ENG1060: COMPUTING FOR ENGINEERS

Lab 10 – Week 11

2020 OCT NOV

Welcome to lab 10. This is the final lab with assessed questions. The next lab is dedicated for assignment marking. Remember that laboratories continuously build on previously learned concepts and lab tasks. Therefore, it is crucial that you complete all previous labs before attempting the current one.

Self-study:
Students are expected to attempt these questions during their own self-study time, prior to this lab session. There may be questions that require functions not covered in the workshops. Remember to use MATLAB's built-in help for documentation and examples.

Learning outcomes:

1. To revise user-defined functions and apply good programming practices
2. To identify the mathematics that represents the problem to be solved
3. To summarise the requirements and limitations of each ODE-solving method
4. To apply ODE-solving methods to solve ODEs both by hand and with MATLAB

Background:
Ordinary differential equations (ODEs) are equations that contain ordinary derivatives of the independent variable. This is different to partial differential equations (PDEs), which contain partial derivatives with respect to two or more independent variables. Both ODEs and PDEs describe phenomena in many disciplines including science and engineering. Computers and numerical techniques are required to solve such equations in real situations as they require a large amount of calculations.

Primary workshops involved:

- Workshop 3: Functions, commenting, debugging and strings
- Workshop 10: Ordinary differential equations

Assessment:
This laboratory comprises **2.5%** of your final grade. The questions are designed to test your recollection of the workshop material and to build upon important programming skills. You will be assessed on the quality of your programming style as well as the results produced by your programs during your laboratory session by the demonstrators. Save your work in **m-files** named ***lab1t1.m***, ***lab2t2.m,*** etc. Inability to answer the demonstrator's questions will result in zero marks, at the demonstrator's discretion.

There is no team task for this lab as the assignment worth 10% is due this week. A consolidation quiz for Part B on Moodle worth 2.5% will be available in week 12. Check Moodle for details.

Lab submission instructions

Follow the instructions below while submitting your lab tasks.

Team tasks:
The team tasks are designed for students to test and demonstrate their understanding of the fundamental concepts specific to that lab. These tasks will occur at the start of the lab and will be assessed on the spot. Demonstrators will advise on how these will be conducted. Most team tasks do not require the use of MATLAB but MATLAB should be used for checking purposes.

Individual tasks:
The individual tasks are designed for students to apply the fundamentals covered in the team tasks in a variety of contexts. These tasks should be completed in separate m-files. There is typically one m-file per task unless the task requires an accompanying function file (lab 3 onwards). Label the files appropriately. E.g. lab6t1.m, lab6t2.m, eridium.m, etc.

Deadline:
The lab tasks are due next Friday at 9am (MYT) or 12pm (AEDT). Late submissions will not be accepted. Students will need to apply for special consideration after this time.

Submission:
Submit your lab tasks by:

1) Answering questions in Google Form, and
2) Submitting one .zip file which includes all individual tasks.

The lab .zip file submission links can be found on Moodle under the weekly sections, namely Post-class: Lab participation & submission. The submission box ("Laboratory 10") will only accept one .zip file. Zipping instructions are dependent on the OS you are using.

Your zip file should include the separate m-files for the individual tasks including function files.

It is good practice to download your own submission and check that the files you have uploaded are correct. Test run your m-files that you download. You are able to update your submission until the deadline. Any update to the submission after the deadline will be considered late.

Grade and feedback:
The team will endeavour to grade your lab files by Tuesday of the following week. Grades and feedback can be viewed through the Moodle Gradebook, which is available on the left side pane on the ENG1060 Moodle site.

 Important: If you are struggling with a task, ensure that you have performed hand-written work (e.g. hand calculations, pseudocode, flow charts) to better understand the processes involved. Do this before asking demonstrators for help and use it to assist with your illustration of the problem.

Lab 10 – Assessed questions

Remember good programming practices for all tasks even if not specifically stated. This includes, but is not limited to:

- using clc, close all, and clear all, where appropriate
- suppressing outputs where appropriate
- labelling all plots, and providing a legend where appropriate
- fprintf statements containing relevant answers

PRELIMINARY

Create function files implementing each of the following ODE solver methods following the function declarations provided. This will help you consolidate your understanding of the techniques involved.

- Euler's method: function [t,y] = euler(dydt,tspan,y0,h)
- Heun's method: function [t,y] = heun(dydt,tspan,y0,h)
- Midpoint method: function [t,y] = midpoint(dydt,tspan,y0,h)

TASK 1

[2 MARKS – L10A]

Consider the following ordinary differential equation (ODE):

$$\frac{\mathrm{d}y}{\mathrm{d}t} = 3e^{t} - \frac{8y}{3}$$

Given the initial condition *y*(0) = 3, find *y*(3) using a step size of *h*=1 by:

A. Euler's method,
B. Midpoint method,
C. Heun's method.

TASK 2

[2 MARKS – L10C]

Solve the equation below using Euler's method over the interval from $t = 0$ to 3 where $y(0) = 3$ using the following step sizes: $h = 1, 0.75, 0.5, 0.001$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = 3e^{t} - \frac{8y}{3}$$

A. For each step size, plot the results at each step starting from y(0)=3 to y(3) on a single figure. **You must use a for loop to go through each value of h.**
B. On the same figure from part A, plot the analytical solution as a continuous line with a thickness value of 3. The analytical solution is given by:

$$y = \frac{9}{11}e^{t} + \frac{24}{11}e^{-\frac{8}{3}t}$$

C. Calculate and print the percentage error between the Euler's method and the analytical result at $y(3)$. The percentage error is calculated as $error\ \% = \left|\frac{y_{approx} - y_{analytical}}{y_{analytical}}\right| \times 100\%$

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An example output is shown below.

```
          h    % error
          1        ???
   7.50e-01        ???
   5.00e-01        ???
   1.00e-03        ???
```

TASK 3

[2 MARKS – L10F]

Solve the following equation over the interval from t=0 to 2 where y(0) = 1 and display your results on the same graph using the methods below. Remember to include a legend for your plot.

$$\frac{\mathrm{d}y}{\mathrm{d}t} = yt^3 - 1.5y$$

A. Euler's method with a step size of 0.5
B. Heun's method with a step size of 0.5
C. Midpoint method with a step size of 0.5
D. Ode45 using adaptive step size

Run your code multiple times with various step sizes. As you decrease the step size, which method (Euler or midpoint) converges faster? Use fprintf to provide a reason for this.

TASK 4

[2 MARKS – L10M]

Consider the following ODE:

$$4xy\frac{\mathrm{d}y}{\mathrm{d}x} = y^2\sin(x) + x^2\cos(y)$$

with $y(20) = 1.6$ for $20 \leq t \leq 70$.

A. Use ode45() to solve the ODE and plot y against x with a resolution of 0.001.
B. Use the midpoint method to solve the ODE with step sizes $h = 0.01, 0.05, 0.1$. Use a for loop to go through each value of h. Plot each of these on the same figure produced in part A. Remember to include a legend for each subplot.

Observe the differences in the solutions with changing step size and how it compares to the ode45() solution. No comments are required.

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TASK 5

[2 MARKS – L010L]

Consider a following ODE

$$\frac{dy}{dt} = \exp(-2t)\cos(y) - 0.1\alpha y^2$$

where $y = 1\text{m}$ at $t = 0\text{s}$.

a. Use Heun's method to solve the ODE for $\alpha = 0.1,\ 0.5, 1, 2, 5, 10$ with increments of 0.01s for the range of $0 \leq t \leq 20$.
b. Plot y against t for the various α cases. Leave the plot formatting up to MATLAB (i.e. don't provide any line specifications). The legend can be hardcoded here.
c. Export the α values and corresponding y values at $t = 20$ (i.e. $y(20)$) to a text file named 'diff_eqn.txt' using fopen, fprintf and fclose. The format should look similar to the following:

```
alpha      y(20)
  0.1       x.xx
  0.5       x.xx
  1.0       x.xx
  …           …
```

2 marks deducted for poor programming practices (missing comments, unnecessary outputs, no axis labels, inefficient coding, etc.)

END OF ASSESSED QUESTIONS

The remainder of this document contains supplementary and exam-type questions for extended learning. Use your allocated lab time wisely!

 Important: If you are struggling with a task, ensure that you have performed hand-written work (e.g. hand calculations, pseudocode, flow charts) to better understand the processes involved. Do this before asking demonstrators for help and use it to assist with your illustration of the problem.

Lab 10 – Supplementary questions

These questions are provided for your additional learning and are not assessed in any way. You may find some of these questions challenging and may need to seek and examine functions that are not taught in this unit. Remember to use the help documentation. Coded solutions will not be provided on Moodle. Ask your demonstrators or use the discussion board to discuss any issues you are encountering.

TASK 1S

Solve the following equation, by hand, over the interval x=0 to x=1 using a step size of 0.25 assuming y(0)=1.

$$\frac{\mathrm{d}y}{\mathrm{d}x} = (1 + 4x)\sqrt{y}$$

Do this:

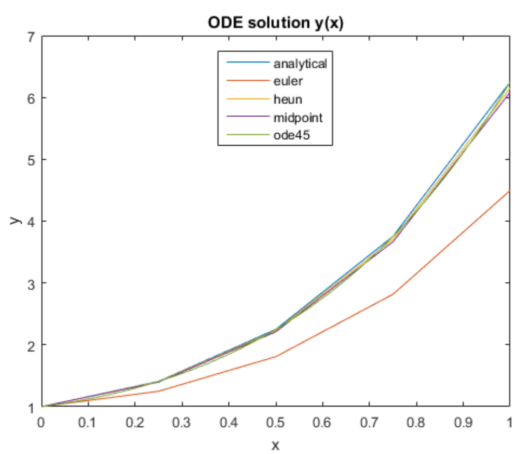
A. analytically,
B. using Euler's method,
C. using Heun's method, and
D. using the midpoint method.

Check your hand-calculated answers in MATLAB using your function files. Plot the solutions on the same plot in addition to MATLAB's ode45 solution.

SOLUTION

```
      x   y_analyt   y_euler    y_heun    y_midpt
 0.0000    1.0000    1.0000    1.0000    1.0000
 0.2500    1.4102    1.2500    1.4045    1.3977
 0.5000    2.2500    1.8090    2.2307    2.2110
 0.7500    3.7539    2.8178    3.7061    3.6670
 1.0000    6.2500    4.4964    6.1518    6.0862

The plot shows Euler's approach to be the worst approximation
```



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TASK 2S

Solve the following equation, by hand, over the interval *t*=0 to *t*=4 using a step size of 1 assuming *y*(0)=1.

$$\frac{\mathrm{d}y}{\mathrm{d}t} = -2y + t^2$$

Use:

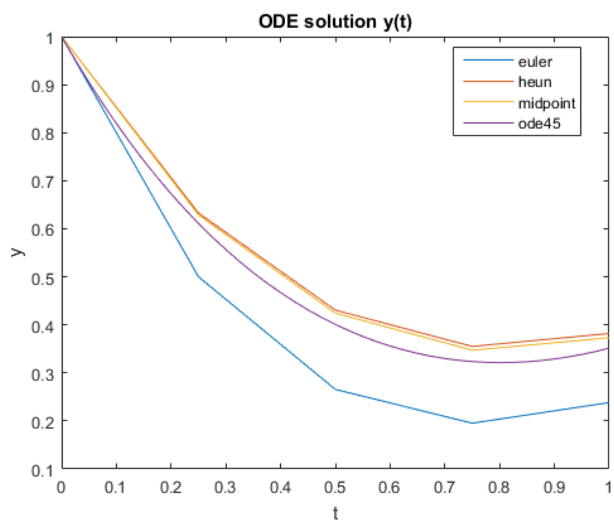
A. Euler's method,
B. Heun's method, and
C. the midpoint method.

Check your hand-calculated answers in MATLAB using your function files. Plot the solutions on the same plot in addition to MATLAB's ode45 solution.

SOLUTION

```
      t    y_euler    y_heun    y_midpt
 0.0000     1.0000    1.0000     1.0000
 0.2500     0.5000    0.6328     0.6289
 0.5000     0.2656    0.4307     0.4243
 0.7500     0.1953    0.3551     0.3472
 1.0000     0.2383    0.3821     0.3733
```

```
The plot shows Euler's approach to be the worst approximation
```



 Important: If you are struggling with a task, ensure that you have performed hand-written work (e.g. hand calculations, pseudocode, flow charts) to better understand the processes involved. Do this before asking demonstrators for help and use it to assist with your illustration of the problem.

TASK 3S

Solve the following equation, by hand, over the interval *t*=0 to *t*=1 using a step size of 0.25 assuming *y*(0)=1.

$$\frac{\mathrm{d}y}{\mathrm{d}t} + y = \sin(t)$$

Use:

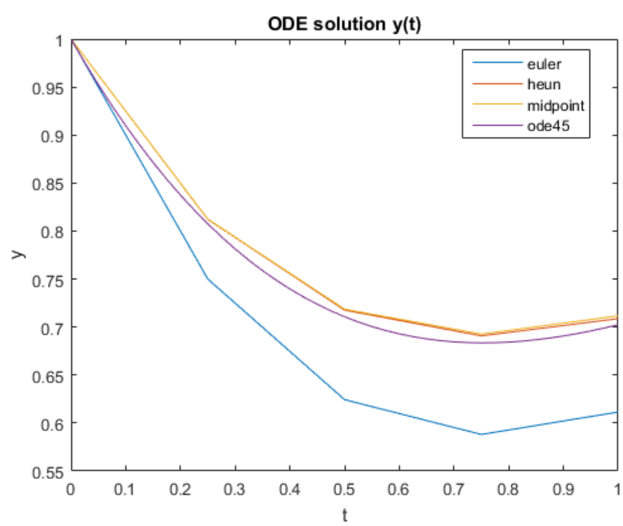
A. Euler's method,
B. Heun's method, and
C. the midpoint method.

Check your hand-calculated answers in MATLAB using your function files. Plot the solutions on the same plot in addition to MATLAB's ode45 solution.

SOLUTION

t	y_euler	y_heun	y_midpt
0.0000	1.0000	1.0000	1.0000
0.2500	0.7500	0.8122	0.8124
0.5000	0.6244	0.7176	0.7185
0.7500	0.5881	0.6908	0.6927
1.0000	0.6115	0.7088	0.7117

```
The plot shows Euler's approach to be the worst approximation
```



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TASK 4S

Solve the following equation, by hand, over the interval t=0 to t=1 using a step size of 0.25 assuming $y(0)$=1.

$$\frac{\mathrm{d}y}{\mathrm{d}t} = (1 - t)y$$

Do this:

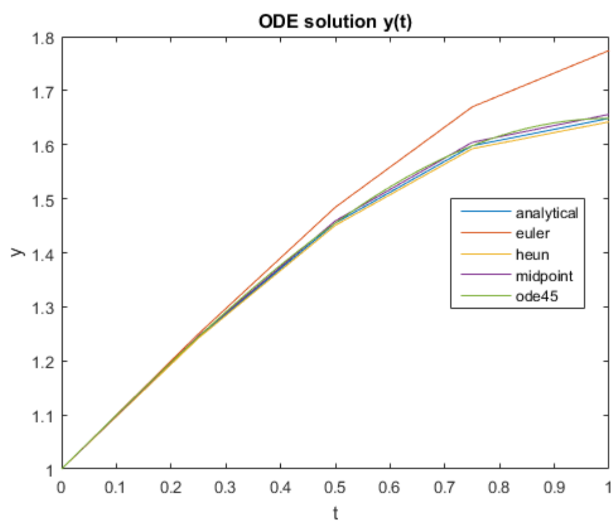
A. analytically,
B. using Euler's method,
C. using Heun's method, and
D. using the midpoint method.

Check your hand-calculated answers in MATLAB using your function files. Plot the solutions on the same plot in addition to MATLAB's ode45 solution.

SOLUTION

t	y_analyt	y_euler	y_heun	y_midpt
0.0000	1.0000	1.0000	1.0000	1.0000
0.2500	1.2445	1.2500	1.2422	1.2461
0.5000	1.4550	1.4844	1.4508	1.4590
0.7500	1.5980	1.6699	1.5925	1.6044
1.0000	1.6487	1.7743	1.6423	1.6561

```
The plot shows Euler's approach to be the worst approximation
```



 Important: If you are struggling with a task, ensure that you have performed hand-written work (e.g. hand calculations, pseudocode, flow charts) to better understand the processes involved. Do this before asking demonstrators for help and use it to assist with your illustration of the problem.

TASK 5S

Solve the following equation, by hand, over the interval t=0 to t=1 using a step size of 0.25 assuming y(0)=1.

$$\frac{\mathrm{d}y}{\mathrm{d}t} = -y\sqrt{t}$$

Do this:

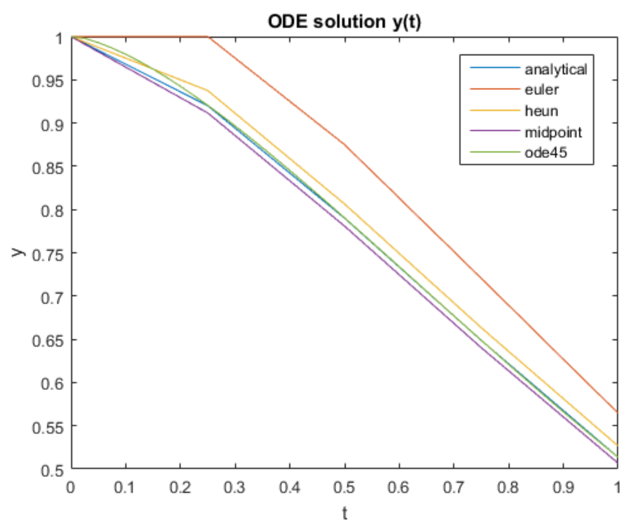
A. analytically,
B. using Euler's method,
C. using Heun's method, and
D. using the midpoint method.

Verify your hand-calculated answers in MATLAB using your function files. Plot the solutions on the same plot in addition to MATLAB's ode45 solution.

SOLUTION

t	y_analyt	y_euler	y_heun	y_midpt
0.0000	1.0000	1.0000	1.0000	1.0000
0.2500	0.9200	1.0000	0.9375	0.9116
0.5000	0.7900	0.8750	0.8064	0.7808
0.7500	0.6486	0.7203	0.6633	0.6401
1.0000	0.5134	0.5644	0.5265	0.5066

```
The plot shows Euler's approach to be the worst approximation
```



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