Department of Electrical and Computer Systems Engineering
Monash University

Information and Networks, ECE3141

Lab 8: Quadrature Amplitude Modulation

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1. Introduction

We have seen in lectures that a signal can be used to change in some way ("modulate") a sinusoidal carrier such that the signal is embedded in the carrier and thereby moved to a suitable frequency for transmission and sharing of the available spectrum with other users. We have also seen that, if we can detect the phase of a received signal, we can double our transmitted data rate by modulating both the in-phase and quadrature components of a carrier independently. Quadrature amplitude modulation (QAM) employs two quadrature carriers, $\cos(2\pi f_c t)$ and $\sin(2\pi f_c t)$, each of which is amplitude modulated by an independent sequence of information bits. The modulator input is a combination of information bits which get mapped onto an amplitude level that is then multiplied by a continuous time waveform that lasts for a symbol period T. The m^{th} output of the modulator is represented by the symbol

$$s_m(t) = A_{mc} g(t) \cos(2\pi f_c t) + A_{ms} g(t) \sin(2\pi f_c t), \tag{1}$$

where $\{A_{mc}\}$ and $\{A_{ms}\}$ are the sets of amplitude levels for $\cos(2\pi f_c t)$ and $\sin(2\pi f_c t)$, respectively, to denote k-bit sequences, and $g(t)$ defines the pulse shape. (Refer to Lab 7 for a description of the different pulse shapes.) An M-QAM waveform can carry $k = log_2(M)$ information bits occurring at a symbol rate of R_b / k, where R_b is the bit rate. E.g., for $M = 4$, $A_{mc} = \{1, -1\}$ and $A_{ms} = \{1, -1\}$ and for $M = 16$, $A_{mc} = \{-3, -1, +1, +3\}$ and $A_{ms} = \{-3, -1, +1, +3\}$[1].

In this laboratory class, you will carry out the following as you investigate M-QAM systems for $M = 4$ and 16:

- Calculate the theoretical bit error rate (BER) over a range of signal to noise ratio (SNR).
- Carry out Monte-Carlo simulations to measure BER for a range of SNR conditions.
- Compare theoretical and simulated BER.
- Use signal space diagrams or scatterplots to understand the effect of noise.

[1] The magnitudes may be scaled. If we wanted to maintain a maximum magnitude of 1, then the values of A would come from the set {-1, -1/3, 1/3, 1}.

- Investigate the effect of timing errors on the resulting BER of various pulse shaped waveforms.

2. Pre-lab

Prior to the laboratory class, you must read through this entire laboratory description and complete the pre-lab online quiz.

You should also familiarise yourself with the flow and processing in the Matlab code. Though the code has been written for you, you will need to change parameters and comment/uncomment certain sections; you cannot compete this lab without some interpretation of what different parts of the code is doing.

3. M-QAM System Model

A generic M-QAM system is shown in ***Figure 1 .*** The incoming bit sequence is first split into two streams of data which are carried in parallel by the in-phase and quadrature components of the carrier after modulation. The information bits pass through the modulator block where a k-bit sequence is mapped on to an amplitude level. The amplitude level acquires a pulse shape (in earlier labs, you have met rectangular, sinc and raised cosine (RC) pulse shapes) based on the specified signal bandwidth. The outputs from modulators undergo carrier modulation (refer to the Physical Layer lecture material on carrier modulation) and the in-phase and quadrature streams are combined, bandpass filtered and transmitted over the wireless channel.

At the receiver, the in-phase and quadrature components of the signal are obtained by multiplying with $\cos(2\pi f_c t)$ and $\sin(2\pi f_c t)$ respectively. The complex baseband signal passes through the M-QAM demodulator. The first step here is to bandlimit the real and imaginary components and hence a low-pass filter is applied. Generally, the receiver low-pass filter is matched to the transmitted pulse shape chosen at the modulator. After filtering, the detector is set to calculate the Euclidean distance between the received signal $r_m(t)$ and the set of possible transmitted messages $s_m(t)$ and outputs a decision that corresponds to the minimum distance. In other words, it outputs the transmitted signal that is closest to what was received.

Minimum ISI, maximum SNR

As you know from lectures, if we have a series of Nyquist pulses, appropriately spaced, then this can minimise the ISI at the point of detection. Raised Cosine pulses (as a more practical alternative to sinc() pulses) are examples and you investigated them in Lab 7.

Complementary to the objective of minimising ISI, though, is to try to maximise SNR at the point of detection, and this can be achieved using a "matched" filter.

A way of achieving both these objectives is to use what is called a "Root Raised Cosine" (RRC) filter for both pulse-shaping at the transmitter <u>and</u> as the matched filter at the receiver. The RRC impulse response is literally the square root of the RC impulse response, such that the overall effect of the cascaded filters in combination is to give a RC pulse shape at the receiver detection point[2]. Thus, we achieve both the objectives mentioned before – maximum SNR with minimum ISI.

However, this means that we do not see a RC pulse at the output of the transmitter or on the transmission line; it is only after the second filter is applied at the receiver that we have a RC (Nyquist) pulse shape. The pulse shaping at the transmitter does not give us minimum ISI, and therefore we cannot expect a nice open eye at this point.

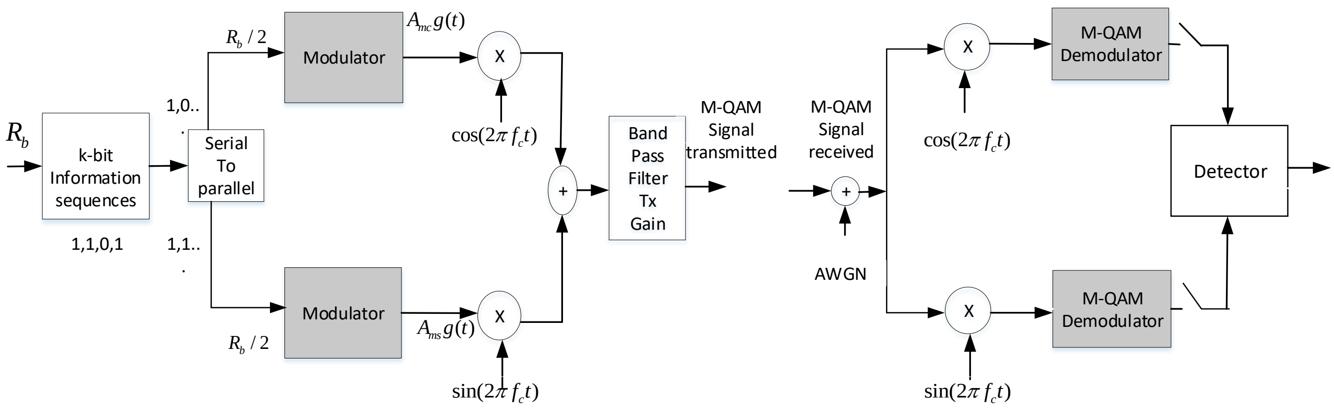


Figure 1 A generic block diagram of an M-QAM system.

In lab experiments we carry out Monte-Carlo (MC) simulations which are frequently used in performance analysis of digital communication systems (you did one of these in Lab 3 on error coding). The sequence of functions involved in MC simulations of an M-QAM system is as shown in ***Figure 2.***

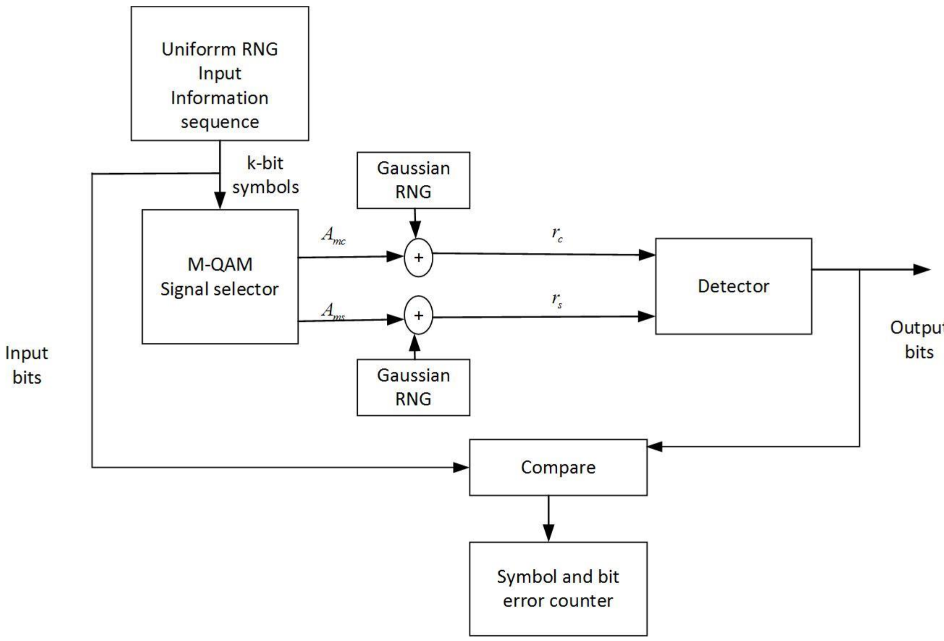


Figure 2 Block diagram of an M-QAM system for the Monte-Carlo Simulation.

[2] We can do this because the RRC pulse shape is symmetrical in time which means that it is its own matched filter. This detail is beyond our scope in this unit, though.

The uniform random number generator (RNG) is used to generate the sequence of information symbols from the set of M possible k-bit combinations of information bits. Each k-bit sequence is mapped to the corresponding signal points (as illustrated in ***Figure 1***) which have the coordinates $[A_{mc}, A_{ms}]$. The Gaussian RNG blocks are used to model the effect of Gaussian noise.

$$\begin{aligned}\mathbf{r} &= [r_c, r_s] \\ &= [A_{mc} + n_c, A_{ms} + n_s]\end{aligned} \tag{2}$$

The detector computes the distance metric between the received vector and the set of possible transmitted constellation points $\mathbf{s}_m$.

$$D(\mathbf{r}, \mathbf{s}_m) = |\mathbf{r} - \mathbf{s}_m|^2 \quad m = 1, 2, \ldots M \tag{3}$$

An estimate of the transmitted symbol is chosen based on the minimum distance criterion. The symbol error counter compares the detector output with the transmitted symbol to calculate the symbol and bit errors in transmission.

4. Scatterplots

In an M-QAM system, there are M distinct k-bit sequences, where $k = \log_2(M)$. Assuming that the average power in the transmission is E_s, the co-ordinates of the QAM signal are expressed as

$$\mathbf{s}_m = \left(\sqrt{E_s} A_{mc} \quad \sqrt{E_s} A_{ms}\right) \quad m = 1, 2, \cdots M \tag{4}$$

If we set $\sqrt{E_s} = 1$ for simplicity, the signal space diagrams (also known as scatterplots) for BPSK, 4-QAM and 16-QAM are shown in ***Figure 3.*** Note that the use of quadrature modulation generates a two-dimensional set of transmitted points in these diagrams, whereas BPSK just has two transmitted values on the real axis.

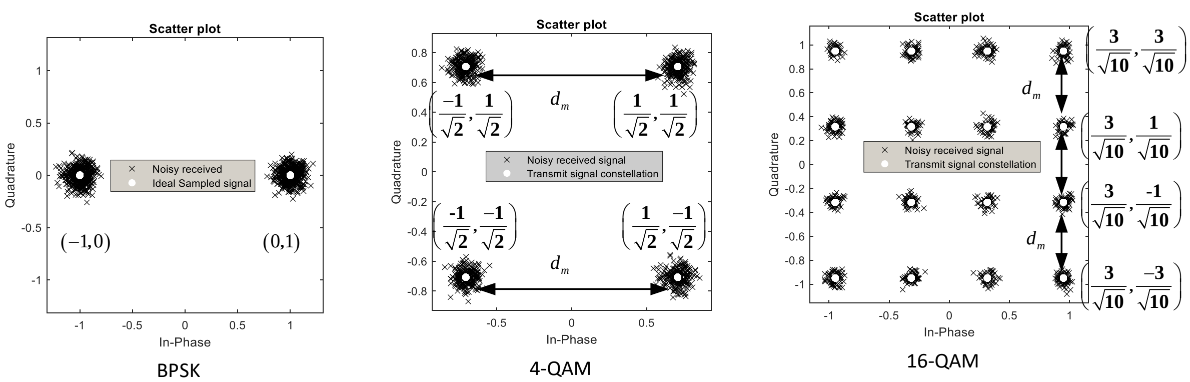


Figure 3: Scatterplots of normalised signal constellations for BPSK, 4-QAM and 16-QAM at SNR=20dB

In practical implementations, it is important to normalise the average power of the digital constellation[3] so that it is independent of the modulation order M. The transmit power then is altered using a gain control mechanism at the transmitter front-end.

[3] Note that this is not the same as fixing the maximum amplitude to 1. Some of the magnitudes of the points in the 16-QAM signal space have magnitudes less than 1. So, if we're to keep the average power equal to 1, we need to take account of the probability of

As the modulation order increases, the distance between any two adjacent points decreases. This means that, with noise in transmission, the chances of a receiver making an error is higher for higher order QAM. While this is obviously not a good thing, we gain a higher bit rate! So, we must balance our requirements to obtain a final design.

Exercises

Download the Matlab zipped folder named Lab 8 from the Moodle page for ECE 3141, unzip it and save it in your working directory. <u>As you work through the exercises below, make sure you copy any plots and results into this Word document so it is a complete record of what you have done and an aid in reviewing the material prior to the exam.</u>

5. Pulse shaped M-QAM system

Eye diagrams and scatterplots will be used in experiments in the Lab to understand the effect of modulation order (e.g., M = 4, 16) and noise on the bit error performance of pulse shaped M-QAM signals.

Open the m-file named "Qam_BER.m". The parameter "M" in the file is used to define the modulation order of a QAM signal. Set M = 4 and uncomment sections of the m-file that plots eye diagrams and scatter plots to answer questions below. (Study the m-file to understand the functions implemented there.) The BER calculated from simulations is displayed in the command window of Matlab.

Observe the eye diagram plot at the output of the transmitter.

a) Why are there two eye diagrams displayed for a QAM signal? How many distinct amplitudes can you see for an M=4-QAM signal?

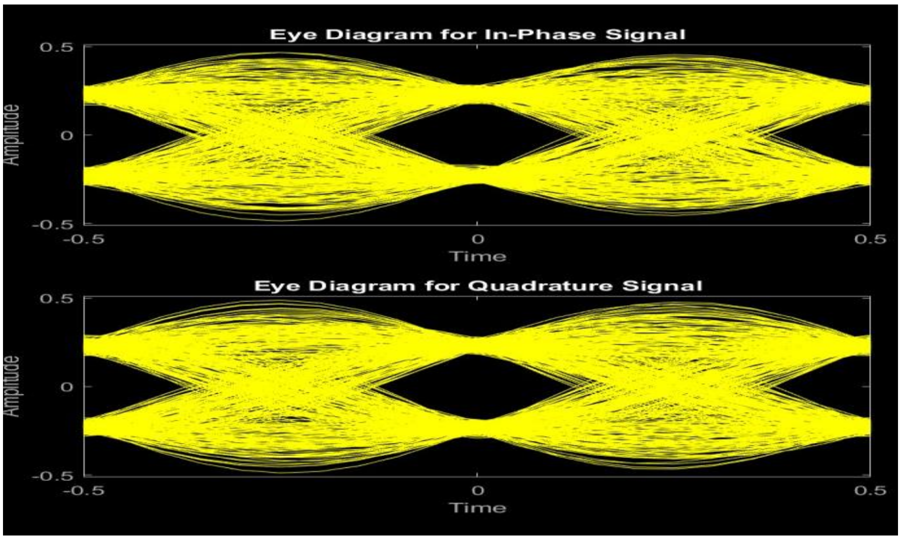


Figure 1: Eye Diagram of the transmitted signal

occurrence of each of the signal points (assumed to be equiprobable) and the power of each when we calculate the average power of the whole signal space.

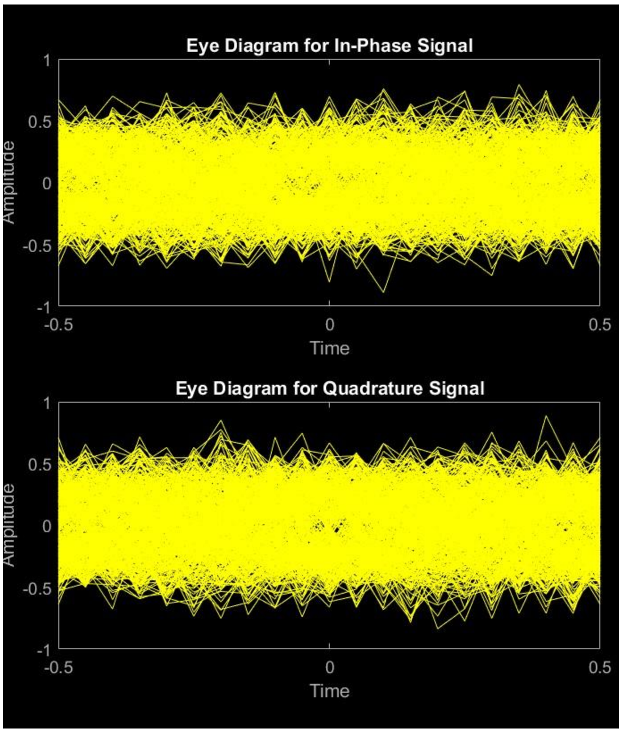


Figure 2: Eye Diagram of the Received Signal

There are two eye diagrams displayed for the QAM signal as QAM has two components the in-phase (cosine component) and the quadrature (sine component). The two diagrams in each of the transmitted and received signal means that one is for the in-phase signal and the other is for the Quadrature signal.

There are 2 distinct amplitudes (1 and -1) that I can see for an M=4 QAM signal in each eye diagram.

b) Are the two plots identical? What would you expect when you generate waveforms using real hardware?

The two plots are nearly identical for both the transmitted and the received pulses. When we generate the waveforms on real hardware, there would be more noise added to the signal due to external factors during transmission. This would cause the eye diagram to be much noisier, making it easily differentiable.

c) Generate transmit and receive eye diagrams for the 4-QAM signal at SNR = 20dB for a RC pulse roll-off factor of 0.1 and 0.5. Compare the two received signal eye diagrams (after the receiver filter). (Copying them into the space below will allow you to compare them side-by-side.) Identify an advantage of using a higher roll-off factor. What would be a disadvantage?

When α = 0.1 at SNR = 20 dB and α = 0.5 at SNR = 20 dB

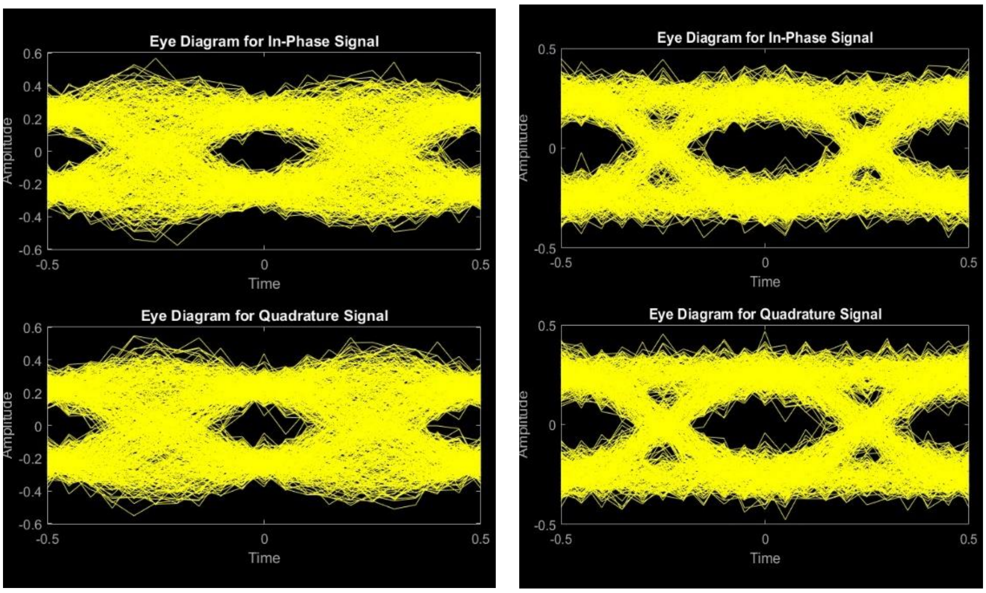
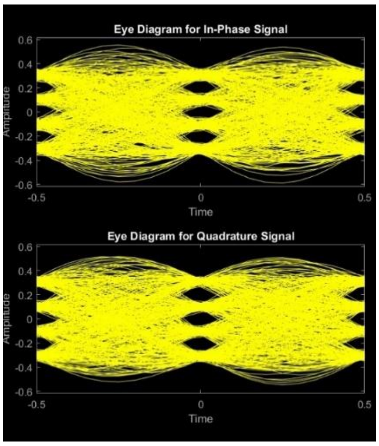


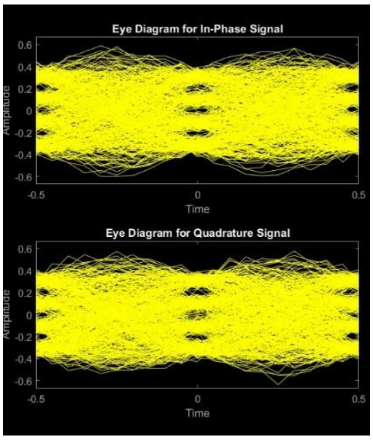
Figure 3 & 4: Received signal eye diagram for both components of 4-QAM, α = 0.1 and the received signal eye diagram for both component of 4-QAM, α = 0.5

As we can see from the figures above, when the value of α = 0.5, the opening of the eye diagram will be larger than when the value of α = 0.1 for both components of the 4-QAM. This would mean that there is less distortion for α = 0.5 at the zero crossing than α = 0.1.

The advantage of using a higher roll-off factors is that it would reduce the synchronisation errors as there will be less distortion at the zero crossing. The disadvantage of using a higher roll-off factor would be higher bandwidth which would allow OOB omission.

d) Change the value of the modulation order to M=16. Run the m-file for roll-off=0.1 and SNR=20dB. Observe transmit and receive signal eye diagrams. From the eye diagrams displayed, how many amplitude levels are there in the in-phase and quadrature components of the 16-QAM signal? Why are the eyes more open after the receiver filter than before it?





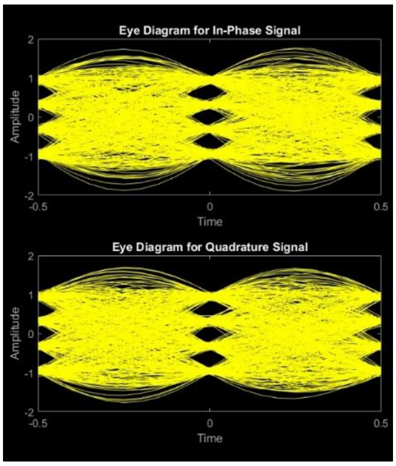


Figure 5, 6 & 7: The transmitted signal eye diagram for both components of 16-QAM, α = 0.1 and the received signal eye diagram for both components before and after Rx filter.

As we can see from the eye diagram displayed, we can see that there are 4 amplitude levels in the eye diagram for both the components.

The reason the eyes are more open after the receiver filter than before it is due to the addition of AWGN noise that cause the noise margin to be smaller before the receiver filter.

e) Uncomment the section of code that draws the scatterplots, leaving the parameter values unchanged. Look at the transmitter eye diagram and the transmitter scatterplot. Why are the points in the scatterplot dispersed from their ideal location (red dots)? Can you relate this to the opening in the eye diagram? Explain.

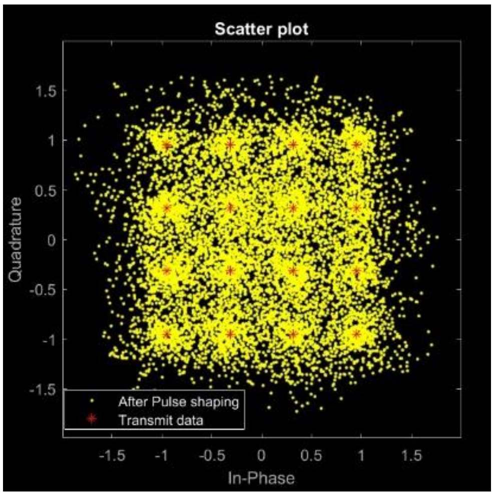


Figure 8: Transmitter scatterplot for 16-QAM at α = 0.1

The yellow points are dispersed from their ideal original transmitted point (red points) due to filtering. As the roll-off factor is increased, there will be fewer errors due to the opening of the eye-diagram being increased. However, as there are more levels right now (4 levels), the sensitivity to noise will increase and the points will shift due to the noise introduced at the transmitter filter of the QAM along with the sensitivity to noise being greater. If the opening of the eye diagram is small (smaller noise margin), the signal will be very susceptible to noise error, causing the samples to be more dispersed.

We can also say that the points in the scatterplot are dispersed from their ideal location is because of the ISI. The zero crossing of the eye is distorted and blurry will show the effect of high OOB (Out of Band) which causes more errors. The eye width is also small as there is only a small period where sampling of the signal will yield the correct result. This high OOB will lead to high ISI which relates the eye opening to the scatterplot.

f) With SNR = 10dB, observe receiver scatterplots for M=4 and M=16 and compare the resulting BERs. Which of the two cases has higher BER? Explain the reason for higher BER using the scatterplots of the two modulations. (Copy them below as a record.)

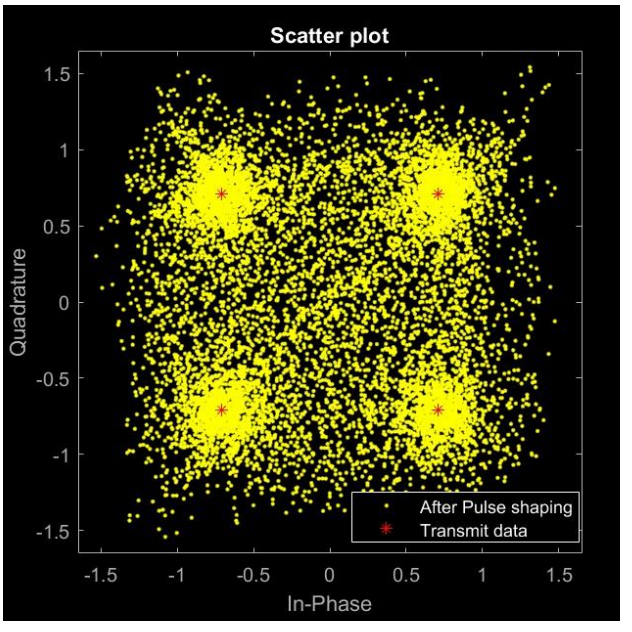


Figure 9: Scatter plot of the transmitted signal at M = 4 and SNR = 10dB

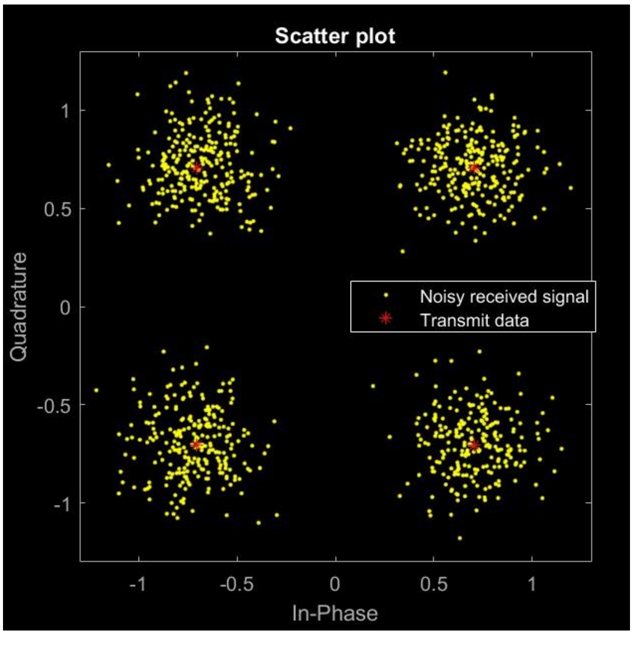


Figure 10 : Scatter plot of the received signal at M = 4 and SNR = 10dB

The BER from the 4-QAM is 0

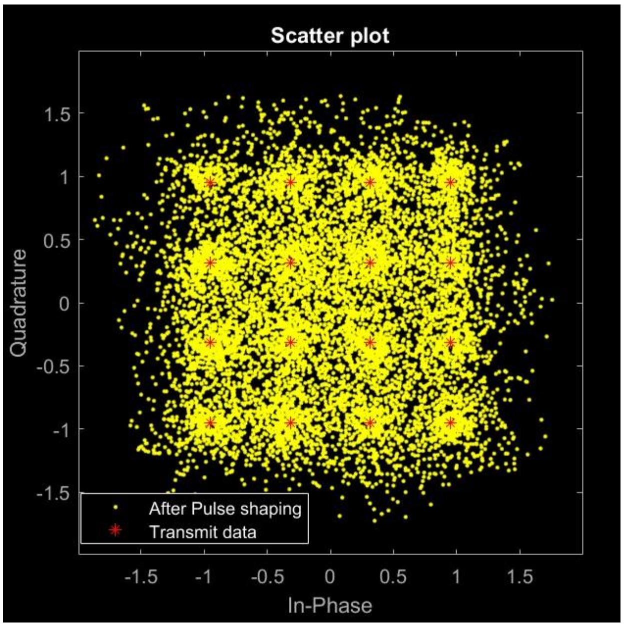


Figure 11: Scatter plot of the transmitted signal at M=16 and SNR = 10 dB

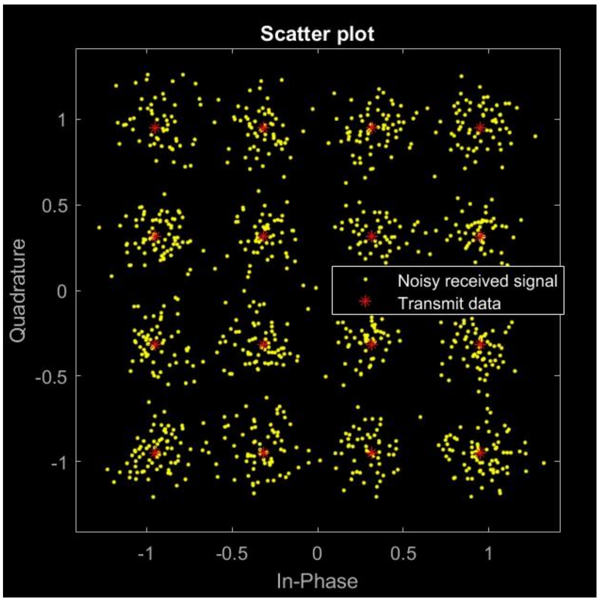


Figure 11: Scatter plot of the received signal at M=16 and SNR = 10 dB

The BER will have a value of 3e-03.

The 16-QAM will have a higher BER. This is due to the larger number of levels. The levels are closer together, so it is easier to detect a wrong level as the level sizes are shorter and the noise is comparably larger to the level sizes which means the threshold is crossed. This would mean that there is a greater sensitivity to noise for the higher M-QAM. The same noise will have more tolerance for the 4-QAM than 16-QAM. Thus, we can conclude that with a higher M, the BER will increase.

6. Theory versus Simulations

Note: In this section we will focus on the error performance and so you should comment out the Matlab code concerning eye diagrams and scatterplots, as we don't need it and it will just leave lots of figures all over your desktop. Use the section of the code in the Matlab file to plot the BER.

In this simulation-based experiment, we will generate plots for bit errors under a variety of channel conditions. We will specifically consider additive white Gaussian noise (AWGN) channels which are commonly used for system analysis. We will carry out Monte Carlo simulations for pulse shaped QAM signals over a wide range of noise levels and compare the measured average BER obtained from simulations with theory.

SNR is generally defined as energy per bit-to-noise power-spectral-density ratio $\left(E_b / N_0\right)$ (refer lecture slides on Physical Layer). For M-QAM, a sequence of $k = \log_2(M)$ bits form an information symbol. We can say that the energy per symbol-to-noise power-spectral-density ratio $\left(E_s / N_0\right)$ is k times the $\left(E_b / N_0\right)$.

$$\frac{E_s}{N_0} = \frac{kE_b}{N_0} \tag{5}$$

Each symbol period is represented with more than one sample value and so the noise added to each symbol should take this into account. Assuming that there are n_{samp} number of samples representing a symbol period, the energy per sample to noise ratio is obtained from

$$\frac{E_{samp}}{N_0} = \frac{E_s}{n_{samp} N_0} \tag{6}$$

Substituting (5) into (6) and converting the ratio of powers into dB scale we have

$$10\log_{10}\left(\frac{E_{samp}}{N_0}\right) = 10\log_{10}\left(\frac{E_b}{N_0}\right) + 10\log_{10}(k) - 10\log_{10}(n_{samp}) \tag{7}$$

Study the Matlab file to understand how AWGN is added to the received signal.

a) Given an SNR equal to $10\log_{10}\left(\frac{E_b}{N_0}\right) = 10dB$, calculate the quantity $10\log_{10}\left(\frac{E_{samp}}{N_0}\right)$ for a 4-QAM signal which is sampled with n_{samp} = 10 samples/symbol. Now calculate $10\log_{10}\left(\frac{E_{samp}}{N_0}\right)$, for a 16-QAM system. Write the difference in the values you observed. Explain the difference.

Given n_{samp} = 10 samples/symbol

$$\text{SNR} = 10\, log_{10}\left(\frac{E_b}{N_0}\right) = 10\, dB$$

$$\left(\frac{E_b}{N_0}\right) = 10$$

$$\left(\frac{E_b}{N_0}\right) = \left(\frac{1}{k}\right)\frac{E_s}{N_0} = 10$$

$$\left(\frac{E_s}{N_0}\right) = n_{samp}\,\frac{E_{samp}}{N_0} = 10k$$

$$\frac{E_{samp}}{N_0} = \frac{10k}{n_{samp}}$$

Using

For 4-QAM signal

K = $log_2(4)$ = 2

10log10 ($\frac{Esamp}{N0}$) = 10 + 10 $log_{10}(2)$ – 10 $log_{10}(10)$ = 3.0103 dB

For 16-QAM signal

K = log2(16) = 4
10log10 ($\frac{Esamp}{N0}$) = 10 + 10log10(4) – 10log10(10) = 6.0206 dB

For a 2-bit increase in the information symbol k, there is a 3dB increase in the SNR. As M is increased, the number of bits used to represent the symbols will also increase. This would cause energy per symbol-to-noise power to increase.

b) Uncomment the section of the code to plot the theoretical error performance curve SNR$\left(E_b / N_0\right)$ versus BER. Use the BER-SNR theoretical plot generated to answer this question. To achieve a BER of 10^{-3}, what value of SNR is required at the receiver when M=4 is used? Is it different for M=16? What is the difference in SNR? (Note: Don't close the Matlab figure, you will be using this to plot more results for comparisons. But you should copy them below as a record.)

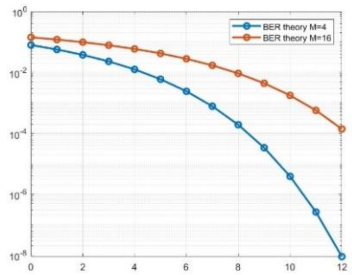


Figure 12: Plot of SNR vs BER

We can see from the graph that the SNR for 10^-3 BER at M = 4 is 6.771 dB

We can also see from the graph that the SNR at 10^-3 BER for M =16 is 10.496 dB

The difference in SNR is approximately 3.725 dB.

c) Uncomment the section of the code that plots the BER versus SNR from simulations in the Matlab file. (Comment the section of the code that plots theoretical BER results for this question along with the `close all` command at the start of the m-file. You will be comparing theoretical curves with the simulated BER.) Run the M file with SNR = [0:1:12] for M=4 and then for M=16. Do your theoretical BER curves match the performance obtained using Monte-Carlo simulations?

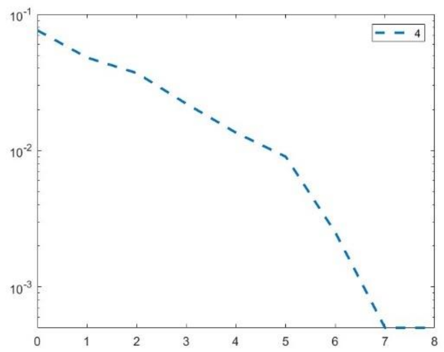


Figure 13: Plot of simulated BER vs SNR at M=4 and α = 0.1

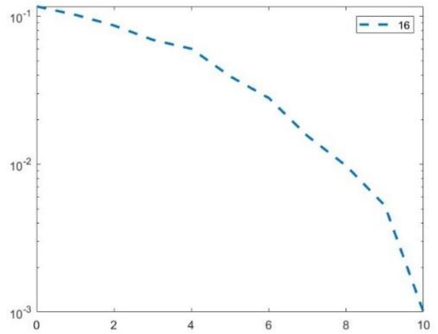


Figure 14: Plot of simulated BER vs SNR at M=16 and α = 0.1

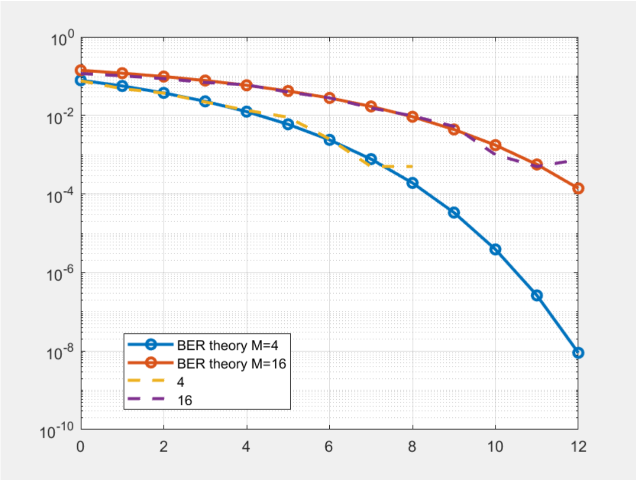


Figure 15: Simulation vs Theoretical BER at α = 0.1

The graph for M = 4 from figure 13 is downward sloping until it reaches an SNR value of 7 dB and after which the BER remains constant. For M = 16, the graph for M =16 from figure 14 is also downward sloping but the BER decreases until an SNR of about 11 dB. Unlike M = 4, the BER for M = 16 is not constant and rises after SNR = 11 dB.

We will see that the theoretical curves closely match the curves generated using the Monte Carlo simulations.

d) Change the roll-off factors and re-run simulations for M = 4, 16. Does the choice of modulation order affect the simulation performance? What can you conclude from these comparisons?

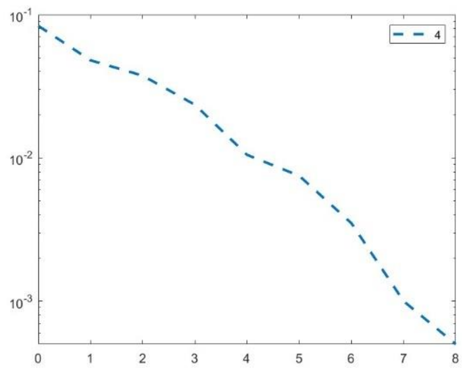


Figure 16: Simulated BER vs SNR at M = 4 and α = 0.5

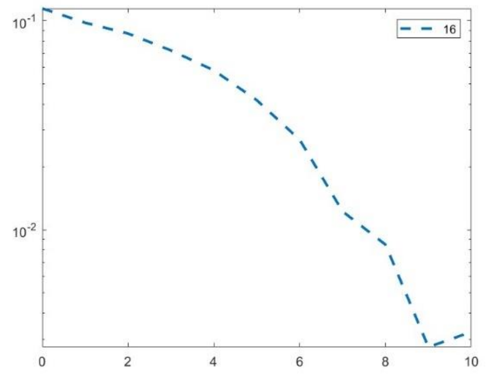


Figure 17: Simulated BER vs SNR at M = 16 and α = 0.5

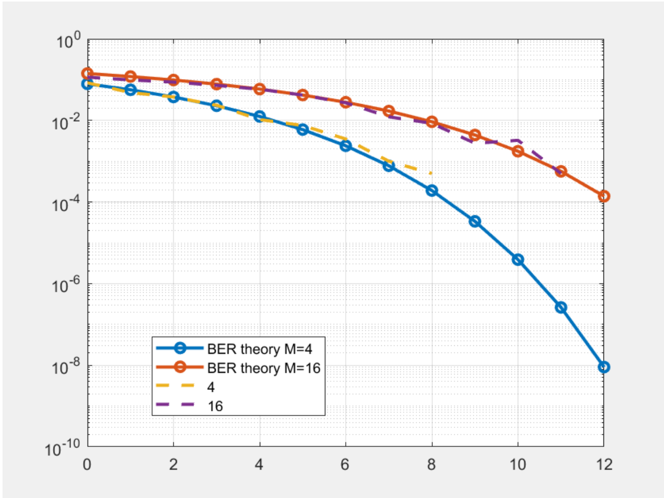


Figure 18: Simulation vs Theoretical BER at α = 0.5

The change in the pulse shaping (changing of the roll-off factor) will not have much effect on the performance of the BER of a system. We can say that for a higher roll-off factor, we do not need a higher SNR to achieve the same BER compared to a lower roll-off factor.

We can notice that the graphs from the simulations have become more like the theoretical curves generated. So, we can conclude that changing the roll-off factor, α will make the two types of graphs more similar.

7. Bandwidth efficiency of RC M-QAM signals

An ideal-brick-wall filter is known to have a sinc pulse shape in the time domain (i.e. the impulse response). The signal bandwidth (i.e. the first null) for the brick-wall filter is $1/T$ with a data rate of $\log_2(\mathrm{M})/T$. RC pulses occupy bandwidth more than the absolute minimum used by the brick-wall filters. This excess bandwidth is controlled by the roll-off factor α (refer back to Lab 7 for information on RC pulse shaping). For RC pulses modulated using M-QAM, the bandwidth is $(1+\alpha)/T$ for a data rate of $\log_2(\mathrm{M})/T$. Bandwidth efficiency is the ratio of data rate to the bandwidth occupied by a pulse shape. Using this, the bandwidth efficiency for the brick wall filter (sinc() impulse response) is η_{sinc} and is given by

$$\eta_{sinc} = log_2(M) \text{ bits/sec/Hz} \tag{8}$$

The bandwidth efficiency of RC pulses is given by

$$\eta_{RC} = \frac{\log_2(M)}{1+\alpha} \text{ bits/sec/Hz} \tag{9}$$

a) Generate a plot of η_{RC} (y-axis) versus M for $M = 4, 16, 64, 256, 1024$ (x-axis) for a $\alpha = 0.1, 0.2, 0.5$. (Use Matlab commands in the script file to generate the plot). Explain the effect of increasing M and also increasing roll-off.

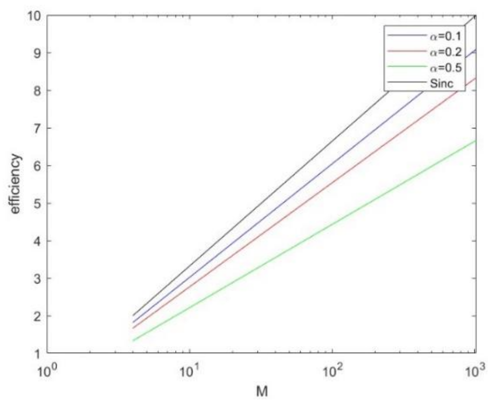


Figure 17: Bandwidth Efficiency of η_{RC} versus M (M = 4,16,64,256,1024) for α = 0.1, 0.2, 0.5

The bandwidth efficiency of the pulses increases with increasing M. However, it is the opposite case for the roll-off factor, α. By increasing α, this would decrease the bandwidth efficiency. A trade-off is seen between the timing jitter and the OOB emissions compared with the bandwidth usage.

8. Conclusion

Digital modulation of data using QAM is utilised in a number of technologies such as Wi-Fi, digital video broadcasting (DVB) and Long Term Evolution (LTE/LTE-Advanced). As seen in experiments, a digital QAM symbol is formed from independent combinations of amplitude and phase representing independent bit sequences. The spectral efficiency of the M-QAM signal increases as M increases. However, it does so at the cost of an increase in the probability of symbol errors. Trade-offs associated with the use of raised cosine pulse shaping in combination with M-QAM system are studied in this lab.

Before finishing, could each student please click on the "Feedback" icon for this laboratory on Moodle, to record some brief, anonymous feedback on this laboratory exercise.

Finally, please submit this completed report via Moodle by the stated deadline. In so doing, please be aware of the following:

- Even if you have had a mark assigned to you during the lab session, this mark will not be registered unless you have also submitted the report.
- Your mark may also not be accepted or may be modified if your report is incomplete or is identical to that of another student.
- By uploading the report, you are agreeing with the following student statement on collusion and plagiarism, and must be aware of the possible consequences.

Student statement:

I have read the University's statement on cheating and plagiarism, as described in the *Student Resource Guide*. This work is original and has not previously been submitted as part of another unit/subject/course. I have taken proper care safeguarding this work and made all reasonable effort to make sure it could not be copied. I understand the consequences for engaging in plagiarism as described in *Statue 4.1 Part III – Academic Misconduct*. **I certify that I have not plagiarised the work of others or engaged in collusion when preparing this submission.**

9. References

[1] Simon Haykin. 2009. Communication Systems (5th ed.). Wiley Publishing.

– END –