HETEROGENEOUS BIG MRA GRADIENT

$$a_y^1 = \frac{1}{n}\sum_{i=0}^{n-1} y[i] \longrightarrow \gamma a_x, \quad a_x^1 = \frac{1}{L}\sum_{i=0}^{L-1} x[i]$$

ALL INDEXING IS ZERO PADDED OUT OF BOUNDS

Let: $M_1 = \frac{1}{L}\gamma^T X^T \mathbb{1}_L$ $\quad\Big|\quad \nabla_X M_1 = \frac{1}{L}\mathbb{1}_L \gamma^T$

$\left(\frac{1}{L}\mathbb{1}_L^T X\right)^T$ $\quad = \frac{1}{L}\langle \gamma, X^T \mathbb{1}_L\rangle \quad\Big|\quad \nabla_\gamma M_1 = \frac{1}{L}X^T\mathbb{1}_L = \text{means of } X.$

$$a_y^2[\ell] = \frac{1}{n}\sum_{i=0}^{n-1} y[i]\,y[i+\ell] \longrightarrow \sum_{k=1}^{K}\gamma_k\, a_{x_k}^2[\ell] + \underline{\text{bias}}$$

$0 \leq \ell \leq L-1$

$\gamma_k = \frac{L m_k}{n}$, $\quad a_{x_k}^2[\ell] = \frac{1}{L}\sum_{i=0}^{L-1} x_k[i]\,x_k[i+\ell]$

$m_k = \#$ occurrences of x_k in y.

bias: if $\ell = 0$: σ^2 in total: we could ignore that entry, or estimate σ.

$$M_2[\ell] = \gamma^T a^2 + \sigma^2\delta_0$$

$a^2 = [\;] \in \mathbb{R}^K$ $\qquad a^2[k] = \frac{1}{L}\langle x_k[I_1], x_k[I_2]\rangle$

$$= \frac{1}{L}\langle I_1 x_k, I_2 x_k\rangle$$

$$\frac{\partial}{\partial x_k}\left(M_2[\ell]\right) = \gamma_k \frac{1}{L}\left(I_1^T I_2 + I_2^T I_1\right) x_k$$

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