

First steps in *BEAST2* and setting priors

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Installation



Beast2

Bayesian evolutionary analysis by sampling trees

<https://www.beast2.org>



FigTree is a program for viewing trees, publication quality figures.

<https://beast.community/figtree>



Tracer is a graphical tool for visualization and diagnostics

<https://beast.community/tracer>



Looking just for Java?

Azul Zulu Builds of OpenJDK

<https://www.azul.com/downloads/?package=jdk-fx>

| Java Version | Operating System | Architecture | Java Package |
|---------------|------------------|--------------|--------------|
| Any | Any | Any | JDK FX |
| Java 24 | | | |
| Java 21 (LTS) | | | |
| Java 17 (LTS) | | | |

Linux x86 64-bit



**The R Project for
Statistical Computing**

<https://www.r-project.org>

<https://posit.co/download/rstudio-desktop>

Installation

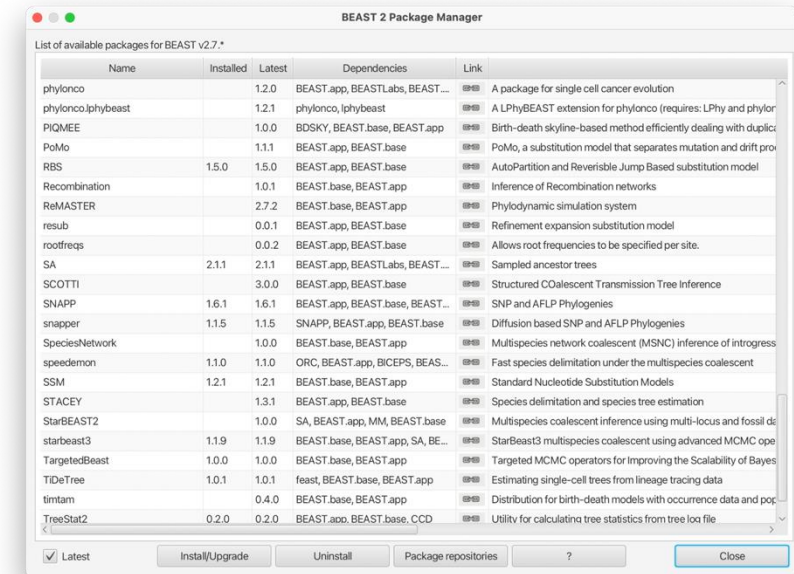
- Managing packages

<https://www.beast2.org/managing-packages/index.html>

- Adding package repositories

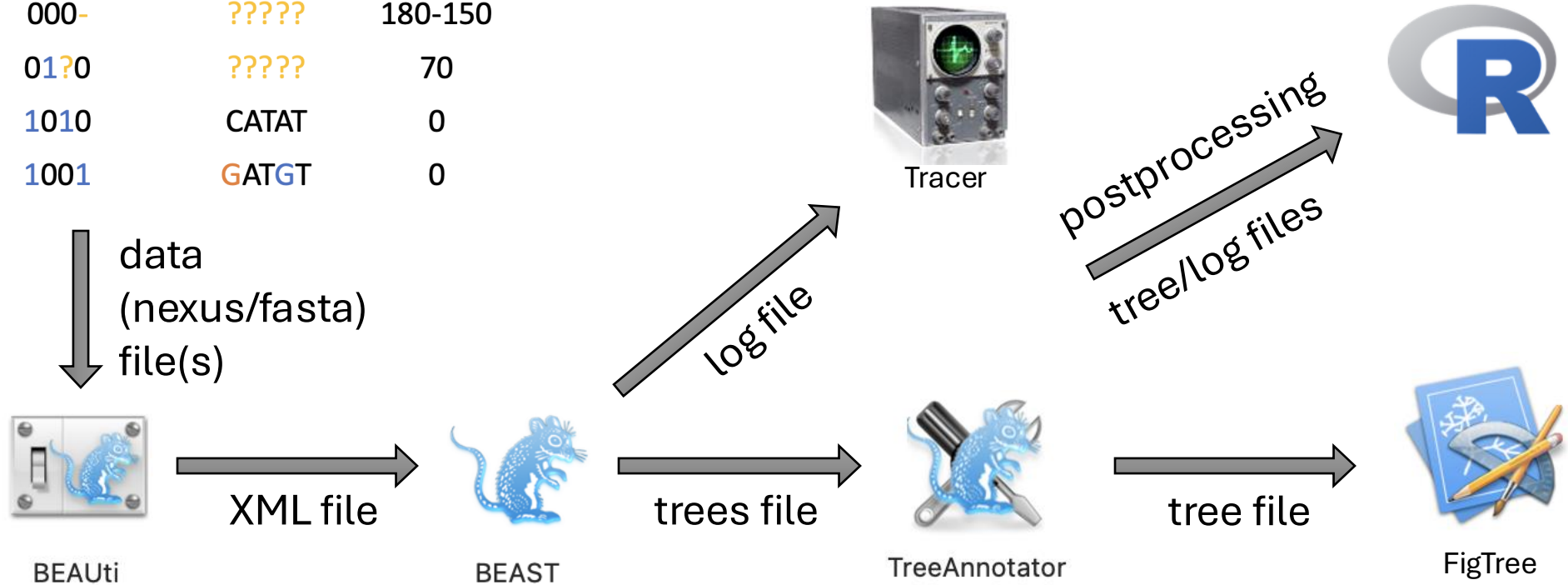
- Installing packages by hand

- Windows: `Users\<YourName>\BEAST\2.7\<PKG>`
- Mac: `/Users/<YourName>/Library/Application Support/BEAST/2.7/<PKG>`
- Linux: `/home/<YourName>/.beast/2.7/<PKG>`



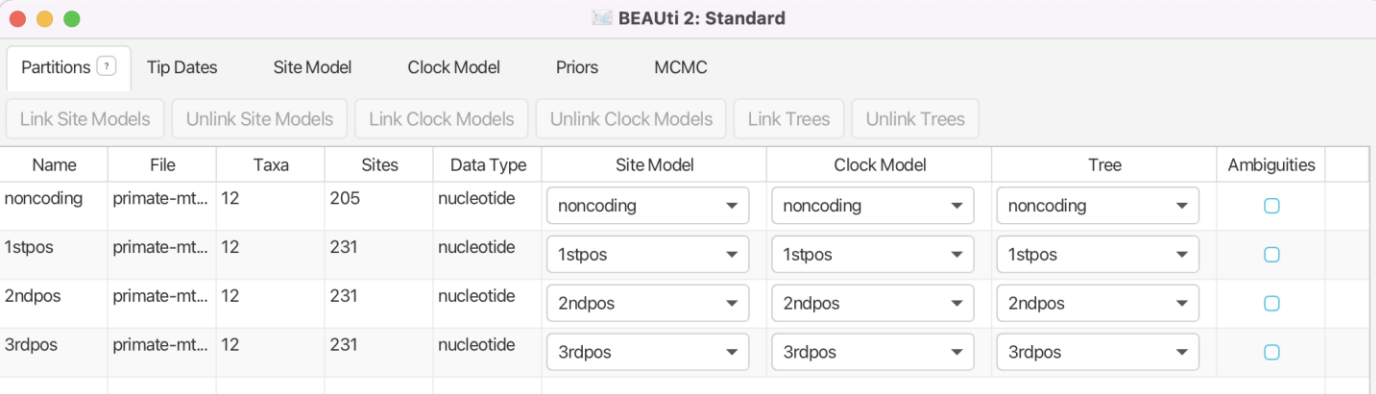
Workflow

| Taxa | Morphological | Molecular | Age(Ma) |
|----------------|---------------|-----------|---------|
| F ₁ | 000- | ????? | 180-150 |
| F ₂ | 01?0 | ????? | 70 |
| S ₁ | 1010 | CATAT | 0 |
| S ₂ | 1001 | GATGT | 0 |



Workflow

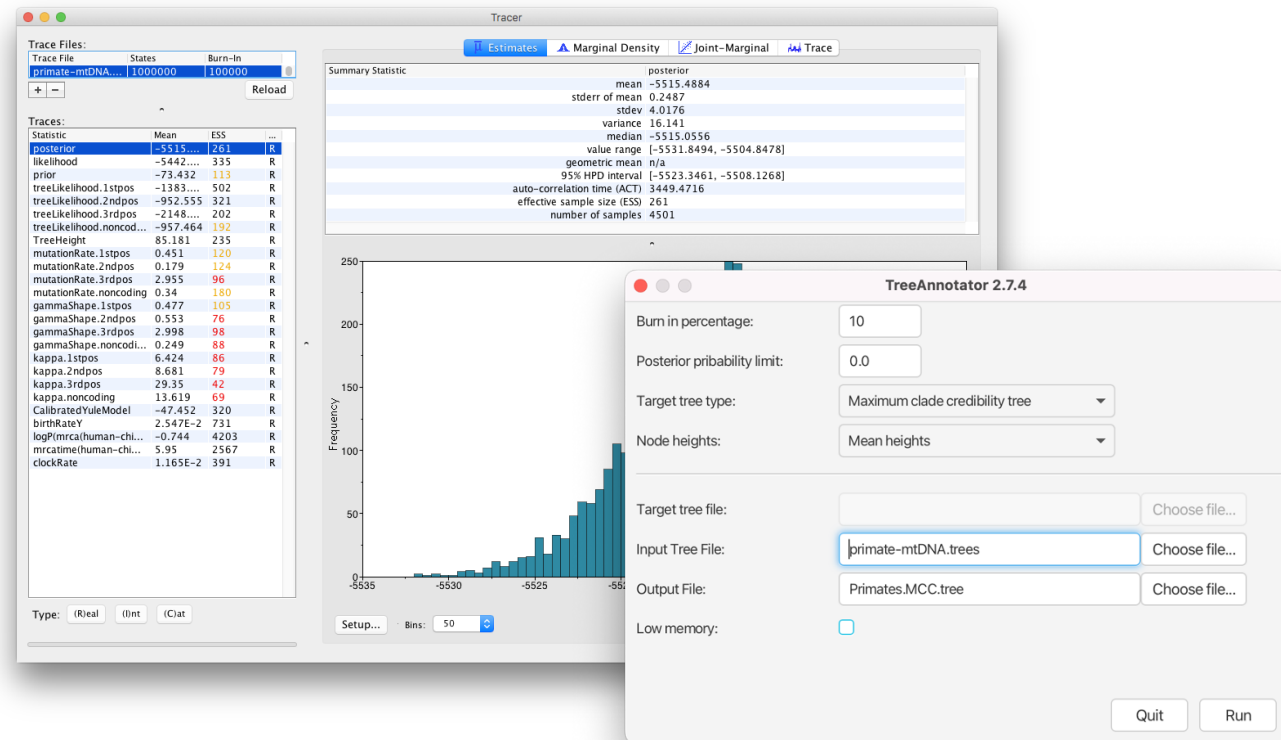
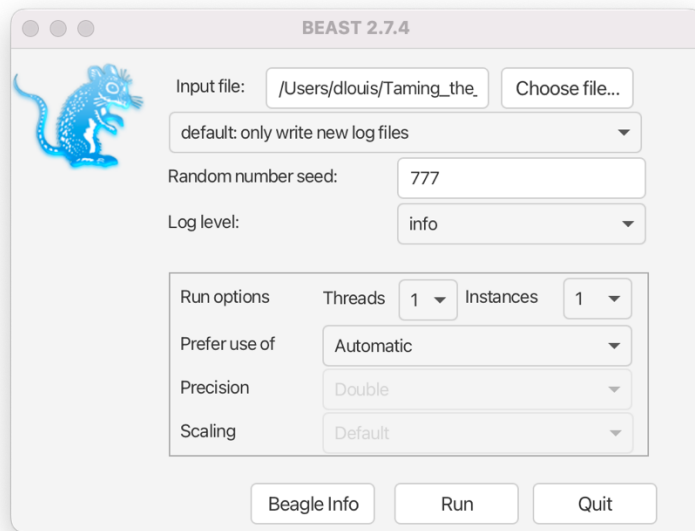
- <https://taming-the-beast.org/tutorials/Introduction-to-BEAST2>
- primate-mtDNA.nex
 - non-coding region
 - 1st codon positions
 - 2nd codon positions
 - 3rd codon positions
- Setting up the analysis
- Generating a XML file



| Name | File | Taxa | Sites | Data Type | Site Model | Clock Model | Tree | Ambiguities |
|-----------|---------------|------|-------|------------|------------|-------------|-----------|--------------------------|
| noncoding | primate-mt... | 12 | 205 | nucleotide | noncoding | noncoding | noncoding | <input type="checkbox"/> |
| 1stpos | primate-mt... | 12 | 231 | nucleotide | 1stpos | 1stpos | 1stpos | <input type="checkbox"/> |
| 2ndpos | primate-mt... | 12 | 231 | nucleotide | 2ndpos | 2ndpos | 2ndpos | <input type="checkbox"/> |
| 3rdpos | primate-mt... | 12 | 231 | nucleotide | 3rdpos | 3rdpos | 3rdpos | <input type="checkbox"/> |

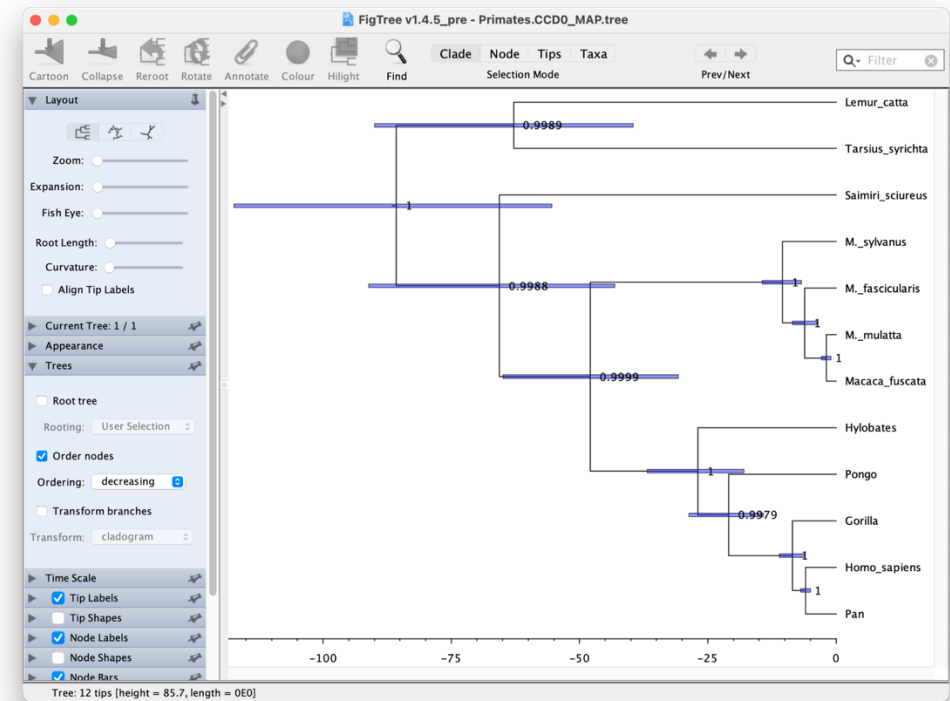
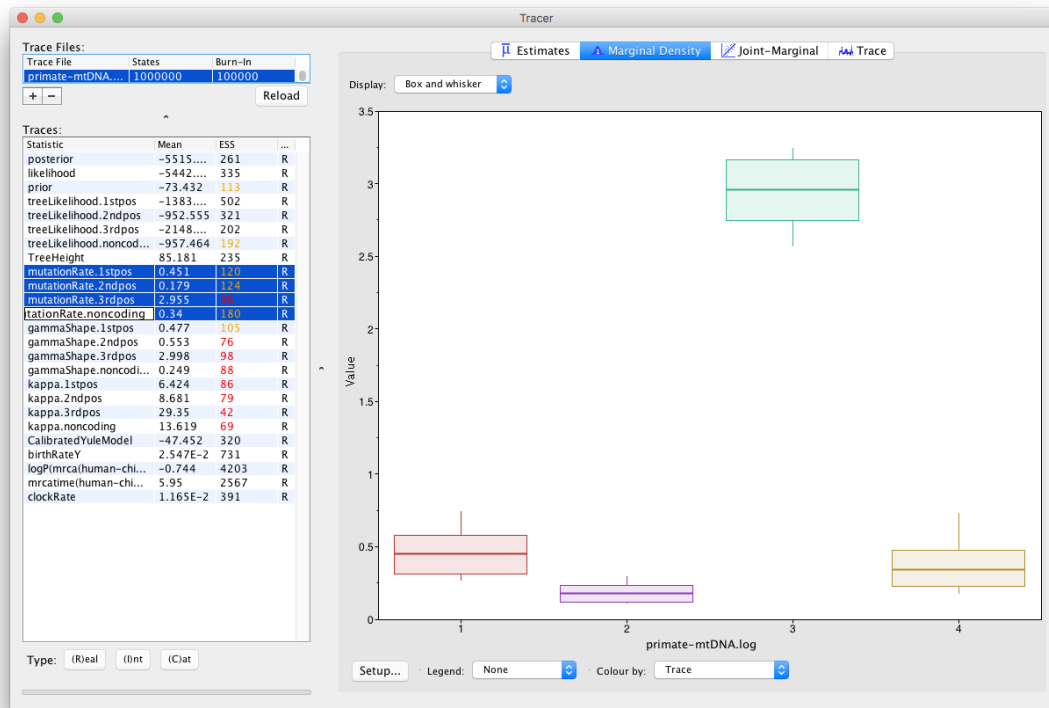
Workflow

- Running the analysis
- Processing the results



Workflow

- Visualizing the results



Bayesian inference

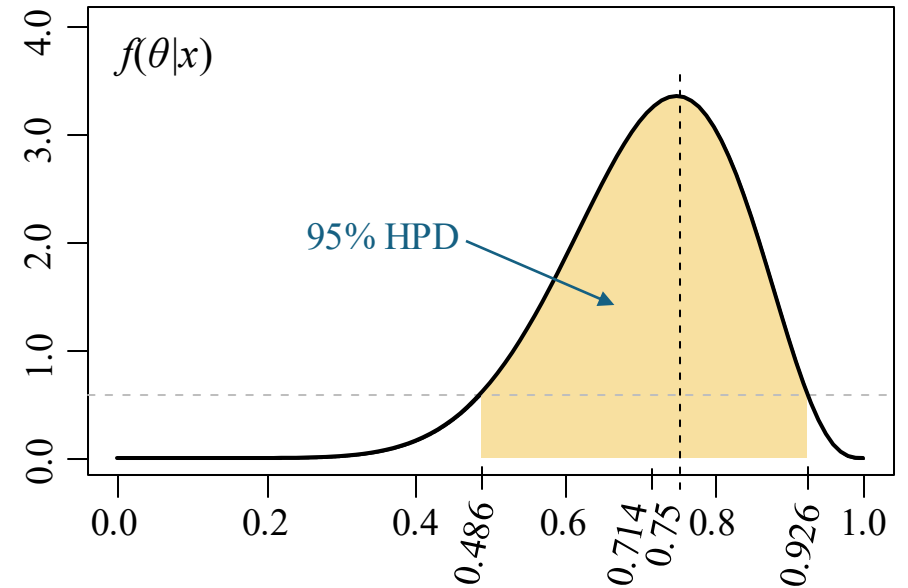
- Example: coin tossing (Yang 2014. p34)
- **Data**: $x = 9$ heads and $r = 3$ tails in $n = 12$ independent tosses
- **Parameter**: probability of head θ
- **Likelihood**: binomial distribution

$$\Pr(x|\theta) = \binom{n}{x} \theta^x (1 - \theta)^{n-x}$$

- **Prior**: $\theta \sim U(0, 1)$
- Posterior: $\theta|x \sim \text{Beta}(10, 4)$

Bayesian inference

- Point estimates:
 - Mean: $10/14 = 0.714$
 - Median: $\approx (10 - 1/3) / (14 - 2/3) = 0.725$
- Credibility interval:
 - 95% equal-tail credibility interval (0.462, 0.909)
 - 95% highest posterior density interval (0.486, 0.926)



Prior

- Represent one's subjective belief about the parameter before seeing or analyzing the data (subjective Bayesian)
- Noninformative priors do not exist (uniform priors are not noninformative)
- Vague or diffuse priors
- Informative priors
- In practice: assess the influence of the prior

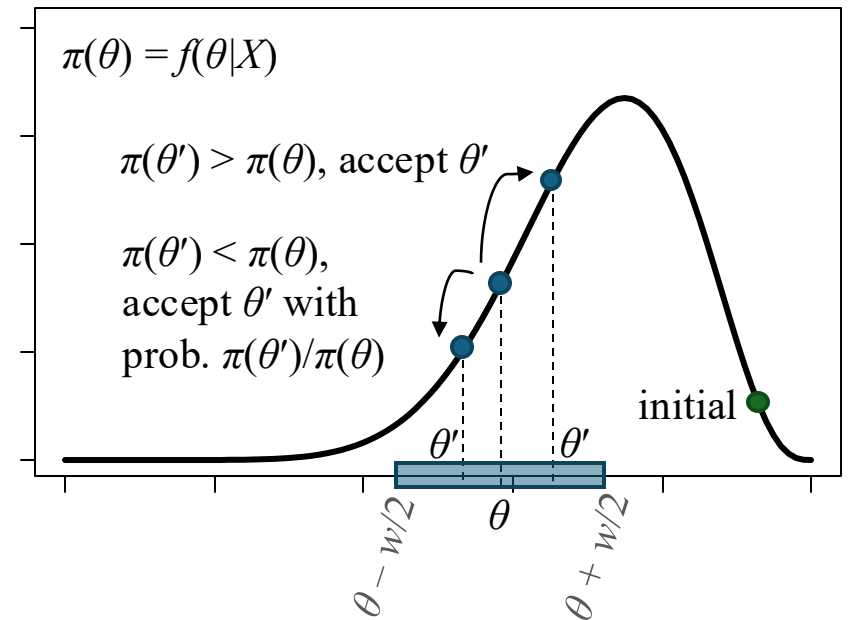
Markov chain Monte Carlo (MCMC)

- Metropolis algorithm (Metropolis et al. 1953)

1. Set initial state of θ
2. Propose a new state $\theta' \sim U(\theta - w/2, \theta + w/2)$
3. If $\pi(\theta') > \pi(\theta)$, accept θ' ; otherwise, accept θ' with probability $\alpha = \pi(\theta')/\pi(\theta)$
4. If the proposal is accepted, set $\theta = \theta'$; otherwise, set $\theta = \theta$. Print out θ
5. Go to step 2

- Acceptance ratio

$$\alpha = \min \left(1, \frac{\pi(\theta')}{\pi(\theta)} \right) = \min \left(1, \frac{f(\theta')f(X|\theta')}{f(\theta)f(X|\theta)} \right)$$



MCMC

- Metropolis-Hastings algorithm (Hastings 1970)

- Acceptance ratio $\alpha = \min \left(1, \frac{f(\theta')f(X|\theta')q(\theta|\theta')}{f(\theta)f(X|\theta)q(\theta'|\theta)} \right)$

prior ratio

likelihood ratio

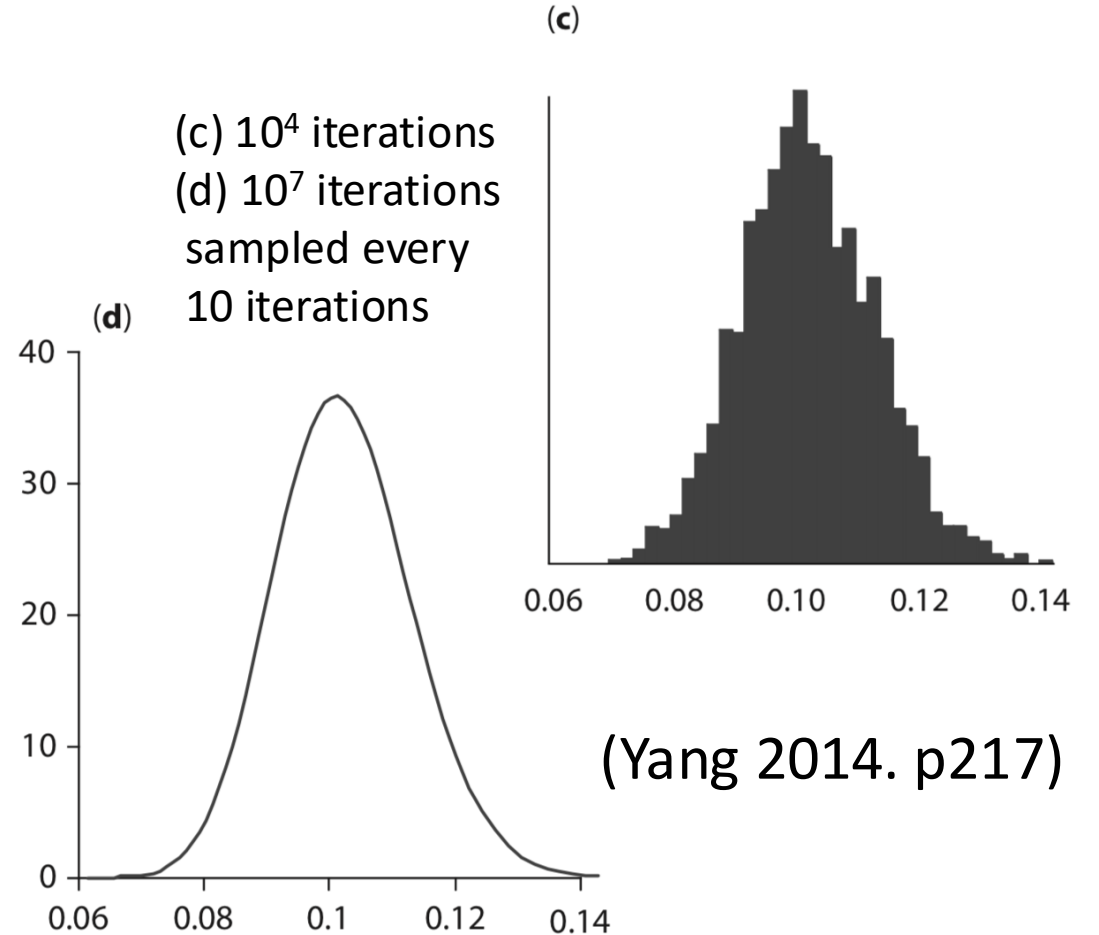
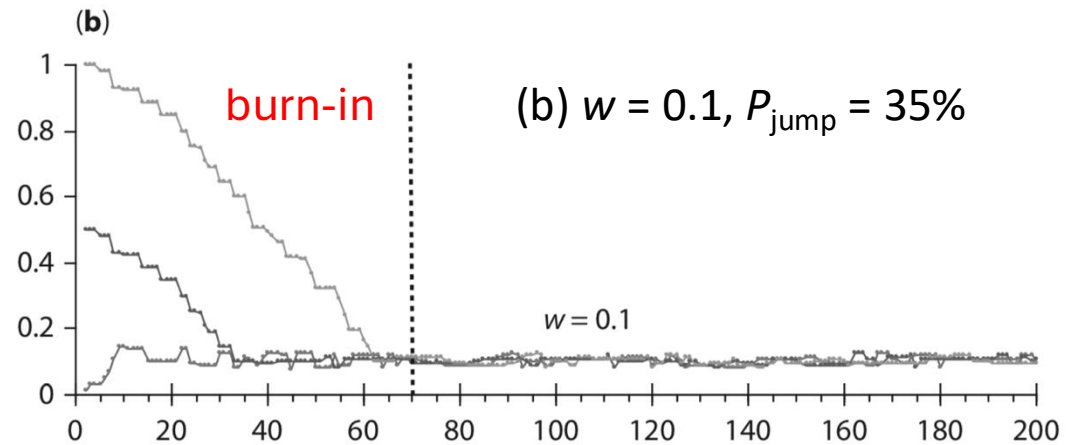
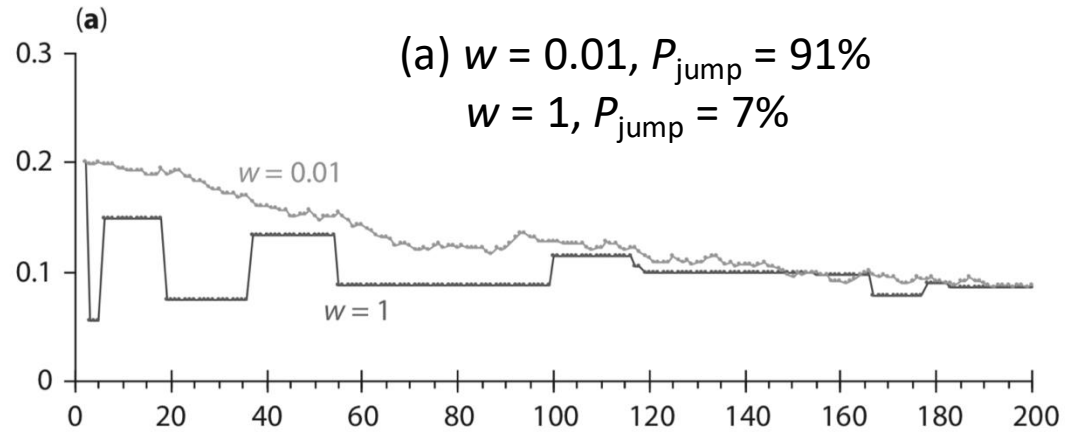
proposal ratio

- Multiplier proposal
 - Propose a new state $\theta' = \theta c = \theta e^{\varepsilon(u - 1/2)}$
 - Proposal ratio is $c = \theta'/\theta$

MCMC

- Example: JC69 distance (Yang 2014. p216)
- Data: human and orangutan 12s rRNA genes ($n = 948, x = 90$)
- Likelihood: $f(x|d) = (p_1/4)^x (p_0/4)^{n-x}$
$$= \left(\frac{1}{16} - \frac{1}{16} e^{-4d/3} \right)^x \left(\frac{1}{16} + \frac{3}{16} e^{-4d/3} \right)^{n-x}$$
- Prior: $f(d) = \lambda e^{-\lambda d}$, with $\lambda = 5$ (exponential distribution)

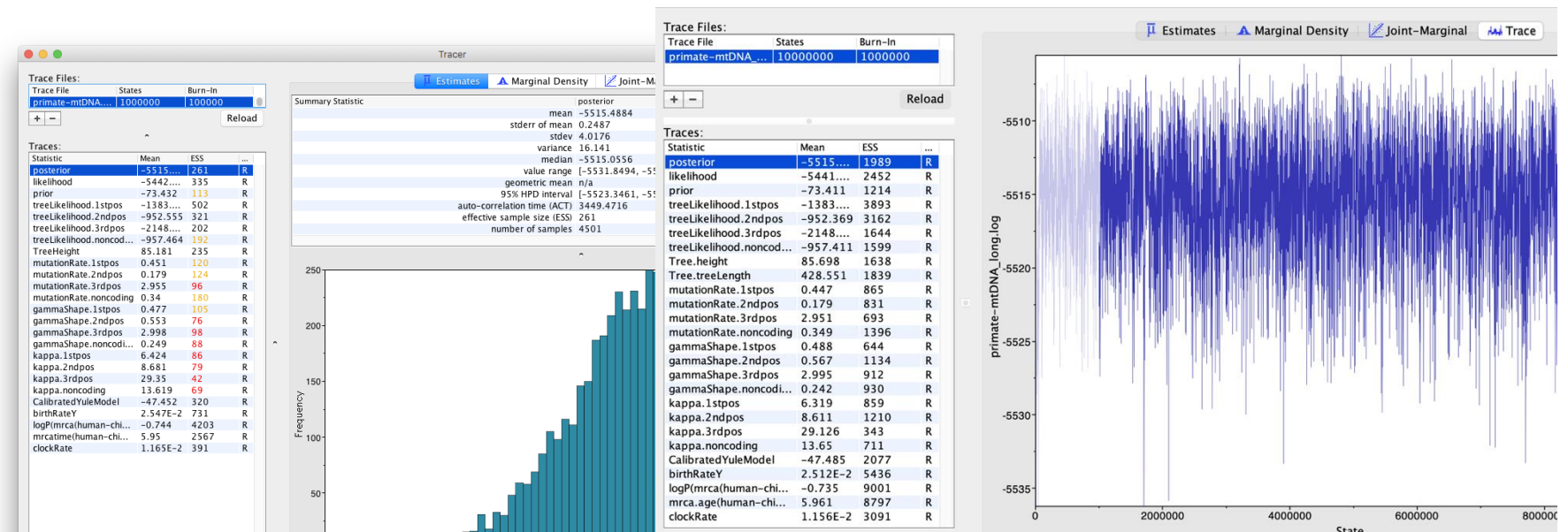
MCMC



MCMC

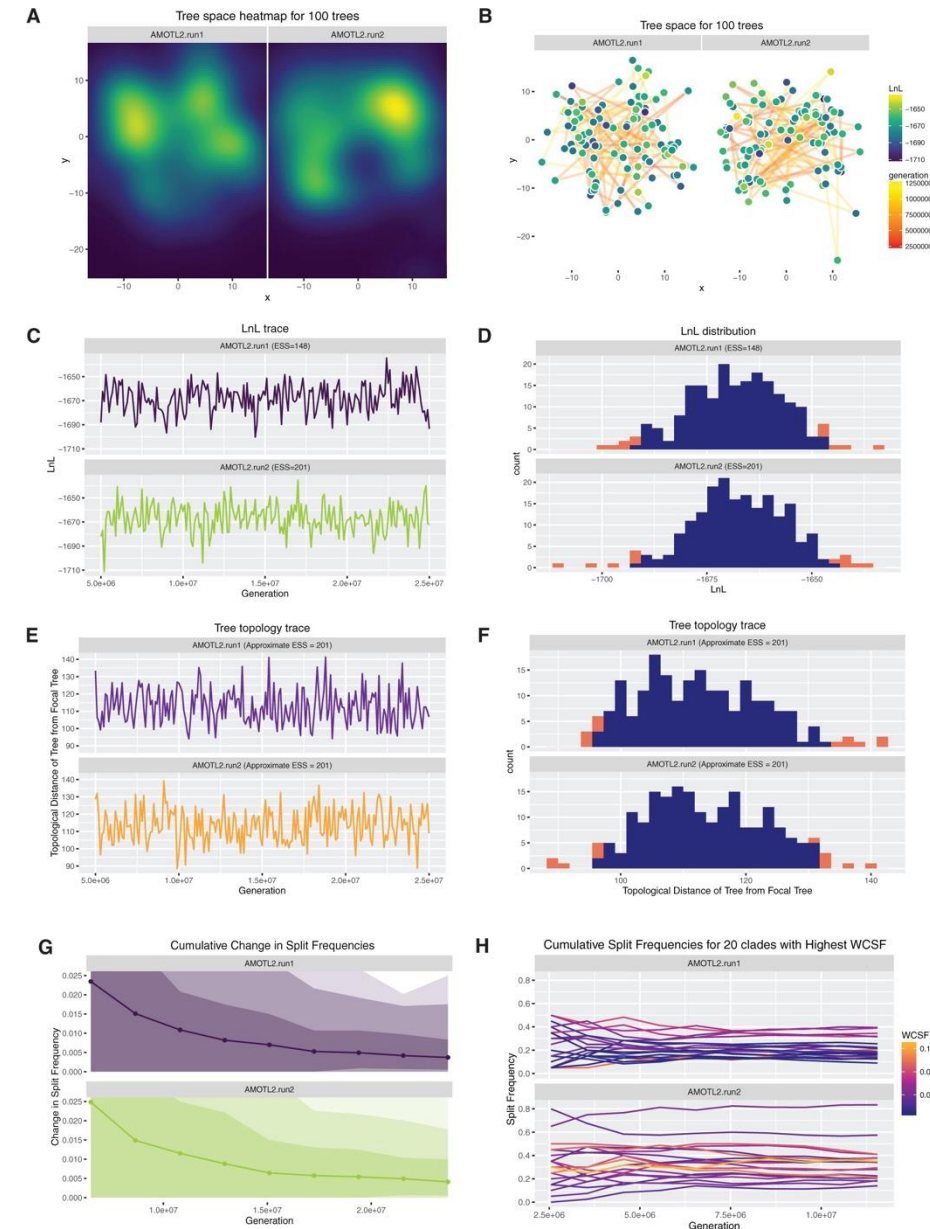
- Effective sample size (ESS)
- A dependent sample of size N is as informative as an independent sample of size $N/[1 + 2(\rho_1 + \rho_2 + \rho_3 + \dots)]$.

- $ESS > 200$



MCMC

- AWTY (R We There Yet) in R
(Warren et al. 2017)
 - trace plots of parameters
 - (approx.) ESS of tree topologies
 - visualizing treespace
 - split frequencies



MCMC

- Run multiple independent chains
- Compare samples among runs
- Check convergence and mixing
- Combine the samples (logcombiner)

Summarizing trees

- Maximum *a posterior* (MAP) tree
 - tree topology with the maximum posterior probability
- Maximum clade credibility (MCC) tree
 - tree with the maximum product of clade probabilities
- Conditional clade distribution (CCD) MAP tree
 - clade frequencies (CCD0)
 - clade split frequencies (CCD1)
 - pairs of clade split frequencies (CCD2)
- **CCD0-MAP tree should be the preferred point estimator** (Berling et al. 2025) [does not work for trees with sampled ancestors]

Probability distributions

- Discrete random variable
 - probability mass function
 - $\Pr(X = x_i) = p_i$ ($i = 1, 2, \dots, k$, and k can be ∞), with $\sum_i p_i = 1$
- Continuous random variable
 - probability density function (PDF)
 - $\Pr(a < X < b) = \int_a^b f(x)dx$
 - area under the entire PDF curve = 1

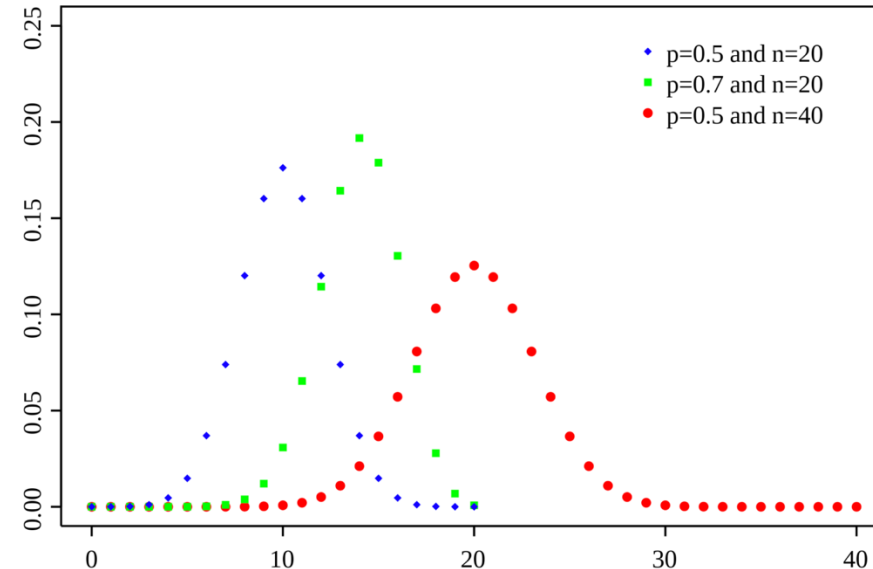
Probability distributions

- Binomial distribution
 - the number of successes in a sequence of n independent experiments, each asking a yes–no question, and each with its own Boolean-valued outcome: success (with probability p) or failure (with probability $q = 1 - p$)

- $\Pr(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$
 $(k = 0, 1, \dots, n)$

- Mean: np

- Variance: npq



Probability distributions

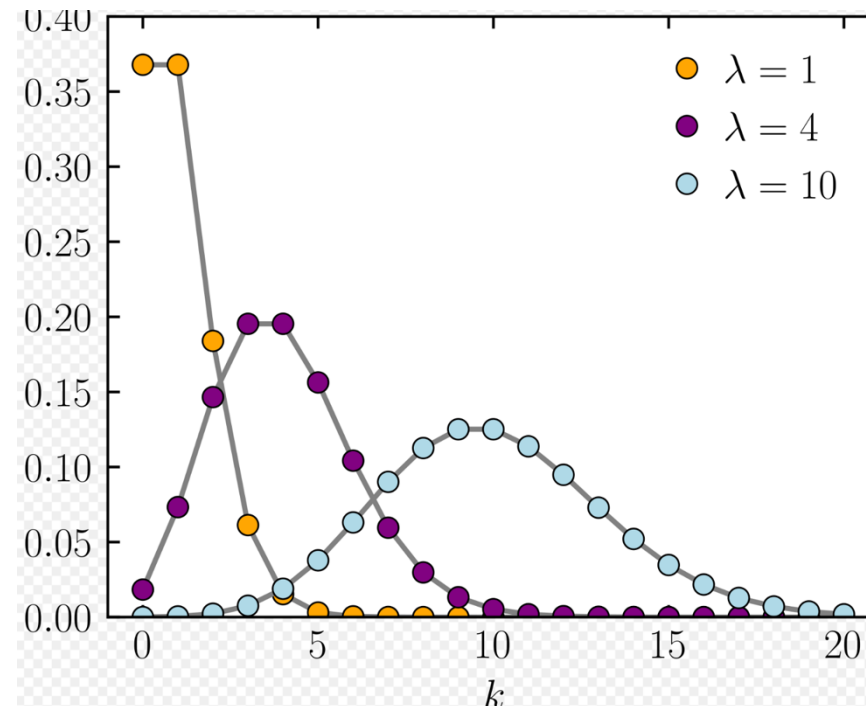
- Poisson distribution

- probability of a given number (k) of events occurring in a fixed interval of time or space

- $\Pr(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$

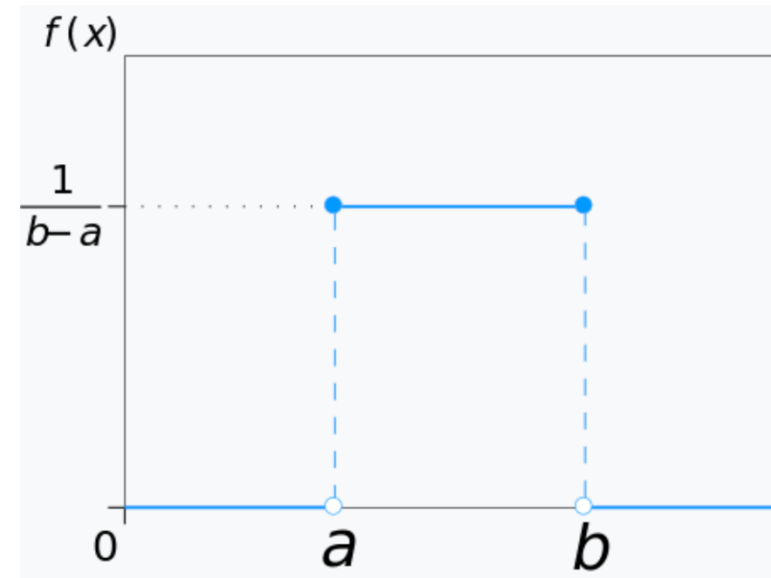
- Mean: λ

- Variance: λ



Probability distributions

- Uniform distribution
- $f(x) = \frac{1}{b-a}, a \leq x \leq b$
- Mean: $(a + b)/2$
- Variance: $(b - a)^2/12$

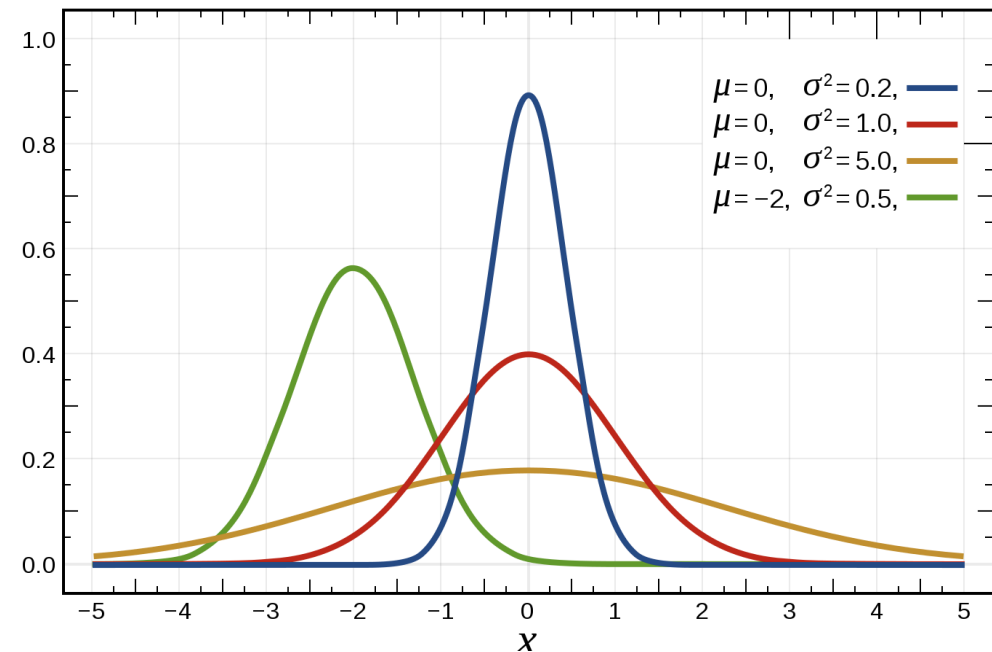
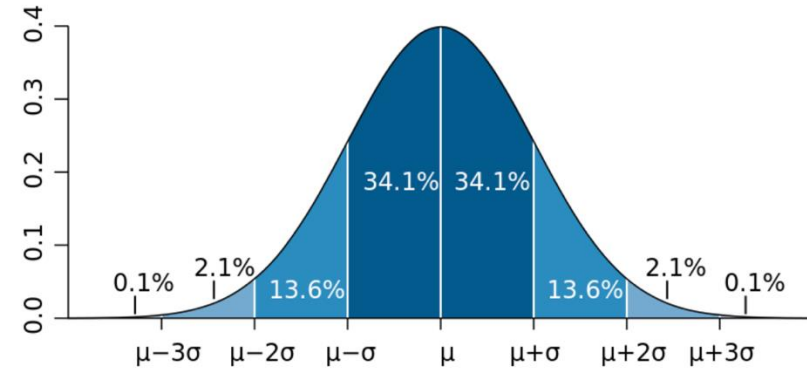


Probability distributions

- Normal distribution

- $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

- Mean: μ
- Variance: σ^2



Probability distributions

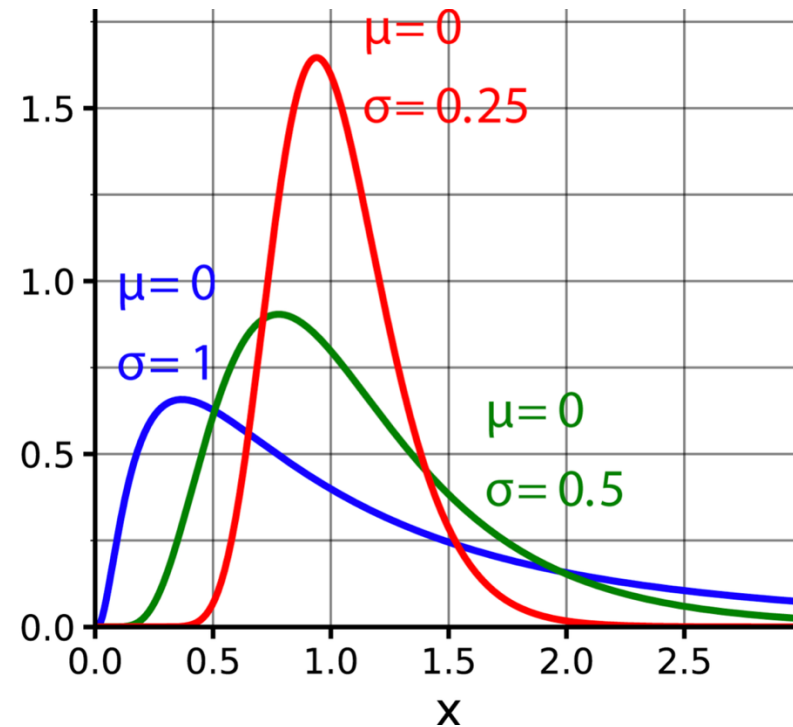
- Lognormal distribution
 - if Y has a normal distribution, then $X = e^Y$ has a lognormal distribution

- $$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{(\ln x - \mu)^2}{2\sigma^2}}, \quad x > 0$$

- Mean: $e^{\mu + \frac{\sigma^2}{2}}$

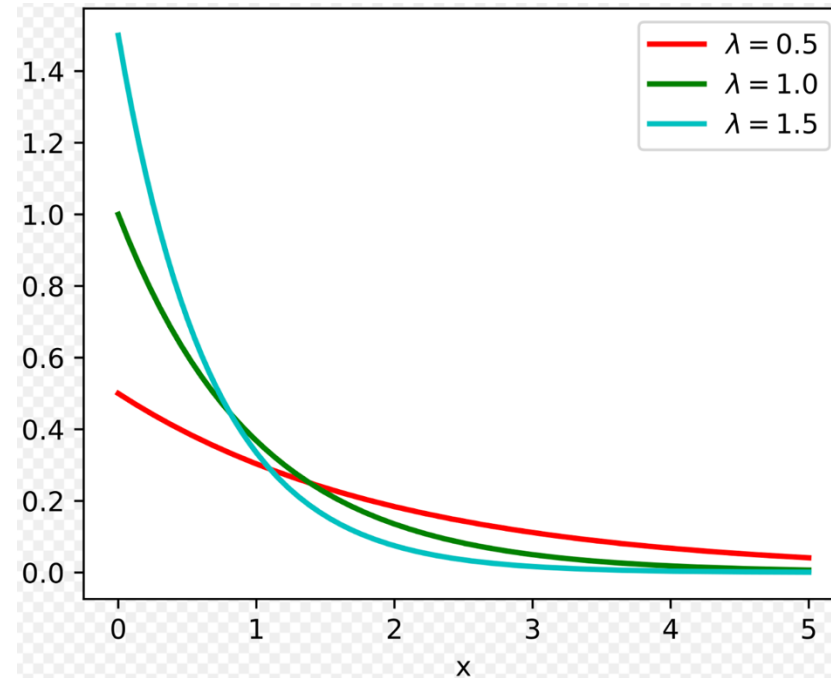
- Median: e^{μ}

- Variance: $(e^{\sigma^2} - 1)e^{2\mu + \sigma^2}$



Probability distributions

- Exponential distribution
- $f(x) = \lambda e^{-\lambda x}, x \geq 0$
- Mean: $1/\lambda$
- Variance: $1/\lambda^2$



Probability distributions

- Gamma distribution

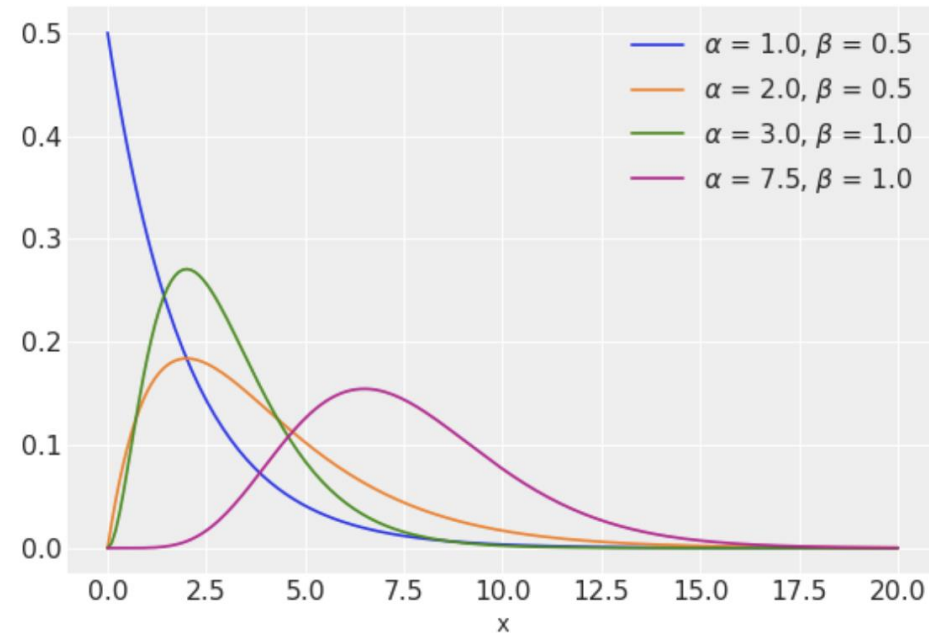
- $f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}, \quad x > 0$

- Mean: α/β

- Variance: α/β^2

- $\text{Exp}(\beta)$ when $\alpha = 1$

- χ_n^2 when $\alpha = n/2$ and $\beta = 1/2$



Probability distributions

- Beta distribution

- $f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1},$
 $0 < x < 1$

- Mean: $\alpha/(\alpha + \beta)$

- Variance: $\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$

- $U(0, 1)$ when $\alpha = \beta = 1$



Probability distributions

- Dirichlet distribution
- $f(x_1, \dots, x_K) = \frac{1}{B(\alpha)} \prod_{i=1}^K x_i^{\alpha_i - 1}$,
 $\sum_{i=1}^K x_i = 1, x_i \in [0, 1]; i \in \{1, \dots, K\}$
- Mean: $\frac{\alpha_i}{\sum_{i=1}^K \alpha_i}$
- It is a multivariate generalization of the beta distribution

