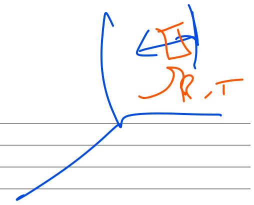
$$\underset{\text{img}}{p_i} = \overset{3\times 4}{M} \underset{\text{world}}{P_i}$$

2dH ... 3dH

$\underset{\text{world}}{P_i} \to \underset{\text{image}}{p_i}$ Projection

$\underset{\text{img}}{p_i} \to \underset{\text{world}}{P_i}$ reconstruction



Perspective projection

$$\underset{3\times4}{M} = \underset{3\times3}{K^*} \underbrace{[\underset{3\times3}{R^*} \mid \underset{3\times1}{T^*}]}_{3\times4}$$

K^*: intrinsic parameters; $[R^* \mid T^*]$: extrinsic parameters

$$K^* = \begin{bmatrix} \alpha_u & s & u_0 \\ 0 & \alpha_v & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\alpha_u = f k_u$

$\alpha_v = f k_v$

$s = \alpha_u \, tg\,\theta$

Calibration

find parameters:

$M \to K^*, R^*, T^*$

$\to R, T, f, k_u, k_v, u_0, v_0, tg\,\theta$

Problem statement

Determine intrinsic/extrinsic camera parameters using image(s) and a calibration object. These parameters are necessary in various algorithms.

Use correspondence between image and world points to form a system of equations that can then be solved for the unknown parameters.

Approaches:

1. Non-planar calibration target (requires a minimum of one view). eg-cube - purchase from industry with known accurate calibration
2. Planar calibration target (requires multiple views). eg-chessboard - with accurate corners, easy to localize

– direct
– indirect (through M)

For extrinsic parameters we estimate R^* and t^* (the rotation of the world with respect to the camera). For the rotation/translation of the camera with respect to the world we have:

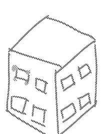
$$R^* = R^T \quad \Rightarrow \quad R = (R^*)^T \tag{55}$$

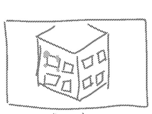
$$t^* = -R^T t \quad \Rightarrow \quad t = -Rt^* = -(R^*)^T t^* \tag{56}$$

View image

Calibration target

given corresponding points $\{\underset{\text{pixels}}{p_i}\}_{i=1}^{m} \leftrightarrow \{\underset{\text{meters}}{\underline{P}_i}\}_{i=1}^{m}$ find parameters





3.2	1.7	1.8	20.0	50.0
7.2	2.2	7.8	30.0	40.0
⋮	⋮	⋮	⋮	⋮

} input for calibration alg.

in pixels, by opencv package, we can find accurate values of the corners in pixels.
It need not be that the point falls into the entire pixel

To get M, we solve below

Solve AX=0

Solving homogeneous linear systems

A is not square

To solve $A^T A x = 0$

solution = zero eigenvalue of $A^T A$

200×12

$2 \times$ # points

unknowns

OR

using SVD: $A = UDV^T$

U,D,V = SVD(A)

Solution = column of V belonging to zero singular value of A

Solving homogeneous linear systems

$$\underset{m \times n}{A} = \underset{\substack{m \times m \\ \text{orthogonal}}}{U} \; \underset{\substack{m \times n \\ \text{diagonal}}}{D} \; \underset{\substack{n \times n \\ \text{orthogonal}}}{V^T}$$

$$\begin{bmatrix} m \times n \end{bmatrix} = \begin{bmatrix} m \times m \end{bmatrix} \begin{bmatrix} m \times n \end{bmatrix} \begin{bmatrix} n \times n \end{bmatrix}$$

singular values

Solving homogeneous linear systems

The singular value decomposition (SVD) of a matrix $A \in \mathbb{R}^{m \times n}$:

$$A = UDV^T \tag{57}$$
$$U \in \mathbb{R}^{m \times m} \tag{58}$$
$$D \in \mathbb{R}^{m \times n} \tag{59}$$
$$V \in \mathbb{R}^{n \times n} \tag{60}$$

Properties:

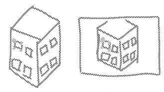
1. D is diagonal with the singular values on its main diagonal.
2. U and V are orthogonal.
3. The columns of U are the eigenvectors of AA^T.
4. The columns of V are the eigenvectors of $A^T A$.

The solution of a linear homogeneous system $Ax = 0$ is given by the column of V belonging to the smallest singular value of A (the last column).

- a matrix is singular if $\frac{s_0}{s_n} < \tau$ (condition number)
- for a symmetric matrix $U = V$ and singular values are squares of eigenvalues.

Non-planar calibration

Given: $\{p_i\}_{i=1}^{n} \leftrightarrow \{\underline{P}_i\}_{i=1}^{n}$ find camera parameters



$\{\underline{P}_i\}$ 3D

$\{p_i\}$ 2D

Steps:

① find projection matrix M

② find parameters from M

Non-planar calibration

2.3.1 Estimating the projection matrix

Dropping the superscript notations from K, R, t:

$$p = K[R|t]P = MP = \begin{bmatrix} m_1^T \\ m_2^T \\ m_3^T \end{bmatrix} P$$

$$p = \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} m_1^T \\ m_2^T \\ m_3^T \end{bmatrix} P$$

For each pair of corresponding points $p \Leftrightarrow P$ we have two equations:

$u = m_1^T \underline{P}$

$v = m_2^T \underline{P}$

$w = m_3^T \underline{P}$

$$\begin{aligned} x &= \frac{u}{w} = \frac{m_1^T P}{m_3^T P} \\ y &= \frac{v}{w} = \frac{m_2^T P}{m_3^T P} \end{aligned}$$

Non-planar calibration

$$\begin{aligned} -x m_3^T P + m_1^T P &= 0 \\ -y m_3^T P + m_2^T P &= 0 \end{aligned}$$

Using $n > 6$ corresponding points $\underbrace{\{p_i\} \Leftrightarrow \{P_i\}}_{n}$ we have a homogeneous linear system:

$$2n \left\{ \begin{bmatrix} P_1^T & 0^T & -x_1 P_1^T \\ 0^T & P_1^T & -y_1 P_1^T \\ . & . & . \\ . & . & . \\ . & . & . \\ P_n^T & 0^T & -x_n P_n^T \\ 0^T & P_n^T & -y_n P_n^T \end{bmatrix} \right. \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ . \\ . \\ . \\ 0 \\ 0 \end{bmatrix}$$

$$\Leftrightarrow Am = 0$$

Using SVD obtain a solution $\hat{m}$. Because this is a homogeneous system the desired solution m is up to a scale ρ:

$\hat{m} = \begin{bmatrix} m_1^T \\ m_2^T \\ m_3^T \end{bmatrix}$

$$m = \rho \hat{m}$$

Non-planar calibration

The scale ρ is irrelevant for projection but is relevant when trying to extract the camera parameters from M. We obtained an estimate of the projection matrix up to an unknown parameter:

$$M = \begin{bmatrix} m_1^T \\ m_2^T \\ m_3^T \end{bmatrix} = \rho \begin{bmatrix} \hat{m}_1^T \\ \hat{m}_2^T \\ \hat{m}_3^T \end{bmatrix} \tag{70}$$

Find ρ so that:

$$M \equiv K^* \left[R^* \mid T^* \right] = \rho \hat{M}$$

find parameters and ρ from $\hat{M}$

Non-planar calibration

2.3.2 Camera parameters from the projection matrix

The projection matrix M has combinations of intrinsic/extrinsic camera parameters. We need to extract the individual parameters.

3×3 3×1

$$M = K[R|t] = [KR|Kt] = \rho\hat{M} \equiv \rho[A|b] \tag{71}$$

Notation:

$$A \equiv \begin{bmatrix} a_1^T \\ a_2^T \\ a_3^T \end{bmatrix} \quad ; \quad R \equiv \begin{bmatrix} r_1^T \\ r_2^T \\ r_3^T \end{bmatrix} \quad ; \quad \rho = \epsilon|\rho| \tag{72}$$

The problem we need to solve:

knowns	a_1^T, a_2^T, a_3^T, b
unknowns	$\epsilon, \lvert\rho\rvert, u_0, v_0, \alpha_u, \alpha_v, s, r_1^T, r_2^T, r_3^T, t$
equations	$\rho A = KR$ $\rho b = Kt$

$$\hat{M} \equiv \rho \left[\begin{matrix} - a_1^T - \\ - a_2^T - \\ - a_3^T - \end{matrix} \middle| \; b \right]$$

Non-planar calibration

$$\rho A = KR = \begin{bmatrix} \alpha_u & s & u_0 \\ 0 & \alpha_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_1^T \\ r_2^T \\ r_3^T \end{bmatrix}$$

$$\rho \begin{bmatrix} a_1^T \\ a_2^T \\ a_3^T \end{bmatrix} = \begin{bmatrix} \alpha_u r_1^T + s r_2^T + u_0 r_3^T \\ \alpha_v r_2^T + v_0 r_3^T \\ r_3^T \end{bmatrix}$$

Isolating individual parameters is done using the fact that r_i are orthonormal. For example:

$r_1 \cdot r_2 = 0$ $\quad r_2 \cdot r_3 = 0$

$r_1 \cdot r_3 = 0$

$r_1 \times r_2 = r_3$ $\quad r_2 \times r_3 = r_1$

$r_3 \times r_1 = r_2$

$$|\rho a_3| = |r_3| \;\Rightarrow\; |\rho| = \frac{1}{|a_3|}$$

$$\begin{aligned} \rho a_1 \cdot \rho a_3 &= (\alpha_u r_1 + s r_2 + u_0 r_3) \cdot r_3 = u_0 \\ &\Rightarrow u_0 = \rho^2 a_1 \cdot a_3 \end{aligned}$$

Non-planar calibration

Finding α_v:

$$\rho a_2 \cdot \rho a_2 = (\alpha_v r_2^T + v_0 r_3^T) \cdot (\alpha_v r_2^T + v_0 r_3^T)$$

$$= \alpha_v^2 + v_0^2$$

$$\boxed{\alpha_v = \sqrt{\rho^2 a_2 \cdot a_2 - v_0^2}}$$

Non-planar calibration

Finding s:

$$\rho a_1 \times \rho a_3 = (\alpha_u r_1^T + s r_2^T + u_0 r_3^T) \times r_3^T$$

$$= -\alpha_u r_2^T + s r_1^T$$

$$\rho a_2 \times \rho a_3 = \alpha_v r_1^T$$

$$(\rho a_1 \times \rho a_3) \cdot (\rho a_2 \times \rho a_3) = (-\alpha_u r_2^T + s r_1^T) \cdot \alpha_v r_1^T = s\alpha_v$$

$$\boxed{s = \frac{1}{\alpha_v} \rho^4 (a_1 \times a_3) \cdot (a_2 \times a_3)}$$

Non-planar calibration

- Finding the unknown sign of ρ

$$\rho \equiv \epsilon |\rho| \qquad \epsilon = \text{sign}(\rho) = ?$$

$$K^* T^* = \rho b = \epsilon |\rho| b$$

$$[K^* T^*]_z = \epsilon |\rho| b_z$$

Positive (object in front of camera)

$$\Rightarrow \boxed{\epsilon = \text{sign}(b_z)}$$

Non-planar calibration

Finding T^*:

Build K^* from recovered intrinsic parameters

$$K^* T^* = \epsilon |\rho| b$$

$$\boxed{T^* = (K^*)^{-1} \epsilon |\rho| b}$$

Non-planar calibration

Summary of parameter values

$$
\begin{aligned}
|\rho| &= 1/|a_3| \\
u_0 &= |\rho|^2 a_1 \cdot a_3 \\
v_0 &= |\rho|^2 a_2 \cdot a_3 \\
\alpha_v &= \sqrt{|\rho|^2 a_2 \cdot a_2 - v_0^2} \\
s &= |\rho|^4/\alpha_v (a_1 \times a_3) \cdot (a_2 \times a_3) \\
\alpha_u &= \sqrt{|\rho|^2 a_1 \cdot a_1 - s^2 - u_0^2} \\
K^* &= \begin{bmatrix} \alpha_u & s & u_0 \\ 0 & \alpha_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \\
\epsilon &= \mathrm{sgn}(b_3) \\
T^* &= \epsilon |\rho| (K^*)^{-1} b \\
r_3 &= \epsilon |\rho| a_3 \\
r_1 &= |\rho|^2/\alpha_v a_2 \times a_3 \\
r_2 &= r_3 \times r_1 \\
R^* &= [r_1^T \; r_2^T \; r_3^T]^T
\end{aligned}
$$

Solve $A m = 0$

Estimate ρ

$$M = \rho M$$

$$\left[\begin{array}{c|c} a_1^T & b_1 \\ a_2^T & b_2 \\ a_3^T & b_3 \end{array}\right]$$

$\epsilon = \mathrm{sign}(b_3)$

Degenerate configurations

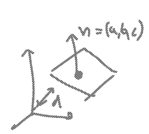
To recover M we need to solve:

$$
\begin{bmatrix}
P_1^T & 0 & -x_1 P_1^T \\
0 & P_1^T & -y_1 P_1^T \\
\vdots & &
\end{bmatrix}
\begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}
$$

when P_i are all on the same plane π

we have $\pi^T P_i = 0 \quad \forall P_i$

$\pi = (\underbrace{a, b, c}_{\text{normal}}, \underset{\text{distance}}{d})$, $\quad P_i = (x, y, z, 1)$



Degenerate configurations

- When P_i are on the same plane π there are many possible solutions of the form:

$$\begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \alpha \begin{bmatrix} \pi \\ 0 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 0 \\ \pi \\ 0 \end{bmatrix} + \gamma \begin{bmatrix} 0 \\ 0 \\ \pi \end{bmatrix}$$

- E.g.:

$$\begin{bmatrix} P_1^T & 0 & -x_1 P_1^T \\ 0 & P_1^T & -y_1 P_1^T \\ \vdots & & \\ \vdots & & \end{bmatrix} \begin{bmatrix} \pi \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Additional degenerate configurations are points on a quadratic surface

Assessing the quality of fit

Given $\{P_i\}_{i=1}^{m} \leftrightarrow \{p_i\}_{i=1}^{m}$ and estimated $M = \begin{bmatrix} - m_1^T - \\ - m_2^T - \\ - m_3^T - \end{bmatrix}$ assess the quality of fit using:

$$E = \frac{1}{m} \sum_i \left(\left\| x_i - \frac{m_1^T P_i}{m_3^T P_i} \right\|^2 + \left\| y_i - \frac{m_2^T P_i}{m_3^T P_i} \right\|^2 \right)$$

distance between known and predicted positions

e.g. $(7.3, 12.1, 15.3) \leftrightarrow (5, 3)$ known → Σ → error

$\longrightarrow (6, 2)$ predicted using M

Recovering R and T

Recovering R and T from R^* and T^*:

$$R^* = R^T \implies \boxed{R = (R^*)^T}$$

$$T^* = -R^T T \implies T = -R T^*$$

$$\implies \boxed{T = -(R^*)^T T^*}$$

Planar calibration

2.4.1 Approach

1. Estimate a 2D homography between image points and planar calibration points.
2. Repeat this for multiple images.
3. Using multiple homography estimates, estimate the intrinsic parameters.
4. Given a specific image estimate the extrinsic parameters for it.
5. Refine the results through optimization.

Planar calibration

The calibration target points are assumed to be on the $z = 0$ plane:

$\{P_i\}_{z=0} \Leftrightarrow P_i = (X_i, Y_i, 0)$

$$
\begin{aligned}
p_i &= K[R|t]P_i \\
&= K[r_1\, r_2\, r_3|t][X_i\, Y_i\, 0\, 1]^T \\
&= K[r_1\, r_2|t][X_i\, Y_i\, 1]^T \\
&= \underset{\text{2D projective map}}{H} P_i^*
\end{aligned}
$$

The matrix H is a 2D homography.

$$\underset{\text{2D H}}{p_i} = \underset{3\times 3}{M} \underset{\text{2D H}}{P_i}$$

Planar calibration

2.4.2 Estimating a 2D homography for a single view

Let $\{P_i\}_{i=1}^n$ ($n \geq 4$) be a set of world points placed on the z plane so that $P_i = [X_i, Y_i, 0, 1]^T$. Define $P_i^* = [X_i, Y_i, 1]^T$. Let $\{p_i\}_{i=1}^n$ be a set of corresponding image points where $p_i = [x_i, y_i]^T$. Compute the homography matrix H between the point sets $\{P_i^*\}_{i=1}^n$ and $\{p_i\}_{i=1}^n$.

$$
\begin{array}{cc}
 & \overbrace{\hphantom{(P_1^*)^T \quad 0^T \quad -x_1(P_1^*)^T}}^{9} \\
2n & \begin{bmatrix} (P_1^*)^T & 0^T & -x_1(P_1^*)^T \\ 0^T & (P_1^*)^T & -y_1(P_1^*)^T \\ (P_2^*)^T & 0^T & -x_2(P_2^*)^T \\ 0^T & (P_2^*)^T & -y_2(P_2^*)^T \\ \dots & & \\ \dots & & \end{bmatrix}
\end{array}
\begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{95}
$$

where h_i^T are the unknown rows of H. Solve for the unknown rows of H.

Homography estimation

$$\underset{2D\,H}{\{p_i\}_{i=1}^n} \leftrightarrow \underset{3D\,H}{\{P_i\}_{i=1}^n} \rightarrow \underset{2D\,H}{\{P_i^*\}_{i=1}^n} \qquad p_i = \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} = H P_i^* \equiv \begin{bmatrix} - h_1^T - \\ - h_2^T - \\ - h_3^T - \end{bmatrix} P_i^*$$

$$x_i = \frac{u_i}{w_i} = \frac{h_1^T P_i^*}{h_3^T P_i^*} \Rightarrow h_1^T P_i^* - x_i h_3^T P_i^* = 0$$

$$y_i = \frac{v_i}{w_i} = \frac{h_2^T P_i^*}{h_3^T P_i^*} \Rightarrow h_2^T P_i^* - y_i h_3^T P_i^* = 0$$

Planar calibration

2.4.3 Estimating the intrinsic parameters matrix

Let $\{H_j\}_{j=1}^m$ be a set of homography matrices estimated for m different views ($m \geq 3$) as described above. Let $H_j = [h_1, h_2, h_3]$ (h_1, h_2, h_3 are the columns of H_j). Each homography matrix H_j provides two constraints regarding the unknown matrix $S = (K^*)^{-T}(K^*)^{-1}$:

$$\begin{bmatrix} V_{12}^T \\ V_{11}^T - V_{22}^T \end{bmatrix} \begin{bmatrix} s_{11} \\ s_{12} \\ s_{22} \\ s_{13} \\ s_{23} \\ s_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{96}$$

where s_{ij} are the elements of the symmetric matrix S, $h_i = [h_{i1}, h_{i2}, h_{i3}]^T$, and $V_{ij} = [h_{i1}h_{j1},\ h_{i1}h_{j2} + h_{i2}h_{j1},$
$h_{i2}h_{j2},\ h_{i3}h_{j1} + h_{i1}h_{j3},\ h_{i3}h_{j2} + h_{i2}h_{j3},\ h_{i3}h_{j3}]^T$

Using all m homography matrices H_j, solve for the unknown rows of the matrix S (s_1, s_2, s_3):

$$\begin{array}{cc} & 6 \\ 2m & \begin{bmatrix} V_{12}^T \\ V_{11}^T - V_{22}^T \\ \cdots \\ \cdots \\ \cdots \\ \cdots \end{bmatrix} \end{array} \begin{bmatrix} s_{11} \\ s_{12} \\ s_{22} \\ s_{13} \\ s_{23} \\ s_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \cdots \\ \cdots \\ \cdots \\ \cdots \end{bmatrix}$$

Planar calibration

Let $\hat{H} = [\hat{h}_1\ \hat{h}_2\ \hat{h}_3]$

note columns of H instead of rows of H before.

$$H = K^*[r_1\ r_2\ T^*] = \alpha[\hat{h}_1\ \hat{h}_2\ \hat{h}_3]$$

$$\Rightarrow [r_1\ r_2\ T^*] = \alpha K^{*-1}[\hat{h}_1\ \hat{h}_2\ \hat{h}_3]$$

$$\Rightarrow \begin{cases} r_1 = \alpha K^{*-1}\hat{h}_1 \\ r_2 = \alpha K^{*-1}\hat{h}_2 \\ T^* = \alpha K^{*-1}\hat{h}_3 \end{cases}$$

Planar calibration

$$r_1 \cdot r_2 = 0 \;\Rightarrow\; (\cancel{\lambda} K^{*-1} \hat{h}_1)^T (\cancel{\lambda} K^{*-1} \hat{h}_2) = 0$$

$$\Rightarrow \hat{h}_1^T K^{*-T} K^{*-1} \hat{h}_2 = 0$$

$$r_1 \cdot r_1 = r_2 \cdot r_2 = 1$$

$$\Rightarrow \hat{h}_1^T K^{*-T} K^{*-1} \hat{h}_1 = \hat{h}_2^T K^{*-T} K^{*-1} \hat{h}_2$$

$$\begin{cases} \hat{h}_1^T S \hat{h}_2^T = 0 \\ \hat{h}_1^T S \hat{h}_1 = \hat{h}_2^T S \hat{h}_2 \end{cases} \qquad S = K^{*-T} K^{*-1}$$

Planar calibration

2.4.4 Estimating the intrinsic parameters

Given the elements s_{ij} of the matrix S as estimated in the previous step, compute the intrinsic parameters:

$$\begin{aligned}
c_1 &= (s_{12}s_{13} - s_{11}s_{23}) && (98)\\
c_2 &= (s_{11}s_{22} - s_{12}^2) && (99)\\
v_0 &= c_1/c_2 && (100)\\
\lambda &= s_{33} - (s_{13}^2 + v_0 c_1)/s_{11} && (101)\\
\alpha_u &= \sqrt{\lambda/s_{11}} && (102)\\
\alpha_v &= \sqrt{\lambda s_{11}/c_2} && (103)\\
s &= -s_{12}\alpha_u^2\alpha_v/\lambda && (104)\\
u_0 &= s v_0/\alpha_u - s_{13}\alpha_u^2/\lambda && (105)
\end{aligned}$$

Build the intrinsic parameters matrix:

$$K^* = \begin{bmatrix} \alpha_u & s & u_0 \\ 0 & \alpha_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (106)$$

Planar calibration

2.4.5 Estimating the extrinsic parameters

Given the homography matrix $H = [h_1, h_2, h_3]$ (h_1, h_2, h_3 are the columns of H) for a specific view, computed as describe before, and given the matrix K^* computed as before, find the extrinsic parameters $R^* = [r_1, r_2, r_3]$, and T^*.

$$\begin{aligned}
|\alpha| &= 1/|(K^*)^{-1}h_1| && (107)\\
\text{sign}(\alpha) &= \text{sign}(((K^*)^{-1}h_3)_z) && (108)\\
\alpha &= |\alpha|\text{sign}(\alpha) && (109)\\
r_1 &= \alpha(K^*)^{-1}h_1 && (110)\\
r_2 &= \alpha(K^*)^{-1}h_2 && (111)\\
r_3 &= r_1 \times r_2 && (112)\\
T^* &= \alpha(K^*)^{-1}h_3 && (113)
\end{aligned}$$

Direct methods

- In indirect methods: find projection matrix M
  then find parameters
- In direct methods: compute parameters directly

- Given $\{p_i\}_{i=1}^{n} \leftrightarrow \{\underline{P}_i\}_{i=1}^{n}$
  the calibration parameters (K^*, R^*, T^*) can be used to project $\underline{P}_i$: $q_i = K^*[R^* | T^*]\, \underline{P}_i$

Measurement of the goodness of calibration

Direct methods

- Objective function:

$$E(K^*, R^*, T^*) = \sum \left\| \left(\frac{(q_i)_x}{(q_i)_z}, \frac{(q_i)_y}{(q_i)_z} \right) - \left((p_i)_x, (p_i)_y \right) \right\|^2$$

(distance between known and predicted projection)

- Minimize:

$$K^*, R^*, T^* = \underset{K,R,T}{\operatorname{argmin}}\ E(K, R, T)$$

non-linear equation $\Rightarrow$ Iterative solution (eg. Tsai)