Parallelizing-Mandelbrot set

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Introduction

The Mandelbrot set is a fractal defined by iterating a complex function (z_{n+1} = z_n^2 + c), where (z) and (c) are complex numbers. If the iterated function remains bounded for all (n), the point (c) belongs to the Mandelbrot set; otherwise, it diverges.

Equation: [z_{n+1} = z_n^2 + c]

Condition: [|z_n|^2 ≤ 4]

How we parallelized the Problem

The Mandelbrot set is particularly convenient t parallelize for a message passing system because each pixel can be computed without any information about the surrounding pixels. When parallelizing the computation of the Mandelbrot set, we employed two distinct approaches to distribute the workload among multiple processes using MPI

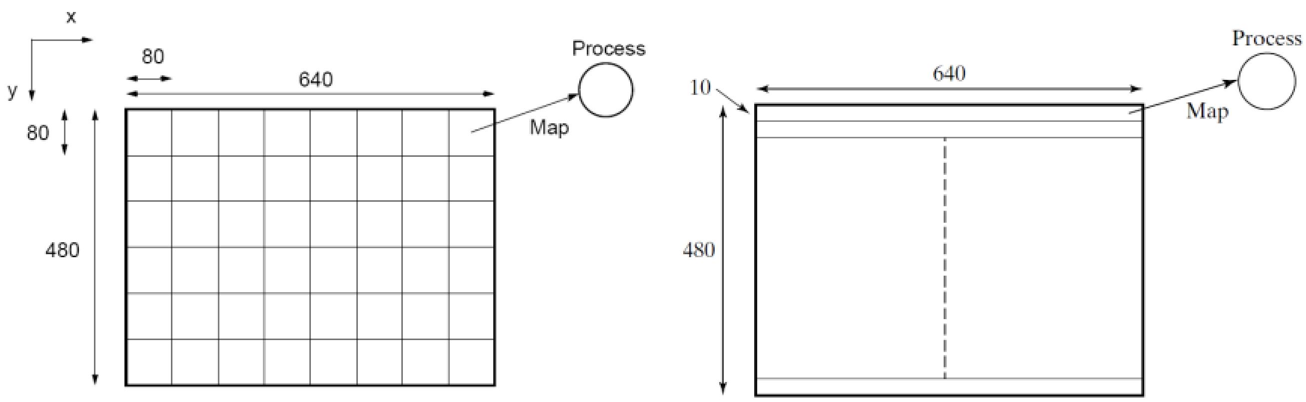
Static

each MPI process is responsible for a fixed portion of the grid, and the workload is evenly divided among processes. The results are gathered at the end for image generation.

-Pseudo code:

```
  Master Process (Pmaster):
Calculate rows_per_process = HEIGHT / num_proc.
Calculate remainder_rows to distribute the remaining rows evenly
For each process (Pi) except the master:
    Calculate the starting row (start_row) for Pi.
    Calculate the ending row (end_row) for Pi based on rows_per_process.

Initialize an empty image array to collect results.
Collect results from all slave processes:
    For each slave process:
        Receive a message containing the portion of the Mandelbrot set computed by the slave.
        Extract the starting row index from the received message to identify where this portion fits in the final image array.
        Store the received portion in the appropriate place in the final image array.

Save the collected image using save_pgm.
```

```
Slave Process (Pslave):
Based on the received rank, calculate the assigned rows to compute (start_row to
end_row).

    For each row assigned:
        For each pixel (x) in the row:
            Calculate the complex coordinates (c.real, c.imag) based on the
pixel's x-coordinate and the row number.
            Compute the color for the pixel using cal_pixel(c) and store it in a
local part of the image.

Send the computed local image part along with its starting row index back to the
master process using MPI_Send.
```

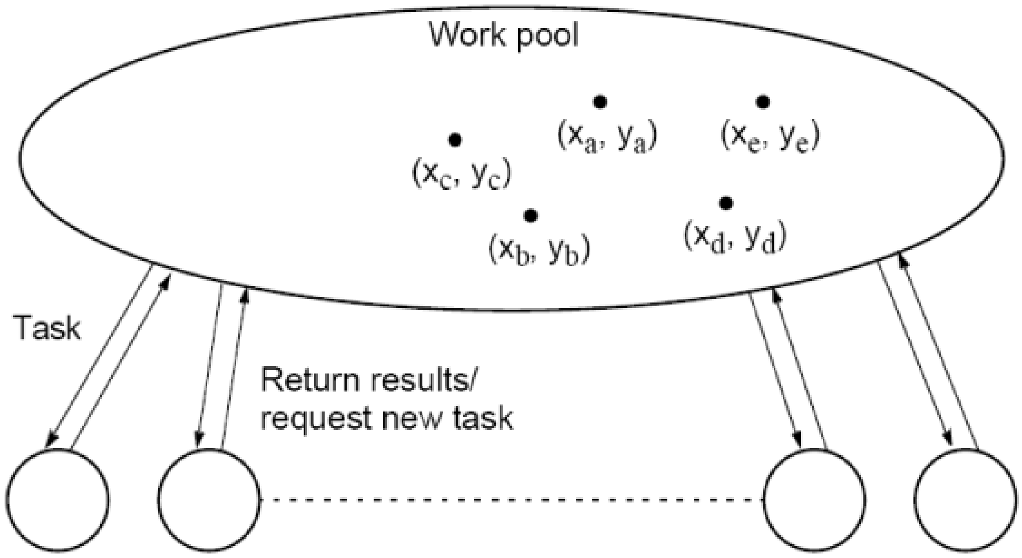


Dynamic

The dynamic approach on the other hands has initially each slave process is given a row to compute. As soon as a process finishes computing its assigned row and sends the result back to the master it is immediately assigned a new row to work on. This process repeats until all rows have been computed. The dynamic method ensures that all processes are kept busy as much as possible dynamically redistributing work based on process availability. This could perform better than the static approach since here we aim to minimize idle time for processors. -Pseudo code:

```
Master Process (P_master):

Initialize an empty 2D array image[HEIGHT][WIDTH] to store the final result.
Send the first batch of row numbers to each slave process, marking the start of
their workload.

While there are rows left to assign:
  Receive a completed row from any slave process along with the row index.
  Update the image array with the received row at the correct index.
  Send a new row number to the slave process that just completed work, if any rows
remain.

After all rows are assigned and received, send a termination signal to all slave
processes
```

```
Save the final image using save_pgm.

Slave Process (P_slave):
Receive the initial row number from the master process.
While not receiving a termination signal:
For each assigned row:
  For each pixel in the row:
    Calculate the complex coordinates (c.real, c.imag) based on the pixel's
coordinates.
    Compute the color for the pixel using cal_pixel(c) and store it in a local
array.
  Send the computed row along with its index back to the master process.
  Wait for the next row number or a termination signal from the master process.
```



Setup:

I used google colab to run the MPI programs for the Mandelbrot Set

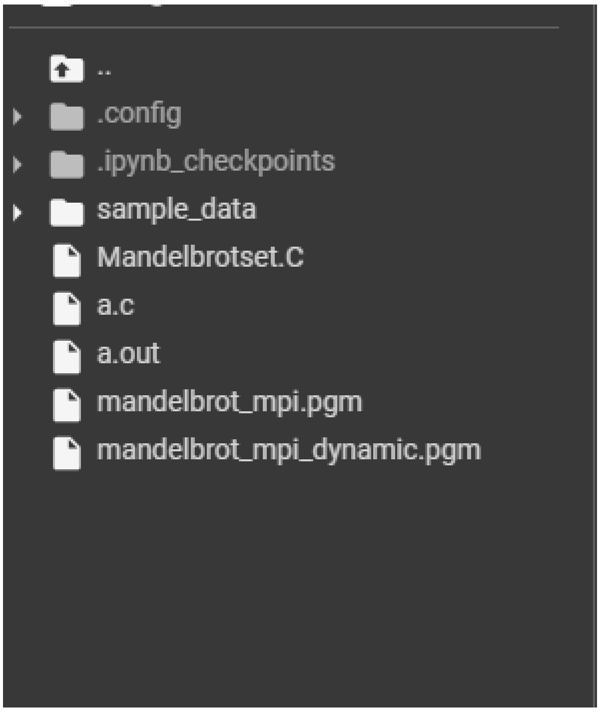
```
                }
                free(image);
            } else {
                MPI_Send(local_image, rows_per_process * WIDTH, MPI_INT, 0, 0, MPI_COM
            }
            free(local_image);
        }

        MPI_Finalize();
        return 0;
    }


    EOF
    ls -l
```

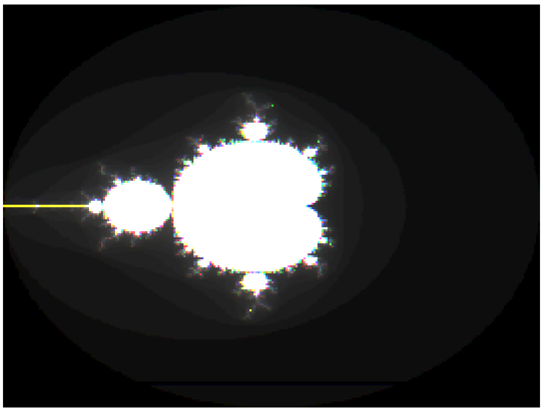
```
total 4916
-rw-r--r-- 1 root root    4213 Feb 18 20:56 a.c
-rwxr-xr-x 1 root root   16680 Feb 18 20:11 a.out
-rw-r--r-- 1 root root 2228993 Feb 18 12:42 mandelbrot2.pgm
-rw-r--r-- 1 root root  681010 Feb 18 20:09 mandelbrot_mpi_dynamic.pgm
-rw-r--r-- 1 root root  702959 Feb 18 19:03 mandelbrot_mpi.pgm
-rw-r--r-- 1 root root  686866 Feb 18 17:44 mandelbrot.pgm
-rw-r--r-- 1 root root    2346 Feb 18 10:21 Mandelbrotset.C
drwxr-xr-x 1 root root    4096 Feb 14 14:28 sample_data
-rw-r--r-- 1 root root  686866 Feb 18 15:33 stat_mandelbrot.pgm
```

```
[420] !mpicc  a.c -lm
```

```
!mpirun --oversubscribe -np 2 --allow-run-as-root a.out
```

```
Average CPU time used (including communication): 0.044838 seconds
Average computation time only: 0.041700 seconds
```





Code

https://github.com/Tamerkobba/Parallelizing-mandelbrotset-using-MPI

Performance

Sequential

Best sequential execution time=0.076644s

Static

Speed up factor

S(2)=0.076644/0.04643 8=1.6507 S(4)=0.076644/0.040987 =1.8699 S(8)=0.076644/0.031549=2.429 S(16)=0.076644/0.018901=4.055 S(32)=0.076644/0.009695=7.9055 S(64)=0.076644/0.005576=13.7453 S(128)=0.076644/0.003884=19.733 S(256)=0.076644/0.004755=16.1186

Efficiency

Efficiency(E) = Execution time using one processor/ Execution time using a multiprocessor x number of processors

Processors	Speedup	Efficiency (%)
2	1.6507	82.535%
4	1.8699	46.7475%
8	2.429	30.3625%

Processors	Speedup	Efficiency (%)
16	4.055	25.34375%
32	7.9055	24.7046875%
64	13.7453	21.47703125%
128	19.733	15.41640625%
256	16.1186	6.296328125%

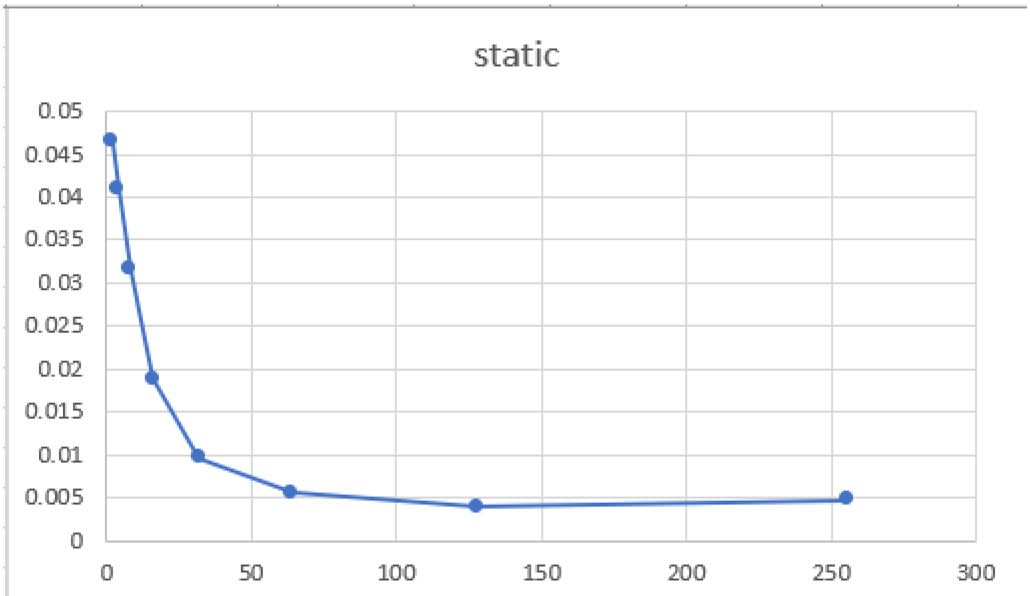
Computation to communication ratio

Number of processors	Total Speed in seconds	Computation time(without communication)
2	0.044838	0.041700
4	0.040987	0.002388
8	0.031549	0.001105
16	0.018901	0.000533
32	0.009695	0.000245
64	0.005576	0.000132
128	0.003884	0.000067
256	0.004755	0.000036

Number of processors	Computation/Communication ratio
2	0.93
4	0.5826
8	0.03502
16	0.02819
32	0.2527
64	0.236728
128	0.017250
256	7.57x10^-3

The computation to communication ratio for the dynamic parallelization of the Mandelbrot set decreases significantly with the increase in the number of processors, indicating a rise in communication overhead relative to computation time. Particularly, the ratio shows a substantial drop as we move from 2 to 256 processors, with the lowest ratios observed at the highest processor counts. This trend suggests that while parallelization reduces total computation time, the cost of communication becomes more pronounced with more processors involved. This trend highlights the importance of carefully balancing computation and communication in the design of parallel algorithm

Scalability



Looking at the results of our parallel algorithm up to 256 processors reveals a pattern of increasing speedup with an initial rise in processor count, peaking at 128 processors before slightly declining at 256 processors. However, efficiency significantly decreases as more processors are added, starting strong at 82.535% with 2 processors and plummeting to 6.296% with 256 processors. This indicates that while the algorithm benefits from parallelization initially, its scalability is limited by diminishing efficiency at higher processor counts.

Dynamic

Speed up factor

S(2)=0.076644/0.099608=0.7694 S(4)=0.076644/0.029533=2.595 S(8)=0.076644/0.00386=19.855 S(16)=0.076644/0.002622=29.2311 S(32)=0.076644/0.0026225=29.225 S(64)=0.076644/0.00239=32.068 S(128)=0.076644/0.002285=33.5422 S(256)=0.076644/0.00194=39.507

Efficiency

Efficiency(E) = Execution time using one processor/ Execution time using a multiprocessor x number of processors

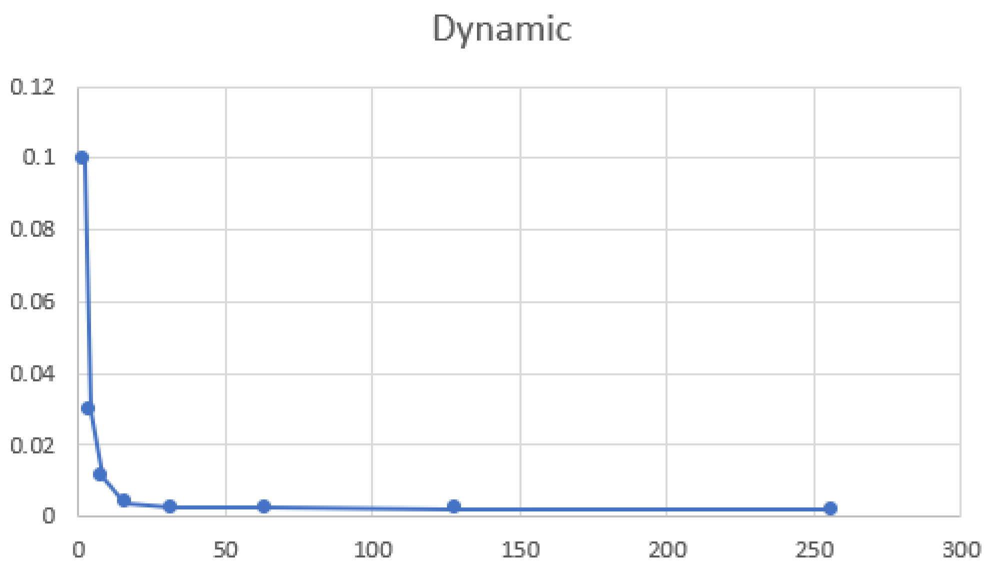
Number of Processors	Speedup	Efficiency (%)
2	0.7694	38.47
4	2.595	64.875
8	19.855	248.1875
16	29.2311	182.694375
32	29.225	91.328125
64	32.068	50.10625
128	33.5422	26.20484375

Number of Processors	Speedup	Efficiency (%)
256	39.507	15.432421875

Computation to communication ratio

Scalability

Number of processors	Speed in seconds
2	0.099608
4	0.029533
8	0.011122
16	0.00386
32	0.002622
64	0.00239
128	0.002285
256	0.00194



We notice as we increase we increase the number of processors it benefits in terms of speed up.However, the efficiency significantly decreases with higher processor counts, particularly beyond 32 processors, suggesting diminishing returns on scalability.

Discussion and Conclusions

We saw both approaches generally lead to better speeds compared to the sequential to varying degrees of speedup. Static implementation while straightforward lead to suboptimal performance due to fixed process assignments, potentially leaving some processors idle even when optimizing this approach by sending back the results from the slave to the master as an array instead of one pixel at a time to reduce communication overhead it didn't perform as well as the dynamic implmentation. In contrast The dynamic approach was notable to achieving superlinear speedup for when processors where between 8 and 16 counterintuitive result can be attributed to factors such as improved cache utilization and dynamic load balancing, which optimizes processor usage by distributing workloads more effectively across processors.This was because dynamic load balancing adapts to the varying computational demands of each task and the differing speeds of processors ensuring a more efficient utilization of resources by assigning tasks to processors as they become available. As the number of processors increases, however, efficiency naturally declines due to the overhead associated with managing more parallel tasks and communication. Nonetheless, the scalability of the dynamic approach is evident, with substantial reductions in computation time as more processors are employed, underscoring the benefits of dynamic workload distribution in parallel computing environments.