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2.1 The Kruskal extension ✓(still questions)

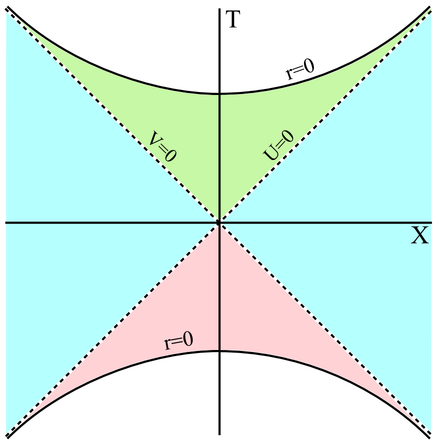

Figure 1. This is the XT plane of the **Kruskal extension**. The horizons are the dashed lines. The blue wedges are the exterior, the green wedge is the future interior and the past interior is in red. Nothing can escape the green wedge into the blue wedges, because there is no radial null geodesic which would connect the wedges.

From here on, we will use $r_s = 2GM = 1$. Instead of using the coordinates (t, r, Ω) like in the previous section, we now introduce the Kruskal-Szekeres coordinates, because they are a better choice for near-horizon physics.

First we parametrize the radial null geodesics in the Schwarzschild geometry as

was sind geodäten? lösungen der geodätengleichung, vielleicht eine fussnote?

$$t = \pm r_* + C, \tag{2.1}$$

where C C is some constant of motion and r_* is a new radial coordinate defined as

$$r_* \equiv r + \log(r - 1). \tag{2.2}$$

also called the *tortoise coordinate*[1], because now we have an infinite coordinate range that fits in a finite geodesic distance.

The Kruskal-Szekeres coordinates are then defined as

$$U \equiv -e^{\frac{r_* - t}{2}} \tag{2.3}$$

[1] The name *"tortoise"* has its origin in the paradox of Achilles and the tortoise.

$$V \equiv e^{\frac{r_* + t}{2}}. \tag{2.4}$$

Their lines of constant U and V are radial null geodesics and these coordinates have the property, that

$$UV = (1 - r)e^r. \tag{2.5}$$

This means we have a singularity at $UV = 1$ and the horizon is at $U = 0$ or $V = 0$. The metric looks now like

$$\mathrm{d}s^2 = -\frac{2}{r}e^{-r}\left(\mathrm{d}U\mathrm{d}V + \mathrm{d}V\mathrm{d}U\right) + r^2\mathrm{d}\Omega_2^2 \tag{2.6}$$

Because this metric still has an off-diagonal tensor, we define another set of coordinates

$$\begin{aligned} U &= T - X \\ V &= T + X \end{aligned} \tag{2.7}$$

Now the metric looks as follows:

$$\mathrm{d}s^2 = \frac{4}{r}e^{-r}\left(-\mathrm{d}T^2 + \mathrm{d}X^2\right) + r^2\mathrm{d}\Omega_2^2 \tag{2.8}$$

Note that there is now no singularity at $r = 1$.

This metric defines a geometry over the full XT plane, which can be seen in Figure 1. The **right blue wedge** is, in the old Schwarzschild coordinates, former $r > 1$, $-\infty < t < \infty$. If one wants to continue to $r < 1$, there is a branch cut defined in (2.3), which allows us to either go to the regions with $X^2 - T^2 < 0$ and $T > 0$ which is the **green wedge**, or the **red wedge** with $T < 0$ and also $X^2 - T^2 < 0$. At last we have the left blue wedge in which we also can have $r \gg 1$. Both blue wedges are asymptotically Minkowski regions.

wie sieht dieser branch cut genau aus?

The singularity $r = 0$, which you find at the top and the bottom of Figure 1, is the hyperboloid $X^2 - T^2 = -1$. It has two connected components, one at each boundary of the green and the red regions. These two regions are also called the future and the past interiors, while the other two are called the original/new exteriors (right/left blue wedges).

All regions together can interpret the full Schwarzschild metric as a wormhole connecting two nearly flat universes, both acting at $r \gg 1$ as if there were a point source of mass M. Signals can not travel through the wormhole, but two observers coming from opposite sides could meet in the middle and compare notes.

how the hell does he know that?

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