EE660

Aug 22, 2018

Discussion # 1

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Submission of HW:

use den D2L

only accept PDF

2 pdf (1 pdf for programming code
1 pdf HW)

Review on Probability:

event A, prob. of event A

$0 \leq P(A) \leq 1$

Discrete RV:

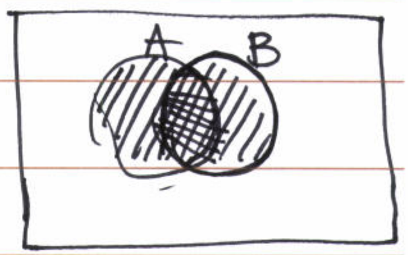
prob. of event $X=x$ denoted as

$P(X=x)$ Prob. Mass Func. PMF

Fundemental Rules:

(1) prob. of union:

A and B



$$P(A \text{ or } B) = P(A) + P(B) - P(A,B)$$

$$= P(A) + P(B)$$

A and B are mutually exclusive

$P(AB)=0$ X

(2)

② prob. of joint

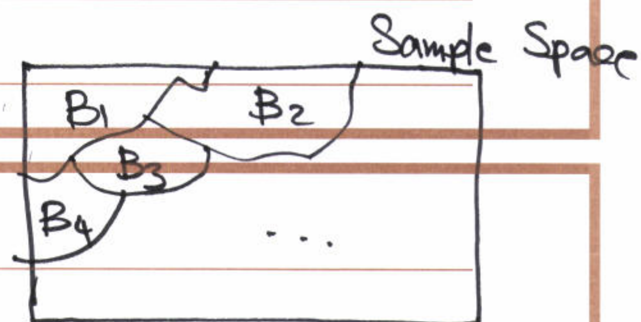
$$P(A,B) = P(A)P(B|A) = P(B)P(A|B)$$

extend

$$P(X_1 X_2 \cdots X_n) = P(X_1)P(X_2|X_1)P(X_3|X_1 X_2) \cdots P(X_n|X_1 \cdots X_{n-1})$$

Suppose we are given the joint prob. $P(A,B)$ how to find marginal prob. ?

$P(A)$



$$P(A) = \sum_i P(A|B_i)P(B_i)$$

(Total Prob. Law)

③ Conditional Prob.

$$P(A|B) = \frac{P(AB)}{P(B)} \qquad P(B) \neq 0$$

④ Baye's Rule :

$$P(X=x \mid Y=y) = \frac{P(Y=y \mid X=x)\,P(X=x)}{P(Y=y)}$$

$$= \frac{P(X=x, Y=y)}{P(Y=y)} = \frac{P(Y=y \mid X=x)\,P(X=x)}{P(Y=y)}$$

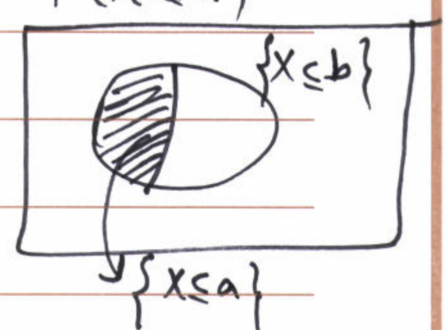
$$= \frac{P(Y=y \mid X=x)\,P(X=x)}{\sum_{x} P(Y=y \mid X=x)\,P(X=x)}$$

Continuous RV :

X can take any values

$$P(X=a) = 0$$

$$P(\underbrace{a < X \leq b}_{\{X \leq b\} - \{X \leq a\}}) = P(X \leq b) - P(X \leq a)$$



$$P(X \leq x) \triangleq F_X(x)$$ cumulative dist. function (CDF)

$$f_X(x) \triangleq \frac{d}{dx} F_X(x)$$: prob. density func. (PDF)

$$P(a < X \leq b) = \int_a^b f_X(x)\,dx$$

Note: $f_X(x) \geqslant 0 \quad \forall x$

$f(x)$ is NOT nece. ≤ 1

$$\int_{-\infty}^{+\infty} f_X(x)\,dx = 1$$

(disc. $\sum_x P(X=x) = 1$)

Properties of distributions:

1. Average: $E(x) = \sum x\, P_X(x)$

$$= \int x\, f_X(x)\,dx$$

2. Variance: $Var(X) = E\left((X - E(x))^2 \right)$

$$= E\left(X^2 - 2X\,E(x) + E^2(x) \right)$$

$$= E(X^2) - E^2(X)$$

3. Covariance between X and Y

$$\mathrm{Cov}(X,Y) \triangleq E\left((X-E(X))(Y-E(Y)) \right)$$

$$= E(XY) - E(X)E(Y)$$

Normalize $\mathrm{Cov}(X,Y)$

$$\mathrm{Corr}(X,Y) \triangleq \frac{\mathrm{Cov}(X,Y)}{\sqrt{\mathrm{Var}(X)\ \mathrm{Var}(Y)}}$$

Some Discrete Distributions:

① Bernouli (p)

$X \in \{0,1\}$

$P(X=1) = p$

$E(X) = p$

$\mathrm{Var}(X) = p(1-p)$

② Binomial (n,p)

$E(X) = np$

$\mathrm{Var}(X) = np(1-p)$

$X \in \{0,1,2,\cdots,n\}$

$$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$X = \sum_{i=1}^{n} X_i$$

③ Poisson Distribution: (with parameter λ)

$$X \in \{0, 1, 2, \dots\}$$

$$P(X = x) = e^{-\lambda} \frac{\lambda^x}{x!}$$

④ Emperical Distribution:

given $\mathcal{D} = \{x_1, x_2, \dots, x_N\}$

$$P_{emp}(A) \triangleq \frac{1}{N} \sum_{i=1}^{N} \delta_{x_i}(A)$$

$$\delta_x(A) \triangleq \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{else} \end{cases}$$

Continuous Distribution:

① Gaussian (μ, σ^2)

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{ \frac{-1}{2\sigma^2} (x - \mu)^2 \right\}$$

$$E(X) = \mu$$
$$Var(X) = \sigma^2$$

② Exponential Distribution (λ)

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & \text{else} \end{cases}$$

$$E(X) = \frac{1}{\lambda} \qquad Var(X) = \frac{1}{\lambda^2}$$

Sofar we had only one RV, we can extent to multivariable scenario,

$P(x_1, x_2, \cdots, x_D)$

Discrete $F_X(X_1 = x_1, X_2 = x_2, \cdots, X_n = x_n)$

$= \text{Prob}(X_1 \leq x_1, \cdots, X_n \leq x_n)$

$$Cov(\underline{X}) \triangleq E\left((\underline{X} - E(\underline{X}))(\underline{X} - E(\underline{X}))^T \right)$$

$$= \begin{pmatrix} Var(X_1) & Cov(X_1, X_2) & \cdots \\ Cov(X_2, X_1) & Var(X_2) & \\ & & \ddots \\ & & \end{pmatrix}$$

Correlation Matrix $R = \begin{pmatrix} Corr(X_1, X_1) & Corr(X_1, X_2) & \cdots \\ Corr(X_2, X_1) & Corr(X_2, X_2) & \\ \vdots & & \ddots \end{pmatrix}$

$$Corr(X_1 X_2) = 1$$

$\Rightarrow$ linear relationship between X_1 & X_2

$$X_2 = aX_1 + b$$

what if $X_1 \perp X_2 \Rightarrow Corr(X_1, X_2) = 0$

?

Proof: $Corr(X_1 X_2) \Rightarrow Cov(X_1 X_2) = E(X_1 X_2) - E(X_1) E(X_2)$

$$E(f(X) g(Y)) \underset{\text{indep.}}{\triangleq} E(f(X)) E(g(Y))$$

Transformation of RV:

if X is a multivariable RV & $\underline{y} = f(\underline{x})$

what prop. does $\underline{y}$ have ?

$$\underline{Y} = A\underline{x} + \underline{b}$$

$$E(\underline{Y}) = A\,E(X) + b$$

$$Cov(\underline{Y}) = E\left((Y - E(Y))(Y - E(Y))^T \right)$$

$$= E\left((A(X - E(X)) + b)(A(X - E(X)) + b)^T \right)$$

$$= A \underbrace{\Sigma}_{Cov(X)} A^T$$