ENG2005
Assignment 1

School of Mathematics
Monash University
Semester 1 2022

Due: **Friday, 8 April 2022, 23:55 (Clayton)/ 21:55 (Malaysia)**

Complete the following questions, scan, upload and submit them in Moodle in a pdf file. ***Late assignments will be penalised*** *at 10% of the maximum mark per day late. Justify all your answers.*

Learning outcomes:

- *Use essential concepts related to $m \times n$ linear systems, including linear independence and basis*
- *Manipulate and evaluate double and triple integrals in Cartesian, cylindrical and spherical coordinates*
- *Express and explain mathematical techniques and arguments clearly in words*

Question 1 (6 marks)
Let

$$A = \begin{bmatrix} 2 & -1 & -1 \\ 0 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix}.$$

Find the eigenvalues and associated eigenvectors of A.

Question 2 (20 marks)
The crew of a roughly cylindrical spaceship is loading a section of the ship, with radius 2m. At times, the spaceship will spin along the cylindrical axis.
A bulkhead extends across the ship, perpendicular to the cylindrical axis. The load must be secured to the bulkhead, and there are operational constraints on the distribution of mass.
The load consists of

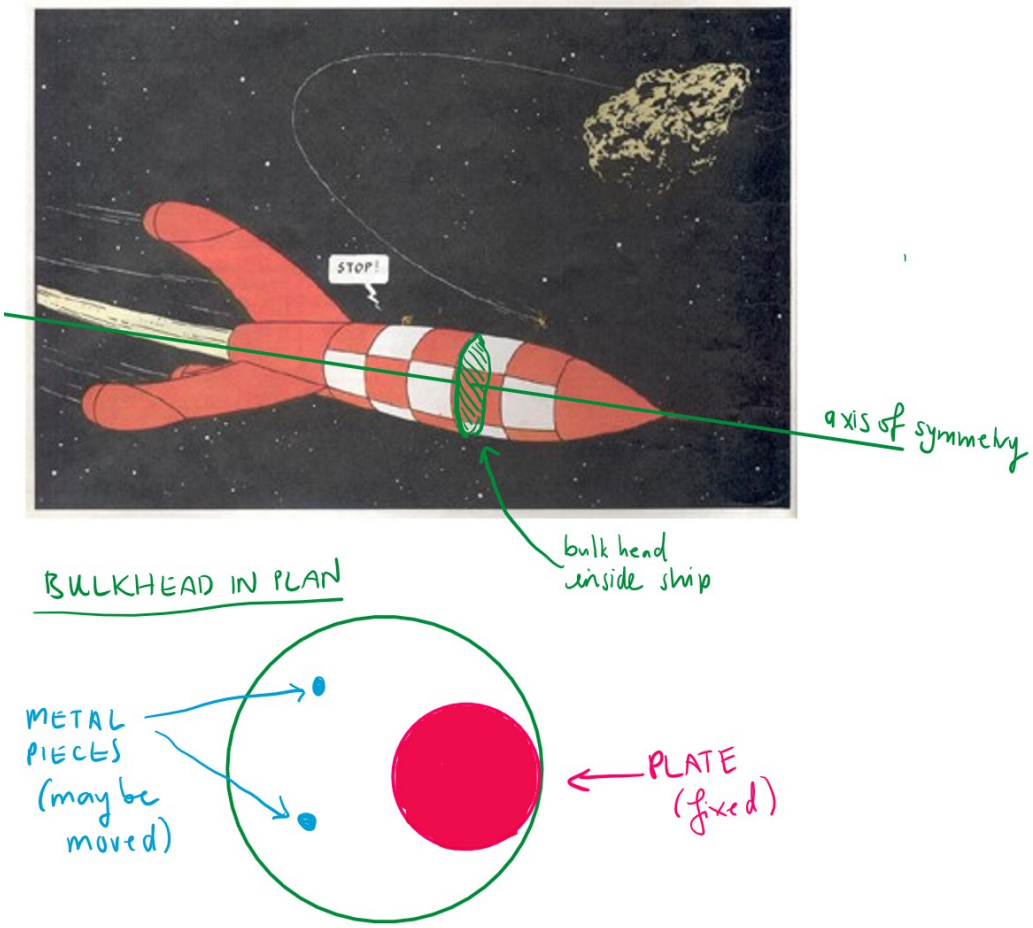
- a circular plate of radius 1. It is in a fixed position, flat against the bulkhead and just touching the outer wall of the ship. It has variable density given by $\rho(r) = \rho_0 r \text{kg}/\text{m}^2$, where r is the distance to the cylindrical axis, and ρ_0 is a dimensionless constant.
- two small, extremely dense pieces of metal, each of which has mass $3\rho_0$kg. Because they are so dense, we will treat them as point masses. These may be secured in any position on the bulkhead.

We can find the moment of inertia for a flat body R of density ρ, perpendicular to the z axis, rotating around the z axis using

$$I_z = \iint_R (x^2 + y^2)\rho(x, y)\, dx\, dy.$$

a) What is the mass, centre of mass, and moment of inertia (relative to the cylindrical axis) of the plate alone?

b) For this question, let $\rho_0 = 1$. Where should the two pieces of metal be attached so that the load (plate and pieces of metal) as a whole has centre of mass located on the cylindrical axis, and moment of inertia 15kg? (Answer to the nearest centimeter.)



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[1]Image credit: *Explorers on the Moon* (1954), Hergé