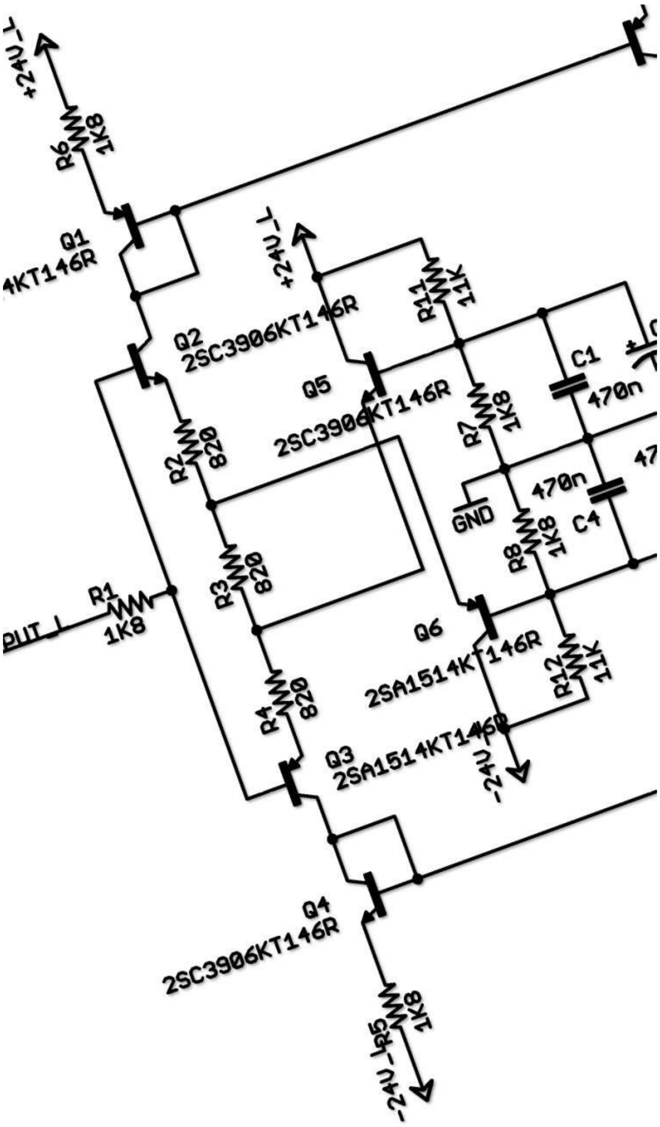
k MONASH University
Engineering

ECE2131

Electrical Circuits

Laboratory Notes



2022 Edition

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Electrical and Computer Systems Engineering, Monash University

2022

6 Frequency Response of 1st and 2nd Order Circuits

6.1 Learning Objectives and Introduction

The sinusoidal steady-state response of both first and second order series circuits as a function of frequency will be investigated in this experiment. Making use of what you learned in previous experiments, the relationship between the transient response and the frequency response of these circuits is to be practically examined.

This laboratory is divided into two parts, Part A and Part B. Part A deals with first-order linear circuits, mainly the RC case, and Part B with a second-order series RLC circuit. Sinusoidal voltages will be used to excite the circuits. This will allow the amplitude and phase of the output voltage in relation to the input voltage to be both predicted and measured.

By the end of this lab you should:

- Relate complex impedances to a frequency-domain representation (AC analysis) of circuits
- Measure the frequency response of a circuit with a network analyser
- Understand how to relate the frequency response to the effect on periodic signals (voltage waveforms …) coming into a circuit

6.2 Reference Equations

Additional equations are available in the lab addendum notes on Moodle, and in the lectures notes, but some key results are summarised below:

First order transfer function magnitude and phase components are given by:

$$\left|\underset{\sim}{A}_V(j\omega)\right| = \frac{1}{\sqrt{1+(\omega RC)^2}} \qquad \arg\left(\underset{\sim}{A}_V(j\omega)\right) = -\tan^{-1}(\omega RC)$$

Break frequency magnitude and phase (cf RC time constant):

$$\underset{\sim}{A}_V(j\omega_b) = \frac{1}{1+j\omega_b RC} = \frac{1}{1+j1} \rightarrow \left|\underset{\sim}{A}_V(j\omega_b)\right| = \frac{1}{\sqrt{2}} \approx 0.7071$$

$$\omega_b = \frac{1}{RC} = \frac{1}{\tau} \qquad \arg\left(\underset{\sim}{A}_V(j\omega_b)\right) = -\tan^{-1}(1) = \frac{-\pi}{4} \text{ radians, or } -45^\circ$$

6.3 Equipment and Components

- Breadboard
- 100 nF (0.1 μF) capacitor
- 100 mH inductor
- 820Ω and 1.6 kΩ resistors
- Potentiometer

6.4 Experimental Work

6.4.1 Setting up for network analyser experiments

6.4.2 Network analyzers

Basically, network analyzers are tools that you can use to measure the AC magnitude and phase response of a circuit. They give you magnitude and phase plots.

Essentially, a swept frequency is input to a circuit, and the output is measured. Timing (i.e. phase) is either done internally, or (in the case of this lab) with an external reference measurement. So, instead of having to analyse a series of measurements of sinusoids, you are able to measure amplitude and phase over thousands of points in matter of seconds.

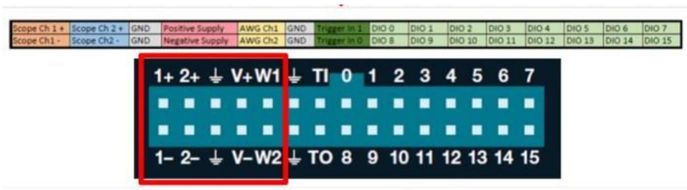
What this builds for you is a frequency-domain transfer function for the circuit, which you can then use to understand the circuit's AC behaviour.

6.4.3 Connect ADALM2000 boards, open Scopy and set-up experiment

a) Connect the ADALM USB board to lab computer & Connect the 2x5 pin connector to the left-most pins on the ADALM2000 module

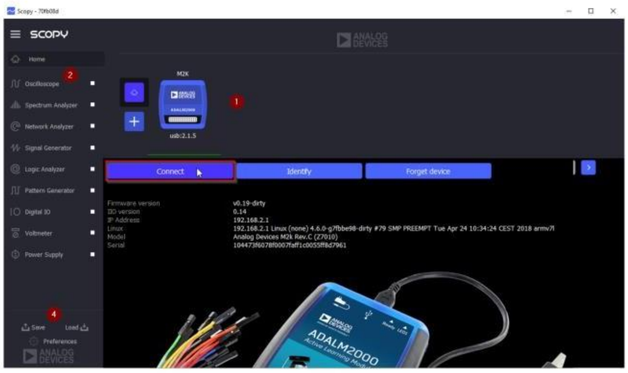
Connect micro-USB


Connect to PC



We only need these 10 pins for our labs

b) Open Scopy, click on device in Home screen and click connect.

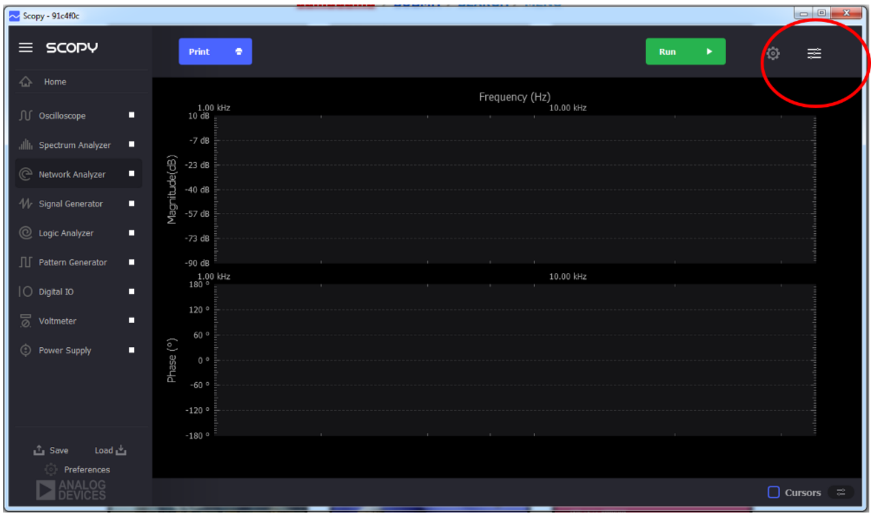


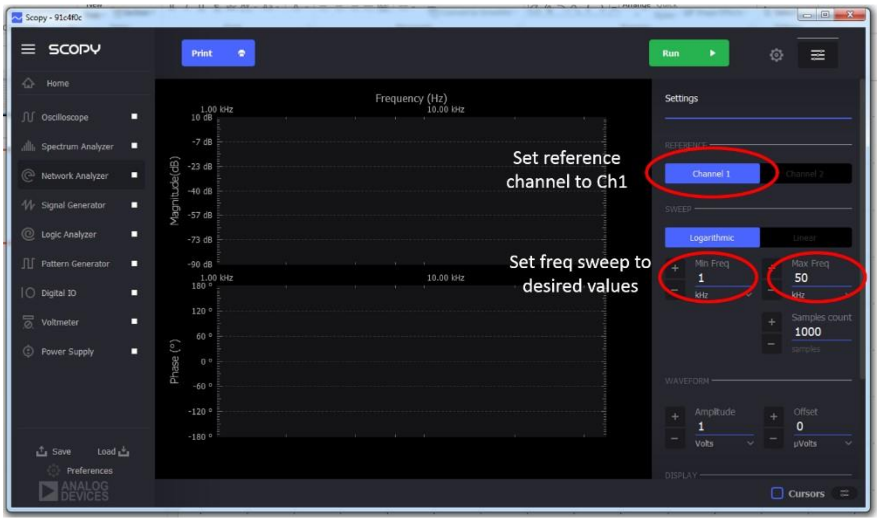
Wait a few seconds for it to connect and calibrate.

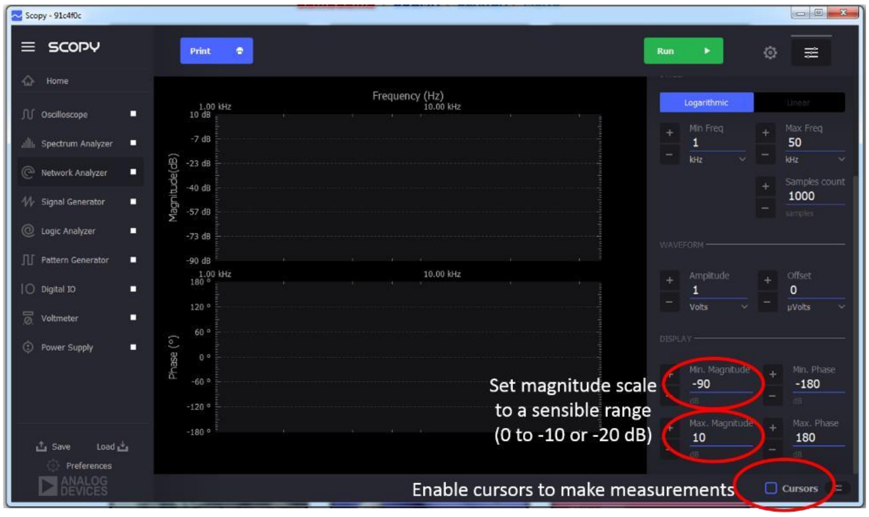
c) Open the Network Analyzer tab in Scopy set **Channel 1** to reference.

d) The ports from ADALM2000 that you will be using are as follows:

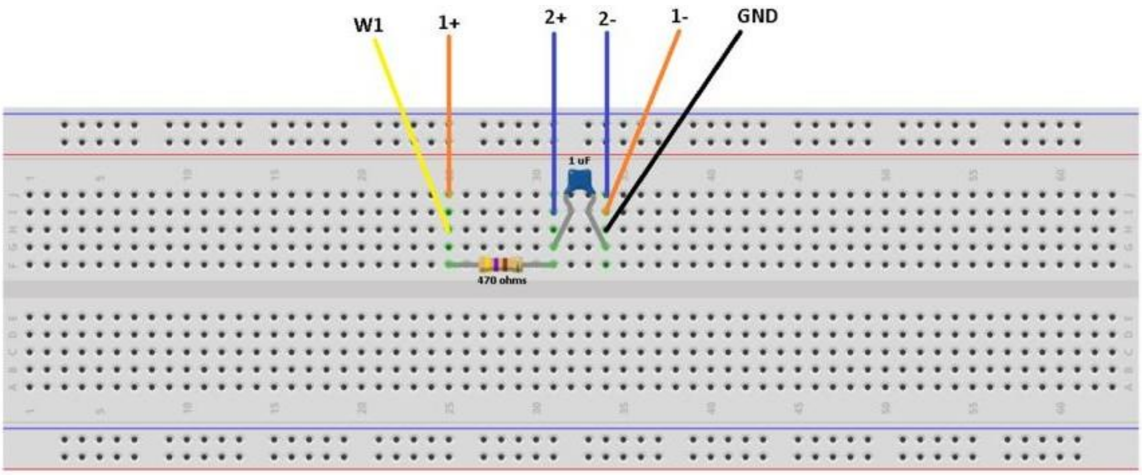
- W1 (yellow) as the signal generator, and use ground (black) as the negative terminal, **not W2**.
- Ports 1+ and 1- (orange wires) will be used as the reference probe (connected to the source) for this experiment. Connect 1- to ground.
- Ports 2+ and 2- (blue wires) of the oscilloscope will be the measurement probe (connected between the measurementnode and ground). Connect 2- to ground.







e) For the first circuit (RC first-order circuit), set out your circuit like so (**ignore values on the schematic image below**). Use the components as listed in section 6.3.



6.4.4 PART A: SERIES FIRST-ORDER CIRCUITS

Construct the circuit of Figure S5.2 with C = 0.1 μF 10% and R = 1,600 Ω.

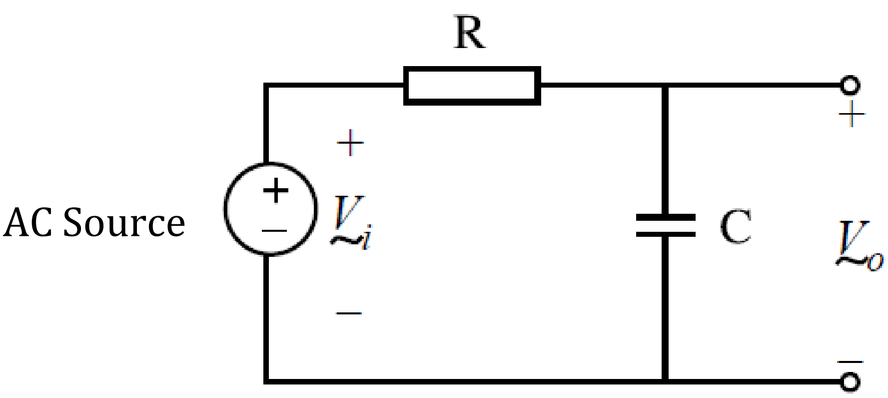


Figure S5.2 A series RC circuit

6.4.4.1 Measure the magnitude and phase of the voltage "gain" transfer function in the frequency range from 20 Hz to 20 kHz.

6.4.4.2 Plot both the amplitude- and phase- frequency responses using a **logarithmic scale for frequency**. It is recommended that a linear scale is used for amplitude and phase, and the units in dB, as per Scopy. **Remember to label your axes, as well as mark any measurement points in your graphs.**

The plot for amplitude and phase frequency response using a logarithmic scale for frequency.

The break frequency will occur when the magnitude of voltage gain is $\frac{1}{\sqrt{2}}$ which equates to about 0.707 V when the phase is about 45°.

0.707 V in terms of decibels can be calculated with the following formula which is -20log(V). This would equate to about 3.01 dB.

From the visual observation of the graph above, we can see that the magnitude of the voltage gain will drop as the frequency is increased and that the phase shift will tend to $-90°$ when the frequency increases. Both of the trend of the graph can be related back to the equation of the magnitude of voltage gain and the equation of the phase shift which is given above under the first order transfer function section.

6.4.4.3 From the plots above, determine the break or corner frequency. Compare the angular corner frequency, $\omega_B = 2\pi f_B$ against the reciprocal of the time constant RC, $1/\tau$. Compare your plots with those of Figure S6.4 from the lab addendum notes available on Moodle.

From the magnitude vs frequency graph shown above, the frequency obtained when the magnitude of the voltage gains in terms of decibels (3.01dB) is about 937.38 Hz.

We can calculate the angular corner frequency by using the formula $\omega_B = 2\pi f_B$

We will get ω_B to be 2 * π * 937.38 Hz which equates to 5889.73 rad/s

When the phase gain is at 45°, the measured break frequency based on the graph above is about 991.33 Hz.

We can calculate the angular corner frequency by using the formula $\omega_B = 2\pi f_B$

We will get ω_B to be 2 * π * 991.33 Hz which equates to 6228.71 rad/s

☐ CHECKPOINT: Get a demonstrator to check your answers, and initial here

From here, we will compare our values with the theoretical value of break frequency which can be calculated using the formula

$$\omega_B = \frac{1}{RC}$$

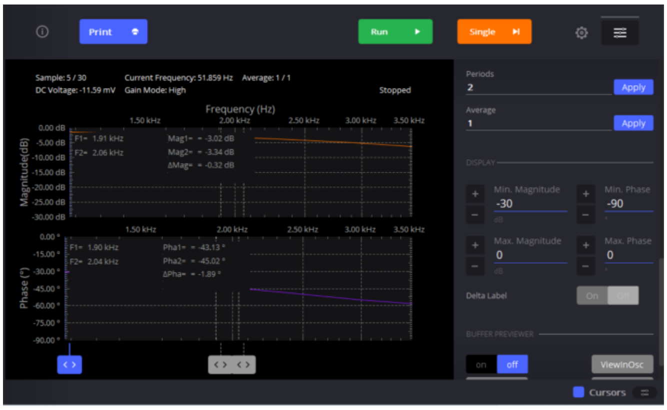
$$\omega_B = \frac{1}{1600\Omega * 0.1\mu C}$$

$$\omega_B = 6250\, rad/s$$

The theoretical value obtained and the measured value will be different due to the systematic error made by humans during the extraction of data from the graph. This could also be due to the inaccuracies of the graph by Scopy itself.

The plots obtained from Scopy for the magnitude of the voltage gain against the frequency will have a slight difference compared with the graph in the lab addendum (Figure S6.3). This is because the magnitude of the voltage gain is in terms of decibels for the graph that we have obtained and the graph in Figure S6.3 is in linear scale as the amplitude frequency response curve starts from 1 and decreasing towards 0. However, for the phase frequency response curve, both of the curve obtained are the same as both of the curve shows that the curve starts from $0° \, and\ decreasing\ towards -90°$.

6.4.4.4 Now change the load resistance to 800Ω. Sweep the frequency of the source to find the new break point frequency. Compare your experimental value against theoretical values.



The amplitude at break frequency will be $\frac{1}{\sqrt{2}}$ which equates to about 0.707 V. The measured break frequency based on the amplitude frequency response graph above is 1.91 kHz.

We can convert the measured frequency above into angular frequency with the given formula $\omega_B = 2\pi f_B$

We will get ω_B to be 2 * π * 1.91kHz which equates to 12000.88 rad/s.

The phase at the break frequency will be 45°. The measured break frequency based on the phase frequency response graph above is 2.04 kHz.

We can convert the measured frequency above into angular frequency with the given formula $\omega_B = 2\pi f_B$.

We will get ω_B to be 2 * π * 2.04kHz which equates to 12817.70 rad/s.

From here, we can obtain the average value between the two which would equate to about 12409.29 rad/s.

The theoretical break frequency can be found using the formula

$$\omega_B = \frac{1}{RC}$$

$$\omega_B = \frac{1}{800\Omega * 0.1\mu C}$$

$$\omega_B = 12500\, rad/s$$

There is a slight difference in the measured break frequency and the calculated break frequency due to the systematic error made by humans during the extraction of data from the graph. This could also be due to the inaccuracies of the graph by Scopy itself.

☐ CHECKPOINT: Get a demonstrator to check your answers, and initial here

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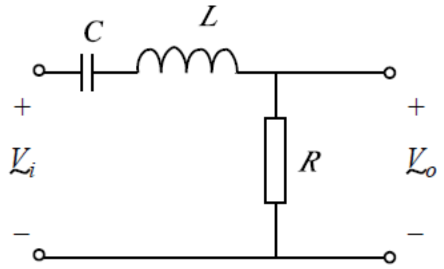
6.4.5 PART B: SERIES RLC CIRCUIT

Construct the circuit of figure S6.6 using L = 100 mH, C = 0.1 μF, and a potentiometer for R.



S6.6 - Series RLC circuit.

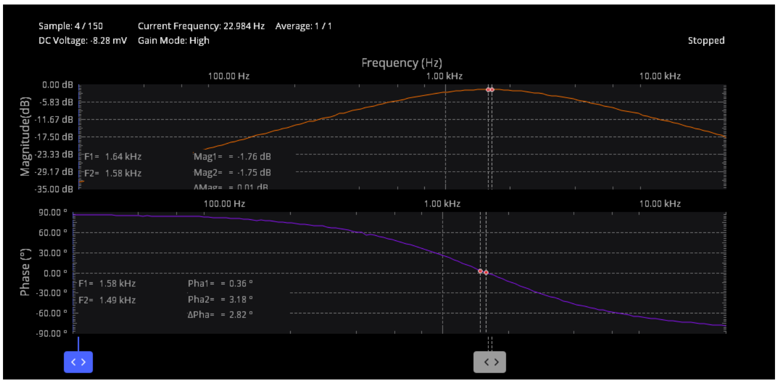
Copy the un-damped natural frequency ω_0 and R_{crit}, the critical circuit resistance from the prelim quiz. Show your working on how you obtained this value.

From the prelim quiz, the value of the un-damped natural frequency, ω_0 would be 10000.00 rad/s and the value of the critical circuit resistance would be 2000Ω.

The value of the un-damped natural frequency can be obtained by using the formula $\omega_0 = \frac{1}{\sqrt{LC}}$

The value of the critical circuit resistance can be obtained by using the formula $R_{crit} = 2 * \sqrt{\frac{L}{C}}$

6.4.5.1 Set R to R_{crit} (to do this, you may either: adjust your potentiometer, or replace your potentiometer with one/some fixed-value resistors). Examine your calculated amplitude of V_o. Test your circuit and check that the circuit response changes significantly around the natural frequency ω_0. This is important, so that the rest of your analysis in this lab makes sense. Plot both the amplitude- and phase- frequency responses of this new circuit.



The theoretical natural frequency in Hz can be found using the formula $\omega_0 = 2\pi f_0$. We can rewrite the equation into this form $f_0 = \frac{\omega 0}{2 * \pi}$.

$$f_0 = \frac{10000}{2 * \pi}$$

We will get f_0 to be 1591.55 Hz.

The measured natural frequency will be around 1.58kHz.

The magnitude of the voltage gain will be the highest at the frequency of Hz based on the above magnitude frequency response graph. This is near to the theoretical natural frequency that we have calculated from above. We can say that the circuit response will change significantly around the natural frequency, ω_0 .

Based on the above phase frequency response graph, we can see that the turning point of the phase shift is at around the frequency of 1.58kHz which is near to the theoretical frequency.

This slight difference could be due to the systematic error made by humans during the extraction of data from the graph. This could also be due to the inaccuracies of the graph by Scopy itself.

Are these responses as expected? What features from the graphs indicate this to you?

The responses are expected for the magnitude of voltage gain and phase gain of a RLC circuit.

As we can see from the graph above, the range of phase goes from -90 to 90 degrees. The peak is at 0 degrees, this is when the circuit purely resistive. The amplitude is also 1V due to there being no reactance. By using the voltage divider rule, the resistor will get all the voltage (1V). The range of phase is as such because when the frequency is low, the impedance of the capacitor is high, causing the circuit to become capacitive cause the phase of the resistor to reach 90 degrees. When the frequency is high, the impedance of the inductor is high, causing the circuit to become inductive cause the phase of the resistor to reach -90 degrees. The resistor phase depends on the current across the circuit, this is what causes this change in phase. Phase of the resistor will be the same as the phase of current. Current is calculated using Ohms Law, I = V/Ztotal.

For example, when the frequency is very low, the circuit is capacitive. Therefore, the phase of the total impedance would be -90. When calculating the current, taking an arbitrary magnitude V for voltage and Z for impedance magnitude, I = V∠0/Z∠ (-90) = (V/Z) ∠ (90). Since the phase of the resistor will be the same as the phase of current, this would explain why the phase of the current is 90 degrees on the far left of the graph (low frequency) (phase of resistor voltage is positive on the left side as it is more capacitive than inductive on the left, causing impedance phase to always be negative on the left of the peak, thus causing current phase/phase of voltage across resistor to be positive). When the frequency is very high, the circuit is inductive, therefore the phase of the total impedance would be 90.

When calculating for current, take an arbitrary magnitude V for voltage and Z for impedance magnitude, I = V∠0/Z∠ (90) = (V/Z) ∠ (-90). Since the phase of the resistor will be the same as the phase of current, this would explain why the phase of the current is -90 degrees on the far right of the graph (high frequency) (phase of resistor voltage is negative on the right side as it is more inductive than capacitive on the right, causing impedance phase to always be positive on the left of the peak, thus causing current phase/phase of voltage across resistor to be negative).

The graph is a normal distribution curve due to the circuit being capacitive on the left of the peak and inductive on the right of the peak. The previous circuit was an S shape due to it not containing an inductor. The previous circuit was from -90 to 0 degrees phase due to the absence of an inductor (because only the inductor causes the phase go from 0 to 90 degrees) and the previous circuit decreased from 1V in an S curve. This is because we were measuring the voltage across the capacitor. When the frequency is low, the impedance of the capacitor is very high. According to the voltage divider rule, it took up almost all of the voltage and therefore it was around 1V in amplitude.

On the left side of the peak, the magnitude drops. This is due to the increase in the capacitor impedance. On the left side of the peak, the frequency is decreasing past the undamped natural frequency, and a decrease in frequency past the undamped natural frequency results in an increase of capacitor impedance, causing the circuit to become more capacitive which causes the magnitude of total impedance to increase and less voltage to flow through the resistor as a result of the voltage divider rule. Therefore, causing the voltage to decrease as the frequency decreases below the natural undamped frequency.

On the right side of the peak, the magnitude drops. This is due to the increase in the inductor impedance. On the right side of the peak, the frequency is increasing past the undamped natural frequency, and a decrease in frequency past the undamped natural frequency results in an increase of inductor impedance, thus causing the circuit to become more inductive, and this causes the magnitude of total impedance to increase and causes less voltage to flow through the resistor as a result of the voltage divider rule. Therefore, causing the voltage to decrease as the frequency increases above the natural undamped frequency.

☐ CHECKPOINT: Get a demonstrator to check your answers, and initial here

6.4.5.2 Measure the magnitude and phase responses of the voltage "gain" transfer function over a range of frequencies from 20Hz to 20 kHz. The gain is V_o/V_i. Perform this with two different resistance values: (a) $2R_{crit}$ and (b) $0.5R_{crit}$.

2 Rcrit

(4000 Ohms)

0.5 Rcrit

(1000 Ohms)

Show your working and the values of the damping ratio ζ and the quality factor Q for each of the two resistance cases measured in 6.4.5.2

2 Rcrit

Using Rcrit of about 2000 Ohms

Given that a = $\frac{2*R_{crit}}{2*L}$ which equates to about 20000 and $\omega_0 = \frac{1}{\sqrt{LC}}$ which equates to about 10000

Damping ratio, ζ can be found with the following formula

$$\zeta = \frac{a}{\omega_0} = \frac{20000}{10000} = 2.0$$

The quality factor Q can be calculated using the formula

$$Q = \frac{\omega_0}{2*a} = \frac{10000}{20000} = 0.5$$

0.5 Rcrit

Given that a = $\frac{0.5 * R_{crit}}{2*L}$ which equates to about 5000 and $\omega_0 = \frac{1}{\sqrt{LC}}$ which equates to about 10000

The damping ratio can be calculated using the formula

$$\zeta = \frac{a}{\omega_0} = \frac{5000}{10000} = 0.5$$

The quality factor Q can be calculated using the formula

$$Q = \frac{\omega_0}{2*a} = \frac{10000}{2*5000} = 1$$

☐ CHECKPOINT: Get a demonstrator to check your answers, and initial here

6.4.5.3 FILTERING PROPERTIES OF SERIES RLC CIRCUITS

Do trend of the graphs plotted in 6.4.5.2 reflect the values for the damping ratio ζ and the quality factor Q that you calculated? How does the width of the observed features of the amplitude frequency response relate to these values?

The measurements plotted relates the values of the damping ratio to the quality factor. The relationship between the damping ratio and the quality factor is that the damping ratio is inversely proportional to the quality factor. When the damping ratio is higher than 1.0, then the circuit is overdamped, when the damping ratio is equal to 1.0, then the circuit is critically damped and when the damping ratio is lower than 1.0, the circuit is underdamped.

When the quality factor is between 0 and 0.5(non-inclusive), the circuit is overdamped. When the quality factor is equal to 1.0, the circuit is critically damped. When the damping ratio is higher than 0.5, the circuit is underdamped.

An overdamped circuit goes to steady state slowly without oscillations. An underdamped circuit goes to steady state faster than an overdamped circuit but slower than a critically damped circuit. A critically damped circuit is one that goes to steady state the fastest.

For R = 2Rcrit, the quality factor is < 0.5 and the damping ratio is > 1.0. This means that it is overdamped. When quality factor decreases, the bandwidth of the circuit increases. The bandwidth of the circuit relates to the width of the amplitude frequency responses. As the quality factor is lower than 0.5Rcrit, it will have a higher circuit bandwidth thus having a wider amplitude frequency response. This means that it has a wider range of frequency values at which the circuit will respond to. Therefore, the graph of the magnitude response of 2Rcrit is wider than the graph of 0.5Rcrit.

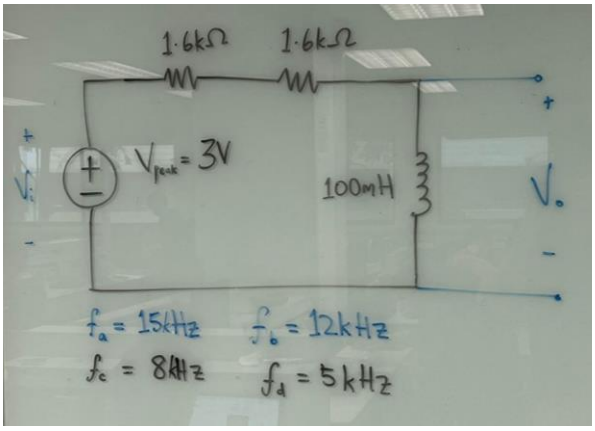
For R = 0.5Rcrit, the quality factor is > 0.5 and the damping ratio is < 1.0. This means that it is underdamped. When quality factor increases, the bandwidth of the circuit decreases. The bandwidth of the circuit relates to the width of the amplitude frequency responses. As the quality factor is higher than 2Rcrit, it will have a lower circuit bandwidth thus having a narrower amplitude frequency response. What it means is that it has a narrower range of frequency values at which the circuit will respond to. Therefore, the graph of the magnitude response of 0.5Rcrit is narrower than the graph of 2Rcrit.

The results from the Rcrit graph also agree with this as it has a quality factor of 0.5 which is between that of the quality factor of 2Rcrit and 0.5Rcrit. The bandwidth of the circuit and the width of the amplitude frequency response is in between that of 2Rcrit and 0.5Rcrit. Therefore, based on the visual observation the graph, the width of the graph of the magnitude response of Rcrit is between that of the graph of 2Rcrit and 0.5Rcrit.

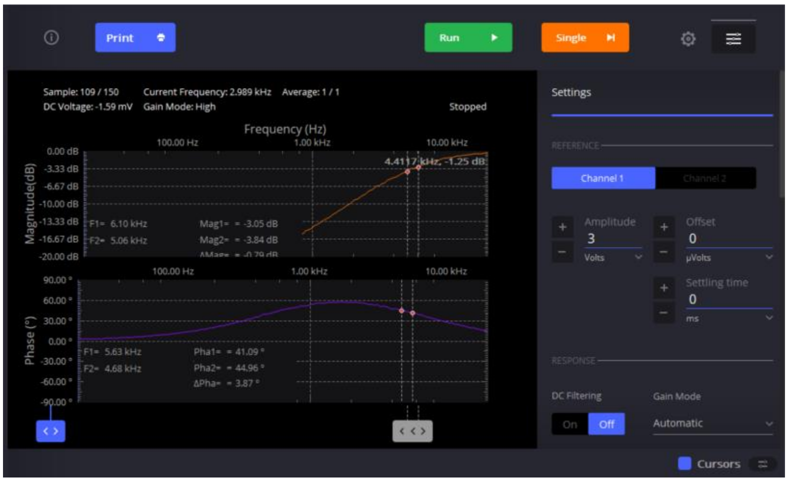
☐ CHECKPOINT: Get a demonstrator to check your answers, and initial here

LAB ASSESSMENT

1. Given Circuit



2. The break frequency measured would be 5092.96 Hz



The value is obtained from the graph above when the graph is 45° and the magnitude of voltage in terms of decibels is 3.01dB. We would get the value of 4.68kHz (from the phase) and 6.10kHz (from the magnitude of the voltage). We will find the average from both of the values which would equate to 5.39kHz.

3. The theoretical break frequency can be calculated using the formula

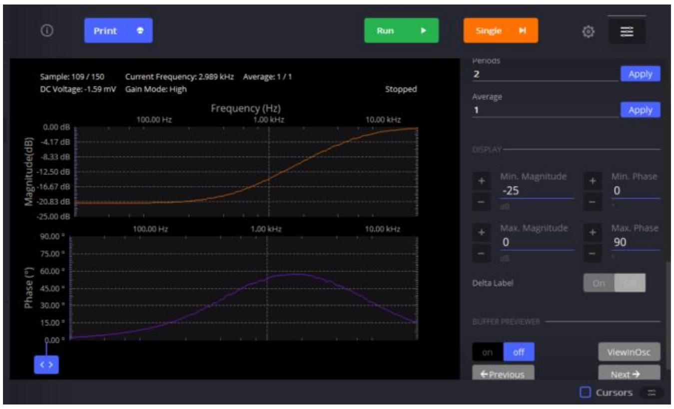
$$\omega_0 = \frac{1}{\tau} \ and \ \tau = \frac{L}{R}$$

We can find $\tau = \frac{100mH}{3200}$ which would equate to 3.125*10^-5. ω_0 would be equal to 32000 radians per second. From here, we can find our theoretical break frequency by using the formula $f_b = \frac{\omega_b}{2*\pi}$ which would equate to 5092.96 Hz.

4. Comparing the experimental and the theoretical break frequencies and commenting on the discrepancies/similarities.

There is a slight difference between the theoretical break frequencies and the experimental break frequencies. This is due to the systematic error made by human and the inaccuracies in reading made during the visual observation of the graph.

5. Explain the trend of the magnitude-frequency graph that you have obtain from Scopy.



When the frequency of the input voltage is high, the impedance of the inductor will be high. The inductor will become an open circuit. Since the impedance is directly proportional to the voltage, the voltage across the inductor will be high as well. When the frequency of the input voltage is low, the impedance of the inductor will be low. The inductor will become a short circuit. The voltage across the inductor will be low as well.

6. What type of filter is the circuit? Explain your answer

The filter will be a high-pass filter. This is because the graph has a slope that goes from a low magnitude to a high magnitude. It can also be seen that the high frequencies are filtered out while the low frequencies are not filtered out.

7. What is the gain of the output when frequency is fa.

Based on the graph above and the given value of fa (15kHz), we can see that when the frequency is 15kHz, the magnitude of the voltage will be -0.68dB. From here, we can use the equation $20 * log_{10}\left(\frac{V_{out}}{V_{in}}\right) = dB$ *and derive the equation* $Gain = 10^{\left(\frac{-0.68}{20}\right)}$. Our gain will equate to 0.925.

8. What is the magnitude of the output voltage when frequency of input is fb.

Based on the graph above, we can obtain the magnitude of the voltage in terms of decibels when the frequency is fb (12kHz) which would equate to -1.01dB. We can use the same equation that we have derived above which is $Gain = 10^{\left(\frac{-1.01}{20}\right)}$. Since Gain $= \frac{Vo}{Vin}$, we can multiply the value of the gain by the input voltage which is 3V. We will get 2.671 V.

9. What is phase difference between output and input when the input frequency is fc.

As we can see in the graph above, the value for the phase angle will be 32.68° when the value of the frequency is 8kHz

10. Write Vout(t) in sinusoidal form when input is fd.

First, we will have to get the value of V_{out} which can be found using the same formula above which is

$20 * log_{10}\left(\frac{V_{out}}{V_{in}}\right) = dB$. When the frequency will have a value of 5kHz, the magnitude of the voltage in terms of decibels will have a value of -3.94 V. From here, we can find the gain by deriving the formula into the following formula $Gain = 10^{\left(\frac{-3.94}{20}\right)}$. The gain would be 0.63533. The output voltage can be obtained 0.63533*3 = 1.906 V. From here, we would need to find the value of $\omega = 2 * \pi * f$ which would equate to 31415.93 rad/s. The phase angle obtained would be 43.81° when the frequency is 5kHz. From here, we can obtain Vout(t) = 1.906 sin (31415.93t + 43.81°) V.

11. Proving theoretically for question 8-part a

We can prove the answer for question 8-part a by taking the voltage divider rule. We will get

$$\frac{j * (\omega) * L}{j * \ (\omega) * L + R_{total}}$$

$$\frac{j * (2 * \pi * 15000) * 100 * 10^{-3}}{j * \ (2 * \pi * 15000) * 100 * 10^{-3} + 3200}$$

$$\frac{j * 9424.78}{j * \ 9424.78 + 3200}$$

We will get

$$0.89664$$

The theoretical value obtained from here is close to the measured value obtain from the question 8-part a. There is a slight difference due to the systematic error made by human and the inaccuracies in reading made during the visual observation of the graph.

ASSESSMENT

Student Statement:

I have read the university's statement on cheating and plagiarism, as described in the *Student Resource Guide*. This work is original and has not previously been submitted as part of another unit/subject/course. I have taken proper care safeguarding this work and made all reasonable effort to make sure it could not be copied. I understand the consequences for engaging in plagiarism as described in *Statue 4.1 Part III – Academic Misconduct*. **I certify that I have not plagiarized the work of others or engaged in collusion when preparing this submission.**

Student signature:___________Tan Jin Chun__________Date: 15 /4/ 2022

TOTAL: ________(/7)

ASSESSOR: ________