Homework 7 for ECS 20

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This assignment was made with ♡ (and LaTeX).

Contents

1 Question 1

For a number to end in 0s, it has to be divisible by both 2 and 5. So 10! has two trailing zeroes becuase $10/5 = 2$ and $10/2 = 5$, $10/4 = 2$, and $10/8 = 1$(4 and 8 each have more than one 2, so divide 10 by them) and 5+2+1 = 8 twos. But there are at most 2 pairs of 5 and 2, hence 2 trailing zeroes. Therefore, for every pair of 5 and 2 in 1000 there will be a trailing zero.
We will focus on 5 because it seems to be the limiting "agent".
So following the same pattern, $\frac{1000}{5} + \frac{1000}{25} + \frac{1000}{125} + \frac{1000}{625} = 249$ zeroes.
.
SOLUTION CHECK: I got this right

2 Question 2

2.1 part a

For $n \in \mathbb{Z}$, where $n > 1$ and n is composite, we know that every composite number is a multiple of prime numbers. We also know that 1 is a divisor of primes and composite numbers. If n is greater than 1 and the smallest composite integer, then it is 4, then the smallest $\sqrt{n}$ is 2. But we said 1 divides n and $1 < 2$, so $d \leq \sqrt{n}$.

SOLUTION CHECK: I'm pretty sure my proof holds.

2.2 part b

for (int i=2; $i < \sqrt{n}$; i++)
if (n%i == 0)
print ("Not prime")

else
print ("Prime")

.
SOLUTION CHECK: I got this right.

3 Question 3

A factorial of a number n is always composite, so a sequence of n! would be composite: n!+1, n!+2, n!+3.
.
SOLUTION CHECK: I was wrong, the sequence starts with (n+1)!+2 and ends with (n+1)!+(n+1). I'm not sure
why they chose (n+1)! when n! would work. But I do understand why they defined k as they did.

4 Question 4

$5^5 \equiv 1 mod 11$
$789 \equiv 5 * 157 + 4$
$5^4 \equiv 9 mod 11$
Then we draw the conclusion that 9 is the remainder when 11 divides 5^{789}
.
SOLUTION CHECK: I was right.

5 Question 5

5.1 part a

Using the Euclidean Algorithm:
$20112340 = 1076 * 18675 + 18040$
$18675 = 1 * 18040 + 635$
$18040 = 28 * 635 + 260$
$635 = 2 * 260 + 115$
$260 = 2 * 115 + 30$
$115 = 3 * 30 + 25$
$30 = 1 * 25 + 5$
gcd = 5 .
SOLUTION CHECK: I was right.

5.2 part b

$18,675 = 3^2 * 5^2 * 83$
$20112340 = 2^2 * 5 * 1005617$
.
SOLUTION CHECK: I was right.

5.3 part c

Wasn't sure how to do this at first, but you add all the factors.
If you multiply $2^2 + 3^2 + 5^2 + 83 + 1005617 = 75119589900$
So apparently the lcm is a very large number
.
SOLUTION CHECK: Looked at answer, thought the the lcm was four different numbers seperated by a comma, I was wrong about that.

6 Question 6

To prove the statement we need to show that there is an natural number n such that it is a common divisor of a,d, and r. So let's realize that 1 is a natural number that can divide a,d, and r. But now let's assume that n is greater than 1 and it divides d and r or is $gcd(d, r)$. If this is true, then it also divides a. Well, if this is the case, then n divides a and d.
$\therefore gcd(a, d) = gcd(d, r)$

SOLUTION CHECK: I should have done a containment proof, it wasn't enough to just show
that a common divisor greater than one exists for a and d and r. A containment
proof solidifies that it indeed is the gcd and it that the gcds are equal.