Nov 21 2017

$y \in \mathbb{R}^N$, $y_i \in \mathbb{R}^W$ is the ith window of length W in y (cyclically):

Contains $x_1, \dots, x_K \in \mathbb{R}^L$ separated by at least $W-1$ 0's ~~+noise~~

$$y_i = \underbrace{y_{i:i+W-1}}_{\text{mod}\, N}, \quad i = 0 \dots N-1$$

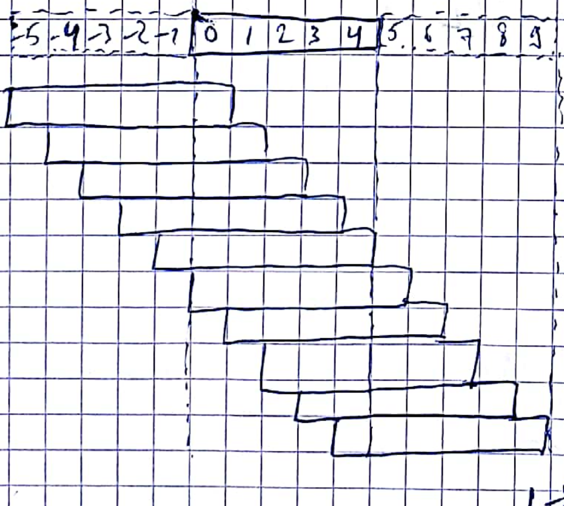
$$M_1 = \frac{1}{N}\sum_{i=0}^{N-1} \mu(y_i)$$

$$\cancel{M_2 = \frac{1}{N}\sum_{i=0}^{N-1} P_{y_i}}$$

for each occurrence of $x_1, \dots, x_K$, we get $W+L-1$ important windows: if $\underset{\sim}{x} = x_k$ appears, we get:

$$\frac{1}{W+L-1}\sum_{i=-W+1}^{L-1} \dots$$

i is position of first entry of window



$W=6$, $L=5$: $W+L-1 = 10$

$$(W-L+1)\,\varphi(\underset{\uparrow}{x_W}) + \sum_{i=0}^{L-2} \varphi((x_{0:i})_W) + \sum_{i=1}^{L-1} \varphi((x_{i:L-1})_W)$$

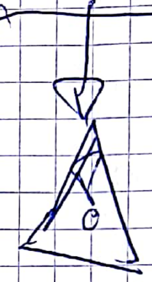
means: zero-pad

And this leaves $N - \underbrace{(\#\text{ sig occurrences})}_{\sum_{k=1}^{K} m_k}(W+L-1)$ noise windows

$\sum_{k=1}^{K} m_k$: m_k is # occ. of x_k.

$$M = \frac{1}{N}\sum_{i=0}^{N-1} \varphi(y_i)$$

CAREFUL: WINDOW NOISE NOT INDEPENDENT DUE TO OVERLAP?



"up to non-independence" $$\left(\frac{N-\left(\sum_{k=1}^{K} m_k\right)(W+L-1)}{N}\right)\mathbb{E}\{\varphi(\text{noise})\}$$

$$+\sum_{k=1}^{K}\frac{m_k}{N}\Bigg[(W-L+1)\,\mathbb{E}\{\varphi((x_k)_W+\text{noise})\}$$

$$+\sum_{i=0}^{L-2}\mathbb{E}\{\varphi(((x_k)_{0:i})_W+\text{noise})\}$$

$$+\sum_{i=1}^{L-1}\mathbb{E}\{\varphi(((x_k)_{i:L-1})_W+\text{noise})\}\Bigg]$$

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if φ = mean:

each entry appears W times "in an average"

$$\text{rhs} = 0 + \sum_{k=1}^{K}\frac{m_k}{N}\,\mathbb{1}_L^T x_k$$

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if φ = power spectrum:

$$\text{rhs} = \sigma^2 W + \sum_{k=1}^{K}\Bigg[\frac{m_k}{N}\Bigg[(W-L+1)\,P_{(x_k)_W} \;\ldots$$

$$\ldots + \sum_{i=0}^{L-2} P_{((x_k)_{0:i})_W}$$

$$+\sum_{i=1}^{L-1} P_{((x_k)_{i:L-1})_W}\Bigg]\Bigg]$$

if φ = bispectrum :

$$\mathbb{E}\left\{ B_{(x_{sub})_W + noise} \right\} = B_{(x_{sub})_W} + \mu((x_{sub})_W)\, \sigma^2 W^2 A .$$

$$\mathbb{E}\left\{ B_{noise} \right\} = 0 .$$

$$rhs = \sigma^2 W^2 A \cdot M_{mean}$$

$$+ \sum_{k=1}^{K} \left[\frac{m_k}{N} \left[(W-L+1)\, B_{(x_k)_W} + \sum_{i=0}^{L-2} B_{((x_k)_{0:i})_W} + \sum_{i=1}^{L-1} B_{((x_k)_{i:L-1})_W} \right] \right]$$

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