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2.1 Eigenstate and Euklidean path integral in general ✓

We now introduce the Euclidean path integral because we want to find the eigenstates of the left and right Rindler wedges.

First of all we have a ground state $\langle\Omega|$ of H and a vacuum state $\langle 0|$, so we can write for a very long time T

$$\begin{aligned} e^{-iHT}\langle 0| &= \sum_n e^{-iE_nT}\langle n|\,\langle n|0\rangle \\ &= e^{-iE_0T}\langle\Omega|\,\langle\Omega|0\rangle + \sum_{n\neq 0} e^{-iE_nT}\langle n|\,\langle n|0\rangle \end{aligned}$$

Now we enclose and get the ground state

$$\langle\Omega| = \lim_{T\to\infty}\frac{e^{iE_0T}}{\langle\Omega|0\rangle}\,e^{-iHT}\langle 0|$$

We can define E_0 with $H_0\langle 0| = \langle 0|$ so

$$\langle\Omega| = \frac{1}{\langle\Omega|0\rangle}\lim_{T\to\infty} e^{-iHT}\langle 0|$$

(see p.86 in [2]) But this equation does still tell us nothing about the entanglement. So let's continue:
Let a time-independent field ϕ act on this ground state:

$$\langle\phi|\Omega\rangle = \frac{1}{\langle\Omega|0\rangle}\lim_{T\to\infty}\langle\phi|e^{-iHT}|0\rangle$$

Now use the Euclidean path integral formalism, rotate t about 90° into the complex plane: $t\to -it_E$ and choose the early boundary condition $\phi = 0$, so that

$$\langle\phi|\Omega\rangle \propto \int_{\hat\phi(t_E=-\infty)=0}^{\hat\phi(t_E=0)=\phi} D\hat\phi e^{-I_E} \tag{2.1}$$

with the Euclidean action for a free massive scalar field

$$I_E[\hat\phi] = \frac{1}{2}\int \mathrm{d}^3x\mathrm{d}t_E\left[(\partial_{t_E}\hat\phi)^2 + (\vec\nabla\hat\phi)^2 + m^2\hat\phi^2\right] \tag{2.2}$$

Note that $\hat\phi$ compared to ϕ is time-dependent.

References

[1] D. Harlow. *Jerusalem Lectures on Black Holes and Quantum Information*. URL: https://arxiv.org/abs/1409.1231.

[2] M.E. Peskin and D.V. Schroeder. *An Introduction to Quantum Field Theory*. 1995.