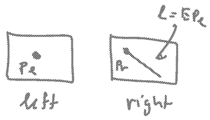
Epipolar constraint

- Right epipolar line:

  Given point P_ℓ in the left image the epipolar line on the right image is given by:

  $\boxed{\ell = E P_\ell}$

  the corresponding point P_r should be on this line.



- Explanation:

  $P_r^T E P_\ell = 0 \Rightarrow P_r^T \ell = 0 \Rightarrow P_r$ is on the line ℓ

  (line eq. $P^T \ell = 0$; $\ell = (\overbrace{a, b, c}^{n}, d)$ ← distance from origin

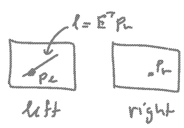
Epipolar constraint

- Left epipolar line:

  Given point P_r in the right image the epipolar line on the left image is given by:

  $\boxed{\ell = E^T P_r}$

  the corresponding point P_ℓ should be on this line.



- Explanation:

  $P_r^T E P_\ell = 0 \Rightarrow (P_r^T E P_\ell)^T = 0 \Rightarrow P_\ell^T E^T P_r = 0$

  $\Rightarrow P_\ell^T \ell = 0 \qquad \ell = E^T P_r$

Fundamental matrix

- So far the epipolar constraint was expressed in camera coordinates. We want to move to image coordinates.

  $\bar{P}_\ell = K_\ell^* P_\ell$ ($\bar{P}_\ell$: image coords, P_ℓ: camera coords)

  $$K_\ell^* = \begin{bmatrix} \alpha_u & s & u_0 \\ 0 & \alpha_v & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$

  $\Rightarrow P_\ell = K_\ell^{*-1} \bar{P}_\ell$

- Likewise

  $P_r = K_r^{*-1} \bar{P}_r$

Fundamental matrix

epipolar constraint:

$$P_r^T E P_l = 0$$

$$\Leftrightarrow (K_r^{*-1} \bar{p}_r)^T E (K_l^{*-1} \bar{p}_l) = 0$$

$$\bar{p}_r^T \underbrace{(K_r^{*-T} E K_l^{*-1})}_{= F \leftarrow \text{fundamental matrix}} \bar{p}_l = 0$$

- The fundamental matrix takes into account both internal and external parameters, whereas the essential matrix takes into account only internal parameters.

F is rank 2 because of E

Summary

In camera coordinates:

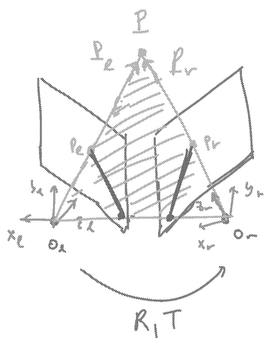
$$p_r^T E p_l = 0$$

← essential matrix

In image coordinates:

$$p_r^T F p_l = 0$$

← fundamental matrix



Summary

$$\underset{3\times3}{E} = R^T [T]_\times$$

$$\underset{3\times3}{F} = K_r^{*-T} E K_l^{*-1}$$

E and F are rank 2

Weak calibration

- Full calibration: R, T, K_ℓ^*, K_r^* $\Longrightarrow$ E, F
- Weak calibration: E, F
- Problem statement:

given $\underbrace{\{p_i'\}_{i=1}^m}_{\text{left points}} \longleftrightarrow \underbrace{\{p_i\}_{i=1}^m}_{\text{right points}}$ $\quad m \geq 8$

find F

(8 unknowns: elements of F)

Eight points algorithm

- One equation for each point pair:

$$p_i^T F p_i' = 0$$

8 unknowns $\Rightarrow$ need at least 8 point pairs

$$[x_i, y_i, 1] \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} x_i' \\ y_i' \\ 1 \end{bmatrix} = 0$$

$$x_i x_i' f_{11} + x_i y_i' f_{12} + x_i f_{13} + y_i x_i' f_{21} + y_i y_i' f_{22} + y_i f_{23} + x_i' f_{31} + y_i' f_{32} + f_{33} = 0$$

Eight points algorithm

$$\underbrace{\begin{bmatrix} x_1 x_1' & x_1 y_1' & x_1 & y_1 x_1' & y_1 y_1' & y_1 & x_1' & y_1' & 1 \\ \vdots & & & & & & & & \\ \vdots & & & & & & & & \\ x_m x_m' & x_m y_m' & x_m & y_m x_m' & y_m y_m' & y_m & x_m' & y_m' & 1 \end{bmatrix}}_{n \times 9} \underbrace{\begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ \vdots \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix}}_{9 \times 1} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}}_{n \times 1}$$

$Ax = 0$ $\Longrightarrow$ solution is right zero eigenvector of A

(SVD $\Rightarrow$ $A = UDV^T$ then take last column of V)

Eight points algorithm

- To enforce rank 2 for the estimated F:

SVD → $F = UDV^T$

$D' = D$ with smallest singular value set to 0
$(D'[3,3] = 0)$

$F' \equiv U D' V^T$ → rank 2 matrix

Normalization

- For a stable solution normalize the point sets.

$$\{p_i\}_{i=1}^{m} \Leftrightarrow \{p_i'\}_{i=1}^{m} \xRightarrow[\text{normalize}]{} \{q_i\}_{i=1}^{m} \Leftrightarrow \{q_i'\}_{i=1}^{m}$$

$$q_i = \frac{p_i - \mu_p}{\sigma_p} \qquad q_i' = \frac{p_i' - \mu_{p'}}{\sigma_{p'}}$$

μ = mean
σ = stdv

Normalization

- In matrix form:

$$q_i = \underbrace{\begin{bmatrix} 1/\sigma_p & & \\ & 1/\sigma_p & \\ & & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -\mu_{p_x} \\ 0 & 1 & -\mu_{p_y} \\ 0 & 0 & 1 \end{bmatrix}}_{\equiv M_p} p_i$$

$$q_i = M_p \, p_i$$

$$q_i' = M_{p'} \, p_i'$$

Normalization

- using $\{q_i\}_{i=1}^m \leftrightarrow \{q_i'\}_{i=1}^m$ and the 8 point algorithm find F':

$$q_i^T F' q_i' = 0$$

$$(M_p p_i)^T F' (M_{p'} p_i') = 0$$

$$\underbrace{p_i^T M_p^T F' M_{p'}}_{=F} p_i' = 0 \qquad \boxed{F = M_p^T F' M_{p'}}$$

Summary

Given $\{p_i\}_{i=1}^m \leftrightarrow \{p_i'\}_{i=1}^m$

① normalize points : $q_i = M_p p_i$
$q_i' = M_{p'} p_i'$

② use 8-points alg $\Rightarrow F'$

③ convert F' to F : $F = M_p^T F' M_{p'}$

Finding epipoles from F

- Epipoles: - intersection of baseline with images
  - place where all epipolar lines intersect

$$p_r^T F p_e = 0$$

- Given p_e we have epipolar line Fp_e in the right image
- every point p on this line satisfies $p^T F p_e = 0$
- specifically : $e_r^T F p_e = 0$
- since this is for all p_e it must be that:

$$e_r^T F = 0$$

Finding epipoles from F

$$e_r^T F = 0 \Rightarrow \underset{3\times 3}{F^T}\ \underset{3\times 1}{e_r} = 0$$

$\Rightarrow e_r$ is the left null space of F

(SVD: $F = UDV^T \rightarrow e_r$ is the last column of U)

- For the left epipole:

$$p_r^T F e_l = 0 \quad \forall p_r \Rightarrow F e_l = 0$$

$\Rightarrow e_l$ is the right null space of F

(SVD: $F = UDV^T \rightarrow e_l$ is the last column of V)

View image