PS5

Tamim
Nekaien

1. The power set ~~was~~ of the set

$\boxed{\{\emptyset, a, \{a\}, \{\{a\}\}\}}$ because it contains $\{\emptyset\}, \{a\}, \{\{a\}\}, \{\{\{a\}\}\}$

2. part a. Prove $\overline{A \cap B} \subseteq \bar{A} \cup \bar{B}$, and

$\bar{A} \cup \bar{B} \subseteq \overline{A \cap B}$

$\overline{A \cap B} \rightarrow x \notin A \cap B \rightarrow x \notin A \vee B$

$\rightarrow x \in \bar{A} \vee \bar{B} \rightarrow \boxed{x \in \bar{A} \cup \bar{B}}$

Now, reverse proof

$\bar{A} \cup \bar{B} \rightarrow x \in \bar{A} \vee \bar{B} \rightarrow \overline{x \notin A \vee B}$

$\rightarrow x \notin A \cap B \rightarrow x \in \overline{A \cap B}$

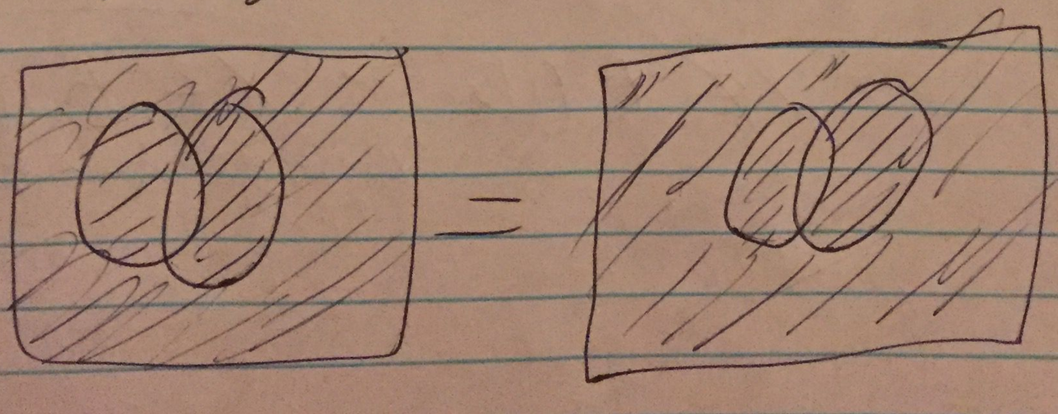
Demorgan's ~~law~~

b. Membership table

A	B	$A \cap B$	$\bar{A}$	$\bar{B}$	$\overline{A \cap B}$	$\bar{A} \cup \bar{B}$
0	0	0	1	1	1	1
0	1	0	1	0	1	1
1	0	0	0	1	1	1
1	1	1	0	0	0	0

c. Venn diagram

$\overline{A \cap B}$ $\qquad\qquad$ $\bar{A} \cup \bar{B}$



3. prove ~~that~~ reflexive, symmetry, and transitive property

Reflexive:

~~[illegible]~~

Let's use (x, y, z)

$$\boxed{\forall x \in X\, R\, X, \quad \forall x \in X\, R^{-1}\, X}$$

Symmetry: (x, y, z)

$\forall x, y \quad xRy \rightarrow yRx,$

$$\boxed{\begin{array}{c} xR^{-1}y \rightarrow yR^{-1}x \\ \text{or} \\ yRx \rightarrow xRy \end{array}}$$

Transitive: (x, y, z)

$xRy \wedge yRz \rightarrow xRz,$

$$\boxed{\begin{array}{c} xR^{-1}y \wedge yR^{-1}z \rightarrow xR^{-1}z \\ (yRx \wedge zRy) \rightarrow zRx \end{array}}$$

4a.

$$f(n) = \begin{cases} -\frac{n}{2} & n \text{ is even, including } 0 \\ \frac{(n-1)}{2} & n \text{ is odd} \end{cases}$$

(either one could be negative)

case 1: Let $f(n_1) = f(n_2)$ for $n = 2k$

$$\frac{-2k_1}{2} = \frac{-2k_2}{2}$$

$$\boxed{k_1 = k_2} \checkmark$$

case 2: Let $f(n_1) = f(n_2)$ for $n = 2k+1$

$$\frac{(2k_1+1)-1}{2} = \frac{(2k_2+1)-1}{2}$$

$$\boxed{k_1 = k_2} \checkmark$$

case 3: Let $f(n_1) = f(n_2)$ for $n_1 = 2k$ $n_2 = 2k+1$

$$\frac{-2k}{2} = \frac{(2k+1)-1}{2}$$

$-k = k$, contradiction

6. Inverse of function

$$f^{-1}(n) = \begin{cases} -2n, & \text{if } n \text{ is negative, including } 0, \text{ which is neither } +, \text{ or } - \\ 2n+1, & \text{if } n \text{ is positive} \end{cases}$$

5. Prove: If f is onto, then g is onto and if g is onto, then f is onto.

If f is onto,

then every element $x \in A \mid f(x) \in P(B)$

then $g(S)$, where $S\ (x \in A)$ maps onto $P(B)$. $\therefore$ g is onto.

And if g is onto,

then $S(x \in A), f(x) \in P(A))$

and $f(x)$ maps onto B.

7. From UC Davis keaben math page:

Rule 2: $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

Rule 4: $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

So $\sum_{j=1}^{n} (j^3 - 2j) = \sum_{j=1}^{n} j^3 - 2\left(\sum_{j=1}^{n} j\right)$

$$\frac{n^2(n+1)^2}{4} - \frac{2n^2+2n}{\cancel{2}}$$

$$(n+1)^2 = (n^2+2n+1)\,n^2$$

$$\frac{n^4+2n^3+n^2}{4} - \frac{4n^2+4n}{4}$$

dominant term $\longrightarrow \frac{n^4+2n^3-3n^2-4n}{4}$

8. Proof by Induction

Base case: I think there was a typo, $F_i = F_{i+2} - 1$, instead of $F_i = F_{n+2} \cancel{*} - 1$

$$F_i = F_{i+2} - 1, \quad i = 1$$

$$F_1 = F_3 - 1 = 1 = 1 \checkmark$$

IH: $\sum_{i=1}^{k} F_i = F_{k+2} - 1$, I was wrong about typo

$$\sum_{i=1}^{k+1} F_i + F_{k+1}$$

$$\sum_{i=1}^{k+1} F_{k+2} - 1 + F_{k+1}$$

$$\sum_{i=1}^{k+1} F_{k+3} - 1 \therefore P(k) \to P(k+1)$$

we have proven Fibonacci's numbers.