This is an open notes exam. You may use up to 1 double sided page (8.5 x 11in) of notes and a simple pocket calculator. All other electronic devices such as cellphones, laptops, smart watches, or tablets must be **turned off** for the duration of the exam. You may **not** use any books and you may **not** share any materials with other students. The weight of this exam is 10% of the final grade.

You must show your work in the space provided below each question (except for true/false questions). If you do not see the exact answer you computed, mark the closest answer. Note that the inverse of a diagonal matrix is also a diagonal matrix with reciprocal diagonal elements.

Question 1 Let $P = (1, 2, 3)$ be a 3D point. Let P' be a point obtained from P by first translating P by $T = (1, -1, -1)$ and then scaling it by a factor of $(2, 2, 2)$ in x,y,z, respectively. Compute the point P' then add its coordinates and subtract 5 to yield the answer.

$$\underset{(3DH)}{P'} = STP$$

$$= \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cdot\cdot \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ 2 \\ 4 \\ 1 \end{bmatrix}$$

$$P' = (4, 2, 4) \rightarrow 10 - 5 = 5$$

A -6	D -7	G -8	J 9	M -5	P 4
B 5	E -2	H 6	K 1	N 2	Q -4
C 8	F -1	I 3	L 7	O -3	R 0

Question 2 Let $f = 4$ be the focal length of a camera. Let $p = (1, 2, 2)$ be a world point. Find the coordinate of the 2D point p when projecting it onto the image. Assume that the projection is done in camera coordinates so there is no need for a transformation between coordinate systems. Assume that the image is rotated so that the object does not appear inverted. Note that the point p is two dimensional and so has two coordinates. Add the coordinates of p and use this sum to mark the answer.

2d point p $(u, v) = \left(\frac{fx}{z}, \frac{fy}{z}\right)$

$p = \left(\frac{4\times 1}{2}, \frac{4\times 2}{2}\right) = (2, 4)$

Ans → 6

p, 2d4 $\alpha \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$= \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \\ 2 \end{bmatrix}$

2d → $\left(\frac{4}{2}, \frac{8}{2}\right)$ → $(2, 4)$

A -5	D 8	G 2	J 5	M -7	P -4
B -1	E -6	H -8	K 7	N 3	Q 0
C 9	F -2	I 1	L 6	O -3	R 4

Question 3 Let C be a camera with a focal length of $f = 10$[mm]. The sensor of this camera has $k_u = 100$[pixels/mm] in the horizontal direction and $k_v = 100$[pixels/mm]. The translation of the optical center is given by $u_0 = 500$[pixels] in the horizontal direction and by $v_0 = 500$[pixels] in the vertical direction. Assume that the pixels are not skewed. Write the intrinsic camera parameters matrix K^* (the part of the projection matrix that depends on the internal parameters of the camera). Add the entries of the matrix K^* then divide the sum by 1000 and round. Use the number you get to mark the answer.

$$K^* = \begin{bmatrix} fk_u & 0 & u_0 \\ 0 & fk_v & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1000 & 0 & 500 \\ 0 & 1000 & 500 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\frac{3001}{1000} \rightarrow 3.001$$

A -2	D 1	G 6	J -8	M -4	P 5
B -6	E -3	H -5	K 2	N 0	Q 8
C 9	F -1	I 3 ✓	L -7	O 7	R 4

Review

Question 4 ♣ Let M be a general 3D perspective projection matrix that projects a 3D point P in 3DH coordinates onto a 2D point p in 2DH coordinates. Let R be the 3D rotation matrix of the camera with respect to the world. Let T be the 3D translation of the camera with respect to the world. Assume that the camera is first rotated and then translated. Let K^* be the intrinsic camera parameters matrix. Mark *all* the statements that are **correct**.

- [A] Accounting for skewing in the image can be achieved by updating some fixed coefficients in the intrinsic camera parameters matrix K^*.
- [B] The $3x4$ projection matrix is not unique.
- [C] Accounting for radial lens distortion can be achieved by updating some fixed coefficients in the intrinsic camera parameters matrix K^*.
- [D] The projection matrix for this camera can be written as $K^*[R^*|T^*]$ where R^* and T^* are the rotation and translation of the world with respect to the camera, respectively.
- [E] The projection matrix M for this camera has the exact elements of the matrix R.
- [F] A weak-perspective camera is a hypothetical camera model that does not occur in real-world cameras.
- [G] Projecting the point P using the projection matrix M is done by first multiplying M by P (MP) and then homogenizing the resulting point in order to find its location in the image.
- [H] The projection matrix M for this camera has the exact elements of the vector T.
- [I] Accounting for non-rectangular pixels can be achieved by updating some fixed coefficients in the intrinsic camera parameters matrix K^*.

Question 5 ♣ Let P_i be points on a 3D object that are projected onto an image plane using a projection transformation M. Mark *all* the statements that are **correct**.

(u,v) = (fx/z,fy/z)

z - dist b/w camera and object

- [A] The fundamental relationship between world and image points is derived using triangle similarity.
- [B] In the projection matrix expression $M = K[I|0]$ the 0 indicates a 3×3 matrix of zeros.

3+1

- [C] The projection of a 3D point P is non-linear because it is not possible to express the coordinates of the projected point as a weighted sum of the coordinates of P.
- [D] The projection of a 3D object gets bigger when increasing its distance from the camera.
- [E] The 3D camera projection matrix M is a 3×4 matrix.
- [F] The 3DH points $P_1 = (1, 2, 3, 1)$ and $P_2 = (2, 4, 6, 1)$ are equivalent in 3D.
- [G] The 3DH point $P = (1, 2, 3, 0)$ does not correspond to an actual point in 3D.
- [H] The projection of a 3D object gets bigger when increasing the focal length of the camera.
- [I] When using the projection matrix $M = K[I|0]$ to project a point with K composed of f, k_u, k_v, u_0, v_0, the result of the projection is given in the same coordinate system using the same units. ~~not sure~~

FALSE

Question 6 Let W be a 3×3 window centered about the pixel $(1,1)$ (coordinates start with 0). Let the gradient vectors in this window (top to bottom, left to right) be $\{(0,1),(0,1),(0,1),(1,0),(1,0),(1,0),(0,0),(0,0),(0,0)\}$. To localize a corner in this window you need to multiply the inverse of the correlation matrix C by a 2x1vector Q. Compute the vector Q and add its entries to form your answer.

$$Q = \sum_i g_i g_i^T x_i$$

$$i=1 \quad \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$i=2 \quad \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$Q = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

A -2	D -4	G -5	J 6 ✓	M -6	P -3
B 4	E 7	H 8	K 9	N -8	Q 0
C -7	F 3	I 2	L 1	O -1	R 5

Question 7 Find a 2D transformation matrix M that will perform rotation by 90 degrees about the point $p = (0, 1)$ in homogeneous coordinates. Hint - to compute M translate the point p to the origin, perform the rotation, then translate back. Compute the sum of the elements of the matrix M you computed and use this sum as the answer. Note that the matrix M is a 3×3 matrix.

$$R = \begin{bmatrix} \cos 90 & -\sin 90 & 0 \\ \sin 90 & \cos 90 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$RT(-p) = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T(p)RT(-p) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

A 3	D -7	G 4	J -5	M 7	P -8
B 1	E -6	H -3	K 9	N 8	Q -1
C 0	F -2	I 6	L -4	O 5	R 2

Question 8 Let I be a grayscale image where all the even rows have a value of 0 and all the odd rows have a value of 100. Note that the first row in the image is assumed to be row 0. Let G be a 3×3 smoothing filter designed to average entries using equal weights in a manner that will not increase the overall image intensity. Convolve the filter G with the image I and find the value at location $(1, 1)$ of the convolved image. Divide this value by 10 and round. Use the resulting value as your answer.

$I: \begin{bmatrix} 0 & 0 & \cdots & \cdots \\ 1 & 1 & \cdots & \cdots \\ 0 & 0 & \cdots & \cdots \end{bmatrix}$ filter $\to \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$\frac{100}{9} + \frac{100}{9} + \frac{100}{9} \Rightarrow 33 \cdots \quad \frac{33}{10} = 3.5$

A -6	D 5	G -7	J -4	M 8	P 3 ✓
B 2	E -3	H 7	K -2	N -8	Q 0
C 1	F 9	I -5	L 4	O -1	R 6

Question 9 Let W be a 3×3 window centered about the pixel $(1, 1)$ (coordinates start with 0). Let the gradient vectors in this window (top to bottom, left to right) be $\{(0, 1), (0, 1), (0, 1), (1, 0), (1, 0), (1, 0), (0, 0), (0, 0), (0, 0)\}$. Compute the correlation matrix C in this window. Compute the sum of of entries in this matrix to form your answer.

$$C = \sum_{i=1}^{9} g_i g_i^T$$

$i = 1$; $\begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

A -2	D 5	G 3	J 8	M -1	P 9
B -5	E 2	H 7	K 6	N 1	Q 0
C -6	F -4	I -7	L 4	O -8	R -3

Question 10 ♣ Let I be an image where edge detection is to be performed. Mark *all* the statements that are **correct.**

- [A] The Canny edge detection algorithm detects edges where the directional derivative along the direction of the gradient has a local maximum.
- [B] Smoothing before edge detection is not commonly employed because it makes the detected edges less accurate.
- [C] A Gaussian image pyramid is a useful way for processing an image at multiple scales.
- [D] When designing a smoothing filter using a Gaussian with a standard deviation of 5, the size of the Gaussian filter should be set to 5×5 to match the rounded value of the standard deviation.

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- [E] The image derivative in the X direction may be computed using two 1D convolutions.
- [F] Smoothing before edge detection is important to reduce noise.
- [G] The Sobel filter can be produced by convolving a 2×2 averaging filter with a 2×2 forward differences filter.
- [H] Edges in the image can be detected using image gradients with high magnitudes.

TRUE

~~not sure~~

- [I] The x derivative of an image can be produced by convolving the image with a 1D Gaussian smoothing filter in the y direction and then convolving the result with a 1D Gaussian derivative filter in the x direction.