ENG1060: COMPUTING FOR ENGINEERS

Lab 9 – Week 10

2020 OCT NOV

Welcome to lab 9. Remember that laboratories continuously build on previously learned concepts and lab tasks. Therefore, it is crucial that you complete all previous labs before attempting the current one.

Self-study:
Students are expected to attempt these questions during their own self-study time, prior to this lab session. There may be questions that require functions not covered in the workshops. Remember to use MATLAB's built-in help for documentation and examples.

Learning outcomes:

1. To revise user-defined functions and apply good programming practices
2. To identify the mathematics that represents the problem to be solved
3. To summarise the requirements and limitations of each integration method
4. To apply numerical integration both by hand and with MATLAB to solve engineering problems

Background:
Numerical integration is ubiquitous in engineering as it can be used to evaluate the total amount or quantity of a given physical variable. Some integrated quantities include: displacement, velocity, volume, area centroids, work, and force. Note that it is not possible to analytically integrate all mathematical expressions. Hence, numerical integration is required to approximate solutions to such problems.

Primary workshops involved:

- Workshop 9: Numerical integration

Assessment:
This laboratory comprises **2.5%** of your final grade. The questions are designed to test your recollection of the workshop material and to build upon important programming skills. You will be assessed on the quality of your programming style as well as the results produced by your programs during your laboratory session by the demonstrators. Save your work in **m-files** named ***lab1t1.m***, ***lab2t2.m,*** etc. Inability to answer the demonstrator's questions will result in zero marks, at the demonstrator's discretion.

Team tasks begin at the start of the lab session so please ensure you arrive on time to form your groups. Students who arrive late will not be able to participate in the team tasks as teams will have already formed and will therefore forfeit all associated marks. These tasks will be assessed during class.

Lab submission instructions

Follow the instructions below while submitting your lab tasks.

Team tasks:
The team tasks are designed for students to test and demonstrate their understanding of the fundamental concepts specific to that lab. These tasks will occur at the start of the lab and will be assessed on the spot. Demonstrators will advise on how these will be conducted. Most team tasks do not require the use of MATLAB but MATLAB should be used for checking purposes.

Individual tasks:
The individual tasks are designed for students to apply the fundamentals covered in the team tasks in a variety of contexts. These tasks should be completed in separate m-files. There is typically one m-file per task unless the task requires an accompanying function file (lab 3 onwards). Label the files appropriately. E.g. lab6t1.m, lab6t2.m, eridium.m, etc.

Deadline:
The lab tasks are due next Friday at 9am (MYT) or 12pm (AEDT). Late submissions will not be accepted. Students will need to apply for special consideration after this time.

Submission:
Submit your lab tasks by:
1) Answering questions in Google Form, and
2) Submitting one .zip file which includes all individual tasks.

The lab .zip file submission links can be found on Moodle under the weekly sections, namely Post-class: Lab participation & submission. The submission box ("Laboratory 9") will only accept one .zip file. Zipping instructions are dependent on the OS you are using.

Your zip file should include the separate m-files for the individual tasks including function files.

It is good practice to download your own submission and check that the files you have uploaded are correct. Test run your m-files that you download. You are able to update your submission until the deadline. Any update to the submission after the deadline will be considered late.

Grade and feedback:
The team will endeavour to grade your lab files by Tuesday of the following week. Grades and feedback can be viewed through the Moodle Gradebook, which is available on the left side pane on the ENG1060 Moodle site.

 Important: If you are struggling with a task, ensure that you have performed hand-written work (e.g. hand calculations, pseudocode, flow charts) to better understand the processes involved. Do this before asking demonstrators for help and use it to assist with your illustration of the problem.

Lab 9 – Assessed questions

TASK 1

[2 MARKS – L09TE]

Note: Team tasks are designed for students to recall material that they should be familiar with through the workshops and practice of the individual questions prior to this lab session.

Students will be split into groups of 3-4 for the team tasks. Students in each group must explain aspects of the question below to receive the marks. Ensure that everyone has equal learning opportunities. Additionally, ask your table for help.

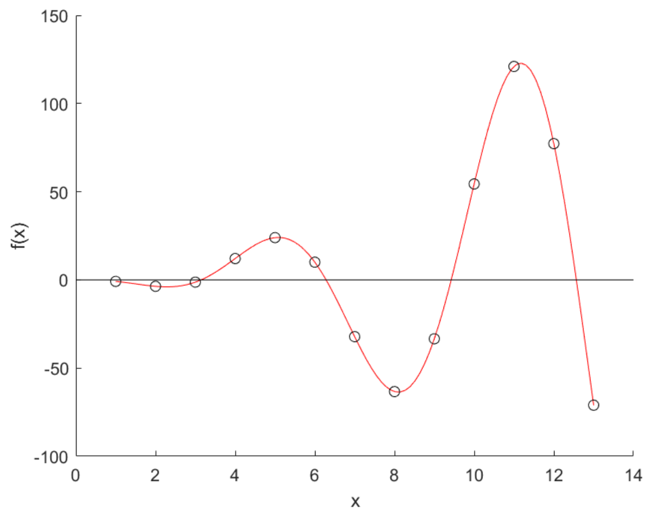
<u>Numerical integration:</u>

Each group will be assigned one of the following numerical integration rules.

- Trapezoidal rule
- Simpson's 1/3 rule
- Simpson's 3/8 rule

$$I_{\text{simp }3/8} = \frac{3h}{8}\left[f(x_1) + 3\left(\sum_{i=2,5,8\ldots}^{n-2} f(x_i)\right) + 3\left(\sum_{i=3,6,9\ldots}^{n-1} f(x_i)\right) + 2\left(\sum_{i=4,7,10\ldots}^{n-3} f(x_i)\right) + f(x_n)\right]$$

The following table contains the $f(x)$ values calculated at several x locations.



x	1	2	3	4	5	6	7
$f(x)$	-1	-4	-1	12	24	10	-32

x	8	9	10	11	12	13
$f(x)$	-63	-33	54	121	77	-71

Complete the following:

1. Copy the plot into the slide
2. Write the name of the numerical integration rule and the equations (single and composite versions)
3. Determine the points needed to perform <u>two</u> applications of the numerical integration rule over the entire range of x values. Colour in these circles on the plot. Note:
   - 1 application of the trapezoidal rule requires 2 points
   - 1 application of Simpson's 1/3 rule requires 3 points
   - 1 application of Simpson's 3/8 rule requires 4 points
4. State the polynomial order that the numerical integration rule uses between the points for each application and draw these as dashed lines using different colours for the two applications.
5. Use the single application rule to calculate the area of each application
6. Verify that the area obtained by the composite rule equates to the sum of areas in step 6
7. Discuss the task and explore any misunderstandings. Also, browse the work of other teams related to the other Codes and ensure that you have understood it as concepts from all sets may be required for the individual tasks.
8. Have a demonstrator assess your understanding

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Remember good programming practices for all tasks even if not specifically stated. This includes, but is not limited to:

- using clc, close all, and clear all, where appropriate
- suppressing outputs where appropriate
- labelling all plots, and providing a legend where appropriate
- fprintf statements containing relevant answers

PRELIMINARY

Create function files implementing each of the following numerical integration methods. This will help you consolidate your understanding of the techniques involved.

- Comp. trapezoidal rule: function I = comp_trap(f,a,b,n)
  function I = comp_trap_vector(x,y)
- Comp. Simpson's 1/3 rule: function I = comp_simp13(f,a,b,n)
  function I = comp_simp13_vector(x,y)
- Simpson's 3/8 rule: function I = comp_simp38(f,a,b,n)
  function I = comp_simp38_vector(x,y)

TASK 2

[4 MARKS]

Consider the following function:

$$I = \int_{-2}^{4} (1 - x - 4x^3 + 2x^5)\,\mathrm{d}x$$

The following table contains function values evaluated at several *x* points.

x	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
$f(x)$	-29	0.8125	4	1.9375	1	0.0625	-2	1.1875	31

x	2.5	3	3.5	4
$f(x)$	131.3125	376	876.4375	1789

Evaluate $f(x)$ for the following cases using **pen and paper**. Express your answers to 4 decimal places.

A. analytically
B. single application of the trapezoidal rule
C. composite trapezoidal rule using 4 segments
D. single application of the Simpson's 1/3 rule
E. composite application of the Simpson's 1/3 rule using 6 segments
F. single application of Simpson's 3/8 rule

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TASK 3

[4 MARKS]

A rocket's mass $m(t)$ is a function of time t because it decreases as it burns fuel. Newton's law states that the equation of motion for a rocket in flight is given by:

$$m(t)\,\frac{\mathrm{d}v}{\mathrm{d}t} = T - m(t)g$$

where T is the rocket's thrust, g is the gravitational acceleration. Hence, the velocity of the rocket can be calculated as:

$$v(t) = \int \frac{T - m(t)g}{m(t)}\,\mathrm{d}t$$

The mass follows the relationship:

$$m(t) = m_0\left(1 - \frac{rt}{b}\right)$$

where m_0 is the rocket's initial mass, b is the burn time, and r is the fraction of the total mass accounted for by the fuel.

Assume the following values: $T = 50000$ N, $m_0 = 2000$ kg, $r = 0.8$, $g = 9.81$ ms^{-2}, $b = 40$ s.

A. Plot the acceleration $\left(\frac{\mathrm{d}v}{\mathrm{d}t}\right)$ of the rocket from t = 0s to 40s as a blue continuous line.
B. Calculate the velocity of the rocket at burnout using
   a. trapezoidal rule with 99 points
   b. Simpson's 1/3 rule with 99 points
   c. MATLAB's integral() function

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TASK 4

[4 MARKS]

The outflow concentration from a reactor is measured at a number of times over a 24-hour period:

t (hr)	0	1	5.5	10	12	14	16	18	20	24
c(*t*) (mg/L)	1	1.5	2.3	2.1	4	5	5.5	5	3	1.2

The flow rate for the outflow in m^3/hr can be computed with the following equation:

$$Q(t) = 20 + 10\sin\left(\frac{2\pi}{24}(t - 10)\right)$$

The flow-weighted average concentration leaving the reactor over time *t* is given by

$$\bar{c}(t) = \frac{\int_0^t Q(t)c(t)\mathrm{d}t}{\int_0^t Q(t)\mathrm{d}t}$$

A. Create a figure with 2 subplot panels vertically stacked. In the top panel, plot *Q*(*t*)**c*(*t*) against *t* with blue circles and a continuous line. In the bottom panel, plot *Q*(*t*) against *t* with red squares and a continuous line.
B. Observe the data and determine which numerical integration methods to use and for which segments to achieve the least amount of error. Hence, determine the flow-weighted averaged concentration after 24 hours using the trapezoidal, Simpson's 1/3 and Simpson's 3/8 rules. Use Simpson's 1/3 at lower values of *t* and Simpson's 3/8 at higher values of *t*. Write an fprintf statement for the flow-weighted average.
C. Determine the maximum *Q*(*t*)**c*(*t*) value and the time that it occurs. Print these values through an fprintf statement.

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TASK 5

[4 MARKS]

An inverted conical water tank is filled with water and a farmer needs to empty the tank by pumping all water out over the top. The total work (Nm) required to pump the water can be calculated as

$$W = \int_0^{h_t} hF dh$$

where h is the height of water (m), h_t is the height of the tank and F is the force exerted on the water per meter (N/m), which is represented by the expression

$$F = g\rho\pi r^2$$

where $g = 9.81\ m/s^2$, $\rho = 1000\ kg/m^3$ and r represents the cross-sectional radius of the tank (m), and can be calculated through

$$r = \frac{r_t}{h_t}(h_t - h)$$

where r_t is the radius at the top of the tank.

Assume that the water tank has a height of 12 meters and a radius of 2 meters at the top of the tank.

A. Plot the force exerted hF (N) against the height of the water h (m)
B. Use the integral() function to calculate the total work required to pump the water, W_{int}. Use fprintf() to print this answer.
C. Determine the minimum number of points required for the composite Simpson's 1/3 rule, W_{sim}, to achieve a percentage error of 0.01% or less compared to the answer in part B for the total work required. Use fprintf() to print the number of points required and comment on the error achieved. State if this was expected and state why.

Note: Percentage error is calculate as $\left|\frac{W_{int} - W_{sim}}{W_{int}}\right| \times 100\%$.

2 marks deducted for poor programming practices (missing comments, unnecessary outputs, no axis labels, inefficient coding, etc.)

END OF ASSESSED QUESTIONS

The remainder of this document contains supplementary and exam-type questions for extended learning. Use your allocated lab time wisely!

 Important: If you are struggling with a task, ensure that you have performed hand-written work (e.g. hand calculations, pseudocode, flow charts) to better understand the processes involved. Do this before asking demonstrators for help and use it to assist with your illustration of the problem.

Lab 9 – Supplementary questions

These questions are provided for your additional learning and are not assessed in any way. You may find some of these questions challenging and may need to seek and examine functions that are not taught in this unit. Remember to use the help documentation. Coded solutions will not be provided on Moodle. Ask your demonstrators or use the discussion board to discuss any issues you are encountering.

TASK 1S

Use the trapezoidal rule to calculate the integral of the quadratic function $x^2 - 3x + 2$ between x=1 and x=3. Experiment with different number of points and check your answers against the exact answer.

SOLUTION

```
Precision = 1E-05:
I_trapz(448 segments) = 0.6666733099
I_analytical = 0.6666666667
Percentage error = 9.9649E-04%
```

TASK 2S

The function

$$f(x) = e^{-x}$$

can be used to generate the following table of unequally spaced data:

x	0	0.1	0.3	0.5	0.7	0.95	1.2
f(*x*)	1	0.9048	0.7408	0.6065	0.4966	0.3867	0.3012

Evaluate the integral from *a*=0 to *b*=1.2

A. analytically
B. by applying the trapezoidal rule once
C. using a combination of the trapezoidal rule and Simpson's rules (single segments) to attain the highest accuracy. Check this by computing the percentage error.

SOLUTION

 Important: If you are struggling with a task, ensure that you have performed hand-written work (e.g. hand calculations, pseudocode, flow charts) to better understand the processes involved. Do this before asking demonstrators for help and use it to assist with your illustration of the problem.

```
I_analytical = 0.6988057881
I_trap = 0.7807165271
I_optimal = 0.6988973739
error = 0.0131%
```

TASK 3S

The luminous efficiency (ratio of the energy in the visible spectrum to the total energy) of a black body radiator may be expressed as a percentage by the formula

$$E = 64.77T^{-4}\int_{4\times10^{-5}}^{7\times10^{-5}} x^{-5}\left(e^{1.432/Tx}-1\right)^{-1}\mathrm{d}x$$

where T is the absolute temperature in degrees Kelvin, x is the wavelength in cm, and the range of integration is over the visible spectrum.

Taking T=3500 Kelvin, compute the integral using the Simpson's 1/3 rule for 11 points, 21 points and 31 points. Compare your results with the value obtained using the quad() function by calculating the percentage error.

SOLUTION

```
I_quad = 14.5126625573
n = 11pts, I_simp13 = 14.5127246104, error = 4.276E-04%
n = 21pts, I_simp13 = 14.5126666781, error = 2.839E-05%
n = 31pts, I_simp13 = 14.5126636471, error = 7.509E-06%
```

TASK 4S

To estimate the amount of water that flows in a river during a year, a section of the river is made to have a rectangular cross section as shown below. In the beginning of every month (starting at January 1st) the height h of the water and the speed v of the water flow are measured. The first day of measurement is taken as 1, and the last day which is January 1st of the next year is day 366. The following data was measured:

Day	1	32	60	91	121	152	182	213	244	274	305	335	366
h (m)	2.0	2.1	2.3	2.4	3.0	2.9	2.7	2.6	2.5	2.3	2.2	2.1	2.0
v (m/s)	2.0	2.2	2.5	2.7	5.0	4.7	4.1	3.8	3.7	2.8	2.5	2.3	2.0

Use the data to calculate the volume flow rate, then integrate the volume flow rate to obtain an estimate of the total amount of water that flowed in the river during the year. Apply applications of a single trapezoid between each data point.

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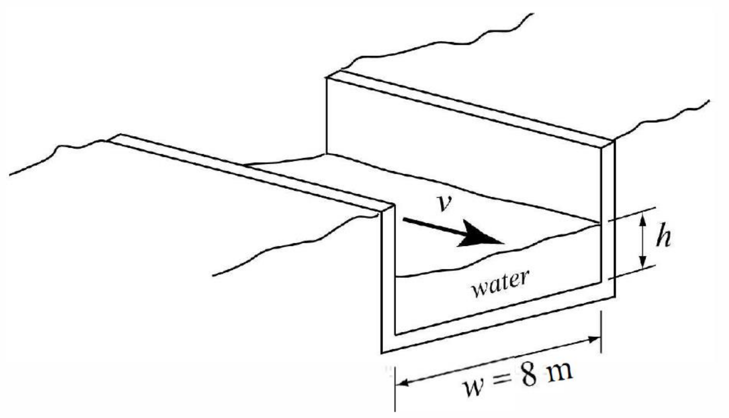

SOLUTION

```
23506.36 m^3 of water passed the river this year.
```

[10] **Important:** If you are struggling with a task, ensure that you have performed hand-written work (e.g. hand calculations, pseudocode, flow charts) to better understand the processes involved. Do this before asking demonstrators for help and use it to assist with your illustration of the problem.