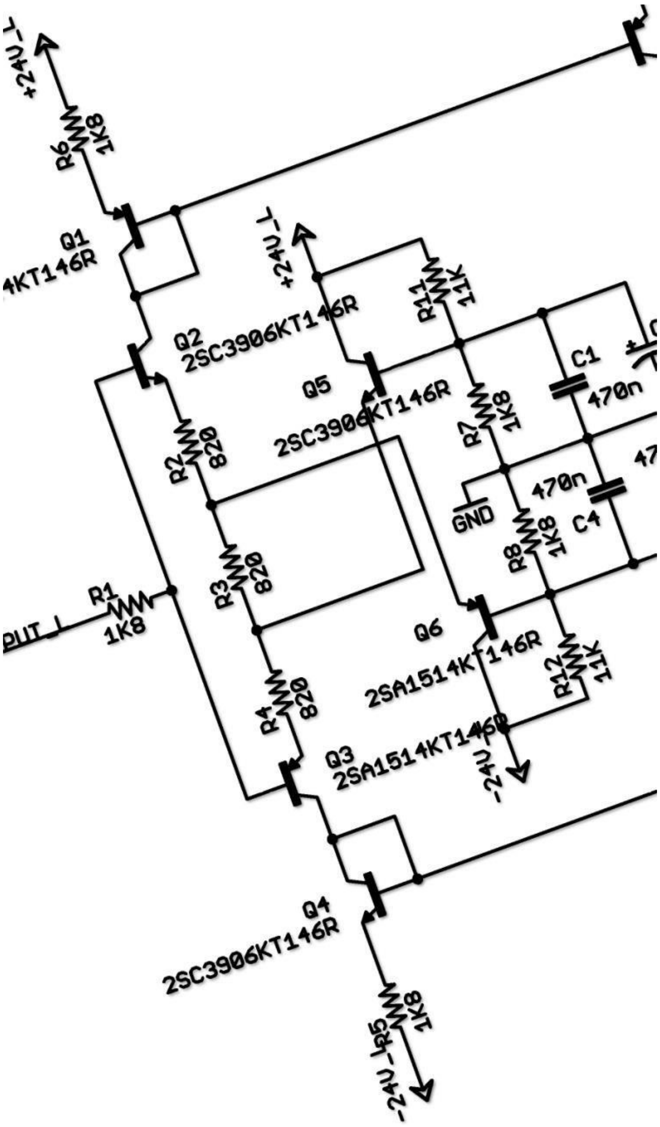
k MONASH University
Engineering

ECE2131

Electrical Circuits

Laboratory Notes



2022 Edition

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2022

5 Network Theorems

5.1 Learning Objectives and Introduction

This experiment provides experimental verification of Norton and Thévenin equivalent circuits. This also provides you with some understanding as to how to use an equivalent circuit, and why that might be preferable to fully understanding the internal layouts of complex circuits (e.g. signal generators, oscilloscopes, general test and measurement equipment, ...).

By the end of this lab you should:

- Understand how equivalent circuits can be included in larger overall circuits
- Generate equivalent circuits for both circuit designs and test equipment attached to them
- Predict the effects of connecting test equipment to circuits by analysing their equivalent circuits

5.1.1 Thévenin's Theorem

This experiment is restricted to circuits and networks operating under sinusoidal steady-state conditions. Under these conditions, consider a linear network N (as shown in Figure S5.1) connected toan arbitrary (not necessarily linear) load between A and B. Further assume that the only interaction between N and the load is via the terminals A and B (this means for example there is no magnetic coupling between N and the load).

Thévenin's theorem then states that:

The terminal current and voltage will be unchanged if N *is replaced by its Thévenin equivalent circuit, consisting of an impedance Z_{eq} (the equivalent impedance) in series with a voltage source V_{oc} (the opencircuit voltage).*

In particular, the short-circuit current, I_{sc}, of the equivalent circuit and hence of N is given by V_{oc}/Z_{eq}. The equivalent circuit can be found experimentally or theoretically from any two of V_{oc}, Z_{eq} and I_{sc}. Four methods of varying practicality can be distinguished.

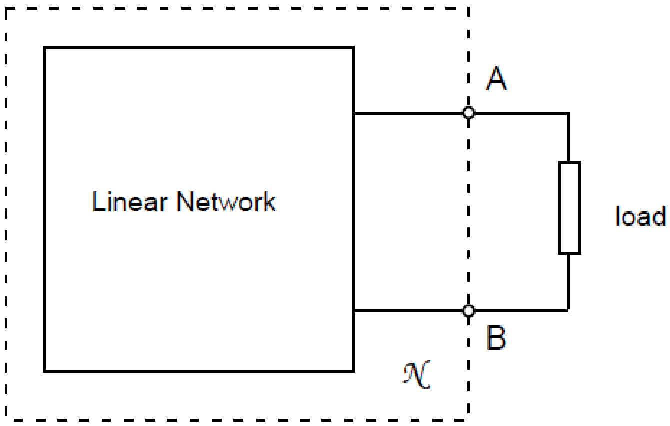


Figure S5.1 Network connected to a load

5.1.2 FINDING THE THÉVENIN'S EQUIVALENT CIRCUIT

Possible methods of measuring the equivalent circuit for network N:

1 Measurement of V_{oc} and output impedance with the sources turned to zero output.

This requires application of an externally applied voltage to measure Z_{eq}. Under laboratory condition, direct measurement of Z_{eq} is not practical because not all generators are capable of safely and linearly "sinking" current, and because turning a generator off may make its impedance totally undefined, due to the internal circuitry of the generator.

2 Measurement of open circuit voltage and short circuit current

This also needs to be applied with extreme care as a short circuit load may damage the equipment, or more likely cause it to reach some overload condition, in which case the network is no longer linear.

3 Apply two different loads to N and in each case measure the voltage across and the current flowing through the load.

If the loads are known impedances, it is sufficient to measure either the voltage or the current. The parameters can be found by solving the simultaneous equations that result.

4 Half voltage method

If Z_{eq} is known to be resistive, we can (a) measure the open circuit voltage and (b) connect a resistive load and reduce the load resistance until the voltage is half its open circuit value. That resistance is equal to Z_{eq} (to be proven in Question 1 of Preliminary Work).

5.1.3 NORTON'S THEOREM

Norton's theorem states that:

A linear network can be represented by a parallel combination of a current generator equal to I_{sc} and the Thévenin equivalent impedance Z_{eq}.

5.2 EQUIPMENT AND COMPONENTS

The components required for this laboratory are:

- Breadboard
- Resistors: 2x 100 Ω
- Capacitors: 2.2 μF
- 1k Ω potentiometer

5.3 EXPERIMENTAL WORK

To illustrate Thévenin's theorem, the circuit in Figure S5.2 is tested in this experiment.

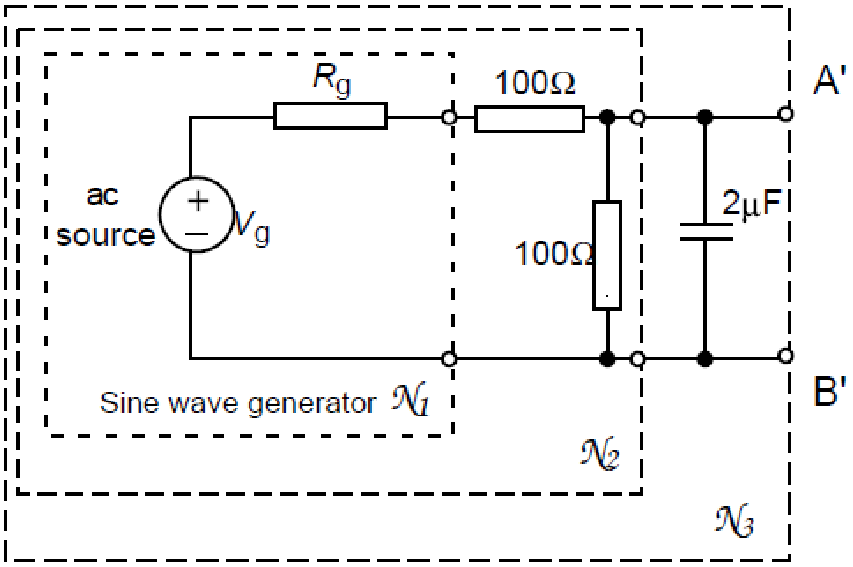


Figure S5.2 Networks under tests

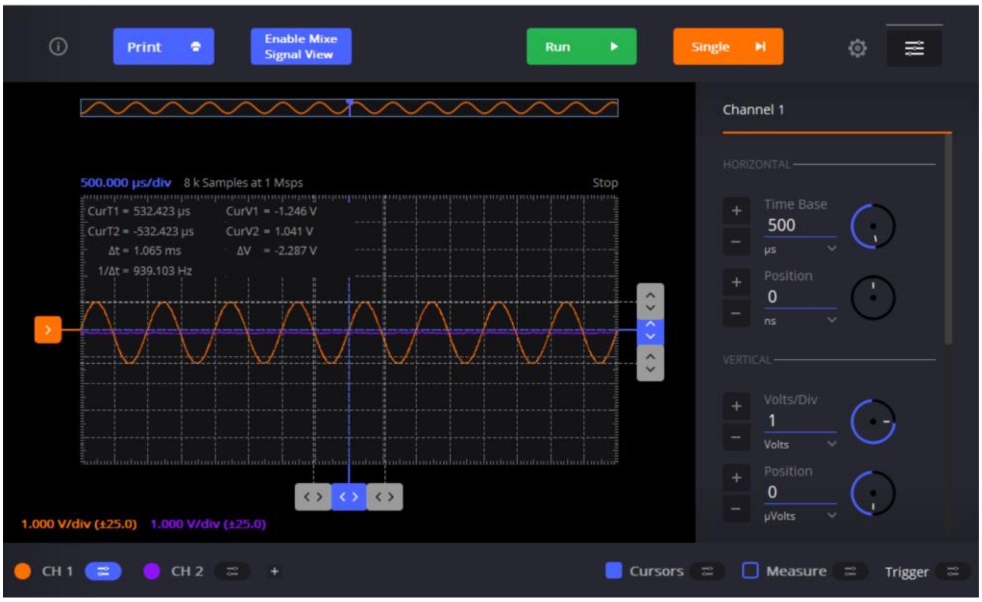
Use the provided components to construct the networks, first N_1, and then N_2. You will first study N_1 and then in the later part of this laboratory you will add resistors to construct N_2, and calculate and measure the Thévenin equivalent of N_2. Finally, a capacitor will be added to construct N_3. The Norton equivalent of this circuit is also then considered.

5.3.1 EQUIVALENT CIRCUIT OF THE SINE WAVE GENERATOR (N_1)

Determine R_g. Set your signal generator to sine wave with an output voltage level of 5.0 V (pp) at an operating frequency of 1.0 kHz. R_g is the internal resistance of the source (signal generator). It is **NOT** a component to connect in your breadboard. In LTSpice, set the voltage series resistance (Rser) to 50Ω.

5.3.1.1 Use a potentiometer as load, and use voltage divider rule to determine R_g. This method is most accurate when the potentiometer is adjusted until the voltage is half of the input. What is the value of R_g?

Based on the voltage measured above, we will get our peak to peak value to be 4.573 V. This is due to the presence of internal resistance that reduces the current of the circuit which would result in the reduction of the voltage flowing through the circuits.



We will now take half of the source voltage across the potentiometer in order to calculate Rg.
Rg can be obtained using the voltage divider rule.
Rg can be calculated using the formula

$$Vleft = \frac{Rg}{Total\ resistance\ across\ the\ potentiometer} * \text{Total voltage across the circuit}$$

$$79.262\ mV\ = \frac{Rg}{1\ kOhms} * 2.021$$

Rg = 39.2 Ohms

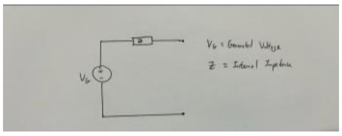
5.3.1.2 What happens when you change between high Z and 50 Ohms (this only applies to the desktop oscilloscope equipment in the laboratory)? What happens to the value the signal generator displays?

Based on theory, When the potentiometer is increased to a very high impedance, the voltage across the oscilloscope will be approaching very close to source voltage of 5V since the internal resistance can be considered to be negligible.

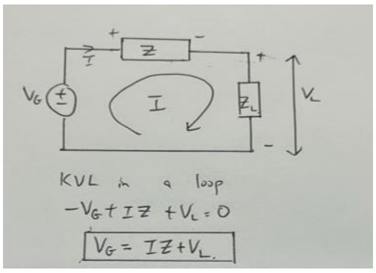
5.3.1.3 How would you prove that the internal impedance of the generator is a linear and constant resistance? Is the internal impedance of the generator linear and constant?

The internal impedance of the generator can be proved to be linear and constant by testing it using multiple values of voltages.

In general, we can represent the generator with voltage source in series with impedance with the following circuit.



We can connect a load across this generator terminal.



From here, we can derive V_G as $V_G = ZI + V_L$.

We can compare the expression above with the straight-line equation.

y = mx + c

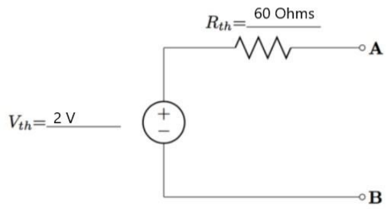
When comparing the above two equations, m would be equal to z. This linear characteristic can only be obtained by resistor only. Hence, we can assume that the internal impedance of the generator is linear and constant.

☐ CHECKPOINT: Get a demonstrator to check your answers, and initial here

5.3.2 THÉVENIN'S EQUIVALENT NETWORK OF N_2

Construct the network N_2 two 100Ω resistors and a signal generator as shown in Figure S4.2. Write the theoretical values in the diagram on the left in the box below and use the space to write your workings. Measure the Thévenin open-circuit voltage of N_2 at the open terminals (the resistors are part of the Thevenin circuit, do not remove them). Write your measurements in the space on the right in the box below. Compare your experimental measurements with the theoretical values. Comment on any discrepancies/similarities.

The theoretical value



The theoretical value obtained for Vth would be 2V and 60 Ohms for Rth.

The value of 2V is obtained by using the Kirchoff's Current Law (KCL).

$$\frac{(V-5)}{100+50}+\frac{V}{100}=0$$

$$2V-10+3V=0$$

$$V=\frac{10}{5}=2V$$

The value of 60 Ohms can be obtained by zeroing the voltage source, making a short circuit.

Requivalent would be equal to (50+100) ||100

Requivalent would be equal to 60 Ohms

The measured value

The measured value of Vth is 1.718 V

The measured value for Rth would be approximately 86.7 Ohms. This value can be found by taking Vth dividing by I.

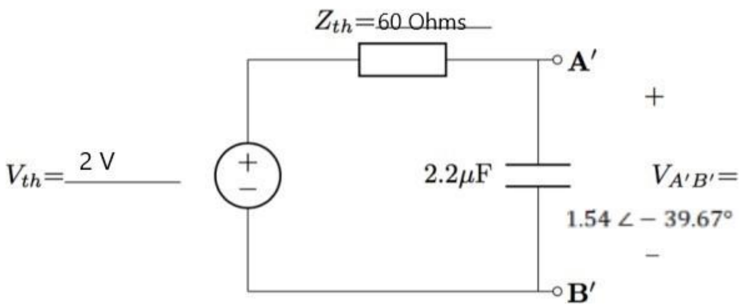
5.3.3 APPLICATION OF THÉVENIN'S THEOREM

Connect a 2.2 μF capacitor as the load for the N_2 Thevenin circuit. Measure $V_{A'B'}$.

The measured value of Vab would be 1.282 V.

The phase angle measured would be 242.67 μs * 1000 * 180 = 43.68°

Write the values of the Thévenin equivalent circuit of N_3 into the diagram below. Compare the theoretical Thevenin open-circuit voltage of N_3 in the diagram below, against the measured $V_{A'B'}$ from the previous question. Explain any discrepancies/similarities.



As we can compare between the theoretical and the measured value, there are some discrepancies between those two values. This could be due to the presence of internal resistance within the circuit. There could also be systematic error made due to the inaccurate reading made by humans or inaccuracies resulting from the graph in Scopy.

☐ CHECKPOINT: Get a demonstrator to check your answers, and initial here

5.3.4 NON-SINUSOIDAL SOURCE

Suppose that the signal generator (source of the circuit) is non-sinusoidal, for example a square wave. What do you <u>expect</u> the output to look like?

Test your conclusion experimentally by switching the output of the generator to a square wave anduse an oscilloscope to examine the voltage waveform across $V_{A'B'}$. <u>Sketch</u> the output waveform below.

Voltage across the resistor

Legend – Green: Sinusoidal Wave Source, Orange: Square Wave Source

Voltage across the capacitor

Legend – Green: Square Wave Source, Orange: Sinusoidal Wave Source

Comment and explain on your observation. Are you able to obtain this using AC analysis? Why/ why not?

No, I was not able to obtain this using AC analysis. When a square wave is used, the source will be constant for every half cycle. This would allow the capacitor to charge up exponentially until it reaches a steady state and discharge when the source goes to 0V. As the source have a constant value every half cycle, the capacitor will charge and discharge as usual until it reaches steady state. When an AC source is used, the source will be constantly changing it's pattern. This will cause the capacitor ot having the opportunity to go to its usual steady state where it remains at a constant value. Thus, the would cause the capacitor to constantly charge and discharge as the source voltage varies.

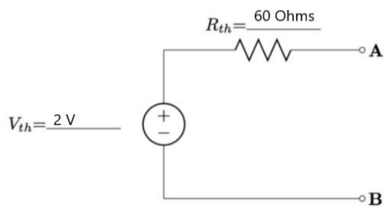
In short, when a capacitor is in DC circuit, it will charge exponentially until about 5 time constant until the voltage across the capacitor is equal to the source voltage and discharges when the source goese to 0V. As the capacitor is fully charged, it will become an open circuit as it is going into the steady state where no current will pass through it.

When a capacitor is in AC circuit, the capacitor will continually to charge and discharge as the voltage of the AC source keeps varying. The capacitor would reverse its charges as the current alternates. This would result in a voltage that lags behind the source. As a result of this, the output waveform will be in the form of a sine wave.

☐ CHECKPOINT: Get a demonstrator to check your answers, and initial here

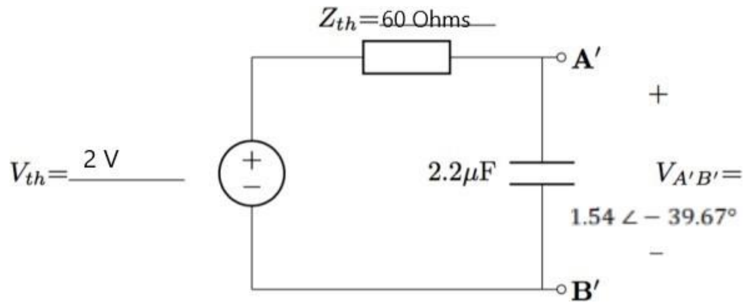
5.3.5 NORTON'S THEOREM

Calculate the theoretical Norton equivalent of N_2 starting from the Thévenin equivalent circuit you found in 5.3.2.



The open circuit voltage (Voc) will have a value of 2 V. The equivalent resistance (Rth) of the circuit above would have a value of 60 Ohms. The short circuit current (Isc) would have a value of Voc / Req = 2 / 60 = 0.033 A

Now calculate the Norton equivalent circuit of the network N_3.



The open circuit voltage (Voc) will have a value of $1.54\angle -39.67°\ V\ peak.$ The equivalent resistance (Req) can be calculated using the formula

Zc (60 Ohms) || (1/jwC) = 60 || ((1/j) * 2π * 1000 * (2.2e-6)).

We will get Zc to be $46.18\angle -39.67°\ \ Ohms.$

Short circuit current (Isc) = Voc /Req = $1.54\angle -39.67°\ V\ /\ 46.18\angle -39.67°\ \ Ohms$

The short circuit current (Isc) will be equal to 0.033 A

Consider connecting a new load to N_3. Find the load impedance required to produce the maximum output power (remember that you can look up appropriate formula from lectures or the internet if required). What values of resistance and capacitance/inductance would need to construct this load?

The load impedance needs to be the complex conjugate of the Thevenin equivalent resistance in order to have maximum power across the load.

Based on the values obtained above, we have the equivalent resistance to be $46.18\angle -39.67°\ \ Ohms$ which would equate to 35.55 – 29.48j Ohms.

Since the load impedance must the complex conjugate of the equivalent resistance, the load impedance would be 35.55 + 29.48j Ohms.

The real part of the load impedance will be provided by the resistor of the same value (35.55 Ohms) while the imaginary part of the load impedance (29.48j) will be provided by an inductor.

The value of the inductor can be calculated using the formula jwL.

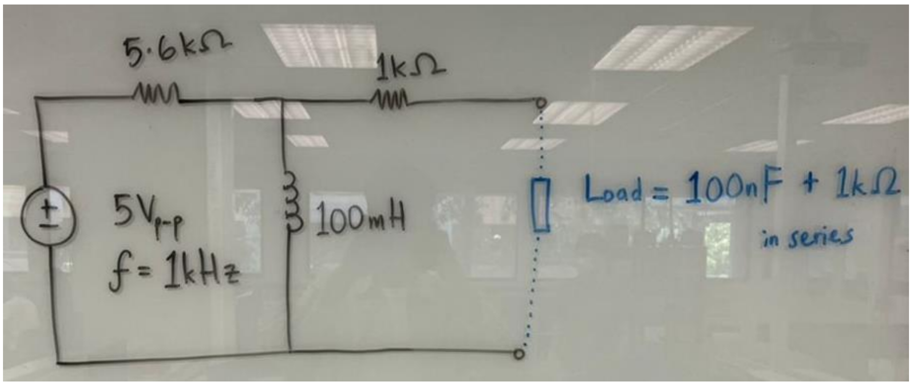
jwL = 29.48j
2*π*1000*L = 29.48
L will have the value of 4.69 mH

☐ CHECKPOINT: Get a demonstrator to check your answers, and initial here

Lab Assessment

For your lab assessment, you will be given a random load to connect to a Thevenin circuit. Use this space below to <u>draw</u> the circuit and <u>write</u> the theoretical values. You should also <u>show</u> your working to obtain these theoretical values. <u>Use</u> the space below also to write all measured experimental values. <u>Comment</u> on any discrepancies/similarities between theoretical and experimental values.

1. Given Circuit



2. Theoretical Thevenin Open Circuit Voltage

The theoretical Thevenin Open Circuit Voltage can be calculated based on the formula

$$\frac{j*\omega*L}{j\ *\omega*L+R1}*2.5\ V$$

$$\frac{j*2*\ \pi*1000*100m}{j\ *2*\ \pi*1000\ *100m+5.6k}*2.5\ V$$

$$V_{oc} = 0.03108 + 0.22701j$$

Converting the cartesian form into phasor form

$$R = \sqrt{(0.03108)^2 + (0.22701)^2}$$

$$R = 0.229$$

$$\theta = \tan^{-1}(\frac{y}{x})$$

$$\theta = 83.6°$$

$$V_{oc} = 0.229 \angle 83.6°\ V$$

3. Equivalent Resistance

$$1000 + \frac{5600 * j * 2 * \pi * 1000 * 100 * 10\text{^}-3}{5600 + j * 2 * \pi * 1000 * 100 * 10^{-3}}$$

$$1000 + \frac{5600 * j * 628.32}{5600 + j * 628.32}$$

$$1000 + (69.6211 + 620.509\ \mathrm{i})$$

$$1069.6211 + 620.509\, i$$

$$R = 1236.58\ \angle\ 30.12°\Omega$$

4. Measure the experimental Thevenin open-circuit voltage source (V_{oc})

The experimental Thevenin open-circuit voltage source measured would be 238.908 mV.

The phase angle can be found using the formula Δt * f * 180

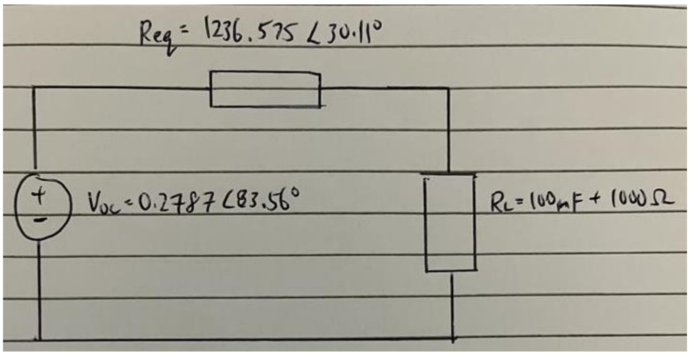
We will get 122.867 µs * 1000 * 180 = 22.12°

5. Compare the experimental and the theoretical values of V_{oc}

The experimental value of V_{oc} is 0.238 V. The theoretical values of V_{oc} is 0.229 V . There is a slight difference due to systematic error made by human and the inaccuracies in reading made during the visual observation of the graph.

6. Calculate the voltage across the load, V_{load}

Thevenin Equivalent Circuit



$$V_{load} = \frac{RL}{RL + Req} * V_{oc} \text{ , where RL} = 1000 + \frac{-j}{2*\pi*f*C}$$

Load = 1000 – 1591.55j

Convert from cartesian form to polar form

RL = 1879.64 ∠ 57.86°

V_{oc} = 0.229 ∠ 83.6° V

RL + Req = 2286.23 ∠ − 25.14°

V_{load} = 0.229 ∠ 50.840°

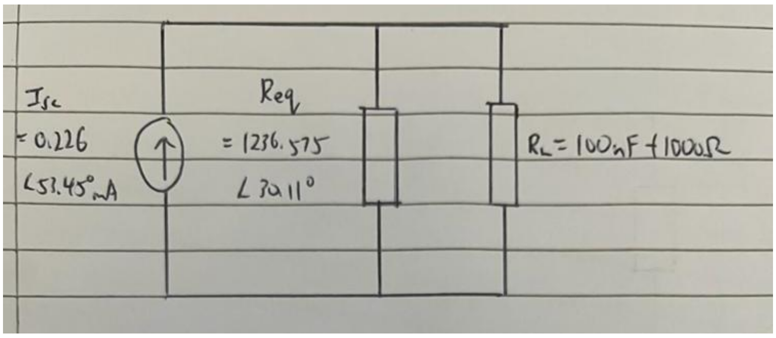
7. The measured value of the load, V_{load} and the measured phase angle of the load

The measured value of the load (V_{load}) would be 0.247 mV
The measured phase angle is 184.44 µs * 1000 * 180 = 33.2°

8. Compare the experimental and theoretical values of V_{load}.

The experimental value of V_{load} would be 0.247 ∠ 33.2° and the theoretical values of the load is 0.229 ∠ 50.840° . There is a slight difference due to systematic error made by human and the inaccuracies in reading made during the visual observation of the graph.

9. Norton Analysis



$$I_{sc} = \frac{V_{OC}}{R_{eq}}$$

$$Isc = \frac{0.229 \angle 83.6° \text{ V}}{1236.575 \angle 30.11° \ \Omega}$$

$$Isc = 0.2259 \angle 53.45° \text{ mA}$$

The formula that is used to calculate the V_{load} would be

$$V_{load} = \frac{R_L * R_{eq}}{R_L + R_{eq}} * I_{SC}$$

Since R_L is 1879.64 ∠ 57.86° and R_{EQ} is 1236.575 ∠ 30.11° ,

$$V_{load} = \frac{(1879.6355 \angle{-57.858°}) * (1236.575 \ \angle 30.11°)}{(2069.6211 - 971.0424j)\Omega} * \ I_{SC}$$

$$V_{load} = 1016.714 \angle - 2.613° \ \Omega \ * \ I_{SC}$$

$$V_{load} = 0.229 \text{ V} \angle - 50.837° \ V$$

ASSESSMENT

Student Statement:

I have read the university's statement on cheating and plagiarism, as described in the *Student Resource Guide*. This work is original and has not previously been submitted as part of another unit/subject/course. I have taken proper care safeguarding this work and made all reasonable effort to make sure it could not be copied. I understand the consequences for engaging in plagiarism as described in *Statue 4.1 Part III – Academic Misconduct*. **I certify that I have not plagiarized the work of others or engaged in collusion when preparing this submission.**

Student signature:_______Tan Jin Chun______________Date:1/4/2022

TOTAL: ________(/7)

ASSESSOR: ________