Blind deconvolution from multiple instances

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We aim to recover a signal x from a set of its convolutions with unknown kernels

$$y_i = x * h_i, \tag{.1}$$

or, equivalently,

$$\hat{y}_i[k] = \hat{x}[k]\hat{h}_i[k], \tag{.2}$$

Suppose we are given N measurements. Then, averaging over them yields:

$$\frac{1}{N}\sum_{i=1}^{N}\hat{y}_i[k] = \hat{x}[k]\frac{1}{N}\sum_{i=1}^{N}\hat{h}_i[k]. \tag{.3}$$

When $N \to \infty$ then

$$\mathbb{E}\left\{\hat{y}[k]\right\} = \hat{x}[k]\mathbb{E}\left\{\hat{h}[k]\right\}. \tag{.4}$$

So, if $\mathbb{E}\left\{h\right\}$ is known and invertible, then we can estimate y. Without loss of generality, we assume that $|\hat{x}[k]| = 1$.

If we have no access to $\mathbb{E}\left\{\hat{h}[k]\right\}$, we go to the second-order matrix. Here, we get

$$y_i y_i^T = C_x h_i h_i^T C_x^T. \tag{.5}$$

Or,

$$\mathbb{E}\left\{yy^T\right\} = C_x \mathbb{E}\left\{hh^T\right\} C_x^T. \tag{.6}$$

Under the assumption h is i.i.d. then this is an eigenvalue problem.