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Q. Proof TSP is NP-complete.

Soln: To prove decision version of TSP is NP-complete we need to proof that TSP is NP, then it is also NP-hard.

For the first part, we can provide a certificate if we have the path given. And time will be ~~be~~ polynomial. And the path will be one of the permutation of the vertices.

Now, to prove NP-hard we use already a known NP-complete Hamiltonian cycle problem.

We need to show,

$$\text{Hamiltonian cycle} \le_P \text{TSP}$$

Let $G(V, E)$ be the graph of TSP.

we create a new graph G_2 where all $e \in E$ is present with cost or weight $c(e) = 1$.

And we add all the edges belonging to inverse graph $G'(V, E')$ with $e' \in E'$ where $c(e') = 2$.

Now, we need to find the solution of TSP.

If we find that the TSP outputs a value n i.e. there a cycle of length n which includes all vertices. But that is essentially the hamiltonian cycle. if TSP generates cycle length $> n$ means that it use some edge not present in G. Thus ~~forcing~~ resulting it to not have any hamiltonian cycle present in graph G.

Thus TSP is NP-complete.