Homework 3 for ECS 20

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This assignment was made with ♡ (and LaTeX).

Contents

1 Question 1

1.1 Part 1

Direct Proof: $x, y, k, m \in \mathbb{Z}, x, y = 2k + 1$(odd), then $x + y = 2m$(even)
$x + y = (2k + 1) + (2k + 1) = 4k + 2 = 2(k + 1), k + 1 = m, 2(k + 1) = 2m$
$2m$ is a multiple of 2 and even, implying x+y is even QED.

1.2 Part 2

Proof by Contradiction: x, y, k, $m \in \mathbb{Z}$, x, $y = 2k + 1$(odd), then $x + y = 2m + 1$(odd)
$x + y = (2k + 1) + (2k + 1) = 4k + 2 = 2(k + 1)$, $k + 1 = m$, $2(k + 1) = 2m$
$2m$ is even, contradiction found, x+y is even QED.

2 Question 2

Direct Proof: k, $m \in \mathbb{Z}$, Two consecutive integers are odd and even, so $2k(2k + 1) = 4k^2 + 2k = 2(2k^2 + k)$, $2k^2 + k = m$, $2(2k^2 + k) = 2m$
The product of two consecutive interger (odd and even) are even QED.

3 Question 3

Direct Proof: x, k, $m \in \mathbb{Z}$, $x^2 = 2k$
Making consecutive integers: $x^2 + x = x(x + 1) = 2k$
$(x^2 + x = 2m) = x = 2m - 2k = 2(k - m)$.
$\therefore$ x is even QED

4 Question 4

Proof by Contradiction: Prove that you need less than 6 games a day for 41 games. Let games= g, $g \in \mathbb{N}$
$g(max) = 5$. There are 7 days in a week. $5 * 7 = 35$, $35 < 41$
Contradiction found, 5 games a day is not enough.

5 Question 5

Direct Proof: Let $n, m \in \mathbb{Z}$, $n + (n + 1) + (n + 2) + (n + 3) + (n + 4) = 5n + 10 = 5(n + 2)$, $(n + 2) = m$, $5m$

The sum of five consecutive numbers is a multiple of 5, QED

6 Question 6

Biconditional Proof: If n is odd then $3n^2 + 8$ is odd.
Let n, k, $m \in \mathbb{Z}$, $n = 2k + 1$, $3(2k + 1)^2 + 8 = 12k^2 + 12k + 11$
$= 2(6k^2 + 6k + 5) + 1$, $(6k^2 + 6k + 5) = m$, $2m + 1$ (odd).
This statement is true, now for the inverse:
If n is even then $3n^2 + 8$ is even.
Let n, k, $m \in \mathbb{Z}$, $n = 2k$, $3(2k)^2 + 8 = 12k^2 + 8$
$2(k^2 + 4)$, $(k^2 + 4) = m$, $2m$ (even).
Statement is true both ways. QED