Notations

x_i

x_i^n

x^n $\hat{y}^n$

No	age	t1	t2	t3	t4	t5	t6	time	Step frequency	n1	n2	n3	n4	n5	n6	px	py	pz	Steps	Gender	TUG	y1	BBS	y2
1	70	1.76	2.64	6.24	7.02	10	12.8	11	2.285	80	120	282	317	453	575	11.67	1.809	-1.99	13	F	11	0	26	0
2	86	1.64	2.6	5.82	7.27	10.4	12.6	11	1.934	75	118	263	328	470	570	11.14	2.302	-4.651	12	F	11	0	24	0
3	76	1.76	2.93	6.27	7.04	10.3	12.8	11	2.109	80	133	283	318	465	575	11.53	2.169	-3.253	14	F	11	0	22	1
4	70	2.38	3.29	5.58	6.47	9.02	10.4	8	2.461	108	149	252	292	407	468	11.6	1.838	-3.138	12	F	8	0	24	0
5	66	3.09	4.07	6.6	7.4	10.2	12.1	9	2.461	140	184	298	334	462	545	11.55	2.531	-2.742	12	F	9	0	26	0
6	79	1.76	2.91	5.87	6.6	10.2	12.8	11	2.109	80	132	265	298	462	575		1.788	-1.349	13	F	11	0	26	0
7	85	1.2	2.33	5.42	8.31	12.1	17.2	16	2.988	55	106	245	375	545	775		2.203	-4.89	17	M	16	1	18	1
8	81	1.64	2.93	5.98	7.47	10.9	13.6	12	1.758	75	133	270	337	493	615	11.1	2.667	-4.594	10	F	12	0	24	0
9	82	0.64	1.47	4.76	5.76	9.36	11.6	11	2.109	30	67	215	260	422	525	11.26	4.1	-2.693	14	M	11	0	24	0
10	69	1.64	2.49	5.02	5.98	9.82	12.6	11	2.637	75	113	227	270	443	570	11.27	3.292	-3.522	13	F	11	0	20	1
11	84	0.64	1.4	5.67	7.29	11.5	14.6	14	1.934	30	64	256	329	520	660	11.53	2.335	-2.999	15	M	14	1	26	0
12	69	1.09	1.98	5	5.62	8.38	10.1	9	2.109	50	90	226	254	378	455	11.15	1.919	-4.608	11	M	9	0	26	0
13	73	1.09	2.13	6.78	8.38	12.4	17.1	16	3.691	50	97	306	378	558	770	11.46	2.264	-3.333	16	F	16	1	14	1
14	81	0.64	1.87	9.24	11.2	19	22.6	22	1.934	30	85	417	507	857	1020	11.58	2.511	-2.157	27	M	22	1	24	0
15	80	0.76	1.71	3.98	5	7.58	9.76	9	2.109	35	78	180	226	342	440	11.33	2.821	-3.595	10	M	9	0	26	0
16	88	0.98	2.13	6.31	7.44	11.5	14	13	1.934	45	97	285	336	518	630	11.38	2.498	-3.702	16	M	14	1	26	0
17	81	1.09	2.09	4.18	5.16	7.76	10.1	9	2.285	50	95	189	233	350	455	11.21	2.241	-4.337	10	M	9	0	28	0
18	76	1.76	2.64	5.87	6.98	9.98	12.8	11	1.406	80	120	265	315	450	575	11.33	2.679	-3.736	10	M	11	0	26	0
19	69	0.36	3.76	13.3	16.7	24.2	29.4	29	3.691	17	170	598	753	1090	1322	11.31	1.361	-4.171	28	F	29	1	10	1
20	75	1.98	2.93	5.98	7.91	12.2	15	13	1.934	90	133	270	357	551	675	11.5	2.202	-1.495	14	M	13	0	28	0
21	87	1.53	3.2	10.9	13.8	21.3	26.5	25	2.9	70	145	492	624	960	1195	11.6	2.199	-2.54	19	F	25	1	16	1
22	72	0.2	1.02	3.36	4.11	7.42	10.2	10	1.758	10	47	152	186	335	460	11.52	2.658	-2.081	9	M	10	0	28	0
23	109	0.64	1.93	5.04	5.71	9.13	10.6	10	2.285	30	88	228	258	412	480	11.51	2.056	-3.158	15	F	10	0	28	0

Learning steps

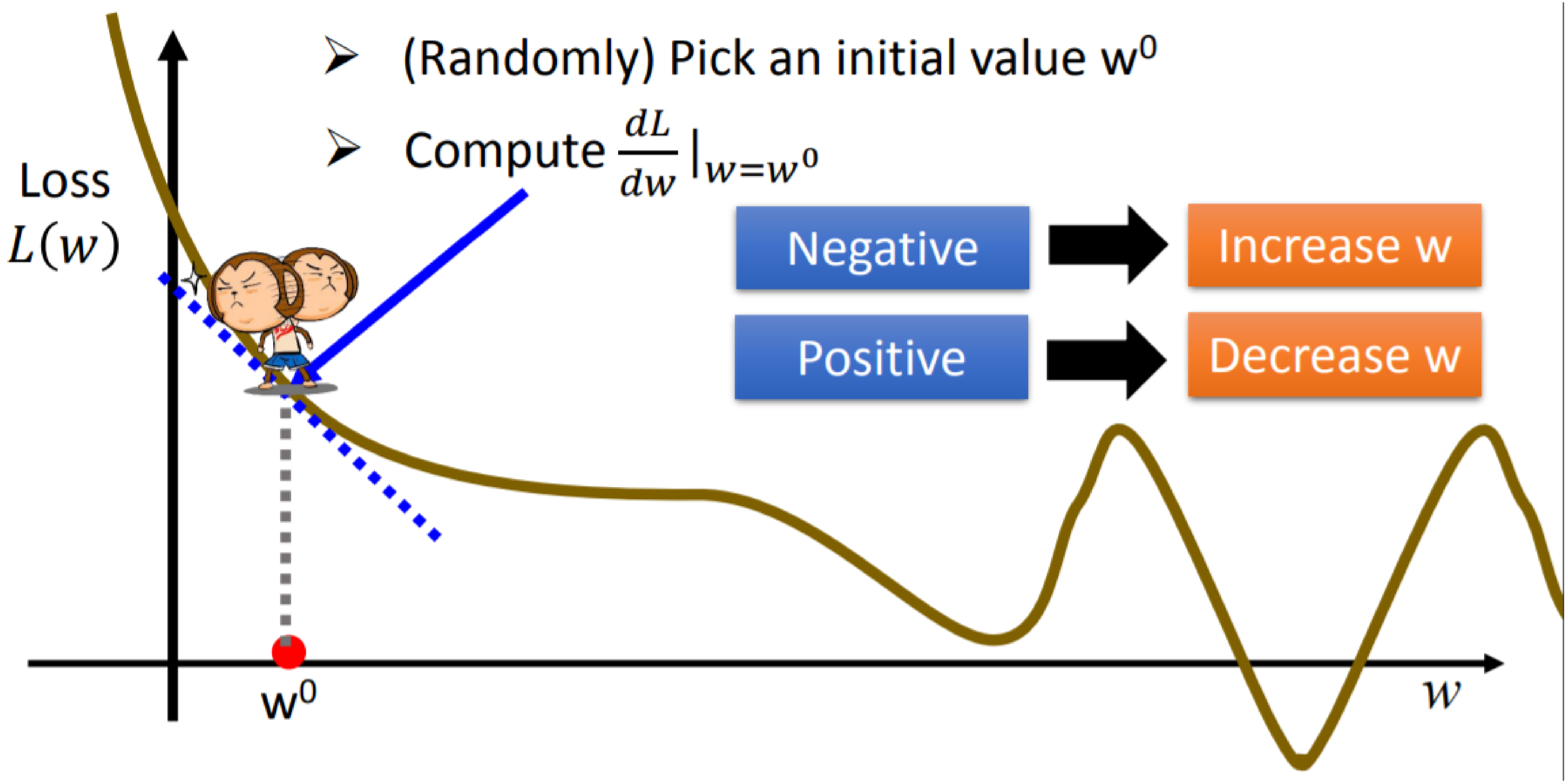
- Define a function to be learned: $y^n = f(x^n)$
- Define a loss function $\mathcal{L}(f)$ to describe the error between y^n and $\hat{y}^n$
- Find the optimal parameters that minimize $\mathcal{L}(f)$

Regression – Linear model

- Linear model: $y^n = \sum_i (w_i x_i^n) + b$
- Loss function: $L(w, b) = \sum_{n=1}^{N} (\hat{y}^n - y^n)^2 = \sum_{n=1}^{N} (\hat{y}^n - (\sum_i (w_i x_i^n) + b))^2$
- Find the optimal parameters that minimize loss: $\arg\min_{w,\, b} L(w, b)$

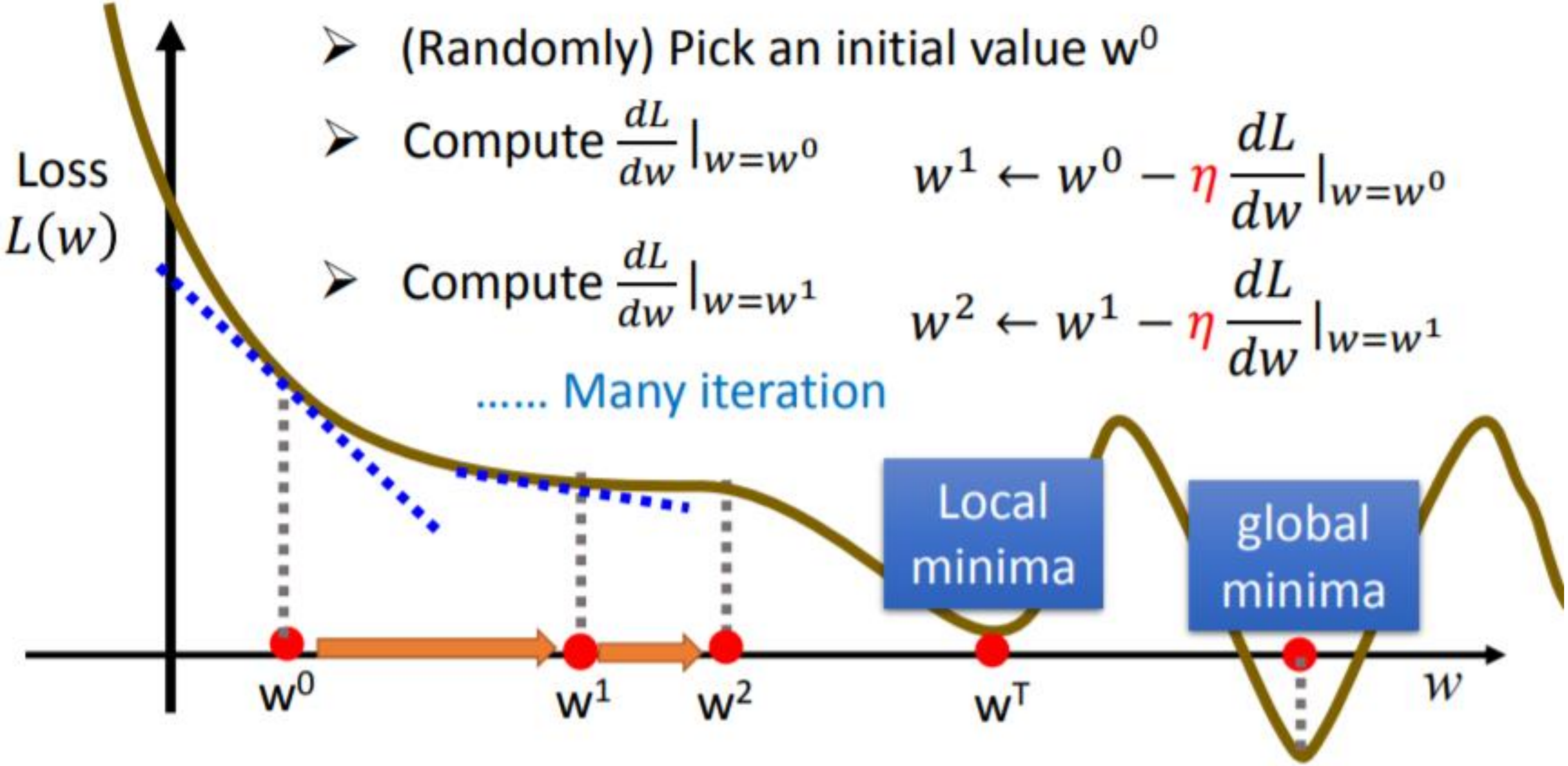
Use gradient decent to find w^*

- (Randomly) Pick an initial value w^0
- Compute $\frac{dL}{dw}|_{w=w^0}$



Reference: 李弘毅 ML Lecture 1 https://youtu.be/CXgbekl66jc

Gradient decent



Gradient decent to find two parameters w^* and b^*

- How about two parameters? $w^*, b^* = arg \min_{w,b} L(w,b)$
  - ➢ (Randomly) Pick an initial value w^0, b^0
  - ➢ Compute $\frac{\partial L}{\partial w}|_{w=w^0,b=b^0}, \frac{\partial L}{\partial b}|_{w=w^0,b=b^0}$

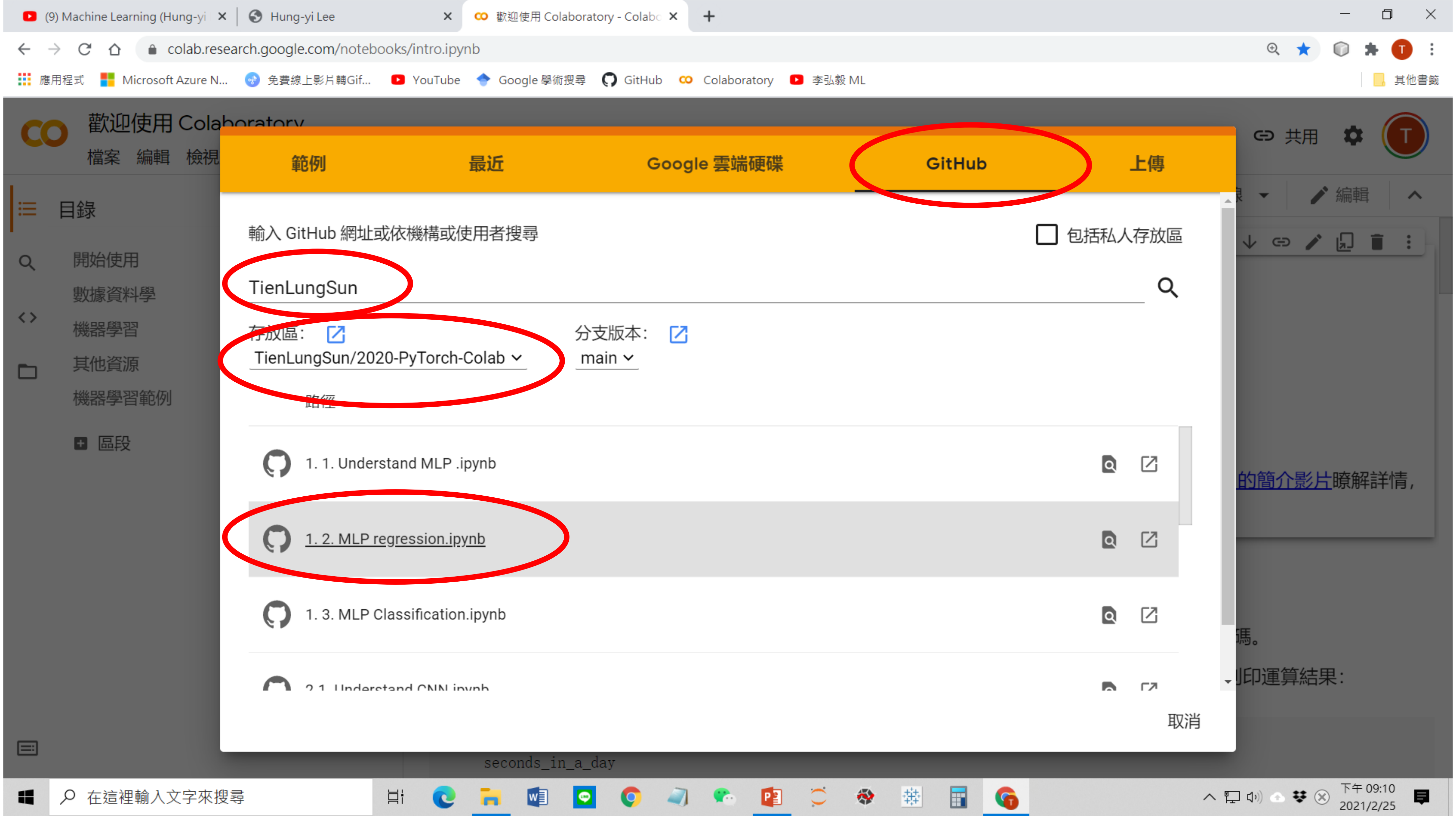
$$w^1 \leftarrow w^0 - \eta \frac{\partial L}{\partial w}|_{w=w^0,b=b^0} \qquad b^1 \leftarrow b^0 - \eta \frac{\partial L}{\partial b}|_{w=w^0,b=b^0}$$

  - ➢ Compute $\frac{\partial L}{\partial w}|_{w=w^1,b=b^1}, \frac{\partial L}{\partial b}|_{w=w^1,b=b^1}$

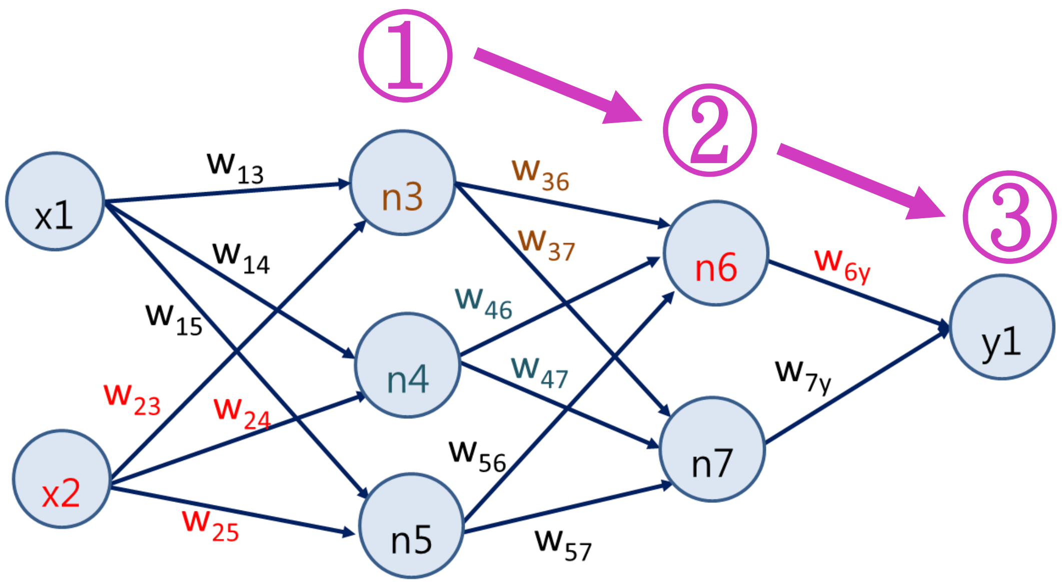
$$w^2 \leftarrow w^1 - \eta \frac{\partial L}{\partial w}|_{w=w^1,b=b^1} \qquad b^2 \leftarrow b^1 - \eta \frac{\partial L}{\partial b}|_{w=w^1,b=b^1}$$

Reference: 李弘毅 ML Lecture 1 https://youtu.be/CXgbekl66jc

Practice – MLP regression



Model = neural network



①
$$n_3 = \sigma(x_1 * w_{13} + x_2 * w_{23} + b_3)$$
$$n_4 = \sigma(x_1 * w_{14} + x_2 * w_{24} + b_4)$$
$$n_5 = \sigma(x_1 * w_{15} + x_2 * w_{25} + b_5)$$

②
$$n_6 = \sigma(n_3 * w_{36} + n_4 * w_{46} + n_5 * w_{56} + b_6)$$
$$n_7 = \sigma(n_3 * w_{37} + n_4 * w_{47} + n_5 * w_{57} + b_7)$$

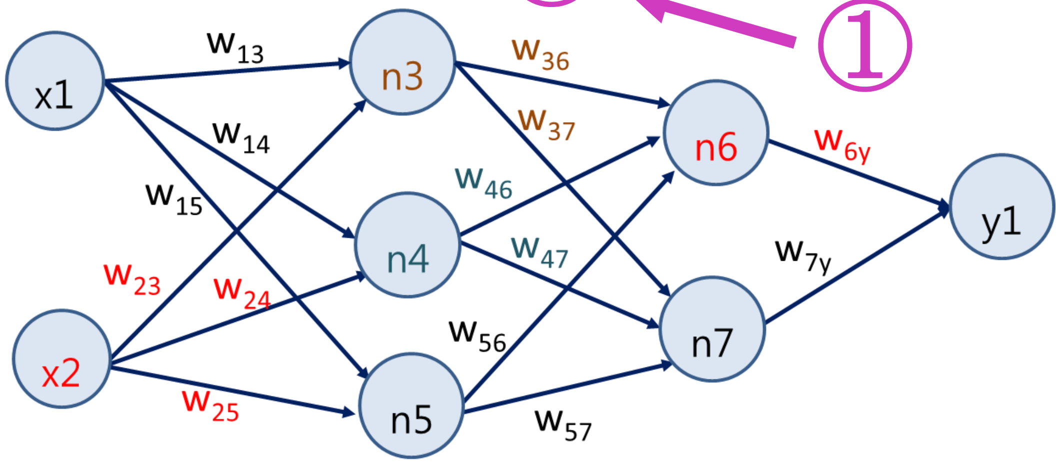
③
$$y_1 = \sigma(n_6 * w_{6y} + n_7 * w_{7y} + b_y)$$

Comparison: linear model in this case only have 3 parameters $y = \sum_i (w_i x_i) + b$

Gradient decent to find optimal parameters

$$e = g(y - y_1) \quad y_1 = \sigma(n_6 * w_{6y} + n_7 * w_{7y} + b_y)$$

③ ← ② ← ①



$$w_i \leftarrow w_i - \eta \frac{\partial e}{\partial w_i}$$

①

$$w_{6y} \leftarrow w_{6y} - \eta \frac{\partial e}{\partial w_{6y}} \qquad \frac{\partial e}{\partial w_{6y}} = \frac{\partial e}{\partial y_1} \frac{\partial y_1}{\partial w_{6y}}$$

$$w_{7y} \leftarrow w_{7y} - \eta \frac{\partial e}{\partial w_{7y}} \qquad \frac{\partial e}{\partial w_{7y}} = \frac{\partial e}{\partial y_1} \frac{\partial y_1}{\partial w_{7y}}$$

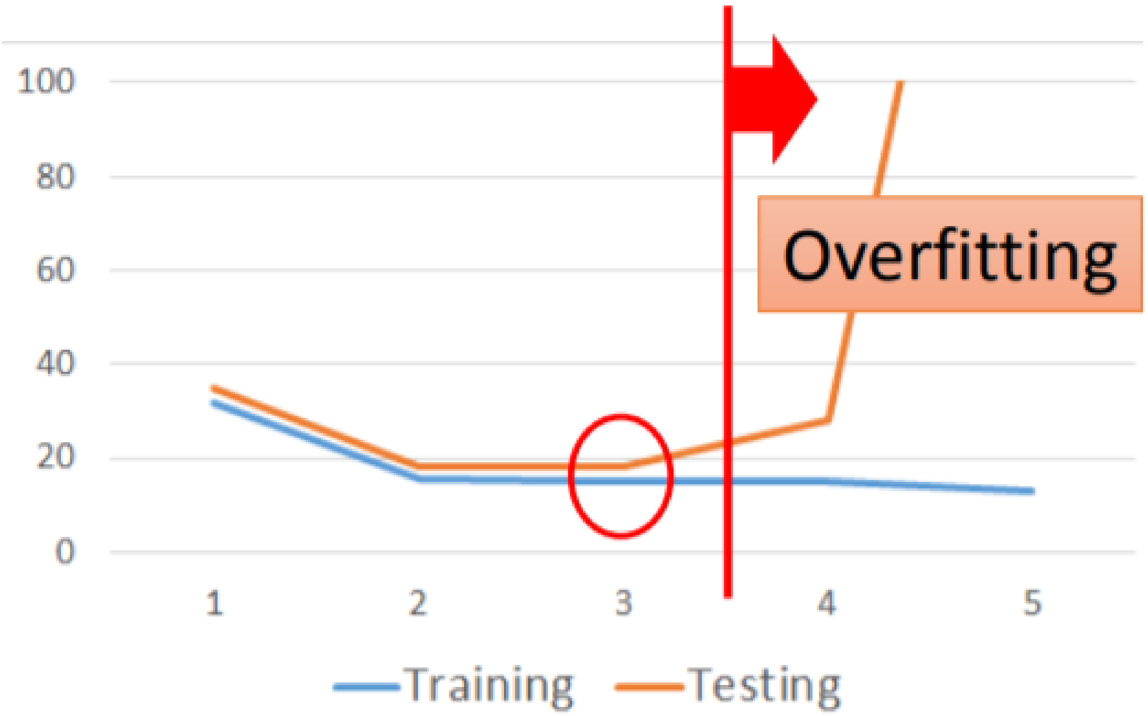
②

$$w_{57} \leftarrow w_{57} - \eta \frac{\partial e}{\partial w_{57}} \qquad \frac{\partial e}{\partial w_{57}} = \frac{\partial e}{\partial y_1} \frac{\partial y_1}{\partial n_7} \frac{\partial n_7}{\partial w_{57}}$$

$$n_7 = f(n_3 * w_{37} + n_4 * w_{47} + n_5 * w_{57} + b_7)$$

After finding optimal parameters that minimize $\mathcal{L}(f)$, we want to test the model on un-seen test data

Overfitting problem

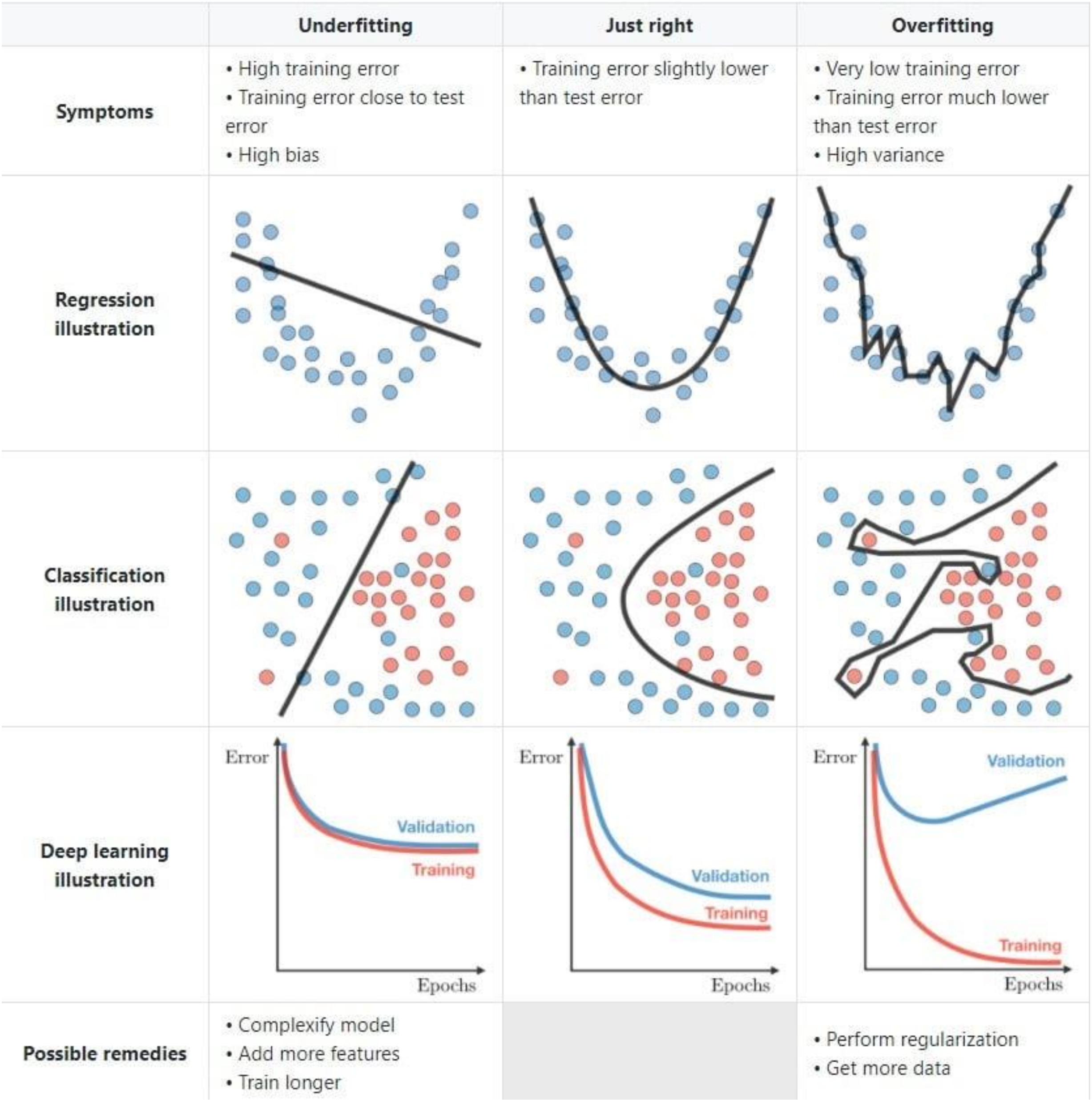


	Training	Testing
1	31.9	35.0
2	15.4	18.4
3	15.3	18.1
4	14.9	28.2
5	12.8	232.1

A more complex model does not always lead to better performance on ***testing data***.

This is ***Overfitting***. → Select suitable model

Reference: 李弘毅 ML Lecture 1 https://youtu.be/CXgbekl66jc



	Underfitting	Just right	Overfitting
Symptoms	• High training error • Training error close to test error • High bias	• Training error slightly lower than test error	• Very low training error • Training error much lower than test error • High variance
Regression illustration			
Classification illustration			
Deep learning illustration	Error Validation Training Epochs	Error Validation Training Epochs	Error Validation Training Epochs
Possible remedies	• Complexify model • Add more features • Train longer		• Perform regularization • Get more data

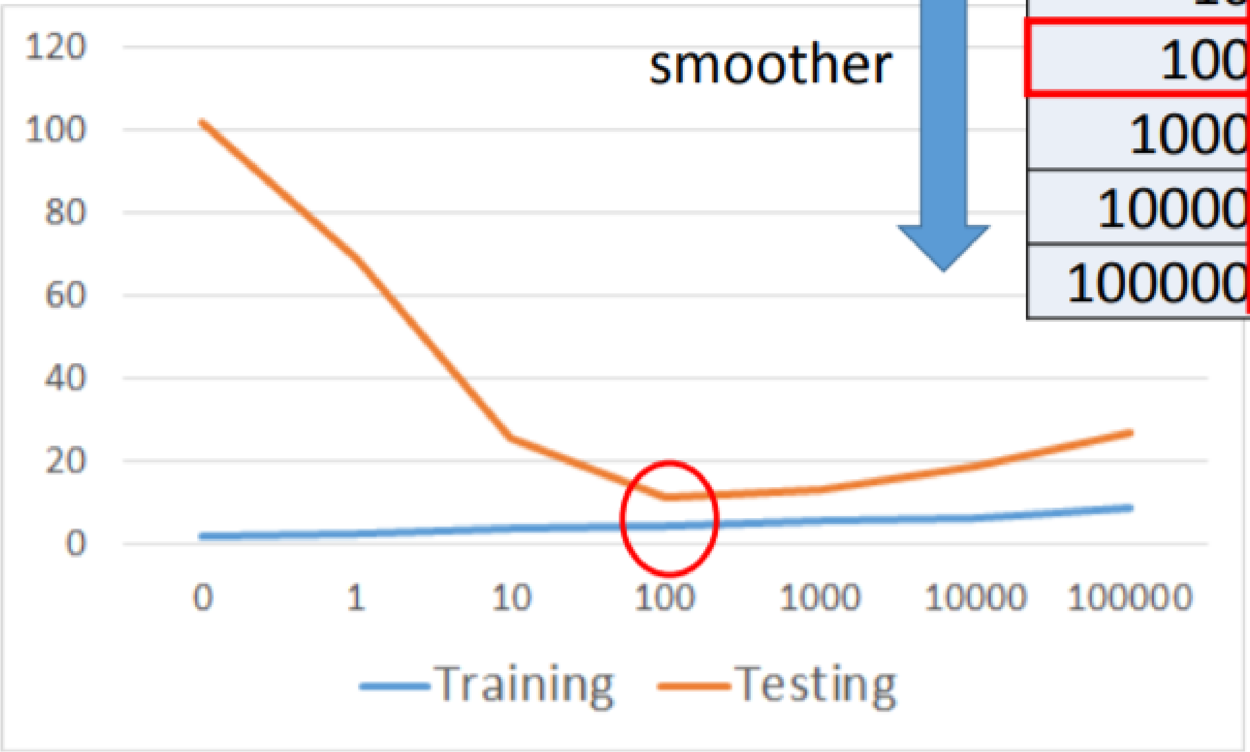
https://www.facebook.com/buggyprogrammers/photos/a.113230983855158/213140590530863/

Regularization

Regularization

smoother

λ	Training	Testing
0	1.9	102.3
1	2.3	68.7
10	3.5	25.7
100	4.1	11.1
1000	5.6	12.8
10000	6.3	18.7
100000	8.5	26.8



How smooth?

Select λ obtaining the best model

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Practice

- Design NN of different complexities (numbers of layers and nodes)
- Use loss plot to observe overfitting problem

Errors of ML models

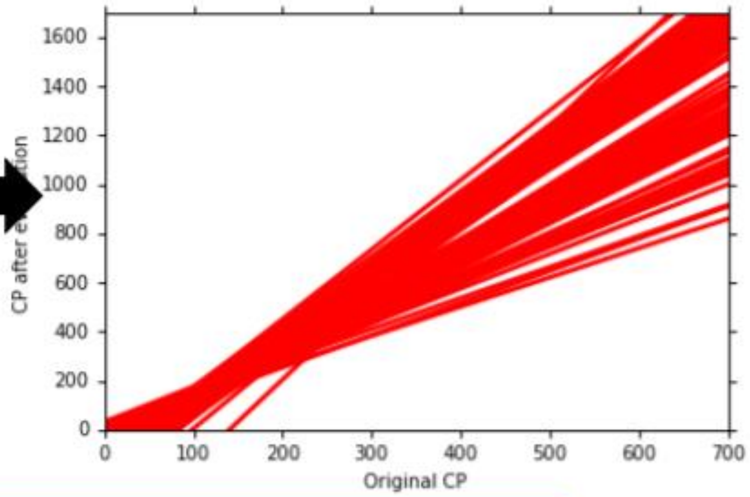
Where does the errors come from?

- ML models learned from training data will have bias and variances

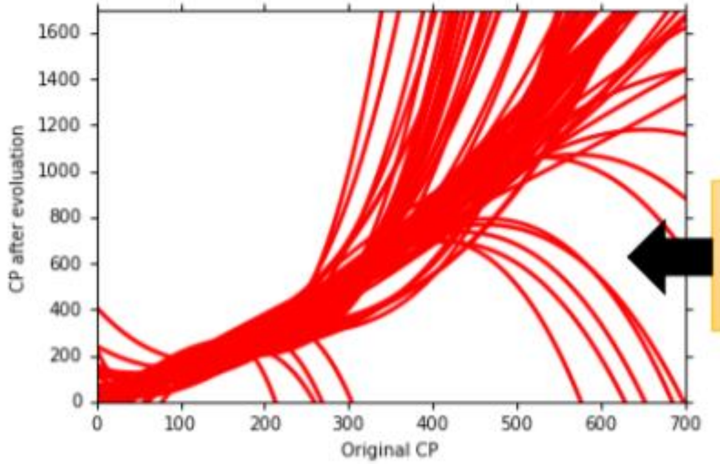
Variances of ML models

Each model is learned from 100 sampled data.

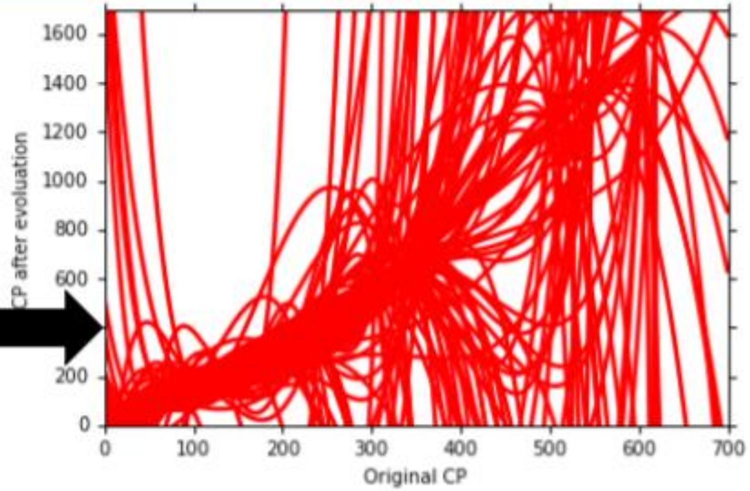
$$y = b + w \cdot x_{cp}$$



$$y = b + w_1 \cdot x_{cp} + w_2 \cdot (x_{cp})^2 + w_3 \cdot (x_{cp})^3$$

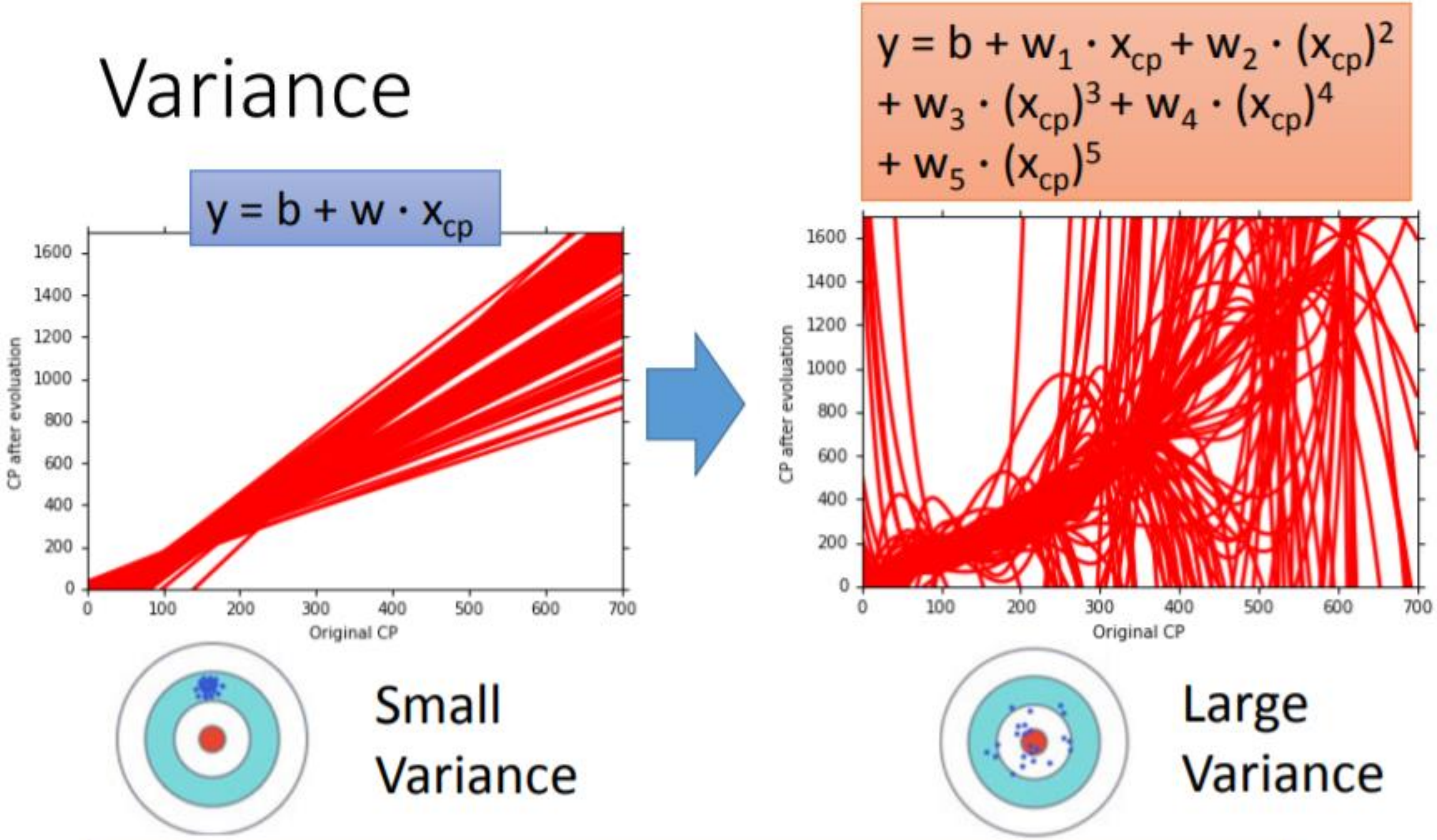


$$y = b + w_1 \cdot x_{cp} + w_2 \cdot (x_{cp})^2 + w_3 \cdot (x_{cp})^3 + w_4 \cdot (x_{cp})^4 + w_5 \cdot (x_{cp})^5$$



Reference: 李弘毅 ML Lecture 2 https://youtu.be/CXgbekl66jc

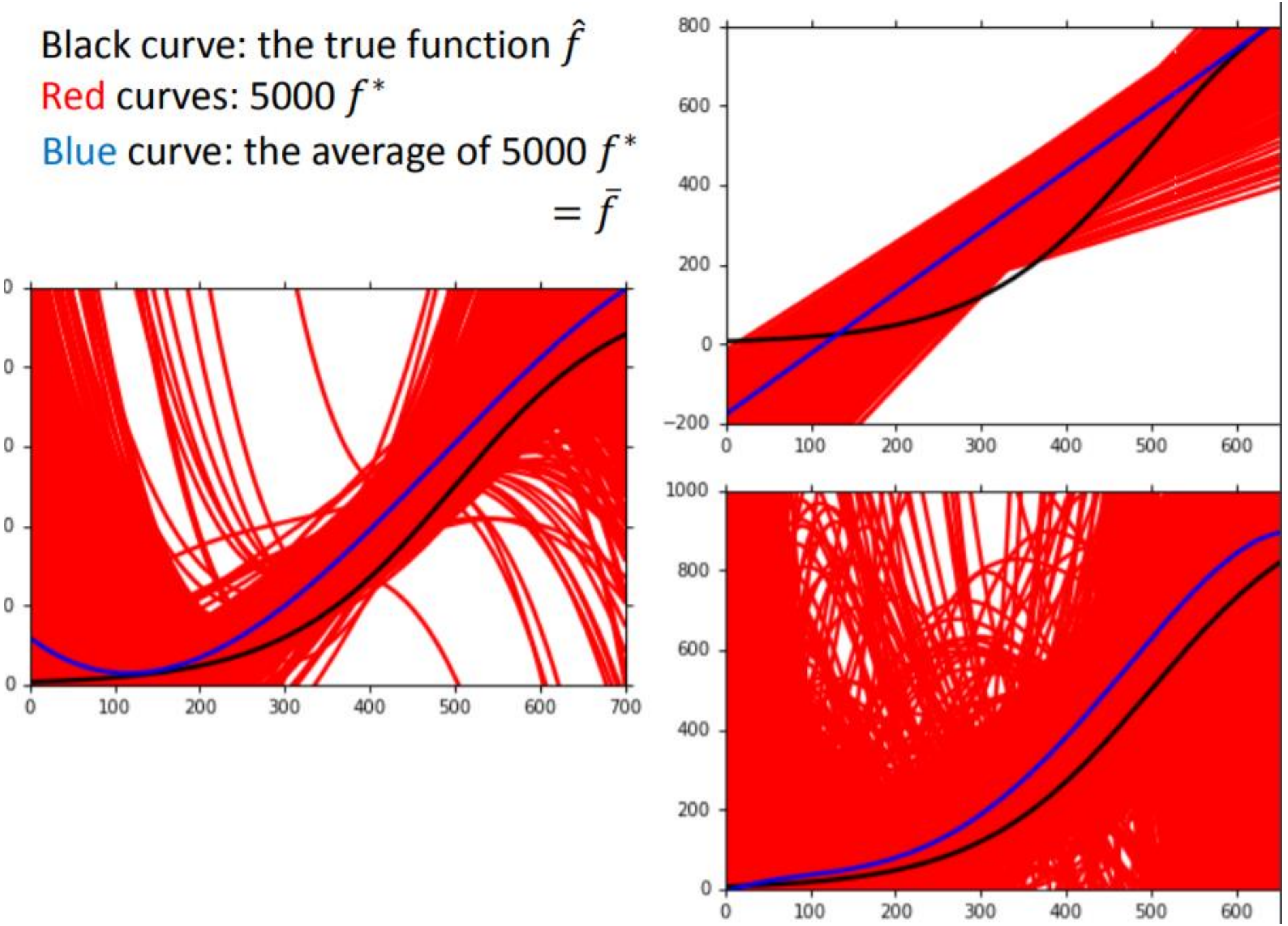
Variance of ML models



Simpler model is less influenced by the sampled data

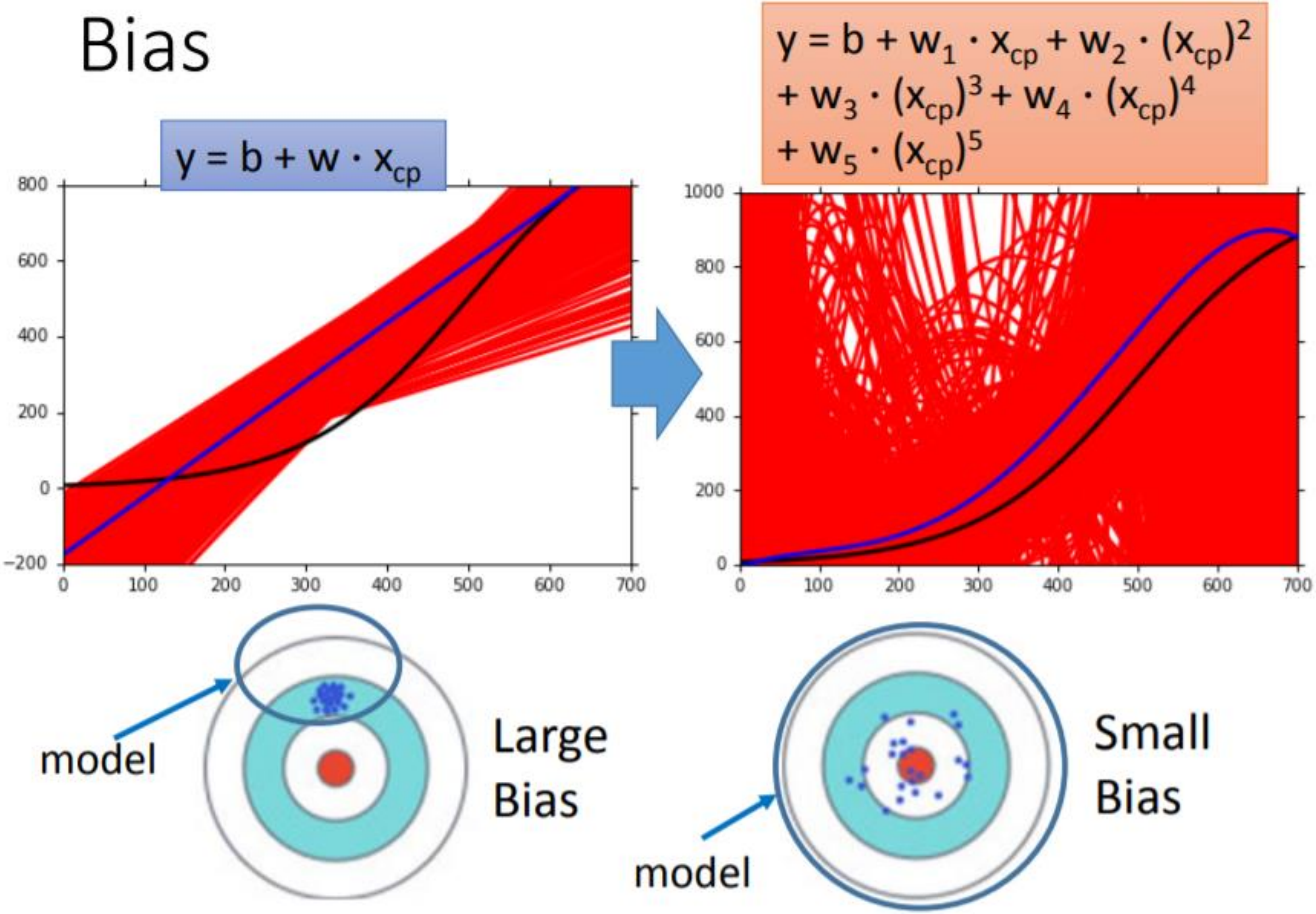
Reference: 李弘毅 ML Lecture 2 https://youtu.be/CXgbekl66jc

Bias and variance of ML models



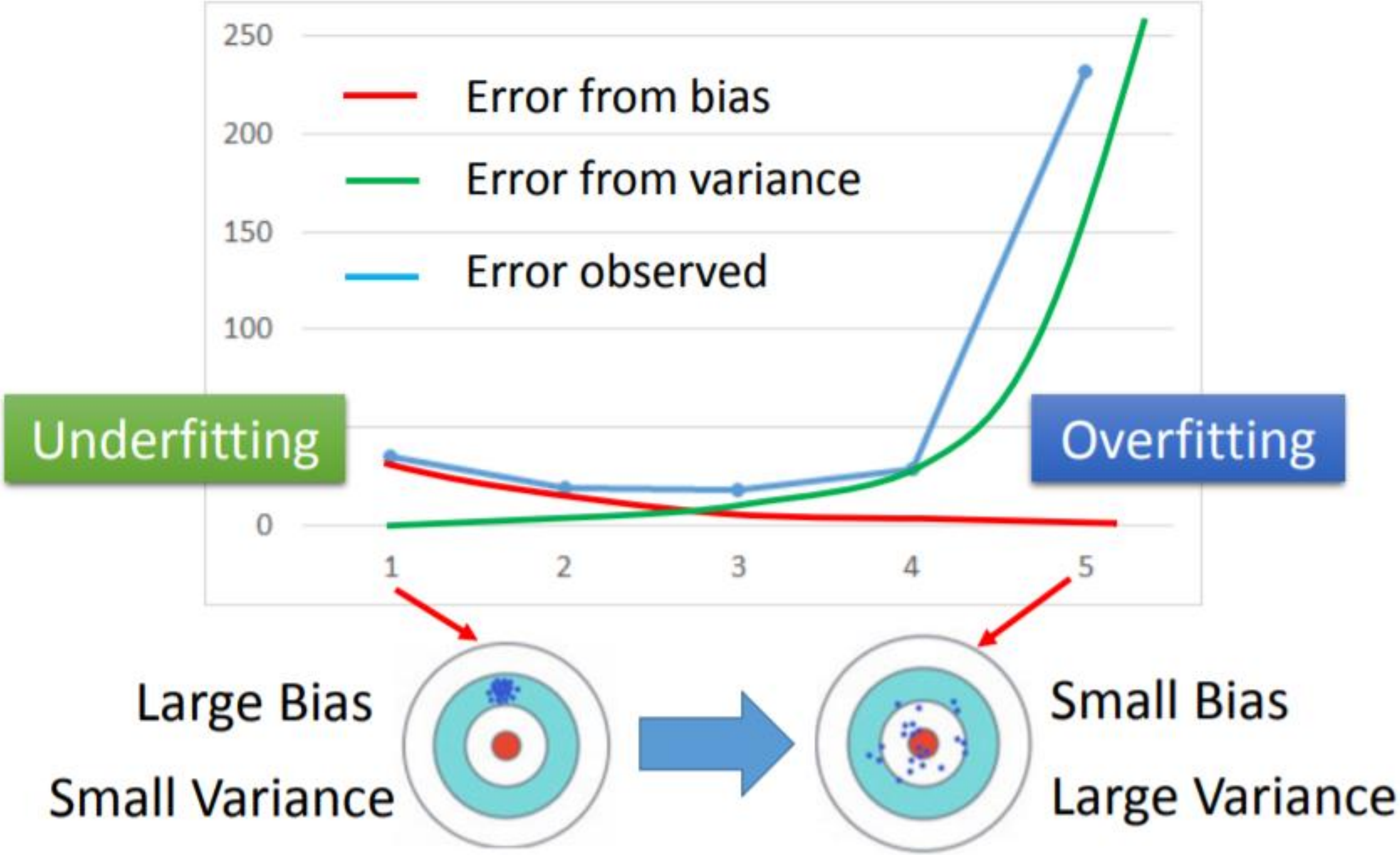
Reference: 李弘毅 ML Lecture 2 https://youtu.be/CXgbekl66jc

Bias and variance of ML models



Reference: 李弘毅 ML Lecture 2 https://youtu.be/CXgbekl66jc

Errors of ML model

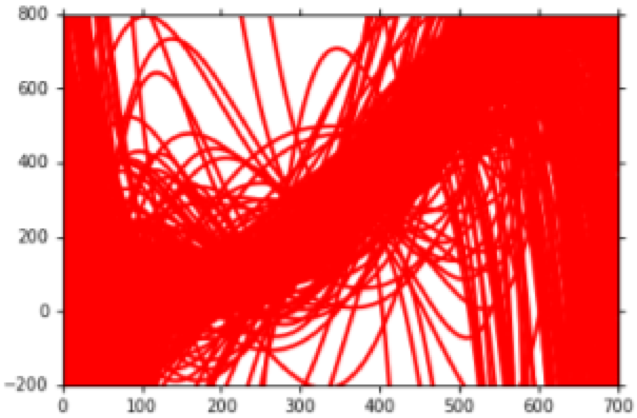


Reference: 李弘毅 ML Lecture 2 https://youtu.be/CXgbekl66jc

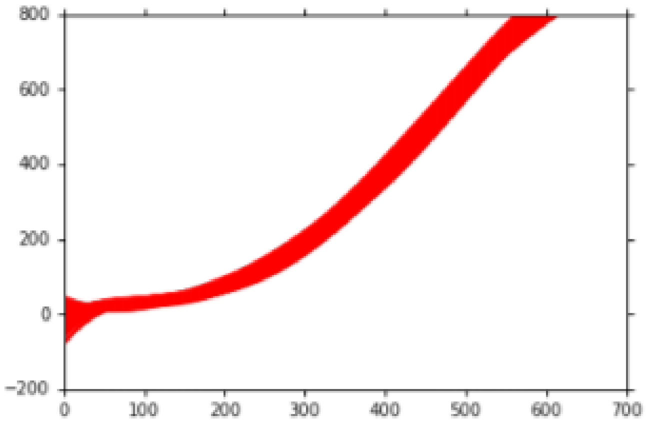
Errors of ML model → Reduce variances

- More data

Very effective,
but not always
practical

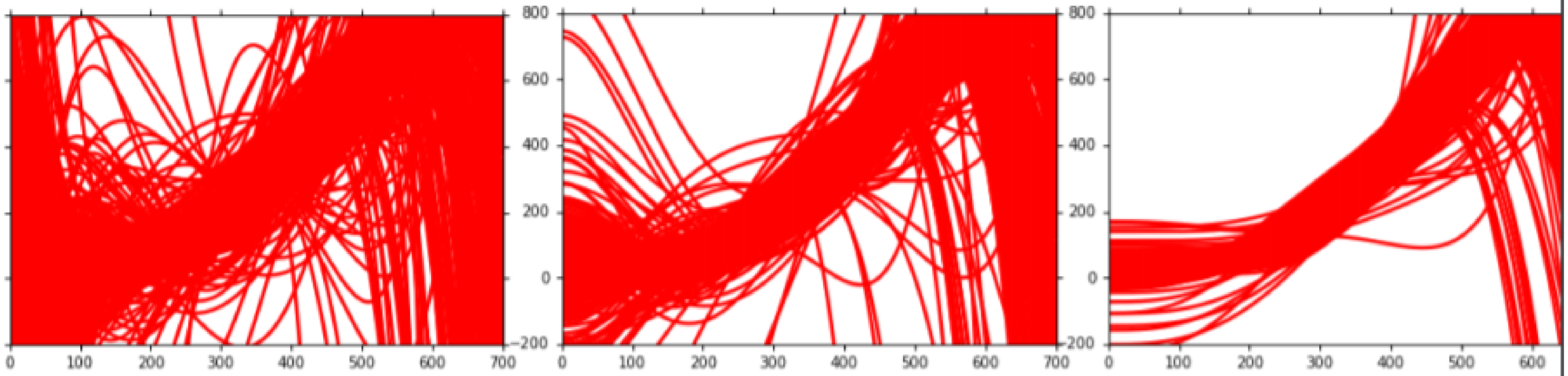


10 examples



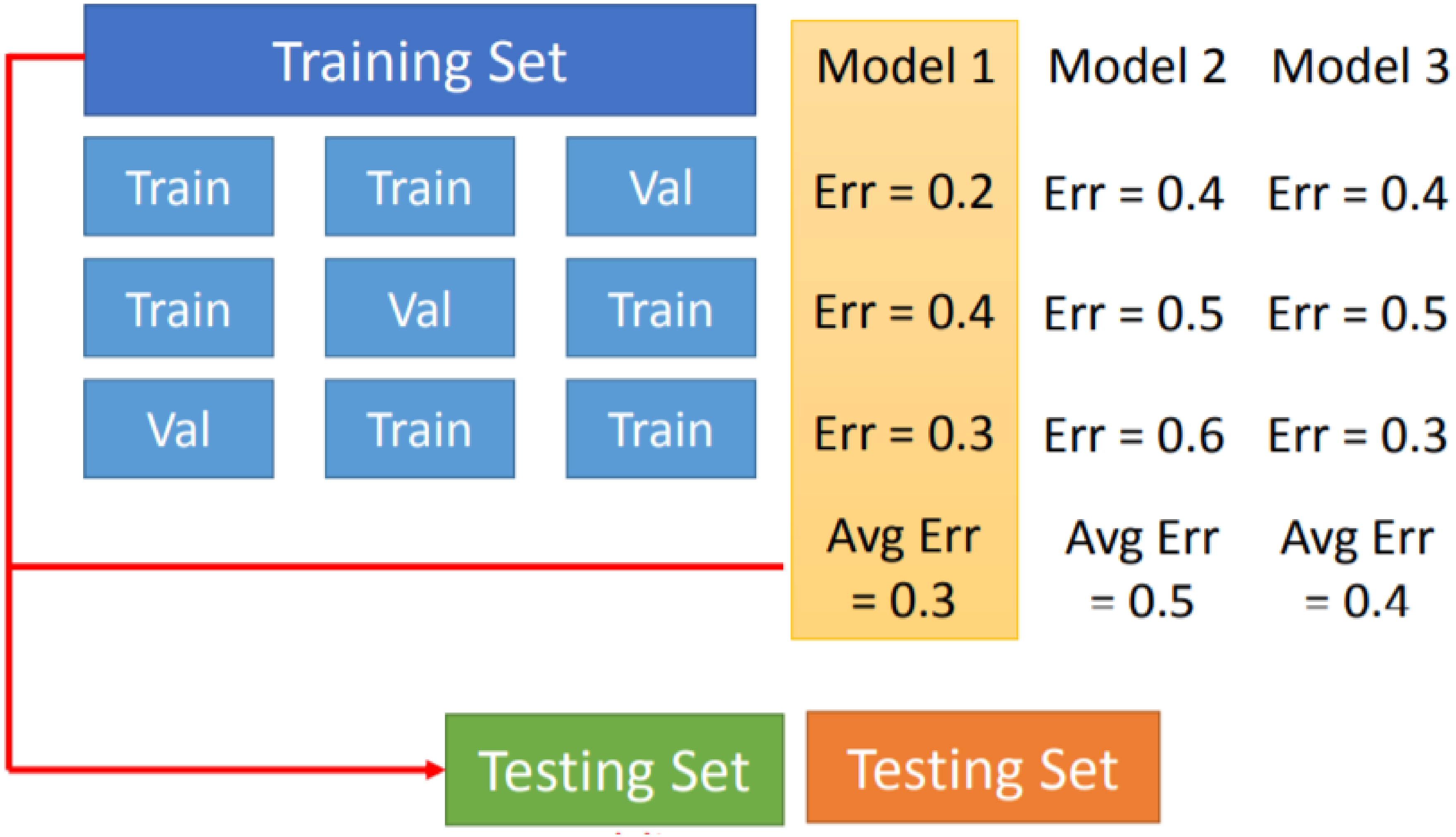
100 examples

- Regularization



Reference: 李弘毅 ML Lecture 2 https://youtu.be/CXgbekl66jc

Errors of ML model → Cross validation



Reference: 李弘毅 ML Lecture 2 https://youtu.be/CXgbekl66jc

Practice

- Use cross-validation and box-plot to examine bias and variances of the regression models learned
- Study the effect of regularization on model variance
- Study the effect of training data numbers on model variance
- Use loss plot to study the effect of learn rates