



# Forecasting cryptocurrencies prices using data driven level set fuzzy models

Leandro Maciel<sup>a,\*</sup>, Rosangela Ballini<sup>b</sup>, Fernando Gomide<sup>c</sup>, Ronald Yager<sup>d</sup>

<sup>a</sup> Department of Business Administration, Faculty of Economics, Business and Accounting, University of São Paulo, São Paulo, Brazil

<sup>b</sup> Department of Economic Theory, Institute of Economics, University of Campinas, São Paulo, Brazil

<sup>c</sup> Department of Computer Engineering and Automation, School of Electrical and Computer Engineering, University of Campinas, São Paulo, Brazil

<sup>d</sup> Machine Intelligence Institute, Iona College, New Rochelle, NY, USA

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## ABSTRACT

The paper develops fuzzy models to forecast cryptocurrencies prices using a data-driven fuzzy modeling procedure based on level set. Data-driven level set is a novel fuzzy modeling method that differs from linguistic and functional fuzzy modeling in how the fuzzy rules are built and processed. The level set-based model outputs the weighted average of output functions of active fuzzy rules. Output functions map the activation levels of the fuzzy rules directly in model outputs. Computational experiments are done to evaluate the level set method in one-step-ahead forecasting of the closing prices of cryptocurrencies. Comparisons are made with the autoregressive integrated moving average, multi layer neural network, and the naïve random walk as a benchmark for Cardano, Binance Coin, Bitcoin, Ethereum, Chainlink, Litecoin, Tron, Stellar, Monero and Ripple. The results suggest that the random walk outperforms most methods addressed in this paper, confirming the Meese–Rogoff puzzle for the case of digital coins, i.e. the difficulty to surpass the naïve random walk in predicting exchange rates. However, when performance is measured by the direction of price change, the level set-based fuzzy modeling performs best amongst the remaining methods.

## 1. Introduction

Since the creation of block chain and Bitcoin (Nakamoto, 2008), one of the most popular cryptocurrencies, a rapid growth of the digital coin market has been verified. According to CoinMarketCap website, the market capitalization of cryptocurrencies on February 14, 2022 was higher than USD 1.91 trillion, with Bitcoin accounting for more than USD 810 billion.<sup>1</sup> In contrast to traditional cash systems, advantages of cryptocurrencies are decentralization, security and privacy, easy transfer of funds, and lower transaction costs (Mukhopadhyay et al., 2016). More than 2000 types of cryptocurrencies are currently available for public trading, which reveals the significance of digital coins as an electronic payment system and financial asset, attracting substantial interest from the general public, investors, and researchers (Balcilar et al., 2017; Zhang et al., 2021).

One particular feature of most cryptocurrencies is the high volatile price dynamic, which directly affects investors and speculators profits and losses. The low correlations of digital coins with conventional assets also make the analysis of future price fluctuations more difficult (Parfenov, 2022). Despite their complexity and risky characteristics, cryptocurrencies are an alternative investment and portfolio

diversification instrument (Sun et al., 2020). For example, Selmi et al. (2018) stated that Bitcoin serves as a hedge, a safe haven, and a diversifier for oil price movements in terms of diversification opportunities and downside risk reductions. The development of accurate price-forecasting models for cryptocurrencies is of key interest by market participants and managers.

The price of cryptocurrencies is influenced by many factors such as the movements of macroeconomic variables, news and fake news, government policies, and social media contents that induce volatility (Philippas et al., 2019). Researchers have developed many models to predict future prices movements of cryptocurrencies, which can be broadly organized into i) time-series models such as Autoregressive Integrated Moving Average (ARIMA) (Tandon et al., 2021), and Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models (Fung et al., 2021), and (ii) machine learning models (Chowdhury et al., 2020), such as Support Vector Machines (SVM) (Hitam et al., 2019), neural networks (Alonso-Monsalve et al., 2020; Parvini et al., 2022; Zhang et al., 2021), and deep neural nets (Lahmiri & Bekiros, 2019; Ortu et al., 2022; Tanwar et al., 2021; Wei et al., 2021).<sup>2</sup>

This paper develops a data-driven fuzzy model to forecast time series of cryptocurrencies prices. The data-driven fuzzy model uses the

\* Corresponding author.

E-mail addresses: [leandromaciel@usp.br](mailto:leandromaciel@usp.br) (L. Maciel), [ballini@unicamp.br](mailto:ballini@unicamp.br) (R. Ballini), [gomide@unicamp.br](mailto:gomide@unicamp.br) (F. Gomide), [yager@panix.com](mailto:yager@panix.com) (R. Yager).

<sup>1</sup> Source: <https://coinmarketcap.com/>. Access on February 14, 2022.

<sup>2</sup> A review of artificial intelligence and machine learning models for cryptocurrency prices is found in Patel et al. (2022).

concept of level sets to shape a novel fuzzy modeling paradigm. It differs from previous fuzzy modeling paradigms in the way that the fuzzy rules are built and processed. The level set method outputs the weighted average of output function values of the active rules. Output functions map the activation levels of the rules directly in the output variable value. It has been shown that the data driven level set method (LSM) is simple, effective, and transparent (Maciel et al., 2022). The efficacy of LSM modeling is evaluated in forecasting the daily closing prices of ten leading cryptocurrencies: Cardano, Binance Coin, Bitcoin, Ethereum, Chainlink, Litecoin, Tron, Stellar, Monero and Ripple. Its performance is compared with the autoregressive integrated moving average (ARIMA), the naïve random walk, and a multilayer neural network (MLP).

Since cryptocurrencies price forecasting is still an emerging topic in the literature, especially due to the particularities of the market, fuzzy techniques appear as a potential modeling tool, because it naturally deals with the imprecision of the digital coin price movements. This aspect comprises the main contribution of this paper. For example, in March 2021, Elon Musk had announced in a series of tweets that people could begin buying Tesla cars with Bitcoin, after which the prices of Bitcoin rose about 5%.<sup>3</sup> The data driven level set method is a potential candidate to model and forecast nonlinear and time-varying dynamics of data such as cryptocurrency prices. The LSM is appropriate to model time series that are affected by intangibles like market sentiments which, because its fuzzy nature, limits the expressiveness of statistical and machine learning techniques. Additionally, this paper also address a large number of cryptocurrencies instead of focusing on Bitcoin solely. This work also evaluates the potential of a fuzzy technique in the cryptocurrency market, a technique that is still scarce in the literature of digital coin prices forecasting.

After this introduction, the paper proceeds as follows. Section 2 overviews the literature on the predictability of cryptocurrency prices. Section 3 summarizes the data driven fuzzy modeling based on the level set framework. Experiments concerning modeling and price forecasting of cryptocurrencies and their performance are reported in Section 3. Finally, Section 4 concludes the paper and suggests topics for future development.

## 2. Literature review

Methods for cryptocurrency price forecasting can be broadly divided into two main categories, time series modeling, and machine learning methods.

The use of econometric models for non-stationary time series prediction of financial data have been extensively explored in the literature, as is the case of digital coins. For instance, Catania et al. (2019) compares several univariate and multivariate models for point and density forecasting of Bitcoin, Litecoin, Ripple and Ethereum. A set of crypto-predictors based on dynamic model averaging to combine a large set of univariate dynamic linear models and several multivariate vector autoregressive models with different forms of time variation is addressed. The authors identify statistically significant improvements in point forecasting when using combinations of univariate models, and in density forecasting when relying on the selection of multivariate models. Both schemes delivered sizeable directional predictability. Forecasting of Bitcoin exchange rates using autoregressive integrated moving average (ARIMA) is accounted in Bakar and Rosbi (2017).

Modeling and forecasting of cryptocurrencies volatility is investigated in Katsiampa (2017) from the point of view of the ability of several competing GARCH-type models to explain the Bitcoin price volatility. The author finds evidence that the optimal model in terms of goodness-of-fit to the data is the AR-CGARCH, a result that suggests

the importance of having both a short-run and a long-run components of the conditional variance.

Most of the current literature focuses on the use of machine learning techniques to forecast in the cryptocurrency prices. The work in Kurbucz (2019) uses a single-layer artificial neural network and the most frequent edges of the Bitcoin (BTC) transaction network to predict its price on a daily basis. The work shows the higher accuracy of the neural network when compared with those that use historical prices.

Different feature selection techniques are used in Mallqui and Fernandes (2019) to find the most relevant attributes to predict Bitcoin prices. Artificial neural networks, Support Vector Machines, and Ensemble algorithms are adopted to predict price directions. The results disclose the selected attributes and the best machine learning model to achieve an accuracy improvement above 10% in price direction predictions compared with state-of-the-art techniques.

The work of Zhang et al. (2021) proposes a weighted and attentive memory channels model to predict the daily close price and the fluctuation of Ethereum and Bitcoin cash. The authors show that the model proposed achieves state-of-the-art performance and outperforms the baseline models from the point of view of prediction error, accuracy, and profitability.

Using intraday data, Gradojevic et al. (2021) used hourly and daily frequencies data to predict Bitcoin returns. Non-linear models were evaluated on a multitude of popular technical indicators that represent market trend, momentum, volume, and sentiment. Results indicate that technical analysis combined with non-linear forecasting models are statistically significantly dominant relative to the random walk model on a daily horizon and that the random forest model as the most accurate to predict Bitcoin prices.

Derivatives market of digital currencies are evaluated in Akyildirim et al. (2021) using various machine learning algorithms to predict mid-price movement for Bitcoin futures prices. Based on high-frequency intraday data, the authors measure the relative forecasting performances across various time frequencies, ranging between 5 and 60 min. They show that the average classification accuracy for five out of the six machine learning algorithms are consistently above the 50% threshold, indicating that they outperform benchmark models such as ARIMA and random walk in forecasting Bitcoin prices.

More recently, Parvini et al. (2022) investigated different predictors of various markets including Gold, Oil, S&P500, VIX, USDI, Ether and Ripple as well as Bitcoin historical prices to predict one-step-ahead Bitcoin returns. A two-stage forecasting method was suggested, comprising the discrete wavelet transform as the decomposition method, and a deep long short-term memory network as the forecaster. The results indicate that it is hard to uncover the best predictor for the period before the spike observed in Bitcoin return in 2018. However, after the 2018 spike, Gold and Oil show the highest statistical accuracy, while S&P500 is the most profit-making predictor.

Few studies have addressed price forecasting of cryptocurrencies using fuzzy models. Exceptions include (Atsalakis et al., 2019) which develops a hybrid neuro-fuzzy model to forecast the direction in the change of the daily price of Bitcoin. The neuro-fuzzy model outperformed two other computational intelligence-based models, a simple neuro-fuzzy, and an artificial neural network. The work of Garcia et al. (2019) addresses Bitcoin price forecasting using an evolving granular fuzzy rule-based model with a modified rule structure that includes reduced-term consequent polynomials, supplied with an incremental learning algorithm that simultaneously impute missing data, and update the model parameters and structure. The work indicates the high accuracy of the evolving model when compared with fuzzy and neuro-fuzzy evolving modeling methods for Bitcoin price prediction.

Most of the works reported in the literature evaluate the forecasting potential of machine learning and fuzzy-related techniques for the case of Bitcoin only. Clearly there is a demand to consider cryptocurrencies other than Bitcoin to produce more robust forecasting methods and their evaluation. This is especially notable in the realm of fuzzy forecasting techniques, as is the case of this paper.

<sup>3</sup> Source: <https://www.cnn.com/2021/03/24/elon-musk-says-people-can-now-buy-a-tesla-with-bitcoin.html>. Access on February 14, 2022.

### 3. Data driven level set modeling

This section summarizes the data driven fuzzy modeling based on the notion of level set. A detailed coverage is given in Leite et al. (2022). Consider a fuzzy model whose fuzzy rules are as follows

$$\mathcal{R}_i : \text{if } x \text{ is } \mathcal{A}_i \text{ then } y \text{ is } \mathcal{B}_i \quad (1)$$

where  $i = 1, 2, \dots, N$  and  $\mathcal{A}_i$  and  $\mathcal{B}_i$  are convex fuzzy sets with membership functions  $\mathcal{A}_i(x) : \mathcal{X} \rightarrow [0, 1]$  and  $\mathcal{B}_i(y) : \mathcal{Y} \rightarrow [0, 1]$ . Given an input  $x \in \mathcal{X}$ , the level set method is as follows Leite et al. (2022), Yager (1991).

1. Compute the activation degree of each rule  $\mathcal{R}_i$  as

$$\tau_i = \mathcal{A}_i(x) \quad (2)$$

2. Find the level set  $\mathcal{B}_{\tau_i}$  for each  $\tau_i$

$$\mathcal{B}_{\tau_i} = \{y | \tau_i \leq \mathcal{B}_i(y)\} = [y_{il}(\tau_i), y_{iu}(\tau_i)] \quad (3)$$

3. Compute the midpoint of the level set

$$m_i(\tau_i) = \frac{y_{il}(\tau_i) + y_{iu}(\tau_i)}{2} \quad (4)$$

4. Compute the model output  $y$  as

$$y(\tau) = \frac{\sum_{i=1}^N \tau_i m_i(\tau_i)}{\sum_{i=1}^N \tau_i} \quad (5)$$

where  $y_{il}(\tau_i)$  is the lower bound, and  $y_{iu}(\tau_i)$  is the upper bound of the  $i$ th level set, and  $\tau = (\tau_1, \dots, \tau_N)$ . When fuzzy set  $\mathcal{B}_i$  is discrete,  $m_i$  is the average of the elements of  $\mathcal{B}_{\tau_i}$ . We assume that there exists an  $i$  such that  $\tau_i > 0$ . Notice that the level set method assumes  $\mathcal{B}_i$  given, what is typical in knowledge-based fuzzy modeling. Often the membership functions of  $\mathcal{A}_i(x)$  can be estimated using fuzzy clustering, but in many circumstances it may not be easy to identify the membership functions of the consequents of the fuzzy rules because they specify the values of the output variables  $y$ , what can be difficult. This task can be alleviated when input-output data are available because we can approximate the output function specified in (5) using the data set and compute the output directly from the output functions. Data-driven level set modeling augments the knowledge-based level set modeling as follows.

Let  $F_i(\tau_i) = m_i(\tau_i)$ , and  $D = \{(x^k, y^k)\}$ ,  $x^k \in R^p$ ,  $y^k \in R$  such that  $y^k = f(x^k)$ ,  $k = 1, 2, \dots, K$  be a data set. The goal is to build a fuzzy model  $F$  to approximate the function  $f$  using  $D$  where

$$F(\tau) = \frac{\sum_{i=1}^N \tau_i F_i(\tau_i)}{\sum_{i=1}^N \tau_i} \quad (6)$$

In the simplest and most common cases  $F_i$  is affine

$$F_i(\tau_i) = v_i \tau_i + w_i \quad (7)$$

Coefficients  $v_i$  and  $w_i$  can be estimated using least squares-based procedures, regularized, recursive, or alternative procedures. For simplicity, we use the pseudo inverse-based solution because of the efficiency and availability of powerful solvers in open-source environments such as R and Python, and in tools such as Matlab. The essential steps are as follows.

For each data pair  $(x^k, y^k)$  compute the activation degrees  $\tau_i^k = \mathcal{A}_i(x^k)$ ,  $i = 1, 2, \dots, N$ , and let  $s^k = \sum_{i=1}^N \tau_i^k$ . From (6) and (7), the corresponding output is

$$z^k = \frac{\tau_1^k (v_1 \tau_1^k + w_1)}{s^k} + \dots + \frac{\tau_N^k (v_N \tau_N^k + w_N)}{s^k} \quad (8)$$

Let  $\mathbf{d}^k = [(\tau_1^k)^2/s^k, \tau_1^k/s^k, \dots, (\tau_N^k)^2/s^k, \tau_N^k/s^k]$  and let the vector of parameters  $\mathbf{u} = [v_1, w_1, \dots, v_N, w_N]^T$ . The expression (8) becomes

$$z^k = \mathbf{d}^k \cdot \mathbf{u}, \quad k = 1, \dots, K \quad (9)$$

If we let  $\mathbf{z} = [z^1, \dots, z^K]^T$ ,  $\mathbf{D} = [\mathbf{d}^1, \dots, \mathbf{d}^K]^T$ , and  $\mathbf{y} = [y^1, \dots, y^K]^T$ , then the set of Eqs. (9) can be expressed compactly as  $\mathbf{z} = \mathbf{D}\mathbf{u}$ . The vector of parameters  $\mathbf{u}$  solution of  $\min_{\mathbf{u}} \|\mathbf{y} - \mathbf{z}\|^2$  is

$$\mathbf{u} = \mathbf{D}^+ \mathbf{z} \quad (10)$$

where  $\mathbf{D}^+$  is the Moore–Penrose pseudo inverse of  $\mathbf{D}$  (Serre, 2010). Recalling that  $\mathbf{d} = \mathbf{d}(\tau_1, \dots, \tau_N)$  and that  $\tau_i = \mathcal{A}_i(x)$ , the model output for input  $x$  is

$$y = \mathbf{d} \cdot \mathbf{u} \quad (11)$$

which is equivalent to (5).

Summing up, the main steps of the data driven level set method (LSM) are as follows:

1. Cluster the data set  $D$  into  $N$  clusters.
2. Assign membership function  $\mathcal{A}_i$  to cluster  $i = 1, \dots, N$ .
3. Find consequent vector of coefficients using (10).
4. Compute model output using (11).

As it is well known in the fuzzy modeling literature, clustering can be done to estimate the membership functions of each  $\mathcal{A}_i$  using clustering algorithms, typically the fuzzy c-means (FCM) or its variations, and alternatives such as adaptive vector quantization, grid, or knowledge-based granulation. The membership function parameters can be further adjusted using context knowledge, or an appropriate parameter search algorithm. Here, FCM is used to cluster the data and assign membership degrees.

## 4. Computational experiments

### 4.1. Data

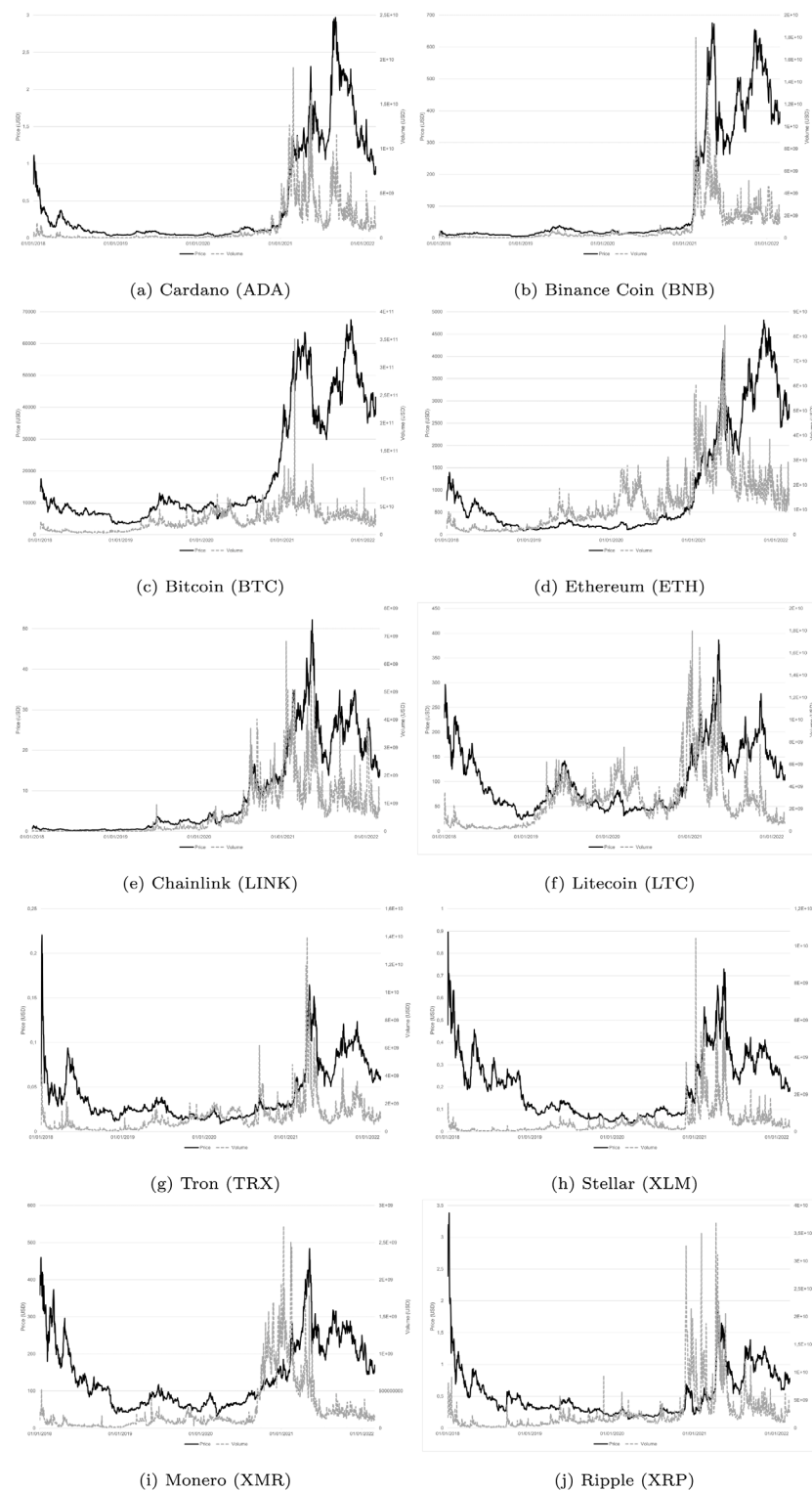
This section reports the computational experiments using daily closing prices data (USD) of ten leading cryptocurrencies: Cardano (ADA), Binance Coin (BNB), Bitcoin (BTC), Ethereum (ETH), Chainlink (LINK), Litecoin (LTC), Tron (TRX), Stellar (XLM), Monero (XMR) and Ripple (XRP).<sup>4</sup>

Distinct from the literature, which focuses on Bitcoin solely or in a reduced number of digital currencies, this paper reports a more comprehensive study, with a larger sample of cryptocurrencies to account for the different price dynamics. Data are from January 1, 2018 to February 28, 2022, totaling 1520 observations for each cryptocurrency.<sup>5</sup> This sample covers periods with low, and mostly high volatility in prices. The idea is to avoid time periods with flat or nearly stable price values because, in these situations, prediction is easier and risk management is less relevant.

The data were divided in in-sample and out-of-sample sets to train and test the models, respectively. A cross-validation approach is done to evaluate how the model developed would perform in practice. Table 1 summarizes the in-sample and out-of-sample sets for the three estimation/forecasting windows considered in this paper. Three out-of-sample sets, within a total of one year of observations, are considered for models evaluation. They correspond to the years of 2019, 2020 and 2021, in general, as they cover the months of January and February of the next corresponding years. Table 1 gives the details. In the table  $T$  is the number of samples (size) of the data set.

<sup>4</sup> Selection done choosing cryptocurrencies with the highest liquidity and market capitalization. A minimum of four years of historical data availability was also considered as a requirement to train and evaluate the models. Data source: <https://coinmarketcap.com/>.

<sup>5</sup> Data start in 2018 because previously to this year the number of digital coins with relevant liquidity and data availability were limited. February 2022 refers to the most recent period of data available when this research was conducted.



**Fig. 1.** Temporal evolution of prices and volumes (USD) of the cryptocurrencies.

Fig. 1 illustrates the temporal evolution of the price (USD) and trading volume (USD) of all digital coins addressed in this paper. For most of them, the end of 2020 and the beginning of 2021 show a trend of significant price appreciation. For example, the price of ADA (Cardano) moved from USD 0.18 in January 1, 2021 to USD 1.38 in January 1, 2022, i.e. an increase of approximately 4.015%. For the same time period, the rate of return of Binance Coin (BNB), Bitcoin (BTC) and

Ethereum (ETH) were approximately 3.752%, 416% and 562%, respectively. The considerable price volatility of all digital currencies is associated with the corresponding trading volume fluctuations. Higher price variability is viewed as an opportunity for investors to get higher returns, resulting in an increasing number of transactions. Additionally, due to the negative consequences of the COVID-19 pandemic in the economies and financial markets worldwide, cryptocurrencies appeared



**Table 1**  
Cross-validation data for cryptocurrency price forecasting.

In-Sample			Out-of-sample		
Start	End	T	Start	End	T
1/1/2018	2/28/2019	424	3/1/2019	2/29/2020	366
1/1/2019	2/29/2020	425	3/1/2020	3/28/2021	365
1/1/2020	2/28/2021	424	3/1/2021	3/28/2022	365

as a competitive asset for portfolio diversification, risk management, and investment in a period of financial crisis, as verified by the trading volume series dynamics of Fig. 1.

#### 4.2. Methods and performance assessment

Forecasting is done for one-step-ahead and performed iteratively. The modeling and forecasting techniques considered for evaluation and comparison are the classic autoregressive moving average (ARIMA) (Box et al., 2016), the naïve random walk (RW), and a multilayer perceptron neural network (Haykin, 2009). Notice that the RW is a very important benchmark for currency price forecasting models. The seminal study (Meese & Rogoff, 1983) compares exchange rate predictions obtained from structural models against a naïve random walk and shows that it is not easy to be outperformed. Subsequently, an extensive literature emerged, confirming the finding of Meese and Rogoff (1983) what became the so-called Meese–Rogoff puzzle. Here in this paper we investigate if the Meese–Rogoff puzzle also holds for cryptocurrencies.

The naïve random walk produces forecasts as follows. Let

$$y_t = y_{t-1} + \epsilon_t \quad (12)$$

where  $y_t$  is the actual price at instant  $t$ , and  $\epsilon_t$  is a random value with zero mean and independent over time. RW assumes the current price as a forecast of the next time step:

$$\hat{y}_t = y_{t-1} \quad (13)$$

where  $\hat{y}_t$  is the price forecast at  $t$ .

RW forecasting also follows the weak form of market efficiency, which states that future securities prices are random and are not affected by past events (Fama, 1970). It assumes that information of stock prices are reflected in the current prices and has no relationship with the past market prices.

Similarly as in time series modeling and forecast approaches, MLP and LSM models predict cryptocurrency prices  $\hat{y}_t$  using lagged values of the series, that is:

$$\hat{y}_t = f(y_{t-1}, y_{t-2}, \dots, y_{t-d}) \quad (14)$$

where  $d$  is the number of lagged closing prices, and  $f(\cdot)$  encodes the modeling method.

Performance evaluation of the methods is done using the root mean squared error (RMSE), the mean absolute error (MAE), and the mean absolute percentage error (MAPE), respectively:

$$\text{RMSE} = \sqrt{\frac{1}{T} \sum_{t=1}^T (y_t - \hat{y}_t)^2} \quad (15)$$

$$\text{MAE} = \frac{1}{T} \sum_{t=1}^T |y_t - \hat{y}_t| \quad (16)$$

$$\text{MAPE} = \frac{1}{T} \sum_{t=1}^T \left| \frac{y_t - \hat{y}_t}{y_t} \right| \quad (17)$$

where  $y_t$  is the actual price at time  $t$ ,  $\hat{y}_t$  the forecasted price at  $t$ , and  $T$  is the size of the out-of-sample data set.

In practice, the direction of price change is as important as, sometimes even more important than, the magnitude of the forecasting error (Burns & Moosa, 2015). A measure of forecast direction is:

$$\text{DA} = \frac{1}{T} \sum_{t=1}^T Z_t, \quad Z_t = \begin{cases} 1, & \text{if } (\hat{y}_{t+1} - y_t)(y_{t+1} - y_t) > 0 \\ 0, & \text{otherwise} \end{cases} \quad (18)$$

Notice that RW is not capable to predict price direction. Further, direction accuracy is a very important metric for agents interested in the application of market trend technical indicators in their investment strategies.

In addition to these performance measures, the model confidence set (MCS) procedure, developed by Hansen et al. (2011), is used to verify whether the models have statistically different predictive abilities.

The MCS considers an initial set of models  $M^0$  of dimension  $m$  encompassing the considered forecasting approaches. The procedure provides, for a given confidence level  $1 - \alpha$ , the so-called *Superior Set of Models* (SSM), a smaller set  $\hat{M}_{1-\alpha}^*$  of dimension  $m^* \leq m$ , for which the null hypothesis of equal predictive ability is not rejected at a certain confidence level.

Let  $l_{i,t} = l(y_t, \hat{y}_t)$  be the loss of model  $i$  at time  $t$ ,  $i = 1, \dots, m$ . The method of Hansen et al. (2011) evaluates the loss differential between models  $i$  and  $j$  using:

$$d_{i,j,t} = (m-1)^{-1} \sum_{j \in M} d_{ij,t} \quad (19)$$

called the simple loss of model  $i$  relative to any other model  $j$  at time  $t$ , with  $d_{ij,t} = l_{i,t} - l_{j,t}$ ,  $i, j = 1, \dots, m$ , and  $t = 1, \dots, T$ .

The equal predictive ability, for a given set of models  $M$  is formulated as:

$$H_{0,M} : E[d_{i,j}] = 0, \text{ for all } i, j = 1, \dots, m$$

$$H_{1,M} : E[d_{i,j}] \neq 0, \text{ for some } i, j = 1, \dots, m \quad (20)$$

The statistics suggested by Hansen et al. (2011) to test the two hypothesis is:

$$t_i = \frac{\bar{d}_i}{\sqrt{\hat{\sigma}^2(\bar{d}_i)}} \text{ for } i, j \in M \quad (21)$$

where  $\bar{d}_i = (m-1)^{-1} \sum_{j \in M} \bar{d}_{ij}$  is the simple loss of the  $i$ th model relative to the averages losses across models in the set  $M$ , and  $\bar{d}_{ij} = T^{-1} \sum_{t=1}^T d_{ij,t}$  measures the relative sample loss between the  $i$ th and  $j$ th models, and  $\hat{\sigma}^2(\bar{d}_i)$  is the bootstrap estimate of  $\sigma^2(\bar{d}_i)$ .

Test statistics is found using:

$$T_{\max,M} = \max_{i,j \in M} t_i. \quad (22)$$

The relevant distribution under the null hypothesis has to be estimated using bootstrapping since it is asymptotic and nonstandard. To obtain the SSM, the MCS proceeds as follows:

1. Set  $M = M^0$ ;
2. Test the equal predictive hypothesis: if it is accepted, then terminate and set  $\hat{M}_{1-\alpha}^* = M$ , otherwise finds the worst model of the current iteration;
3. Remove the worst model and go to step 2.

The elimination rule to remove the worst model uses the valuation:

$$e_{\max,M} = \arg \max_{i \in M} \frac{\bar{d}_i}{\hat{\sigma}^2(\bar{d}_i)} \quad (23)$$

This paper uses the squared error (SE) and the absolute error (AE) as loss functions in the MCS procedure. They are computed as follows:

$$l_{i,t}^{\text{SE}} = (y_t - \hat{y}_t)^2 \quad \text{and} \quad l_{i,t}^{\text{AE}} = |y_t - \hat{y}_t| \quad (24)$$

Here the MCS uses  $\alpha = 10\%$  as suggested by Hansen et al. (2011). Computations are performed using the MCS package for R developed by Bernardi and Catania (2011).

**Table 2**  
Structure of the forecasting models.

Crypto	ARIMA	MLP	LSM
ADA	(1,1,6)	(21,11,1)	(1,2)
BNB	(2,1,4)	(11;6,1)	(1,2)
BTC	(5,1,2)	(2;2,1)	(1,2)
ETH	(0,1,3)	(12;6,1)	(1,2)
LINK	(1,1,6)	(20;10,1)	(1,2)
LTC	(3,1,3)	(16;8,1)	(1,2)
TRX	(1,1,4)	(14;8,1)	(1,2)
XLM	(0,1,5)	(23;12,1)	(1,2)
XMR	(5,1,2)	(19;10,1)	(1,2)
XRP	(4,1,8)	(25;13,1)	(1,2)

### 4.3. Results

Evaluation and comparison of LSM with ARIMA, RW and MLP in one-ste-ahead cryptocurrencies closing prices forecasting use out-of-sample data sets as testing data. For our cross-validation analysis, three out-of-sample sets are considered, comprising the years of 2019 (Mar 2019 to Feb 2020), 2020 (Mar 2020 to Feb 2021) and 2021 (Mar 2021 to Feb 2022).

**Table 3**  
Performance of the cryptocurrencies price models.

Model	RMSE			MAE			MAPE		
	2019	2020	2021	2019	2020	2021	2019	2020	2021
<i>Panel A: Cardano (ADA)</i>									
RW	0.0029 <sup>a</sup>	0.0244 <sup>a</sup>	0.0985 <sup>a</sup>	0.00202 <sup>a</sup>	0.0091 <sup>a</sup>	0.0679 <sup>a</sup>	3.4013 <sup>a</sup>	4.5177 <sup>a</sup>	4.2608 <sup>a</sup>
ARIMA	0.0032	0.0246 <sup>b</sup>	0.1087	0.00225	0.0132	0.0727	3.8138	4.5277 <sup>b</sup>	4.6166
MLP	0.0032	0.2379	0.4392	0.00219	0.0946	0.2958	3.6337	24.9366	15.2953
LSM	0.0030 <sup>b</sup>	0.0317	0.1033 <sup>b</sup>	0.00204 <sup>b</sup>	0.0111 <sup>b</sup>	0.0699 <sup>b</sup>	3.4277 <sup>b</sup>	4.9359	4.3717 <sup>b</sup>
<i>Panel B: Binance Coin (BNB)</i>									
RW	0.9248 <sup>a</sup>	9.9197 <sup>a</sup>	24.5068 <sup>a</sup>	0.6559 <sup>a</sup>	2.3895 <sup>a</sup>	17.1364 <sup>a</sup>	2.9752 <sup>a</sup>	3.8826 <sup>a</sup>	4.0569 <sup>a</sup>
ARIMA	0.9478	10.3374	32.9171	0.6913	2.3832 <sup>b</sup>	25.4969	3.1452	3.9771 <sup>b</sup>	5.9868
MLP	4.8256	31.7958	225.5101	3.2584	7.5175	200.8152	11.8272	6.5152	42.3970
LSM	0.9484 <sup>b</sup>	10.0511 <sup>b</sup>	27.1162 <sup>b</sup>	0.6775 <sup>b</sup>	4.7935	18.1365 <sup>b</sup>	3.0832 <sup>b</sup>	4.9825	4.3475 <sup>b</sup>
<i>Panel C: Bitcoin (BTC)</i>									
RW	328.0145 <sup>a</sup>	975.7554 <sup>a</sup>	1785.6320 <sup>a</sup>	208.4262 <sup>a</sup>	493.1740 <sup>a</sup>	1313.8272 <sup>a</sup>	3.8188	2.6685 <sup>a</sup>	2.8320 <sup>a</sup>
ARIMA	361.0601	976.4539 <sup>b</sup>	1845.4967	248.3610	498.9268 <sup>b</sup>	1367.9117	3.8150 <sup>b</sup>	2.7415 <sup>b</sup>	2.9603
MLP	331.2505 <sup>b</sup>	9796.2812	7212.2915	212.9706 <sup>b</sup>	4483.1064	5593.3379	5.1722	13.7220	11.1114
LSM	337.0599	1012.7705	1802.6325 <sup>b</sup>	217.7696	518.7797	1329.1934 <sup>b</sup>	2.4838 <sup>a</sup>	2.7956	2.8692 <sup>b</sup>
<i>Panel D: Ethereum (ETH)</i>									
RW	8.6914 <sup>a</sup>	39.8736 <sup>a</sup>	154.0182 <sup>a</sup>	5.6956 <sup>a</sup>	20.5105 <sup>a</sup>	112.5590 <sup>a</sup>	2.7956 <sup>a</sup>	3.6639 <sup>a</sup>	3.8188 <sup>a</sup>
ARIMA	8.7356	39.8959 <sup>b</sup>	154.0233 <sup>b</sup>	5.7898	22.6145	112.5670 <sup>b</sup>	2.8464	3.8485	3.9150
MLP	9.5938	426.9995	1040.0563	6.6952	198.1576	833.7559	3.3009	18.7882	23.7322
LSM	8.7102 <sup>b</sup>	42.5030	158.2301	5.7339 <sup>b</sup>	21.4853 <sup>b</sup>	115.0453	2.8087 <sup>b</sup>	3.7918 <sup>b</sup>	3.9076 <sup>b</sup>
<i>Panel E: Chainlink (LINK)</i>									
RW	0.1317 <sup>a</sup>	0.9560 <sup>a</sup>	1.9938 <sup>a</sup>	0.0820 <sup>b</sup>	0.5747 <sup>a</sup>	1.3596 <sup>a</sup>	4.1078 <sup>a</sup>	4.9955 <sup>a</sup>	5.1712 <sup>a</sup>
ARIMA	0.1331 <sup>b</sup>	0.9688 <sup>b</sup>	2.0028 <sup>b</sup>	0.0818 <sup>a</sup>	0.5926 <sup>b</sup>	1.3757	4.1151 <sup>b</sup>	5.1480 <sup>b</sup>	5.2520
MLP	1.3622	6.9217	2.9732	1.0972	4.1538	1.8659	43.8748	25.0548	6.6243
LSM	0.1348	1.0149	2.0128	0.0847	0.6105	1.3656 <sup>b</sup>	4.2289	5.2667	5.1992 <sup>b</sup>
<i>Panel F: Litecoin (LTC)</i>									
RW	3.7005 <sup>b</sup>	5.5146 <sup>b</sup>	12.4843 <sup>a</sup>	2.4712 <sup>a</sup>	3.0551 <sup>a</sup>	7.6655 <sup>a</sup>	3.2245 <sup>a</sup>	3.6931 <sup>a</sup>	4.1021 <sup>a</sup>
ARIMA	3.7248	5.4426 <sup>a</sup>	12.6780 <sup>b</sup>	2.5286	3.1478	7.9459	3.3041	3.8276	4.2468
MLP	4.0612	6.5871	33.0928	2.6352	3.4366	21.8387	3.3577	3.8825	10.6769
LSM	3.6996 <sup>a</sup>	6.0135	12.8272	2.4742 <sup>b</sup>	3.0986 <sup>b</sup>	7.8667 <sup>b</sup>	3.2348 <sup>b</sup>	3.7743 <sup>b</sup>	4.1980 <sup>b</sup>
<i>Panel G: Tron (TRX)</i>									
RW	0.00116 <sup>a</sup>	0.00164 <sup>a</sup>	0.0059 <sup>a</sup>	0.00078 <sup>a</sup>	0.00095 <sup>a</sup>	0.0036 <sup>a</sup>	3.5133 <sup>a</sup>	3.6918 <sup>b</sup>	4.0524 <sup>a</sup>
ARIMA	0.00146	0.00165 <sup>b</sup>	0.0060	0.00105	0.00115	0.0040	4.8045	3.7896	4.5450
MLP	0.00127 <sup>b</sup>	0.00245	0.0326	0.00088	0.00119	0.0251	4.0736	4.1485	25.2508
LSM	0.00117	0.00188	0.0060 <sup>b</sup>	0.00079 <sup>b</sup>	0.00102 <sup>b</sup>	0.0038 <sup>b</sup>	3.5624 <sup>b</sup>	3.6760 <sup>a</sup>	4.2757 <sup>b</sup>
<i>Panel H: Stellar (XLM)</i>									
RW	0.00380 <sup>a</sup>	0.0144 <sup>a</sup>	0.0248 <sup>a</sup>	0.00252 <sup>a</sup>	0.0065 <sup>a</sup>	0.0152 <sup>a</sup>	3.0111 <sup>a</sup>	4.2026 <sup>a</sup>	4.1625 <sup>a</sup>
ARIMA	0.00389	0.0146	0.0252	0.00257	0.0068 <sup>b</sup>	0.0154	3.0698	4.4120	4.2194 <sup>b</sup>
MLP	0.00653	0.0853	0.0633	0.00535	0.0346	0.0429	7.9790	12.0145	11.2692
LSM	0.00387 <sup>b</sup>	0.0102 <sup>b</sup>	0.0251 <sup>b</sup>	0.0024 <sup>b</sup>	0.0079	0.0150 <sup>b</sup>	3.0082 <sup>b</sup>	4.2954 <sup>b</sup>	4.2690

(continued on next page)

Table 3 (continued).

Model	RMSE			MAE			MAPE		
	2019	2020	2021	2019	2020	2021	2019	2020	2021
<i>Panel I: Monero (XMR)</i>									
RW	3.2969 <sup>a</sup>	6.5493 <sup>a</sup>	15.9728 <sup>a</sup>	2.3086 <sup>a</sup>	3.8554 <sup>a</sup>	10.2099 <sup>b</sup>	3.1298 <sup>a</sup>	3.5894 <sup>a</sup>	4.0489 <sup>a</sup>
ARIMA	3.3243	7.4994	16.5087	2.3982	3.8909 <sup>b</sup>	10.082 <sup>a</sup>	3.2226	3.7500	4.2245
MLP	3.6508	18.7389	45.7885	2.6115	8.9292	29.1011	3.5149	6.3158	10.0164
LSM	3.2977 <sup>b</sup>	7.1634 <sup>b</sup>	16.3386 <sup>b</sup>	2.3153 <sup>b</sup>	4.0305	10.4420	3.1400 <sup>b</sup>	3.6595 <sup>b</sup>	4.1344 <sup>b</sup>
<i>Panel J: Ripple (XRP)</i>									
RW	0.0123 <sup>a</sup>	0.0268 <sup>a</sup>	0.0735 <sup>a</sup>	0.0079 <sup>a</sup>	0.0129 <sup>a</sup>	0.0449 <sup>a</sup>	2.5548 <sup>a</sup>	3.8926 <sup>a</sup>	4.6291 <sup>a</sup>
ARIMA	0.0160	0.0377	0.0770	0.0109	0.0157	0.0520	3.5015	4.2073	5.4148
MLP	0.0248	0.0416	0.3318	0.0197	0.0196	0.2482	7.5668	5.1488	22.3249
LSM	0.0125 <sup>b</sup>	0.0314 <sup>b</sup>	0.0718 <sup>b</sup>	0.0084 <sup>b</sup>	0.0139 <sup>b</sup>	0.0482 <sup>b</sup>	2.7580 <sup>b</sup>	4.1077 <sup>b</sup>	4.9698 <sup>b</sup>

<sup>a</sup>Indicates the best model.<sup>b</sup>Indicates the second best model.

Table 4

Superior Set of Models (SSM) of MCS test for squared error (SE) and absolute error (AE) loss functions.  $\alpha = 10\%$ . p-values are associated with the null hypothesis of equal predictive ability.

Crypto	2019				2020				2021			
	SE		AE		SE		AE		SE		AE	
	SSM	p-value	SSM	p-value	SSM	p-value	SSM	p-value	SSM	p-value	SSM	p-value
ADA	RW, LSM	0.1878	RW, LSM	0.3678	RW, ARIMA, LSM	0.1286	RW, LSM	0.1798	RW, ARIMA, LSM	0.586	RW	0.0182
BNB	RW, ARIMA, LSM	0.2698	RW	0.0000	RW, ARIMA, MLP, LSM	0.6264	RW, LSM	0.1328	RW	0.0422	RW, LSM	0.0142
BTC	RW, LSM	0.0808	RW	0.0000	RW, ARIMA, LSM	0.2096	RW, LSM	0.4448	RW, LSM	0.2348	RW, LSM	0.1352
ETH	RW, ARIMA, LSM	0.6164	RW, ARIMA, LSM	0.1310	RW, ARIMA, LSM	0.1168	RW, ARIMA	0.2944	RW, LSM	0.9854	RW, ARIMA, LSM	0.1694
LINK	RW, LSM	0.3422	RW, LSM	0.7596	RW, LSM	0.2210	RW	0.0012	RW, ARIMA, LSM	0.5744	RW, ARIMA, LSM	0.4656
LTC	RW, LSM	0.4132	RW, LSM	0.1106	RW, LSM	0.4808	RW, LSM	0.1234	RW, LSM	0.26	RW	0.0808
TRX	RW, LSM	0.8686	RW	0.0452	RW, LSM	0.2226	RW, ARIMA, LSM	0.1924	RW, LSM	0.1762	RW, LSM	0.3888
XLN	RW, ARIMA, LSM	0.3868	RW, LSM	0.1056	RW, ARIMA, LSM	0.1120	RW, LSM	0.2120	RW, ARIMA, LSM	0.6064	RW, LSM	0.1736
XMR	RW, LSM	0.4902	RW, LSM	0.4848	RW, LSM	0.2728	RW, LSM	0.3134	RW, LSM	0.3196	RW, ARIMA, LSM	0.1178
XRP	RW, LSM	0.9556	RW, LSM	0.4684	RW, ARIMA, LSM	0.7458	RW, ARIMA, LSM	0.9682	RW, LSM	0.5048	RW, LSM	0.3324

Table 3 summarizes the forecasting performance of the models in terms of the RMSE, MAE and MAPE. The symbols (\*) and (\*\*) indicate the best model (lowest value) and the second best model, respectively.

Table 3 shows that the naïve random walk outperforms all competitors from the point of view of RMSE, MAE, and MAPE measures, except for the following 6 cases out of 90: MAPE value for Bitcoin in the 2019 out-of-sample set, MAE value for Chainlink in the 2019 out-of-sample set, RMSE values for Litecoin in the 2019 and 2020 out-of-sample sets, MAPE value for Tron in the 2020 out-of-sample set and MAE value for the 2021 out-of-sample set. In three of these cases the LSM achieved the best results, and in the other three, ARIMA appears as the most accurate method (see Table 3). These results are consistent with the *Meese–Rogoff puzzle* (Meese & Rogoff, 1983) because no forecasting model outperforms the random walk. Interestingly, the simulations show that the *Meese–Rogoff puzzle* holds for cryptocurrencies as well.

After random walk, LSM reaches the highest accuracy for most of the measures (RMSE, MAE and MAPE) and out-of-sample data sets (2019, 2020 and 2021) for all considered cryptocurrencies, except for Bitcoin (BTC) and Chainlink (LINK). For Bitcoin the runner up model is not easily identifiable, as LSM, ARIMA and MLP performed well for some particular metrics and data sets. For Chainlink (LINK) ARIMA performs highest after the RW approach. In general, LSM forecasts are either the closest, or better than those of RW.

Additionally to the accuracy evaluation, the model confidence set approach is used to verify whether the models have statistically different predictive abilities. As discussed previously, the MCS test develops *Superior Set of Models*, a set of models whose forecasting abilities are equally accurate. Table 4 summarizes the results for  $\alpha = 0.10$ , the squared error (SE), and the absolute error (AE) loss functions. Table 4 indicates that the LSM model performs well when predicting the cryptocurrency prices considered in this paper. As expected, the random walk model is a member of all the superior sets of models. ARIMA is a member of 32% of the superior set of models (19 out of 60), whereas LSM is in approximately 83% of the superior set of models (50 out of 60). Hence, from the point of view of MCS model confidence test, LSM has superior predictive ability than ARIMA, and its predictive ability can be considered statistically equal to random walk.

It is worth to mention that sometimes LSM provided a better performance than ARIMA and vice-versa. To explain this we can come across the well-known no free-lunch theorem (Wolpert & Macready, 1997): if an algorithm performs well on a certain class of problems, then it necessarily pays for that with degraded performance on the set of all remaining problems. In this paper, we have evaluated a linear and nonlinear techniques, respectively, ARIMA and LSM. Additionally, to improve robustness, three different out-of-sample sets are considered, to account for different price dynamics for the evaluated cryptocurrencies. Generally, the sets differ in terms of the level of price volatility. From Fig. 1, we can see that in 2021 there is an increase in prices for all assets, with significant data variability. Analyzing the results in Table 3 we can note that in the price appreciation, in general, the LSM model had the second best performance. It is also noted that in the 2020 period, assets, in general, had a lower variability (see Fig. 1), and the ARIMA model presented the second best performance. That is, for series, or periods, with high volatility, the proposed model has a performance that is similar to the RW model. Hence in periods of high volatility, LSM provides the second best performance, meaning that when price dynamics presents a high variation, nonlinear techniques do provide a better tool for forecasting the corresponding future prices, as the volatility behavior of financial prices are nonlinear and asymmetric (investors respond differently for good and bad news, or positive and negative returns, respectively). Otherwise, during the periods of lower volatility, ARIMA showed a good performance, revealing that a linear structure is able to produce adequate forecasts.

Recently, Burns and Moosa (2015) and Moosa and Burns (2014) found that forecasting models can outperform the naïve random walk for out-of-sample data if performance is measured by economic/financial measures such as the direction of change and/or profitability as in forecast-based trading operations. Table 5 reports the direction accuracy (DA) values of ARIMA, MLP and LSM. We notice that for the 2019 out-of-sample data, the LSM outperforms ARIMA and MLP for all digital coins, except for Trol (TRX) and Monero (XMR), when MLP and ARIMA achieve the highest DA values, respectively. LSM also outperforms the competitors for 6 cryptocurrencies for the 2020 out-of-sample data set. For the remaining four cases and 2020 dat, MLP

**Table 5**

Cryptocurrencies price forecasting performance measured by the direction accuracy (DA).

Crypto	2019			2020			2021		
	ARIMA	MLP	LSM	ARIMA	MLP	LSM	ARIMA	MLP	LSM
ADA	49.59%	49.32%	53.97% <sup>a</sup>	55.22%	50.55%	58.66% <sup>a</sup>	52.62%	53.72%	53.24% <sup>a</sup>
BNB	49.04%	52.33%	58.22% <sup>a</sup>	45.33%	53.02%	54.12% <sup>a</sup>	52.62% <sup>a</sup>	47.93%	50.14%
BTC	53.15%	50.68%	54.84% <sup>a</sup>	51.92%	48.63%	53.35% <sup>a</sup>	49.04%	51.52% <sup>a</sup>	50.96%
ETH	46.03%	54.25%	55.60% <sup>a</sup>	51.37%	48.08%	53.80% <sup>a</sup>	51.24% <sup>a</sup>	47.38%	50.96%
LINK	51.51%	50.41%	51.93% <sup>a</sup>	46.15%	48.35%	50.27% <sup>a</sup>	50.82%	53.85%	54.40% <sup>a</sup>
LTC	46.85%	48.68%	49.04% <sup>a</sup>	51.37%	53.30% <sup>a</sup>	48.08%	45.18%	47.11%	50.96% <sup>a</sup>
TRX	49.32%	52.05% <sup>a</sup>	49.04%	55.22%	54.12%	58.35% <sup>a</sup>	50.00%	46.70%	56.59% <sup>a</sup>
XLM	49.86%	49.59%	51.58% <sup>a</sup>	48.90%	56.32% <sup>a</sup>	55.70%	53.30%	50.82%	54.45% <sup>a</sup>
XMR	53.70% <sup>a</sup>	51.51%	48.49%	55.77% <sup>a</sup>	43.96%	49.18%	52.75%	44.78%	53.82% <sup>a</sup>
XRP	66.58%	61.37%	69.86% <sup>a</sup>	50.27%	55.49% <sup>a</sup>	51.92%	71.70%	56.87%	74.45% <sup>a</sup>

<sup>a</sup>Indicates the best model.**Fig. 2.** Actual Cardano (ADA) prices and their LSM forecasts for the out-of-sample sets of 2019, 2020 and 2021.**Fig. 3.** Actual Binance Coin (BNB) prices and their LSM forecasts for the out-of-sample sets of 2019, 2020 and 2021.

is the best model in three of them, and ARIMA the best for Monero (XMR) only (see Table 5). For 2021 out-of-sample data sets LSM gives the highest accuracy for seven digital coins. Notice that the random walk underperforms all the methods when performance is measured using the direction accuracy. This is of utmost importance for trading strategies that use direction, once the potential to anticipate price change is crucial for the investment success.

To further illustrate the efficiency of LSM in cryptocurrencies forecast, Figs. 2–11 show the actual closing prices and the corresponding LSM forecasts developed for Cardano (ADA), Binance Coin (BNB), Bitcoin (BTC), Ethereum (ETH), Chainlink (LINK), Litecoin (LTC), Tron (TRX), Stellar (XLM), Monero (XMR) and Ripple (XRP) using the corresponding out-of-sample sets, respectively. The figures reveals that LSM accurately predict price dynamics of the digital coins considered in



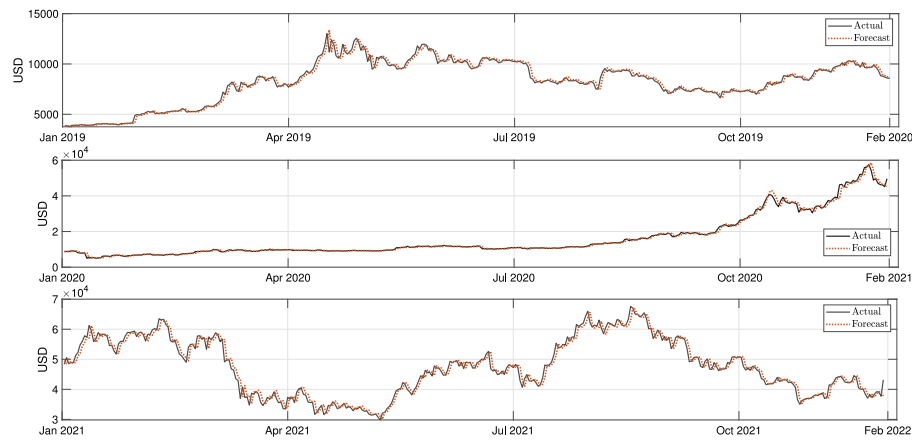


Fig. 4. Actual Bitcoin (BTC) prices and their LSM forecasts for the out-of-sample sets of 2019, 2020 and 2021.

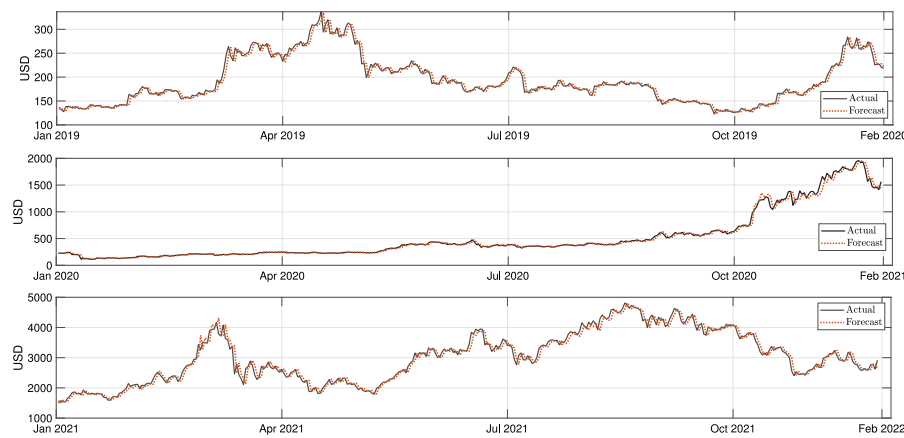


Fig. 5. Actual Ethereum (ETH) prices and their LSM forecasts for the out-of-sample sets of 2019, 2020 and 2021.

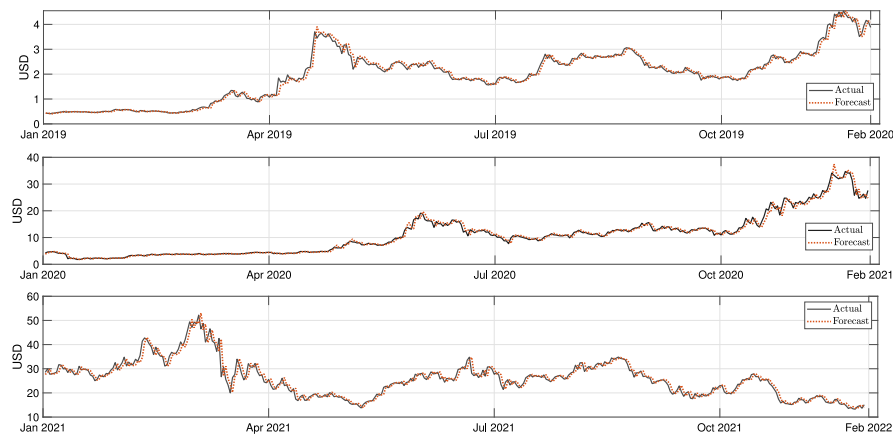


Fig. 6. Actual Chainlink (LINK) prices and their LSM forecasts for the out-of-sample sets of 2019, 2020 and 2021.

this work, and unfolds as a potential forecast tool to develop trading strategies in cryptocurrency market.

## 5. Conclusion

This paper has developed data-driven fuzzy level set-based models to forecast cryptocurrencies prices. They differ from knowledge-based

and functional fuzzy models because the rule base is constructed from data and is processed using the notion of level set. The output of a level set-based model is the weighted average of the active rules output functions, mapping the activation levels of the rules directly in the model output. Computational experiments were done to develop fuzzy models for one-step-ahead forecasting of the closing prices of Cardano, Binance Coin, Bitcoin, Ethereum, Chainlink, Litecoin, Tron, Stellar, Monero and

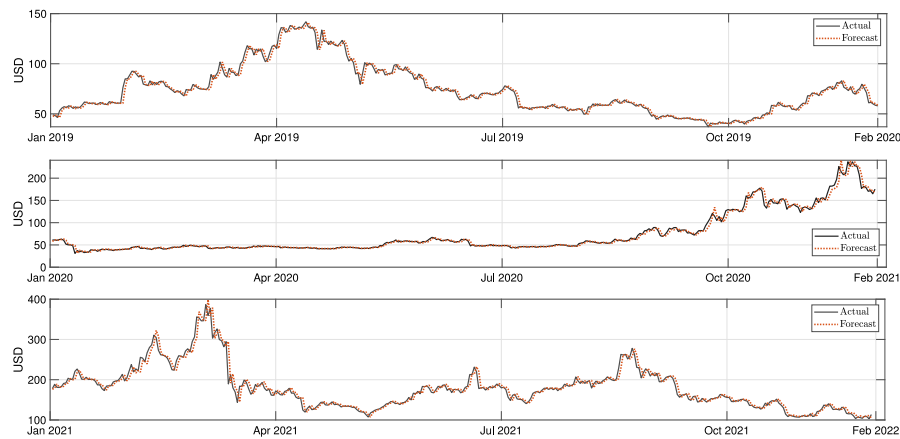


Fig. 7. Actual Litecoin (LTC) prices and their LSM forecasts for the out-of-sample sets of 2019, 2020 and 2021.

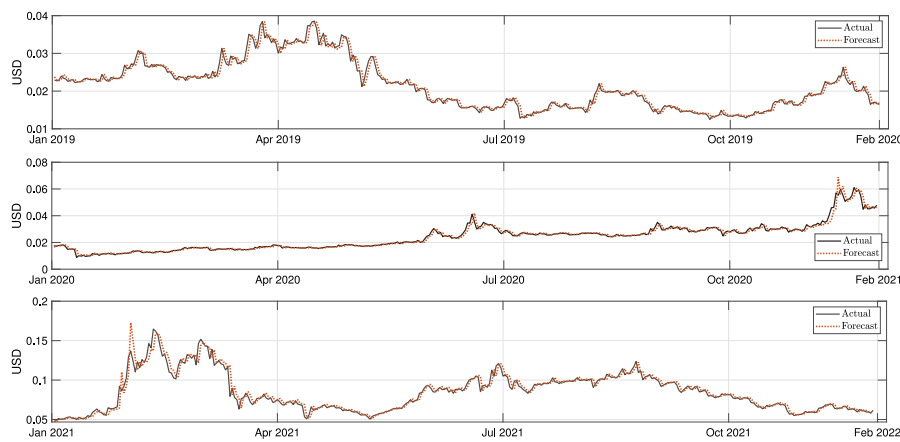


Fig. 8. Actual Tron (TRX) prices and their LSM forecasts for the out-of-sample sets of 2019, 2020 and 2021.

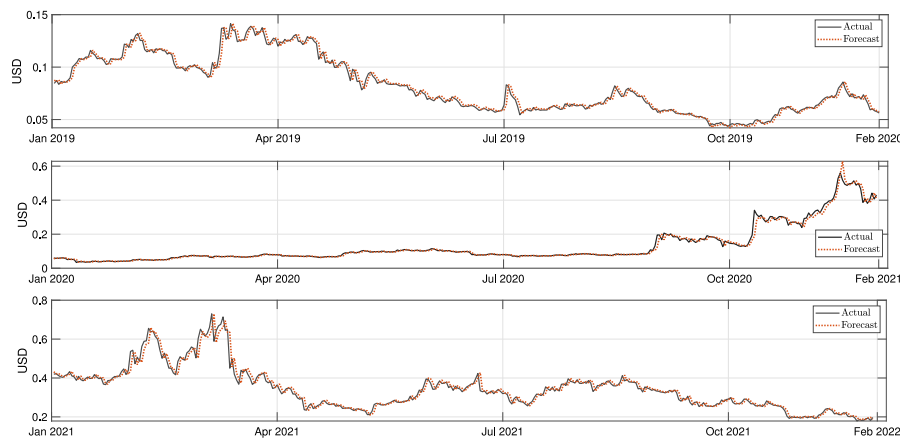


Fig. 9. Actual Stellar (XLM) prices and their LSM forecasts for the out-of-sample sets of 2019, 2020 and 2021.

Ripple. Evaluation and comparisons of the data-driven level set-based models was done with ARIMA, MLP and naïve random walk. The results indicate that random walk outperforms all the competitors addressed in this paper in terms of accuracy. However, the model confidence set test shows that LSM and RW can be considered statistically equal to forecast digital coins prices. Moreover, when performance is measured by the direction of price change, the level set-based fuzzy models rank the

highest. Future work shall consider the use and evaluation of the data-driven level set models in trading strategies to evaluate the forecast performance in terms of profitability, the consideration of combining forecasts, and measuring the level of nonlinearities of the series and relate this aspect to the predictability performance from linear and nonlinear techniques.

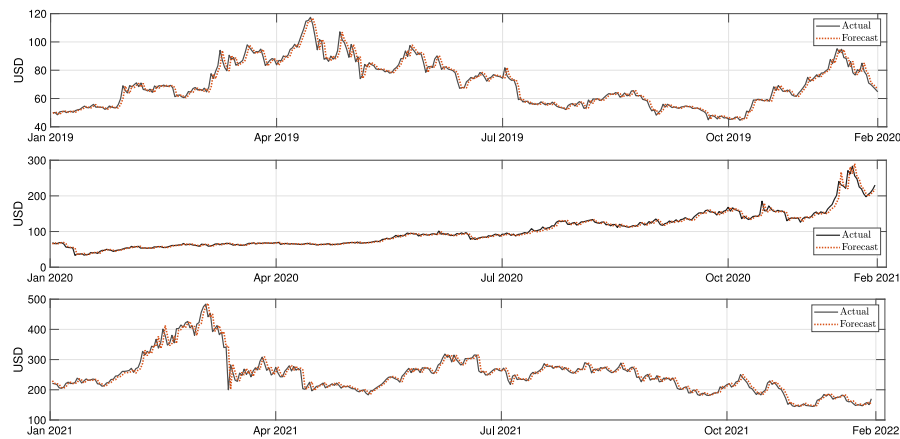


Fig. 10. Actual Monero (XMR) prices and their LSM forecasts for the out-of-sample sets of 2019, 2020 and 2021.

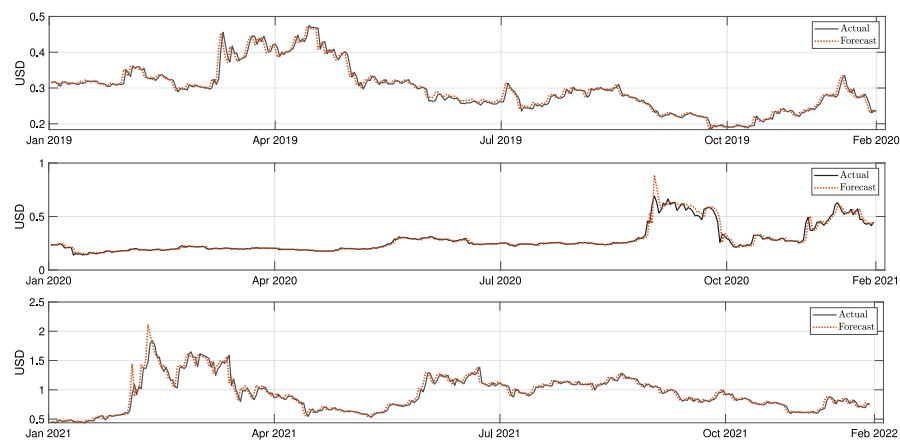


Fig. 11. Actual Ripple (XRP) prices and their LSM forecasts for the out-of-sample sets of 2019, 2020 and 2021.

### CRedit authorship contribution statement

**Leandro Maciel:** Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Software, Validation, Visualization, Writing – original draft, Writing – review & editing. **Rosangela Ballini:** Conceptualization, Investigation, Methodology, Writing – original draft, Writing – review & editing. **Fernando Gomide:** Conceptualization, Formal analysis, Investigation, Methodology, Software, Validation, Writing – original draft, Writing – review & editing. **Ronald Yager:** Conceptualization, Methodology.

### Declaration of competing interest

One or more of the authors of this paper have disclosed potential or pertinent conflicts of interest, which may include receipt of payment, either direct or indirect, institutional support, or association with an entity in the biomedical field which may be perceived to have potential conflict of interest with this work. For full disclosure statements refer to <https://doi.org/10.1016/j.eswa.2022.118387>. Leandro Maciel reports financial support was provided by Brazilian National Council for Scientific and Technological Development (CNPq). Leandro Maciel reports financial support was provided by Ripple Impact Fund. Fernando Gomide reports financial support was provided by Brazilian National Council for Scientific and Technological Development (CNPq). Rosangela Ballini reports financial support was provided by Brazilian National Council for Scientific and Technological Development (CNPq). Rosangela Ballini reports financial support was provided by Research Foundation of the State of São Paulo (FAPESP).

### Data availability

Data will be made available on request.

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