J. Econ. Theory 220 (2024) 105867

Contents lists available at ScienceDirect

Journal of Economic Theory

journal homepage: www.elsevier.com/locate/jet

Influencer marketing and product competition ☆

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ARTICLE INFO

JEL classification:
M31
M37
L11
L51
L82

Keywords:
Creator economy
Digital economy
Internet celebrity
Industrial Organization
Streaming

ABSTRACT

With the rise of social platforms and digital media (e.g., videos, (live)streaming, and multi-media blogs), firms and brand owners increasingly depend on influencers to attract consumers, who care about both product quality and consumer-influencer interaction. Sellers thus compete in both influencer and product markets. We develop a theory to understand the influencer labor market and its interaction with product competition, under a plausible specification of influencers' value added. We find: (i) more powerful influencers sell better quality products for sellers with weaker direct-sale capacity, facilitating seller competition; (ii) advances in the intermediation technology can lead to non-monotonic changes in influencers' payoff and income inequality; (iii) style pluralism mitigates market concentration by horizontally differentiating the consumer experience but serves as either complements (intermediate style dispersion) or substitutes (small or large dispersions) to vertical product differentiation; and (iv) influencers may inefficiently under-invest in consumer outreach and unidirectional exclusivity can improve welfare in less competitive markets. Collectively, our findings highlight the novelty of the influencer economy and establish several theoretical baselines for future studies.

1. Introduction

The past decade has witnessed the rise of the influencer economy (also known as "Wang Hong economy" and more recently dubbed by the media as the "creator economy") in both developed and emerging economies, including the United States, Europe, China, Latin America, etc. It prominently features large-scale, wide reaching digital marketing through social media (e.g., short videos and live-streaming), testimonial endorsements, and precise product placements and distributions by people and organizations who have a purported expertise or influence. This phenomenal growth has been driven by the exponential growth of e-commerce

☆ The authors are especially grateful to Gary Biglaiser, Tomasso Bondi, Jaden Chen, Yi Chen, Itay Fainmesser, Hanming Fang, Ruobing Han, Justin Johnson, Fei Li, David Ng, Peter Norman, Marzena Rostek (the editor), Ji Shen, Jian Sun, Can Tian, Liyan Yang, and Mao Ye for helpful discussions and comments. They thank Jussi Keppo, Chao Ma, Desheng Ma, Simon Mayer, Imke Reimers, Fahad Saleh, Haokun Sun, Yu Tong, Marianne Verdier, Jing Wu, Kairong Xiao, Alex Yang, and seminar and conference participants at the 2022 Annual Conference in Digital Economics, the Asian Meeting of the Econometric Society in East and South-East Asia (AMES2022), the 35th Australasian Finance & Banking Conference, Cornell Johnson Research Salon, China Fintech Research Conference (CFRC2022), China International Conference in Finance (CICF2022), Financial Management Association Annual Conference (FMA2022), NBER Chinese Economy Working Group 2024 Spring Conference, New Zealand Finance Meeting (NZFM2022), Southwestern University of Finance and Economics and the UNC at Chapel Hill for constructive feedback. Previously circulated under the title "A Model of Influencer Economy," this paper received generous research support from the FinTech Initiative at Cornell, Ripple's University Blockchain Research Initiative (UBRI), Guangzhou Municipal Science and Technology Project (2023A03J0691) and Guangzhou-HKUST (GZ) Joint Funding Program (2024A03J0630).

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https://doi.org/10.1016/j.jet.2024.105867
Received 14 May 2023; Received in revised form 2 June 2024; Accepted 7 June 2024

Available online 20 June 2024

and digital platforms and was further accelerated by the recent COVID-19 pandemic (e.g., Sinha, 2021). In this new digital economy, influencers come from a diverse background, including content creators, celebrities, idols, and key opinion leaders (KOL, Williams, 2016). They manage their own fan bases, who are drawn to their talent, charisma, wisdom, appearance, etc., and profit by helping brand owners and service providers promote various products to potential consumers. The thriving influencer marketing and creator economy not only generates sizable revenues but also unleashes huge potential in job creation.[1]

Clearly, despite having to share revenues with influencers, sellers increasingly adopt influencer-intermediated sales. Examples include beauty and cosmetic companies such as Estee Lauder, Revolution Beauty, and REFY Beauty, which have led the way in their use of influencers with success stories. Furthermore, given the explosion of product varieties in online retail sales (Baslandze et al., 2023), influencers can still effectively reach niche markets and local demographics, even without increasing product varieties (Han, 2023). Several questions naturally arise: (a) Why can influencer marketing grow so quickly? (b) What is its impact on product competition? (c) How does the influencer labor market evolve? We provide initial answers to these questions by developing a novel game-theoretic model in which sellers depend on influencers to acquire customers and compete in both the product market and the influencer labor market, with potentially endogenous and heterogeneous influence.

Specifically, we model three important groups of agents—sellers (who are also producers), influencers, and consumers—and allow pairwise group interactions through the product market, influencers' labor market, and social media platforms (for influencers to connect with consumers). Sellers or brand owners hire influencers to sell products to consumers, which may interact with direct sales. Consumers are uniformly located on a unit circle in the type space ("style" or "taste") with consumption utilities determined by both the underlying quality of the product and the style, status, identity, etc.—features that draw consumers toward influencers selling the product on social media like Instagram or Alibaba (known as Da Ren). Agents interact in sequential stages: (i) sellers make production decisions, (ii) sellers hire and match with influencer(s) in the labor market, and (iii) consumers choose which influencer to follow and consume the products the influencer promotes. We solve the model backward and discuss economic insights and predictions in each stage. We also discuss endogenous influence acquisition.

In our baseline setting, two sellers compete in the labor market of influencers, each hiring one influencer, in addition to competing in the product market. We find that influencer-intermediated sales feature assortative matching under Nash bargaining: sellers of better products work with more powerful influencers due to the complementarity between influencer power and product quality. Moreover, influencer-intermediated sales generally enjoy local monopoly power regardless of product quality when influencer styles are sufficiently distinct. For one seller-influencer group to crowd out the rival group, the style difference between influencers needs to be sufficiently small, and both the influencers' power gap and the product quality gap must be sufficiently large. Finally, technological advances in intermediation can affect influencers' payoffs and income inequality, first benefiting them by scaling their customer outreach but eventually hurting them in the labor market as the crowded intermediation space transforms into more fierce price competition.

We then move one stage back to endogenize sellers' production decisions. Specifically, we investigate how influencer heterogeneity affects horizontal and vertical product differentiations compared to traditional economies. We find that the style difference of influencers is a substitute for horizontal product differentiation. Specifically, when the influencer style difference is small, sellers differentiate products to reduce competition. When the difference is large, sellers hire influencers and have no incentive to differentiate products because divergent influencers give sellers less elastic demand for their products. However, both the product quality gap and the influencer power gap complement horizontal differentiation because the stronger seller competes more aggressively when selling superior products or working with a more powerful influencer. This leads to lower profits for the weaker seller, spurring her to differentiate horizontally.

Sellers can also vertically differentiate by improving product quality to win market shares. In general, when the style difference between influencers increases, the return on investment in quality also increases. When the style difference is sufficiently large, both groups can break even and choose high product quality, generating minimal vertical differentiation. When the style difference is sufficiently small, both groups choose low product quality, resulting in minimal vertical differentiation again. Only for intermediate style differences does vertical differentiation occurs because the investment profit allows one and only one seller-influencer pair investing in quality to break even.

Next, we introduce direct sales (which include sales through conventional commercials) and analyze how they compete and interact with influencer-intermediated sales. We generalize the model so that sellers are positioned in the center of the circle with a significant transportation cost in direct sales to reflect the fundamental friction that companies traditionally do not participate in the users' and customers' social lives. We show that when sellers have comparable capacity in traditional marketing and sales, all main results remain robust in the presence of direct sales. However, this general setting provides two additional insights: First, influencer marketing can foster seller entry and competition by allowing sellers with weak traditional sales to bypass the transportation cost barrier. Second, sellers with weak traditional marketing channels have a stronger incentive to hire influencers, leading to negative assortative matching between direct sales capacity and influencer power (for any given product quality). Both contribute to the rapid growth of influencer marketing.

We also extend the model to allow for endogenous influence style and power. Although the assortative matching between sellers and influencers remains under endogenous style selection, socially inefficient under-investment in power can arise due to: (i) influencers do not internalize the externality on consumer welfare when acquiring influencer power; and (ii) the threat of intense price pushback further discourages influencers from investing in power, and any potential gain in the consumer base is dominated

[1] See Section 2 for more institutional details on influencers and influencer economy.

by lowered product prices.[2] We further consider "unbalanced matching" to better understand the welfare implications of exclusivity contracts and regulatory policies. We find that requiring balanced seller-influencer matching can encourage seller competition, whereas unidirectional exclusivity contracts are welfare-improving for sufficiently differentiated products and uncontested influencer markets.

Overall, our theory explains and predicts several trends in the influencer market. First, on the one hand, influencer marketing favors large brands that can afford to invest more in improving product quality and match with more powerful influencers; on the other hand, it also benefits small and less established firms by facilitating the entry of sellers and enabling those with weak direct sales capacity to bypass the transportation cost barrier to access niche markets and accumulate reputation. As such, it creates both market concentration and greater entry at the same time. Second, influencers improve the consumer experience because their style pluralism effectively differentiates products horizontally and enhances investments in product improvements. Third, as digital technology continues to improve, the aggregate wage and income inequality for influencers will first increase and then decrease, while the growth of the influencer market always benefits the seller and the total welfare.[3] Finally, regarding policy interventions and designs, mutual exclusivity can foster competition, help prevent "arms race" among influencers in power acquisition, and thus improve total welfare in a congested influencer market. In the long run, establishing creator-friendly platforms and providing additional support to less established influencers will likely boost the growth of the influencer economy.

Literature Our study adds foremost to the nascent literature on the influencer or creator economy. Previous studies have focused on the relationship between influencers and digital platforms or multichannel networks (MCNs), especially the rules of revenue sharing (Bhargava, 2022; Jain and Qian, 2021), disclosure by internet influencers (Mitchell, 2021), search technology and advice transparency (Fainmesser and Galeotti, 2021), influencer cartels (Hinnosaar and Hinnosaar, 2021), and firms' optimal affiliation with influencers (Pei and Mayzlin, 2019). Instead, we analyze seller competition and seller-influencer matching, which in turn affect product differentiation and endogenous influence acquisition. Recently, Bian et al. (2023) provides empirical evidence for our model predictions about the influencer labor market evolution and its dependence on technology, whereas Cong an Lin (2024) examine the negative consequences of digital influence on education and occupational choices.

Our study is thus related to the broad literature on marketing and industrial organization (e.g., Salop, 1979). We analyze their interaction in the context of the fast-emerging influencer economy. Studies on advertising have focused on the aggregate and cross-sectional levels of advertising and its welfare implications (Becker and Murphy, 1993; Spence and Owen, 1977; Butters, 1978; Dixit and Norman, 1978; Grossman and Shapiro, 1984; Nichols, 1985; Stegeman, 1991; Nelson, 1974; Johnson and Myatt, 2006). Most models assume no media or focus only on the informational effects or nuisance costs on viewers of advertisements (e.g., Johnson, 2013). Moreover, most studies either do not endogenize the locations of media stations or take the differentiation of sellers' products as exogenous (e.g., Gal-Or and Dukes, 2003; Dukes, 2004). We study influencers whose matching with sellers is affected by the consumer base and analyze endogenous product differentiation and influencers' style choices simultaneously. We consider the level of advertising and its informational role (in reduced-form) through influence, focusing on the complementarity between the two dimensions of consumers' utility from following influencers and consuming products.

More recently, Amaldoss and He (2010) study how firms strategically target consumers to avoid intense price competition. Several studies in marketing analyze how firms compete in the effort to hire influencers, including the intensity of advertisements, the competitive targeting of influencers in a network, and the structure of the network and its influence on prices, firm profits and consumer surplus (Galeotti and Goyal, 2009; Katona, 2018). In particular, Fainmesser and Galeotti (2021) analyze search quality, transparency of advice, and influencer strategy in the market for online influence. We differ by analyzing the interaction of seller-influencer matching and product market competition. Furthermore, we add to the discussion on exclusivity contracting and the link between unidirectional exclusivity contracts and bargaining (e.g., Gal-Or, 1997; Dukes and Gal-Or, 2003) by contrasting unidirectional exclusivity contracts with mutual exclusivity contracts in their impact on welfare in the influencer economy.

Finally, our paper is related to studies on the endogenous emergence of intermediation. Middlemen can emerge for various reasons, including concentrating delegated monitoring incentives (Diamond, 1984), reducing search frictions (Rubinstein and Wolinsky, 1987; Duffie et al., 2005), absorbing excess inventory demand or supply in the financial market (Glosten and Milgrom, 1985), facilitating information production in asset management (Gârleanu and Heje Pedersen, 2018). Recent articles have also explored the measurement of financial intermediation (Philippon, 2015), efficient intermediation chains (Glode and Opp, 2016; Glode et al., 2019), and the self-selection of intermediaries (Zhong, 2023). Judge (2022) shows how overgrown and opaque middlemen became the backbone of modern capitalism and the cause of many of its ailments and fragility. Our study complements this literature by focusing on influencer intermediaries on social media platforms and their impact on product competition.

The remainder of the paper is organized as follows: Section 2 presents institutional details on influencers and the influencer economy. Section 3 presents the baseline model. Section 4 studies seller competition and influencer hiring. Section 5 explores influencer heterogeneity and product differentiations. Section 6 introduces direct sales and compares them with influencer-intermediated sales. Section 7 investigates endogenous influence and exclusivity contracting. Section 8 discusses the empirical implications and concludes. The appendix and the online appendix contain the proofs and extended discussions.

[2] Over-investment may ensue under a monopolist seller. See Remark 4 for a detailed discussion.

[3] See Section OA.4 for a detailed discussion of influencer competition.

2. Institutional details

This section briefly describes influencers and the creator/influencer economy. Readers familiar with the emergence of the influencer economy may want to skip this section.

The influencer or creator economy generally refers to independent businesses and side hustles launched by self-employed individuals who make money off of their knowledge, skills, and following. CB Insights (2021) provides an excellent introduction to the industry. Many influencers generate up to seven-figure income. Wei Ya, once the most famous influencer, made a fortune of over 1 billion CNY in live-streamed presales on Single's Day (November 11, the black Friday equivalent in China) alone in 2019 (China Business Industry Research Institute, 2020). Famous Instagram influencers like Huda Kattan or Eleonora Pons net up to 6 figures per post. The top writers on Substack can make up to $1 million annually. YouTube paid $30 billion to creators between 2019 and 2021 (CB Insights, 2021).

Who are the influencers? Loosely speaking, they may include content creators, celebrities and idols, and key opinion leaders (KOL), and they touch almost all aspects of life, including entertainment, fashion, food, movies, music, sports, etc., and increasingly utilize low-cost and easy-to-spread short videos. Content creators derive from "YouTube stars" marketed by YouTube in as early as 2011 (Lorenz, 2019). Now, it can be anyone who creates any form of content online, including TikTok videos and Clubhouse audio. For example, on Twitch, daily users can watch live streams of video games played by others, and tips paid to creators on Twitch alone are estimated to be $141 million. Unlike live streaming creators, Internet celebrities on Instagram and the like can post about or live stream special travel, dining experiences, or their simple daily routines. Many even rely on physical attributes alone without actively creating content. For example, Instagram enables brand owners to sell products through idols that attract consumers who simply seek to see them. Similarly, KOLs can target a specific demographic in an interactive manner, making product sales more engaging by sharing their own thoughts and ideas.

Furthermore, the last decade has witnessed rapid growth in influencer marketing activities. For example, global influencer marketing has grown from a mere US $1.7 billion in 2016, to a size of $16.4 billion in 2022, and was expected to reach $21 billion in 2023 (Geyser, 2023). For comparison, global television advertising spending was valued at $141 billion in 2021, and global print advertising spend is estimated at $50.38 billion in 2022 (Okoronkwo, 2022). Goldman-Sachs (2023) predicts that the total addressable market of the creator economy is estimated to double in size from the current $250 billion to half a trillion dollars by 2027, comparable to the current total revenue from the e-Commerce market. Meanwhile, regarding job creation, Influencity (2023) estimates more than 10.2 million Instagram creators and 10.1 million TikTok creators in the United States, and 10.2 million and 18.9 million Instagram influencers for Europe and Latin America. PJdaren (2023) has identified 10.1 million influencers in China whose fan base exceeds 10K followers. According to Goldman-Sachs (2023), the current number of 50 million influencers will continue to grow at a compound annual rate of 10-20% in the next five years.

Finally, we provide several useful statistics on marketers and consumers. For marketers, more than 80% intend to have a delicate budget for influencer marketing, with 23% spending more than 40% of their budget on influencer marketing (Geyser, 2023). For consumers, 36.4% of all respondents prefer influencer posts as the primary way to try new products, with 56% of them purchasing a product after seeing it used by an influencer (IZEA, 2022).

3. Model setup

Two risk-neutral, profit-maximizing sellers indexed by $k \in \{1,2\}$ each sell a unit consumption product/service of a common value y_k. Let U_k denote the utility of the kth seller. They work with influencers for interactive marketing and outreach (and we allow direct sales later). Two representative influencers with "style" $\theta_j \in \mathbf{S}^1$, where $j \in \{1,2\}$ indexes them and $\mathbf{S}^1 := \{s \in \mathbb{R}^2 : s_1^2 + s_2^2 = 1\}$. Style could refer to identity, fashion taste, and other characteristics that attract consumers to influencers on Instagram, TikTok, Alibaba, etc., among the recent proliferation of social networks, digital platforms, and broadcast channels. Let $I_j \in \mathbb{R}_+$ denote the influencer power of the jth influencer, which captures her persuasiveness and charisma among followers, as well as general technological advances in digital social platforms that facilitate influencer-follower interactions and allow influencers to create products or content with more appeal to follower consumers.

Traditional advertising channels through television, newspapers, etc., can be viewed as having low persuasiveness since TV commercials, for example, entail limited interaction with the audience, and capital-intensive advertising makes it difficult to instill authenticity in product recommendations. In contrast, social interactions can dramatically increase the authenticity of influencers and the consumer experience. Differential power then reflects distinctions among celebrities, macro-influencers, and micro-influencers, and influencer attributes and intermediation technologies jointly determine the effective consumer base.

A continuum of consumers with a measure 2π is uniformly located on $\mathbf{S}^1$. They derive utilities from the baseline quality of the products and from having similar styles or affinity with certain influencers (McDonald, 2019). A greater affinity can be interpreted as how effectively an influencer helps a consumer understand, select, and enjoy the product purchased, potentially due to the trust built over time or the influencer's expertise or influencer-specific product demo.[4] $\forall x_1, x_2 \in \mathbf{S}^1$, we define $\|x_1 - x_2\|$ to be the distance along the short arc.

[4] Malmendier and Veldkamp (2022) provide a microfoundation through information resonance, that is, information resonates with recipients when they identify with the person who communicates it.

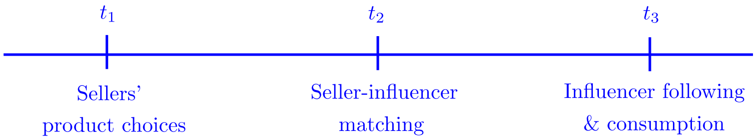


Fig. 1. Timeline.

Consumer i's utility buying from Seller k through Influencer j of type θ_j is:

$$u_i^j(x_i, \theta_j, y_k) = \begin{cases} y_k * \left(1 - \|x_i - \theta_j\|/I_j\right) - p_k, & \text{if a unit good is consumed,} \\ 0, & \text{otherwise,} \end{cases} \tag{1}$$

where p_k is the unit price charged for the consumption good. We suppress the superscript j whenever there is no confusion. Here, brand owners and product sellers depend on influencers to sell goods, with only consumers having $\|x_i - \theta_j\| \leq I_j$ entering the aggregate demand. Importantly, the utility of the consumer depends on the affinity for style. Our specification aims to adequately capture the typical reasons for engaging celebrities or influencers in advertising campaigns: attention-grabbing, persuasion through expertise, and global outreach (Moeran, 2003). The influencer power includes direct attention grabbing, either through vacuous "human pseudo-events" in the words of American historian Daniel Boorstin or through skills or entertainment. Expertise and global, cross-cultural outreach can manifest through the combination of location and power.

Timeline Influencers' type and power are the accumulation of knowledge and skills since childhood and are therefore taken as given in the baseline model. Sellers first decide on the products and subsequently hire influencers. The consumers then choose which influencer to follow and consume the products offered (see Fig. 1). In Section 4, we take as given influencers' type and power, as well as the products of sellers, to focus on the sellers' hiring of influencers and influencers' impact on consumption. In Section 5, we allow sellers to endogenize the products for sale. Our main findings are robust to having product decisions following seller-influencer matching. In practice, firms decide on their business operations before exploring marketing channels, which is what our setup captures.[5]

Matching and bargaining protocols We use a general (bilateral) Nash bargaining protocol to model negotiation once influencers are hired by sellers. This is standard for modeling in commercial media and healthcare (Gal-Or, 1999; Dukes and Gal-Or, 2003). Let γ and $(1-\gamma)$ denote the bargaining power assigned to the seller and the influencer, respectively.[6] Once sellers and influencers are matched, they have exogenous outside options, e.g., from revisiting the influencer market, which we normalize to zero for simplicity. Anticipating such bargaining processes, sellers and influencers endogenously match. Our baseline setup focuses on one-to-one match—often the case in practice, the seller-influencer contracts either feature mutual exclusivity clauses or they are all allowed to have multiple relationships so that the matching is balanced. This negotiation-based approach is realistic and popular for setting the advertising price in the media industry (Dukes and Gal-Or, 2003; Gal-Or, 1997).

The joint matching and bargaining problem is nontrivial. In specifying the protocols, we strive to balance tractability, transparency, convention in the literature, coherence with our non-repeated game set-up, and realism. In fact, many key results are independent of how the surplus is divided between matched sellers and influencers, as long as they care about the group surplus. We discuss unbalanced matching and the welfare implications of contract exclusivity in Section 7.2 where sellers can require exclusive relationship and impose non-compete clauses, as seen in many nascent markets for influencers.

Influencers vs. traditional advertisers In some sense, influencers are intermediaries who place products into consumers' hands, while adding to consumers' net utility. We shall clarify the role of influencers and interpret the additional utility induced in Equation (1) as broadly capturing realistic scenarios.

Intuitively, the specification represents how the interaction with influencers enhances the experience of the product/service. For example, Alessandra Sales, the vice president of growth at Ipsy, once told McKinsey that transgender creators helped the beauty brand connect with the transgender community. It is also common for followers to derive sustained benefits from purchasing through influencers because they can coordinate followers who made purchases to form a community to discuss additional usage of products, user experience, etc. Such communities are typically difficult for sellers to foster directly.[7]

An influencer connects products with his followers (that is, customers within his range $\|x - \theta\|/I$), generating variations in the customers' willingness to pay (that is, the ones more loyal to influencers are more willing to pay). This differentiates influencers sharply from traditional advertisers, who tend to speak to the general audience without "personal" social interactions. In Section 6, traditional advertisers are modeled as agents in the center of the circle, and customers derive identical utility $y(1-c)-p$ when buying

[5] In Section 7.1, we endogenize the influencer power (potentially interpreted as skill training in the intermediate term) and type (potentially interpreted as culture, talent or interest cultivated in the long run).

[6] Our Nash bargaining setup is consistent with the convention in the sales agents literature that the seller sets the price, and agents take commission fees. Since all agents have identical bargaining power in our symmetric setting, it is equivalent to maximize sales revenue. Moreover, our modeling of product pricing is also consistent with the practice of how influencers get paid. In 2023, 74% of influencers are paid by percentage of sales and product levels (Geyser, 2023).

[7] Influencers may also benefit followers via getting them occasional large discounts or customized products.

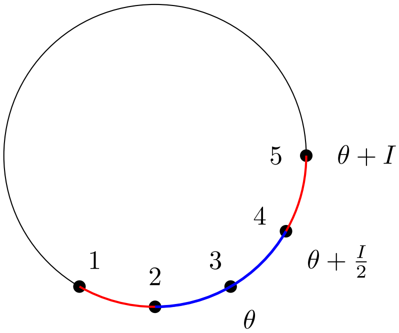


Fig. 2. The circular market. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

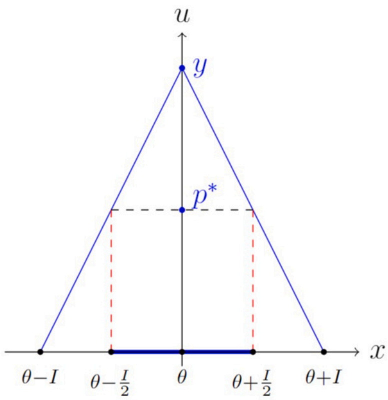


Fig. 3. The monopolist pricing.

from an advertiser, where c is a transportation cost parameter. Therefore, if a seller can hire an influencer on top of a traditional advertiser, it allows her to charge a higher price to the more loyal customers of the influencer and a lower price to other customers who bought from the advertiser. To see this, given the price charged by the influencer p^i and that charged by the advertiser p^d, the utility gap between buying from an influencer versus an advertiser is equal to $(c - \|x - \theta\|/I) - (p^i - p^d)$, which decreases with the distance of the consumer from the influencer $\|x - \theta\|$. Thus, customers with greater influencer affinity are willing to pay more.

A single-influencer benchmark Consider a monopolist seller working with a single influencer. When $I < \pi$, not all consumers are within reach of the influencer. Fix the price p and the quality of the product y, only consumers with a non-negative utility are served, which determines the demand $D(p) = 2(1 - p/y)I$. When $I \geq \pi$, the influencer can reach all consumers. Specifically, if p is sufficiently low such that $p \leq y(1 - \pi/I)$, $D(p) = 2\pi$; otherwise, $D(p) = 2(1 - p/y)I$. We can further analyze the monopolist pricing strategy and the profit as follows. For any given values of (I, y), the optimal pricing strategy p^* and the resulting profit Π for a monopolist seller can be derived (see Appendix A.1):

$$p^* = \begin{cases} \frac{y}{2}, & \text{if } I < 2\pi \\ y\left(1 - \frac{\pi}{I}\right), & \text{if } I \geq 2\pi \end{cases} \quad \textit{and} \quad \Pi = \begin{cases} \frac{yI}{2}, & \text{if } I < 2\pi \\ 2\pi y\left(1 - \frac{\pi}{I}\right), & \text{if } I \geq 2\pi \end{cases} \tag{2}$$

The seller's payoff U_1 and the influencer's wage w_1 are given by $U_1 = \gamma\Pi$ and $w_1 = (1 - \gamma)\Pi$.

Depending on I, a monopolist seller can choose to target a subpopulation or the whole demographic of consumers. Figs. 2 and 3 illustrate the potential (the red arc between point 1 and 5) and actual (the blue arc between point 2 and 4) consumer base and the monopolist pricing strategy in Equation (2) for an influencer located at θ. The vertical axis corresponds to the consumer utility with the optimal price p^* marked, and the consumers served are indicated by the thick (blue) line segment.

4. Seller competition and influencer hiring

We now consider seller competition in both the influencers' labor market and the product market, with implications for influencer-intermediated sales. A seller-influencer group is characterized by the 3-tuple (y_m, θ_m, I_m) for $m = 1, 2$. Without loss of generality, we assume that $y_1 \geq y_2$ and $I_1 \geq I_2$. Unless otherwise stated, we consider balanced matching in which each seller is matched with at most one influencer and vice versa. Denote by w_j Influencer j's wage, and by U_k Seller k's payoff. Let $\beta := \|\theta_1 - \theta_2\|$ denote the style dispersion between the two influencers. Denote by $k(j)$ the identity of the matched seller for Influencer j.

4.1. Assortative matching between sellers and influencers

Do brands of higher quality hire more powerful influencers? Indeed, given style locations for the influencers, positive assortative matching typically ensues in equilibrium. This echoes practitioners' concern that bigger brands crowd out smaller ones in hiring better influencers. That said, positive assortative matching also provides a rationale for the rise of influencer marketing. By amplifying the outreach of better products, influencer marketing can potentially place better products into consumers' hands and improve the total welfare.

Proposition 1 *(Positive assortative matching). $k(j)=j$ for $j=1,2$ when one of the following holds: (i) $\beta \geq \frac{1}{2}I_1+\frac{1}{2}I_2$, that is, the market is uncongested; (ii) a seller dominates the entire market; (iii) influencers are maximally distant and sellers compete on both sides, that is, $\beta=\pi$ and $I_j \geq \pi$; or (iv) sellers compete only on one side and $y_1 \leq 4y_2$.*

Proposition 1 presents fairly general sufficient conditions for assortative matching to arise, which also constitutes an important middle step to characterize equilibrium outcomes with product competition. Specifically, (i) shows the emergence of assortative matching when the influence power is relatively small compared to the style dispersion. The seller with a more valuable good can offer to hire a more powerful influencer by proposing a higher wage because the total profit of the seller-influencer group is supermodular in influencer power and product quality.[8] Since influencers do not increase the common value of consumption y, the product with better quality is always priced higher.

Furthermore, in (ii), assortative matching emerges when market competition is intense and a single seller can dominate all other sellers. If only a single seller can survive, it has to be the strong seller because consumers who are sufficiently loyal to her will always keep purchasing from her. Intuitively, if the strong seller can defeat the other seller by hiring the relatively weak influencer, it only makes the strong seller more powerful by hiring the strong influencer in the product market. These two facts jointly lead to assortative matching.

In addition, in (iii), assortative matching also occurs when influencers are maximally distanced (that is, $\beta=\pi$), which can be an outcome of influencers' endogenous choices of style, as we discuss later. The condition $I_2 \geq \pi$, combined with $I_1 \geq I_2$, means that the market is crowded and influencers compete on both sides. Finally, in (iv), assortative matching also ensues when a unique equilibrium exists in which the two sellers compete only along the short arc and the gap in product quality is relatively small. The intuition remains. The stronger seller, by hiring a more powerful influencer, enjoys an advantage in competing against the other seller and gaining a bigger market share and thus a larger joint profit.

4.2. Market dominance in the influencer economy

Despite positive assortative matching, influencer marketing can empower smaller brands and endow them with market power in niche markets and subsequently maintain seller competition instead of having one seller dominate. To see this, we first establish a set of sufficient conditions under which a single seller-influencer group grabs the entire market. Compared to traditional markets without influencers, it is more difficult to achieve such market dominance, which now requires relatively large gaps in both product quality and influencer power, as well as a small style dispersion between influencers.

Proposition 2 *(Market dominance). Fix $I_1 \leq 2\pi$. If $\frac{\beta}{I_1} \leq \min\{\frac{1}{2}-\frac{y_2}{y_1}, \frac{1}{2}-\frac{I_2}{I_1}\}$, then Seller 1 hires Influencer 1 and sets $p_1^*=\frac{y_1}{2}$ to completely force Seller 2 out of the market. Payoffs for sellers and influencers are given by: $U_1=\frac{\gamma y_1 I_1}{2}$, $w_1=\frac{(1-\gamma)y_1 I_1}{2}$, and $U_2=w_2=0$.*

Proposition 2 establishes the conditions in which the best seller dominates the market and gets the entire market share, which is the standard prediction in traditional economies without influencers. However, unlike direct price competition, it is not enough to have greater influencer power or a much better product to force the rival out in product competition. Suppose that Seller 1 has a superior product. Seller 2 can still compete to gain some market share because Influencer 2, who works with Seller 2, creates a sufficiently large "product differentiation" ($\theta_1 \neq \theta_2$). Here, the sufficient conditions in Proposition 2 involve multidimensional heterogeneity and are stringent, as illustrated next.

4.3. Heterogeneity and market power

We now study how influencers affect product pricing, sellers' profits, and consumer welfare in novel ways. We first present three examples, each involving a distinct form of heterogeneity, and then study general properties and comparative statics of equilibrium outcomes.

Three examples The three examples below not only show that sufficient conditions in Proposition 2 are easily violated, but also provide insights on how sellers compete in the presence of product quality difference and influencer heterogeneity.

[8] A twice-differentiable function $f: X \times Y \to \mathbb{R}$ is supermodular iff $\frac{\partial^2 f}{\partial x \partial y} \geq 0$ for all $(x,y) \in X \times Y$.

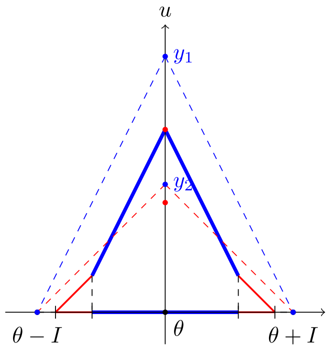


Fig. 4. Heterogeneous product quality.

Example 1 *(Heterogeneous product quality).* Consider $y_1 \geq y_2$, $\beta = 0$, and $I_1 = I_2 = I \leq \pi$. In equilibrium, $k(j) = j$ for $j = 1, 2$. After matching, the two groups choose $(p_1^C, p_2^C) = \left(\frac{2y_1(y_1-y_2)}{4y_1-y_2}, \frac{y_2(y_1-y_2)}{4y_1-y_2}\right)$. Seller 1 targets consumers with $\|x-\theta\| \leq \frac{2Iy_1}{4y_1-y_2}$ and Seller 2 targets those with $\frac{2Iy_1}{4y_1-y_2} < \|x-\theta\| \leq \frac{3Iy_1}{4y_1-y_2}$. Furthermore, the group profits are given by:

$$\Pi_1^C = \frac{8Iy_1^2(y_1-y_2)}{(4y_1-y_2)^2}, \quad \text{and} \quad \Pi_2^C = \frac{2Iy_1y_2(y_1-y_2)}{(4y_1-y_2)^2}. \tag{3}$$

Payoffs for sellers and influencers are given by:

$$U_1 = \gamma\Pi_1^C,\ U_2 = \gamma\Pi_2^C,\ w_1 = (1-\gamma)\Pi_1^C, \text{ and } w_2 = (1-\gamma)\Pi_2^C.$$

Fig. 4 illustrates the equilibrium. Specifically, Seller 1, who offers a relatively superior product and works with Influencer 1, targets consumers with high loyalty, and thus a high willingness to pay. We illustrate these clients with the blue line. In contrast, Seller 2, which offers a relatively inferior product, attracts those consumers not targeted by Seller 1, and we illustrate them with the red line. The intuition is that when $y_1 > y_2$, the product offered by Seller 1 is still more attractive to consumers with high loyalty (that is, $\|x-\theta\| \to 0$) even if Seller 2 sets $p_2^* = 0$. Therefore, Seller 1 has incentives to set a positive price $p_1^* > 0$, which implies that Seller 2 can make a profit by attracting consumers not targeted by Seller 1.

Furthermore, when the quality gap shrinks (that is, $y_1 \downarrow y_2$), it converges to the market outcome of Bertrand competition with $(p_1^C, p_2^C) = (0, 0)$. In contrast, when $y_1 \gg y_2$, it converges to an equilibrium with $(p_1^C, p_2^C) = (y_1/2, y_2/4)$, which means that the high quality product is priced at its monopoly price, while the low quality product is priced at a monopoly price in the residual market after removing the market share taken by the strong seller. Note that the sufficient conditions in Proposition 2 are violated in Example 1.

To further appreciate the content of Example 1, we compare it with traditional economies in which two sellers with quality indices $y_1 > y_2$ engage in vertical product competition. Bertrand price competition implies that $p_1 = y_1 - y_2$, $p_2 = 0$ and that all consumers buy from Seller 1. Compared to the benchmark economy without influencers in which Seller 2 has a zero market share, she has a strictly positive market share in the presence of influencers, even if there is no horizontal style dispersion between the two influencers. The reason is that the consumer's utility difference between buying two products, $(y_1 - y_2)(1 - \|x-\theta\|/I) - (p_1 - p_2)$, decreases in the distance to the influencer. Therefore, even with identical influencers, consumers are effectively facing vertical product differentiation. Such a phenomenon does not arise without influencers, because all consumers then have the same willingness to pay.

Without influencers, all consumers are concentrated in a mass, which always leads to the dominance of the superior product. In contrast, with influencers, even without horizontal differentiation, consumers are heterogeneous and distributed in a nontrivial decentralized way. In particular, consumers' willingness to pay for the product differs greatly when located near the influencer's style location, whereas consumers are more homogeneous but less valuable when located away from the center. Hence, the strong seller, by targeting the more valuable consumers, can establish a great advantage in revenue through influencer marketing. However, this revenue advantage also deteriorates when it comes to distant consumers, creating an incentive conflict with attracting the most valuable consumer for the dominant seller. This prevents perfect price competition as in traditional economies. Hence, as long as there exists a difference in product quality, both sellers can survive and secure a share of client, and thus a positive profit.

Example 2 *(Heterogeneous influencer power).* Consider $I_2 \leq I_1 \leq \pi$, $\beta = 0$ and $y_1 = y_2 = y$. Again, $k(j) = j$ for $j = 1, 2$ and equilibrium prices are set at $(p_1^C, p_2^C) = \left(\frac{2y(I_1-I_2)}{4I_1-I_2}, \frac{y(I_1-I_2)}{4I_1-I2}\right)$. Seller 1 targets consumers with $\frac{I_1I_2}{4I_1-I_2} < \|x-\theta\| \leq \frac{I_1(2I_1+I_2)}{4I_1-I_2}$ and Seller 2 targets those with $\|x-\theta\| \leq \frac{I_1I_2}{4I_1-I_2}$. Group profits are given by:

$$\Pi_1^C = \frac{4I_1^2(I_1-I_2)y}{(4I_1-I_2)^2}, \quad \text{and} \quad \Pi_2^C = \frac{I_1I_2(I_1-I_2)y}{(4I_1-I_2)^2},$$

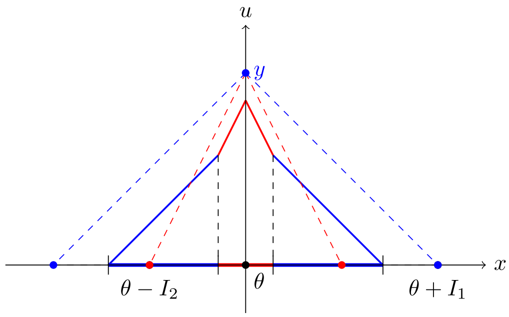


Fig. 5. Heterogenous influencer power.

and payoffs for sellers and influencers are given by:

$$U_1 = \gamma\Pi_1^C,\ U_2 = \gamma\Pi_2^C,\ w_1 = (1-\gamma)\Pi_1^C, \text{ and } w_2 = (1-\gamma)\Pi_2^C.$$

Fig. 5 illustrates the equilibrium. Specifically, Seller 1, who works with Influencer 1, targets consumers sufficiently distant from the influencer's style, as illustrated with blue lines, mainly to avoid tough price competition in the product market. Indeed, they can afford this because the strong influencer power alleviates the utility loss from consumers. In contrast, Seller 2, who works with Influencer 2, targets consumers with high loyalty, and we depict it with red lines. For the equilibrium, note that $p_1 \geq p_2$. Otherwise, Seller 2 will be forced out of the market. However, $p_1 = 0$ is sub-optimal for Seller 1 because consumers with $I_2 < \|x-\theta\| \leq I_1$ prefer Seller 1. Thus, Seller 1 can be better off by charging a low and positive price, which further implies that Seller 2 can also make a positive profit by attracting consumers close to θ. Furthermore, when $\frac{I_1}{I_2} \to \infty$, prices satisfy $(p_1^C, p_2^C) \to (\frac{y}{2}, \frac{y}{4})$, and when $\frac{I_1}{I_2} \to 1$, $(p_1^C, p_2^C) \to (0,0)$, which coincides with outcomes in standard Bertrand competitions.

By comparing Example 1 and 2, we can see that the competition mode depends on the way heterogeneity arises. Specifically, when it is driven by the consumption value of the product, the stronger group focuses on attracting consumers with a taste similar to that of the influencer. In contrast, when it is related to how easily the influencer attracts followers, the stronger group focuses on those consumers not reachable by the weaker group and sacrifices the loyal followers in the sense of taste proximity.

Example 3 *(Heterogeneous influencer style).* Consider $\beta > 0$, $I_1 = I_2 = I$ and $y_1 = y_2 = y$. Define $\beta_0 := \frac{2}{67}(-7+5\sqrt{10})I \approx 0.263I$. Again, $k(j) = j$ for $j = 1,2$. After matching, the kth seller-influencer group's price p_k^C and total profit Π_k^C satisfy:

(i) when $\beta > I$ holds, $p_k^C = \frac{y}{2}$ and $\Pi_k^C(\beta, y) = \frac{yI}{2}$;

(ii) when $\frac{6}{7}I < \beta \leq I$ holds, $p_k^C = y * \left(1 - \frac{\beta}{2I}\right)$ and $\Pi_k^C(\beta, y) = y * \beta\left(1 - \frac{\beta}{2I}\right)$[9];

(iii) when $\beta_0 \leq \beta \leq \frac{6}{7}I$ holds, $p_k^C = \frac{y}{5I}(2I+\beta)$ and $\Pi_k^C(\beta, y) = \frac{3y}{50I} * (2I+\beta)^2$.

Furthermore, $U_1 = \gamma\Pi_1^C$, $U_2 = \gamma\Pi_2^C$, $w_1 = (1-\gamma)\Pi_1^C$, and $w_2 = (1-\gamma)\Pi_2^C$.

Note that when $\beta < \beta_0$, there exists no pure strategy equilibrium. Once again, the stronger seller cannot dominate the market. In summary, the three examples above jointly show that it is insufficient to achieve market dominance by introducing one-dimensional heterogeneity in an influencer economy.

Market outcomes with influencers Next, we explore some general properties and provide a comparative static analysis, focusing on the case in which two seller-influencer groups both survive and compete only on one side.[10] Let W and CS denote total welfare and consumer welfare, respectively. Define $\widetilde{W} = \frac{W}{y_2 I_2}$ and $\widetilde{CS} = \frac{CS}{y_2 I_2}$.

Proposition 3 *(Market outcomes with influencers). There exist parameters under which a unique equilibrium exists and two sellers compete only on one side. Furthermore, if $y_1 \leq 4y_2$:*

(i) Positive assortative matching ensues and $\Pi_1^C \geq \Pi_2^C$;

[9] Note that there exist multiple equilibria when $\frac{6I}{7} \leq \beta \leq I$ and we focus on symmetric equilibrium such that $p_1^* = p_2^*$ in Example 3. Specifically, all equilibria are characterized as follows: (i) when $\beta \in [6I/7, 13I/14]$, $p_1^* + p_2^* = (2 - \beta/I)y$ with $p_k^* \in [10y/7 - \beta y/I, 4y/7]$ and $\Pi_k^C \in [24yI/49, 2y(10I-7\beta)(7\beta-3I)/(49I)]$; and (ii) when $\beta \in (13I/14, I]$, $p_1^* + p_2^* = (2-\beta/I)y$ with $p_k^* \in [y/2, 3y/I - \beta y/I]$ and $\Pi_k^C \in (4\beta y - 2y\beta^2/I - 3yI/2, yI/2]$.

[10] When the influencer market is uncongested, the comparative statics are simple. Specifically, the style dispersion becomes irrelevant. The price ratio is proportional to the magnitude of y_1/y_2 and independent of I_1/I_2. The ratio of sellers' profits, as well as total welfare and consumer welfare, increases in the ratios of y_1/y_2 and I_1/I_2. This also holds when both average quality and influencer power are held fixed.

(ii) $\frac{\partial\left(\Pi_1^C/\Pi_2^C\right)}{\partial\beta} \leq 0$, $\frac{\partial\left(P_1^C/P_2^C\right)}{\partial\beta} \leq 0$, $\frac{\partial\widetilde{W}}{\partial\beta} > 0$, *and* $\frac{\partial\widetilde{CS}}{\partial\beta} > 0$ *for* β *small (and vice versa);*

(iii) $\frac{\partial\left(\Pi_1^C/\Pi_2^C\right)}{\partial(y_1/y_2)} > 0$, $\frac{\partial\left(P_1^C/P_2^C\right)}{\partial(y_1/y_2)} > 0$, $\frac{\partial\widetilde{W}}{\partial(y_1/y_2)} > 0$, *and* $\frac{\partial\widetilde{CS}}{\partial(y_1/y_2)} > 0$*;*

(iv) $\frac{\partial\left(\Pi_1^C/\Pi_2^C\right)}{\partial(I_1/I_2)} > 0$, $\frac{\partial\left(P_1^C/P_2^C\right)}{\partial(I_1/I_2)} > 0$, $\frac{\partial\widetilde{W}}{\partial(I_1/I_2)} > 0$ *and* $\frac{\partial\widetilde{CS}}{\partial(I_1/I_2)} > 0$.

Proposition 3 yields the following insights: First, greater style dispersion across influencers results in lower ratios in sellers' profits, prices charged, and higher total welfare, as well as a non-monotonic change in consumer surplus. Although a higher style dispersion benefits both sellers by mitigating the intense competition, it generates an even larger increase in the price and the profit for the weaker seller, lowering the relative ratios of prices and profits. However, greater style dispersion can both increase consumer welfare by expanding the consumer base (between the two influencers along the short arc) and decrease consumer welfare by inducing more aggressive pricing by both sellers. When it is small, a higher style dispersion improves consumer welfare because the implied intense competition greatly limits the ability of sellers to exploit consumers. Thus, the positive welfare effect dominates. In contrast, when it is large, sellers price products aggressively, and therefore the negative welfare effect dominates.

Second, a larger gap in product quality, measured by y_1/y_2, increases the ratios of profits and prices between the two sellers. For welfare impacts, we fix both product quality and influencer power of the weak group.[11] A higher ratio of y_1/y_2 affects welfare by improving the average quality of products (with a superior good 1) and empowering the superior good to dominate the market. Since two forces reinforce each other, both total welfare and consumer welfare improve when y_1/y_2 increases. A similar pattern can be observed with a higher ratio of power difference I_1/I_2.

Remark 1 *(Ambiguous welfare effects)*. Alternatively, we can fix the competitiveness of the strong seller-influencer group, including y_1 and I_1, and then analyze the welfare impacts of increasing y_1/y_2 and I_1/I_2. It generates two opposite forces, including: (i) empowering the superior good 1 to dominate the market; and (ii) decreasing the average quality of products. Therefore, the overall impact on welfare can be ambiguous. However, when the product quality indices of the two goods are close (that is, $y_1/y_2 \to 1$), the reduction in average product quality dominates, decreasing both total welfare and consumer welfare.

Furthermore, even when the average quality of the products or the average influencer power remains unchanged, an increase in the relative ratios of y_1/y_2 and I_1/I_2 has an ambiguous effect on total welfare. On the positive side, making one product more superior can increase the average quality of products faced by consumers, especially when there is a large overlap in the consumer base for a small β. Meanwhile, increasing the power difference helps to empower the superior product to dominate the market. On the negative side, introducing more asymmetry in product quality or influencer power can decrease the total number of consumers served in equilibrium. More discussion can be found in Section OA.1.

4.4. Intermediation technology and labor market outcomes

How do advances in intermediation technologies, such as digital platforms, as well as exogenous shocks to their adoption (e.g., after the Covid-19 pandemic) affect influencers' labor market? For arbitrarily $\gamma < 1$, $\beta > 0$, and $I_1 > I_2$, we broadly define advances in the intermediation technology as $\Delta I_1 \geq \Delta I_2 > 0$.

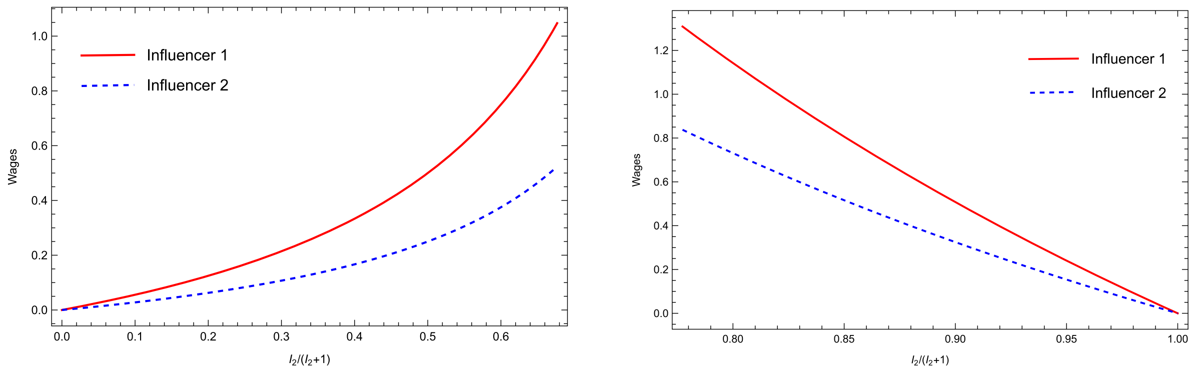
Proposition 4 *(Technological advances and influencer labor market). When* $(I_1 - I_2)y_1 > 4\pi(y_1 - y_2)$ *and* $I_1 < \beta$*, we have:*

(i) The influencers' aggregate payoff is non-monotonic in the intermediation technology (i.e., $\Delta(w_1 + w_2) > 0$ *for small* $\Delta I_1 < \beta - I_1$*, and* $\Delta(w_1 + w_2) < 0$ *for some large* ΔI_2*); and*

(ii) The wage gap between influencers is non-monotonic in the intermediation technology (i.e., $\Delta(w_1 - w_2) > 0$ *for small* $\Delta I_1 < \beta - I_1$*, and* $\Delta(w_1 - w_2) < 0$ *for some large* ΔI_2*).*

Proposition 4 studies the implications on the influencer labor market when technological advances allow more powerful intermediation under certain conditions (that is, asymmetry in influencer power, a small gap in product quality, and relatively small initial influencer power). We find that such advances in intermediation first benefit influencers and then hurt them in the labor market. A similar pattern also emerges for the influencer wage gap. Intuitively, when the influencer market is not congested, stronger intermediation always transforms into larger revenues. However, when the market becomes crowded, stronger intermediation transforms into more fierce price competition, which harms the capacity to generate revenues. Figs. 6a and 6b illustrate this insight. The parameters are given by: $\gamma = 0.5$, $\beta = \pi$, $y_1 = y_2 = 1$ and $I_1 = 2I_2$. Specifically, the solid red line is the wage for Influencer 1, and the dashed blue line is the wage for Influencer 2. Fig. 6a shows that technological advances in intermediation generate an increase in both total wages and the wage gap when the influencer market is not congested, while Fig. 6b shows an opposite effect in a congested influencer market.

[11] Both y_1/y_2 and I_1/I_2 only affect the ratios of sellers' profits and prices through their magnitudes.



(a) Uncongested market (b) Congested market

Fig. 6. Intermediation Advances and Influencers' Labor Market.

5. Influencer style and product differentiations

Having derived how sellers hire influencers and price products, we now endogenize sellers' production. Specifically, we investigate how influencer heterogeneity affects horizontal and vertical product differentiations compared to traditional economies.

5.1. Horizontal product differentiation

We investigate how style dispersion affects the horizontal product differentiation by sellers. Our key observation, perhaps quite intuitively, is that influencers' style difference and horizontal product specialization are substitutes. However, both the product quality gap and the influencer power difference complement horizontal differentiation.

To model horizontal differentiation, we stipulate that Seller k can either choose to hire an influencer with a given style or pay a cost f_E to nurture a new influencer with power $\widehat{I}_k$ and select any style $\theta \in \mathbf{S}^1$. This is equivalent to producing a differentiated product of style θ with an effective "transportation cost" $\widehat{I}_k$. To avoid trivial cases and allow for horizontal differentiation, let $I := \max\{I_1, I_2, \widehat{I}_1, \widehat{I}_2\} \le \frac{\pi}{2}$. Denote by β^* the optimal style dispersion after horizontal differentiation, and by $\Pi_k(\beta|\widehat{I}_1, \widehat{I}_2, I_1, I_2, y_1, y_2)$ Seller k's profit when the initial style dispersion is β. Then, Seller k engages in horizontal differentiation if

$$f_E < \Pi_k(\beta^*|\widehat{I}_1, \widehat{I}_2, I_1, I_2, y_1, y_2) - \Pi_k(\beta|\widehat{I}_1, \widehat{I}_2, I_1, I_2, y_1, y_2).$$

Otherwise, Seller k hires one influencer and accepts his style location. Intuitively, we expect that $\Pi_k(\beta|I_1, I_2, y_1, y_2)$ increases in β, and therefore the incentive for horizontal differentiation is negatively associated with the influencer style dispersion β.

To isolate the effects of influencer marketing on product differentiation from influencers' labor matching, we focus on $I_j = \widehat{I}_j = I \le \frac{\pi}{2}$ for $j \in \{1, 2\}$, which is relaxed in the online appendix OA.2. For $l \neq k$, we define:

$$\Pi_k(\beta|y_1, y_2, I) = \frac{y_k(2y_k + y_l)(\beta y_l(3y_k + 4y_l) + 2I(2y_k^2 + 3y_k y_l + y_l^2))^2}{I(y_k + y_l)(8y_k^2 + 19y_k y_l + 8y_l^2)^2}.$$

Denote $b = \beta/I$ and $t = y_1/y_2$ where $t \ge 1$. Then, $\Pi_k(\beta|y_1, y_2, I) = H_k(b, t)y_k I$, where

$$H_1(b,t) = \frac{(1+2t)(2+8t+4t^2+b(4+3t))^2}{(1+t)(8+19t+8t^2)^2},$$
$$H_2(b,t) = \frac{(2+t)(4+(8+3b)t+(2+4b)t^2)^2}{(1+t)(8+19t+8t^2)^2}.$$

Define: $\tau = y_2/y_1$, $a = \frac{3}{10}$, and $c = \frac{10(1+t)^2+2t}{12(1+t)^2+t} \in (5/6, 6/7]$. Obviously, $H_1(b,t) = H_2(b,\tau)$. We first show how $H_k(b,t)$, or equivalently $\Pi_k(\beta|y_1, y_2, I)$, depends on b and t (or τ).

$$\frac{\partial H_k(b,t)}{\partial b} > 0, \quad \frac{\partial H_2(b,t)}{\partial t} < 0, \quad \text{and} \quad \frac{\partial H_1(b,\tau)}{\partial \tau} < 0. \tag{4}$$

Equation (4), verified in the proof of Proposition 5, provides two important insights. First, a larger β, or more dispersed style locations, lead to higher profits for both sellers and reduces the need for horizontal differentiation. Second, a larger t, or a more superior good 1, lowers the profit of Seller 2 and thus motivates her to use more horizontal differentiation.

Proposition 5. *The influencer style dispersion is a substitute for horizontal product differentiation. For* $\frac{y_1 I}{2} - H_1(c,t)y_1 I < f_E < \frac{y_2 I}{2} - H_2(a,t)y_2 I$*, we have:*

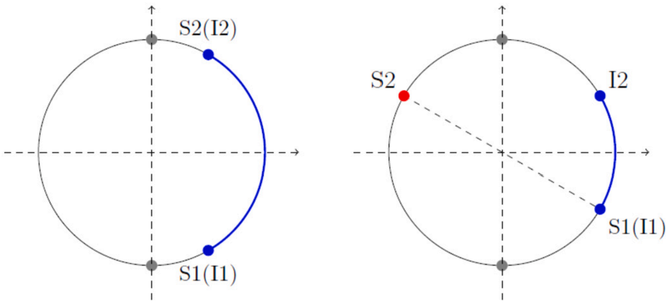


Fig. 7. Style dispersion and horizontal differentiation.

(i) Seller 1 has a stronger incentive to differentiate products horizontally for $aI \le \beta < \frac{I}{2}$; otherwise, for $\frac{I}{2} \le \beta \le cI$, Seller 2 has a stronger incentive to differentiate products.

(ii) There exists $\widetilde{\beta} \in (aI, cI)$ such that each seller hires an influencer and accepts his style location when $\widetilde{\beta} \le \beta \le cI$, and (only) one seller pays f_E and selects a location such that $\beta^* \ge I$ when $aI \le \beta < \widetilde{\beta}$.

(iii) Sellers are more willing to differentiate horizontally when faced with a more powerful opponent. Specifically, with y_2 fixed, the cutoff $\widetilde{\beta}$ increases strictly in the ratio $t = \frac{y_1}{y_2}$ when Seller 2 chooses horizontal differentiation (that is, $\frac{\partial \widetilde{\beta}}{\partial t} > 0$). Similarly, with y_1 fixed, $\widetilde{\beta}$ increases strictly in $\tau = \frac{y_2}{y_1}$ when Seller 1 chooses horizontal differentiation (that is, $\frac{\partial \widetilde{\beta}}{\partial \tau} > 0$).

Fig. 7 illustrates the equilibrium in Proposition 5. The left sub-figure entails a case with a large style difference. The style locations for Influencers 1 and 2 are marked in blue, while the two gray nodes illustrate an optimal style separation. The blue arc corresponds to β. In this case, both sellers hire influencers to save the fixed cost in product differentiation and adapt to the influencers' styles. Therefore, the dispersion of style between influencers serves as a substitute for horizontal differentiation. A sufficiently large dispersion, albeit not maximal, removes the incentive to horizontally differentiate. Furthermore, the right sub-figure illustrates the equilibrium when influencers' style dispersion is small so that once a seller works with an influencer, the other seller differentiates in style (red dot) to reduce competition, leading to maximum style differentiation between the seller-influencer groups.

Remark 2 *(Power difference complements horizontal differentiation)*. It is tempting to generalize the substitution effect in Proposition 5 and argue that influencer heterogeneity always substitutes horizontal differentiation. However, this is wrong when influencer heterogeneity manifests itself in influencer power difference. Similarly to the product quality gap, a power difference complements horizontal differentiation (that is, $\frac{\partial \widetilde{\beta}}{\partial (I_1/I_2)} > 0$), as illustrated in Proposition A.1 in Section OA.2, which implies that a larger influencer power gap strengthens the incentive to differentiate horizontally by allowing the stronger seller to compete more aggressively, lowering both prices and profits for the seller who works with the weaker influencer. This, in turn, motivates sellers to differentiate more.

Remark 3 *(Perfectly symmetric case)*. An undesirable limitation of Proposition 5 is $c < \frac{6}{7}$. Ideally and intuitively, we could expect $c = 1$, so there exists a single cutoff point $\widetilde{\beta}$ such that no seller wants to differentiate products for all $\beta > \widetilde{\beta}$, rather than for $\widetilde{\beta} < \beta < cI$. When $\beta \in (cI, I)$, the profit functions may not be differentiable and there exists a continuum of equilibria that are characterized by a system of inequalities. However, in the perfectly symmetric case where $y_1 = y_2$ and $I_1 = I_2$, we can verify this single cutoff property (that is, $c = 1$) for consistent equilibrium selection rules including the highest-payoff, the lowest-payoff and the symmetric-payoff equilibrium. For example, we select the symmetric equilibrium for both sellers in Proposition A.2.

5.2. Vertical product differentiation

To model the relationship between influencers' style dispersion and vertical product differentiation, we allow sellers to incur a cost to improve product quality and potentially gain larger market shares. In general, the endogenous vertical differentiation is non-monotonic in influencers' style dispersion, with an endogenous quality gap only observed for intermediate style dispersion. Specifically, consider two sellers who initially make products of identical quality $y_k = \underline{y}$, but $\beta \ge 0$. We normalize $\underline{y} = 1$ and assume $I_1 = I_2 = 1$ to remove confounding issues in labor matching. Influencers can pay a fixed R&D cost F_V to improve quality to $\bar{y} = y > 1$. We also use "L" and "H" to denote "Low quality" and "High quality." Let $V_1(\beta, y)$ denote the increment profit change from quality investment when the competing group selects low quality,

$$V_1(\beta, y) \equiv \Pi^1_{H,L} - \Pi_{L,L} = \frac{y(1+2y)\left(2+8y+4y^2+\beta(4+3y)\right)^2}{(1+y)\left(8+19y+8y^2\right)^2} - \frac{3}{50}(2+\beta)^2,$$

and by $V_2(\beta, y)$ the investment benefit when the opponent also selects high quality,

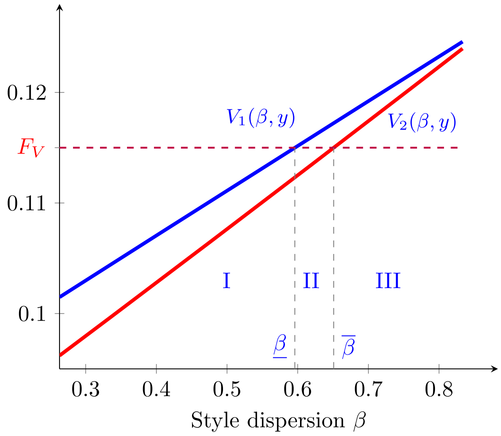


Fig. 8. Vertical product differentiation.

$$V_2(\beta, y) \equiv \Pi_{H,H} - \Pi^1_{L,H} = \frac{3y}{50}(2+\beta)^2 - \frac{(2+y)\big(4+8y+2y^2+\beta y(3+4y)\big)^2}{(1+y)\left(8+19y+8y^2\right)^2}.$$

We first derive two properties of $V_1(\beta, y)$ and $V_2(\beta, y)$ in the proof for Proposition 6: (i) $V_1(\beta, y) > V_2(\beta, y)$; and (ii) both $V_1(\beta, y)$ and $V_2(\beta, y)$ strictly increase in β. Intuitively, (i) implies that a superior product provided by the competitor reduces the return on vertical differentiation, and (ii) implies that a higher β increases the return on vertical differentiation. These two forces jointly shape the incentives for vertical differentiation.

Next, define $\bar{\beta}$ and $\overline{\beta}$ such that $V_1(\underline{\beta}, y) = F_V$ and $V_2(\overline{\beta}, y) = F_V$. Note that both $V_1(\beta, y)$ and $V_2(\beta, y)$ increase in style dispersion. Thus, $\underline{\beta}$ is the break-even point for quality investment, given that the opponent selects low quality. Similarly, $\overline{\beta}$ is the break-even point for investment when the opponent selects high quality. Recall that $\beta_0 = \frac{2}{67}(-7+5\sqrt{10})I_2 \approx 0.263$, a number we use in the figures for illustration.

Proposition 6 *(Non-monotonic vertical differentiation). Assume that: (i) $\beta_0 \le \beta \le \frac{5}{6}$; (ii) $\overline{y}(1-\beta_0) \le \underline{y}$; and (iii) $V_1(\beta_0, y) < F_V < V_2(\frac{5}{6}, y)$. Then, there exist $\overline{\beta}$ and $\underline{\beta}$ such that:*

(a) when $\beta \ge \overline{\beta}$, there exists a unique Nash equilibrium (H, H);
(b) when $\underline{\beta} \le \beta < \overline{\beta}$, there are two asymmetric Nash equilibria: (H, L) and (L, H);
(c) when $\overline{\beta}_0 \le \beta < \underline{\beta}$, there exists a unique Nash equilibrium (L, L).

Proposition 6 characterizes a non-monotonic relationship between influencer style dispersion and vertical product differentiation. In particular, vertical differentiation arises only for intermediate values of β. Note that: (i) ensures the existence of pure strategy equilibrium and simplifies the algebra in the proof; (ii) ensures that under asymmetric investment (H, L), both groups can survive in the product market; and (iii) focuses on the most interesting cost range in which non-monotonicity arises. Note that for $F_V \le V_1(\beta_0, y)$, (H, H) is the unique Nash equilibrium, so is (L, L) for $F_V > V_1(\frac{5}{6}, y)$.

Fig. 8 illustrates the equilibrium configuration in Proposition 6 with $y = \frac{5}{4}$. Specifically, the horizontal axis β corresponds to style dispersion. The two functions, $V_1(\beta, y)$ and $V_2(\beta, y)$, corresponding to the blue and red solid lines in the figure, measure the profit gap between "high quality" and "low quality," given the other group's choice. Fix the investment cost $F_V > 0$ (dashed purple line). There are three regions, "I", "II" and "III," separated by two cutoffs, $\underline{\beta}$ and $\overline{\beta}$. In Region III, $V_2(\beta, y) > F_V$, and thus both groups choose high quality. In Region II, $V_2(\beta, y) < F_V$ and $V_1(\beta, y) > F_V$, which means that only one group chooses high quality. In Region I, both groups choose low quality.

Proposition 6 implies that, when style dispersion (β) increases from low values, we initially see more vertical differentiation (and thus influencers' style dispersion and vertical product differentiation are complements). In contrast, as β further increases from a large value, we see less vertical differentiation (and thus these two are substitutes). Intuitively, for a small β, intense competition greatly limits the return on investment in high quality. Thus, both groups choose low quality, and the vertical differentiation is minimal. For an intermediate β, the competition is less intense, which improves the investment return provided that the opponent does not invest. Finally, for a large β, the competition is minimal, allowing both groups to at least break even with their investments. Given that investment profit strictly increases in influencer-style dispersion, we no longer observe vertical differentiation in equilibrium.

6. Direct sales VS. influencer-intermediated sales

To further understand why influencers emerge and assess their economic impact, we introduce direct sales to capture real-life situations in which either sellers sell products directly or a seller uses a conventional advertisement without targeting a particular audience. The corresponding friction is then the relatively high transportation cost for direct sales because consumers do not interact socially with sellers or traditional marketers. In contrast, influencers' advantage lies in their close interaction with followers on digital social platforms.

With direct sales, the ith consumer's utility can be written as:

$$u_i(x_i, y, p) = y * \max\{(1-c), 0\} - p. \tag{5}$$

Here, y and p are the quality index and the price. The transportation cost parameter $c \in (0, 1]$ captures brand reputation, product perception, consumer goodwill, product acceptance rate, and market penetration power. The baseline model can be viewed as a special case with a sufficiently high transportation cost of $c \geq 1$ to rule out direct sales. Furthermore, Equation (5) implies that all consumers are equally distant from the seller, and thus all get an identical utility (that is, the seller is located in the center of the circle). We consider two distinct cases, *price discrimination* and *uniform pricing*, which differ in the sellers' ability to charge direct sales prices differently from influencer-intermediated prices.

6.1. Equilibrium with price discrimination

We start with the case of price discrimination. Denote by p_k^d and p_k^i the prices charged by Seller k in direct sales and influencer-intermediated sales. First, we consider a single seller and a single influencer and assume that $I_1 \leq 2\pi$. Under the optimal pricing strategy, the monopolist seller charges $p_1^d = (1-c)y_1$ and $p_1^i = (1-c/2)y_1$.[12] Specifically, the seller targets consumers with $\|x - \theta_1\| \leq \frac{c_1 I_1}{2}$ through influencer-intermediated sales and other consumers through direct sales. Now, the effective power, measured by the size of consumers who prefer influencer-intermediated sales to direct sales before paying the prices, decreases to $c_1 I_1$. Then, influencer-intermediated sales can be viewed as monopoly pricing after charging the maximum price in direct sales, and thus only consumers within the effective power range are targeted. This generates a total profit of $\Pi = 2\pi * (1-c_1)y_1 + \frac{1}{2}c_1^2 y_1 I_1$.

Next, we analyze the setup with two sellers and two influencers.

Lemma 1 *(Equilibrium with direct sales). Consider $y_1 \geq y_2$, $I_1 \geq I_2$, $\beta \geq \frac{c_1 I_1 + c_2 I_2}{2}$ and $(1-c_1)y_1 = (1-c_2)y_2$. In equilibrium, $k(j) = j$ for $j \in \{1, 2\}$. After matching, Seller k sets $p_k^d = 0$ for direct sales and sets $p_k^i = \frac{c_k y_k}{2}$. Influencer j, who works with Seller $k(j)$, targets consumers with $\|x - \theta_j\| \leq \frac{c_{k(j)} I_j}{2}$ for influencer-intermediated sales. Furthermore, the group profits are given by $\Pi_{k(j),j} = \frac{c_{k(j)}^2 y_{k(j)} I_j}{2}$ for $j = 1, 2$. Payoffs for sellers and influencers are given by $U_k = \gamma \Pi_k$ and $w_j = (1-\gamma)\Pi_{k(j)}$ for $k = 1, 2$ and $j = 1, 2$.*

Lemma 1 first verifies the robustness of our main results when sellers have comparable traditional marketing capacity in direct sales. Indeed, when $(1-c_1)y_1 = (1-c_2)y_2$ holds, Equation (5) reduces to:

$$u(x, \theta_j | c_k, y_k, p_k) = y_k(c_k - \|x - \theta_j\|/I_j) - p_k, \tag{6}$$

after we remove the impact of direct sales. The similarity between Equations (6) and (1) implies that our main results on assortative matching, market dominance, and product differentiation, etc., remain robust in the presence of direct sales.

Two new insights emerge from Lemma 1 once we consider heterogeneous marketing capacity in direct sales. First, compared to traditional economies with only direct sales, influencer marketing can foster seller entry and competition by allowing sellers with weak traditional sales to bypass the transportation cost barrier. Consider $(1-c_1)y_1 > (1-c_2)y_2$ and $y_2 > y_1$. With only direct sales, Seller 1, say, the incumbent, by leveraging the low transportation cost advantage, can effectively force Seller 2, the potential entrant with a superior good, out of the product market. In contrast, with influencer marketing, Seller 2 can effectively bypass the transportation cost barrier by employing influencers to gain access to niche markets.

Second, because sellers with weak traditional marketing channels have a stronger incentive to hire influencers, the transportation cost parameter c_k and product quality jointly determine sellers' incentives to hire influencers. Consequently, negative assortative matching occurs, in a spirit similar to that of Proposition 1, between direct sales capacity and influencer power when the quality of the product is fixed (that is, $y_1 = y_2 = y$). Proposition 7 below considers a representative case such that $c_1 < c_2$ and $I_1 > I_2$ (that is, Seller 1 has stronger direct sales capacity and Influencer 1 has a larger influencer power).

Proposition 7 *(Negative assortative matching). Consider that $y_1 = y_2 = y$, $\beta > \frac{c_2(I_1+I_2)}{2}$ and $\frac{\gamma c_2}{2} < (c_2 - c_1)$. Then: $k(j) = 2 - j$, that is, Seller 2, who has weaker direct sales capacity, is matched with the more powerful influencer, Influencer 1; Furthermore, in equilibrium, $p_1^d = (c_2 - c_1)y$, $p_2^d = 0$ and $p_1^i > p_2^i = \frac{c_1 y}{2}$.*

Similar to Proposition 1, Proposition 7 also helps explain the fast growth of influencer marketing despite the direct sales channel. Negative assortative matching between direct sales capacity and influencer power encourages product competition and efficient

[12] To derive the optimal pricing, note that $p_1^i \geq (1-c_1)y_1$. Otherwise, if $p_1^i < (1-c_1)y_1$, the seller prefers direct sales alone and sets $p_1^d = (1-c_1)y_1$. Note also that $p_1^d = (1-c_1)y_1$. If not, suppose that $p_1^d < (1-c_1)y_1$. Then we can increase both p_1^d and p_1^i by a small amount $\delta > 0$ without altering consumers' purchase decisions, resulting in a profitable deviation. We now optimize over p_1^i the profit $\Pi = (2\pi - 2s_1) * p_1^d + 2s_1 * p_1^i$, where $s_1 = I_1 * (1 - p_1^i/y_1)$ is derived from the indifference condition that $y(1 - s_1/I_1) - p_1^i = (1-c_1)y_1 - p_1^d = 0$, we get $p_1^i = (1 - c_1/2)y_1$.

product placement, which improves consumer welfare. To see this, note that consumers always receive a baseline surplus equal to $(1-c_2)y$ from direct sales and a heterogeneous extra surplus rent when buying from influencers with similar styles. Seller 1, which has stronger direct sales capacity, charges a higher price in influencer-intermediated sales compared to Seller 2. This is because Seller 1 earns a positive profit from direct sales (that is, $p_1^d > 0$) and thus faces an additional incentive compatibility condition $\gamma p_1^i > p_1^d$ when placing products by influencers. Therefore, more consumers receive an extra surplus from influencer marketing under negative assortative matching, leading to higher consumer welfare.

Interestingly, a large difference in direct sales capacity can weaken and even reverse the positive assortative seller-influencer matching involving product quality and influencer power in our baseline model, as the next example illustrates.

Example 4 *(Reversed seller-influencer match).* Fix a small $\epsilon > 0$. Consider $y_1 = y_2 + \epsilon$, $2\pi \gg I_1 > I_2$, $\beta > \frac{I_1+I_2}{2}$, $c_2 = 1$, and $\frac{y_2}{2} > (1-c_1)y_1 > \frac{1}{2}y_1 - \epsilon$. In short, Good 1 slightly outperforms Good 2 in quality. In direct sales, Seller 2 cannot reach consumers who buy only from Seller 1. Thus, $p_2^d = 0$ and $p_1^d \le (1-c_1)y_1$. Furthermore, Seller 1 will charge a price $p_1^d \to (1-c_1)y_1$ because direct sales are far too important when $2\pi \gg I_1$. This, in turn, implies that Seller 2 will charge a monopolist price $p_2^i \to \frac{y_2}{2}$, and the total profit from working with Influencer 1 approaches $\frac{1}{2}y_2 I_1$. In contrast, Influencer 1, when working with Seller 1, can generate a total profit about $\frac{1}{2}c_1^2 y_1 I_1 \approx \frac{1}{8}y_1 I_1$, which is less than $\frac{1}{2}y_2 I_1$.[13] Therefore, under Nash bargaining, Influencer 1 chooses to work with Seller 2. Negative assortative matching implies that influencer marketing is more useful for those with limited access to traditional marketing channels.

6.2. Equilibrium under uniform pricing

We next consider uniform pricing between direct sales and influencer-intermediated sales. Again, we start with a single seller and an influencer and assume that $I \le 2\pi$ and $0 \le c < 1$. When $p^d \le (1-c)y$, direct sales occur and the total profit is $\Pi(p^d) = 2\pi p^d$. In contrast, in influencer-intermediated sales, when $p^i \ge (1-c)y$, the demand is $I(1-p^i/y)$, leading to a profit of $\Pi(p^i) = 2I(1-p^i/y)p^i$. Note that $\Pi(p)$ might be discontinuous at $p = (1-c)y$. We can further optimize over p to simplify the total profit as $\Pi^* = \max\left\{\frac{yI}{2}, 2\pi(1-c)y\right\}$. Thus, the seller uses direct sales only when $2\pi(1-c)y \ge \frac{\gamma y I}{2}$. More precisely, the seller chooses direct sales only when: (i) the transportation cost c is small; (ii) the influencer power I is small; and (iii) the seller's bargaining power γ is small. Note that the choice of marketing channel is independent of the quality of the product.

Next, we analyze the case with two sellers and two influencers with balanced matching. For simplicity, we assume that $y_k = y$, $c_k = c \in [\frac{1}{2}, 1]$ and $I_j = I \ll \beta$ where $\beta = \|\theta_1 - \theta_2\|$. Denote by q and $(1-q)$ the probability of using influencer-intermediated sales and direct sales, and by p the price charged in direct sales and $f(p)$ the corresponding density function.

Lemma 2. *With uniform pricing, (i) when $\frac{\gamma y I}{2} \ge (2\pi - I)(1-c)y$, both sellers use influencer-intermediated sales and charge the price $p_k = \frac{y}{2}$; (ii) when $(2\pi - I)(1-c)y > \frac{\gamma y I}{2}$, both sellers use a symmetric mixed strategy $(q, f(p))$ with $p \in [\underline{P}, \bar{P}]$, where $\bar{P} = (1-c)y$,*

$$q = \frac{\gamma I}{2(1-c)(2\pi - I)}, \quad f(p) = \frac{\gamma y I}{4\pi(1-q)p^2}, \quad \text{and} \quad \underline{P} = \frac{2\gamma y I}{2q(2\pi - I) + 4\pi(1-q)}.$$

Under uniform pricing, there are two insights. First, an increase in transportation costs or influencers' intermediation power induces sellers to use more influencer marketing. When this occurs, all the main results hold with slight modifications. As with price discrimination, influencer marketing facilitates seller entry and competition. Moreover, unlike the case of price discrimination, influencer marketing can alleviate competition in direct sales and help avoid Bertrand competition. Now, sellers randomize between influencer-intermediated sales and direct sales, even when influencer marketing is not sufficiently desirable. However, this might harm welfare as fewer consumers are served.

7. Endogenous influence and exclusivity contracting

We now extend the baseline model of influencer marketing (Sections 3 and 4) to consider: (i) influencers may endogenize their influence, either in power or style, and (ii) sellers and influencers may not have one-on-one matching. Both have important welfare implications.

7.1. Endogenous influence acquisition

Influencers sometimes incur a cost to boost their influencer power, e.g., through livestreaming training or multi-homing on various media platforms. We gain novel insights considering such endogenous influence: First, socially insufficient power acquisition can

[13] Note that given that $\beta > \frac{I_1+I_2}{2}$, the pricing strategy for Seller 1 in influencer-intermediated sales is independent of that of Seller 2. Thus, when $p_1^d \to (1-c_1)y_1$, we have $p_1^i \to (1-c_1/2)y_1$ with an effective client base approximately given by $c_1 I_1$, just as shown in the case of a monopolist seller.

arise due to externalities and intense price competition under balanced matching. Second, we show that assortative seller-influencer matching holds in the long run when we allow for style selection.

Note that the utility from consumption is bounded above, which means that many influencers might spend effort to acquire power, and too many endogenously become influencers in practice. We show that this concern about the arms race among influencers is not justified in a duopoly product market. Instead, socially insufficient power acquisition can arise due to externalities and intense price competition under balanced matching. For simplicity, consider two influencers with maximum distance $\beta = \pi$ and initial power I. Either can pay a fixed cost $C_P > 0$ to increase the influencer power to $kI > I$.

Proposition 8 *(Socially insufficient power acquisition). Fix $\beta = \pi$. There exists under-investment in influencer power when: (i) $kI \leq \pi/2$ and $\frac{1}{2}(1-\gamma)(k-1)yI < C_P < 2(k-1)yI$; or (ii) $I \geq \frac{3}{2}\pi$ and $0 \leq C_P < \frac{\pi^2 y(k-1)(7k+2)}{18k(k+1)I}$.*

There are two forces driving this result. First, influencers do not internalize the externality on consumer welfare when acquiring influencer power. When the market for influencers is not crowded, power acquisition can increase consumer utility, and it can exhibit under-investment when the positive externality on consumer welfare is not internalized. The bargaining power clearly matters, and a large γ reduces the incentive for power acquisition and causes under-investment because condition (i) is more likely to hold. Second, the intense price competition between the two sellers acts as a disincentive for influencers to invest in power when the influencer market is congested because it further intensifies the existing competition in the product market. In particular, any potential gain in consumers is dominated by lowered product prices, even when power acquisition costs very little.

Remark 4 *(Monopolist and over-investment in influence).* The endogenous acquisition of influencer power also depends on the structure of the product market. In Proposition 8, both forces induce under-investment. In contrast, when there is a monopolist seller, intense price pushback no longer exists. However, a third channel arises: influencers exert a negative externality on other influencers when hired by the same monopoly seller. Depending on the congestion of the influencer market and the bargaining power, these three forces can generate over-investment, efficient investment, and under-investment in power acquisition. More discussion can be found in Section OA.5.

Remark 5 *(Endogenous style selection).* Our main results are robust when we allow endogenous style selection. We only prove a special case of an uncongested influencer market in which $I_1 + I_2 \leq 2\pi$. Let β^* denote the optimal style dispersion for influencers. When selecting the style location is costless, $\beta^* \geq \frac{I_1+I_2}{2}$. Then, by Proposition 1(i), the assortative matching result follows. This implies that influencers, whenever possible, have an incentive to minimize competition and target niche markets.

7.2. Exclusivity contracting and welfare

In practice, a seller sometimes hires multiple influencers and requires them not to advertise the products of rival sellers (e.g., Zietek, 2016). For example, a large influencer survey by Mavrck (Katz, 2019) shows that the majority of influencers (61%) are receiving exclusivity requests from brands. In fact, exclusivity contracts have been prevalent in industries such as healthcare and insurance and have led to many antitrust cases (Gal-Or, 1999). However, policies have recently been introduced to better protect influencers and reduce market concentration by encouraging competition. The awareness has also grown in the industry that exclusivity should be mutual.[14] This means that either both sides can contract with multiple counterparties or both sides must exclusively collaborate, which is exactly what our balanced matching set-up aims to capture.

However, to better understand the welfare implications of exclusivity contracts in this emerging industry and to guide regulatory policies geared toward balancing the power of influencers and sellers, we extend the analysis to "unbalanced matching." In this setting, a seller can hire multiple influencers, but not the other way around, which is consistent with the contracting landscape in the early stages of the influencer industry (e.g., Zietek, 2016). Proposition 9 compares the efficiency between balanced and unbalanced matching to offer two key insights. First, balanced matching (mutual exclusivity contracting) is optimal under one-dimensional heterogeneity even when we allow sellers to compete for multiple influencers. Second, unbalanced matching (unidirectional exclusivity) can be optimal when influencer style locations are sufficiently unique in uncongested influencer markets.

Proposition 9 *(Exclusivity contracting and welfare). With exclusivity clauses:*

(i) (Congested influencer market or homogeneous product market). Unbalanced matching decreases total welfare under one-dimensional heterogeneity, including heterogeneous product quality, heterogeneous influencer power, and heterogeneous influencers' style locations.

(ii) (Uncongested influencer market). When $\beta > \frac{I_1+I_2}{2}$ and $y_1 > y_2$, unbalanced matching dominates balanced matching in total welfare.

[14] Influencers increasingly value long-term partnerships with brands rather than one-off exclusivity requests; they also expect to be compensated more when exclusivity is required. As early as 2008, the entertainment industry began to see the value behind full-time creators building multi-platform brands, and influencers started getting Hollywood agents to help negotiate bilateral exclusive contracts (Collectively, 2020). New York State Regulators, China's State Administration of Market Regulation, and the British Federal Trade Commissions are all increasing regulations on influencer contracting (Federal Trade Commission, 2019; Zerbo, 2017; China's State Administration for Market Regulation, 2021).

The key messages in Proposition 9 are intuitive. Both the quality gap of products and the style dispersion between the influencers affect the intensity of the seller's competition. When the market for influencers is not crowded and influencers' styles are distinct, regulation on mutual exclusivity contracting does not help encourage competition because of the inevitable local market power derived from influencer heterogeneity. Given that the economy features monopoly pricing anyway, uni-directional exclusivity improves welfare because it allows the high-quality product to dominate. However, when products are quite homogeneous or influencers are very similar in style, requiring mutual exclusivity and balanced matching can improve consumer welfare. Note that unbalanced matching always features joint profit maximization and the dominance of the higher-quality product, while balanced matching features greater price competition. Simply put, the quality improvement channel is shut down in a homogeneous product market, while regulation can improve competition when the influencer market is crowded.

Remark 6 *(The general case with more than two influencers)*. In reality, there are not just two influencers and there could be low entry costs into the influencer market. Indeed, imagining a situation in which a brand hires all influencers is not easy. Typically, it is feasible for a large brand to have many influencers to persuade consumers in many locations. Furthermore, allowing for more flexible pricing through many influencers gives sellers a bigger advantage in competition. Thus, the natural question is whether it is restrictive to have only two influencers. Proposition A.3 answers this question and extends the insights to a more general case with more than two influencers, that is, with unbalanced matching, a seller might now hire many influencers, but not necessarily all influencers. The basic insight remains. Specifically, when there is a limited supply of influencers and the quality of products differs, unbalanced matching is strictly dominant in total welfare since products with low quality are forced out of the market. In contrast, with homogeneous products, equally distanced influencers, and a crowded market for influencers, balanced matching always weakly dominates unbalanced matching because it can enhance competition among sellers. The intuition is that the symmetry in the quality of products and the features of influencers requires identical prices among sellers. Balanced matching is always the most natural way to go (with the highest consumer welfare), although sometimes it might not be the only way. However, as long as asymmetric market power and thus asymmetric pricing arise, unbalanced matching always harms total welfare. A more detailed discussion can be found in Section OA.3.

8. Conclusion

We develop a theory of the influencer economy that focuses on influencer marketing and product competition. Our model captures several salient, important, and realistic features: (i) sellers produce goods and compete for consumers through influencers, (ii) sellers and influencers are matched in influencers' labor market and engage in Nash bargaining, and (iii) influencers acquire influence to attract consumers who identify with their style and enjoy the social interaction, in addition to value the products they promote. We derive four key insights to better understand this new business-social phenomenon, with empirical implications for future studies.

First, more powerful influencers match with sellers with better product quality and weaker direct sales capacity. Empirically, this can be verified using data from e-commerce and digital media platforms. For example, Amazon and Taobao reviews can be used to measure product quality, the gross merchandise value (GMV) of live streaming sales or the number of followers on digital media platforms for influencer power, and the share of revenue from direct sales or in-store sales for direct sales capacity. Depending on the assortative matching, influencer marketing may favor large brands, but can also facilitate seller entry and revive niche markets. As larger companies hire more influencers, the market may become too saturated for small brands to differentiate and survive through influencer-intermediated sales.

Second, technological advances in intermediation can lead to non-monotonic changes in influencers' payoffs and income inequality. Empirically, digital technology advances can be measured using the average cost for influencers to connect and interact with potential consumers, or commissions/fees charged by digital platforms. Then we could possibly explore the event of the Covid-19 pandemic, which drastically made the influencer market more congested. In particular, we anticipate a positive (negative) association between intermediation technology and influencers' payoff and income inequality before (after) the pandemic.

Third, compared to traditional economies, influencer style pluralism mitigates market concentration by effectively differentiating the consumer experience horizontally. In contrast, the dispersion in influencer power intensifies product competition and increases market concentration. Meanwhile, only intermediate style differences complement vertical differentiation. Empirically, one can measure influencers' style difference by collecting data on influencers' educational background, age, style, category of topic-based channels on social media platforms, etc. The construction of product differentiation measures is subtle, but can be done through surveys, focus groups, or customer interviews.

Fourth, influencers may inefficiently under-invest in consumer outreach to avoid exacerbating price competition, and balanced matching and mutual exclusivity can be socially beneficial by fostering competition when the influencer market is congested. In fact, we expect a congested influencer market for the following two reasons: One, advances in digital technologies and platforms greatly facilitate influencer entry and allow digital creators to bypass the institutional gatekeepers in various industries. Two, machine learning algorithms facilitate the identification and growth of influencers, which may quickly make niche markets saturated. Regulation toward balanced matching may improve welfare.

To focus on influencer marketing and its industrial organization, (and for tractability), we have largely abstracted away from the inner workings of platforms and MCNs. The symbiotic relationship between influencers and platforms remains crucial in this new digital economy. The organization and heterogeneity of platforms such as Instagram and TikTok, as well as the general impact of influencers on society, also constitute interesting future research. As such, our findings establish initial theoretical benchmarks rather than foregone conclusions.

CRediT authorship contribution statement

Lin William Cong: Writing – review & editing, Writing – original draft, Investigation, Conceptualization. **Siguang Li:** Writing – review & editing, Investigation, Formal analysis.

Declaration of competing interest

Declarations of interest: none

Data availability

No data was used for the research described in the article.

Appendix A. Derivations and proofs

A.1. Derivation of Equation (2)

Proof. First, when $I < \pi$, the demand $D(p) \le 2\pi$ for all $p \ge 0$ and thus $D(p) = 2(1 - p/y) * I$, which further implies that $\Pi(p) = D(p) * p$ and thus $p^* = \frac{y}{2}$ and $\Pi(p^*) = \frac{yI}{2}$. Second, consider $I \ge \pi$. In this case, $\Pi(p) = 2\pi * p$ if $p \le y(1 - \pi/I)$, and $\Pi(p) = 2p(1 - p/y) * I$ if $p > y(1 - \pi/I)$. Note that $\Pi(p)$ is continuous at $p = y(1 - \pi/I)$. Again, depending on the value of I, there are two cases. One, when $I \in [\pi, 2\pi]$, $y(1 - \pi/I) \le \frac{y}{2}$. The quadratic term implies that $\Pi(p)$ is strictly increasing for all $p \in [y(1 - \pi/I), y/2]$ and strictly decreasing for $p > y/2$. Therefore, $\Pi(p)$ is maximized at $p^* = \frac{y}{2}$, which yields $\Pi(p^*) = \frac{yI}{2}$. Two, when $I > 2\pi$, we have $y(1 - \pi/I) > \frac{y}{2}$. Thus, $\Pi(p)$ is strictly increasing for $p \le y(1 - \pi/I)$ and strictly decreasing for $p > y(1 - \pi/I)$ because the quadratic term is strictly decreasing for all $p > \frac{y}{2}$. Therefore, $\Pi(p)$ is maximized at $p = y(1 - \pi/I)$ and $\Pi(p^*) = 2\pi y(1 - \pi/I)$. Third, we can combine the two cases that $I < \pi$ and $I \in [\pi, 2\pi]$ to simplify the formula. □

A.2. Proof of Proposition 1

Proof. All the skipped coefficients in this proof can be found in Section OA.7.1.

Let $\Pi(i, j)$ denote the group profit when Seller i works with Influencer j. The bilateral Nash bargaining further implies that $U_i(i, j) = \gamma\Pi(i, j)$ and $w_j(i, j) = (1 - \gamma)\Pi(i, j)$. Thus, it suffices to show that $U_1(1, 1) \ge U_1(1, 2)$ and $w_1(1, 1) \ge w_1(2, 1)$ to establish assortative matching (i.e., $k(j) = j$), which further reduce to

$$\Pi(1,1) \ge \Pi(1,2) \quad \text{and} \quad \Pi(1,1) \ge \Pi(2,1). \tag{A.1}$$

Now, we verify this condition case by case under balanced matching.

Case (i). Here, we first prove the case when the market is not crowded, that is, $\beta \ge \frac{I_1+I_2}{2}$. Then, Seller $j \in \{1, 2\}$ always charges $p_j^* = \frac{y_j}{2}$, regardless of the matching outcome. Thus, $\Pi(i, j) = \frac{y_i I_j}{2}$. Obviously, equation (A.1) holds.

Case (ii). First, given that $y_1 > y_2$, Seller 1 can always get a positive profit by attracting consumers close to the influencer who works with Seller 1. Thus, when a single seller dominates, it must be Seller 1. Second, we show that negative assortative matching (i.e., $k(j) = 3 - j$ for $j = 1, 2$) never happens. If not, suppose that Seller 1 hires Influencer 2 and sets a price p_1 and beats Seller 2 who hires Influencer 1 and sets any arbitrary price p_2. Now, consider the case where Seller 1 hires Influencer 1 and use the original pricing strategy. For all consumers, they either stay outside of the reach of Seller 2, or fall within the consumer base of Seller 2, but are attracted by Seller 1. In the latter case,

$$\begin{aligned} y_1(1 - \|x - \theta_1\|/I_1) - p_1 &> y_1(1 - \|x - \theta_1\|/I_2) - p_1 \\ &\ge y_2(1 - \|x - \theta_2\|/I_1) - p_2 > y_2(1 - \|x - \theta_2\|/I_2) - p_2 \end{aligned}$$

We used the fact that $I_1 \ge I_2$ in the first and the third inequality, and the second inequality follows from the assumed market dominance. This implies that Seller 1 and Influencer 1 will jointly deviate and form a new group, and thus negative assortative matching is sub-optimal.

Case (iii). First, we compute $\Pi(i, j)$ and start with $\Pi(1, 1)$. Note that $\beta = \pi$ implies that the share of consumers is symmetric around θ_j for Influencer j. Fix prices (p_1, p_2), the cutoff type is given by the indifference condition that $y_1(1 - s_1/I_1) - p_1 = y_2(1 - s_2/I_2) - p_2$ where $s_1 = \pi - s_2$ is the share of Seller 1 on one side, and we can solve it to get $s_1 = \frac{I_1(I_2(p_2 - p_1) + I_2(y_1 - y_2) + \pi y_2)}{y_1 I_2 + y_2 I_1}$. Thus, $\Pi(1, 1) = 2 s_1 p_1$ and $\Pi(2, 2) = 2(\pi - s_1)p_2$. By differentiating $\Pi(1, 1)$ and $\Pi(2, 2)$ over p_1 and p_2, we get:

$$\frac{\partial\Pi(1,1)}{\partial p_1} = \frac{2I_1 I_2(y_1 - y_2 + p_2 - 2p_1) + 2\pi I_1 y_2}{y_1 I_2 + y_2 I_1}$$

and

$$\frac{\partial \Pi(2,2)}{\partial p_2} = \frac{2I_1 I_2(y_2 - y_1 + p_1 - 2p_2) + 2\pi I_2 y_1}{y_1 I_2 + y_2 I_1}.$$

Solving first-order conditions yields $p_1^* = \frac{I_1 I_2(y_1-y_2)+\pi(I_2 y_1+2I_1 y_2)}{3I_1 I_2}$ and $p_2^* = \frac{I_1 I_2(y_2-y_1)+\pi(I_1 y_2+2I_2 y_1)}{3I_1 I_2}$. Furthermore, second-order conditions can be checked directly.[15] Thus, we can compute $\Pi(1,1) = \frac{a_1}{b_1}$ and $\Pi(2,2) = \frac{a_2}{b_1}$, where $b_1 = 9I_1 I_2(y_1 I_2 + y_2 I_1)$ and

$$a_1 = 2\big(I_1 I_2(y_1 - y_2) + \pi(y_1 I_2 + 2y_2 I_1)\big)^2, \quad a_2 = 2\big(I_1 I_2(y_2 - y_1) + \pi(y_2 I_1 + 2y_1 I_2)\big)^2.$$

Similarly, we calculate $\Pi(1,2)$ and $\Pi(2,1)$ in the alternative matching $k(j) = 2 - j$. Specifically, $\Pi(1,2) = \frac{a_3}{b_2}$ and $\Pi(2,1) = \frac{a_4}{b_2}$, where $b_2 = 9I_1 I_2(y_1 I_1 + y_2 I_2)$ and

$$a_3 = 2\big(I_1 I_2(y_1 - y_2) + \pi(y_1 I_1 + 2y_2 I_2)\big)^2, \quad a_4 = 2\big(I_1 I_2(y_2 - y_1) + \pi(y_2 I_2 + 2y_1 I_1)\big)^2.$$

To verify $\Pi(1,1) \ge \Pi(1,2)$, it suffices to show that $a_1 b_2 - a_3 b_1 \ge 0$. Denote $I_1 = \kappa I_2$, $y_1 = t y_2$ and $I_2 = d\pi$, where $\kappa \ge 1, t \ge 1$ and $d \ge 1$. Then, $a_1 b_2 - a_3 b_1 = 9\kappa d^5 \pi^7 y_2^3 * (g_0 + g_1(d-1) + g_2(d^2-1))$, where $g_2 = (\kappa-1)\kappa^2(t-1)^3$, $g_1 = 2\kappa(\kappa^2-1)(t-1)t$ and $g_0 = (4-t)t + \kappa^2(-1+5t-2t^2+t^3) + \kappa(4-2t+5t^2-t^3)$. Obviously, $g_1 \ge 0$ and $g_2 \ge 0$. Note that $g_0 = (\kappa-1)(\kappa t(t-1)^2 + t^2) + \kappa(t(2t-1) + (4t-1) + 4) + 4t > 0$. Similarly, to verify $\Pi(1,1) \ge \Pi(2,1)$, it suffices to show that $a_1 - a_4 \ge 0$ because $b_1 \le b_2$. Again, note that $(a_1 - a_4) \propto 2\pi(y_1 + y_2)(2I_1 + I_2)(y_1 - y_2)\big(\pi I_2 + 2I_1(I_2 - \pi)\big) \ge 0$ when $I_2 \ge \pi$.

Case (iv). First, note that the existence of a unique equilibrium under certain parameters in which the two sellers compete only on one side directly follows from Example 3. Since payoffs are continuous, there is also a unique equilibrium when $\max\{I_1/I_2, y_1/y_2\} \to 1$. Next, we establish positive assortative matching, given that a unique equilibrium exists and $y_1 \le 4y_2$. Under positive assortative matching, the profits satisfy $\Pi(k,k) = \Pi_k^C$, where Π_k^C are defined in Equation (A.2) below. Specifically, with fixed prices (p_1, p_2), the cutoff type is given by the indifference condition that $y_1(1 - s_1/I_1) - p_1 = y_2(1 - s_2/I_2) - p_2$ where $s_1 = \beta - s_2$ is the share of Seller 1 along the short arc, which implies that: $s_1 = \frac{I_1(I_2(p_2-p_1)+I_2(y_1-y_2)+\beta y_2)}{y_1 I_2 + y_2 I_1}$. Since there is no competition along the long arc, the market shares of two groups are given by $\widehat{s}_j = I_j * (1 - p_j/y_j)$. We can set the profits $\Pi_k^c = p_k(s_k + \widehat{s}_k)$ for $k \in \{1,2\}$, and solve the joint equations $\frac{\partial \pi_k}{\partial p_k} = 0$ to get $p_k = \frac{y_k(I_l^2 y_k(4y_k - y_l) + 4I_k(\beta + I_k)y_l^2 + I_l y_l(3\beta y_k + 9I_k y_k - 2I_k y_l))}{8I_2^2 y_1^2 + 19 I_1 I_2 y_1 y_2 + 8I_1^2 y_2^2}$. Furthermore, for $k \ne l$, we have:

$$\Pi_k^C = \frac{I_k y_k(2I_l y_k + I_k y_l)(I_l^2 y_k(4y_k - y_l) + 4I_k(\beta + I_k)y_l^2 + I_l y_l(3\beta y_k + 9I_k y_k - 2I_k yl))^2}{(I_l y_k + I_k y_l)(8I_l^2 y_k^2 + 19 I_k I_l y_k y_l + 8I_k^2 y_l^2)^2} \tag{A.2}$$

Similarly, under negative assortative matching, we can compute:

$$\Pi(k,l) = \frac{I_l y_k(2I_k y_k + I_l y_l)(I_k^2 y_k(4y_k - y_l) + 4I_l(\beta + I_l)y_l^2 + I_k y_l(3\beta y_k + 9I_l y_k - 2I_l y_l))^2}{(I_k y_k + I_l y_l)(8I_k^2 y_k^2 + 19 I_k I_l y_k y_l + 8I_l^2 y_l^2)^2}$$

Then, it suffices to show that $\frac{\Pi(1,1)}{\Pi(1,2)} \ge 1$ and $\frac{\Pi(1,1)}{\Pi(2,1)} \ge 1$.

Let $t = y_1/y_2$, $\kappa = I_1/I_2$ and $b = \beta/I_2$. Then, $\frac{\Pi(1,1)}{\Pi(2,1)} = \frac{A_0}{B_0}$ and $A_0 - B_0 = (t-1)(C_0 + D_0 * b + E_0 * b^2)$. We can directly verify that $C_0 > 0$, $D_0 > 0$, and E_0 is undetermined. If $E_0 \ge 0$, then it is done. If $E_0 < 0$, then $F_0 = D_0 + E_0 * (\kappa+1)/2$ is minimized because $b < (\kappa+1)/2$ (which follows from $\beta < \frac{I_1+I_2}{2}$). Again, if $F_0 \ge 0$, then it is done. Otherwise, if $F_0 < 0$, then $(A_0 - B_0)/(t-1) \ge C_0 + F_0 * (\kappa+1)/2 =: G_0$. We can directly verify that $G_0 > 0$ when $t < 4$.

Similarly, we can express $\frac{\Pi(1,1)}{\Pi(1,2)} = \frac{\widehat{A}_0}{\widehat{B}_0}$ and $\widehat{A}_0 - \widehat{B}_0 = (\kappa-1)(\widehat{C}_0 + \widehat{D}_0 * b + \widehat{E}_0 * b^2)$. Given that $\kappa \ge 1$ and $t \ge 1$, we can show that $\widehat{C}_0$, $\widehat{D}_0$ and $\widehat{E}_0$ are all positive. □

A.3. Proof of Proposition 2

Proof. Consider the equilibrium conjecture: (i) $k(j) = j$ for $j \in \{1,2\}$; and (ii) Seller 1, who works with Influencer 1, charges a price at $p_1^* = \frac{y_1}{2}$ and makes a total profit of $\Pi_1 = \frac{y_1 I_1}{2}$. Seller 2 sets $p_2^* = 0$ and effectively stays out of the market.

Next, we verify that this forms an equilibrium. First, given $p_1^* = \frac{y_1}{2}$, all potential consumers of Influencer 2 (that is, $\|x - \theta_2\| \le I_2$) buy from Seller 1 for all $p_2 \ge 0$ whenever consumers with type $x = \theta_2$ and type $\|x - \theta_2\| = I_2$ & $\|x - \theta_1\| = \beta + I_2$ both choose to buy from Seller 1. This requires $y_1 * \big(1 - \beta/I_1\big) - p_1^* \ge y_2$ and $y_1 * \big(1 - (\beta + I_2)/I_1\big) - p_1^* \ge 0$. Simplifying these two equations yields $\frac{\beta}{I_1} \le \min\{\frac{1}{2} - \frac{y_2}{y_1}, \frac{1}{2} - \frac{I_2}{I_1}\}$. Second, given the equilibrium strategies of Seller 2 and all consumers, Seller 1 always charges the monopolist price. Seller 2 cannot attract any consumer by setting $p_2^* > 0$ and it is optimal to set $p_2^* = 0$. Third, Influencer 1 gets a wage of $w_1 = \frac{(1-\gamma)y_1 I_1}{2}$ when working with Seller 1, which dominates the wage $\hat{w}_1 = \frac{(1-\gamma)y_2 I_1}{2} < w_1$ when working with Seller 2. Given Influencer 1's equilibrium matching choice, Influencer 2 can only match with Seller 2. The proof concludes. □

[15] By plugging p_j^* and s_j, we can verify that the consumer indifferent between buying from Seller 1 and 2 gets a non-negative utility.

A.4. Proofs of examples in Section 4.3

Here, we prove the examples in Section 4.3.

(i) Proof of Example 1

Proof. First, the assortative matching holds trivially with balanced matching, since the two influencers are identical for sellers. Second, we construct an equilibrium with $p_1^C \geq p_2^C \geq 0$ because $y_1 \geq y_2$. In particular, Seller 1 targets consumers sufficiently loyal to Influencer 1, and the cutoff type x^* is pinned down by $y_1(1 - \|x^* - \theta\|/I) - p_1 = y_2(1 - \|x^* - \theta\|/I) - p_2$. Note that all consumers with $\|x - \theta\| < \|x^* - \theta\|$ buy from Seller 1. Given this, Seller 2 serves consumers with $\|x - \theta\| \geq \|x^* - \theta\|$ and $\|x - \theta\| \leq \|x^{**} - \theta\|$ where $y_2(1 - \|x^{**} - \theta\|/I) - p_2 = 0$. Thus, the demand for Seller 1 is $q_1 = 2\|x^* - \theta\| = 2I * (1 - \frac{p_1 - p_2}{y_1 - y_2})$, and that for Seller 2 is $q_2 = 2(\|x^{**} - \theta\| - \|x^* - \theta\|) = 2I * (\frac{p_1 - p_2}{y_1 - y_2} - \frac{p_2}{y_2})$. Then, we can further calculate profits as $\Pi_1 = 2I * p_1 * (1 - \frac{p_1 - p_2}{y_1 - y_2})$ and $\Pi_2 = 2I * p_2 * (\frac{p_1 - p_2}{y_1 - y_2} - \frac{p_2}{y_2})$. Taking derivatives over Π_m with respect to p_m for $m = 1, 2$,

$$\frac{2I(-2p_1^C + p_2^C + y_1 - y_2)}{y_1 - y_2} = 0, \text{ and } \frac{I(-4p_2^C y_1 + 2p_1^C y_2)}{(y_1 - y_2)y_2} = 0$$

Solving these two equations yields the desired solution. Moreover, the second-order conditions are trivially true, and thus the F.O.C.s fully characterize the optimal solution. Inserting $(p_1^C, p_2^C) = (\frac{2y_1(y_1 - y_2)}{4y_1 - y_2}, \frac{y_2(y_1 - y_2)}{4y_1 - y_2})$ into the profits, we obtain all the profit functions after simple algebraic manipulations. Finally, we impose $I \leq \pi$ so that the market shares of both sellers are well defined (that is, $3Iy_1/(4y_1 - y_2) \leq \pi$). $\square$

(ii) Proof of Example 2

Proof. Again, the matching part is trivial. We construct an equilibrium with $p_1^C \geq p_2^C \geq 0$. Otherwise, if $p_1 < p_2$, Seller 2 is priced out of the market because $\beta = 0$, $y_1 = y_2$ and $I_1 \geq I_2$. First, we calculate the market shares for both sellers. When $p_1 \geq p_2$, the consumer with $x \to \theta$ buys from Seller 2 because the utility gap, compared to that of Seller 1, is equal to $(p_1 - p_2) - y * \|x - \theta\| * (I_1 - I_2)/I_1 I_2$. This further implies that there exists a cutoff type x^* such that all consumers with $\|x - \theta\| < \|x^* - \theta\|$ buy from Seller 2, that is, $(p_1 - p_2) = y * \|x - \theta\| * (I_1 - I_2)/I_1 I_2$. Solving it yields $\|x^* - \theta\| = \frac{I_1 I_2 (p_1 - p_2)}{y(I_1 - I_2)}$. Meanwhile, consumers with $\|x - \theta\| > \|x^* - \theta\|$ buy from Seller 1 when it generates a positive utility, which gives a second cutoff type x^{**} given by $y\left(1 - \|x^{**} - \theta\|/I_1\right) - p_1 = 0$.

Second, we compute the profits as $\Pi_1 = p_1 * \left((1 - p_1/y)I_1 - \frac{I_1 I_2 (p_1 - p_2)}{y(I_1 - I_2)}\right)$ and $\Pi_2 = p_2 * \frac{I_1 I_2 (p_1 - p_2)}{y(I_1 - I_2)}$. Obviously, Π_j is concave in p_j for $j = 1, 2$. Thus, the equilibrium is pinned down by first-order conditions that $\frac{I_1}{y(I_1 - I_2)}\left(I_2(p_2 - y) + I_1(y - 2p_1)\right) = 0$ and $\frac{I_1 I_2}{y(I_1 - I_2)}(p_1 - 2p_2) = 0$. By solving these two equations, we can compute (p_1^C, p_2^C) and the group profits by simple algebra. Note that market shares are well defined (that is, $\|x^{**} - \theta\| \leq \pi$) when $I_1 \leq \pi$. $\square$

(iii) Proof of Example 3

Proof. Here, we only construct the equilibrium price. First, we calculate the group profits Π_k^C for $k = 1, 2$. Fix p_2 and consider seller 1's profit. First, if p_1 and p_2 are close and not too small (to be discussed shortly), we expect that two sellers share consumers along the short arc between θ_1 and θ_2, and the cutoff type x^* satisfies $p_2 - p_1 = \left(\|x^* - \theta_1\| - \|x^* - \theta_2\|\right) y/I$. Define $s_k = \|x^* - \theta_k\|$ for $j = 1, 2$. All consumers are served along the short arc between θ_1 and θ_2 only when $I * (1 - p_1/y) + I * (1 - p_2/y) \geq \|\theta_1 - \theta_2\|$ is satisfied or, equivalently, $p_1 + p_2 \leq (2 - \beta/I) * y$. When these conditions are verified, $s_1 + s_2 = \beta$ holds. Thus, we have $s_k = \frac{1}{2}(\beta + (p_{3-k} - p_k) * I/y)$ and $\Pi_1^C = p_1 * (s_1 + I(1 - p_1/y))$. However, when p_1 is sufficiently low compared to p_2, then $s_1 > \beta$ may occur, which implies $\|x - \theta_1\| - \|x - \theta_2\| = \beta$. In particular, when $p_1 \leq p_2 - y\beta/I$, Seller 1 beats Seller 2 and takes the entire market, and thus $\Pi_1^C = 2p_1 * (1 - p_1/y)I$. In contrast, when $p_1 \geq p_2 + y\beta/I$, Seller 1 loses all consumers, and thus $\Pi_1^C = 0$. Note that there are two discontinuity points for Π_1^C.

Second, we solve the equilibrium price. Obviously, it is sub-optimal for Seller 1 to set $p_1 \geq p_2 + y\beta/I$, which leads to a zero profit. Then, we start with the case in which p_1 and p_2 are sufficiently close. We can plug in the formula for s_j and derive first-order conditions: $\frac{y\beta + I(p_2 - 6p_1 + 2y)}{2y} = 0$ and $\frac{y\beta + I(p_1 - 6p_2 + 2y)}{2y} = 0$. Solving these two equations yields $p_1^* = p_2^* = \frac{y(2I + \beta)}{5I}$, and we can further plug p_j^* into Π_k^C to solve $\Pi_1^C = \frac{3y(2I + \beta)^2}{50I}$, as long as the condition $p_2^* - y\beta/I < p_1^* < p_2^* + y\beta/I$ is satisfied.

However, we also need to verify $p_1^* + p_2^* \leq (2 - \beta/I) * y$, which reduces to $\beta \leq 6I/7$. When $\beta > \frac{6}{7}I$, we define $\widehat{p}_1^* = \widehat{p}_2^* = (1 - \beta/2I) * y$. Note that given $p_2 = \widehat{p}_2^*$, the term $p_1 * \left((1 - p_1/y) * I + s_1\right)$ is strictly increasing in p_1 for $p_1 \leq p_1^*$. Thus, the first (influencer-seller) group has no incentive to deviate downward. Meanwhile, for $p_1 > \widehat{p}_1^*$, the term $p_1 * (1 - p_1/y) * I$ is strictly decreasing for $p_1 > \widehat{p}_1^*$ since $\widehat{p}_1^* \geq \frac{y}{2}$. This implies that the first group does not have an incentive to deviate upward. Thus, $(\widehat{p}_1^*, \widehat{p}_2^*)$ constitutes an equilibrium when $\frac{6}{7}I < \beta < I$, as long as no seller is priced out of the market.

Third, we need to ensure that, for p_1 and p_2 sufficiently close, Seller 1 has no incentive to undercut Seller 2 and force Seller 2 out of the market. In this deviation, Seller 1 only needs to set $p_1 = p_2^* - y\beta/I$, which leads to a profit of $\widehat{\Pi}_1 = 2I * (p_2^* - y\beta/I) *$

$(1-(p_2^*-y\beta/I)/y)$. This is also the most profitable deviation, since $\Pi_1 = 2I(1-p_1/y)p_1$ is strictly decreasing for all $p_1 \le p_2^* - y\beta/I$. Simplifying $\widehat{\Pi}_1 \le \Pi_1^C$ leads to the assumed condition on β_0. $\square$

A.5. *Proof of Proposition 3*

Proof. All the coefficients skipped can be found in Section OA.7.1.

Part (i). The positive assortative matching result follows from Proposition 1. Now, we show that $\Pi_1^C \ge \Pi_2^C$. Recall that $t = y_1/y_2$, $\kappa = I_1/I_2$, and $b = \beta/I_2$. Then: $\frac{\Pi_1^C}{\Pi_2^C} = \frac{A_1}{B_1}$, and it suffices to show that $A_1 \ge B_1$. Specifically, $A_1 - B_1 = (C_1 + D_1 * b + E_1 * b^2) * F_1$. Since $\kappa \ge 1$ and $t \ge 1$, we can verify that $C_1 \ge 0$, $D_1 \ge 0$ and $F_1 > 0$. If $E_1 \ge 0$, then it is done. Otherwise, if $E_1 < 0$, then we can verify that $D_1 + E_1 * \frac{(\kappa+1)}{2} \ge 0$ because $b \le (\kappa+1)/2$ (or equivalently, $\beta \le \frac{I_1+I_2}{2}$).

We now check the derivatives in the proposition one by one.

Derivatives of $\frac{\Pi_1^C}{\Pi_2^C}$. By Equation (A.2), we get: $\frac{\partial\left(\Pi_1^C/\Pi_2^C\right)}{\partial\beta} = \frac{A_2}{B_2 I_2}$, $\frac{\partial\left(\Pi_1^C/\Pi_2^C\right)}{\partial(y_1/y_2)} = -\frac{A_3}{B_3}$ and $\frac{\partial\left(\Pi_1^C/\Pi_2^C\right)}{\partial(I_1/I_2)} = -\frac{A_4}{B_3}$. First, we can show that $A_2 \ge 0$ and $B_2 < 0$. Therefore, $\frac{\partial\left(\Pi_1^C/\Pi_2^C\right)}{\partial\beta} < 0$. Second, we can show that $A_3 = (C_3 + D_3 * b + E_3 * b^2) * F_3$ with $C_3 > 0$, $D_3 \ge -8\kappa^5 z - 8t^5$ and $E_3 < 0$. Hence, $A_3/F_3 \ge C_3 + D_3 * (\kappa+1)/2 + E_3 * (\kappa+1)^2/4 =: G_1 > 0$ and $B_3 = B_2 * (2\kappa + t) < 0$. Together, this implies: $\frac{\partial\left(\Pi_1^C/\Pi_2^C\right)}{\partial(y_1/y_2)} > 0$. Third, we can show that $A_4 > 0$, together with $B_3 < 0$, $\frac{\partial\left(\Pi_1^C/\Pi_2^C\right)}{\partial(I_1/I_2)} > 0$.

Derivatives of $\frac{P_1^C}{P_2^C}$. By Equation (A.2), $\frac{\partial\left(P_1^C/P_2^C\right)}{\partial\beta} = \frac{A_5}{B_5 I_2}$, $\frac{\partial\left(P_1^C/P_2^C\right)}{\partial(y_1/y_2)} = \frac{A_6}{B_5}$ and $\frac{\partial\left(P_1^C/P_2^C\right)}{\partial(I_1/I_2)} = \frac{A_7}{B_5}$. First, we can verify that $A_5 \le 0$ and $B_5 > 0$. Therefore, $\frac{\partial\left(P_1^C/P_2^C\right)}{\partial\beta} \le 0$. Second, we can show that $A_6 > 0$. Hence, $\frac{\partial\left(P_1^C/P_2^C\right)}{\partial(y_1/y_2)} > 0$. Third, we can show that $A_7 = (C_7 + D_7 * b + E_7 * b^2) * F_7$, where $C_7 > 0$, $D_7 > 0$ and $E_7 < 0$. Note that $D_7 + E_7 * (\kappa+1)/2 = \frac{1}{2}\left(32\kappa^3 + \kappa^2(25t+48)t + \kappa(48t+25)t^2 + 32t^4\right) > 0$. Therefore, $A_7 > 0$ and $\frac{\partial\left(P_1^C/P_2^C\right)}{\partial(I_1/I_2)} > 0$.

Derivatives of $\widetilde{W}$. The total welfare is: $W = \sum_{k=1}^2 \hat{s}_k y_k(1-\hat{s}_k/(2I_k)) + \sum_{k=1}^2 s_k y_k(1-s_k/(2I_k))$, where $s_k = \frac{I_k(I_l(p_l-p_k)+I_l(y_k-y_l)+\beta y_l)}{y_k I_l + y_l I_k}$ and $\hat{s}_k = I_k * (1-p_k/y_k)$. Furthermore, we fix y_2 and I_2 when analyzing the comparative statics over y_1/y_2 and I_1/I_2. Define $\widetilde{W} = \frac{W}{y_2 I_2}$. Then, we get: $\frac{\partial\widetilde{W}}{\partial\beta} = \frac{A_8}{B_8 I_2}$, $\frac{\partial\widetilde{W}}{\partial(I_1/I_2)} = \frac{A_9}{B_9}$, and $\frac{\partial\widetilde{W}}{\partial(y_1/y_2)} = \frac{A_{10}}{B_9}$.

First, we can show that $A_8 = t * (C_8 + D_8 * b)$ and $B_8 > 0$. Here, $C_8 > 0$ and $D_8 < 0$. However, $C_8 + D_8 * (\kappa+1)/2 = G_8 > 0$. Therefore, $\frac{\partial\left(\widetilde{W}\right)}{\partial\beta} > 0$. Second, we can show that $A_9 = t * (C_9 + D_9 * b + E_9 * b^2)$ and $B_9 > 0$. Here, $C_9 > 0$ and $E_9 > 0$. Note that in D_9, only the term $32\kappa^6(-32+19t)$ can be negative. However, it can be further decomposed as $32\kappa^6 * (-13) < 0$ plus $32\kappa^6 * (t-1) > 0$. Now, we can combine it with a term of the form $\kappa^6 b^2$ from E_9 and a term of κ^6 (or κ^7, κ^8) from C_9 to form a squared term. Consequently, we can show that $A_9 > 0$. Therefore, $\frac{\partial\widetilde{W}}{\partial(I_1/I_2)} > 0$. Third, we can show that $A_{10} = j * (C_{10} + b * (D_{10} + E_{10} * b))$, where $C_{10} > 0$, $D_{10} > 0$ and $E_{10} < 0$. Since $\beta < \frac{I_1+I_2}{2}$, $d < \frac{\kappa+1}{2}$. Also note that $D_{10} + E_{10} * (\kappa+1)/2 > 0$. Therefore, $A_{10} > 0$ and thus $\frac{\partial\widetilde{W}}{\partial(y_1/y_2)} > 0$.

Derivatives of $\widetilde{CS}$. The consumer welfare is: $CS = \sum_{k=1}^2 (y_k\hat{s}_k/I_k) * t_k + \sum_{k=1}^2 (y_k\hat{s}_k/I_k - y_k s_k/(2I_k)) * s_k$. Furthermore, we fix y_2 and I_2 when analyzing the comparative statics over y_1/y_2 and I_1/I_2. Define $\widetilde{CS} = \frac{CS}{y_2 I_2}$. Then, we get: $\frac{\partial\widetilde{CS}}{\partial\beta} = \frac{A_{11}}{B_8 I_2}$, $\frac{\partial\widetilde{CS}}{\partial(I_1/I_2)} = \frac{A_{12}}{B_9}$, and $\frac{\partial\widetilde{CS}}{\partial(y_1/y_2)} = \frac{A_{13}}{B_9}$.

First, we can show that $A_{11} = t * (C_{11} + D_{11} * b)$ and $B_8 > 0$. Here, $C_{11} > 0$ and $D_{11} < 0$. Therefore, when $\beta < -\frac{C_{11}}{D_{11}}$, $\frac{\partial\widetilde{CS}}{\partial\beta} > 0$ and vice versa. Note that for $t = \kappa = 1$, $\frac{C_{11}}{D_{11}} \approx 0.44$. Second, we can show that $A_{12} = t * (C_{12} + D_{12} * b + E_{12} * b^2) > 0$. Here, $C_{12} > 0$ and $E_{12} > 0$. Note that in D_{12}, we have three terms which might be negative, including: $32\kappa^6(-16+t)$, $4\kappa^5 t(-736+183t)$ and $8\kappa^4 t^2(-802+429t)$. Then, we can use the trick to make perfect square terms. In this way, we can prove that $A_{12} > 0$, which implies that $\frac{\partial\widetilde{CS}}{\partial(I_1/I_2)} > 0$. Third, we can show that $A_{13} = \kappa * (C_{13} + b * (D_{13} + E_{13} * b))$, where $C_{10} > 0$, $D_{13} > 0$ and $E_{13} < 0$. Note that if $D_{13} + E_{13} * (\kappa+1)/2 = G_{13}$ is positive, then it is done. If not, then A_{10} is minimized at $d = (\kappa+1)/2$, that is, $A_{12}/\kappa \ge C_{13} + D_{13} * (\kappa+1)/2 + E_{13}^{(}(\kappa+1)/2)^2 = G_{13} > 0$ and thus $\frac{\partial\widetilde{CS}}{\partial(y_1/y_2)} > 0$. $\square$

A.6. Proof of Proposition 4

Proof. The proof consists of two steps. First, we verify equilibrium matching and calculate the profits for both groups. When $\frac{I_1+I_2}{2} < \beta$, Proposition 1(i) implies $k(j) = j$ for $j \in \{1,2\}$. Then, monopolist pricing implies that $\Pi_j = \frac{y_j I_j}{2}$ for Seller j. On the other hand, as $I_2 \to \infty$, it approaches the Bertrand price competition (that is, $p_1 = y_1 - y_2$ and $y_2 = 0$) and thus $\Pi_1 = 2\pi(y_1 - y_2)$ and $\Pi_2 = 0$, regardless of $k(j)$. This also implies $k(j) = j$ for $j \in \{1,2\}$.

Second, we show the non-monotonicities for influencers' wages and income gap. When $I_1 < \beta$, we have $\frac{I_1+I_2}{2} < \beta$. Then, Nash bargaining implies $w_j = \frac{(1-\gamma)y_j I_j}{2}$. We can easily show that $w_1 + w_2 = \frac{(1-\gamma)(y_1 I_1 + y_2 I_2)}{2}$ and $w_1 - w_2 = \frac{(1-\gamma)(y_1 I_1 - y_2 I_2)}{2}$. Since $\Delta I_1 \geq \Delta I_2 > 0$, we have $\Delta(w_1 + w_2) > 0$. Furthermore, $\Delta(y_1 I_1 - y_2 I_2) = y_1 \Delta(I_1 - I_2) + (y_1 - y_2)\Delta I_2$ and thus $\Delta(w_1 - w_2) > 0$ also holds. Finally, given that $(I_1 - I_2)y_1 > 4\pi(y_1 - y_2)$, $w_1 + w_2 = w_1 - w_2 = 2\pi(1-\gamma)(y_1 - y_2)$ as $\Delta I_2 \to \infty$, which is smaller than $\frac{(1-\gamma)(y_1 I_1 - y_2 I_2)}{2}$. This implies that $(w_1 \pm w_2)$ indeed decreases for some ΔI_2 sufficiently large. □

A.7. Proof of Proposition 5

Proof. The proof consists of two parts.

Part (i). Calculate the equilibrium profits before differentiation.

We construct an equilibrium such that the equilibrium prices are characterized by the F.O.C.s of the profit functions. We first derive equilibrium prices and profits using the F.O.C.s and then verify the equilibrium. Note that matching is trivial because $I_1 = I_2$.

First, consider a pair of (p_1, p_2) such that no seller dominates (to be verified later). Then we have an indifference condition along the short arc whose length is β, that is, $y_1(1 - s_1/I) - p_1 = y_2(1 - s_2/I) - p_2$, where s_j is Seller k's market share along the short arc, whose identity is used to differentiate the two identical sellers. Note that s_1 can be larger than β or negative, depending on the prices p_k. By imposing $s_2 = \beta - s_1$, we can solve $s_1 = \frac{I(p_2-p_1)+I(y_1-y_2)+\beta y_2}{y_1+y_2}$ and $s_2 = \beta - s_1$. We can further set up the profit $\Pi_k = (s_k + (1 - p_k/y)I) * p_k$ and get the FOC's as below:

$$\frac{\beta y_k y_l + I(y_k p_l + 2y_k^2 - 4p_k y_k - 2p_k y_l)}{y_k(y_k + y_l)} = 0$$

Solving the FOC's yields $p_k^* = \frac{y_k(\beta y_l(3y_k+4y_l)+2I(2y_k^2+4y_k y_l+y_l^2))}{I(8y_k^2+19y_k y_l+8y_l^2)}$ and

$$\Pi_k = \frac{y_k(2y_k + y_l)(\beta y_l(3y_k + 4y_l) + 2I(2y_k^2 + 4y_k y_l + y_l^2))^2}{I(y_k + y_l)(8y_k^2 + 19y_k y_l + 8y_l^2)^2}.$$

Second, we verify the equilibrium payoffs shown above under the assumed conditions, including: (1) All consumers are served along the short arc, that is, $I(1 - p_1^*/y_1) + I(1 - p_2^*/y_2) \geq \beta$, which yields

$$\beta \leq \frac{2I(5y_1^2 + 11y_1 y_2 + 5y_2^2)}{12y_1^2 + 25y_1 y_2 + 12y_2^2} =: cI$$

Note that $I \leq \frac{\pi}{2}$ implies no competition along the long arc; and (2) Sellers have no incentive to undercut and force competitors out of the product market. To force Seller 2 to quit, Seller 1 needs to induce the farthest consumer of Seller 2 to switch by setting $\hat{p}_1 = y_1(1-(\beta+\hat{s}_2)/I)$, where $\hat{s}_2 = I(1-p_2^*/y_2)$. Meanwhile, to force out Seller 1, Seller 2 must set a price $\hat{p}_2 = y_2(1-\beta/I)-(y_1-p_1^*)$. These deviations lead to profits given by $\hat{\Pi}_k = 2I_j\hat{p}_k(1-\hat{p}_k/y_k)$. We can verify $\hat{p}_k \leq \frac{y_k}{2}$, which constitutes the most profitable deviation because $2Ip_k(1-p_k/y_k)$ is strictly decreasing when $p_k < \frac{y_k}{2}$. Thus, the nonexistence of profitable deviations reduces to $\Pi_k \geq \hat{\Pi}_k$ for $j = 1,2$. In particular,

$$\Pi_1 - \hat{\Pi}_1 = -\frac{t y_2 I h_1(b,t)}{(1+t)(8+19t+8t^2)^2} \quad \text{and} \quad \Pi_2 - \hat{\Pi}_2 = -\frac{y I_2 h_2(b,t)}{(1+t)(8+19t+8t^2)^2}$$

where

$$\begin{aligned} h_1(b,t) &= 4(1+2t)(-7-22t-18t^2-3t^3+t^4) + 4b(4+39t+110t^2+124t^3+58t^4+8t^5) \\ &\quad + b^2(144+696t+1209t^2+914t^3+288t^4+32t^5) \\ h_2(b,t) &= b^2(128+608t+1108t^2+967t^3+406t^4+66t^5) + (8+8t-55t^2-98t^3-17t^4+40t^5+16t^6) \\ &\quad \times(4+2t) + 2b(-64-216t-117t^2+322t^3+483t^4+238t^5+40t^6) \end{aligned}$$

Note that $t \geq 1$ and thus $h_k(b,t)$ is strictly increasing in b for $b > 0$. Therefore

$$h_1(b,t) \geq h_1(a,t) = \frac{1}{100}(-1024 - 3456t - 719t^2 + 7506t^3 + 7552t^4 + 2048t^5)$$

$$h_2(b,t) \geq h_2(a,t) = \frac{1}{100}(512 - 2688t - 17448t^2 - 22177t^3 + 6234t^4 + 27474t^5 + 16800t^6 + 3200t^7)$$

Obviously, $h_k(a,t) > 0$ because $t \geq 1$. Thus, the pricing strategies and profits characterized by the F.O.C.s are optimal.

Part (ii). Verify the optimality of Sellers' differentiation choices.

We first verify statement (i). We measure the incentive to differentiate by the net return, that is, $R_k(\beta) = \frac{y_k I}{2} - \Pi_k(\beta|y_1, y_2, I) - f_E$, where $\frac{y_k I}{2}$ is the profit when $\beta^* \geq I$. Then:

$$R_1 - R_2 = \frac{(3-2b)(1-2b)I(t-1)ty2}{2(8+19t+8t^2)}$$

Thus, $R_1 - R_2 \leq 0$ if and only if $\frac{1}{2} \leq b \leq \frac{3}{2}$, which further implies that $R_1 \leq R_2$ for $\beta \in [I/2, cI]$ and $R_1 < R_2$ for $\beta \in [aI, I/2]$.

We then verify Statement (ii). By the continuity and monotonicity of $H_1(b,t)$ and $H_2(b,t)$, which increases strictly with b, $R(\beta) = \max\{R_1(\beta), R_2(\beta)\}$ increases strictly with β and is continuous at $\beta = \frac{I}{2}$. Then, the condition $\frac{y_1 I}{2} - H_1(c,t) * y_1 I < f_E < \frac{y_2 I}{2} - H_2(a,t) * y_2 I$ is equivalent to $R(aI) > 0$ and $R(cI) < 0$, which further implies that there is a unique $\widetilde{\beta} \in (aI, cI)$ such that $R(\widetilde{\beta}) = 0$. Thus, for $\beta \in [aI, \widetilde{\beta})$, one seller engages in horizontal differentiation; otherwise, for $\beta \in [\widetilde{\beta}, bI]$, no seller differentiates the product horizontally.

Finally, we verify statement (iii). Denote $\widetilde{b} = \widetilde{\beta}/I$. We first verify statement (iii) for $\beta \in [\frac{I}{2}, cI]$. By (i), Seller 2 chooses horizontal differentiation. It suffices to verify $\frac{\partial \widetilde{\beta}}{\partial t} > 0$. To see it, we use implicit function differentiation for $R_k(\widetilde{\beta}, t) = \frac{y_k I}{2} - H_k(\widetilde{b}, t) * y_k I - f_E = 0$, that is,

$$\frac{\partial \widetilde{\beta}}{\partial t} = -I * \frac{\partial R_k/\partial t}{\partial R_k/\partial \widetilde{\beta}} = -\frac{\partial H_2/\partial t}{\partial H_2/\partial \widetilde{b}}$$

Now, the result follows from Equation (4). Similarly, when Seller 1 chooses to differentiate, then $\frac{\partial \tilde{\beta}}{\partial \tau}$ can be proved similarly once we redefine $R_1(\tilde{\beta}, \tau) = \frac{y_1 I}{2} - H_2(\tilde{b}, \tau) y_1 I - f_E$ and apply the implicit function differentiation. Therefore, it suffices to verify Equation (4).

Verification of Equation (4). Obviously, $\partial H_2/\partial \widetilde{b} > 0$. Next, we verify $\partial H_2/\partial t < 0$. Note that $\frac{\partial H_2}{\partial t} = \frac{\left(\sum_{i=0}^{6} m_i t^i\right)}{(1+t)^2(8+19t+8t^2)^3}$, where $m_0 = -320 + 384b$, $m_1 = -2000 + 2032b + 288b^2$, $m_2 = -4960 + 4064b + 1512b^2$, $m_3 = -6128 + 3600b + 3157b^2$, $m_4 = -3888 + 1064b + 3224b^2$, $m_5 = -1188 - 236b + 1576b^2$ and $m_6 = -136 - 128b + 288b^2$. It is easy to check that $m_i(b) < 0$ for $1 \leq i \leq 6$ for $b \in [0, c] \subseteq [0, 6/7]$. Furthermore, since $t \geq 1$, $\sum_{i=0}^{6} m_i(b) t^i < \sum_{i=0}^{6} m_i(b) = 245(-76 + 44b + 41b^2) < 0$ for all $b \in [0, 6/7]$.

The proof concludes. □

A.8. Proof of Proposition 6

Proof. The proof consists of two parts.

Step (1). Compute $\Pi_{H,H}$, $\Pi_{L,L}$, $\Pi^j_{L,H}$ and $\Pi^j_{H,L}$ for $j = 1, 2$.

First, by Example 3 and Condition (i), we have $\Pi_{H,H} = yA(\beta)$ and $\Pi_{L,L} = A(\beta)$ for $j = 1, 2$, where $A(\beta) = \frac{3}{50}(2I + \beta)^2$. Second, we calculate the profits under asymmetric investments in quality, say (H, L), which means that only Group 1 chooses high quality. Given Condition (ii) assumed, Group 2 can still attract some consumers even if Group 1 sets a price at $p_1 = 0$. Now, denote by p_j the price charged by Group $j = 1, 2$. For Group 1, the size of the consumers served along the long arc is $y_1(1 - \|x - \theta_1\|) - p_1 \geq 0$ or equivalently $(1 - p_1/y)$. Meanwhile, along the short arc, the cutoff type of the consumer x^* satisfies $y(1 - \|x^* - \theta_1\|) - p_1 = 1 - (\beta - \|x^* - \theta_2\|) - p_2$, which yields $s_1 := \|x^* - \theta_1\| = \frac{p_2 - p_1 + (y-1) + \beta}{y+1}$ and $s_2 = \beta - s_1$. We can also calculate the group profits as $\Pi^j_{H,L} = (1 - p_j/y_j) * p_j + p_1 s_j$ for $j = 1, 2$. Using the first-order conditions, we get $p_1^* = \frac{2(1+2\beta)y + (8+3\beta)y^2 + 4y^3}{8+19y+8y^2}$ and $p_2^* = \frac{4 + (8+3\beta)y + 2(1+2\beta)y^2}{8+19y+8y^2}$. Meanwhile, we need to ensure that the cut-off type x^* gets a non-negative utility, that is, $y(1 - \|x^* - \theta\|) - p_1^* \geq 0$, which reduces to $\frac{y\left(2(5-6\beta)(1+y^2) + (22-25\beta)y\right)}{(1+y)(8+19y+8y^2)} \geq 0$, which trivially holds under condition i). Furthermore, we can directly verify that the second-order conditions are satisfied. Thus, we can compute $\Pi^1_{H,L} = \frac{y(1+2y)\left(2+8y+4y^2+\beta(4+3y)\right)^2}{(1+y)\left(8+19y+8y^2\right)^2}$ and $\Pi^2_{H,L} = \frac{y(2+y)\left(4+8y+2y^2+\beta(3y+4y^2)\right)^2}{(1+y)\left(8+19y+8y^2\right)^2}$. Finally, note that $\Pi^j_{H,L} = \Pi^i_{L,H}$ for $i \neq j$.

Step (2). Nash equilibrium construction

We first verify the equilibrium under the following two properties, including: (1) $V_1(\beta, y) > V_2(\beta, y)$; and (2) both $V_1(\beta, y)$ and $V_2(\beta, y)$ strictly increase in β.

First, by condition (iii), $V_1(\beta_0, y) < F_V < V_2(5/6, y)$ and property (1), we have

$$V_1(5/6, y) > V_2(5/6, y) > F_V > V_1(\beta_0, y) > V_2(\beta, y)$$

which, together with property (2), the strict monotonicity of $V_1(\beta, y)$ and $V_2(\beta, y)$, implies that there exists $\underline{\beta}, \overline{\beta} \in (\beta_0, 5/6)$ such that $F_V = V_1(\underline{\beta}, y) = V_2(\overline{\beta}, y)$ and $\overline{\beta} > \underline{\beta}$.

To summarize: (a) For $\beta \geq \overline{\beta}$, $V_2(\beta, y) \geq F_V$, or equivalently, $\Pi_{H,H} \geq F_V + \Pi^1_{L,H}$. Therefore, since Influencer 2 chooses high quality, it is optimal for Influencer 1 to invest. By symmetry, Influencer 2 also chooses to invest, and thus (H, H) is a Nash equilibrium.

(b) For $\underline{\beta} \leq \beta < \overline{\beta}$, we have both $V_1(\beta, y) \geq F_V$ and $V_2(\beta, y) < F_V$, that is, $\Pi^1_{H,L} - \Pi_{L,L} \geq F_V$ and $\Pi_{H,H} - \Pi^2_{H,L} < F_V$ since $\Pi^2_{H,L} = \Pi^1_{L,H}$. These two conditions read as follows. One, given that Influencer 2 chooses low quality, it is optimal for Influencer 1 to invest. Two, given that Influencer 1 chooses high quality, it is optimal for Influencer 2 to choose low quality. Thus, (H, L) is a Nash equilibrium, so is (L, H) by symmetry.

(c) For $\beta < \underline{\beta}$, $V_1(\beta, y) < F_V$, or equivalently, $\Pi^1_{H,L} - \Pi_{L,L} < F_V$, which implies that even if Influencer 2 does not invest, it is optimal for Influencer 1 not to invest. By symmetry, (L, L) is a Nash equilibrium.

Verification of Properties (1) and (2) for $V_1(\beta, y)$ and $V_2(\beta, y)$. To this end, we write down the formulas and check them one by one.

First, with y fixed, $V_1(\beta, y)$ and $V_2(\beta, y)$ are strictly increasing in β. Note that

$$V_1(\beta, y) = \frac{(y-1) * \left(1600y^5 + 32M_1y^4 + 4M_2y^3 + 25M_3y^2 + 8M_4y + 192(2+\beta)^2\right)}{50(y+1)(8+19y+8y^2)}$$

$$V_2(\beta, y) = \frac{(y-1) * \left(1600 + 32M_1y + 4M_2y^2 + 25M_3y^3 + 8M_4y^4 + 192(2+\beta)^2y^5\right)}{50(y+1)(8+19y+8y^2)}$$

where $M_1 = 251 + 51\beta - 6\beta^2$, $M_2 = 3704 + 1604\beta - 99\beta^2$, $M_3 = 500 + 340\beta + 3\beta^2$ and $M_4 = 623 + 548\beta + 62\beta^2$. After simple algebra, we can show that all quadratic terms $M_j (j = 1, 2, 3, 4)$ are positive and strictly increasing for $\beta \in (\beta_0, 5/6)$, which verifies the monotonicity of both $V_1(\beta, y)$ and $V_2(\beta, y)$.

Second, with y fixed, $V_1(\beta, y) > V_2(\beta, y)$ for all $\beta \in (\beta_0, 5/6)$.

$$V_1(\beta, y) - V_2(\beta, y) = \frac{(y-1)^2 * (64 * M_5 + 40 * M_6y + M_7 * y^2 + 40 * M_6y^3 + 64 * M_5y^4)}{50(y+1)(8+19y+8y^2)}$$

where $M_5 = 13 - 12\beta - 3\beta^2$, $M_6 = 97 - 88\beta - 22\beta^2$, and $M_7 = 6196 - 5604\beta - 1351\beta^2$.

To see that $M_5 > 0$, note that there are two solutions $\beta_1 \approx -4.89$ and $\beta_2 \approx 0.89$. Hence, $M_5 > 0$ for all $\beta \in (-4.89, 0.89)$, and thus $M_5 > 0$ for all $\beta \in (\beta_0, 5/6)$. We can prove $M_6 > 0$ and $M_7 > 0$ in a similar way. □

A.9. Proof of Proposition 7

Proof. We construct an equilibrium such that: (1) $k(j) = 2 - j$; (2) $(p^d_1)^* = (c_2 - c_1)y$, $(p^d_2)^* = 0$, and consumers buy from Seller 1; and (3) $(p^i_2)^* = \frac{c_1 y}{2}$ and $(p^i_1)^* > (p^i_2)^*$.

First, we consider Seller 2 working with Influencer j. When $p^d_1 = (c_2 - c_1)y$ and all consumers buy from Seller 1, Seller 2 is forced out of the direct sales market because $(1 - c_1)y - p^d_1 \geq (1 - c_2)y$, which means that all consumers prefer Seller 1 even $p^d_2 = 0$. Then Seller 2 makes a profit of $\Pi^i_2 = 2s_2p^i_2$ by charging p^i_2 in influencer-intermediated sales, where $2s_2$ is the market share of Seller 2, which is further defined by the indifference condition $y(1 - s_2/I_j) - p^i_2 = (1 - c_2)y$. Solving it yields $(p^i_2)^* = \frac{c_2 y}{2}$, $(p^d_2)^* = 0$ and $\Pi^i_2 = \frac{1}{2}c^2_2 I_j y$.

Next, we consider Seller 1 working with Influencer $i \neq j$. Given Seller 2's pricing strategies (p^d_2, p^i_2), Seller 1 makes a total profit $\Pi_1 = \Pi^d_1 + \Pi^i_1$, where $\Pi^d_1 = p^d_2(2\pi - 2s_2 - 2s_1)$ and $\Pi^i_2 = \gamma p^i_1 * 2s_1$. Here, $2s_1$ is the market share of Seller 1 in influencer-intermediated sales, which is defined by $y(1 - s_1/I_{i\neq j}) - p^i_1 = (1 - c_1)y - p^d_1$. Note that $p^d_1 \leq (c_2 - c_1)y$; otherwise, Seller 1 is forced out of the direct sales market by Seller 2. Furthermore, $\gamma p^i_1 \geq p^d_1$, which means that Seller 1 gets more from selling through influencers compared to direct sales. Then, we get $\frac{d\Pi_1}{dp^d_1} = (2\pi - 2s_2 - 2s_2) + \frac{2I_2}{y}(\gamma p^i_1 - p^d_1) \geq 0$, and thus $(p^d_1)^* = (c_2 - c_1)y$. Now, Seller 1 faces an identical problem as that of Seller 2 under the additional constraint $\gamma p^i_1 \geq p^d_1$. Thus, $(p^i_1)^* = \frac{c_2 y}{2}$ if $\gamma p^i_1 \geq p^d_1$ holds; otherwise, $(p^i_1)^* = (p^d_1)^*/\gamma$ since Π^i_1 is a quadratic function of p^i_1. Obviously, in the latter case, $\Pi^i_1 < \frac{1}{2}c^2_2 I_i y$, whenever $\gamma * \frac{c_2 y}{2} < (c_2 - c_1)y$, which is $\frac{\gamma c_2}{2} < (c_2 - c_1)$. Also note that $\beta > \frac{c_2(I_1 + I_2)}{2}$ ensures that there is no overlap in the consumer base in influencer-intermediated sales.

Finally, we consider the incentive for influencers. Influencer 1 earns a salary of $w_1 = \frac{\gamma}{2}c^2_2 I_j y$ when working with Seller 2, which strictly dominates $\hat{w}_1 < \frac{\gamma}{2}c^2_2 I_j y$ when working with Seller 1. Thus, $k(j) = 2 - j$. The proof concludes. □

A.10. Proof of Proposition 8

Proof. We use the superscript $l \in \{a, b\}$ to index "after" and "before" power acquisition. Let SW denote total welfare. Define $\Delta w_j = w^a_j - w^b_j$, and $\Delta SW = SW^a - SW^b$. Note that Influencer j invests if and only if $C_P < \Delta w_j$ holds. Thus, under-investment arises if $\Delta SW > \#\{j : C_P > \Delta w_j\} * C_P$, and over-investment arises if $\Delta SW < \#\{j : C_P > \Delta w_j\} * C_P$.

Case (i). The condition that $kI < \frac{\pi}{2}$ guarantees the feasibility of monopolist pricing (i.e., $p^*_1 = p^*_2 = \frac{y}{2}$), and thus $\Pi^b_j = \frac{yI}{2}$ and $\Pi^a_j = \frac{kyI}{2}$. Note that $w^l_j = (1 - \gamma)\Pi^l_j$ for $j \in \{1, 2\}$ and $l \in \{a, b\}$. Therefore, Influencer j will invest iff $C_P \leq w^a_j - w^b_j = \frac{(1-\gamma)(k-1)yI}{2}$.

Meanwhile, note that the total welfare, before power acquisition, is $SW^b = 2\int_{y(1-\|x-\theta_1\|/I)\geq 0} y(1-\|x-\theta_1\|)dx = 2yI$ and similarly $SW^a = 2kyI$, and thus $\Delta SW = SW^a - SW^b = 2(k-1)yI$. Thus, there exists under-investment if $\frac{(1-\gamma)(k-1)yI}{2} < C_P < 2(k-1)yI$.

Case (ii). First, note that both sellers can get positive profits by attracting consumers sufficiently loyal to their own influencers. Meanwhile, since $I \geq \frac{3}{2}\pi \geq \pi$, all consumers are served. The condition $I \geq \frac{3}{2}\pi$ ensures that the cutoff consumer who is indifferent between purchasing from the first seller and the second one can get a positive utility. Since some influencer(s) might acquire additional power, we use I_1 and I_2 in the equilibrium construction.

Second, given prices (p_1, p_2), the cutoff type x^* is determined by the equation $y(1-s_1/I_1) - p_1 = y(1-s_2/I_2) - p_2$, where $s_j := \|x^* - \theta_j\|$ is the size of consumers attracted by Seller j on one side. We can solve it to get $s_j = \frac{\pi y + I_1 I_2(p_{3-j}-p_j)}{y(I_1+I_2)}$ and thus Seller j's profit is given by $\Pi_j = 2s_j p_j$. By taking derivatives w.r.t. p_j over Π_j, we get $\frac{\partial \Pi_j}{\partial p_j} = \frac{2I_1 I_2(p_{3-j}-2p_j)+2I_j\pi y}{y(I_1+I_2)}$. We can solve it to get $p_1 = \frac{\beta y(2I_1+I_2)}{3I_1 I_2}$ and $p_2 = \frac{\beta y(I_1+2I_2)}{3I_1 I_2}$. The profits are given by

$$\Pi_j = \frac{2\pi^2 y(2I_j + I_{3-j})^2}{9I_1 I_2(I_1+I_2)} \tag{A.3}$$

and the cutoff type consumer gets a utility of $u(x^*) = \frac{y\left(3I_1I_2(I_1+I_2)-2\pi(I_1^2+I_2^2)-5\pi I_1 I_2\right)}{3I_1I_2(I_1+I_2)}$. We can verify that the cutoff type consumer indeed gets a positive utility. To see it, when $I_1 = I_2 = I$, it reduces to $y - \frac{3\pi y}{2I} \geq 0$, which holds if and only if $I \geq \frac{3}{2}\pi$. When $I_1 = I_2 = kI$, it reduces to $kI \geq \frac{3}{2}\pi$. When $I_1 = kI$ and $I_2 = I$, it reduces to $u(x^*) = y - \frac{\pi y(2+5k+2k^2)}{3Ik(1+k)}$. Note that $\frac{du(x^*)}{dk} = \frac{\pi y(2+4k+3k^2)}{3k^2(1+k)^2} > 0$, and thus $u(x^*) \geq 0$ for any $k \geq 1$ as long as $I \geq \frac{3}{2}\pi$.

Third, we analyze the incentive for power acquisition. Obviously, before power acquisition, $w_1^b = w_2^b = \frac{(1-\gamma)\pi^2 y}{I}$. Next, we consider the incentive for power acquisition when the opponent influencer, say Influencer 2, does not acquire additional power. This means $I_1 = kI$ and $I_2 = I$. After power acquisition by Influencer 1, by the profit formula, equation (A.3), we have $w_1^a = \frac{2\pi^2 y(1-\gamma)(1+2k)^2}{9Ik(1+k)}$ and $w_2^a = \frac{2\pi^2 y(1-\gamma)(2+k)^2}{9Ik(1+k)}$. This further implies $\Delta w_1 = \frac{\pi^2 y(1-\gamma)(2+k)(1-k)}{9Ik(1+k)} < 0$, so Influencer 1 has no incentive to acquire additional power. Finally, we consider the incentive for power acquisition when the opponent influencer acquires power. By symmetry, we consider Influencer 1. If he chooses not to acquire power, he gets $\hat{w}_1^b = \frac{2\pi^2 y(1-\gamma)(2+k)^2}{9Ik(1+k)}$. If he acquires power, he gets $\hat{w}_1^a = \frac{(1-\gamma)\pi^2 y}{kI}$. Thus, $\Delta \hat{w}_1 = \frac{\pi^2 y(1-\gamma)(1-k)(1+2k)}{9Ik(1+k)} < 0$.

Fourth, we compute the cost range of C_P where power acquisition improves welfare. Since the transfer between consumers and sellers does not affect aggregate welfare, we can measure social welfare as $SW = \sum_{j=1}^{2}\int_{\|x-\theta_j\|\leq s_j} y * (1-\|x-\theta_j\|/I_j)dx$. Then we can compute social welfare SW_0 when no influencer acquires power, SW_1 when only one influencer acquires power, and SW_2 when both influencers acquire power as follows: $SW_0 = \frac{\pi(4I-\pi)y}{2I}$, $SW_1 = \frac{1}{9}\pi y\left(18 - \frac{\pi(k^2+7k+1)}{I(k^2+k)}\right)$, and $SW_2 = \frac{\pi(4kI-\pi)y}{2kI}$. It is easy to show that $SW_2 > SW_1 > SW_0$ and $2(SW_1 - SW_0) > SW_2 - SW_0$, and thus power acquisition improves social welfare as long as $C_P < \min\{SW_1 - SW_0, \frac{1}{2}(SW_2 - SW_0)\} = \frac{\pi^2 y(7k+2)(k-1)}{18Ik(k+1)}$. The proof concludes. $\square$

A.11. Proof of Proposition 9

Proof. Here, we first present Lemma 3, which computes the profit when one seller hires the two influencers in a parameterized setting. The proofs can be found in Section OA.7.2.

Lemma 3. *Assume $I_1 = I_2 = I \leq \pi$. In equilibrium, the profit $\Pi_{\{1,2\}}$ and prices satisfy: (i) when $\beta \geq I$, $\Pi_{\{1,2\}} = yI$ and $p_j^* = \frac{y}{2}$; (ii) when $2I/3 \leq \beta < I$, $\Pi_{\{1,2\}} = (2I-\beta)\beta y/I$ and $p_j^* = y(1-\beta/2I)$; and (iii) when $0 \leq \beta < 2I/3$, $\Pi_{\{1,2\}} = (2I+\beta)^2 y/(8I)$ and $p_j^* = \frac{y}{2} + \frac{y\beta}{4I}$.*

Since internal transfers do not affect total welfare, when products are homogeneous (that is, $y_1 = y_2$), efficiency only depends on the size of the consumers served, which further depends on equilibrium prices. The proof consists of two parts.

Part (i). Let W_U and W_B denote the total welfare under unbalanced matching and balanced matching. Now, it suffices to show that $W_U \leq W_B$ and we have three cases:

Case (1). Heterogeneous influencer power (that is, $y_1 = y_2 = y$, $\beta = 0$ and $I_1 \geq I_2$). Under unbalanced matching, $p_1^* = p_2^* = \frac{y}{2}$, and the total size of consumers served is just I_1 (i.e., $\{x \in \mathbf{S}^1 : y\left(1 - \frac{1}{I_1}\|x-\theta\|\right) - p_1^* \geq 0\}$). In contrast, under balanced matching, the marginal consumer faces an equilibrium price given by $p_1^C = \frac{2y(I_1-I_2)}{4I_1-I_2}$ (see Example 2), which implies that the total size of consumers is greater than I_1 because $\frac{2y(I_1-I_2)}{4I_1-I_2} \leq \frac{y}{2}$. Thus, $W_U \leq W_B$.

Case (2). Heterogeneous influencers' style type (that is, $y_1 = y_2$, $I_1 = I_2$ and $\beta > 0$). Here, we only focus on the case in which a pure strategy equilibrium exists (that is, $\beta \geq \beta_0$) and the equilibrium prices are p_j^C in Example 3. In contrast, under unbalanced matching, the equilibrium prices are given by p_j^* in Lemma 3. We can directly verify that $p_1^C = p_2^C \leq p_1^* = p_2^*$ for $\beta \geq \beta_0$. Thus, total welfare is higher under regulated matching, because a consumer who purchases the product under unmatched matching is also willing to buy it under regulated matching. Hence, $W_U \leq W_B$.

Case (3). Heterogeneous product quality (i.e., $I_1 = I_2 = I$, $\beta = 0$ and $y_1 \geq y_2$). Under unbalanced matching, seller 1 hires both influencers and sets a price at $p_j^* = \frac{y_j}{2}$. Influencer 2 is not active in the market. Therefore, $W_U = \int_{\left\{x\in\mathbf{S}^1 : \|x-\theta\|\leq\frac{1}{2}I\right\}} y(1 - \|x - \theta\|/I)dx = \frac{3}{4}y_1 I$.

In contrast, under balanced matching, the equilibrium outcome is given by Example 1, and thus $W_B = \int_{R_1} y(1 - \|x-\theta\|/I_1)dx + \int_{R_2} y(1 - \|x - \theta\|/I_2)dx$, where $R_1 = \{x \in \mathbf{S}^1 : \|x - \theta\| \leq \frac{2Iy_1}{4y_1 - y_2}\}$ and $R_2 = \{x \in \mathbf{S}^1 : \frac{2Iy_1}{4y_1-y_2} < \|x - \theta\| \leq \frac{3Iy_1}{4y_1-y_2}\}$. We can further compute $W_B = \frac{Iy_1(14y_1^2-4y_1y_2-y_2^2)}{(4y_1-y_2)^2} = Iy_1 * f(x)$ where $f(x) = (14x^2 - x - 1)/(4x-1)^2$ and $x := y_1/y_2$.[16] Note that $f'(x) = -\frac{12(x-1)}{(4x-1)^3} < 0$, and thus $f(x)$ strictly decreases for $x > 1$. Moreover, since $\lim_{x\to 1} f(x) = 1$ and $\lim_{x\to\infty} f(x) = \frac{7}{8}$, we have $W_B \geq \frac{7}{8}Iy_1 > \frac{3}{4}Iy_1 = W_U$.

Part (ii). First, note that when $\beta \geq \frac{I_1+I_2}{2}$, each influencer charges a monopolist price $p_j^* = \frac{y}{2}$ when hired by a seller with product quality y, and serves a sub-population such that $x \in R_j$ where $R_j := \left\{x \in \mathbf{S}^1 : \|x - \theta_j\| \leq \frac{I}{2}\right\}$. Hence, there is zero overlap among consumers served by the two influencers.

Under unbalanced matching, both influencers are hired by Seller 1. Furthermore, similar to that of unbalanced matching in the heterogeneous product quality case, we can compute that $W_U = \frac{3}{4}y_1(I_1 + I_2)$. Similarly, under balanced matching, $W_B = \frac{3}{4}y_1 I_1 + \frac{3}{4}y_2 I_2$. Obviously, $W_U > W_B$ as long as $y_1 > y_2$. All the proofs conclude. □

A.12. Proof of Lemma 1

Proof. First, note that when $(1-c_1)y_1 = (1-c_2)y_2$ is satisfied, $p_k^d = 0$ for direct sales, regardless of the matches. To see it, note that if Seller 2, charges a price $p_2^d > 0$, then it is sub-optimal for Seller 1 to charge $p_1^d > p_2^d$, regardless of p_k^i. Similarly, $p_1^d = p_2^d$ is also sub-optimal unless all consumers buy from Seller 1 in direct sales. However, this implies that Seller 2 has an incentive to undercut Seller 1 if it were the case.

Second, we come to verify the optimality of monopolist pricing by both sellers. The indifference condition is given by $y_k(1 - s_j/I_j) - p_k \geq (1 - c_k)y_k$, where $s_j = \|x - \theta_j\|$. We can solve $s_j = I_j(c_k - p_k/y_k)$ and thus $\Pi_k = 2s_j * p_k = 2I_j p_k(c_k - p_k/y_k)$, which further implies that $p_k^d = \frac{c_k y_k}{2}$ and $s_j = \frac{c_k I_j}{2}$. Furthermore, we need to verify that $\beta \geq s_1 + s_2 = \frac{c_1 I_1 + c_2 I_2}{2}$. Henceforth, $\Pi_{k,j} = \frac{c^2 y_k I_j}{2}$ for any match $(j, k(j))$.

Third, we turn to equilibrium matching. Given the joint profit $\Pi_{k,j}$ and the bilateral Nash bargaining protocol, $k(j) = j$ is stable because U_1 and w_1 are maximized and therefore they have no incentive to deviate. □

A.13. Proof of Lemma 2

Proof. Note that influencers are identical for sellers under balanced matching.

Case (i). Given that Seller 2 uses influencer-intermediated sales, it is optimal for Seller 1 to use influencer-intermediated sales when $\frac{\gamma y I}{2} \geq (2\pi - I)(1 - c)y$ is satisfied.

Case (ii). When $(2\pi - I)(1 - c)y > \frac{\gamma y I}{2}$ holds, there exists no pure strategy equilibrium. We consider a symmetric mixed strategy equilibrium as described in Lemma 2. Given that Seller 2 follows the prescribed strategy, the indifference condition for Seller 1 to charge a direct-sale price p_1 requires:

$$q * (2\pi - I)p_1 + (1-q) * \int_{p_1}^{(1-c)y} 2\pi p_1 f(p_2)dp_2 = \frac{1}{2}\gamma y I \tag{A.4}$$

Here, we used the fact that $(1-c)y \leq \frac{1}{2}y$, that is, the maximum price under direct sales is smaller than the monopolist price under influencer-intermediated sales. This further implies that Seller 2 always sells to a measure of I consumers under influencer-intermediated sales.

By plugging $p_1 = (1-c)y$ into equation (A.4), we get $q * (2\pi - I)(1-c)y = \frac{1}{2}\gamma y I$. Then, by plugging $p_1 = \underline{P}$ into equation (A.4), we get $q * (2\pi - I)\underline{P} + (1-q) * 2\pi\underline{P} = \frac{1}{2}\gamma y I$, which pins down the lower bound $\underline{P}$. Note that Seller 1 gets a payoff strictly less than $\frac{1}{2}\gamma y I$ when setting $P < \underline{P}$. Finally, rearranging equation (A.4) yields $q * (2\pi - I)p_1 + (1-q) * 2\pi p_1(1 - F(p_1)) = \frac{1}{2}\gamma y I$, where $F(\cdot)$ is the CDF of $f(\cdot)$.

Differentiating with respect to p_1, we get $q * (2\pi - I) + (1-q) * 2\pi(1 - F(p_1)) - (1-q) * 2\pi p_1 f(p_1) = 0$. We can then multiply p_1 on both sides and get $f(p)$ after simple algebra manipulation. □

[16] The skipped algebra is available upon request.

Appendix B. Supplementary material

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.jet.2024.105867.

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