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# Improving Data Utility Through Game Theory in Personalized Differential Privacy

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**Abstract** Due to dramatically increasing information published in social networks, privacy issues have given rise to public concerns. Although the presence of differential privacy provides privacy protection with theoretical foundations, the trade-off between privacy and data utility still demands further improvement. However, most existing studies do not consider the quantitative impact of the adversary when measuring data utility. In this paper, we firstly propose a personalized differential privacy method based on social distance. Then, we analyze the maximum data utility when users and adversaries are blind to the strategy sets of each other. We formalize all the payoff functions in the differential privacy sense, which is followed by the establishment of a static Bayesian game. The trade-off is calculated by deriving the Bayesian Nash equilibrium with a modified reinforcement learning algorithm. The proposed method achieves fast convergence by reducing the cardinality from $n$ to 2. In addition, the in-place trade-off can maximize the user's data utility if the action sets of the user and the adversary are public while the strategy sets are unrevealed. Our extensive experiments on the real-world dataset prove the proposed model is effective and feasible.

**Keywords** personalized privacy protection, game theory, trade-off, reinforcement learning

## 1 Introduction

In this big data era, the proliferation of mobile devices results in massive information being published over various social networks. Users tend to publish or share their information, including sensitive data, over social networks[1]. The utility and the attraction of the social network services origin from data sharing among users[2]. For example, a follower can browse the tweets of the other users in Twitter to learn recent developments[3]. However, the improper collection and abuse of the published data are negatively influencing the service quality of social networks.

Privacy may leak in various perspectives, for instance, identity privacy, social tie privacy, location privacy, interest privacy, and medial content privacy[4]. Therefore, privacy issues are critical while adversaries keep tracing users' sensitive information, especially in social networks[5,6]. The adversaries continuously launch structural-based attacks, background knowledge

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attacks, and collusion attacks to steal sensitive information from users[7]. The adversaries are never tired of devising new attacks or combining a variety of attacks, which put privacy protection under great threats.

Data utility is equivalently necessary as privacy protection in privacy-preserving models. To achieve better data utility, statistical properties are being discussed in different aspects[8]. In accordance with differential privacy, global sensitivity[9] is firstly proposed to maximize the data utility. Then, sampling Laplace noise[10] and machine learning based method come into existence successively. Moreover, the employment of game theory based method[11] captures the characters of the desired trade-off in a more appropriate way. However, data utility is still not satisfying because of insufficient optimization.

The trade-off between personalized privacy and improved data utility arises extensive concerns[12]. Lack of protection results in privacy leakage while over-protection brings about data utility degradation[13]. Normally, the trade-off is being demonstrated by simply splitting privacy level and data utility, which lacks measurement and optimization. This is not practical and feasible due to that trade-off can be optimized from different perspectives[14]. With an optimized trade-off, the proposed model can improve its performance based on the pre-set constraints. Therefore, deriving an optimized trade-off is a necessity in a personalized privacy protection model.

Existing privacy protection models have two main branches including clustering-based methods and differential privacy. Clustering-based methods are first put into use, such as $K$-anonymity[15], $L$-diversity[16], $T$-closeness, and their extensions. Clustering-based methods are feasible under the scenario of data publishing in the sense of dataset. It requires enough data to satisfy the amount, diversity, and distribution requirements. But the data shared in social networks are relative sparse and therefore the clustering-based methods are not practical. Nowadays, differential privacy has been implemented in more and more scenarios, for example, dataset correlations[9], location-based services[17], and so on. Differential privacy[18] and its extensions provide the privacy preservation with solid theoretical foundations[19]. Most existing differential privacy-based mechanisms are used in a data publishing scenario as well. The employment in social networks still requires for modification[20]. In addition, uniform privacy is another big issue. Nevertheless, uniform privacy protection cannot satisfy the ever increasing demands[21]. Built upon the new requirements, personalized differential privacy-preserving models are presented for various circumstances.

We have an observation that existing studies consider the privacy level to be uniform while they do not consider the improved data utility problem. Normally, the adversary is assumed to be static and both the adversary and the attack are measured qualitatively rather than quantitatively[22]. Base on this assumption, the data utility is measured in a way where the impact of the adversary is not taken into consideration. However, this is not practical, especially in a personalized differential private model.

To address the trade-off problem, we propose a game theoretical model aiming to maximize the data utility which takes the adversary effect into account. A personalized differential privacy protection model is proposed based on the social distance in social networks, that is, we measure the distance between the users in social networks with the relationship rather than the locational distance. Although this idea is discussed under the scenario of social networks, it can be extended to multiple situations such as IoT and machine learning. Firstly, we observe that users and adversaries are not sure of each other's payoff. Thus, we employ a static Bayesian game to model the confront between users and adversaries. Secondly, we define the two strategy sets in differential privacy sense, which measures the effect of both sides. As an extension of our conference paper[23] published on IEEE ICC2018, we further define payoff functions of each side and employ a modified reinforcement learning method to fast derive the Nash equilibrium, which accurately denotes the maximum data utility the users can achieve. The fast convergence is guaranteed by cardinality reduction from $n$ to 2. Finally, the evaluation results verify the effectiveness and feasibility of the proposed model.

The contributions are summarized as follows.

• We propose a personalized differential privacy protection model based on social distance. By personalizing the privacy level, we solve the problem of protecting privacy uniformly. Therefore, the personalized differential privacy leads to less overall privacy budget and higher data utility.

• We establish a static Bayesian game to capture the real-world confront characters and take the effect of both the user and the adversary into consideration. We formalize all the parameters in differential privacy sense and thereby measure the adversary and the attack quantitatively. The proposed model eliminates the

uncertainty of data utility measurement. Furthermore, we obtain the improved data utility by deriving the Bayesian Nash equilibrium.

• We derive the Bayesian Nash equilibrium rapidly with a modified Q-learning algorithm. We reduce the cardinality of the data and thereby reduce the updating rules, which brings about fast convergence. We also derive the best static strategy for users to achieve maximum data utility based on static Bayesian game.

• We evaluate the proposed model with extensive experiments on the real-world dataset. The outcomes prove the effectiveness and feasibility of the proposed model.

The rest of this paper is organized as below. In Section 2, we present related work and the preliminaries. We then present the framework of the proposed model in Section 3. Section 4 depicts the statistic Bayesian game while Section 5 demonstrates the Nash equilibrium derivation with reinforcement learning. We illustrate the system analysis in Section 6, which is followed by the performance and evaluation in Section 7. Finally, this paper is concluded in Section 8.

## 2 Related Work

Despite the advantages social networks have brought to us, the problem of privacy leakage arises extensive concerns[24], for example, identity privacy[25], authenticated data redaction[26], and so on. Clustering-based methods were proposed to deal with this, including $K$-anonymity[15], sensitive attributes grouping[27] and so on. These methods are practical with a small database size[12]. However, the privacy concern is highly reinforced in big data era[13]. Motivated by this, differential privacy[18] arises with higher performance and solid theoretical foundations[28]. Differential privacy functions well in statistic query and a lot of extensions have been presented to serve different scenarios[19]. In [29], Du *et al.* proposed a cryptography method which sheds light on privacy protection. Multiple social network privacy has been discussed in recent years[30].

Laplace mechanism provides random noisy responses which realize differential privacy in real-valued data sense[28,31]. After privacy is protected, researchers pay more attention to the optimal trade-off between privacy protection and data utility[14]. Wang and Zhang[11] used game theory and machine learning to obtain the optimal trade-off.

Jorgensen *et al.*[32] argued that not all users require the same level of privacy and introduced a new privacy framework called personalized differential privacy, in which the privacy requirements are specified at the user level, rather than by a single privacy parameter. In the scenario of crowdsourcing data aggregation, Wang *et al.*[33] proposed an aggregation scheme for histogram estimation, which enables participants to publish data at personalized differential-privacy levels. In [34], He *et al.* proposed a latent data privacy preserving model which can achieve personalized data utility in social networks. The authors tried to optimize the trade-off between latent data privacy protection and personalized data utility. Both prediction data utility loss and structure data utility loss are taken into consideration in this paper. Moreover, attribute sanitization and link sanitization are collaboratively solved rather than discussed separately. They proposed an attribute-link sanitization strategy which can provide satisfying the quality of service while protecting the sensitive latent information with personalized data utility. In addition, a powerful adversary with maximum inference attack ability is also employed to test the model which demonstrates its superior performance. Recently, Nie *et al.*[35] personalized the traditional definition of local differential privacy and first proposed a utility optimization framework for histogram estimation with personalized multi-level privacy.

## 3 System Modeling

In this section, the system mode is presented in detail. We propose a personalized differential privacy model based on social distance, which is followed by the establishment of a static Bayesian game.

We use a graph $G = \{n_i, e_i, m_i | n \in N, e \in E, m \in M\}$ to represent a social network in the proposed model. In the graph $G$, $n_i \in N$ denotes the set of nodes and $e_{ij} \in E$ is the set of edges, which are user set and relationship set between users respectively. $M$ indicates the set of sensitive information $m_i \in M$ that the users in $N$ may publish. For a pair of users $(i, j)$, if there is at least one edge $\{e_{ik_1}, e_{k_1k_n}, e_{k_nj}\} \in E$ between them, we conclude that user $i$ and user $j$ have a relationship. Based on the relationship, we also have $d_{ij} \in D$ to denote the social distance between the two users.

To better clarify, we assume the graph $G$ to be an undirected graph. However, this assumption can be removed. In addition, we also assume that there is a trusted central authority which processes the data with

$\epsilon$-differential privacy and transmits the data with secure communication. In a trusted central authority, the privacy budget is a constant $B$. For all the published data, the sum of all privacy budget equals $B$.

### 3.1 Personalized Differential Privacy

Using $\epsilon$-differential privacy, we implement the personalized privacy protection system based on social distance. In social networks, the users tend to share their messages to people with close relationships and do not care about the strangers. When a user $i$ diffuses a sensitive message $m$ over the social networks, it is intuitive that recipients with different social distances should obtain $m$ with different privacy protection.

To personalize differential privacy, we use social distance $d_{ij} \in D$ to decide the privacy budget $\epsilon$. If the social distance is relatively short, user $i$ wants the other users to see the message with high-level privacy protection. On the contrary, user $i$ does not care about people with relative long social distances and spends less privacy budget $\epsilon$ on them. Moreover, we use $\epsilon(\frac{1}{d_{ij}})$-differential privacy to personalize the privacy level as below.

**Definition 1** ($\epsilon(\frac{1}{d_{ij}})$-Differential Privacy (DP)). *Given $\epsilon(\frac{1}{d_{ij}})$ to be a positive privacy budget function decided by social distance $d_{ij}$, $D_1$ and $D_2$ are two datasets with an adjacent relationship, and $\mathcal{M}$ to be a randomized algorithm that sanitizes the dataset, the algorithm $M$ is $\epsilon(\frac{1}{d_{ij}})$-differential private on $D_1$ and $D_2$ if*

$$\Pr[\mathcal{M}(D_1) \in \Omega] \leqslant \mathrm{Exp}(\epsilon(\frac{1}{d_{ij}})) \times \Pr[\mathcal{M}(D_2) \in \Omega],$$

*where the probability space $\Omega$ is taken over the randomness used by $\mathcal{M}$.*

In the case of social distance, we measure the distance between the users in social networks rather than the locational distance. Normally, the social distance is calculated by hop counts based on the graph theory. Therefore, we take hop as the social distance as an example. This model can be further extended to other social distances, such as the shortest social distance, effective social distance, and so on. We use a logarithm function to describe the mapping function between $\epsilon$ and $d_{ij}$ where $\epsilon = -\ln \frac{1}{d_{ij}}$.

In this personalized privacy protection model, users with long social distance are allocated with more privacy budget $\epsilon(\frac{1}{d_{ij}})$ while users with short social distance are allocated with less one. $\epsilon(\frac{1}{d_{ij}})$ denotes the mapping function that decides the privacy budget allocation based on social distance. Moreover, the sum of all privacy levels' budgets equals the fixed budget $B$.

$$B = \sum \epsilon(\frac{1}{d_{ij}})\{(i,j)|1 \leqslant i,j \leqslant n, i \neq j\}.$$

### 3.2 Adversary Model

We consider a static adversary $\mathcal{A}$ who actions after considering the possible payoff functions of the user. Usually, the adversary $\mathcal{A}$ has a prior belief of the sensitive information. The data type of sensitive information in our study is shown as Table 1. The prior belief can be formalized as $\epsilon_p$-differential privacy. If $\epsilon_p \leqslant \epsilon(\frac{1}{d_{ij}})$, the adversary already has enough background knowledge of this sensitive information and he/she can successfully launch an inference attack. If $\epsilon_p > \epsilon(\frac{1}{d_{ij}})$, the privacy protection is strong enough and the adversary has to get more information to launch attacks, for example, collusion. Given the adversary's prior belief $\epsilon_p$ and the obtained information $\epsilon_b$, we formalize the adversary's behavior as

$$DP(\epsilon_p) + DP(\epsilon_b) = DP(\epsilon_p + \epsilon_b),$$

where $DP()$ is short for differential privacy. This equation holds because of the composition character of differential privacy scheme.

$$\begin{cases} DP(\epsilon_p + \epsilon_b) > DP(B), \\ DP(\epsilon_p + \epsilon_b) > DP(\sum_{i,j}^{n,n} \epsilon(\frac{1}{d_{ij}})), \\ DP(\epsilon_p + \epsilon_b) \leqslant DP(\sum_{i,j}^{n,n} \epsilon(\frac{1}{d_{ij}})). \end{cases} \tag{1}$$

**Table 1.** Sensitive Data Type

| Data Type | Explanation |
|---|---|
| Binary state | $s_i \in \{0,1\}$ indicates a binary status like online or not |
| Location | $(longitude_i, latitude_i, height_i) \in R_3$ is the GPS coordinates of a certain entity |
| Time stamp | $t_i \in R_+$ denotes positive real number such as time or date |
| Text data | $d_i \in ABC$ represents alphabetic instance like light-weighted encryption cipher |
| Media data | Media data includes videos, images, sound, and so forth |

Based on (1), we have the following observations. Firstly, the sum of $\epsilon_p$ and $\epsilon_b$ is smaller than $\epsilon(\frac{1}{d_{ij}})$, and the adversary breaches the privacy of a certain level $\frac{1}{d_{ij}}$. Secondly, the sum of $\epsilon_p$ and $\epsilon_b$ is smaller than $B$,

the adversary breaches the privacy of the whole system. Thirdly, the sum of $\epsilon_p$ and $\epsilon_b$ is larger than $\epsilon(\frac{1}{d_{i1}})$, the adversary fails the attack.

### 3.3 Static Bayesian Game

In the established personalized privacy protection model, the confrontation between the user $i$ and an adversary captures the features of the game theory. Before successfully breaching the privacy, the adversary is not sure about how much more information he/she can obtain, and the user has no idea about how much background the adversary holds. Therefore, the confrontation between the user and the adversary can be formalized as a static Bayesian game.

This static Bayesian game includes action space $A$, type space $T$, and their inference $P$, and payoff function $U$. The type of user $i$ is regarded as the personal information, which decides user $i$'s payoff function $u_i(a_1, ..., a_n; t_i)$. Meanwhile, $i$ is an element in the possible type set $T_i$. The user $i$'s inference $p_i = (t_{-i}|t_i)$ describes the uncertainty of the other $(n-1)$ users' possible type $t_{-i}$ when $t_i$ is the type of user $i$. Given all the above conditions, we define the static Bayesian game as (2).

$$G = \{A_1, ..., A_n; T_1, ..., T_n; P_1, ..., P_n; U_1, ..., U_n\}. \quad (2)$$

In Algorithm 1, we can express an incomplete information game as an imperfect information game by the virtue of step 8, which assigns $t_i$ "naturally". The involved players are not aware of the previous game process.

**Algorithm 1.** Game-Based Optimal Data Utility Derivation Algorithm

**Input:** static Bayesian game $G$;
**Output:** optimal data utility $U_o$;
1: Implement $\epsilon(\frac{1}{d_{ij}})$-differential privacy;
2: Formulize the adversary behavior as $(\epsilon_p + \epsilon_b)$-differential privacy;
3: Initialize the action space $A$;
4: Initialize the type space $T$;
5: Define the inference space $P$ based on $(A, T)$;
6: Define the payoff function set $U$ based on $(A, T, P)$;
7: **while** until convergence **do**
8: Players are assigned with $t_i$ but aware of $t_{-i}$ naturally;
9: Players choose their actions $\exists a_i \in A$ at the same time;
10: Payoff generation $u_i(a_1, ..., a_n; t_i)$;
11: Update action $a_{i+1}$ and $t_{i+1}$ based on $p_i$ ;
12: Update payoff $u_{i+1}$;
13: **end while**
14: Nash equilibrium $NE$ obtained;
15: Derive the optimal data utility $U_o$ based on $NE$

We further discuss how to calculate the reference $p(t_{-i}|t_i)$. We first assume the transcendental probability distribution $p(\boldsymbol{t})$ and the type vector $\boldsymbol{t} = (t_1, ..., t_n)$ are common knowledge. After the player $i$ is assigned with $t_i$, he/she can calculate the reference $p(t_{-i}|t_i)$ according to all the other players' conditional probability with Bayes rules.

$$p(t_{-i}|t_i) = \frac{p(t_{-i}, t_i)}{p(t_i)} = \frac{p(t_{-i}, t_i)}{\sum_{t_{-i} \in T_{-i}} p(t_{-i}, t_i)}.$$

In addition, the other players can obtain $i$'s various inferences based on $t_i$, that is, $p(t_{-i}|t_i)$ can be derived for $\forall t_i \in T_i$. Normally, the types of players are assumed to be independent, which means $p(t_{-i})$ does not depend on $t_i$. But $p(t_{-i})$ is still derived from the transcendental probability distribution $p(t)$. In this situation, the other players realize $i$'s inferences on their types.

In this algorithm, as we have iteration process to update and optimize the final output, the total iteration times would be $n$ for each cardinality. Therefore, the computation complexity of each cardinality would be $O(n)$. Since the cardinality of the proposed model is reduced from $n$ to 2 as analyzed above, we conclude the final computation complexity is $O(2n)$, which enjoys a significant reduction compared with the traditional models of $O(n^2)$.

## 4 Bayesian Nash Equilibrium

Built upon the proposed model, we present the Bayesian Nash equilibrium in this part. We first derive the Bayesian Nash equilibrium of the static Bayesian game. The Bayesian Nash equilibrium denotes the optimal data utility of the personalized privacy protection system. We then analyze the confrontation between two players, including one user and one adversary. In addition, the results can be further extended to multiple adversaries. On the one hand, if the multiple adversaries do not collude, then the course of offense-defense can be formulated into multiple independent static Bayesian games. On the other hand, if the multiple adversaries collude with one another, we can formulate them into one collusion adversary according to the composition theorem of differential privacy.

### 4.1 Bayesian Nash Equilibrium

In order to acquire the Bayesian Nash equilibrium, we must define the strategy space of the players. The strategy is a package plan of actions, including every

possible action corresponding to all the possible situations. Given the time sequence of static Bayesian game and the pre-assigned type $t_i \in T_i$, a strategy of player $i$ has to contain a feasible action of every possible type $t_i \in T_i$.

Therefore, in a static Bayesian game $G$, a strategy of player $i$ is a function $s_i(t_i)$. With regard to $\forall t_i \in T_i$, $s_i(t_i)$ contains the possible actions $\exists a_i \in A_i$ when $t_i$ is the type of user $i$.

Unlike complete information game, no strategy space is defined in (2). As a substitute, we can construct the strategy space from the action space $A_i$ and the type space $T_i$. Player $i$'s feasible strategy set $S_i$ is the function set whose definition domain is $T_i$ and whose value domain is $A_i$. For example, every $t_i \in T_i$ chooses a different action $a_i \in A_i$ in a separating strategy while all $t_i \in T_i$ choose the same $a_i \in A_i$ in a pooling strategy.

Based on the analysis, we conclude every player's strategy must be other players' optimal responses of strategy, that is, the following definition of Bayesian Nash equilibrium is the Nash equilibrium of static Bayesian game.

**Definition 2** (Bayesian Nash Equilibrium). *Given $n$ players and a static Bayesian game $G = \{A_1, ..., A_n; T_1, ..., T_n; P_1, ..., P_n; U_1, ..., U_n\}$, a strategy set is a pure strategy Bayesian Nash equilibrium if for $\forall i$ and the type set $\forall t_i \in T_i$, $s_i^*(t_i)$ satisfies*

$$\max_{a_i \in A_i} \sum_{t_{-i} \in T_{-i}} u_i(s_i^*(t_1), ..., s_{i-1}^*(t_{i-1}), a_i, s_{i+1}^*(t_{i+1}), ..., s_n^*(t_n); t) \times p_i(t_{-i}|t_i).$$

Namely, no player wants to change his/her strategy even if this change only involves a single action of a specific type.

### 4.2 Two-Player Game Analysis

In this subsection, we consider the static Bayesian game between one adversary and user $i$. This can be further extended to multiple adversaries scenario. Firstly, we analyze the action-type based strategy space $S_i(a_i, t_i)$. Secondly, the payoff function $U_i$ is presented based on strategy space. Thirdly, we derive the Bayesian Nash equilibrium which could be seen as the optimal trade-off with maximum data utility.

The player $i$ has two actions, which is actually performed by the trusted central authority. The first action $a_i^1$ is to publish the data without privacy protection. Without privacy protection, we use $\epsilon \to \infty$ to denote the action in differential privacy sense. The second action $a_i^2$ is to publish the data with certain privacy protection, which is described as $\epsilon(\frac{1}{d_{ij}})$ based on social distance.

The type of specification action is the personal information that the other player does not know. Therefore, $a_i^1$'s type $t_i^1$ depends on whether player $i$ adopts any protection measures. In the case of $a_i^2$, its type $t_i^2$ can be regarded as how the mapping function $\epsilon(\frac{1}{d_{ij}})$ functions. Namely, the adversary does not know how the function maps social distance $d_{ij}$ into privacy budget $\epsilon$.

Talking about the adversary $j$, there are also two actions. The first action $a_j^1$ means that the adversary launches the attack with background knowledge only, which can be formalized as $\epsilon_p$. The second action $a_j^2$ is that the adversary launches the attack with background knowledge and other supplemental information. We use $\epsilon_p + \epsilon_b$ to denote $a_j^2$.

In the case of types corresponding to the actions, we also consider the uncertainty. Thus, $a_j^1$'s type $t_j^1$ is that user $i$ has no idea of how much background knowledge the adversary has, while $a_j^2$'s type $t_j^2$ is that user $i$ has no idea of how much more information the adversary can obtain.

As we formulate all the strategies in the differential privacy sense, the payoff function $U_i$ can be easily obtained by minus operation. Moreover, the payoffs of the user $i$ and the adversary $j$ are reciprocal numbers. The detailed $U_i$ is shown as Table 2.

**Table 2.** Two-Player Static Bayesian Game

| A | $P(i)$ | |
|---|---|---|
| | $\epsilon \to \infty$ | $\epsilon(\frac{1}{d_{ij}})$ |
| $\epsilon_p$ | $\pm(\epsilon_p - \epsilon)$ | $\pm(\epsilon_p - \epsilon(\frac{1}{d_{ij}}))$ |
| $\epsilon_p + \epsilon_b$ | $\pm(\epsilon_p + \epsilon_b - \epsilon)$ | $\pm(\epsilon_p + \epsilon_b - \epsilon(\frac{1}{d_{ij}}))$ |

## 5 Bayesian Nash Equilibrium Derivation with Reinforcement Learning

In this section, we derive the Bayesian Nash equilibrium built upon Markov decision process with reinforcement learning. We first model the actions of users and adversaries and state transmission. Based on these, we formulate users payoff function and thereby the fast convergence learning algorithm. At last, we present the best strategy of users with optimized data utility.

### 5.1 Actions of Users and Adversaries

In social networks, users tend to publish data (including sensitive information) on their homepage and

share the data with the other users. Therefore, we formulate the action of users $AC_u^i$ as the granularity of the published data.

We have an assumption that adversaries have limited computing resources, which is feasible and practical in real-world application. Therefore, the adversaries can only access and process $n$ pieces of data published by a certain user.

$$\sum_{i \in n} AC_u^i \leqslant \Upsilon, \quad 0 \leqslant AC_u^i \leqslant 1. \tag{3}$$

In (3), $\Upsilon$ denotes the maximum computing power of adversaries while the granularity of $AC_u^i$ is unified into the range $[0, 1]$. When $\Upsilon$ is larger than $n$, the computing power of the adversaries can be regarded as unlimited and they can post-process all the data published by users. In opposite, if $\Upsilon$ is smaller than $n$, we know the adversaries have limited computing power.

From the perspective of adversaries, we formulate the action $AC_{ad}$ as the probability of an adversary re-identifying a specific user. It is intuitive that $0 \leqslant AC_{ad} \leqslant 1$. We can learn the probability by observing the attacking results from adversaries, for example, spam emails, personalized advertisements, and so on.

### 5.2 State Transmission

To model a Markov decision process, we need to investigate the state transmission of the attack and defense confrontation.

From the perspective of the system state, there are two components including the current piece of data and the last attack response. As mentioned above, users can observe the attack results by receiving spam emails with personalized advertisements. This is a good way for a user to measure the degree of sensitive information disclosure. Therefore, the user may change his/her strategy and publish the next piece of data with another granularity. For better data utility, the granularity will be more fine-grained while coarse-grained data can result in higher privacy protection level.

In terms of attack response $AR$, we make $AR_i = 1$ if the adversary successfully obtains the sensitive data $M_{i-1}$ while $AR_i = 0$ if the adversary fails to get any useful information from $M_{i-1}$. Therefore, the system status can be formulated as $S = \{D_i, AR_{i-1}\}$.

To model the state transmission of the confrontation between users and adversaries, we define the system states as $S = \{S_u^i, S_{ad}^i\}$. The state is determined by sensitive data and attack response while they are further decided by actions of users and adversaries, respectively. Therefore, we define the system state transmission as

$$\begin{aligned} &\Pr[(S_u^i, S_{ad}^i)|(S_1^{i-1}, S_2^{i-1}), A_u^{i-1}, A_{ad}^{i-1}] \\ =\ &\Pr[D_i|D_{i-1}] \Pr[AR^i|AR^{i-1}, A_u^{i-1}, A_{ad}^{i-1}] \\ =\ &\Pr[D_i|D_{i-1}] \Pr[AR^i|A_u^{i-1}, A_{ad}^{i-1}]. \end{aligned}$$

We derive the second equation because the attack response $AR_i$ at time slot $i$ only depends on the actions of the user and adversaries at time slot $i-1$. That is the reason why $AR_I$ is not impacted by $AR_{i-1}$.

### 5.3 Payoff Function and Nash Equilibrium

We are aiming to maximize data utility in this article. Therefore, we establish a service quality-based data utility measurement as an index. The measurement is as below.

$$R((S_u^i, S_{ad}^i), A_u^i, A_{ad}^i) = Q(A_u^i) - \omega \times L(S_u^i, S_{ad}^i),$$

where $Q(A_u^i)$ is the quality of service and $L(S_u^i, S_{ad}^i)$ denotes the privacy loss.

We use $\chi$ to denote the strategy. Thus, we use $\chi^u : S_u \longmapsto \delta(A_u)$ to denote the strategy of the user, and $\chi^{ad} : S_{ad} \longmapsto \delta(A_{ad})$ to denote the strategy of adversaries, where $(S_u, S_{ad})$ denotes the state space, and $\delta(A_u)$ and $\delta(A_{ad})$ the probability distribution over $A_u$ and $A_{ad}$, which represents the action spaces of users and adversaries.

Given the initial time slot $i = 0$ and the initial state $s_0 \in S$, we re-formulate the payoff function as

$$\begin{aligned} R^{\chi}(s) \ &= \sum_{i=0} \mathrm{E}[R((S_u^i, S_{ad}^i), A_u^i, A_{ad}^i)|\chi_u, \tau_{ad}, s_0 = s] \\ &= R(s, A_u^i, A_{ad}^i) + \sum_{\widehat{s}} \Pr[\widehat{s}|s, A_s^i, A_{ad}^i] R^{\tau}(\widehat{s}). \end{aligned}$$

As both of the adversary and the user want to follow the best strategy of their own, there is a confrontation between them. To model the confrontation, we first define the best strategy as $\chi_u^*$ and $\chi_{ad}^*$, and the best strategy pair is $\chi^* = (\chi_u^*, \chi_{ad}^*)$. Given a multi-slot game and system state $s \in S$, the Nash equilibrium of $\chi^*$ is

$$\begin{cases} R^{\chi^*}(s) \geqslant R^{\chi_{ad}^*}(s), \\ R^{\chi^*}(s) \leqslant R^{\chi_u^*}(s), \end{cases}$$

where $\chi_{ad} = \{\chi_u, {\chi_{ad}}^*\}$, and $\chi_u = \{{\chi_u}^*, \chi_{ad}\}$, for all $\chi_{ad}$ and $\chi_u$.

### 5.4 Fast Convergence Learning Algorithm

For the sake of fast derivation of Nash equilibrium, we employ a modified Q-learning method to achieve fast convergence. We define the equivalent $\hat{R}_u^{\chi^*}(AR)$ as the expected value $R_u^{\chi^*}(s)$ where $s = \{AR, D\}$, through which we can get rid of $D$. We express $\hat{R}_u^{\chi^*}(AR)$ as

$$\begin{aligned}&\widehat{R}_u^{\chi^*}(AR)\\ &= \mathrm{E}[R_u(s, A^{\chi^*}) + \sum_{AR'}(\Pr[AR'|A^{\chi^*}]\hat{PU}^{\chi^*}(AR'))],\end{aligned}$$

where $A^{\chi^*} = \{A_u^{\chi^*}, A_{ad}^{\chi^*}\}$ is the best action following the best strategy $\chi^*$.

Built upon this, we can derive $\chi^*$ with an equivalent problem as below. The equivalent problem can reduce the cardinality from $n$ to 2 and thereby accelerate the derivation process.

$$\begin{aligned}\chi^* = &\max_{\chi_u}\min_{\chi_{ad}}\mathrm{E}[R_u(s, A^{\chi^*}) + \\ &\sum_{AR'}(\Pr[AR'|A^{\chi^*}]\hat{PU}^{\chi^*}(AR'))].\end{aligned} \tag{4}$$

According to (4), we further leverage $\hat{R}^{\chi^*}$ to accomplish the following updating rule.

$$\begin{aligned}&\hat{R}^{t+1}(AR)\\ &= (1-\alpha_{i+1})\hat{R}^i(AR) + \alpha_{i+1}\mathrm{E}[R(s, A_u^i, A_{ad}^i) + \\ &\widehat{R}^i(AR')],\end{aligned}$$

where $\alpha_i \in [0, 1]$ denotes the learning rate of the algorithm. We set $\alpha_i$ to decrease with time in order to get a deterministic convergence, which is $\alpha_i = 1/i$. In this update step, $\hat{R}^{i+1}(AR)$ is regarded as the approximate value of $\hat{R}^{\chi^*}(AR)$ and it finally converges to $\hat{R}^{\chi^*}(AR)$ after finite rounds of updates.

### 5.5 Best Strategy Generation with Optimized Utility

In our model, we need to derive the Nash equilibrium of the multi-slot confrontation to obtain $\hat{R}^i(AR)$. Built upon the above equations, we can reshape the Nash equilibrium modelling as

$$\begin{aligned}&\min_{\chi_{ad}}\max_{\chi_u}\{\frac{2\mathrm{Exp}(-\rho(A_u^i-\sigma))}{1+\mathrm{Exp}(-\rho(A_u^i-\sigma))} - D(D)A_{ad}^iA_u^i - 1\},\\ &\text{s.t.}\\ &\sum_i A_{ad}^i \leqslant \Psi,\\ &0 \leqslant A_{ad}^i \leqslant 1, \forall i,\\ &0 \leqslant A_u^i \leqslant 1, \forall i,\end{aligned} \tag{5}$$

where $D(d)$ is the function of the piece of data $d$ that $D(d) = \omega DSen(d) + (\widehat{R}^{\chi^*}(AR'=0) - \hat{PU}^{\chi^*}(AR'=1))$. As $\chi^*(AR'=0)$ and $\hat{R}^{\chi^*}(AR'=1)$ maintain the same, $D(d)$ simply depends on $d$. In above analysis, we have the observation that $\hat{R}^{\chi^*}(AR'=0) > \hat{R}^{\chi^*}(AR'=1)$. As $DSen \geqslant 0$, we conclude that the function of message $D(d) > 0$.

To solve (5), we firstly eliminate the effects of the adversary. As we focus on stationary strategy in this paper, $\chi_u$ is fixed to a constant value. As we have proved $D(d) > 0$, we assume the adversary to eavesdrop $\Psi$ messages to launch the attack to minimize the value in (5). Therefore, we re-formulate the problem as

$$\begin{aligned}&\max_{\chi_u,\Theta,T'}\{\frac{2\mathrm{Exp}(-\rho(a_u^i-\sigma))}{1+\mathrm{Exp}(-\rho(A_u^i-\sigma))} - D(d)A_u^i - 1\},\\ &\text{s.t.}\\ &0 \leqslant A_u^i \leqslant 1, \forall i,\\ &A_{ad}^i \leqslant \Theta, \forall i \in T',\\ &A_u^i \geqslant \Theta, \forall i \in \{T/T'\},\end{aligned}$$

where $T'$ is one subset of $T$ consisting of $\Theta$ messages. Given a certain $T'$, we can easily derive the closed form of the best strategy $\chi_u$.

## 6 System Analysis

In this section, we demonstrate the system analysis in terms of collusion attack-proof and optimized data utility.

### 6.1 Optimized Data Utility Analysis

Among all the differentially private mechanisms, the most widely used one is the Laplace mechanism. Laplace mechanism adds controllable noise to the outputs, where the noise generation process complies with differential privacy.

Given the mechanism $\mathcal{M} : R^n \to \Delta(R^n)$ which adds Laplace distributed noise $\mathcal{N}$ as

$$\begin{aligned}&\mathcal{M}(\mathcal{D}) = \mathcal{D} + \mathcal{N},\\ &\text{s.t.}\\ &N \sim Lap(\frac{\delta}{\epsilon}),\\ &Lap(b) \sim d\Pr[N=n] = \mathrm{Exp}(-\frac{||n||_2}{b}),\end{aligned} \tag{6}$$

where $d\Pr[N = n]$ is the density of $Lap(b)$. Formulated by this, we regard $\mathcal{M}$ as a $\epsilon$-differential private mechanism under adjacency relation.

However, the Laplace mechanism cannot be optimal in terms of minimum mean-squared error. Therefore, we target on achieving optimum Laplace mechanism for both minimum entropy and minimum mean-squared error through designing the noise properly.

**Theorem 1** (Optimum Laplace Mechanism). *Given the $\epsilon$-differential private mechanism $A : R^n \to \Delta(R^n)$, $A$ satisfies $y_{ij}^K = d_{ij} + N$, where $N \sim \rho(N) \in \Delta(R^n)$. The mean-squared error is minimized when the noise density $f$ complies with*

$$f_1^n(v) = (\frac{\epsilon}{2})\mathrm{Exp}(-\epsilon||v||_1),$$

*where $f_1^n(v)$ denotes the density of noise in the sense of $v$. Thus, we have*

$$\mathrm{E}||y_{ij}^t - d_{ij}||_2^2 = \mathrm{E}_{V\sim\rho}||V||^2 \geqslant \mathrm{E}_{V\sim f_1^n}||V||_2^2 = \tfrac{2n}{\epsilon^2}.$$

The optimum Laplace mechanism provides the solution to achieve optimized data utility when the privacy level is fixed. We further prove the proposed method can satisfy optimum Laplace mechanism, which makes the proposed model more feasible and practical.

### 6.2 Collusion Attack-Proof Analysis

We present an example of two-fold customized privacy levels for clarity. Given two privacy levels $\epsilon_i$ and $\epsilon_{i+1}$, where $\epsilon_{i+1} > \epsilon_i$, there is a mechanism $\mathcal{M}_{\epsilon_i \to \epsilon_{i+1}} : \mathcal{D} \to \Delta(\mathcal{Y}^2)$ which releases the data in two different social networks. At first, $u_i$ publishes a noisy outcome $y_{ij}^1$ to $u_j$. $y_{ij}^1$ satisfies $\epsilon_i$-DP. Afterwards, the privacy level is relaxed to $\epsilon_{i+1}$-DP. In case that users collude with others to obtain more precise output, the proposed mechanism should at least satisfy

$$\mathcal{M}_{DP}(\epsilon_i + \epsilon'_{i+1}) = \mathcal{M}_{DP}(\epsilon_{i+1}),$$

where $\epsilon'_{i+1}$ is the privacy level of the second noisy response and $\mathcal{M}_{DP}$ denotes the differentially private mechanism. As the upper bound of composition mechanism indicates, we have

$$\mathcal{M}_{DP}(\epsilon'_{i+1}) = \mathcal{M}_{DP}(\epsilon_{i+1} - \epsilon_i),$$

where we can conclude $\epsilon'_{i+1} < \epsilon_{i+1}$. This conclusion implies the second noisy response cannot relax the privacy level to a satisfying degree. Especially in the case that $\epsilon(1) < \epsilon_{i+1} \ll 1$, the privacy level may even be upgraded. The data utility degrades so that it is not suitable for practical applications.

**Theorem 2** (Collusion Attack-Proof Mechanism). *Two privacy levels $\epsilon_1, \epsilon_2$, which are short for $\epsilon_1(\frac{1}{d_{ij}}, t), \epsilon_2(\frac{1}{d_{ij}}, t)$ and $0 < \epsilon_1(\frac{1}{d_{ij}}, t) < \epsilon_2(\frac{1}{d_{ij}}, t)$, are given. Then, the form of the mechanism can be formulated as*

$$y_{i1}^t = d + V_1, \quad y_{i2}^t = d + V_2, \quad (V_1, V_2) \sim \rho\Delta(R^2).$$

*Moreover, the density $f_{\epsilon_1(\frac{1}{d_{ij}},t),\epsilon_2(\frac{1}{d_{ij}},t)}$ is*

$$f_{\epsilon_1,\epsilon_2}(x,y) = \frac{\epsilon_1^2}{2\epsilon_2}\mathrm{Exp}(-\epsilon_2|y|)\delta(x-y) + \frac{\epsilon_1(\epsilon_2^2 - \epsilon_1^2)}{4\epsilon_2}\mathrm{Exp}(-\epsilon_1|x-y| - \epsilon_2|y|).$$

The rationale behind noises complying with Markov stochastic process is that the Markov process requires the current state is only related to the last state. That means the current state is not impacted by the other states before the last state. In this case, the current noise is only decided by the last noise. In the proposed model, the privacy level increases with the trust distance and the noise increases as well. Therefore, the current user has no incentive to collude with the next user who has an inaccurate output with more noises. The proof is presented in Appendix.

## 7 Performance Evaluation

In addition to theoretical analysis, we testify the proposed model with a real-world dataset. The evaluation results are satisfying and confirm the significance of this work.

We evaluate our model on the "Google+" dataset collected by McAuley and Leskovec[36]. This dataset contains 107 614 nodes and 13 673 453 edges. We also use the shortest distance to be the distance parameter. This can be extended to other distance metrics.

We randomly capture a piece of the dataset with 4 000 nodes and 56 352 edges. Speaking of privacy budget $B$, current studies leveraged a method to set the upper boundaries of $\epsilon$ since the lower boundary should be 0. The method is formulated as $B \leqslant \frac{\Delta q}{\Delta v}\ln\{\frac{(n-1)p}{1-p}\}$, where $B = \sum_i^n$ is the privacy budget, $\Delta q$ is global sensitivity, $n$ is the size of dataset, and $p$ is the probability of being successfully attacked. In this paper, we use $n = 5\,000$ nodes, $p = 1\%$, $\Delta q/\Delta v = 1$. Therefore, the value range of $B$ would be $[0, 4]$. Therefore, we use the largest $\epsilon$ as the parameter to test the protection level of the proposed model. Firstly, we illustrate the privacy-level advantages of personalized differential privacy. Secondly, we testify the improved trade-off with maximum data utility based on various parameters.

### 7.1 Privacy Level

We consider an 8-level privacy protection model. For a fixed privacy budget $B = 4$, the classic differential privacy divides the budget equally $\{\forall d_{ij} \in D | \epsilon(\frac{1}{d_{ij}}) = 0.5\}$ while personalized differential privacy customizes the privacy levels according to the distance. In the personalized privacy model, only social distance $d_{ij} \leqslant 6$ is protected. We use the simplest logarithm function to denote this mapping function.

Then, given another 8-level privacy protection model with changing privacy budget $B$, we still assume social distance $d_{ij} \leqslant 6$ is protected in personalized privacy model.

In Fig.1, we compare the privacy protection differentiation between classic differential privacy and personalized privacy. The classic differential privacy is also known as uniform differential privacy. It equally divides the privacy budget into $n$ parts and assigns them to each piece of data. However, in the proposed personalized privacy, the privacy level is personalized based on social distance. Social distance is the distance between the users in social networks with the relationship rather than the locational distance. The details are as follows.

From Fig.1(a), the first thing we can tell is that personalized differential privacy is more flexible than the classic differential privacy. Then second one is that personalized differential privacy can protect privacy in a more accurate way over social networks. The privacy level keeps decreasing with the increment of social distance. When a certain social distance is reached, the users beyond the social distance are not protected anymore, which is feasible and practical in social networks.

Fig.1(b) illustrates how privacy levels increase with the increase of the fixed privacy budget $B$. Fig.1(b) shows that privacy levels increase almost lineally which is easy to handle. In the proposed personalized privacy protection model, privacy levels are derived by the privacy requirements. The flexibility confirms that there is no over-protection or data utility degradation.

### 7.2 Data Utility

We further consider an 8-level privacy protection model. For a fixed privacy budget $B = 4$, the classic differential privacy divides the budget equally $\{\forall d_{ij} \in D | \epsilon(\frac{1}{d_{ij}}) = 0.5\}$ while personalized differential privacy customizes the privacy levels according to the distance. In the personalized privacy model, only social distance $d_{ij} \leqslant 6$ is protected. We use the simplest logarithm function to denote this mapping function and the data utility can be measured with RMSE.

In Fig.2, we compare the data utility tendencies in terms of all three methods, including differential privacy (Classic DP), normal personalized differential privacy (Personalized DP), and the proposed model (OTO). From the overall trends, we can tell that the proposed model has a superior performance from the perspective of data utility improvement.

In Fig.2(a), the data utility is shown for the proposed model (OTO), common personalized differential privacy, and classic differential privacy. The data utility remains a constant for classic differential privacy, which

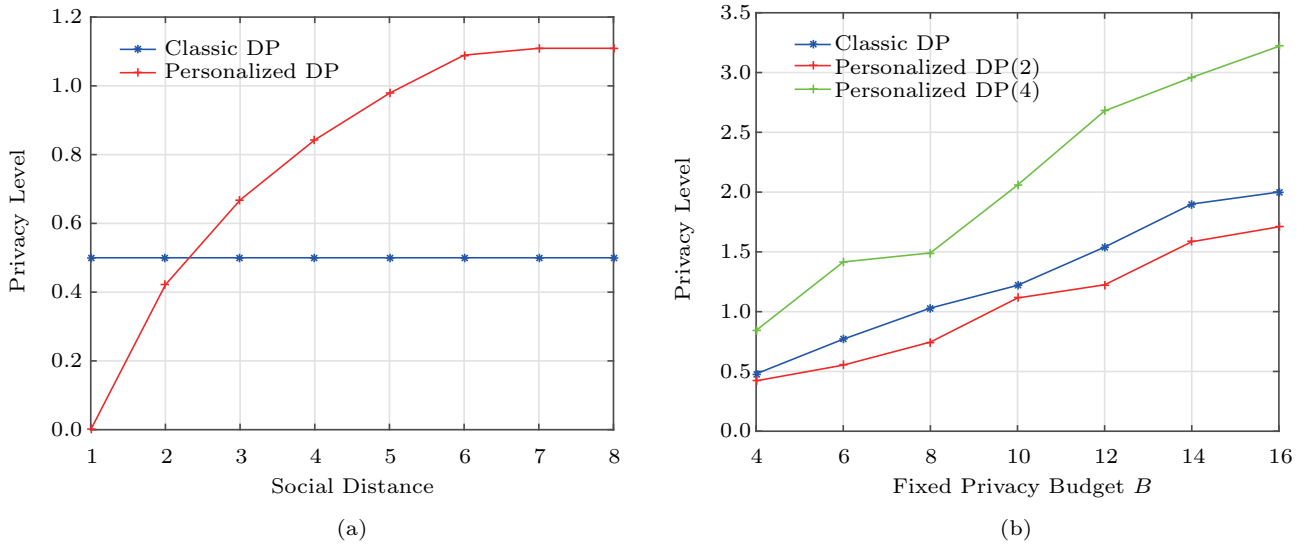


Fig.1. Privacy level comparison in single and multiple social networks. DP($x$) means the social distance of DP is $x$.

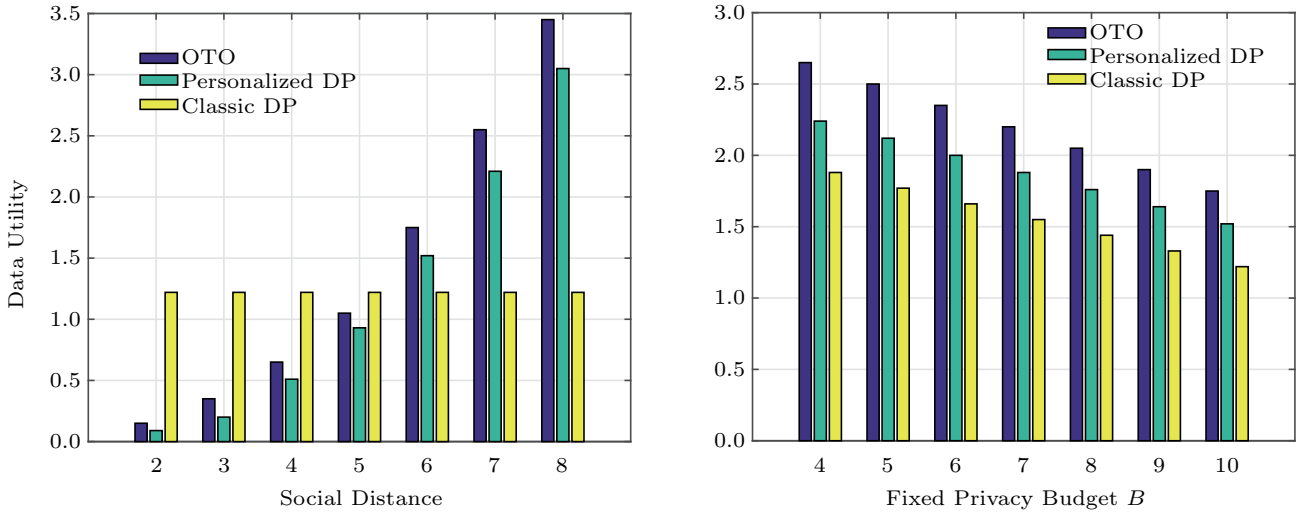


Fig.2. Data utility comparison in single and multiple social networks.

is decided by the fixed privacy-level feature. In the case of the proposed model and common personalized differential privacy, the utility increases with the increment of social distance. But in the proposed model, the starting point and the increasing rate are higher than those of common personalized differential privacy respectively, which proves that the proposed model is more feasible.

Fig.2(b) illustrates the trends of how data utility decreases with the increase of the fixed privacy budget. In all three models, data utility decreases linearly, which is decided by the trade-off feature. However, the proposed model has the maximum data utility from the beginning to the end.

All in all, the data utility of the proposed model is improved compared with classic differential privacy and personalized differential privacy without the game model, which complies with the above analysis.

### 7.3 Efficiency Evaluation

We introduce a modified Q-learning algorithm to achieve fast convergence and thereby obtain the Nash equilibrium. The reason why we can limit iteration times is that we reduce the cardinality from $n$ to 2. Built upon cardinality deduction, we obtain faster convergence as shown in Fig.3.

As shown in Fig.3(a) and Fig.3(b), we demonstrate that the efficiency of the proposed method outperforms the classic one. The proposed method converges between $10^2$ and $10^3$ iteration times while the classic method converges between $10^5$ and $10^6$ iteration times. Therefore, the magnitude order of the proposed idea is roughly two times less than that of the classic one, which means the proposed idea cost around 1% time the classic one.

Due to the limited iteration times, the proposed idea can derive the Nash equilibrium in a very short time. Furthermore, it can ensure that the system can implement the differentially private algorithm in real time.

## 8 Conclusions

In this work, we first established a personalized differential privacy model based on social distance. The privacy budgets $\epsilon$ increase with the increment of social distance. Building upon this privacy-preserving model, we further modeled the confrontation between the user and the adversary as a static Bayesian game because both sides are not aware of each other's strategy sets. We then formalized the action sets, typesets, strategy sets, and payoff functions in the differential privacy sense, which takes the effect of the adversary into consideration. In addition, the Bayesian Nash equilibrium is derived based on the payoff functions. Finally, we obtained the optimal trade-off with maximum data utility with the theoretical analysis. Extensive experiments were implemented to compare existing work and the proposed model. Evaluation results demonstrated the superior performance of our model.

In the case of future work, we plan to extend this model to multiple adversaries and consider the situation of collusion attack. Besides, we also consider tak-

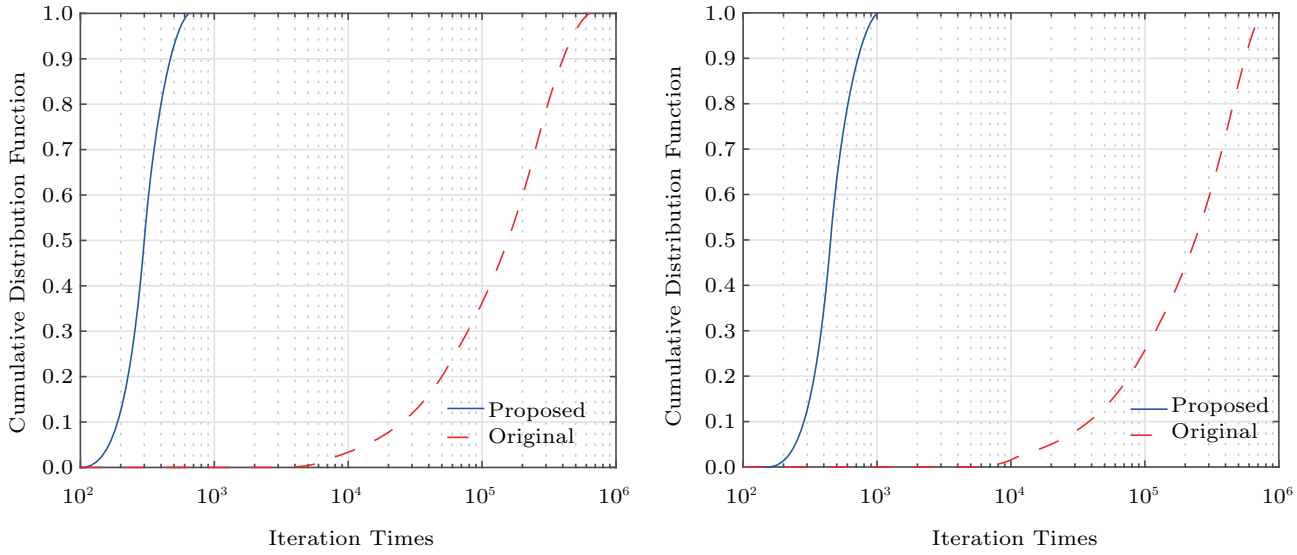


Fig.3. Efficiency comparison in single and multiple social networks.

ing the dynamic continuous data publishing into account to make the model more practical. Furthermore, changing the fixed privacy budget into a varying one could adapt the model into other scenarios.

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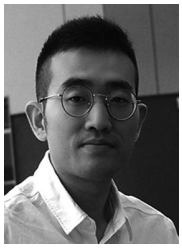

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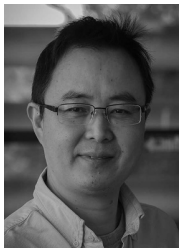

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## Appendix

*Proof of Theorem* 2. The noise of mechanism $\mathcal{M} = (\mathcal{M}_1, \mathcal{M}_2)$ is defined by (6). According to this, we prove the propose mechanism satisfies all the required properties.

1) The first coordinate is Laplacian-distributed with parameter $\frac{1}{\epsilon_1}$. When $x > 0$, we can derive (A1).

$$\begin{aligned}
&\Pr(V_1 = x)\\
=&\int_R \rho(x,y)\mathrm{d}y\\
=&\frac{\epsilon_1^2}{2\epsilon_2}\mathrm{Exp}(-\epsilon_2 x) +\\
&\frac{\epsilon_1(\epsilon_2^2-\epsilon_1^2)}{4\epsilon_2}\int_R \mathrm{Exp}(-\epsilon_1|x-y|-\epsilon_2|y|)\mathrm{d}y\\
=&\frac{\epsilon_1^2}{2\epsilon_2}\mathrm{Exp}(-\epsilon_2 x) +\\
&\frac{\epsilon_1(\epsilon_2^2-\epsilon_1^2)}{4\epsilon_2}(\int_{-\infty}^{0}\mathrm{Exp}(-\epsilon_1 x+(\epsilon_1+\epsilon_2)y)\mathrm{d}y +\\
&\int_0^x \mathrm{Exp}(-\epsilon_1 x+(\epsilon_2-\epsilon_1)\mathrm{d}y)\mathrm{d}y +\\
&\int_0^{+\infty}\mathrm{Exp}(-\epsilon_1 x+(\epsilon_1+\epsilon_2)y)\mathrm{d}y)\\
=&\frac{\epsilon_1^2}{2\epsilon_2}\mathrm{Exp}(-\epsilon_2 x) +\\
&\frac{\epsilon_1(\epsilon_2-\epsilon_1)}{4\epsilon_2}\mathrm{Exp}(-\epsilon_1 x)\mathrm{Exp}((\epsilon_1+\epsilon_2)y)|_{-\infty}^{0} +\\
&\frac{\epsilon_1(\epsilon_2+\epsilon_1)}{4\epsilon_2}\mathrm{Exp}(-\epsilon_1 x)\mathrm{Exp}((\epsilon_2-\epsilon_1)y)|_{0}^{x} +\\
&\frac{\epsilon_1(\epsilon_2-\epsilon_1)}{4\epsilon_2}\mathrm{Exp}((\epsilon_1+\epsilon_2)y)|_{x}^{+\infty}\\
=&\frac{\epsilon_1}{2}\mathrm{Exp}(-\epsilon_1 x). \qquad \text{(A1)}
\end{aligned}$$

When $x < 0$, the equation follows the symmetry $(x, y) \to (-x, -y)$. Thus we can conclude $\mathcal{M}_\infty$ is $\epsilon_1$-differential private and obtains the best data utility.

2) The second coordinate is Laplacian-distributed with parameter $\frac{1}{\epsilon_2}$. We can derive (A2).

$$\begin{aligned}
&\Pr(V_2 = y)\\
=&\int_R \rho(x,y)\mathrm{d}x = \frac{\epsilon_1^2}{2\epsilon_2}\mathrm{Exp}(-\epsilon_2|y|) +\\
&\frac{\epsilon_1(\epsilon_2^2-\epsilon_1^2)}{4\epsilon_2}\mathrm{Exp}(-\epsilon_2|y|)\int_R \mathrm{Exp}(-\epsilon_1|x-y|)\mathrm{d}x\\
=&\frac{\epsilon_1^2}{2\epsilon_2}\mathrm{Exp}(-\epsilon_2|y|) +\\
&\frac{\epsilon_1(\epsilon_2^2-\epsilon_1^2)}{4\epsilon_2}\mathrm{Exp}(-\epsilon_2|y|)\int_R \mathrm{Exp}(-\epsilon_1|x|)\mathrm{d}x\\
=&\frac{\epsilon_1^2}{2\epsilon_2}\mathrm{Exp}(-\epsilon_2|y|) + \frac{\epsilon_2^2-\epsilon_1^2}{4\epsilon_2}\mathrm{Exp}(-\epsilon_2|y|)\\
=&\frac{\epsilon_2}{2}\mathrm{Exp}(-\epsilon_2|y|). \qquad \text{(A2)}
\end{aligned}$$

Therefore, we prove $\mathcal{M}_{\in}$ is $\epsilon_2$-differential private and obtains the best data utility.

3) At last, we need to prove the composition mechanism maintains $\epsilon_2$-differential private. The delta part is separately handled by defining $L = \{x : (x, x) \in \Omega\}$. The probability of landing in $\Omega$ is represented by

$$\begin{aligned}
&\Pr(\mathcal{M} \in S) \\
=\ &\frac{\epsilon_1^2}{2\epsilon_2} \int_D \mathrm{Exp}(-\epsilon_2|x-d|)\mathrm{d}x + \\
&\frac{\epsilon_1(\epsilon_2^2-\epsilon_1^2)}{4\epsilon_2} \iint_S \mathrm{Exp}(-\epsilon_1|(x-d)-(y-d)| - \\
&\epsilon_2|y-d|)\mathrm{d}x\mathrm{d}y.
\end{aligned}$$

We take the derivative and use Fubini's theorem to exchange the derivative with integral as (A3).

$$\begin{aligned}
&\frac{d}{du}\Pr(\mathcal{M} \in S) \\
=\ &\frac{\epsilon_1^2}{2\epsilon_2} \int_D \epsilon_2 \mathrm{sgn}(x-d)\mathrm{Exp}(-\epsilon_2|x-d|)\mathrm{d}x + \\
&\frac{\epsilon_1(\epsilon_2^2-\epsilon_1^2)}{4\epsilon_2} \iint_S \epsilon_2 \mathrm{sgn}(x-d) \times \\
&\mathrm{Exp}(-\epsilon_1|x-y| - \epsilon_2|y-d|)\mathrm{d}x\mathrm{d}y \\
\Rightarrow\ &|\frac{d}{du}\Pr(\mathcal{M} \in S)| \leqslant \frac{\epsilon_1^2}{2\epsilon_2} \int_D \epsilon_2 \mathrm{Exp}(-\epsilon_2|x-d|)\mathrm{d}x + \\
&\frac{\epsilon_1(\epsilon_2^2-\epsilon_1^2)}{4\epsilon_2} \iint_S \epsilon_2 \mathrm{Exp}(-\epsilon_1|x-y| - \epsilon_2|y-d|)\mathrm{d}x\mathrm{d}y \\
\Rightarrow\ &|\frac{d}{du}\Pr(\mathcal{M} \in S)| \leqslant \epsilon_2 \Pr(\mathcal{M} \in S) \\
\Rightarrow\ &|\frac{d}{du}\ln\Pr(\mathcal{M} \in S)| \leqslant \epsilon_2. \qquad \text{(A3)}
\end{aligned}$$