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Supply chain delivery performance improvement: a white-box perspective

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ABSTRACT

This paper proposes a white-box perspective that portrays a supply chain delivery process as a network of related activities which remains to be improved. It addresses a critical disadvantage of supply chain delivery performance models, namely considering a delivery process as a whole and ignoring characteristics and relationships between activities in the delivery process. A delivery process is modeled using the Graphical Evaluation and Review Technique based on the characteristic function (CF-GERT). Based on the CF-GERT model, a framework for applying managerial effort to activities to improve overall delivery performance is proposed. Then, particle swarm optimization (PSO) based on the penalty function is used to solve the delivery performance improvement framework. Finally, a numerical case shows how applying efforts to activities can effectively improve delivery performance and demonstrates the influence of several parameters on the related costs.

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Supply chain management; delivery performance; delivery window; graphical evaluation and review technique; particle swarm optimization

1. Introduction

In today's highly competitive global business environment, meeting customer needs in a timely and reliable manner is of great importance for companies to enhance market competitiveness (Tracey, Lim, and Vonderembse 2005). To increase market competition, many organizations adopt supply chain management as the basis of their competitive strategies (Cooper, Lambert, and Pagh 1997; Kouvelis, Chambers, and Wang 2006). With the support of supply chain management techniques, competition among companies has changed into competition among supply chains and effective supply chain management has become a tool to enhance the overall competitive advantage (Tracey, Lim, and Vonderembse 2005; Li et al. 2006). Supply chain performance evaluation and improvement play an important role in pursuing a high competitive advantage. Many researchers have pointed out the importance of performance evaluation in supply chain management (Martin and Patterson 2009; Nakandala, Samaranayake, and Lau 2013). Within the hierarchy of supply chain performance indicators, delivery performance is considered to be one of the key indicators driving supply chain excellence (Guiffrida and Nagi 2006; Ngniatedema et al. 2018). Logistics managers regard the delivery process as a key measure of strategic performance since the delivery performance directly affects customer satisfaction and the choice of suppliers. Moreover, in the long run, supplier delivery performance will also affect customer buying behavior (Peng and Lu 2017). In addition, the demand of customers for shorter delivery times and timely deliveries enables suppliers to redesign the supply chain process and improve delivery performance (Collin, Eloranta, and Holmström 2009; Calleja et al. 2018).

Many researchers have studied the problem of delivery performance improvement (Bhattacharyya and Guiffrida 2015; Bushuev 2018). Although these papers provide several insightful and useful management methods, they do have a significant limitation: the delivery process is considered a black-box. Most of the existing studies assume a predetermined delivery time distribution form and then explore how changes in distribution parameters influence delivery performance. This perspective regards the supply chain as an overall system, neglecting all subactivities that constitute the supply chain delivery process. That significantly limits opportunities for delivery performance improvement because supply chain managers have no clear idea of which supply chain activity to improve and how to allocate the investment. Hence, we define this perspective as a black-box perspective that provides fewer management insights.

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In reality, a supply chain delivery process consists of several related activities aimed to fulfill customer needs. Therefore, supply chain delivery performance improvement should rely on the improvements in each of the activities, e.g. duration reduction and variance reduction. Also, due to the uncertainty associated with some complex elements of a delivery process, there might be probability branches and rework required in the process. Therefore, an appropriate tool is needed for the description of a complex supply chain delivery process. Moreover, the tool should be able to analyze and improve the delivery process. In this study, the supply chain delivery process will be portrayed as a network of related activities which remains to be improved. So, we call this perspective a white-box perspective.

The white-box perspective defines a supply chain delivery process as a set of mutually related activities that should be performed in a certain specific order. Each activity is considered to be a simple undivided task that has specific characteristics such as duration or its distribution. These characteristics might be changed with a certain amount of managerial effort (cost) that leads to changes in the characteristics of the delivery process as a whole. Thus, the delivery performance improvement required the identification of key activities in the delivery process and the improvement of their characteristics.

The current study aims to build a white-box framework to improve supply chain delivery performance based on the concept of delivery window and the Graphical Evaluation and Review Technique based on the characteristic function (CF-GERT) model (Pritsker 1966; Tao et al. 2017). The research herein allows determining whether it is beneficial to invest in delivery performance improvement and the amount of resources (or applied effort) that should be allocated to each activity. Particularly, the resources (i.e. managerial efforts and monetary assets) will be applied to reduce the variance of supply chain activities. The resources are measured in monetary values. To achieve this goal, the delivery time distribution is deduced by the analysis algorithm of the CF-GERT model. Then, considering the relationship between the applied effort and reduced uncertainty, the expected cost model is established. Finally, the optimal solution is found by finding the optimal position of the delivery window and the percentage of applied effort of each activity that minimizes the expected cost.

The contribution of the paper is twofold. First, the proposed white-box perspective contributes to the area of supply chain delivery performance improvement. Unlike the traditionally used black-box perspective, the white-box perspective takes all related activities within the supply chain and their relationships into full consideration. That significantly increases opportunities for delivery performance evaluation and improvement. Second, the paper herein extends GERT usage from process evaluation to process improvement. Traditionally, GERT techniques including CF-GERT are used only for process evaluation. The proposed general approach optimizes the parameters of CF-GERT to allocate additional investment and managerial efforts. The proposed procedure minimizes the delivery-related costs in the resource allocation model and improves a complex supply chain delivery process.

The rest of this paper is organized as follows. Section 2 provides a literature review. Based on the GERT and CF-GERT models, the concept of optimal position of the delivery window (OPDW), and the relationship between applied effort and uncertainty reduction, section 3 constructs a cost-based supply chain delivery performance improvement model with a white-box perspective. Following this, the model solution based on the particle swarm algorithm (PSO) is developed in section 4. Finally, section 5 conducts the numerical studies and section 6 gives the conclusion and future research directions.

2. Literature review

2.1. Graphical evaluation and review technique

The main differences between the black-box perspective and the white-box perspective are presented in Figure 1.

Both black-box and white-box perspectives have similar inputs, the only difference is that delivery time distribution is provided for the entire delivery process (black-box) or for each activity individually (white-box). Also, the white-box perspective breaks down a delivery process into activities and provides the relationship between the activities in form of a network. In addition to the same outputs (the effect of different parameters and the OPDW), the white-box perspective provides several more outputs. First, the delivery time distribution for the whole process should be deduced based on the delivery time distributions of all activities. Second, an analysis should define what activities should be improved and what amount (managerial effort) should be invested into the improvement of each activity.

The Graphical Evaluation and Review Technique (GERT), proposed by Pritsker (1966) is suitable for an analysis of a delivery process, because of its capability of handling probabilistic activity durations and probabilistic representation of network logic, and rework (Tao et al. 2017). GERT has been widely used in the fields of supply chain management (Zhou et al. 2016; Wang et al. 2020; Zhao et al. 2021). To improve the quality of the perishable food supply chain, Wang et al. (2020) put forward a new technology integrating the GERT and

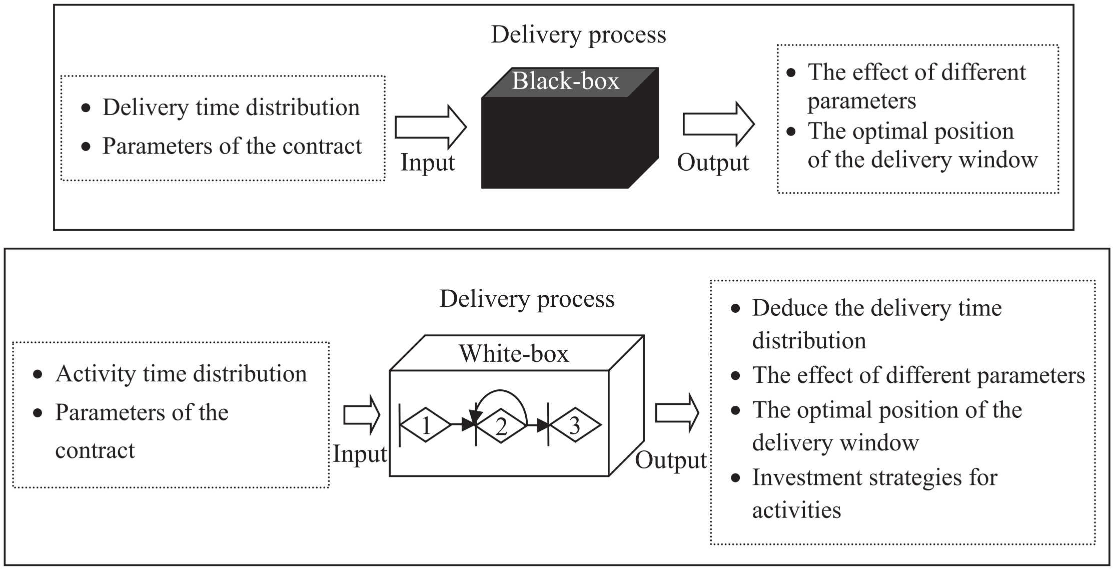

Figure 1. Black-box and white-box perspectives.

Bayesian approach, in which GERT is used to forecast the expected trends of quality, time, and carbon emissions. Zhou et al. (2016) translated the remanufacturing operational process into a stochastic GERT network and developed a forecasting model that can predict the quantity, time, and probability of product return, recyclable parts/components/materials, and disposal. However, GERT can only provide the mean and the variance of several network parameters. To obtain the PDF and CDF of the delivery time, a characteristic function based on GERT (i.e. CF-GERT), proposed by Tao et al. (2017), will be employed to portray the delivery process. It can not only describe the relationship between the completion time of activities and the entire process completion time but also deduce the distribution of the whole delivery process.

2.2. Supply chain delivery performance improvement

The delivery window concept used in supply chain delivery performance models evolved from the general class of time interval constrained models for vehicle routing and scheduling found in the operations research and operations management literature. Supply chain delivery performance models that use delivery windows can be categorized into two groups (Bushuev and Guiffrida 2012): (i) index-based measures that convert the probability of on-time delivery into a delivery capability index, and (ii) cost-based measures that utilize costs incurred by untimely delivery to reveal delivery performance. The present paper extends research in the area of delivery performance improvement. For delivery performance improvement, cost-based models have a huge advantage over index-based models because improvement is easier to understand in cost values than in index values. Moreover, a decision can be easily made if the monetary advantage of the decision is known (Bushuev 2018). Therefore, this study adopts the cost-based delivery performance expression to analyze the delivery optimization of the supply chain.

The concept of the delivery window was introduced by Guiffrida and Nagi (2006), which is defined as the difference between the earliest acceptable delivery date and the latest acceptable delivery date (Figure 2). Thus, the delivery may be classified as early, on-time, or late. When the delivery is completed within a given delivery window, the delivery is regarded as on time. Otherwise, the delivery will be early or late, which will result in additional costs for a buyer which will pass the costs to the supplier in form of penalties for untimely delivery. For delivery performance evaluation using a delivery window, Guiffrida and Nagi (2006) developed a model that incorporates the variability found in the individual stages of the supply chain into a financial measure. They discussed the effect of the reduction of delivery variance and delivery window on the improvement of the overall system performance, financially justifying investments for delivery performance improvement.

Based on the cost-based model, Bushuev and Guiffrida (2012) introduced the concept of the optimal position of the delivery window (OPDW) which defines a time when the delivery process should begin to minimize the expected penalties paid for untimely delivery by the supplier. Further, Guiffrida and Jaber (2008) and Guiffrida, Jaber, and Rzepka (2008) developed cost-based models

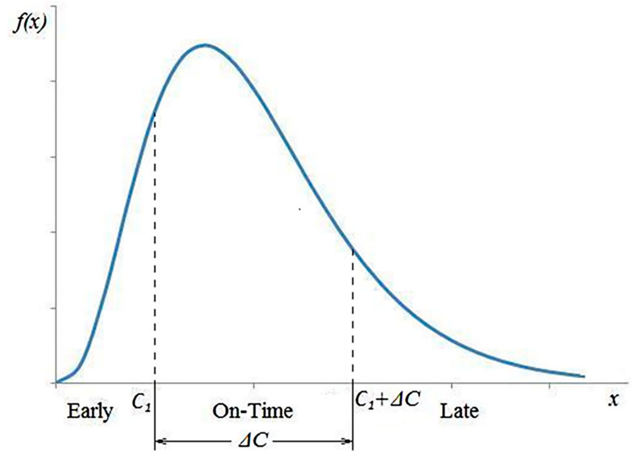

Figure 2. Illustration of a delivery window.

based on the concept of the delivery window and financially quantified the benefit of reducing delivery variance. Hsu, Hsu, and Shu (2013) investigated the effect of further reduction of lead time variability on delivery performance. Ngniatedema, Chen, and Guiffrida (2016) developed a variance reduction modeling approach, which directly incorporates the uncertainty in the delivery time distribution into a financial delivery performance metric; the framework provides a bound on the financial investment required to improve the delivery process. Considering the relationship between buyer and supplier, Bhattacharyya and Guiffrida (2015) demonstrated an optimization framework for improving supplier delivery performance which investigates the buyer's investment decision to reduce the supplier's untimely delivery.

The influences of different delivery time distributions and other model parameters (i.e. the width of the delivery window) were investigated by several researchers. Tanai and Guiffrida (2015) presented a robust asymmetrical Laplace delivery performance model with a set of propositions for modeling delivery improvement in terms of both the mean and variance of a delivery time distribution. Bushuev et al. (2018a) investigated strategies for improving supply chain delivery timeliness when the delivery time follows an asymmetric Laplace distribution and the optimal position of the delivery window (OPDW) concept is implemented. They analyzed the effect of changes to the parameters of the delivery time distribution on the expected penalty cost for untimely delivery. Furthermore, Bushuev et al. (2018b) explored strategies for improving delivery performance utilizing the reduction of the mean and variance of the delivery time when the delivery time follows uniform, exponential, and logistic distributions. Additionally, for the general delivery time distribution, Bushuev (2018) analyzed the effect of the width of the delivery window and penalty costs for early and late deliveries on the optimal position of the delivery window, as well as explored the expected penalty cost. Madadi and Iranmanesh (2012) proposed an indicator and a method to identify important activities and the amount of effort that should be assigned to them. The impact of variability reduction has been reviewed by reducing the variability of the supply chain activities. Incorporating the risk-aversion degree of a decision-maker, Tao et al. (2021) proposed a Conditional Value-at-Risk (CVaR) measure of the penalty for untimely delivery. They introduced the concept of risk-averse optimal position of the delivery window into a cost-based delivery performance model. The influence of the degree of risk aversion, the width of the delivery window, and the ratio of the penalties per unit time late to unit time early were explored. Table 1 provides a summary of papers on supply chain delivery improvement that use delivery windows.

Table 1 shows that all previous research mostly focused on 2-stage supply chains where the delivery process is considered as a whole and not divided into activities (black-box models). That limits researchers in their abilities to analyze and improve a delivery process. The research cannot identify what part of the delivery process should be improved and how.

White-box models separate a delivery process into activities that provide a significant advantage for delivery performance improvement. At the same time, the previous research with a white-box perspective has several limitations. First, all previous research uses the normal distribution to describe the duration of each activity. The use of normal delivery time in supply chain delivery performance models suggests an immediate limitation since the delivery time distribution under these models is always symmetric. Second, all previous research focuses on a serial supply chain and as a result, assumes a serial form of relationship between activities within the delivery process. In general, a delivery process is significantly more complex and might include parallel, serial, and loop forms of connections. Third, all previous research assumes that the position of the delivery window is fixed or in other words, the time when the product should be shipped is predefined which leads to non-optimal decisions. In reality, there is an optimal time to ship the product which is defined in the optimal position of the delivery window (OPDW) concept and this time changes when the delivery process is changed.

Overall, the highlighted limitations do not allow to analyze the complexity of a delivery process and significantly limit opportunities for delivery performance improvement.

The paper herein bridges the research gap by addressing these limitations:

Table 1. A summary of supply chain delivery performance papers.

				model input		model output
	Papers	supply chain network	OPDW concept	Delivery time distribution	Activity time distribution	The effect of different parameters
Black – box model	Guiffrida and Nagi (2006)	2-stage serial	✗	hyperbolic and exponential	✗	Δc, mean and variance of delivery time
	Bushuev and Guiffrida (2012)	2-stage serial	✓	general form	✗	✗
	Ngniatedema, Chen, and Guiffrida (2016)	2-stage serial	✗	uniform and Gaussian	✗	variance of delivery time
	Bhattacharyya and Guiffrida (2015)	2-stage serial	✗	general form	✗	✗
	Tanai and Guiffrida (2015)	2-stage serial	✗	asymmetrical Laplace	✗	mean and variance of delivery time
	Bushuev et al. (2018a)	2-stage serial	✓	asymmetric Laplace	✗	delivery time distribution parameters
	Bushuev et al. (2018b)	2-stage serial	✓	uniform, exponential, and logistic	✗	delivery time distribution parameters
	Bushuev (2018)	2-stage serial	✓	general form	✗	Δc, QH, K, delivery time distribution parameters
	Tao et al. (2021)	–	✗	general form	✗	Δc, QH, K, degree of risk-aversion
	Roy and Sarker (2021)	–	✗	general form	✗	✗
White-box model	Guiffrida and Jaber (2008)	serial	✗	n-fold convolution of normal distribution	✓	variance of delivery time
	Guiffrida, Jaber, and Rzepka (2008)	serial	✗	n-fold convolution of normal distribution	✓	variance of delivery time
	Hsu, Hsu, and Shu (2013)	serial	✗	normal	✓	✗
	Current study	general form	✓	general form	✓	Δc, QH, K, activity uncertainty

1. The paper herein is using a general form of delivery time distribution.
2. The paper herein assumes a general form of a network of delivery process activities with parallel, serial, and loop forms of connections.
3. The paper herein exploits the optimal position of the delivery window (OPDW) concept to find an optimal time to ship the product and minimize the penalty costs.

3. Modelling supply chain delivery process

In this section, the supply chain delivery process is modeled using the white-box perspective. The following assumptions are used:

1. The delivery contract is already signed by the supplier and the contract parameters (delivery window and penalties for untimely delivery) are known.
2. The supplier uses the concept of OPDW to define a time when the delivery process should be initiated.
3. Information about each activity of the delivery process is available. That includes the duration of the activity (distribution and its parameters), the amount of effort (cost) required to reduce the duration, and the minimal possible duration.
4. Relationships between activities are known and fixed.
5. Each activity is independent, thus effort applied to one activity does not affect the other activities.

For the sake of convenience, the notations used in the study are provided in Table 2.

3.1. Optimal position of the delivery window

Guiffrida and Nagi (2006) proposed the following expected penalty cost per period when deliveries are classified as early and late according to a delivery window.

$$Y_0 = QH\int_0^{c_1}(c_1 - x)f(x)dx + K\int_{c_1+\Delta c}^{\infty}(x-(c_1+\Delta c))f(x)dx, \qquad (1)$$

where Y_0 = expected penalty cost of untimely delivery.

Although the earliest acceptable delivery time is predefined by the contract, the supplier can define the time when delivery begins, therefore changing the value of c_1. For example, if the supplier decides to ship the product

Table 2. Notation list.

Y_0	Expected penalty cost of untimely delivery
x	Random delivery time
$f(\cdot)$	Probability density function
c_1	Beginning of the on-time portion of the delivery window / difference between the time the delivery process is initiated and the earliest acceptable delivery time
c_1^*	Optimal position of the delivery window
Δc	Width of the on-time portion of the delivery window
$c_1 + \Delta c$	Ending of the on-time portion of the delivery window
QH	Penalty cost per time unit early (levied by the buyer)
K	Penalty cost per time unit late (levied by the buyer)
Section 3.1	
i, j	Activity from node i to node j
x_{ij}	Stochastic duration of activity (i,j)
$E(\cdot)$	Expectation operator
p_{ij}	Probability of activity i, j occuring
p_{ij}^E	Equivalent probability of activity i, j occuring
W_{ij}	Transfer function concerning activity (i,j)
W_{ij}^E	Equivalent transfer function concerning activity (i,j)
$M_{ij}(s)$	Moment-generating function (MGF) of the activity duration
M_{ij}^E	Equivalent MGF of the activity duration
$\varphi()$	Characteristic function
$\varphi^E()$	Equivalent characteristic function
$p^E()$	Equivalent probability density function
a and b	Smallest and largest duration of the delivery process, respectively
$Re\{\cdot\}$	Operation of taking the real part of a complex number
$\sum'$	Operation means that the weight of the first item is 0.5;
Section 3.3	
σ_{ij}	Original uncertainty of activity (i,j)
σ'_{ij}	Reduced uncertainty of activity (i,j)
γ_{ij}	Applied effort as a percentage of the planned cost of the activity (i,j)
ϵ	Theoretical minimal activity uncertainty as a percentage of the original variability
τ_{ij}	Distance from ϵ_{ij} when 100% effort is applied to the activity (i,j)
Section 3.4	
Y	Expected costs for the delivery
Y_{ij}	Planned cost of the activity (i,j)
Section 3.5	
$\hat{C}$	Expected costs for the delivery after managerial efforts are applied
$\hat{C}_{ij}$	Cost of the activity (i,j) after managerial efforts are applied
$\hat{x}_{ij}$	Stochastic duration of activity (i,j) after managerial efforts are applied
$\hat{x}$	Random delivery time after managerial efforts are applied
$\hat{c}_1^*$	The optimal position of the delivery window after managerial efforts are applied
χ	Maximum additional investment

10 hours before the earliest accepted delivery time, c_1 is equal to 10 hours.

As demonstrated in Bushuev and Guiffrida (2012), Y is a convex function of c_1, and the optimal value of c_1 (which is defined as $c_1{}^*$) that minimizes Y can be determined by evaluating

$$K \cdot p_{late} = QH \cdot p_{early}, \tag{2}$$

where $p_{late} = \int_{c_1+\Delta c}^{\infty} f(x)dx$ and $p_{early} = \int_0^{c_1} f(x)dx$ are the probabilities of late and early deliveries.

3.2. GERT and CF-GERT models

The delivery time is closely related to the activity duration of different stages, so the uncertainty of each activity will affect the entire delivery process. Therefore, we need a tool to derive the distribution function of the delivery time. To this end, this paper reviews the GERT model and its extension, the CF-GERT model, which is extensively adopted to portray complex processes, such as supply chain processes (Li and Liu 2012; Li 2014; Zhou et al. 2016; Wang et al. 2020), product development processes (Nelson, Azaron, and Aref 2016; Tao et al. 2017).

3.2.1. GERT model

The GERT network (Pritsker 1966) belongs to the *Activity on Arc (AOA)* type. Nodes represent the system status, and arrows between nodes represent the activity or transfer relationship. Therefore, the GERT network can be represented by $G = (N, A)$, where N is the set of network nodes indicating the states, and A is the set of network arrows indicating the activity.

Features of the GERT network include (Nelson, Azaron, and Aref 2016):

(i) Probability branch: The GERT network may contain probability branches, deterministic branches, or a combination of both.
(ii) Network loops: GERT networks allow to contain loops. Operational uncertainties associated with the supply chain, such as supplies not meeting the specifications (Mogre, Wong, and Lalwani 2014) always lead to rework. In this setting, a loop in the GERT networks will be employed to describe this event.
(iii) Node implementation logic: A node in GERT network consists of two parts: the input segment and the output segment. Particularly, there are three input types (i.e. Exclusive OR, OR, and AND) and two output types (i.e. deterministic and stochastic) which result in six different node types (Table 3). An exclusive OR-type node will be realized once any of all activities entering the node is completed, furthermore, only one entering activity can be performed. AND node will be released only if all entering activities are performed. Similarly, an OR node can be realized when one or more entering activities are completed. The output segment concerns the probability of activity leaving the node. If all outcoming activities from the node are performed with probability 1, then the output segment is deterministic, otherwise, it belongs to the probabilistic type. In this paper, we focus on the node with Exclusive OR input and probabilistic output.

Table 3. GERT logic.

	Input		
Output	Exclusive OR	OR	AND
Deterministic			
Probabilistic			

(iv) Activity time distribution: GERT network is convenient to handle almost all types of probability distributions (normal, β, γ, etc.).

The general GERT technique includes the following steps (Zhou et al. 2016):

Step 1: Transfer the delivery process to a stochastic network based on the GERT logic presented in Table 3.

Step 2: Estimate parameters of each activity, i.e. delivery time probability p_{ij} and probability density function $f(x_{ij})$, where the subscript ij means the activity is from node i to node j.

Step 3: Integrate the two parameters of each activity (i,j) into one transfer function $W_{ij}(s)$: $W_{ij}(s) = p_{ij}M_{ij}(s)$, where $M_{ij}(s) = E(e^{x_{ij}s}) = \int_{-\infty}^{+\infty} e^{x_{ij}s} f(x_{ij})dx_{ij}$ is the moment-generating function (MGF) of the activity duration;

Step 4: Apply Mason's rule (Mason 1956) to calculate the total equivalent transfer function $W_{0,n}^{E}$ from the source node 0 to the sink node n based on the network structure and the value of $W_{ij}(s)$. Mason's rule is a method to derive a discrete network's transfer function by identifying various forward paths from the input node to the output node of a discrete network, and the various feedback paths that may, or may not, share common signal nodes with those feedforward paths;

Step 5: Derive the probability and the equivalent MGF. According to the definition of the transfer function, the probability $p_{0,n}^{E}$ is $p_{0,n} = W_{0,n}(s)|_{s=0}$, while the equivalent MGF is $M_{0,n} = \frac{W_{0,n}(s)}{p_{0,n}}$;

Step 6: Calculate the moments (mean, variance, etc.). According to the characteristics of MGF, the expected time from the initial node 0 to node n is: $E(t) = \frac{\partial}{\partial s}[W_{0,n}^{E}(s)]|_{s=0}$. A similar approach is used for variance and other moments.

The GERT method is used to describe a delivery process represented as a set of activities. Using the required parameters for each activity, the analysis of the GERT network can be used to calculate the expected delivery time and variance.

3.2.2. CF-GERT model

CF-GERT is an extension of the typical GERT network model. The core idea of CF-GERT (Tao et al. 2017) is to replace the moment generating function (MGF) of GERT with a characteristic function. Then the expected delivery time, variance, etc. are obtained by using the properties of the characteristic function. More importantly, the Cumulative Distribution Function (CDF) and the Probability Density Function (PDF) can be obtained via the inverse Fourier transform of the characteristic function. The following are some key definitions and theorems of CF-GERT, please refer to Tao et al. (2017) for more information.

Definition 1: Characteristic function. Given that the PDF of supply chain activity duration x_{ij} is $f(x_{ij})$, then its characteristic function $\varphi(x_{ij})$ is $\varphi(x_{ij}) = E\left[e^{\sqrt{-1}tx_{ij}}\right] = \int_{-\infty}^{\infty} e^{\sqrt{-1}tx_{ij}} f(x_{ij})dx_{ij}$, if x_{ij} is a continuous random variable, $\varphi(x_{ij}) = E\left[e^{\sqrt{-1}tx_{ij}}\right] = \sum e^{\sqrt{-1}tx_{ij}} p(x_{ij})$, if x_{ij} is a discrete random variable, (3) where $t \in R$ is a real number.

Combining the characteristic function and the occurring probability p_{ij} of activity (i,j), we define the transfer function W_{ij} in the CF-GERT network.

Definition 2: Transfer function W_{ij}. Given the characteristic function $\varphi(x_{ij})$ of the stochastic duration x_{ij} and the occurring probability p_{ij}, the transfer function W_{ij} is defined as $W_{ij} = p_{ij}\varphi_{x_{ij}}$.

Using the transfer function W_{ij}, Mason's formula can be utilized to obtain the equivalent transfer function $W^{E}(t)$ of a GERT network. The related network performance measure can be obtained by the following theorem.

Theorem 1: *If the equivalent transfer function of the CF-GERT network is $W^{E}(t)$, then the equivalent probability p^{E} is $p^{E} = W^{E}(0)$, and the equivalent characteristic function of the CF-GERT network is $\varphi^{E}(t) = \frac{W^{E}(t)}{W^{E}(0)}$.*

See Tao et al. (2017) for the proof of Theorem 1.

Last, we present the method to derive the PDF from the equivalent characteristic function of the CF-GERT network. Since the exact PDF cannot be derived, we obtain an approximation of the probability density function using the Fourier-cosine series. The Fourier-cosine series expansion method proposed by Fang and Oosterlee (2009) is utilized. Furthermore, the PDF $f^{E}(x)$ between the interval $[a, b]$ is

$$f^{E}(x) = {\sum_{k=0}^{N-1}}{}' A_k \cos\left(k\pi \frac{x-a}{b-a}\right), \tag{4}$$

where $A_k = \frac{2}{b-a}\left\{\varphi^{E}\left(\frac{k\pi}{b-a}\right) exp\left(-i\frac{ka\pi}{b-a}\right)\right\}$; k represents harmonics in the Fourier series; N is a positive integer.

The PDF and CDF of the delivery time can be obtained at the hand of the CF-GERT. Consequently, we can explore the optimal position of the delivery window.

3.3. Relation between applied effort and uncertainty reduction

To improve delivery performance, the manager has to reduce the uncertainty of supply chain activities, which means that additional investments are needed. The relation between applied effort and uncertainty reduction needs to be quantified. According to Martens and Vanhoucke (2019), the relationship between applied effort and uncertainty reduction is defined as

$$\frac{\sigma'}{\sigma} = (1-\epsilon) \times \left(\frac{1-\epsilon}{\tau}\right)^{-\gamma} + \epsilon, \tag{5}$$

where σ' is the reduced uncertainty, σ is the original activity uncertainty, γ is the percentage of applied effort.

That is to say γ reflects the applied effort as a percentage of the planned activity cost, ϵ ($\epsilon \in [0,1]$) reflects the theoretical minimal activity uncertainty as a percentage of the original variability, and τ ($\tau \in [0, 1-\epsilon]$) is the distance from the theoretical minimum ϵ as a percentage point when 100% effort is applied. Accordingly, τ reflects how difficult it is (e.g. how much effort is required) to reduce the activity variability. Since ϵ reflects the theoretical minimal activity uncertainty that can be reached, ϵ indicates the capability to reduce activity uncertainty.

According to Eq. (5), we can quantify the relationship between applied effort and uncertainty reduction and quantify the expected penalty cost after applying effort. Inspired by this method, we can further study the improvement of delivery performance in a supply chain.

3.4. Delivery performance improvement without managerial efforts

The incorporation of GERT in the supply chain delivery process evaluation sheds light on the improvement of delivery process activities. Based on the procedure presented in Section 2.1.1 and Eq. (4), the probability density function of the delivery time is derived as $f(x) = \sum_{k=0}^{N-1}{}' F_k \cos(k\pi \frac{x-a}{b-a})$. Accordingly, combined with Eq. (1), the expected penalty cost is defined as.

$$\begin{aligned} Y &= \sum_{(i,j)\in A} Y_{ij} + Y_0 = \\ &= \sum_{(i,j)\in A} Y_{ij} + QH \int_0^{c_1} (c_1 - x) f(x)dx \\ &\quad + K \int_{c_1+\Delta c}^{\infty} (x - (c_1 + \Delta c)) f(x)dx \end{aligned} \tag{6}$$

In this set, the effort for uncertainty reduction is not considered and the total cost is composed of two parts. The first part is the overall fixed costs of each activity, while the second part is the penalty for untimely delivery. Consequently, we can achieve the lowest total cost by determining the optimal value c_1^*, in the sense that.

$$\begin{aligned} \textbf{Model I} c_1^* &= \arg\min_{c_1} Y = \arg\min_{c_1} \left(\sum_{(i,j)\in A} Y_{ij} + Y_0 \right) = \\ &= \arg\min_{c_1} \left(\sum_{(i,j)\in A} Y_{ij} + QH \int_0^{c_1} (c_1 - x) \right. \\ &\quad \times f(x)dx \\ &\quad \left. + K \int_{c_1+\Delta c}^{\infty} (x - (c_1 + \Delta c)) f(x)dx \right) \end{aligned} \tag{7}$$

3.5. Delivery performance improvement with managerial efforts

To decrease the uncertainty of a supply chain delivery process and optimize the delivery performance of the supply chain, managerial efforts can be applied to reduce the variance of activities within CF-GERT network. Specifically, the adopted relation (see Eq. (5)) between applied efforts and uncertainty reduction for each activity (i,j) is $\frac{\sigma'_{ij}}{\sigma_{ij}} = (1-\epsilon_{ij}) \times \left(\frac{1-\epsilon_{ij}}{\tau_{ij}}\right)^{-\gamma_{ij}} + \epsilon_{ij}$.

Different levels of effort should be applied to each activity to reduce their variances and thus change the PDF of each activity. Accordingly, the delivery time distribution $f(x)$ and the total expected cost C will also change. Concerning the effort, the total expected cost is defined as follows.

$$\begin{aligned} \hat{Y} &= \sum_{(i,j)\in A} \hat{Y}_{ij} + \hat{Y}_0 = \\ &= \sum_{(i,j)\in A} \hat{Y}_{ij} + QH \int_0^{\hat{c}_1} (\hat{c}_1 - \hat{x}) f(\hat{x})d\hat{x} \\ &\quad + K \int_{\hat{c}_1+\Delta c}^{\infty} (\hat{x} - (\hat{c}_1 + \Delta c)) f(\hat{x})d\hat{x}, \end{aligned} \tag{8}$$

where $\hat{Y}_{ij} = Y_{ij} + \gamma_{ij} Y_{ij}$ represents the sum of the fixed activity cost and the applied effort, the new delivery time PDF with applied efforts is $f(\hat{x})$, and $\hat{c}_1$ is the changed optimal position of the delivery window.

To improve delivery performance, we need to minimize the total cost $\hat{Y}$ by determining the optimal value of $\hat{c}_1^*$ and the applied efforts γ_{ij}^*. That can be defined as.

Model II $\hat{c}_1^*, \gamma_{ij}^*$

$$= \arg\min_{c_1,\gamma_{ij}} \hat{Y}$$
$$= \arg\min_{c_1,\gamma_{ij}} \left(\sum_{(i,j)\in A} \hat{Y}_{ij} + \hat{Y}_0 \right) =$$
$$= \arg\min_{c_1,\gamma_{ij}} \left(\begin{array}{l} \sum_{(i,j)\in A} (Y_{ij} + \gamma_{ij} Y_{ij}) \\ +QH \int_0^{\hat{c}_1} (\hat{c}_1 - \hat{x}) f(\hat{x}) d\hat{x} \\ +K \int_{\hat{c}_1+\Delta c}^{\infty} (\hat{x} - (\hat{c}_1 + \Delta c)) f(\hat{x}) d\hat{x} \end{array} \right) \tag{9}$$

Moreover, the total budget is limited, thus the budget constraint is added as.

$$s.t. \sum_{(i,j)\in A} \hat{Y}_{ij} < \chi \tag{10}$$

4. Optimal solution

Delivery performance improvement can be completed in 3 steps.

Step 1. The probability density function *f(x)* of delivery time is derived by using the CF-GERT model. The complex and concrete supply chain delivery process is decomposed into activities, which are described by a GERT network. Then the delivery time pdf *f(x)* is found using Eq. (4).

Step 2. Assuming that no effort is applied to improve delivery performance, the optimal position of the delivery window c_1^* and the related expected total cost $C^*$are calculated based on Model I. Bushuev and Guiffrida (2012) showed that Eq. (1) is convex on c_1 and provided the optimal solution (Eq. 2). Compared to Eq. (1), Model I has an additional component, a sum of the planned costs of the activities (C_{ij}). Since the sum of the planned costs is not a function of c_1, Eq. (2) that allows finding an optimal position of the delivery window in Eq. (1) can be directly used for Model I.

The results of step 2 are used as a starting point in step 3; they also allow to analyze the monetary effect of the delivery performance improvement.

Step 3. When managerial efforts are applied, a delivery performance improvement model is proposed to minimize the total cost$\hat{C}$ under the limited budget (see Model II). The decision variables of this model are the position of the delivery window $\hat{c}_1^*$, and the applied effort for each activity γ_{ij}^* (the solution is discussed in the next section).

A solution to Model II is more complicated. The delivery time distribution $f(x)$ obtained by the analytical method of the CF-GERT network has a complex expression, it is difficult to analyze the formula of the expected cost, thus c_1 and γ_{ij} cannot be solved by analytical derivation. Therefore, a heuristic algorithm is required to solve Model II. The solution algorithm adopted in this paper is the particle swarm optimization (PSO) algorithm.

PSO (Parsopoulos and Vrahatis 2002; Bonyadi and Michalewicz 2017; Houssein et al. 2021) is easy to implement. In PSO, every particle in the search space is regarded as the potential solution to the problem to be optimized. In the optimization problem proposed in this paper, the particles are the applied effort γ_{ij} and the delivery position c_1.

The PSO only needs to handle two equations when updating the position and velocity in each generation. Particularly, the velocity and position of each particle i can be expressed as ${v_i}^{k+1}$ and ${x_i}^{k+1}$, respectively, the formulas are as follows:

$${v_i}^{k+1} = w{v_i}^k + c_1 r_1(\text{pbest}_i{}^k - {x_i}^k) + c_2 r_2(\text{gbest}^k - {x_i}^k) \tag{11}$$

$${x_i}^{k+1} = {x_i}^k + {v_i}^{k+1}, \tag{12}$$

where ${x_i}^k$ represents the current position of the particle i in the kth iteration, ${v_i}^k$ represents the velocity of the particle i in the kth iteration, ${pbest_i}^k$ represents the optimal value searched by the particle i, and $gbest^k$ represents the optimal value searched by the entire cluster in the kth iteration. In addition, w refers to the inertia weight, r_1, r_2 are the random numbers of the uniform distribution within the range of $[0, 1]$, and c_1, c_2 represents learning factors used to control the significance of the best solution.

Since Model II is a constrained multivariable nonlinear optimization, classical PSO fails to handle the constraint. Therefore, the particle swarm algorithm based on the penalty function (Parsopoulos and Vrahatis 2002; Coath and Halgamuge 2003) is adopted to solve the optimal window position $\hat{c}_1^*$ and the percentage of applied effort γ_{ij}^*. The algorithm flow is

5. Illustrative case

Make-to-Order (MTO), also referred to as a 'Pull Supply Chain' strategy, is a production or assembly strategy, in which the processes, such as production, assembly, and distribution are driven by customer orders. Therefore, the company, that adopts the Pull Supply Chain strategy, has to provide commodities according to the customer's specific requirements and instructions. In such a case, the delivery times are prolonged since the customization of the product takes longer than generic goods. Company Q is a medium-sized agricultural machinery manufacturer that starts production only after a customer place

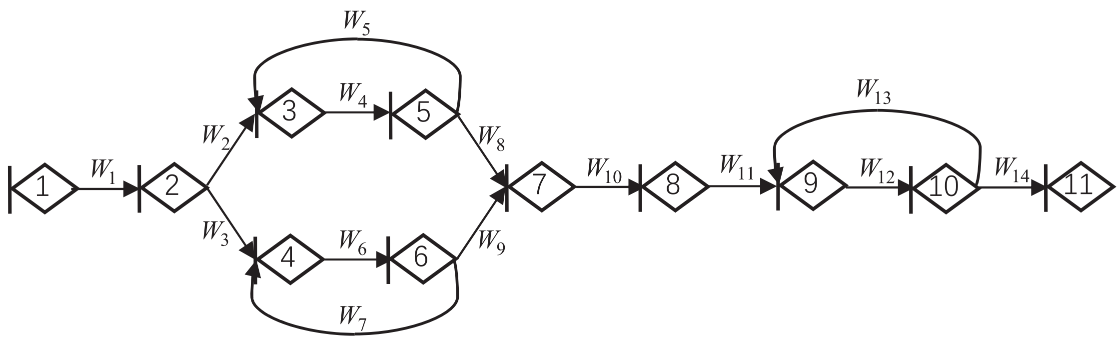

Figure 3. CF-GERT network of a delivery process for a specific product.

Table 4. Parameters of the CF-GERT network.

arc	Activity	p_{ij}	$\varphi_{ij}(t)$	W function	Fixed Cost
1-2	Product design	1	$exp(3it)$	$W_1 = exp(3it)$	10
2-3	production of diesel engines	0.6	$exp(7it-4t^2)$	$W_2 = exp(7it-4t^2)$	10
2-4	production of generator	0.4	$exp(7it-3t^2)$	$W_3 = exp(7it-3t^2)$	10
3-5	Testing of diesel engine	1	$exp(6it)$	$W_5 = exp(6it)$	10
5-3	Repair of diesel engine	0.05	$exp(5it-3t^2)$	$W_5 = 0.05exp(5it-3t^2)$	10
4-6	Testing of generator	1	$exp(6it)$	$W_6 = exp(6it)$	10
6-4	Repair of generator	0.03	$exp(5it-2t^2)$	$W_7 = 0.03exp(5it-2t^2)$	10
5-7	Transportation of diesel engine	0.95	$exp(6it-3t^2)$	$W_8 = 0.95exp(6it-3t^2)$	10
6-7	Transportation of generator	0.97	$exp(7it-4t^2)$	$W_9 = 0.97exp(7it-4t^2)$	10
7-8	Preparation of all assembled parts	1	$exp(3it)$	$W_{10} = exp(3it)$	10
8-9	Production assembly	1	$exp(6it-3t^2)$	$W_{11} = exp(6it-3t^2)$	10
9-10	Product Inspection	1	$exp(14it-7t^2)$	$W_{12} = exp(14it-7t^2)$	10
10-9	Repair	0.02	$exp(7it-4t^2)$	$W_{13} = 0.02exp(7it-4t^2)$	10
10-11	Delivery to customer	0.98	$exp(14it-7t^2)$	$W_{14} = 0.98exp(14it-7t^2)$	10

an order. Suppose company Q has to improve the delivery performance for type D agricultural machinery. The specific assembly and delivery process is described by a GERT network in Figure 3, and the detailed information is in Table 4.

Parameters are as follows. For the sake of convenience, the cost of all activities is set at 10.

5.1. Delivery performance improvement

The 3 steps of the delivery performance improvement process are provided below.

Step 1. Calculate the probability density function $f(x)$ for the original supply chain delivery time before uncertainty reduction.

From the CF-GERT analytical algorithm, Mason's rule is employed to obtain the equivalent transfer function W_E of the network. After Mason's rule is used, each transfer function (W) is substituted by its expression from Table 4.

$$W_E = \frac{\begin{array}{c} W_1W_2W_4W_8W_{10}W_{11}W_{12}W_{14} \\ \times(1-W_6W_7) + W_1W_3W_6W_9W_{10}W_{11} \\ \times W_{12}W_{14}(1-W_4W_5) \end{array}}{\begin{array}{c} 1 - W_4W_5 - W_6W_7 - W_{12}W_{13} + W_4W_5 \\ \times W_6W_7 + W_4W_5W_{12}W_{13} + W_6W_7W_{12}W_{13} \\ -W_4W_5W_6W_7W_{12}W_{13} \end{array}}$$

$$\begin{aligned} &= (0.5586exp(59it-24t^2)(1-0.03exp(11it-2t^2)) \\ &\quad + 0.38024exp(60it-24t^2) \\ &\quad \times (1-0.05exp(11it-3t^2))/ \\ &\quad \times (1-0.05exp(11it-3t^2) \\ &\quad - 0.03exp(11it-2t^2) \\ &\quad - 0.02exp(21it-11t^2) + 0.0015exp(22it-5t^2) \\ &\quad + 0.001exp(32it-14t^2) + 0.0006exp(32it \\ &\quad - 13t^2) - 0.00003exp(43it-16t^2)) \end{aligned}$$

Then Theorem 1 indicates that the success probability is $P_E = W_E(0) = \frac{0.90307}{0.90307} = 1$, and the equivalent characteristic function of the CF-GERT network is $\varphi_E = \frac{W_E}{P_E} = W_E$.

According to Eq. (4), the probability density function $f(x)$ of the delivery time can be obtained from the equivalent characteristic function (φ_E). Let N, a, and b be 10,000, 0, and 120, respectively, then the probability density function of the delivery process can be derived as follows.

$$f(x) = {\sum_{k=0}^{N-1}}' \frac{2}{b-a} \mathrm{Re}\left\{\varphi_E\left(\frac{k\pi}{b-a}\right)\exp\left(-i\frac{ka\pi}{b-a}\right)\right\}$$

Algorithm 1: Penalty Function PSO algorithm solving $\hat{c}_1^*$ and γ_{ij}^*

Input: Population size NP, characteristic function $\hat{\varphi}_E$, the maximum number of iterations G, the constrain function h derived from Eq. (10)
the upper bound of variables $U = \{c_1, \gamma_{ij}\} = \{u(1), u(2), ..., u(n)\}$
and lower bound of variables $L = \{c_1, \gamma_{ij}\} = \{l(1), l(2), ..., l(n)\}$
the parameters N, a, b in Eq. (4)
Output: $\hat{c}_1^*$ γ_{ij}^* $F_{best} = \hat{C}^*$
for k=1: N **do**
  calculate $f(x)$ using Eq. (4)
  calculate $\hat{C}$ using Eq. (8) based on $f(x)$
Return the penalty function-based cost function $F = \hat{C} + 1000\max(0, h)$
Create and initialize an N D-dimensional swarm.
Repeat
  for each particle $i = 1, 2, ..., N$ **do**
    if $h(X_i) \le 0$ **then**
      if $F(X_i) < F(P_{best_i})$ **then**
        $P_{best_i} = X_i$;
      end
      if $F(P_{best_i}) < F(G_{best})$ **then**
        $G_{best} = P_{best_i}$;
      end
    end
  end
  for each particle $i = 1, 2, ..., N$ **do**
    Update particle's velocity using Eq. (11).
    Update particle's position using Eq. (12).
  end
Until maximum iteration is reached;
Return F_{best}

$$\times \cos\left(k\pi \frac{x-a}{b-a}\right)$$

The expected completion time, variance, and standard deviation from node 1 to node 11 are calculated as follows:

$$E(x) = \frac{1}{i}\frac{\partial}{\partial t}[\varphi_E(t)]|_{t=0} = \frac{\partial}{\partial t}\left[\frac{W_E(t)}{W_E(0)}\right]\Bigg|_{t=0} = 60.31$$

$V(x) = \frac{\partial^2}{\partial t^2}[\frac{W_E(t)}{W_E(0)}]|_{t=0} - \{E(x)\}^2 = 63.57$ and $\sqrt{V(x)} = 7.97$.

Step 2. Eq. (2) is utilized to solve the optimal position of the delivery window c_1^* and the expected total cost C^* without applying effort (Model I).

Let $\Delta c = 5$, the final expected total cost C^* tends to be 1092.71 and the final optimal delivery window is $(57.43, 62.43)$.

Step 3. Algorithm 1 is employed to find the optimal position of the delivery window $\hat{c}_1^*$ and the related total cost $\hat{C}^*$ when managerial efforts are applied (Model II).

The method of controlling variables is used to adjust the parameters of the PSO algorithm. First, fix the values of NP and c_1, c_2, and observe the effect of w on the results. When NP $= 100$, $c_1, c_2 = 2$, the value of w is taken from 0.4 to 0.9 at intervals of 0.1. The results show that when $w = 0.8$, the total expected cost is the smallest. Second, fix the values of NP and w and observe the effect of different values of c_1 and c_2 on the results. When NP $= 100$ and $w = 0.8$, the different values of c_1 and c_2 are shown in the following table. The results show that when c_1 and c_2 are 2, the total expected cost is the smallest, which is consistent with the values suggested in Zomaya (2006). Finally, the influence of NP on the results is studied by fixing the values of w and c_1, c_2. When c_1, $c_2 = 2$, and $w = 0.8$, the value of NP is taken as 50, 100, 150, and 200 respectively, and it is found that the best value of NP

Table 5. Comparison of PSO algorithm results under different parameter settings.

NP	w	c_1, c_2	Delivery window	γ_{ij} for each activity	Cost
100	0.8	2, 2	57.20, 62.20	[1.91, 0.59, 0, 0.88, 0.65, 1.43, 1.48, 1.82, 0.44, 2.03]	795.50
100	0.4	2, 2	57.35, 62.35	[1.47, 1.70, 2.11, 0, 1.06, 1.53, 1.35, 1.48, 0.90, 1.87]	810.61
100	0.5	2, 2	57.07, 62.07	[2.42, 1.91, 0.42, 0.40, 2.86, 1.45, 0.69, 1.71, 0.46, 1.85]	821.39
100	0.6	2, 2	57.24, 62.24	[0.65, 2.77, 0.76, 0.20, 1.19, 2.10, 1.16, 2.11, 0.07, 2.85]	817.93
100	0.7	2, 2	57.29, 62.29	[1.41, 0, 0, 1.03, 1.03, 1.97, 1.18, 1.86, 0, 3]	814.13
100	0.9	2, 2	57.19, 62.19	[1.07, 1.59, 0, 1.70, 1.57, 1.48, 1.44, 1.36, 1.13, 1.97]	810.35
100	0.8	0.2, 4	57.02, 62.02	[1.15, 1.80, 1.73, 1.79, 1.18, 1.65, 1.18, 1.22, 0, 1.28]	821.32
100	0.8	4, 0.2	57.93, 62.93	[1.91, 1.05, 1.38, 0, 1.05, 1.15, 3, 1.45, 0, 3]	818.92
100	0.8	1, 2	57.45, 62.45	[1.14, 1.00, 1.52, 0, 1.32, 1.31, 1.29, 2.27, 1.37, 2.15]	807.83
100	0.8	2, 1	57.20, 62.20	[2.52, 1.26, 0, 0.78, 0.87, 2.03, 2.33, 2.25, 0.82, 1.51]	814.50
100	0.8	1.49445, 1.49445	56.65, 61.65	[1.15, 3, 0, 1.87, 0.61, 0.89, 1.70, 0.98, 0, 1.11]	829.69
50	0.8	2, 2	57.87, 62.87	[0.67, 1.46, 0, 0.96, 2.09, 1.74, 1.64, 1.44, 0, 1.77]	805.63
150	0.8	2, 2	57.23, 62.23	[1.30, 1.08, 1.21, 0.38, 2.89, 0.65, 1.15, 1.87, 0, 2.70]	810.96
200	0.8	2, 2	57.35, 62.35	[1.31, 1.98, 0, 0, 3, 0.33, 1.31, 3, 0, 1.53]	811.78

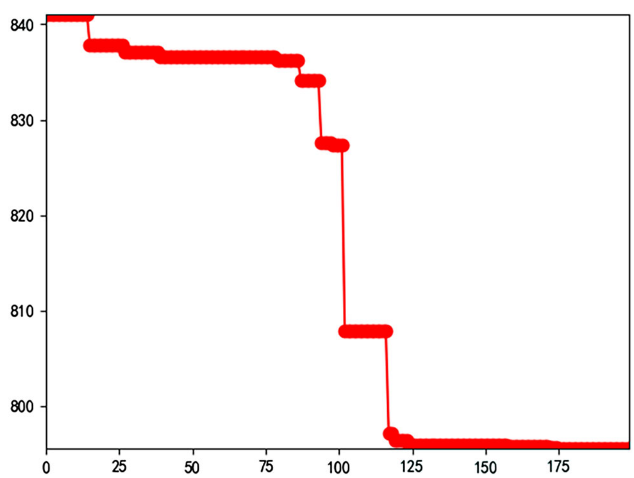

Figure 4. Fitness value of $\hat{C}^*$.

is 100. Therefore, to sum up, when NP $= 100$, $w = 0.8$, $c_1, c_2 = 2$, the particle swarm algorithm provides the best solution effect, which is underlined in Table 5.

The analysis shows that the managerial efforts can be taken for activities 2-3, 2-4, 5-3, 6-4, 5-7, 6-7, 8-9, 9-10, 10-9 and 10-11, while durations of other activities are constant. Algorithm 1 is used to solve the optimal percentage of applied effort γ_{ij} for those activities and the optimal $\hat{c}_1^*$ to minimize the total cost.

Let $\Delta c = 5$, and the parameters of particle swarm algorithm $w = 0.8$, $c_1 = 2$, $c_2 = 2$, $NP = 100$, $G = 200$, $U = \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}$, $L = \{100, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3\}$. After 200 iterations, the fitness function, i.e. the final expected total cost $\hat{C}^*$ tends to 795.50, the percentage of applied effort γ_{ij} taken for each activity is (1.91, 0.59, 0, 0.88, 0.65, 1.43, 1.48, 1.82, 0.44, 2.03) and the final optimal delivery window is (57.20, 62.20). The optimal individual fitness value is shown in Figure 4.

From the above analysis, we can find that the final expected total cost with applied effort is reduced by 297.21 or 27%. Therefore, under the condition that other parameters remain unchanged, applying effort to reduce the variance of activities can effectively reduce the expected cost of the delivery.

Table 6. Comparison of Monte Carlo simulation result and the proposed model solution.

Categories	Mean time	Standard deviation	Probability	Total cost
Monte Carlo simulation result	60.33	4.80	1	799.44
Proposed model	60.31	4.71	1	795.50
Relative error	0.04%	2.02%	0%	0.49%

5.2. Model validation

To verify the effectiveness of the proposed model, we established a Monte Carlo simulation model using Python software. According to the information about each activity in Model II, the delivery time is simulated 10,000 times using the Monte Carlo method. The comparison between the results of the Monte Carlo simulation and the proposed model is shown in Table 6. It can be seen that all relative errors are within 2.02%. Simulation results demonstrate the effectiveness and feasibility of the proposed model in improving supply chain delivery performance at the activity level. Specifically, the relative error of the total cost is 0.49%.

A comparison of the delivery time PDF and CDF of the Monte Carlo simulation model and the proposed method is shown in Figure 5.

5.3. Sensitivity analysis

There are many adjustable parameters in the model, including the width of the delivery window Δc, the penalty cost per time unit early QH and late K (levied by the buyer), and parameters of administrative measures

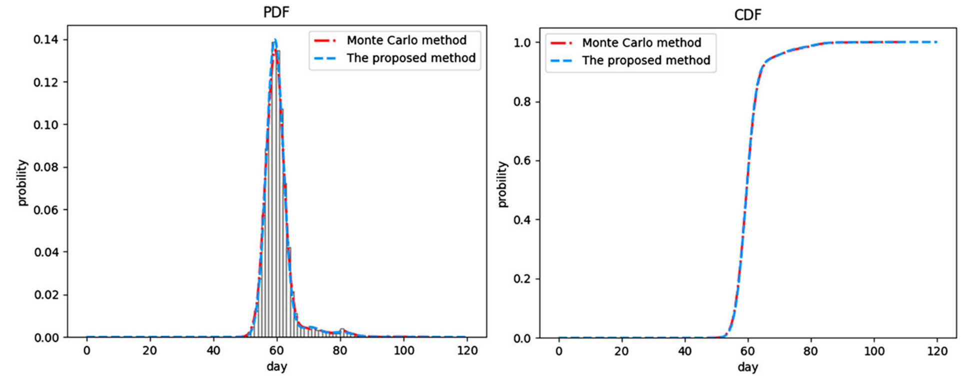

Figure 5. PDF and CDF comparison.

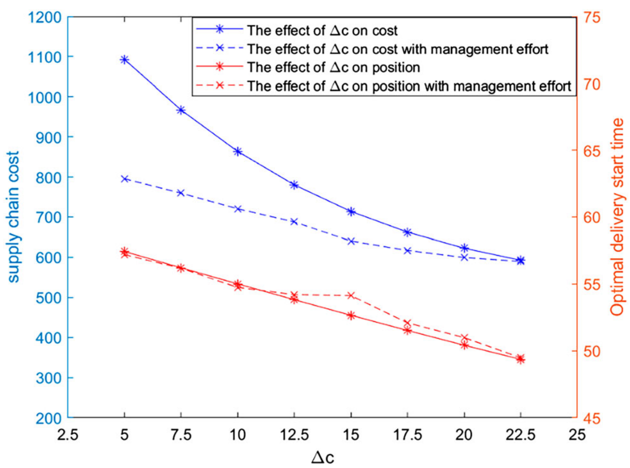

Figure 6. The effect ofΔc on cost and position.

ϵ_{ij}, τ_{ij}. Next, we will present the effects of these parameters on the optimal position of the delivery window and the optimal cost.

5.3.1. The effect of delivery window width Δc

In this section, let $\frac{QH}{K} = \frac{150}{150}$, $\chi = 150$, and $\epsilon_{ij}, \tau_{ij} = 0.1$ for each activity (i,j). Then we analyze the effect of the width of the optimal position of the delivery window (Δc) and the excepted costs. More precisely, the value of Δc is selected from the set $\{5, 7.5, 10, 12, 5, 15, 17.5, 20, 22.5, 25\}$, we calculate the optimal position and the excepted cost without applying effort using Eq. (2) and calculate the optimal position and the excepted cost after applying effort using Algorithm 1. Finally, we plot the optimal positions and costs both with and without applying efforts (represented by the vertical axis) for different values of Δc (represented by the horizon axis) in Figure 6.

Through the analysis of the above figure, we can draw the following conclusions:

(i) With no applied effort to activities (Model I), the expected cost has a decreasing trend, and the optimal position of the delivery window has also decreased. Bushuev (2018) came up with the same conclusion.

(ii) Compared to the cost without applied efforts, the total cost after variance reduction is lower even though the applied efforts result in additional investment. This indicates that the proposed approach is beneficial to the delivery performance improvement for different values of Δc.

(iii) The difference between the original cost and the one with applied efforts depends on the width of the delivery window. More precisely, the difference increases as the width Δc decreases. In other words, the proposed approach is more efficient for tight delivery windows.

(iv) The optimal positions of the delivery window for the two models share a similar decline trend with the increase of the width. Besides, the optimal positions in Model I and II are similar.

5.3.2. The effect of QH and K

In this section, let $\Delta c = 5$, $\chi = 150$, and $\epsilon_{ij}, \tau_{ij} = 0.1$ for each activity (i,j). Then we analyze the effect of QH and Kon the optimal position of the delivery window and the excepted costs for $QH \in \{110, 120, 130, 140, 150, 160, 170, 180, 190\}$, $K \in \{110, 120, 130, 140, 150, 160, 170, 180, 190\}$. When exploring the effect of QH (or K), K (or QH) is 150. We calculate the optimal position and the excepted cost without applying effort using Eq. (2) and calculate the optimal position and the excepted cost after applying effort using Algorithm 1. Finally, we plot the optimal positions and costs both with and without applying efforts (represented by the vertical axis) for different values of QH and K (represented by the horizon axis) in Figure 7.

Through the analysis of the above Figure 7, we can draw the following conclusions:

(i) With no applied effort, Figure 7 reveals that there has been a marked increase in the value of total cost

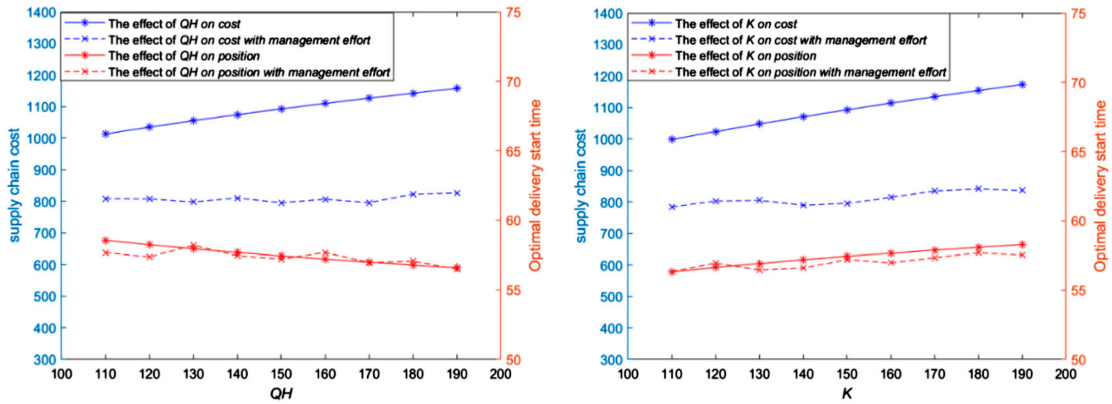

Figure 7. The effect of QH and K on cost and position.

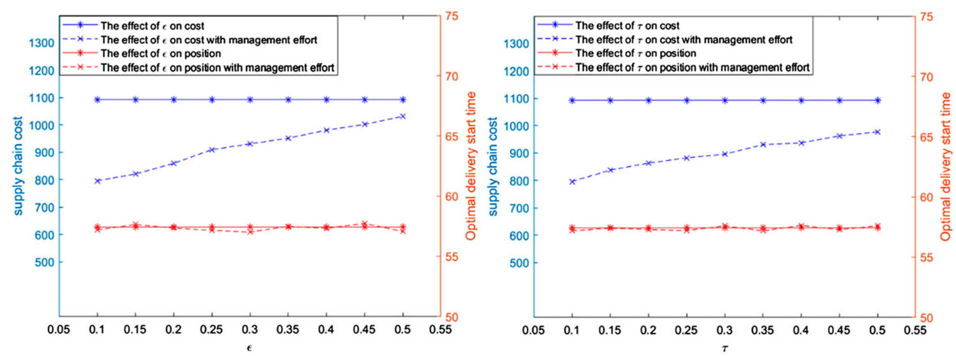

Figure 8. The effect ϵ_{ij} and τ_{ij} on cost and position.

when the QH increases. By contrast, the total cost for the model with applied effort tends to be at a relatively stable level, which is lower than that of the original cost without applied effort. Likewise, the effect of K on total cost operates similarly. More precisely, the cost with managerial effort under the influence of different values of QH is around 810, and the deviation is within 3%, while the cost with managerial effort under the influence of different values of K is around 810, and the deviation is within 5%.

(ii) No matter whether the variance reduction is considered, the increase of QH leads to a smaller optimal position of the delivery window. On the other hand, a larger K always results in a larger optimal position. This finding is comparable to the one in Bushuev (2018).

(iii) Figure 7 shows that there is a gradual rise in the gap between the cost without variance reduction and the one with applied efforts when QH or K increases. This phenomenon indicates that it is more effective to apply managerial effort to improve delivery performance when the penalties per unit time of untimely delivery are higher.

5.3.3. The effect of ϵ_{ij}, τ_{ij}

In this section, let $\frac{QH}{K} = \frac{150}{150}$, $\Delta c = 5$, and $\chi = 150$ for each activity (i, j). Then we analyze the effect of ϵ_{ij} and τ_{ij} on the optimal position of the delivery window and the excepted costs. When $\epsilon_{ij} \in \{0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5\}$, $\tau_{ij} \in \{0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5\}$, we calculate the optimal position and the excepted cost without applying effort using Eq. (2) and calculate the counterparts after applying effort using Algorithm 1. Finally, we plot the optimal positions of the delivery window and costs both with and without applying efforts (represented by the vertical axis) for different values of ϵ_{ij} and τ_{ij} (represented by the horizon axis) in Figure 8.

Through the analysis of the above figure, we find that with the increase of ϵ_{ij} and τ_{ij}, the cost in Model II tends to increase continuously. And with the increase of ϵ_{ij} and τ_{ij}, the magnitude of the cost reduction is reduced when managerial efforts are applied compared to Model I (no applied effort), which indicates that the greater values of ϵ_{ij} and τ_{ij} increase the difficulty of applying effort to reduce the variability of activity time.

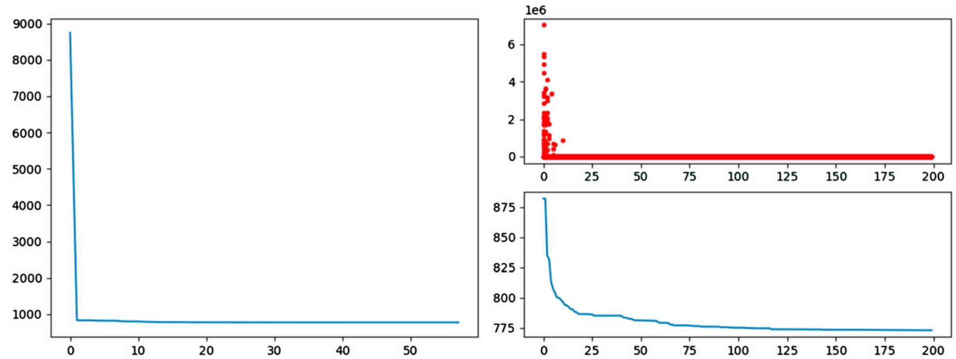

Figure 9. The iterative process of the two algorithms.

Table 7. Comparison of simulated annealing algorithm, genetic algorithm, and PSO algorithm.

Algorithm	Delivery window	Applied effort γ_{ij} for each activity	Cost	$\|\Delta\text{cost}\|/\text{cost}^*$
Simulated annealing	57.24, 62.24	[1.15, 0.85, 0, 0, 1.03, 0.98, 1.27, 1.65, 0, 1.65]	771.71	0%
Genetic algorithm	57.24, 62.24	[1.22, 0.86, 0, 0.01, 0.75, 0.99, 1.26, 1.65, 0, 1.66]	772.94	0.16%
PSO	57.20, 62.20	[1.91, 0.59, 0, 0.88, 0.65, 1.43, 1.48, 1.82, 0.44, 2.03]	795.50	3.08%

5.4. Comparison with other algorithms

The simulated annealing algorithm and the genetic algorithm are used for comparison. The results are shown in the following Table 7 and the iterative process of the two algorithms is shown in Figure 9. It can be seen that the positions of the delivery window obtained by the three algorithms are almost the same. The difference lies in the applied effort taken for each activity and the final expected total cost. The different applied effort taken for each activity results in different final expected total costs. The final expected total costs of simulated annealing and the genetic algorithm are lower than that of the PSO algorithm, but it can be seen from the last column that the deviation of the final expected total cost of the PSO algorithm is controlled within 3.08%. $|\Delta\text{cost}|$ in the last column represents the difference between the cost in the current row and the lowest cost (represented as cost*) in Table 7.

6. Conclusion

For any enterprise, the supply chain delivery process is one of the core processes that deserve attention. Timely delivery can not only allow to avoid extra penalty costs but also build a good reputation for the enterprise and provide a competitive advantage. Enterprises need to evaluate and improve their supply chain delivery processes to achieve the goal of timely delivery. For any enterprise eager to improve its delivery performance, it is inevitable to optimize the supply chain delivery process.

For supply chain management, time and cost are two important factors that are closely watched. Therefore, this paper adopts a cost-based delivery performance model and proposes a three-step approach for the evaluation and improvement of supply chain delivery performance. In the first step, a delivery process is divided into various activities, the CF-GERT model is used to describe the whole process, and the distribution of the delivery time is derived from the delivery time distributions of all activities. In the second step, an optimal position of the delivery window which defines when the product should be shipped is calculated for the initial (not improved) delivery process. The information derived in steps 1 and 2 is used in the third step for delivery performance improvement. Step 3 applies the particle swarm algorithm to obtain the optimal amount of applied effort adopted for each activity and the optimal position of the delivery window.

The illustrative case shows that the expected cost can be reduced by reducing the variance of activity time. In addition, the effects of different parameters on the expected cost of the supply chain and the position of the delivery window are studied. The numerical results show an effect of different parameters on the optimal position of the delivery window and the related cost. Specifically, with the increase of the width of the delivery window (Δc), the cost after applying effort tends to decrease. However, the downward trend is declining, which means that when the value Δc is relatively large, efforts to reduce the activities variance seem to have less and less impact on the cost, and people have to find other ways to reduce the total cost. With the increase of penalties for untimely delivery (QH and K), the cost with managerial effort is sharply reduced compared to the cost without managerial effort. This phenomenon indicates that with the increase of QH and K, it is more effective to apply effort to optimize delivery performance. And with the increase of ϵ_{ij} and τ_{ij}, the magnitude of the cost reduction is reduced when managerial efforts are applied compared to the situation when no managerial efforts are applied. It indicates that the greater ϵ_{ij} and τ_{ij} values increase the difficulty of applying effort to reduce the variability of activity time. Overall, the numerical example shows that the smaller the width of the delivery window and the higher the penalties for untimely delivery, the more beneficial for a supplier to invest in delivery performance improvement. Given that buyers' expectations for timely deliveries increase, suppliers need to improve delivery performance.

Compared to the black-box models, the proposed approach breaks down the delivery process into activities and provides a detailed analysis of the effect of each activity on delivery performance and related costs. That provides deeper insight into a supply chain delivery process and offers more opportunities for delivery performance improvement. Moreover, the proposed approach is applicable to a wide range of delivery processes. It is not limited to a specific form of delivery time distribution and can be used with any form of a network of delivery process activities including parallel, serial, and loop forms of connections. The sensitivity analysis shows that the proposed approach can be used for different values of the model parameters such as the width of the delivery window and the penalties for untimely delivery.

The proposed approach for improving delivery performance provides managerial insights for delivery performance improvement. The paper can serve as guidance for practitioners undertaking a program to improve delivery performance. A supplier can analyze its delivery process and identify what activities are the most critical in the delivery process in terms of their effect on the delivery related costs. That should help the supplier to understand what activities in the delivery process should be improved, estimate the amount of savings, and justify the financial benefits of the improvements. Moreover, the approach determines specific amounts of managerial effort that should be applied to each activity to minimize the cost. It provides managers with a specific plan for delivery performance improvement and helps them to formulate effective strategies to improve supply chain delivery performance.

At the same time, the proposed methodology has several shortcomings. First, it assumes that information about each activity of the delivery process is available including the duration of the activity and the amount of effort (cost) required to reduce the duration. In reality, it might be hard to know in advance how much afford will be required to reduce the duration of an activity. Second, relationships between activities should be known and fixed, but that is not always the case. Third, activities are independent of each other, thus effort applied to one activity does not affect the other activities. In fact, the same effort might affect more than one activity that is not considered in this research.

From future research, we can draw several research directions. Firstly, besides paying attention to the activity time variances, we should also study the influence of the expected values of the activity times on the delivery performance. Second, a more robust algorithm for solving this multivariable nonlinear optimization problem should be considered.

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This research was previously published as conference proceedings (Tao et al. 2022). This paper differs from the proceedings in several ways. First, section 3 is extended to provide additional details about the supply chain delivery performance model. Second, section 5 was added. The section includes an illustrative case with model validation, sensitivity analysis, and comparison of PSO to other algorithms. Third, section 6 was extended to discuss the results of the illustrative case analysis, provide additional managerial insights, and outline shortcomings of the proposed methodology.

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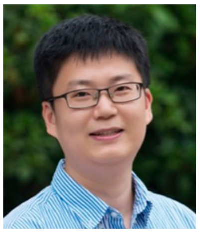

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Data Availability Statement

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

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