cs 512 F24: Assignment 0 - Review

Due by 9/1/2024 (0%)

Answer the following by computing and showing the steps involved

Part A: Vector Operations

Let:

$$p = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}, q = \begin{bmatrix} 0 \\ 3 \\ 5 \end{bmatrix}, r = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$$

Find:

1. $3p + 2q$
2. $\hat{p}$*: a unit vector in the direction of* p
3. $\| p \|$ *and the angle of* p *relative to the positive* y*-axis*
4. *The direction cosines of* p
5. *The angle between* p *and* q
6. $p \cdot q$ *and* $q \cdot p$
7. $p \cdot q$ *using the angle between* p *and* q
8. *The scalar projection of* q *onto* $\hat{p}$
9. *A vector that is perpendicular to* p
10. $p \times q$ *and* $q \times p$
11. *A vector that is perpendicular to both* p *and* q
12. *The linear dependency between* p, q, *and* r
13. $p^T q$ *and* $p q^T$

Part B: Matrix Operations

Let:

$$X=\begin{bmatrix} 2 & 1 & 0 \\ -1 & 3 & 4 \\ 3 & 2 & -2 \end{bmatrix}, Y=\begin{bmatrix} 4 & -1 & 2 \\ 3 & 0 & -3 \\ 1 & 2 & 1 \end{bmatrix}, Z=\begin{bmatrix} 2 & 0 & -1 \\ 1 & 4 & 5 \\ 3 & 1 & 2 \end{bmatrix}, s=\begin{bmatrix} -1 \\ 4 \\ 0 \end{bmatrix}$$

Find:

1. $X+2Y$
2. XY *and* YX
3. $(XY)^T$ *and* $Y^T X^T$
4. $|X|$ *and* $|Z|$ *(see question A-12)*
5. *The matrix (either* X*,* Y*, or* Z*) in which the row vectors form an orthogonal set*
6. X^{-1} *and* Y^{-1} *(see question B-5)* Rework
7. Z^{-1} *(see question B-4)* Rework
8. *The product* Xs
9. *The scalar projection of the rows of* X *onto the vector* s *(with* s *normalized)*
10. *The vector projection of the rows of* X *onto the vector* s *(with* s *normalized)*
11. *The linear combination of the columns of* X *using the elements of* s
12. *The solution* t *for the equation* $Yt=s$
13. *The solution* t *for the equation* $Zt=s$ *and the reason for it (see question B-7)* Check what about B-7

Part C: Eigenvalues and Eigenvectors

Let:

$$M=\begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix}, N=\begin{bmatrix} 5 & -3 \\ -3 & 6 \end{bmatrix}, P=\begin{bmatrix} 2 & 4 \\ 4 & 8 \end{bmatrix}$$

Find:

1. *The eigenvalues and corresponding eigenvectors of* M
2. *The dot product between the eigenvectors of* M
3. *The dot product between the eigenvectors of* N

4. *The property of the eigenvectors of* N *and the reason for it (see question C-4)* Check

5. *The value of a trivial solution* t *to the equation* $Pt=0$

6. *The value of two non-trivial solutions* t *to the equation* $Pt=0$

7. *The only solution* t *to the equation* $Mt=0$ *and the reason for having a single solution*

Part D: Neural Networks Basics

1. *Given a neuron with input values* $x_1=0.5$, $x_2=-0.8$, *and* $x_3=0.3$, *weights* $w_1=0.4$, $w_2=-0.6$, *and* $w_3=0.9$, *and a bias* $b=0.2$, *compute the output of the neuron before applying any activation function.*

2. *Using the output computed in the previous question, apply the sigmoid activation function to find the activation value.*

3. *Given the same input values and weights as in question 1, compute the output of the neuron after applying a ReLU activation function.*

4. *Given a neural network layer with weights* W_h, *bias* b_h, *and input* x, *as follows:*

   $$W_h=\begin{bmatrix} 0.4 & 0.3 \\ 0.2 & 0.7 \end{bmatrix}, b_h=\begin{bmatrix} 0.1 \\ -0.1 \end{bmatrix}, x=\begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

   compute the output of the layer using ReLU activation.

5. *Feed the output of the layer from the previous question into an output layer with weights* W_o, *bias* b_o, *as follows:*

   $$W_o=\begin{bmatrix} 0.5 & -0.3 \end{bmatrix}, b_o=\begin{bmatrix} 0.1 \end{bmatrix}$$

   compute the output of the layer using sigmoid activation.

6. *Assume a neural network with a single hidden layer and an output layer. The hidden layer consists of 2 units, with weights* W_h, *biases* b_h, *and outputs from the hidden layer* z *(after activation). The output layer has 1 unit with weights* W_o *bias* b_o, *and output* y *(after activation). The input to the network is a 2-dimensional vector* $x=[x_1, x_2]$. *Let the loss function be denoted by* L. *Write the chain rule to compute the gradient of the loss* L *with respect to the input components* x_1, *i.e.,* $\frac{\partial L}{\partial x_1}$.

Part E: Gradient Calculations (Do Not Submit in Python)

Let:

$f(x)=2x^2-1, g(x)=3x^2+4, h(x,y)=x^2+y^2+xy$

Find:

1. *The first and second derivatives of* $f(x)$ *with respect to* x*:* $f'(x)$ *and* $f''(x)$
2. *The partial derivatives* $\frac{\partial h}{\partial x}$ *and* $\frac{\partial h}{\partial y}$
3. *The gradient vector* $\nabla h(x,y)$
4. *The derivative* $\frac{d}{dx} f(g(x))$ *with and without using the chain rule for derivatives*

Part F: Python implementation

Repeat questions A, B, C, and D (except for question 6) using Python and prepare a Python notebook showing your computations.

Submission Instructions

1. **Step 1: Prepare your solution**
   - *Prepare your solution in a PDF file (either typed and exported to PDF, or handwritten and scanned/photographed).*
   - *Prepare a Python notebook showing the Python solution.*
   - *Your name, student ID, course number, and semester must be clearly shown at the beginning of your report and the Python notebook.*
2. **Step 2: Create a private Bitbucket repository**
   - *Create a free Bitbucket account or use your existing account.*
   - *Create a* **PRIVATE** *repository named* **cs577-f24-LAST-FIRST** *where* **LAST/FIRST** *are your last/first name. Ensure the repository is private and named correctly.*
   - *Share the repository with* **cs512@cs.iit.edu***.*

- *Clone the repository using the provided clone command and paste it into the following form.*
- *Ensure you are logged into your IIT Google account when accessing the form.*

3. **Step 3: Clone your repository**
   - *Clone the repository locally using the clone command. For example:*

     ```
     git clone
     https://USER_NAME@bitbucket.org/USER_NAME/REPOSITORY_NAME.git
     ```

   - *Modify and push your files back to Bitbucket as needed.*

4. **Step 4: Push your submission**
   - *Inside the cloned repository, create a folder named* **AS0** *and place your submission files (PDF and Python notebook) there.*
   - *Commit and push the files. For example:*

     ```
     git add AS0
     git commit -m "assignment 0 submission"
     git push -u origin master
     ```

   - *Verify your files were pushed correctly by checking your Bitbucket account through a web browser.*
   - *Note: Making any changes to your submission by adding or modifying files inside AS0 and pushing them to bitbucket will change your submission date, however, creating another folder (e.g., AS1) and updating files there will not change the submission date for AS0.*