ENG1060: COMPUTING FOR ENGINEERS

PASS 10 – Week 12

2020 OCT NOV

TASK 1

Solve the following ODE:

$$\frac{\mathrm{d}y}{\mathrm{d}t} = -y - 5e^{-t}\sin(5t)$$

with y(0)=1 and t between 0 and 3 (inclusive) using the Euler, Heun and midpoint methods. Create a 1-by-3 subplot arrangement where each subplot contains the solutions of the 3 methods for h=0.5 (first subplot), h=0.2 (second subplot) and h=0.01 (third subplot). In addition, each subplot should plot the analytical solution of

$$y = e^{-t}\cos(5t)$$

using a step size of 0.001 and a line thickness of 3. Remember to include a legend for each subplot.

Use fprintf to display the step size, the midpoint method solution for t=3 and the absolute difference between the midpoint method and the analytical solution at t=3. The table should look similar to the following.

Step size	Y(3)	Absolute difference
0.5	----	----
0.2	----	----
0.01	----	----

TASK 2

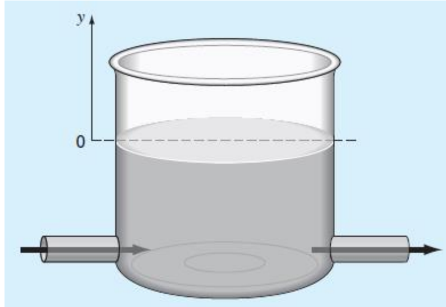
The amount of a uniformly distributed radioactive contaminant contained in a closed reactor is measured by its concentration *c* (Becquerel/litre or Bq/L). The contaminant decreases at a decay rate proportional to its concentration; that is decay rate = $-kc$, where k is a constant with units of day^{-1}. Given that $k = 0.175\text{day}^{-1}$, determine the contaminant concentrations from t=0 to t=5 days with a step size of 0.1 days using:

A. Euler's method
B. Heun's method
C. Midpoint method
D. MATLAB's ode45

Assume the concentration at t=0 is 100 Bq/L. Plot the solutions of all parts in a single figure and print out the concentration value at t=5 days for all parts.

TASK 3

A storage tank (shown below) contains a liquid at depth y where y=0 when the tank is half full. Liquid is withdrawn at a constant flow rate Q to meet demands. The contents are resupplied at a sinusoidal rate $3Q\sin^2(t)$.



A. The change in volume can be written as

$$\frac{\mathrm{d}(y)}{\mathrm{d}t} = \frac{3Q\sin^2(t) - Q}{A}$$

where A is the surface area and is assumed to be constant. Use Euler's method to solve for the depth y from t=0 to t=10 days with a step size of 0.1 days. The parameter values are A=1250 m^2 and Q=450 m^3/day. Assume that the initial condition is y=0. Plot the solution.

B. For the same storage tank, suppose that the outflow is not constant but rather depends on the depth. For this case, the differential equation for the change in volume can be written as

$$\frac{\mathrm{d}(y)}{\mathrm{d}t} = \frac{3Q\sin^2(t) - \alpha(1+y)^{1.5}}{A}$$

Use Euler's method to solve for the depth y from t=0 to 10 days with a step size of 0.1 days. The parameter values are A=1250 m^2, Q=450 m^3/day and α=150. Assume that the initial condition is y=0. Plot this solution on the previous figure. Remember to include a legend and place it in the north-west corner of the figure.

TASK 4

Newton's law describes the motion of a rocket-propelled sled as

$$m\frac{\mathrm{d}v}{\mathrm{d}t} = f - cv$$

where m is the sled mass, f is force of the rocket thrust, and c is the drag constant due to air resistance. Assume m = 1000kg, f = 75000 N and v(0) = 0 m/s. Determine the velocity of the sled from t = 0s to 10s (increments of 0.1s) using the midpoint method for c = 50, 100, 500, 1000 and 2000 Ns/m. On a single figure, plot the velocity against time for each c and provide a statement that contains the velocity of the sled at 10s for each c. E.g.

v=???m/s at t=10s if c=50
v=???m/s at t=10s if c=100