Report CEA internship

Tanguy Marsault

August 12, 2024

Notations

1 Theoretical Aspects

The Standard Model of Particle Physics is the general framework in which the laws of subatomic particles are described. It is widely based on symmetry principles that give rise to particles and interactions between them. This first section aims at defining precisely the main characters of particle physics, that are the elementary particles as well as the interactions between them (Sections 1.3 and 1.4). In order to make this as rigorous as possible, a brief overview of Quantum Field Theory is presented in Section 1.2. This is the mathematical framework in which the Standard Model is described. Finally, the last paragraph of this section si dedicated to CP violation in the Higgs sector, which is the main topic of the analysis presented in this report.

1.1 Poincaré invariance and particles

It is natural to describe the Universe by first describing which kind of particles lives in it. Historically, this has been the first step in the study of the Universe. After discovering the electrons, protons and neutrons, the physicists have been able to describe the interactions between them, the way they can gather to form atoms, and the way atoms can form molecules. It still raises an important question that was left unanswered for a long time, why do we observe these kind of elementary particles. This is rather the modern way to look at the Universe. One first tries to understand the mathematical structure of the Universe and its laws and then determines which kind of particles can evolve in it. The mathematical structure of the Universe is based on symmetry and invariance principles.

A symmetry is any kind of transformation of the state of the Universe that leaves the laws of physics invariant. For example, the laws of physics should be the same wherever one stands in the Universe. This is the translation symmetry. These symmetry principles alone can constrain greatly the kind of particles one may encounter in the Universe.

The first thing to consider when studying the Universe is *spacetime*. This is the utmost basis of the mathematical structure of the Universe. For particles to exist, one needs to have a spacetime in which they can evolve. Mathematically speaking we shall consider spacetime as a 4-dimensional manifold and for the purposes of particles physics one may assume it is flat meaning that the metric is the Minkowski metric,

$$\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1) \tag{1}$$

We shall not delve into the mathematical details of the Minkowski metric and explain really what a manifold is. A good description for this topic can be found in Ref. [2].

Having setup a spacetime, one can already think of very natural transformation that should leave the laws of physics invariant. As mentionned above, the most intutive one is a translation, namely what happens if one translates the origin of the coordinates of spacetime. Translations correspond to transformations of the form

$$x^{\mu} \to x^{\mu} + a^{\mu} \tag{2}$$

where a^{μ} is a constant vector. Hence these transformations can be indexed by some $4-$vector a. It only seems natural that the law of physics should be invariant under such transformations, it means that the Universe looks exactly the same wherever one stands still.

The second kind of symmetry one can think of is the rotation symmetry. But we shall rather take a look at special case of rotations, namely the Lorentz group. The Lorentz group is the group of transformations that leave the Minkowski metric invariant. It can be described as follows,

$$O(1,3) = \{\Lambda \in \mathrm{GL}(4,\mathbb{R}) \mid \Lambda^T \eta \Lambda = \eta\} \tag{3}$$

where η is the Minkowski metric introduced in (1) and is considered here as a 4×4 matrix. A detailed description of the Lorentz group and its infinitesimal structure is given in Appendix B.

Lorentz transformations are far less natural and their origin comes from special relativity. Indeed, if one aims at describing the laws of physics for elementary particles, one should take into account the fact that these may be extremely small quantum-behaved objects, as well as extremely fast relativistic objects. Special relativity indicates that the law of physics should be invariant under Lorentz transformations (see Ref. [7]).

Lorentz transformations can be seen as rotations in spacetime. The combination of Lorentz transformations and translations give rise to the Poincaré transformations,

$$\mathcal{P} = O(1,3) \ltimes R^4 = \{(\Lambda, a) \mid \Lambda \in O(1,3), a \in \mathbb{R}^4\} \tag{4}$$

The set of Poincaré transformations is a group, the *Poincaré group*, with group multiplication law,

$$(\Lambda_1, a_1) \cdot (\Lambda_2, a_2) = (\Lambda_1 \Lambda_2, \Lambda_1 a_2 + a_1)$$

In general one assumes that the laws of physics are invariant under Poincaré transformations. It is not obvious what this should mean in the context of quantum mechanics. In quantum mechanics, the states of a system are described by rays in a Hilbert space. Under a Poincaré transformation of spacetime, the state of the system must transform in a unitary way so that the probabilistic interpretation of quantum mechanics holds [19]. Then the kinds of particles that can be realized in nature are determined by the projective unitary representations of the Poincaré group (Chapter 2 of Ref. [17]). These representations have been classified by Wigner (Ref. [18]) using two characteristics numbers that are the mass m of the particle and its spin s (or its helicity when the particle is massless).

The Poincaré invariance of the laws of physics is a very strong constraint that allows only for specific mathematical objects to evolve in the Universe. All theories of the Universe must satisfy Poincaré invariance because it is the starting point of the description of particles in the first place. Any particle is primarily defined by two numbers that are its mass and its spin.

1.2 Quantum field theory

Before describing the Standard Model of Particle Physics, we shall first give a brief overview of Quantum Field Theory (QFT). QFT is the mathematical framework in which the laws of physics are described in the Standard Model. This theory succeeds in uniting special relativity with quantum mechanics. Its great success is celebrated because of the incredibly precise prediction it is able to make. The most famous example is the prediction of the anomalous magnetic moment [15] of the electron.

In this section, we shall first describe its Lagrangian formulation before explaining the quantization process along with the apparition of quantum fields. Finally, we'll give an introduction to the diagrammatic representation QFT introduced by Feynman.

1.2.1 Lagrangian formulation

The discussion of this section mostly follow Chapter 7 of Ref. [17].

Let us first recall a few important features of the Lagrangian formalism. The Lagrangian is a functional

$$L[q(t),\dot{q}(t)]$$

It depends on generalized coordinates q_i, that we denoted simply with q in the above notation, that themselves evolve in time. For instance, these can be the positions along the three Cartesian axis for a single particle system. When the number of degree of freedom increases one obtains a continuum so that every space point $\mathbf{x}$ has a degree of freedom. This is exaclty the paradigm of QFT, where the q_i becomes fields $\Psi^l(x)$. This way the Lagrangian reads,

$$L[\Psi(t),\dot{\Psi}(t)] \tag{5}$$

where the notation $\Psi(t)$ mean that the Lagrangian can depend on the values of the fields Ψ^l at any point of space $\mathbf{x}$ at time t.

The Lagrangian is not yet the main character of the story. In fact, the more fundamental quantity is the *action* defined as,

$$S[\Psi(t),\dot{\Psi}(t)] = \int \mathrm{d}t L[\Psi(t),\dot{\Psi}(t)] \tag{6}$$

Classically, the field configurations that are realised in nature (without considering the quantum fluctuations that shall be taken into account in the path integral formalism, see Chapter 9 of Ref. [17]), are those that extremizes the action, that is $\delta S = 0$ under any variation of the fields.

In most cases, the Lagrangian is obtained from a *Lagrangian density*,

$$L[\Psi(t),\dot{\Psi}(t)] = \int \mathrm{d}^3\mathbf{x}\mathcal{L}(\Psi^l(x),\partial_\mu\Psi^l(x)) \tag{7}$$

whence the action reads,

$$S[\Psi(t),\dot{\Psi}(t)] = \int \mathrm{d}^4x\mathcal{L}(\Psi^l(x),\partial_\mu\Psi^l(x)) \tag{8}$$

This way, the fundamental equation $\delta S = 0$ reduces to the Euler Lagrange equations,

$$\frac{\partial \mathcal{L}}{\partial \Psi^l(x)} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \Psi^l(x))} = 0 \tag{9}$$

Let us give a few example of Lagrangians that we might refer to in the following paragraphs. The first one is the Lagrangian of a free scalar field,

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2}m^2\phi^2 \tag{10}$$

The second one is the Lagrangian of a free fermion field (a bispinor see Appendix C),

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi \tag{11}$$

The last one is the Lagrangian of a free vector field,

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A_\mu A^\mu \tag{12}$$

where $F_{\mu\nu}$ is the field strength tensor of the vector fields (see in the following sections).

In all of these equations, the constant m corresponds to the mass of the particle (and antiparticle) created by the field. It is interesting to see how the mass term in the Lagrangian appears in different fashions depending on the field considered. This is an important feature that we shall refer to when we study symmetry breaking in Section 1.4.3 and 1.4.4 and Appendix D.

In general, one can perform a unitary transformations on the fields and work with different fields (it is what is done in Appendix D). However, these might not leave the Lagrangian in a fashion similar to the one just exposed in (10), (11) and (12). The fields that allow to write the Lagrangians in this way are called the *mass eigenstates* of the theory.

1.2.2 Variety of fields

The equations of motion obtained above allow to quantize the field $\Psi^l(x)$. This way, the fields can be written in terms of creation and annihilation operators that create and destroy particles (so called *quanta* of the fields). The modern view of physics is to say that particles are highly excited quanta of the field. These fields, associated with the correct mass, allow for the creation of the particles described in Section 1.1, with the correct transformations properties.

For symmetry and invariance reasons, the fields are determined by two quantum numbers (j_1, j_2) that are half integers and determine the spin of a particle $s = j_1 + j_2$ (see Chapter 9 of Ref. [17]). These two numbers correspond precisely to the classification of irreducible unitary representation of the Lorentz group (see Appendix B).

These numbers determine the behaviour of the fields under Lorentz transformations, that is to say if a particle is a boson of a fermion. Bosons are particles that have an integer spin while fermions have a half-integer spin.

There are a few important representations of the Lorentz group that we shall use in the following. The first one is the *scalar* representation that is a singlet under Lorentz transformations

$$U(\Lambda)\phi(x)U(\Lambda) = \phi(\Lambda^{-1}x) \tag{13}$$

It corresponds to the representation $(j_1, j_2) = (0, 0)$. The second one is the *vector* representation that corresponds to $(j_1, j_2) = (1/2, 1/2)$,

$$U(\Lambda)\phi^{\mu}(x)U(\Lambda) = \Lambda^{\mu}{}_{\nu}\phi^{\nu}(\Lambda^{-1}x) \tag{14}$$

The last one we are interested in are the *spinor* representations that are the representations $(j_1, j_2) = (1/2, 0)$ and $(j_1, j_2) = (0, 1/2)$. Their behaviour is much less conventional and the details are exposed in Appendix C.

1.2.3 Gauge theory

Gauge theory is a very important aspect of Quantum Field Theory, and is widely used in the Standard Model. In this paragraph we describe the principles of gauge theory.

Let us consider some Lie group G (see Appendix A) that acts on the the Lagrangian through some representation. Let us suppose the Lagrangian is invariant under this action of the group, that is to say,

$$\mathcal{L}[g \cdot \phi(x)] = \mathcal{L}[\phi(x)] \tag{15}$$

Such a theory is said to be globally symmetric with respect to the group G.

This is to be compared with the case where the group G acts on the fields in a space-time dependent way,

$$\phi(x) \to g(x) \cdot \phi(x) \tag{16}$$

This is the case of a *local* symmetry. In general, Lagrangian that are globally invariant are not locally invariant. However, there is a general recipe that works for all the QFT we shall study in this report, and that naturally makes the Lagrangian locally invariant. A theory with a local symmetry is called a *gauge theory*.

The recipe is to introduce a new fields, the *gauge fields* and to make them transform in a way that cancels the transformation of the fields. The number of gauge fields that needs to be introduced is equal to the number of generators of the group G (see Appendix A). Let us notice how the difference between global and local invariance can only appear in terms that involve derivatives of the fields. This makes natural to introduce the gauge fields through a covariant derivative,

$$D_{\mu} = \partial_{\mu} + T^a A^a_{\mu}(x) \tag{17}$$

where $A_{\mu}(x)$ is the gauge field and T^a are the generators of the gauge group. This way, one can notice how the gauge fields and the field Φ interact in the Lagrangian (see Section 1.2.4).

Introducing the gauge fields enforces us to consider the free part of the gauge fields, namely their kinetic terms (one can show that these can't be massive to ensure gauge invariance), in the Lagrangian. This is done by adding the term,

$$\mathcal{L}_{gauge} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \tag{18}$$

where $F_{\mu\nu}$ is the field strength tensor of the gauge fields defined by,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu] \tag{19}$$

The representation under which the gauge fields should transform is the adjoint representation (see Chapter 2 of Ref. [9]). It means that under a transformation of the group G, the gauge fields transform as,

$$A_\mu(x) \to g(x)A_\mu(x)g(x)^{-1} + g(x)\partial_\mu g(x)^{-1} \tag{20}$$

The group G is ultimately called the gauge group.

To render this discussion more concrete, let us consider the simplest case where the gauge group is $U(1)$, the group of unitary 1×1 matrices. Let us consider the Lagrangian of a free complex scalar field,

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi$$

This is obviously invariant under transformations,

$$\phi(x) \to e^{i\alpha}\phi(x)$$

with α a real constant. However it can be checked that it is not invariant under local transformations,

$$\phi(x) \to e^{i\alpha(x)}\phi(x)$$

The way to make it locally invariant is to introduce a new field $A_\mu(x)$ ($U(1)$ has only one generator) that transform as,

$$A_\mu(x) \to A_\mu(x) + \partial_\mu \alpha(x)$$

The final Lagrangian reads,

$$\mathcal{L} = D_\mu \phi^* D^\mu \phi - m^2 \phi^* \phi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \tag{21}$$

and one can check that this is invariant under local gauge transformations.

1.2.4 Feynman diagrams

As explained above, the Lagrangian is written in terms of fields that are determined by two quantum numbers. In general one can consider polynomial terms of the fields and their derivative in the Lagrangian. Usually, and in everything that we will consider in this report, the Lagrangian reads,

$$\mathcal{L} = \mathcal{L}_{free} + \sum_{i_1,\dots,i_n} g_{i_1,\dots,i_n} \varphi_{i_1} \cdots \varphi_{i_n} \tag{22}$$

where the φ_i are the fields and the $g_{i_1,\dots,i_n}$ are the coupling constants that shall rule the interactions between the fields. The free part of the Lagrangian is the one that contains the kinetic terms of the fields along with the mass terms.

In order to study these theories pertubatively, that is to say when the coupling constants are small, Feynman developped a diagrammatic method that allows to expose which processes can occur. The recipe is simple, each term in the interaction part of the Lagrangian corresponds to a vertex in the diagram with coupling constant $g_{i_1,\dots,i_n}$. each field attached to

(a) $g\phi^3$ (b) $g'\psi^\dagger\phi\psi$

Figure 1: Examples of vertex diagrams

this vertex is a line in the diagram. We sketch examples in Figure 1.

In this way, each field is associated with a line style in the diagram. For our purposes, we shall only be using a few ones that we dscribe below,

- A solid line represents a fermion field ψ (*e.g* the electron or positron field)
- A dashed line represents a scalar field ϕ (*e.g* the Higgs field)
- A wavy line represents a vector field A_μ (*e.g* the photon field)

To each diagram an amplitude is associated (see [16]). The probability of a process to take place, is obtained by summing the amplitudes of all diagrams that can lead to the same final state. This is the essence of the Feynman diagrammatic method.

1.3 The Standard Model of Particle Physics

In Section 1.1 we explained how the Poincaré invariance allows only for specific types of particles to evolve in spacetime. Now we shall actually describe the ones that have been observed and that are gathered in the so called Standard Model (SM) of Particle Physics. The Standard Model is a quantum field theory that describes the interactions between the elementary particles of the Universe. In this paragraph we just give a brief overview of the Standard Model and detail the specific interactions in the next section.

A general summary of the Standard Model is given in Figure 2. The different kind of particles can be divided into 2 categories, the fermions and the bosons. The fermions are the particles that have a half-integer spin while boson have an integer spin. Fermions are typically the particles that are the building blocks of matter while boson are the carrier of interactions between the fermions. Each fermion has an associated antifermion that has the same mass but opposite charge. The fermions are divided into 2 categories, the quarks and the leptons. The quarks are particles that interact through the strong force and form the protons and neutrons in the atomic nucleus. There are 6 quarks and 6 leptons. On Figure 2, one can also take notice of the spin of each particle.

In the Standard Model, the interactions between the particles are described by exchange of bosons. There are three kind of interactions. The first one is the electromagnetic interaction that is described by the exchange of photons. The second one is the weak interaction that is described by the exchange of $W^\pm$ and Z bosons. The third one is the strong interaction that is described by the exchange of gluons. Not all fermions are subjected to all interactions. Only quarks interact through strong interactions, while quarks and leptons interact through weak interactions. Neutrinos have no electric charge and are insensitive to the electromagnetic interaction. Figure 3 gives a summary of the interactions between the particles of the

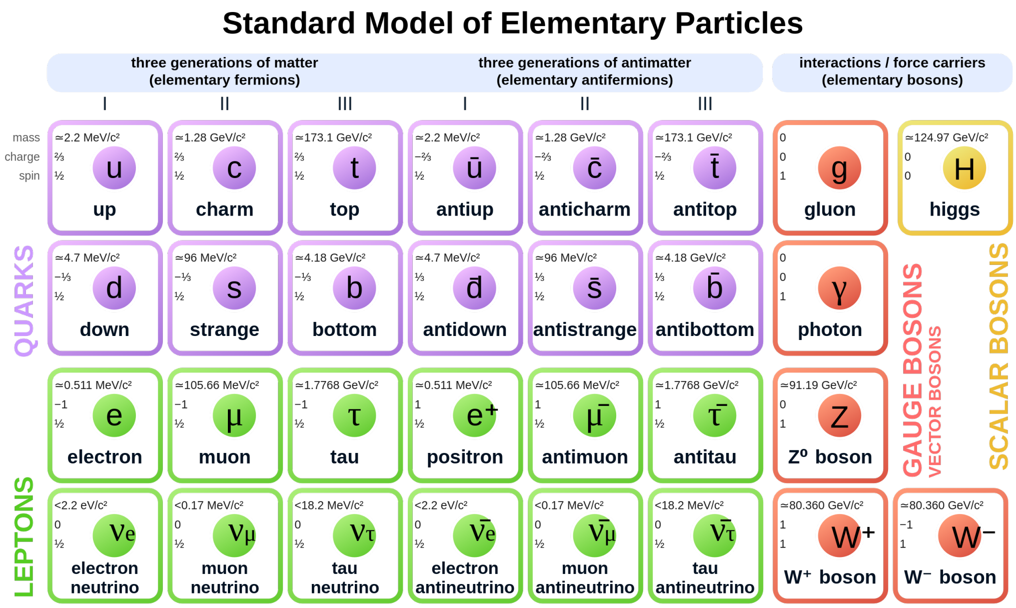


Figure 2: The Standard Model of Particle Physics

Standard Model.

The Higgs particle is classified as a boson eventhough it does not actually carry a force. Through the Weinberg-Salam mechanism (see Chapter 47 of Ref. [11] and Appendix D), the Higgs boson gives mass to the $W^{\pm}$ and Z bosons (Section 1.4.3). The mass of the other particles is given by the Yukawa couplings to the Higgs field (Section 1.4.4). We describe these phenomena in the next section. The Higgs boson is the last particle of the SM that has been discovered in 2012 at the LHC [3, 4].

1.4 Interactions between particles

The mathematical formulation of the Standard Model is based on Quantum Field Theory, which is entirely determined by a Lagrangian (Section 1.2). The Lagrangian of the SM is divided into 4 different parts that represent the Quantum Chromodynamics (QCD), the Electroweak interaction (EW), the Higgs boson Yukawa couplings and the Higgs boson potential. The Lagrangian can be decomposed as,

$$\mathcal{L} = \mathcal{L}_{QCD} + \mathcal{L}_{EW} + \mathcal{L}_{YW} + \mathcal{L}_{Higgs} \tag{23}$$

In the following paragraphs, we describe each of these interactions as well as the symmetry breaking mechanism through which the Higgs boson gives mass to the massive particles of the Standard Model.

1.4.1 Quantum Chromodynamics

Quantum Chromodynamics is the theory that describes the strong interaction, that is to say the interactions between quarks and gluons. As mentioned earlier, there are 6 quarks (and 6 antiquarks), that are interacting through gluons. The gauge group associated with this interaction is $SU(3)$, the group of unitary 3×3 matrices with determinant 1,

$$SU(3) = \{U \in \mathrm{GL}(3, \mathbb{C}) \,|\, U^{\dagger}U = 1, \det(U) = 1\} \tag{24}$$

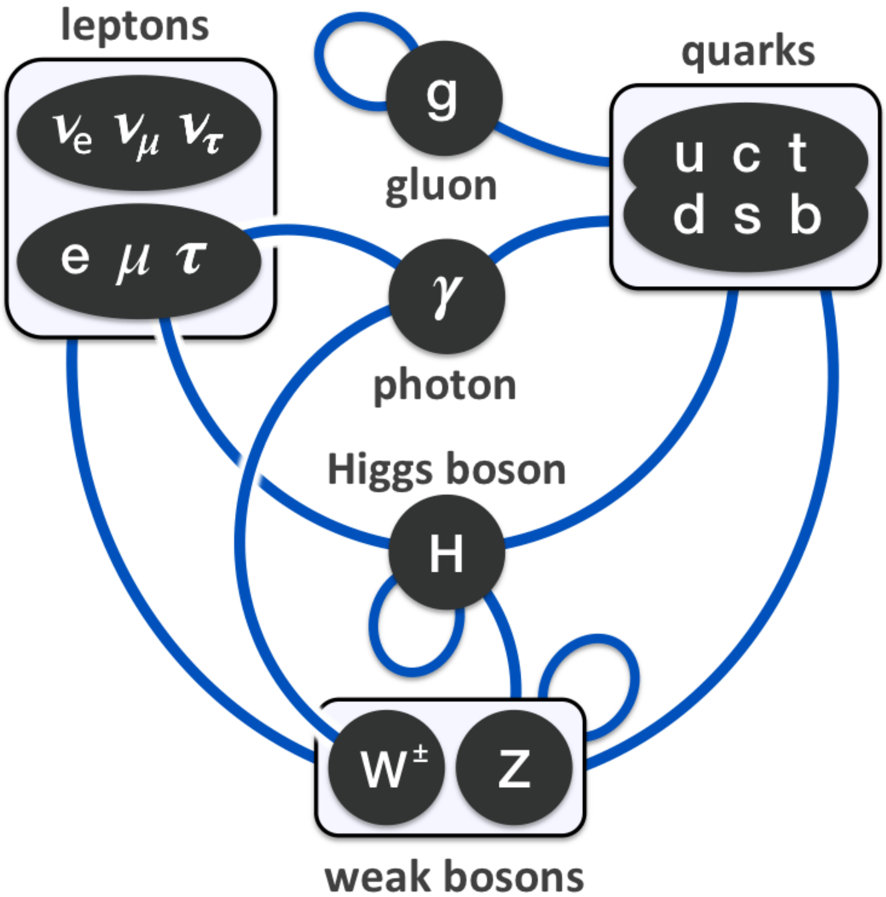


Figure 3: Interactions between the particles of the Standard Model

The $SU(3)$ symmetry is associated with a conserved charge known as *color*. This color has nothing to do with the color we see in everyday life but is named as such because it is formed of 3 different charges, that can be summed in a similar fashion than RGB colors. The color charges are called *red, blue* and *green* for obvious reasons and the associated anticolors, corresponding to the antiparticles, are called *antired, antiblue* and *antigreen*. These anticolors are the analogues of the negative electric charge in classical electrodynamics.

Gluons carry interactions between quarks (or antiquarks) and thus couple different colors. As such gluons carry two color charges. This would add up to a total of $3 \times 3 = 9$ different gluons. However the infinitesimal group structure of $SU(3)$ shows that one of these 9 gluons can be expressed as a linear combination of the others. This leaves us with 8 different gluons.

Since gluon carry color charges, they can interact with themselves. This is a very peculiar feature of the strong interaction. The self-interaction of gluons is responsible for the confinement of quarks. This means that quarks are never to be found alone in nature. They are always found in bound states called hadrons.

All of what we just described can be encapsulated in a very nice mathematical formulation using a Lagrangian. It reads,

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \sum_q \bar{\psi}_q \gamma^\mu D_\mu \psi_q \tag{25}$$

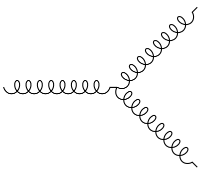
where $G^a_{\mu\nu}$ is the field strength tensor of the gluons, ψ_q is the quark field and D_μ is the covariant derivative that accounts for the color charge. The field strength tensor is defined as,

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f^{abc} A^b_\mu A^c_\nu \tag{26}$$

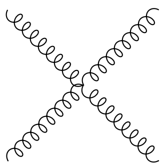
where A^a_μ is the gluon field, g_s is the strong coupling constant and f^{abc} are the structure constants of the $SU(3)$ group. The covariant derivative is defined as,

$$D_\mu = \partial_\mu - i g_s \frac{\lambda^a}{2} A^a_\mu \tag{27}$$

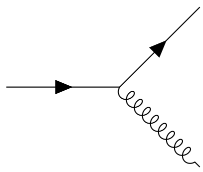
where λ^a are the Gell-Mann matrices that are the generators of the $SU(3)$ group. Expanding the Lagrangian completely, one finds exactly which interaction can be realised. Namely, each term in the Lagrangian corresponds to a different kind of Feynman diagram. We represent them in the Figure (4). The term involving three fields A correspond to the three gluons vertex (4a), the term involving four fields A correspond to the four gluons vertex (4b) and the term involving a quark field, an antiquark field and a gluon field correspond to the quark-antiquark and gluon vertex (4c).



(a) Three gluons vertex



(b) Four gluons vertex



(c) Quark-antiquark and gluon vertex

Figure 4: Interaction diagrams for QCD

In quantum field theory, the coupling constant g_s is not a fixed number but rather a function of the energy scale at which the interaction takes place. At low energy scales, the coupling constant is large, $O(g_S) \sim 1$ and the perturbative expansion of QFT may break, making the theory non-perturbative. This is why QCD is such a difficult theory to study. At high energy scales, the coupling constant becomes low, $O(g_s) \sim 0$ and the perturbative expansion is valid. This decreasing behaviour of the coupling constant with the energy scale is called *asymptotic freedom* [8], and allows for non-neutral colored particle to evolve freely.

1.4.2 Electroweak interaction

One usually says that there are 4 fundamental forces in nature, the strong force (see Section 1.4.1), the electromagnetic force, the weak force and gravity. Particle physics has nothing to say about the fourth one whose effect is so little that it can be neglected at the scale of elementary particles. In the SM the electromagnetic and weak forces are unified into a single force, the electroweak interaction.

The electroweak interaction is described by the exchange of $W^{\pm}$ and Z bosons and photons γ. The gauge group associated with this interaction is $SU(2)_L \times U(1)_Y$. Let us see what these groups mean and how they are related to the $W^{\pm}, Z, \gamma$ bosons.

First let us recall the general definitions of these two groups,

$$SU(2) = \{U \in \mathrm{GL}(2,\mathbb{C}) \mid U^{\dagger}U = 1, \det(U) = 1\} \tag{28}$$
$$U(1) = \{U \in \mathrm{GL}(1,\mathbb{C}) \mid U^{\dagger}U = 1, \det(U) = 1\} \tag{29}$$

In order to describe properly weak interaction, one needs to introduce the concept of *chirality*. Chirality is a specific representation of the bispinor fields (see Appendix C). In the chiral representation, bispinor fields are decomposed into left-handed and right-handed components defined by,

$$\psi_L = \frac{1}{2}(1-\gamma^5)\psi \tag{30}$$
$$\psi_R = \frac{1}{2}(1+\gamma^5)\psi \tag{31}$$

This expression is very general and does not depend on the basis chosen for the γ matrices. A fermion field can thus always be written uniquely as,

$$\psi = \psi_L + \psi_R \tag{32}$$

The $SU(2)_L$ group is associated with the left-handed components of the fermion fields. In its simplest non trivial representation (see Chapter 3 of Ref. [6]), the group $SU(2)$ tells use that two left-handed components of fields are related through an $SU(2)$ transformation. These doublets are,

$$\begin{pmatrix}(\nu_e)_L\\ e_L\end{pmatrix} \quad \begin{pmatrix}(\nu_\mu)_L\\ \mu_L\end{pmatrix} \quad \begin{pmatrix}(\nu_\tau)_L\\ \tau_L\end{pmatrix} \quad \begin{pmatrix}u_L\\ d_L\end{pmatrix} \quad \begin{pmatrix}c_L\\ s_L\end{pmatrix} \quad \begin{pmatrix}t_L\\ b_L\end{pmatrix} \tag{33}$$

The subscript L indicated that only the left-handed components are considered. In fact the right handed components are put into singlet representation of the $SU(2)$ group,

$$(e_R) \quad (\mu_R) \quad (\tau_R) \quad (u_R) \quad (c_R) \quad (t_R) \quad (d_R) \quad (s_R) \quad (b_R) \tag{34}$$

Particles transforming under the $SU(2)$ symmetry belong to representations indexed by a weak isospin I_W and its third component of weak isospin $I_W^{(3)}$ (in the same fashion than spin). By convention the particle in the upper component of the doublet representation has weak isospin third component $I_W^{(3)} = +\frac{1}{2}$ and the one in the lower component has $I_W^{(3)} = -\frac{1}{2}$ while the isospin is I_W for both particles. The right-handed particles are singlets under the $SU(2)$ group, meaning $I_W = 0$ and $I_W^{(3)} = 0$.

The $SU(2)$ group has 3 generators, which leads to three gauge bosons, the W^1, W^2 and W^3 bosons. Note how these aren't the usual W and Z bosons.

The $U(1)$ group is associated with the hypercharge Y of the particles. This behaves like the classical electric charge, but is not quite equal to it, since the particle associated with the generator of this group is B but not the photon, as we shall explain below.

Before describing the electroweak mixing phenomenom, let us notice that the above description is not quite right since there are no reason for particles created through weak interaction to be the mass eigenstate of the theory. This means that eventhough the free particles may be the quarks and leptons, as described above, there is no reason for these to be the ones that are created through weak processes. Keeping an open mind, in general, the weak interaction will mix the mass eigensate of the theory. This translates in 2 unitary matrices that are the *Cabibbo-Kobayashi-Maskawa* (CKM) matrix [1, 10] for quarks and the PMNS matrix for neutrinos [14, 12]. Let us denote them by U_{CKM} and U_{PMNS} respectively. In general, one can then define the weak eigenstates as,

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = U_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \tag{35}$$

and

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{PMNS} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \tag{36}$$

The ν_i are the mass eigenstates of neutrinos. As a consequence, it makes no sense to speak of the masses of ν_e for instance. The correct weak doublets are obtained by replacing the corresponding quarks by their *primed* counterpart. This problem should not appear for ν since the MS expect the neutrinos to be massless. If this last assumption is a very good approximation, it has been proven that neutrinos have a really small non-zero mass (see Section 17.8 of [16]) and so the matter of mass eigenstates mixing is of relevance.

Let us now come back to the matter of electroweak mixing. The $SU(2)_L \times U(1)_Y$ group is broken into the electromagnetic $U(1)_{EM}$ group by the Higgs boson mechanism. This is the so called *electroweak symmetry breaking*, this shall be described in Section 1.4.2. This way the W^1, W^2 and W^3 bosons along wiht the B boson mix to form the $W^\pm$, Z and γ bosons.

The $W^\pm$ comes from the complexficiation of the W^1 and W^2 bosons,

$$W^\pm = \frac{1}{\sqrt{2}}(W^1 \mp iW^2) \tag{37}$$

while the Z and γ (denoted by A) bosons come from the mixing of the W^3 and B through the Weinberg angle,

$$\begin{pmatrix} Z \\ A \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W^3 \\ B \end{pmatrix} \tag{38}$$

The value taken by the Weinberg angle will be determined in the following paragraph.

To compute the resulting electromagnetic charge of the particles, one needs to refer to the hypercharge Y and the weak isospin third component $I_W^{(3)}$ through the Gell-Mann-Nishijima formula [5, 13],

$$Q = I_W^{(3)} + \frac{Y}{2} \tag{39}$$

Finally one writes down the Lagrangian of the electroweak interaction as,

$$\mathcal{L}_{EW} = -\frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \sum_f \bar{\psi}_f \gamma^\mu D_\mu \psi_f \tag{40}$$

where $W^a_{\mu\nu}$ and $B_{\mu\nu}$ are the field strength tensors of the W and B bosons, defined in a fashion similar to the one for gluons. The index f ranges over all fermions. The covariant derivative is defined as,

$$D_\mu = \partial_\mu - igI_W\frac{\sigma^a}{2}W^a_\mu - ig'\frac{Y}{2}B_\mu \tag{41}$$

Let us quickly remark that the derivative term, ∂_μ should not be included twice when suming with the QCD Lagrangian.

Contrary to what we did for QCD, we shall not expand the Lagrangian of the EW interaction and draw the corresponding diagrams since these are not the ones used in practice. We shall provide the diagrams only after the next section when we explain the electroweak symmetry breaking, which leads to the natural apparition of the masses for the W and Z bosons.

1.4.3 Electroweak symmetry breaking

Symmetry breaking refers to the fact that eventhough laws of physics are invariant under some symmetry group, the vacuum state of the theory is not. This is a very important concept in particle physics and is at the heart of the Higgs boson mechanism.

We shall push back the explanation of the Higgs boson mechanism to Appendix D and rather give a simple example of symmetry breaking in the context of quantum field theory in this section.

Let us consider the simplest gauge group, that is $U(1)$, with a complex scalar field φ and an electromagnetic vector field A_μ. The gauge invariant Lagrangian reads,

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\varphi)^\dagger(D^\mu\varphi) - \mu^2\varphi^\dagger\varphi - \lambda(\varphi^\dagger\varphi)^2$$

with covariant derivative

$$D_\mu = \partial_\mu + igA_\mu$$

and field strength tensor

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

The gauge invariant Lagrangian reads,

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\varphi)^\dagger(D^\mu\varphi) - \mu^2\varphi^\dagger\varphi - \lambda(\varphi^\dagger\varphi)^2$$

where the field strength tensor is given by,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

The gauge field and the complex scalar field interact through the covariant derivatives. In fact, if we expand it out, we find that,

$$(D_\mu\varphi)^\dagger(D^\mu\varphi) = \partial_\mu\varphi^\dagger\partial^\mu\varphi + igA_\mu(\varphi^\dagger\partial^\mu\varphi - \partial^\mu\varphi^\dagger\varphi) + g^2A_\mu A^\mu\varphi^\dagger\varphi$$

This Lagrangian is invariant under the simultaneous transformations,

$$\begin{aligned} \varphi(x) &\longrightarrow e^{i\alpha(x)}\varphi(x) \\ A_\mu(x) &\longrightarrow A_\mu(x) + \frac{1}{g}\partial_\mu\alpha(x) \end{aligned}$$

It is easy to check that these transformations represent the gauge group $U(1)$. Let us write the complex scalar field as,

$$\varphi(x) = \rho(x)e^{i\theta(x)} \tag{42}$$

where $\rho(x)$ and $\theta(x)$ are two real-valued scalar fields. In the context of symmetry breaking, the field $\rho(x)$ acquires a *vacuum expectation value*, that is to say, it is not set to zero in the vacuum state of the theory. We write,

$$\rho(x) = \frac{1}{\sqrt{2}}(v + \chi(x)) \tag{43}$$

In terms of ρ and θ, one expands the Lagrangian as,

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \rho^2\left(\partial_\mu\theta - gA_\mu\right)^2 + (\partial\rho)^2 - \mu^2\rho^2 - \lambda\rho^4$$

One can absorb the θ field into the gauge field by defining a new field B_μ as,

$$B_\mu = A_\mu - \frac{1}{g}\partial_\mu\theta \tag{44}$$

It can be shown that the strength tensor associated to A and B are the same so that, one can write the Lagrangian as,

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}(\partial_\mu\chi)^2 + \frac{1}{g^2}\left(\frac{v+\chi}{\sqrt{2}}\right)^2(B_\mu)^2 - \mu^2\left(\frac{v+\chi}{\sqrt{2}}\right)^2 - \lambda\left(\frac{v+\chi}{\sqrt{2}}\right)^4 \tag{45}$$

where we replaced the ρ field by its vacuum expectation value v and the associated field χ.

Expanding the Lagrangian, one finds that,

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}g^2v^2B_\mu^2 + \frac{1}{2}\left(\partial\chi\right)^2 + g^2v\chi B_\mu^2 + \frac{1}{2}g^2\chi^2B_\mu^2 - \mu^2\chi^2 - \sqrt{\lambda}\mu\chi^3 - \frac{\lambda}{4}\chi^4 \tag{46}$$

The computation are somewhat blurring away the main point to be made, that is that the field φ has lost a degree of freedom that has been absorbed by the EM field A which in turn has become massive. One can also notice how various interaction terms have appeared in the Lagrangian.

This process is responsible for the masses of $W^\pm$ and Z bosons. If it weren't for it, these would be massless as emphasized in Section 1.4.2.

In the standard model, the Higgs doublet is defined as

$$\Phi = \begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} \tag{47}$$

and is a doublet with weak isospin $I_W = \frac{1}{2}$ and hypercharge $Y = 1$. After symmetry breaking, gauge invariance allows to write it in the form,

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v+h\end{pmatrix} \tag{48}$$

where $h(x)$ is a real scalar field referred to as the Higgs boson.

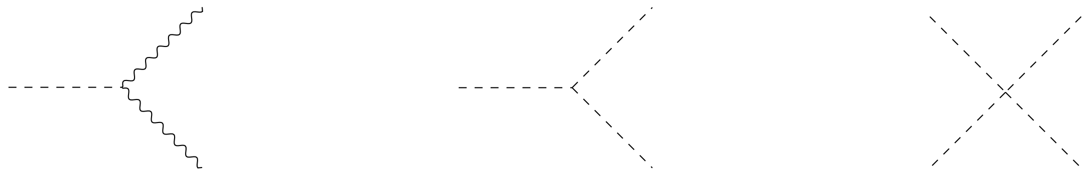
In Appendix D, we shall give a more detailed explanation of the Higgs boson, we only quote the main results here. The Weinberg angle is determined by,

$$\cos\theta_W = \frac{g}{\sqrt{g^2 + g'^2}} \qquad \tan\theta_W = \frac{g'}{g} \tag{49}$$

While the masses of the $W^{\pm}$ and Z bosons are given by,

$$m_Z = \frac{1}{2} v \sqrt{g^2 + g'^2} \quad m_W = \cos\theta_W m_Z \tag{50}$$

To conclude this section, let us draw the diagrammatic interaction this phenomena gives rise to. The ones involving the Higgs boson are represented on Figure 5. There are 3 main diagrams, representing the decays of the Higgs to two $W^{\pm}, Z$ (5a) or γ bosons and the Higgs self-interactions (5b), (5c).



(a) Higgs decay to $W^{\pm}, Z, \gamma$ (b) Three Higgs vertex (c) Four Higgs vertex

Figure 5: Interaction diagrams for EW involving the Higgs boson

One can also draw the diagrams involving the coupling of $W^{\pm}, Z$ and γ to fermions, see Figure 6.

(a) W to lepton and neutrino (b) Z, γ to 2 leptons

Figure 6: Fermion and $W^{\pm}, Z, \gamma$ vertex

1.4.4 Higgs boson and particle masses

It is usually stated that the Higgs boson gives mass to the particles of the SM. Let us make this statement clear in this paragraph.

The first thing one should notice is that in the usual Dirac theory, the mass term appear in the Lagrangian as,

$$-m\bar{\psi}\psi = -m(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L) \tag{51}$$

which does not respect the $SU(2) \times U(1)$ symmetry of the SM. Therefore it can not appear in our theory. To make the mass term appear, the trick is to use the Higgs doublet. Let us go back to our previous notations and denote L one of the doublet of (33) R the associated singlet of (34). Under an infinitesimal $SU(2)$ transformation the Higgs doublet is transformed as,

$$\Phi \to \Phi + i\varepsilon^a(x)\frac{\sigma^a}{2}\Phi \tag{52}$$

and $\bar{L} = L^\dagger\gamma^0$ is transformed as,

$$\bar{L} \to \bar{L} - i\varepsilon^a(x)\bar{L}\frac{\sigma^a}{2} \tag{53}$$

So that the combinations, $\bar{L}\phi$ and $\phi^\dagger L$ are invariant under $SU(2)$ transformations. One can include terms such as,

$$-g_f\left(\bar{L}\phi R + \bar{R}\phi^\dagger L\right) \tag{54}$$

g_f is a coupling constant depending on the fermions considered. Such a term is indeed invariant under $SU(2) \times U(1)$ transformations.

Let us now observe what happens under the Higgs symmetry breaking. One writes,

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix} \tag{55}$$

Let us be precise in the following and settle on $L = \begin{pmatrix} (\nu_e)_L \\ e_L \end{pmatrix}$ and $R = e_R$. Equation (54) becomes,

$$-\frac{g_e}{\sqrt{2}}v\left(\bar{e}_L e_R + \bar{e_R} e_L\right) - \frac{g_e}{\sqrt{2}}h\left(\bar{e}_L e_R + \bar{e_R} e_L\right) \tag{56}$$

The first term is precisely what we wrote in Equation (51) and indicates that fermions have acquired mass,

$$m_f = \frac{g_f}{\sqrt{2}}v \tag{57}$$

The second term correspond to the Yukawa coupling between the Higgs boson and the fermions. Let us remark that this coupling is ruled by the coupling constant g_f to which the mass is proportional. This indicated that the higher the mass of a particle, the higher its coupling constant to the Higgs should be.

Finally let us remark that the neutrinos have not acquired masses. With the same process, one would expect the up-type quarks not to acquire mass as well, eventhough it is well-known that they are massive. A similar process can be built to give masses to up-type quark (see Section 17.5 of Ref. [16]). We shall not detail the process here which is very similar to what we just exposed. The main point is that the result of Equation (57) remain true for all fermions.

To conlcude, let us draw the diagram corresponding to the Yukawa coupling in Figure 7.

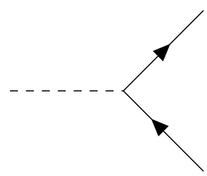


Figure 7: Yukawa coupling diagram

1.5 CP violation

Above we discussed different kind of symmetries, namely spacetime symmetries such as Poincaré transformations which act on spacetime coordinates, and gauge transformations which mix fields in the Lagrangian.

There is yet another kind of transformation that is of interest, it corresponds to the discrete transformations, C, P and T. P stands for parity and corresponds to the spacetime transformation,

$$\mathbf{x} \to -\mathbf{x} \tag{58}$$

T stands for time reversal and corresponds to the transformation,

$$t \to -t \tag{59}$$

C stands for charge conjugation and correspond to the transformation of particles into their antiparticles by reversing their quantum numbers.

The C, P and T transformations are not symmetries of the Standard Model, but their is an utmost famous result that state that the combination of C, P and T transformations should be a symmetry of the laws of physics in a causal theory with Minkowski spacetime. This is the CPT theorem for which a proof can be found in [17].

If the CPT theorem holds, there is no reason for the CP symmetry to hold. As a matter of fact, the CP symmetry is violated in the Standard Model, and more precisely in the weak interaction. Violation could actually have been guessed since the Universe surrounding us is made of matter and not of antimatter. This suggests a difference in behaviour between matter and antimatter. This difference is yet to be understood and is a major open question in particle physics.

For these reasons, CP violation is widely studied in particle physics. It shall be the main topic of the $t\bar{t}H$ analysis presented in Section 1.

The simplest way to implement a CP violating Yukawa coupling is to introduce a phase,

$$\frac{g_f}{\sqrt{2}}\bar{f}hf \longrightarrow \frac{g_f}{\sqrt{2}}\bar{f}e^{i\alpha\gamma^5}hf$$

where α is a real number in $[0, 2\pi)$, that we shall refer to as the $CP-$*phase*. We shall come back to this in due time when studying the $t\bar{t}H$ process.

Appendices

A Lie groups and representations

The elements presented in this section are mostly exposed in Ref. [9].

A.1 Lie groups

Lie groups are groups that are also manifolds and for which the group operations are smooth. This means that the group elements can be locally parametrized by a finite number of real parameters. The group operations are then smooth functions of these parameters.

A representation of a group G is a group homomorphism from G to the group of invertible linear operators on a vector space V,

$$\rho : G \to \mathrm{GL}(V) \tag{60}$$

This ensures that the group operations are preserved by the linear operators. An irreducible representation is a representation that has no non-trivial invariant subspaces. It is an important result that any representation can be decomposed into a direct sum of irreducible representations. This specifically explain why one considers particles as irreducible representations of the Poincaré group (see Section 1.1).

When their is no ambiguity, one shall simply denote the representations by the group element, *i.e* $\rho(g)$ shall be denoted g.

A.2 Lie algebras

As a manifold, a Lie group can be studied infinitesimally by looking at the Lie algebra of the group. The Lie algebra is the tangent space at the identity element of the group T_eG usually denoted by $\mathfrak{g}$. It is a vector space and can be equipped with a Lie bracket operation that is the commutator of the group operation. The dimension of the Lie algebra is determined by the dimension of the group as a manifold, meaning the dimension of the real vector space $\mathbb{R}^l$ with which it islocally diffeomorphic.

The generators of the group always refer to a basis of the Lie algebra. As a vector space, in order to fully grasp the nature of the Lie algebra, one only needs to know the commutation relations between the generators. These can be expressed through structure constants f^{abc} defined by,

$$[T^a, T^b] = i f^{abc} T^c \tag{61}$$

where the settle $\{T_a\}$ is a basis of the Lie algebra.

A representation of the Lie algebra is a vector space morphism from the Lie algebra to the space of linear operators on a vector space V,

$$\rho : \mathfrak{g} \to \mathrm{End}(V) \tag{62}$$

A.3 Exponential map

Using the exponential map, one can relate the Lie algebra to the Lie group. The exponential map is defined as,

$$\exp : \mathfrak{g} \to G \tag{63}$$

It explains why in physics we usually work with the Lie algebra of a group rather than the group itself. The Lie algebra is a vector space and can be studied using linear algebra techniques. The Lie group is a manifold and is much more complicated to study.

It is not entirely clear when one is able to consider only the Lie algebra on a Lie group. We state a few results that may help us grasp the validity of what one can usually see in physics.

The exponential map of a connected Lie group is surjective and so the previous statement would hold for such a group. In anycase, the exponential map is always surjective in some neighbourhood of the identity but is not even surjective on the connected component of the identity.

The most general statement is that the exponential map *generates* the identity component of the group. This means that any element of the identity component can be written as the finite product of exponential of some elements of the Lie algebra.

Let us denote by G_e the identity component of G. Then, for any $x \in G_e$, there exists some vectors $X_i \in \mathfrak{g}$ such that,

$$x = \exp(X_1)\exp(X_2)\ldots\exp(X_k)$$

Let us denote by G_x the connected component of G containing x. The translation map defined by,

$$\begin{aligned} L_x :\ G_e &\longrightarrow G_x \\ y &\longmapsto xy \end{aligned}$$

is smooth and invertible with inverse L_x^{-1} so that G_x is isomorphic to G_e.

This in turn means that if $y \in G_x \neq G_e$, then

$$y = xx^{-1}y = xL_x^{-1}(y) = x\exp(Y_1)\exp(Y_2)\ldots\exp(Y_k)$$

for some $Y_i \in \mathfrak{g}$. This explains why we can only focus on the identity component of the group and include the other components if necessary.

In this report we consider only elements of the group that can be written as exponentials of elements of the Lie algebra,

$$G \ni g = \exp(X),\ X \in \mathfrak{g}$$

* This explain the infinitesimal transformations considered in Section 1.4.4.

As emphasized before, one doesn't really mind about the group G itself but rather about its representation ρ. A representation of the group induces a representation on the Lie algebra, $\rho_* : \mathfrak{g} \longrightarrow \mathfrak{gl}(V) = \mathrm{End}(V)$. The main feature of this induced representation is that it is compatible with the exponential map,

$$\rho(\exp(X)) = \exp(\rho_*(X))$$

This means that were one to consider the action of group elements on the field, $\rho(\exp X)\Phi$, one might as well consider the action of the induced representation ρ_*.

B The Lorentz group

As emphasized in Section 1.1, particles are represented by irreducible representations of the Poincaré group. The Poincaré group is the semi-direct product of the Lorentz group and the translation group. The Lorentz group is the group of transformations that leaves the Minkowski metric invariant. It is the group of rotations and boosts. The Lorentz group is defined as,

$$O(1,3) = \{\Lambda \in M_4(\mathbb{R}) \mid \Lambda^T \eta \Lambda = \eta\} \tag{64}$$

When translated into the field language, Poincaré representation become Lorentz representation of fields. This is why we shall focus on Lorentz representations.

B.1 Group structure

First of all, let us give a brief overview of the Lorentz group. Let us remark that it is not connected, in fact it has four connected components. These are determined by the sign of the determinant of the Lorentz transformation as well as the sign of the time component of the Lorentz transformation. The connected component of the identity is called the proper orthochronous Lorentz group and is denoted by $SO^+(1,3)$. It is defined as,

$$SO^+(1,3) = \{\Lambda \in O(1,3) \mid \Lambda^0_0 > 0,\ \det(\Lambda) = 1\} \tag{65}$$

The other components are obtained by considering $\det \Lambda = -1$ or $\Lambda^0_0 < 0$.

The physical interpretation of the proper orthochronous Lorentz group is that it preserves the orientation of spacetime and the direction of time, which justifies *a posteriori* its name. By introducing the parity and time reversal operators,

$$P = \mathrm{diag}(1,-1,-1,-1) \quad T = \mathrm{diag}(-1,1,1,1), \tag{66}$$

one can check that the other components of the Lorentz group are obtained by applying P, T or PT to the proper orthochronous Lorentz group $SO^+(1,3)$.

In order not to deal with the different components of the Lorentz group, one usually works with the double cover of the proper orthochronous Lorentz group, that is $SL(2,\mathbb{C})$. This group is defined as,

$$SL(2,\mathbb{C}) = \{\Lambda \in M_2(\mathbb{C}) \mid \det(\Lambda) = 1\} \tag{67}$$

For more information on this see Section 2.7 of Ref. [17].

B.2 Representations of the Lorentz group

The previous discussion emphasizes that one can usually work with $SO^+(1,3)$ and use parity or time reversal when needed to move to other components. It means that one only needs representations of $SO^+(1,3)$ along with representation of parity and time reversal.

As explained in Appendix A, we might as well work with infinitesimal representation of the Lie algebra of the Lorentz group and then take an exponential to promote them to representations of the Lorentz group. A nice feature of double covers is that they have the same Lie algebra as the group they cover. In our case,

$$\mathfrak{so}(1,3) = \mathfrak{sl}(2,\mathbb{C}) \tag{68}$$

This means that we can work with the Lie algebra of $SL(2,\mathbb{C})$ that is already greatly studied. The main result is that the Lie algebra of $SL(2,\mathbb{C})$ is,

$$\mathfrak{sl}(2,\mathbb{C}) = \mathfrak{su}(2)_{\mathbb{C}} \oplus \mathfrak{su}(2)_{\mathbb{C}} \tag{69}$$

This in turn proves that the representations are determined by two half integers (j_1, j_2) (see Chapter 3 of Ref. [6] for the representations of $SU(2)$).

C Spinors

Spinors are specific representation of the Lorentz group, namely the $(1/2, 0)$ and $(0, 1/2)$ representations. These are the representations that are used to describe fermions. Is the Standard Model we are rather using the bispinor representation, that is the $(1/2, 0) \oplus (0, 1/2)$ representation. It is the one we shall focus on here.

C.1 Representation

What defines the spinor representation is the transformation of the spinor under Lorentz transformations. Let us denote the spinor by ψ and the Lorentz transformation by Λ. The transformation of the spinor is given by,

$$\psi(x) \to S(\Lambda)\psi(\Lambda^{-1}x) \tag{70}$$

where $S(\Lambda)$ is defined by,

$$S(\Lambda) = \exp\left(\frac{i}{2}\omega_{\mu\nu}S^{\mu\nu}\right) \tag{71}$$

where $\omega_{\mu\nu}$ are the infinitesimal parameters of the Lorentz transformations (there are 6 of them, ω is an anitsymmetric matrix) and $S^{\mu\nu}$ are the generators of the Lorentz group in the spinor representation defined by,

$$S^{\mu\nu} = \frac{i}{4}[\gamma^\mu, \gamma^\nu] \tag{72}$$

The γ^μ are the Dirac matrices. These are defined by the fundamental relation of the Clifford algebra (see Chapter),

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu} \tag{73}$$

where $\{\cdot, \cdot\}$ is the anticommutator.

Any set of matrices that satisfies this relation can be used as the Dirac matrices. The most common representation is the Dirac representation,

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} \tag{74}$$

where σ^i are the Pauli matrices.

One defines the γ^5 matrix by,

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 \tag{75}$$

The γ^5 matrix is used to define the chirality of the spinor. The chirality is defined as the eigenvalue of the γ^5 matrices,

$$\mathrm{Sp}\ \Gamma^5 = \{\pm 1\} \tag{76}$$

The $\pm$ sign indicates the chirality of the spinor. The left-handed spinors are the ones that satisfy $\gamma^5\psi = -\psi$ and the right-handed spinors are the ones that satisfy $\gamma^5\psi = \psi$.

The chirality operator is used to define the projection operators,

$$P_{L,R} = \frac{1}{2}(1 \mp \gamma^5) \tag{77}$$

The left-handed projection operator projects the spinor on its left-handed component and the right-handed projection operator projects the spinor on its right-handed component. Any spinor can be uniquely decomposed into its left and right-handed components,

$$\psi = \psi_L + \psi_R \tag{78}$$

where $\psi_L = P_L\psi$ and $\psi_R = P_R\psi$.

C.2 Dirac equation and Lagrangian

One needs to define the lagrangian for such spinors. To do so, one needs to consider the conjugate spinor, $\bar{\psi} = \psi^\dagger\gamma^0$. The Dirac Lagrangian is then defined as,

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi \tag{79}$$

It yields the Dirac equation by using the Euler Lagrange equations (9),

$$(i\gamma^\mu\partial_\mu - m)\psi = 0 \tag{80}$$

D Salem-Weinberg mechanism

In this section we describe more precisely the Higgs boson mechanism in the context of the Standard Model. As explained in Section 1.4.3, the Higgs doublet is defined as,

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

and is doublet with $I_W = \frac{1}{2}$ with hypercharge $Y = 1$, along with the W and B bosons, the Lagrangian of the system reads,

$$\mathcal{L} = -\frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \left(D_\mu\Phi\right)^\dagger\left(D^\mu\Phi\right) - \mu^2\Phi^\dagger\Phi - \lambda\left(\Phi^\dagger\Phi\right)^2$$

where the covariant derivative is defined as,

$$D_\mu = \partial_\mu - ig\frac{\sigma^a}{2}W^a_\mu - ig'\frac{1}{2}B_\mu$$

Under symmetry breaking and using gauge invariance, one can write the Higgs doublet as,

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

This is called the unitary gauge. Let us focus on the covariant derivative term coupling,

$$-\left[\frac{ig}{2}\begin{pmatrix} W^3_\mu & W^1_\mu - iW^2_\mu \\ W^1_\mu + iW^2_\mu & -W^3_\mu \end{pmatrix} + i\frac{g'}{2}B_\mu\right]\begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

Multiplying this by the adjoint yields terms,

$$\frac{g^2v^2}{4}\left[(W_\mu^1)^2 + (W_\mu^2)^2\right] + \frac{v^2}{4}\left[(gW_\mu^3 - g'B_\mu)^2\right]$$

where the other terms are of no interest for this discussion. The W^1 and W^2 bosons can be packed up into the $W^\pm$ bosons,

$$W^\pm = \frac{1}{\sqrt{2}}(W^1 \mp iW^2)$$

which directly shows they acquired a mass,

$$m_W = \frac{1}{2}v^2g^2$$

We see that we shall transform the W^3 and B bosons into the Z and γ bosons so that the the Z boson is proportional to $gW_\mu^3 - g'B_\mu$. A way to achieve this is through a unitary transformation,

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}$$

where θ_W is the Weinberg angle. The Z boson acquires a mass,

$$m_Z = \frac{gv}{\sqrt{2}|\cos\theta_W|}$$

and the γ boson remains massless. This proves the relations given in Section 1.4.3.

1 $t\bar{t}H$ multilepton CP analysis

In this section we present the scope of the $t\bar{t}H$ multilepton CP analysis. We shall first describe the theoretical framework that parametrizes CP violation in this process. Then we will give a general overview of the statistical language used in the following sections. Finally, we will describe

1.1 CP violating Yukawa coupling

In the SM the coupling of the Higgs with the top quark is given by the Lagrangian:

$$\mathcal{L} = \frac{y_t}{\sqrt{2}}\bar{t}th \tag{81}$$

where, y_t is the top Yukawa coupling, t is the top quark bispinor field and h is the Higgs field. In the SM, the Yukawa coupling is a real number. When can then directly verify that this Lagrangian in invariant under CP transformation. In fact, choosing the P and C representation on spinors to be [**untruc**],

$$P\psi P^\dagger = \gamma^0\psi$$
$$C\psi C^\dagger = i\gamma^2\psi^*$$

Using $\bar{t} = t^\dagger\gamma^0$, one can check that,

$$P\bar{t}tP^\dagger = \bar{t}t$$
$$C\bar{t}tC^\dagger = \bar{t}t$$

which proves that the lagrangian is indeed invariant under CP transformation,

$$(CP)\mathcal{L}(CP)^\dagger = \mathcal{L}$$

However, in the presence of new physics, the Yukawa coupling can be complex and non scalar. A simple model to introduce this is to introduce a CP phase α. Let us parametrize the Lagrangian as,

$$\mathcal{L} = \frac{y_t}{\sqrt{2}}\kappa_t'\bar{t}e^{i\alpha\gamma^5}th$$

Let us recall some useful commutation relations between gamma matrices,

$$(\gamma^5)^2 = 1 \tag{82}$$
$$\{\gamma^5, \gamma^\mu\} = 0 \tag{83}$$

Using these, one shows easily that the Lagragian is not invariant under CP but rather,

$$(CP)\mathcal{L}(CP)^\dagger = \frac{y_t}{\sqrt{2}}\kappa_t'\bar{t}e^{-i\alpha\gamma^5}th$$

Along with this parametrization, the SM case corresponds to $\alpha = 0$ and $\kappa_t' = 1$. Alternatively, one can work with an equivalent parametrization with $\kappa_t = \kappa_t' \cos\alpha$ and $\tilde{\kappa}_t = \kappa_t' \sin\alpha$. The former term is called the CP-even term while the later is the CP-odd term. In the case of the SM, $\kappa_t = 1$ and $\tilde{\kappa}_t = 0$.

It remains to be understood where this phase could come from. In the SM, the Yukawa couling come from the symmetry breaking of the Higgs doublet. One could wonder if it is possible to introduce the CP-phase even before the symmetry breaking. One can convince himself that this is not possible since it would also modify the mass term of the top quark.

The remaining solution it to consider higher order terms in the Lagrangian. This term are ususally discarded since these have a spacetime dimension greater than 4. Such terms are called non-renormalizable. We shall not dive deeper into this matter here. A good discussion of renormalizability can be met in Ref. [**renorm**].

A dimension 6 term invariant under $SU(2) \times U(1)$ is [**baryogenesis**],

$$\mathcal{L}_Y = K \left(\Phi^\dagger \Phi\right) \left(\bar{t}_L \tilde{\Phi} t_R\right)$$

where K is a complex number. This term, along with the usual Yukawa coupling, can be used to find the above Lagrangian.

The dependance of the Lagragian in the alpha phase can be used to probe the CP violating nature of the Yukawa coupling. This is the main goal of the analysis we are going to present.

1.2 Analysis strategy and channels

The goal of the analysis is to botain a confidence interval on the CP-phase α. In this section we describe the strategy employed to constrain this CP-phase as well as the two different channels that shall be used. Along with the CP-phase, the analysis also aims at constraining the value of the scaling parameter κ_t. This means that even if the CP-phase is found to be zero, the SM value of the Yukawa coupling is not necessarily recovered.

In the first section we shall give a very general overview of the analysis. Explaining the strategy

1.2.1 General considerations

1.2.2 0τ channel

1.2.3 2τ channel

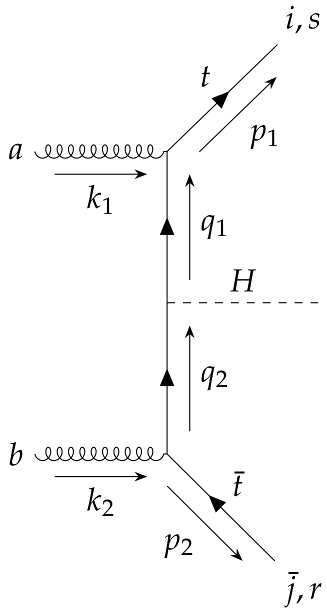
A Matrix elements for $t\bar{t}H$

The main diagram that contributes to $t\bar{t}H$ production is presented in Figure **??**. In this appendix we compute the matrix element for this diagram.

Using the Feynman rules, the matrix element for this diagram is given by,

$$\mathcal{M} = \sum_l \bar{u}^s(p_1) \left(-i\frac{g_s}{2}\right) \lambda^a_{ik} \not{\epsilon}^\sigma(k_1) i\delta_{lk} \frac{\not{q}_1 + m}{q_1^2 - m^2} \frac{y_t}{\sqrt{2}} e^{i\alpha\gamma^5} i\delta_{lp} \frac{\not{q}_2 + m}{q_2^2 - m^2} \left(-i\frac{g_s}{2}\right) \lambda^b_{jp} \not{\epsilon}^\rho(k_2) v^r(p_2)$$

where we used the Feynman notation, $\not{a} = \gamma^\mu a_\mu$, m refers to the mass of the top quark, g_s is the strong coupling constant, λ^a are the Gell-Mann matrices and ϵ are the gluon polarization



vectors. The sum over l is here to account for the color of the virtual top quark.

Let us simplify this expression by denoting,

$$A = \frac{g_s^2 y_t}{4\sqrt{2}} \frac{1}{q_1^2 - m^2} \frac{1}{q_2^2 - m^2}$$

This way, the matrix element reads,

$$\mathcal{M} = A \sum_l \bar{u}^s(p_1) \lambda^a_{il} \not{\epsilon}^\sigma(k_1)(\not{q}_1 + m) e^{i\alpha\gamma^5} (\not{q}_2 + m) \lambda^b_{jl} \not{\epsilon}^\rho(k_2) v^r(p_2)$$

Let us make even more compact and denote by,

$$\begin{aligned} \Gamma^{(1)\mu} &= \gamma^\mu(\not{q}_1 + m) \\ \Gamma^{(2)\nu} &= (\not{q}_2 + m)\gamma^\nu \end{aligned}$$

so that,

$$\mathcal{M} = A \sum_l \bar{u}^s(p_1) \lambda^a_{il} \epsilon^\rho_\mu(k_1) \Gamma^{(1)\mu} e^{i\alpha\gamma^5} \Gamma^{(2)\nu} \lambda^b_{jl} \epsilon^\rho_\nu(k_2) v^r(p_2)$$

What matters to compute physical quantities is the squared norm of the matrix element. This leads to the apparition of the multiplicative factor,

$$K^{ab}_{ij} = \left(\sum_l \lambda^a_{il} \lambda^b_{jl} \right) \left(\sum_{l'} (\lambda^a_{il'})^* (\lambda^b_{jl'})^* \right)$$

The Gell-Mann matrices are Hemitean so that, $(\lambda^a_{il})^* = \lambda^a_{li}$. This wya the previous multiplicative factor reads,

$$K^{ab}_{ij} = \left(\sum_l \lambda^a_{il} \lambda^b_{jl} \right) \left(\sum_{l'} \lambda^a_{l'i} \lambda^b_{l'j} \right)$$

Now one can sum over every possible outgoing colors of the $t\bar{t}$ pair to get a multiplicative coefficient,

$$\begin{aligned}
K^{ab} &= \sum_{i,j} K^{ab}_{ij} \\
&= \sum_{i,j} \left(\sum_{l} \lambda^a_{il} \lambda^b_{jl} \right) \left(\sum_{l'} \lambda^a_{l'i} \lambda^b_{l'j} \right) \\
&= \sum_{l,l'} \left(\sum_{i} \lambda^a_{il} \lambda^a_{l'i} \right) \left(\sum_{j} \lambda^b_{jl} \lambda^b_{l'j} \right) \\
&= \sum_{l,l'} \left[(\lambda^a)^2 \right]_{ll'} \left[(\lambda^b)^2 \right]_{ll'}
\end{aligned}$$

Now Gell-Man matrices are either symmetric $\lambda^T = \lambda$ or antisymmetric $\lambda^T = -\lambda$. So that their square is always symmetric. This way, one can recast K as,

$$K^{ab} = \sum_{l,l'} \left[(\lambda^a)^2 \right]_{l'l} \left[(\lambda^b)^2 \right]_{ll'} = \mathrm{Tr}(\lambda^a \lambda^a \lambda^b \lambda^b)$$

Finally, one can average over the gluon colors, to find,

$$K = \frac{1}{64} \sum_{ab} K^{ab} = \frac{1}{64} \mathrm{Tr} \left[\left(\sum_a \lambda^a \lambda^a \right) \left(\sum_b \lambda^b \lambda^b \right) \right]$$

Using the relation,

$$\sum_a \lambda^a \lambda^a = \frac{16}{3} I$$

one finds,

$$K = \left(\frac{16}{3} \right)^2 \frac{1}{64} \mathrm{Tr}\ I = \frac{8}{3}$$

In a similar fashion, another term related to gluon polarization appears,

$$P^{\sigma\rho}_{\mu\nu'_\mu\nu'} = \epsilon^\rho_\mu(k_1) \epsilon^\rho_{\mu'}(k_1)^* \epsilon^\rho_\nu(k_2) \epsilon^\rho_{\nu'}(k_2)^*$$

One can average this over photon polarizations, so that the coefficient becomes,

$$P_{\mu\nu\mu'\nu'} = \frac{1}{4} \sum_{\rho\sigma} P^{\sigma\rho}_{\mu\nu'_\mu\nu'}$$

using the relation (see Appendix D of Ref. [16])$\sum_{\sigma,\rho} \epsilon^\mu_\sigma(k_1) \epsilon^\nu_\rho(k_2) = -\eta_{\mu\nu}$, one finds,

$$P_{\mu\nu\mu'\nu'} = \frac{1}{4} \eta_{\mu\mu'} \eta_{\nu\nu'}$$

Up to these simplifcations, the squared matrix element reads,

$$|\mathcal{M}|^2 = P_{\mu\nu\mu'\nu'} K |A|^2 \bar{u}^s(p_1) \Gamma^{(1)\mu} e^{i\alpha\gamma^5} \Gamma^{(2)\nu} v^r(p_2) (v^r(p_2))^\dagger \left(\Gamma^{(2)\nu'} \right)^\dagger e^{-i\alpha\gamma^5} \left(\Gamma^{(1)\mu'} \right)^\dagger (\bar{u}^s(p_1))^\dagger$$

Let us recall that $\gamma^0 \gamma^\mu \gamma^0 = \gamma^{\mu\dagger}$ and that $(\gamma^0)^2 = I$. This can be used ot show that,

$$\gamma^0 (\Gamma^{(i)\mu})^\dagger \gamma^0 = \Gamma^{(i)\mu}$$

for $i = 1, 2$.

Since γ^0 is Heremitean, one can write,

$$(\bar{u}^s(p_1))^\dagger = \gamma^0 u^s(p_1)$$

Using these, one shows that the squared matrix element can be written as,

$$|\mathcal{M}|^2 = P_{\mu\nu\mu'\nu'} K|A|^2 \bar{u}^s(p_1)\Gamma^{(1)\mu} e^{i\alpha\gamma^5}\Gamma^{(2)\nu} v^r(p_2)\bar{v}^r(p_2)\Gamma^{(2)\nu'}\gamma^0 e^{-i\alpha\gamma^5}\gamma^0\Gamma^{(1)\mu'} u^s(p_1)$$

Using the precise expression of $P_{\mu\nu\mu'\nu'}$, it yields,

$$|\mathcal{M}|^2 = \frac{K|A|^2}{4}\bar{u}^s(p_1)\Gamma^{(1)\mu} e^{i\alpha\gamma^5}\Gamma^{(2)\nu} v^r(p_2)\bar{v}^r(p_2)\Gamma^{(2)}_\nu\gamma^0 e^{-i\alpha\gamma^5}\gamma^0\Gamma^{(1)}_\mu u^s(p_1)$$

Since $\{\gamma^0, \gamma^5\} = 0$, one can show that,

$$\gamma^0 e^{-i\alpha\gamma^5}\gamma^0 = e^{i\alpha\gamma^5}$$

One can now sum over the helicity states of the outgoing $t\bar{t}$ pair. If one expands the matrix product precisely and use the completeness relation for the spinors (See Section 6.5 of [16]),

$$\sum_s u^s(p_1)\bar{u}^s(p_1) = \not{p}_1 + m \quad \text{and} \quad \sum_r v^r(p_2)\bar{v}^r(p_2) = \not{p}_2 - m$$

one shows that the fully average squared matrix element is,

$$\left\langle |\mathcal{M}|^2 \right\rangle = \frac{K|A|^2}{4}\mathrm{Tr}\left[(\not{p}_1 + m)\Gamma^{(1)} e^{i\alpha\gamma^5}\Gamma^{(2)}(\not{p}_2 - m)\Gamma^{(2)} e^{i\alpha\gamma^5}\Gamma^{(1)}\right]$$

References

[1] Nicola Cabibbo. "Unitary Symmetry and Leptonic Decays". In: *Physical Review Letters* 10.12 (June 1963). Publisher: American Physical Society, pp. 531–533. DOI: 10.1103/PhysRevLett.10.531. URL: https://link.aps.org/doi/10.1103/PhysRevLett.10.531 (visited on 07/22/2024).

[2] Sean Carroll and Sean M. Carroll. *Spacetime and Geometry: An Introduction to General Relativity*. en. Google-Books-ID: 1SKFQgAACAAJ. Addison Wesley, 2004. ISBN: 978-0-8053-8732-2.

[3] ATLAS Collaboration. "Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC". In: *Physics Letters B* 716.1 (Sept. 2012), pp. 1–29. ISSN: 0370-2693. DOI: 10.1016/j.physletb.2012.08.020. URL: https://www.sciencedirect.com/science/article/pii/S037026931200857X (visited on 07/22/2024).

[4] CMS Collaboration. "Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC". In: *Physics Letters B* 716.1 (Sept. 2012), pp. 30–61. ISSN: 0370-2693. DOI: 10.1016/j.physletb.2012.08.021. URL: https://www.sciencedirect.com/science/article/pii/S0370269312008581 (visited on 07/22/2024).

[5] M. Gell-Mann. "The interpretation of the new particles as displaced charge multiplets". In: *Nuovo Cim.* 4.S2 (1956), pp. 848–866. DOI: 10.1007/BF02748000.

[6] Howard Georgi. *Lie algebras in particle physics*. 2nd ed. Vol. 54. Reading, MA: Perseus Books, 1999.

[7] David Griffiths. *Introduction to Elementary Particles, Second Edition*. Wiley-VCH, 2010.

[8] David J. Gross and Frank Wilczek. "Ultraviolet Behavior of Non-Abelian Gauge Theories". In: *Physical Review Letters* 30.26 (June 1973). Publisher: American Physical Society, pp. 1343–1346. DOI: 10.1103/PhysRevLett.30.1343. URL: https://link.aps.org/doi/10.1103/PhysRevLett.30.1343 (visited on 07/22/2024).

[9] Mark J.D. Hamilton. *Mathematical Gauge Theory*. Universitext. Cham: Springer International Publishing, 2017. ISBN: 978-3-319-68438-3 978-3-319-68439-0. DOI: 10.1007/978-3-319-68439-0. URL: http://link.springer.com/10.1007/978-3-319-68439-0 (visited on 07/22/2024).

[10] Makoto Kobayashi and Toshihide Maskawa. "CP Violation in the Renormalizable Theory of Weak Interaction". In: *Prog. Theor. Phys.* 49 (1973), pp. 652–657. DOI: 10.1143/PTP.49.652.

[11] Tom Lancaster and Stephen J. Blundell. *Quantum Field Theory for the Gifted Amateur*. Oxford University Press, 2021.

[12] Ziro Maki, Masami Nakagawa, and Shoichi Sakata. "Remarks on the unified model of elementary particles". In: *Prog. Theor. Phys.* 28 (1962), pp. 870–880. DOI: 10.1143/PTP.28.870.

[13] Kazuhiko Nishijima. "Charge Independence Theory of V Particles". In: *Prog. Theor. Phys.* 13.3 (1955), pp. 285–304. DOI: 10.1143/PTP.13.285.

[14] B. Pontecorvo. "Inverse beta processes and nonconservation of lepton charge". In: *Zh. Eksp. Teor. Fiz.* 34 (1957), p. 247.

[15] Julian Schwinger. "On Quantum-Electrodynamics and the Magnetic Moment of the Electron". In: *Physical Review* 73.4 (Feb. 1948). Publisher: American Physical Society, pp. 416–417. DOI: 10.1103/PhysRev.73.416. URL: https://link.aps.org/doi/10.1103/PhysRev.73.416 (visited on 07/22/2024).

[16] Mark Thomson. *Modern particle physics*. New York: Cambridge University Press, 2013. ISBN: 978-1-107-03426-6. DOI: 10.1017/CBO9781139525367.

[17] Steven Weinberg. *The Quantum Theory of Fields*. Vol. 1. Cambridge University Press, 1995.

[18] E. Wigner. "On Unitary Representations of the Inhomogeneous Lorentz Group". In: *Annals of Mathematics* 40.1 (Jan. 1939), pp. 149–204. DOI: 10.2307/1968551.

[19] E.P. Wigner. *Gruppentheorie und ihre Anwendung auf die Quantenmechanik der Atomspektren*. Die Wissenschaft. F. Vieweg & sogn akt.-ges, 1944. URL: https://books.google.fr/books?id=qWdIAQAAIAAJ.