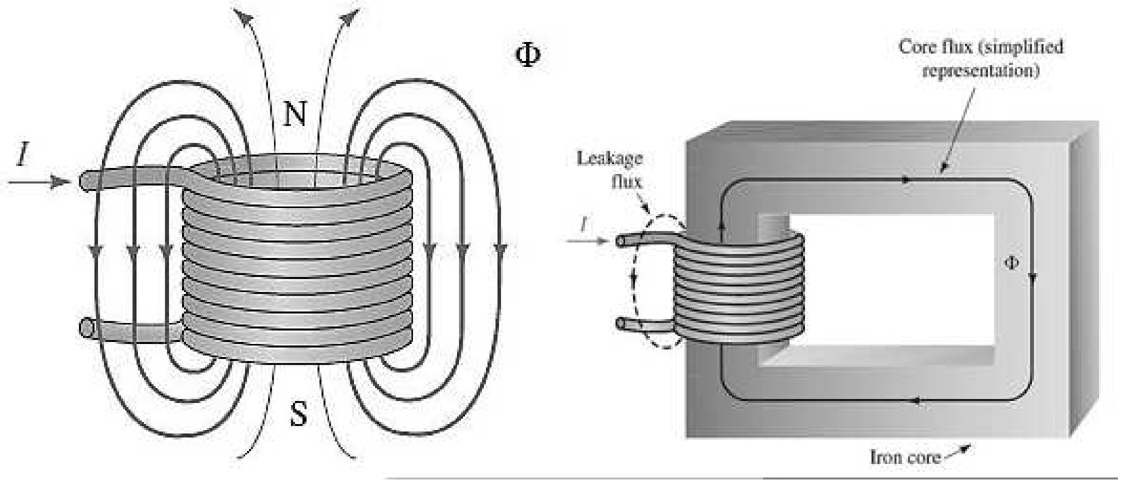
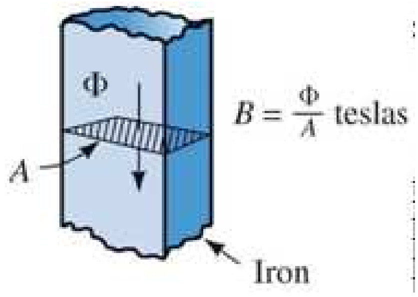


Figure 1.4: *Flux in a ferromagnetic core*

If the conductor is wound into a coil (Provided no ferromagnetic material is present), the fields of its individual turns combine, producing a resultant field as in Fig. 1.4. The direction of the coil flux can also be remembered by means of a simple rule: curl the fingers of your right hand around the coil in the direction of the current and your thumb will point in the direction of the field. If the direction of the current is reversed, the field also reverses. Provided no ferromagnetic material is present, the strength of the coil's field is directly proportional to its current.
If the coil is wound on a ferromagnetic core as in Figure 1.4, almost all flux is confined to the core, although a small amount (called stray or leakage flux) passes through the surrounding air. However, now that ferromagnetic material is present, the core flux is no longer proportional to current.

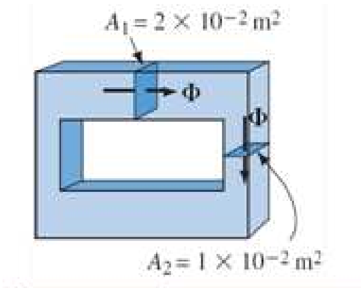
1.3 Magnetic flux and flux Density

The magnetic flux is represented by the symbol Φ. In the SI system, the unit of flux is the weber (Wb). However, we are often more interested in flux density B (i.e., flux per unit area) than in total flux Φ. Since flux Φ is measured in Wb and area A in m^2, flux density is measured as Wb/m^2, the unit of flux density is called the tesla (T) where $1T = 1Wb/m^2$. Flux density is found by dividing the total flux passing perpendicularly through an area by the size of the area,



Example: Given the flux density at cross section 1 is $B_1 = 0.4T$, determine B_2.

$$\Phi = B_1 \times A_1 = B_2 \times A_2 \tag{1.1}$$



Since all flux is confined to the core, the flux at cross section 2 is the same as at cross section 1. Therefore

$$B_2 = \frac{B_1 \times A_1}{A_2} = \frac{0.4 \times 2.10^{-2}}{1.10^{-2}} = 0.8T$$

1.4 Permeability

If cores of different materials with the same physical dimensions are used in the electromagnet, the strength of the magnet will vary in accordance with the core used. This variation in strength is due to the greater or lesser number of flux lines passing through the core. Materials in which flux lines can readily be set up are said to be magnetic and to have high permeability. The **permeability** (μ) of a material is similar in many respects to conductivity in electric circuits. The permeability of free space (vacuum) is

$$\mu_0 = 4\pi \times 10^{-7} Wb/A.m \tag{1.2}$$

The permeability of all nonmagnetic materials, such as copper, aluminum, wood, glass, and air, is the same as that for free space. Materials that have permeabilities slightly less than that of free space are said to be **diamagnetic**, and those with permeabilities slightly greater than that of free space are said to be **paramagnetic**. Materials with these very high permeabilities are referred to as **ferromagnetic**.
The ratio of the permeability of a material to that of free space is called its relative permeability; that is

$$\mu_r = \frac{\mu}{\mu_0} \tag{1.3}$$

In general, for ferromagnetic materials, $\mu_r \geq 100$, and for nonmagnetic materials, $\mu_r \simeq 1$.

1.5 Reluctance

The resistance of a material to the flow of charge (current) is determined for electric circuits by the equation

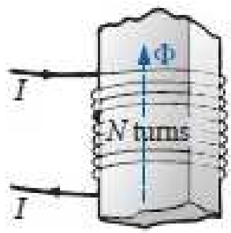
$$R = \frac{\rho l}{A}(Ohms) \tag{1.4}$$

The reluctance of a material to the setting up of magnetic flux lines in the material is determined by the following equation:

$$\mathtt{R} = \frac{l}{\mu A}(At/Wb), \tag{1.5}$$

where R is the reluctance, l is the length of the magnetic path, and A is the cross-sectional area. The units At/Wb is the number of turns of the applied winding. More is said about ampere-turns (At)

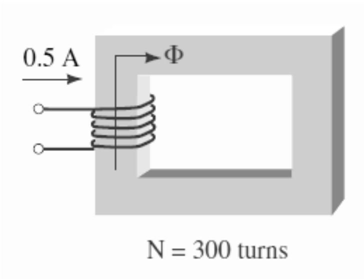
1.6 Ohm's Law for Magnetic circuits



The relationship between flux Φ, magnetomotive force (MMF) $\mathtt{F} = NI(At)$, and reluctance $\mathtt{R}$ is

$$\Phi = \frac{\mathtt{F}}{\mathtt{R}}(Wb). \tag{1.6}$$

EXAMPLE : For Figure shown below, if the reluctance of the magnetic circuit is $\mathtt{R} = 12 \times 10^4 (At/Wb)$, what is the flux in the circuit?



Solution:

$$\mathtt{F} = NI = 300 \times 0.5 = 150At$$
$$\Phi = \frac{\mathtt{F}}{\mathtt{R}} = \frac{300}{12\times 10^4} = 12.5 \times 10^{-4} Wb$$

1.7 Magnetic Field Intensity

We now look at a more practical approach to analyzing magnetic circuits. First, we require a quantity called magnetic field intensity, H (also known as magnetizing force). It is a measure of the MMF per unit length of a circuit. We can define magnetic field intensity as the ratio of applied MMF to the length of path that it acts over. Thus,

$$H = \frac{\mathtt{F}}{\mathtt{l}} = \frac{\mathtt{NI}}{\mathtt{l}}(At/m). \tag{1.7}$$

Rearranging Equation 1.7 yields an important result:

$$NI = Hl. \tag{1.8}$$

In an analogy with electric circuits, the NI product is an MMF source, while the Hl product is an MMF drop.

1.8 The Relationship between B and H

The flux density and the magnetizing force are related by the following equation:

$$B = \mu H \tag{1.9}$$

where μ is the permeability of the core, therefore, the larger the value of μ, the larger the flux density for a given magnetizing current. For all practical purposes, the permeability of air and other nonmagnetic materials is the same as for a vacuum. Thus, in air gaps,

$$\begin{aligned} B_g &= \mu_0 H_g \\ H_g &= \frac{B_g}{4\pi \times 10^{-7}} = 7.96 \times 10^5 B_g (At/m). \end{aligned} \tag{1.10}$$

1.9 Magnetic circuits

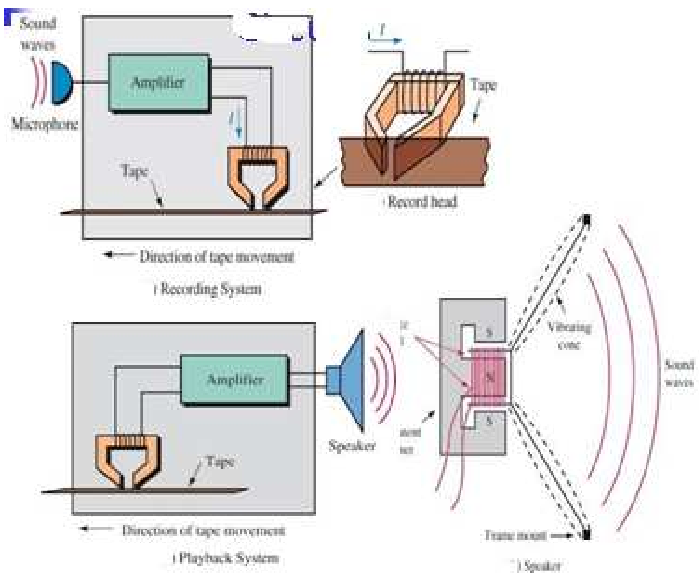


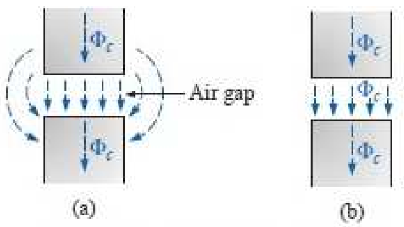
Figure 1.5: *Application of magnetic circuits*

Found in motors, generators, speakers, transformers, magnetic fields can be created by electric currents and permanent magnets. Magnetic stripe contains information used in bank ATM cards, library cards, etc. Magnetic patterns encode information. Reader sees varying magnetic field induces a voltage in the pickup winding; voltage is amplified and sent to decoding circuitry.

Applications of magnetic circuits: Practical applications use structures to guide and shape magnetic flux called magnetic circuits. Magnetic circuit guides flux to an air gap. This provides field for the voice coil, playback heads on tape recorders (see Figure 2.1), VCRs and disk drives.

1.10 Air Gaps, Fringing, and Laminated Cores

The spreading of the flux lines outside the common area of the core for the air gap in Figure 1.6(a) is known as fringing. For our purposes, we shall neglect this effect and assume the flux distribution to be as in Figure 1.6(b). For magnetic circuits with air gaps, fringing occurs, causing a decrease in flux density in the gap. For short gaps, fringing can usually be neglected. Alternatively, correction can be made by increasing each cross-sectional dimension of the gap by the size of the gap to approximate the decrease in flux density.



Air gaps: (a) with fringing; (b) ideal.

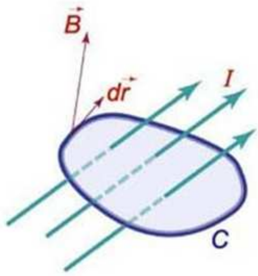
Figure 1.6: *Air gaps*

Chapter 2

Ampere Theorem

Ampere's law is a useful law that relates the net magnetic field along a closed loop to the electric current passing through the loop.

2.1 Definition



The integral around a closed path of the component of the magnetic field tangent to the direction of the path equals μ_0 times the current intercepted by the area within the path:

$$\oint \vec{B} \cdot d\vec{S} = \mu_0 I. \tag{2.1}$$

Or, in a simplified scalar form,

$$\oint B_{||} \cdot dS = \mu_0 I. \tag{2.2}$$

Thus the line integral (circulation) of the magnetic field around some arbitrary closed curve is proportional to the total current enclosed by that curve.

Important Notes:

- In order to apply Ampere's law all currents have to be steady
- Only currents crossing the area inside the path are taken into account and have some contribution to the magnetic field
- Currents have to be taken with their algebraic signs (those going "out" of the surface are positive, those going "in" are negative). Use right hand's rule to determine directions and signs
- The total magnetic circulation is zero only in the following cases:

  the enclosed net current is zero

  the magnetic field is normal to the selected path at any point

  the magnetic field is zero

- Ampere's Law can be useful when calculating magnetic fields of current distributions with a high degree of symmetry (similar to symmetrical charge distributions in the case of Gauss' Law)

2.2 Applications of the Ampere's theorem

2.2.1 Example 1: Calculating Line Integrals

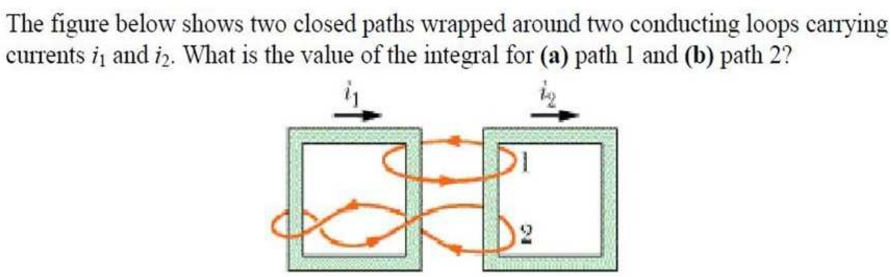


Figure 2.1: *Two conducting loops having currents*

To do this, you have to use the right hand rule to check whether the currents are positive or negative relative to the path.
On path 1, i_1 penetrates in the negative direction while i_2 penetrates in the positive direction, so:

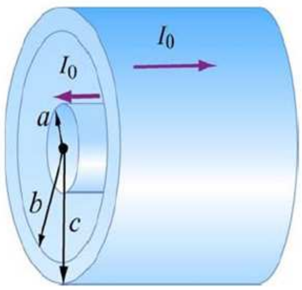
$$\oint B \cdot dS = \mu_0(i_2 - i_1).$$

On path 2, i_1 penetrates twice in the negative direction and i_2 penetrates once in the negative direction.

$$\oint B \cdot dS = -\mu_0(i_2 + 2i_1).$$

2.2.2 Example 2: Co-axial cable

A coaxial cable consists of a solid inner conductor of radius a, surrounded by a concentric cylindrical tube of inner radius b and outer radius c. The conductors carry equal and opposite currents I_0 *distributed uniformly across their cross-sections.* Determine the magnitude and direction of the magnetic field at a distance r from the axis. Make a graph of the magnitude of the magnetic field as a function of the distance r from the axis.



(a) $r < a$;

(b) $a < r < b$;

(c) $b < r < c$;

(d) $r > c$.

Figure 2.2: *Co-axial cable*

(a) The enclosed current is $I_{enc} = I_0(\frac{\pi r^2}{\pi a^2}) = I_0\frac{r^2}{a^2}$.
Applying Ampere's law, we have $B(2\pi r) = \mu_0\frac{I_0 r^2}{a^2}$ or $B = \mu_0\frac{I_0 r}{2\pi a^2}$, running counterclockwise when viewed from left.
(b) The enclosed current is $I_{enc} = I_0$.
Applying Ampere's law, we have $B(2\pi r) = \mu_0 I_0$ or $B = \frac{\mu_0 I_0}{2\pi r}$, running counterclockwise when viewed from left.
(c) The enclosed current is $I_{enc} = I_0 - I_0(\frac{\pi r^2 - \pi b^2}{\pi c^2 - \pi b^2}) = I_0\frac{c^2 - r^2}{c^2 - b^2}$.
Applying Ampere's law, we have $B(2\pi r) = \mu_0 I_0\frac{c^2 - r^2}{c^2 - b^2}$ or $B = \frac{\mu_0 I_0}{2\pi r}\frac{c^2 - r^2}{c^2 - b^2}$, running counterclockwise when viewed from left.
(d) The magnitude of the magnetic field is $B = 0$ since the total enclosed current is $I_{enc} = 0$.

2.2.3 Example 3: Cylindrical Conductor

Consider an infinitely long, cylindrical conductor of radius R carrying a current I with a non-uniform density $J = \alpha r^2$, where α is a constant and r the distance from the center of the cylinder
(a) Find the magnetic field everywhere
(b) Plot the magnitude of the magnetic field as a function of r
Solution:
(a) The enclosed current is given by:

$$I_0 = \int \vec{J} d\vec{A} = \int (\alpha r'^2)(2\pi r' dr') = \int 2\pi\alpha r'^3 dr'.$$

For $r < R$,

$$I_{enc} = \int_0^r 2\pi\alpha r'^3 dr' = \frac{\pi\alpha r^4}{2}.$$

Applying Ampere's law, the magnitude field is given by:

$$B(2\pi r) = \frac{\mu_0 \pi\alpha r^4}{2} \text{ or } B = \frac{\mu_0 \alpha r^3}{4}.$$

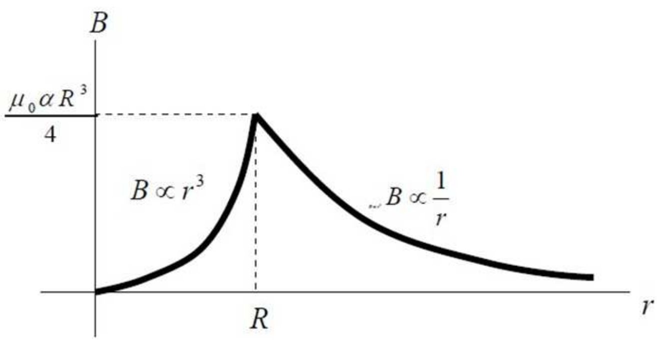
For $r > R$,

$$I_{enc} = \int_0^R 2\pi\alpha r'^3 dr' = \frac{\pi\alpha R^4}{2}.$$

Applying Ampere's law, the magnitude field is given by:

$$B(2\pi r) = \frac{\mu_0 \pi\alpha R^4}{2} \text{ or } B = \frac{\mu_0 \alpha R^4}{4r}.$$

(b) Plot the magnitude of the magnetic field as a function of r.



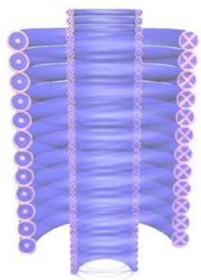


Figure 2.3: *Two long solenoids nested on the same axis*

2.2.4 Example 4: Two long solenoids

Two long solenoids are nested on the same axis as in the figure 2.3. The inner solenoid has radius R_1 and n_1 turns per unit length. The outer solenoid has radius R_2 and n_2 turns per unit length. Each solenoid carries the same current I flowing in each turn, but in opposite directions as indicated on the sketch. Use Ampere's law to find the direction and magnitude of the magnetic field in the following regions. Be sure to show your Amperian loops and all calculations.

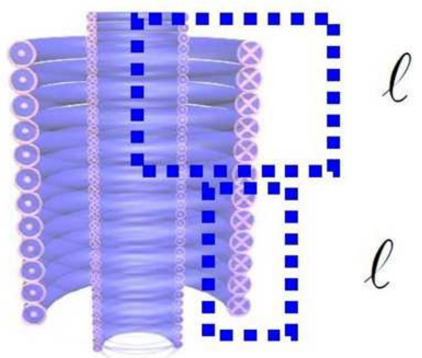
(a) $0 < r < R_1$

(b) $R_1 < r < R_2$

(c) $R_2 < r$

Solution:

(a) $0 < r < R_1$



To solve the magnetic field in this case, we take the top rectangular loop shown on the figure. The current through the loop is:

$$I_{enc} = -n_1 lI + n_2 lI = (-n_1 + n_2)lI.$$

The loop has four segments. Along two of those (top and bottom, horizontal), $\vec{B}$ is perpendicular to $d\vec{S}$ and $\vec{B} \cdot d\vec{S} = 0$. On the other hand, along the outer vertical segment $\vec{B} = \vec{0}$. Thus, using Ampere's law $\oint \vec{B} \cdot d\vec{S} = \mu_0 I_{enc}$ we have

$$\oint \vec{B} \cdot d\vec{S} = Bl + 0 + 0 + 0 = Bl = (-n_1 + n_2)\mu_0 lI \Rightarrow \vec{B} = (-n_1 + n_2)\mu_0 I\vec{k}.$$

(b) $R_1 < r < R_2$

To solve the magnetic field in this case, we take the bottom rectangular loop shown on the figure. The current through the loop is:

$$I_{enc} = n_2 lI.$$

The loop has four segments. Along two of those (top and bottom, horizontal), $\vec{B}$ is perpendicular to $d\vec{S}$ and $\vec{B} \cdot d\vec{S} = 0$. On the other hand, along the outer vertical segment $\vec{B} = \vec{0}$. Thus, using Ampere's law $\oint \vec{B} \cdot d\vec{S} = \mu_0 I_{enc}$ we have

$$\oint \vec{B} \cdot d\vec{S} = Bl + 0 + 0 + 0 = Bl = \mu_0 n_2 lI \Rightarrow \vec{B} = \mu_0 n_2 I\vec{k}.$$

(c) $R_2 < r$

Since the net current enclosed by the Amperian loop is zero, the magnetic field is zero in this region.

Chapter 3

Biot and Savart law

Biot and Savart law is deduced from many experiments on B field produced by currents, including B field around a very long wire.

3.1 Statement of the Biot and Savart law

The magnetic field at any point P located at a distance r due to a steady current in an infinite long straight wire is directly proportional to the current (i) and inversely proportional to the square of the distance.

3.2 Determination of a magnetic field using Biot and Savart law

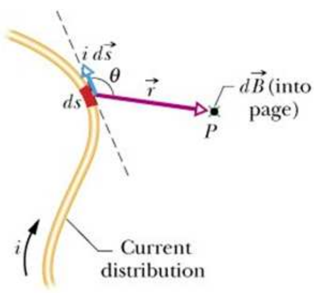


Figure 3.1: *Schematic representation of a magnetic field produced by current distribution*

3.2.1 Vector notation

The magnetic field produced by an infinite small length of a conductor in any point of space located at a distance r of it is

$$d\vec{B} = \frac{\mu_0 i}{4\pi}\frac{\vec{ds} \times \vec{r}}{r^3}, \tag{3.1}$$

Direction of the magnetic field is perpendicular to the plane formed $(\vec{ds}, \vec{r})$ so that $(\vec{ds}, \vec{r}, d\vec{B})$ form a direct trihedron.

3.2.2 Magnitude

$$dB = \frac{\mu_0 i}{4\pi}\frac{ds \sin\theta}{r^2}, \tag{3.2}$$

θ is the angle between $\vec{ds}$ and $\vec{r}$.

3.3 Applications of the Biot and Savart law

To determine the magnetic field, one has the choice between the Ampere's law and the Biot and Savart law.

3.3.1 B Field on axis of circular current loop

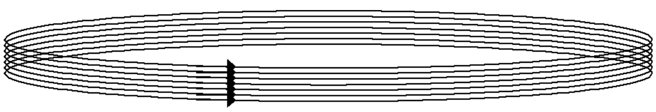


Figure 3.2: *Schematic representation of a circular current loop with N turns*

Consider circular current loop of radius R and current i. Find
(a) B field at center of loop
(b) B field on axis, including center

Solution:

(a) From Biot and Savart (B-S) law by integration, one obtains:

$$B = \frac{\mu_0 i}{2R} \tag{3.3}$$

If N turns close together:

$$B = \frac{N\mu_0 i}{2R} \tag{3.4}$$

Example: $i = 500A$, $R = 5cm$, $N = 20$

$$B = \frac{20\times(4\pi 10^{-7})\times 500}{2\times 0.05} = 1.26T$$

(b) From B-S law by integration, one obtains:

$$B = \frac{N\mu_0 i}{2}\frac{R^2}{(R^2+z^2)^{\frac{3}{2}}}, \tag{3.5}$$

z being the distance from the axis to the center.

Note that:

$z = 0$, one find the same result of (a) i.e $B = \frac{N\mu_0 i}{2R}$

$z \gg R$, one obtains $B = \frac{N\mu_0 i}{2}\frac{R^2}{z^3}$ ($\sim \frac{1}{z^3}$ like E field around electric dipole!)

3.3.2 Field at Center of Partial Loop

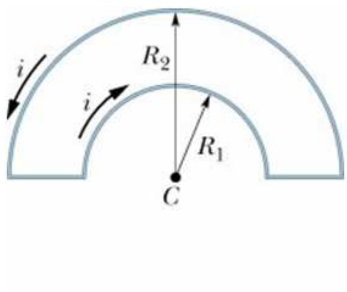


Figure 3.3: *Schematic representation of a circular partial loop with N turns*

Consider circular partial loop of radii R_1, R_2 and current i.

Suppose partial loop covers angle Φ. Calculate B field from proportion of full circle

Solution:

Direction of B: perpendicular to the cross section of the loop

$B = \frac{\mu_0 i}{2R}(\frac{\Phi}{2\pi})$

Use example where $\Phi = \pi$ **(half circles)**

$B = \frac{\mu_0 i}{2R_1}(\frac{\pi}{2\pi}) - \frac{\mu_0 i}{2R_2}(\frac{\pi}{2\pi}) = \frac{\mu_0 i}{4}(\frac{1}{R_1} - \frac{1}{R_2})$

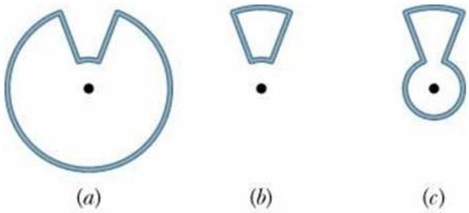


Figure 3.4: *Partial loops*

3.3.3 Partial Loops

Note on problems when you have to evaluate a B field at a point from several partial loops

- Only loop parts contribute, proportional to angle
- Straight sections aimed at point contribute exactly 0
- Be careful about signs, e.g. in (b) fields partially cancel, whereas in (a) and (c) they add