Tutorial 2

1. Rewrite each statement below in "if – then" form.
   i) q I am on time for lecture if I catch the 7 am bus. p
   ii) ~p David studies hard or he fails the examination. q $\sim p \vee q$: $p \rightarrow q$
   iii) p The program is readable only if it is well structured. q
   iv) q This door will not open unless a security code is entered. ~p
   v) p $2x - 5 = 11$ implies $x = 8$. q

i)If i can catch the 7 am bus then, i am on time for lecture. ✓ ✓
ii) If David did not study hard, then he will fail the examination. ✓
Iii)If the program is readable, then it is well structured. ✓
iv)If the security code is not entered then, the door will not open.
v) If 2x-5=11, then x=8. ✓

- Alternatively, $p \rightarrow q$ can also said as
  i) if p then q
  ii) p is sufficient for q
  iii) p is a sufficient condition for q
  iv) p only if q
  v) q is necessary for p
  vi) q is a necessary condition for p
  vii) q if p
  viii) q unless $\sim p$

$$p \rightarrow q \equiv \sim p \vee q$$

If p and q are statement variables, the conditional of q by p is "if p then q" or "p implies q"

vi) Having two 45° angles is a sufficient condition for this triangle to be a right triangle.

vii) Solving all tutorial's questions is a necessary condition for Alan to pass this subject.

viii) To be a citizen in this country, it is sufficient that you were born in this country.

ix) It is necessary to have a valid password to log on to the server.

vi)If a triangle is having 2 45° angles, then it is a right triangle.

vii) If Alan wants to pass this subject then he has to solve all tutorial's questions.

viii)If you born in this country, then you are a citizen in this country.

ix)If you want to log on to the server, then you need to have a valid password.

2. Using truth tables, verify the following,

i) $(p \vee q) \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$

ii) $(p \wedge (p \rightarrow q)) \rightarrow q \equiv t$

iii) $(p \rightarrow q) \wedge (\sim p \rightarrow q) \wedge \sim q \equiv c$

2 $(p \vee q) \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$

i)

p	q	r	$(p \vee q)$	$(p \vee q) \rightarrow r$	$(p \rightarrow r)$	$(q \rightarrow r)$	$(p \rightarrow r) \wedge (q \rightarrow r)$
0	0	0	0	1	1	1	1
0	0	1	0	1	1	1	1
0	1	0	1	0	1	0	0
0	1	1	1	1	1	1	1
1	0	0	1	0	0	1	0
1	0	1	1	1	1	1	1
1	1	0	1	0	0	0	0
1	1	1	1	1	1	1	1

equal

ii) $(p \wedge (p \rightarrow q)) \rightarrow q \equiv t$

equal

p	q	$p \rightarrow q$	$p \wedge (p \rightarrow q)$	$(p \wedge (p \rightarrow q)) \rightarrow q$	t
0	0	1	0	1	1
0	1	1	0	1	1
1	0	0	0	~~0~~ 1	1
1	1	1	1	1	1

iii) $(p \rightarrow q) \wedge (\sim p \rightarrow q) \wedge \sim q \equiv c$

equal

p	q	$(p \rightarrow q)$	$\sim p$	$(\sim p \rightarrow q)$	$\sim q$	$(p \rightarrow q) \wedge (\sim p \rightarrow q)$	$(p \rightarrow q) \wedge (\sim p \rightarrow q) \wedge \sim q$	c
0	0	1	1	0	1	0	0	0
0	1	1	1	1	0	1	0	0
1	0	0	0	1	1	0	0	0
1	1	1	0	1	0	1	0	0

$\sim(p \to q) \equiv \sim(\sim p \vee q)$
$\equiv p \wedge \sim q$

3. Write negations for each of the following statements.

i) If $\overset{p}{(P \text{ is a square})}$, then $\overset{q}{(\text{it is a rectangle.})}$ $p \to q$

$p \to (q \vee r)$ ii) If $\overset{p}{(\text{the sun is shining})}$, then (I shall play $\overset{q}{\text{tennis}}$) or $\overset{r}{(\text{swimming})}$ this afternoon.

$(p \wedge q) \to r$ iii) If $\overset{p}{(\text{I am free})}$ and $\overset{q}{(\text{I am not tired})}$, then $\overset{r}{(\text{I will go to the supermarket.})}$

iv) If $\underset{p}{(x = 17)}$ or $\underset{q}{(x^3 = 8)}$, then $\underset{r}{(x \text{ is prime.})}$ $(p \vee q) \to r$

i) P is a square and is not a rectangle ✓

ii) ~~Even if~~ the sun is shining, ~~then~~ and I shall not play tennis or swimming this afternoon ✓

iii) ~~If~~ I am ~~not~~ free and I am not tired, ~~then~~ but I will not go to the supermarket ✓

iv) ~~If~~ x=17 or x^3=8, and the x is not prime ✓

ii) $\sim(p \to (q \vee r)) \equiv \sim(\sim p \vee (q \vee r)) \equiv p \wedge \sim(q \vee r)$

iii) $\sim((p \wedge q) \to r) \equiv \sim(\sim(p \wedge q) \vee r) \equiv (p \wedge q) \wedge \sim r$

iv) $\sim((p \vee q) \to r) \equiv \sim(\sim(p \vee q) \vee r) \equiv (p \vee q) \wedge \sim r$

4. State the converses, inverses and contrapositives for each of the following implications.

i) If I am late, then I will not take the train to work. $p \to q$

ii) If I have enough money, then I will buy a car and I will buy a house. $p \to (q \wedge r)$

iii) A positive integer is a prime only if it has no divisors other than 1 and itself.

iv) If x is nonnegative, then x is positive or x is 0. $p \to (q \vee r)$

The converse of $p \to q$ (if p then q) is $\boxed{q \to p}$ (if q then p).

The inverse of $p \to q$ (if p then q) is $\boxed{\sim p \to \sim q}$ (if $\sim p$ then $\sim q$).

The contrapositive of a conditional statement of the form "if p then q" is "if $\boxed{\sim q \text{ then } \sim p}$".

4i. If I am late, then I will not take the train to work.

- Converse: If I do not take the train to work, then I am late.
- Inverse: If I am not late, then I will take the train to work.
- Contrapositive: If I take the train to work, then I am not late.

4ii. If I have enough money, then I will buy a car and I will buy a house.

- Converse: If I will buy a car and I will buy a house, then I have enough money. $(q \wedge r) \to p$
- Inverse: If I will not have enough money, then I will not buy a car and I will not buy a house. $\sim p \to \sim(q \wedge r)$
- Contrapositive: If I will not buy a car and I will not buy a house, then I will not have enough money. $\sim(q \wedge r) \to \sim p$

4iii. A positive integer is a prime only if it has no divisors other than 1 and itself.

- Converse: If a positive integer has no divisors other than 1 and itself, then it is a ~~positive integer is a~~ prime.
- Inverse: If A positive integer is not a prime ~~only if~~ then it has divisors other than 1 and itself.
- Contrapositive: If a positive integer has divisors other than 1 and itself, then it is not a ~~positive integer is~~ a prime.

4iv. If x is nonnegative, then x is positive or x is 0.

- Converse: If x is positive or x is 0, then x is nonnegative.
- Inverse: If x is negative, then x is not positive or x is not 0.
- Contrapositive: If x is not positive or x is not 0, then x is negative.

$p \rightarrow q \equiv {\sim}q \rightarrow {\sim}p$ ✓

$\not\equiv q \rightarrow p$

$\not\equiv {\sim}p \rightarrow {\sim}q$

$\equiv {\sim}p \vee q$

5. "If Jim studies hard, then he will pass his final examination." Assuming that this statement is true, which of the following must also be true?

p q $p \rightarrow q$

i) Jim passed his final examination implies he studies hard. : q implies p ($q \rightarrow p$)

ii) p Jim studied hard $\boxed{\text{or}}$ he failed his final examination. ${\sim}q$

iii) Jim will fail his final examination only if he does not study hard. ${\sim}p$; ${\sim}q$ only if ${\sim}p$

$\equiv {\sim}q \rightarrow {\sim}p$

5) If Jim studies hard, then he will pass his final examination

Let p: Jim studies hard ✓

q: Jim pass final exam ✓

$p \rightarrow q$ ✓

i) $q \rightarrow p$ false ✓

ii) $p \vee {\sim}q$ ✓ false ✓

iii) $p \rightarrow q$ True ✓

$\equiv {\sim}q \rightarrow {\sim}p$

Alternatively, $p \rightarrow q$ can also said as

i) if p then q

ii) p is sufficient for q

iii) p is a sufficient condition for q

iv) p only if q

v) q is necessary for p

vi) q is a necessary condition for p

vii) q if p

viii) q unless ${\sim}p$

5. "If Jim studies hard, then he will pass his final examination." Assuming that this statement is true, which of the following must also be true?

iv) (Jim will fail his final examination) unless (he studied hard.) : $\sim q$ unless p

v) A necessary condition for Jim to pass his final examination is that he studied hard. : p is necessary for q

vi) Studying hard is sufficient for Jim to pass his final examination. : p is suff. for q ✓

Let p: Jim studies hard ✓

q: Jim will pass his final examination ✓

p -> q ✓

(iv) ~q -> p (False) ✓

(v) p is a necessary condition for q (q -> p) (False) ✓

(vi) p is sufficient for q (p -> q) (True) ✓

Alternatively, $\boxed{p \to q}$ can also said as

i) if p then q

ii) p is $\boxed{\text{sufficient for}}$ q

iii) p is a sufficient condition for q

iv) p only if q

v) q is $\boxed{\text{necessary for}}$ p ✓

vi) q is a necessary condition for p

vii) q if p

viii) q $\boxed{\text{unless}}$ $\sim p$

6. Given $p \to q \equiv \sim p \vee q$ and $p \leftrightarrow q \equiv (\sim p \vee q) \wedge (\sim q \vee p)$.
Thus, rewrite the following statement form without using $\to$ or $\leftrightarrow$.
i) $p \wedge \sim q \to r$ ii) $(p \to (q \to r)) \leftrightarrow ((p \wedge q) \to r)$

6 i) $p \wedge \sim q \to r$

$p \wedge \sim q \to r \equiv (p \wedge \sim q) \to r$ ✓

$\equiv \sim(p \wedge \sim q) \vee r$

$\equiv \sim p \vee q \vee r$ ✓

ii) $(p \to (q \to r)) \leftrightarrow ((p \wedge q) \to r)$

$(p \to (q \to r)) \leftrightarrow ((p \wedge q) \to r) \equiv (\sim p \vee (q \to r)) \leftrightarrow (\sim (p \wedge q) \vee r)$

$\equiv (\sim p \vee \sim q \vee r) \leftrightarrow (\sim p \vee \sim q \vee r)$ ✓

Let $a \equiv \sim p \vee \sim q \vee r$

$\equiv [\sim(\sim p \vee \sim q \vee r) \vee (\sim (p \wedge q) \vee r)] \wedge [\sim(\sim(p \wedge q) \vee r) \vee (\sim p \vee \sim q \vee r)]$ ✓

$\equiv [\sim(\sim p \vee \sim q \vee r) \vee (\sim p \vee \sim q \vee r)] \wedge [\sim(\sim p \vee \sim q \vee r) \vee (\sim p \vee \sim q \vee r)]$ ✓

$(\sim a \vee a) \wedge (\sim a \vee a)$

$\equiv t \wedge t$

$\equiv t$

$\equiv \sim(\sim p \vee \sim q \vee r) \vee c \vee (\sim p \vee \sim q \vee r)$

$\equiv \sim(\sim p \vee \sim q \vee r) \vee (\sim p \vee \sim q \vee r) \vee c$

$\equiv t \vee c$

$\equiv t$

7. Obtain the PDNF and PCNF of each of the following expressions:

i) $A \equiv \sim(p \vee q)$ ii) $\sim(p \wedge q)$

PDNF: $\bar{p}\bar{q} + 0 + \ldots$

Q7. i. $\sim(p \vee q) \equiv \sim p \wedge \sim q$ ✓

PDNF of A $\equiv \bar{p}\,\bar{q}$ ✓ $\xrightarrow{\text{negate}}$ PCNF of $\sim A \equiv p + q$

~~negation~~

PCNF of A $\equiv (p \vee q) \wedge (\sim p \vee q) \wedge (p \vee \sim q)$ remaining

$\equiv (\bar{p} + \bar{q})(\bar{p} + q)(p + \bar{q})$

ii. $\sim(p \wedge q) \equiv \sim p \vee \sim q$ ✓ PCNF of A

negation

PCNF of A $\equiv (\bar{p} + \bar{q})$ ✓

PDNF $\equiv (p \wedge q) \vee (\sim p \wedge q) \vee (p \wedge \sim q)$

$\equiv (p + q)(\bar{p} + q)(p + \bar{q})$

$\boxed{2^2 = 4}$

PCNF	PDNF
$p+q$	pq ✓
$\bar{p}+q$	$\bar{p}q$
$p+\bar{q}$	$p\bar{q}$
$\bar{p}+\bar{q}$	$\bar{p}\bar{q}$

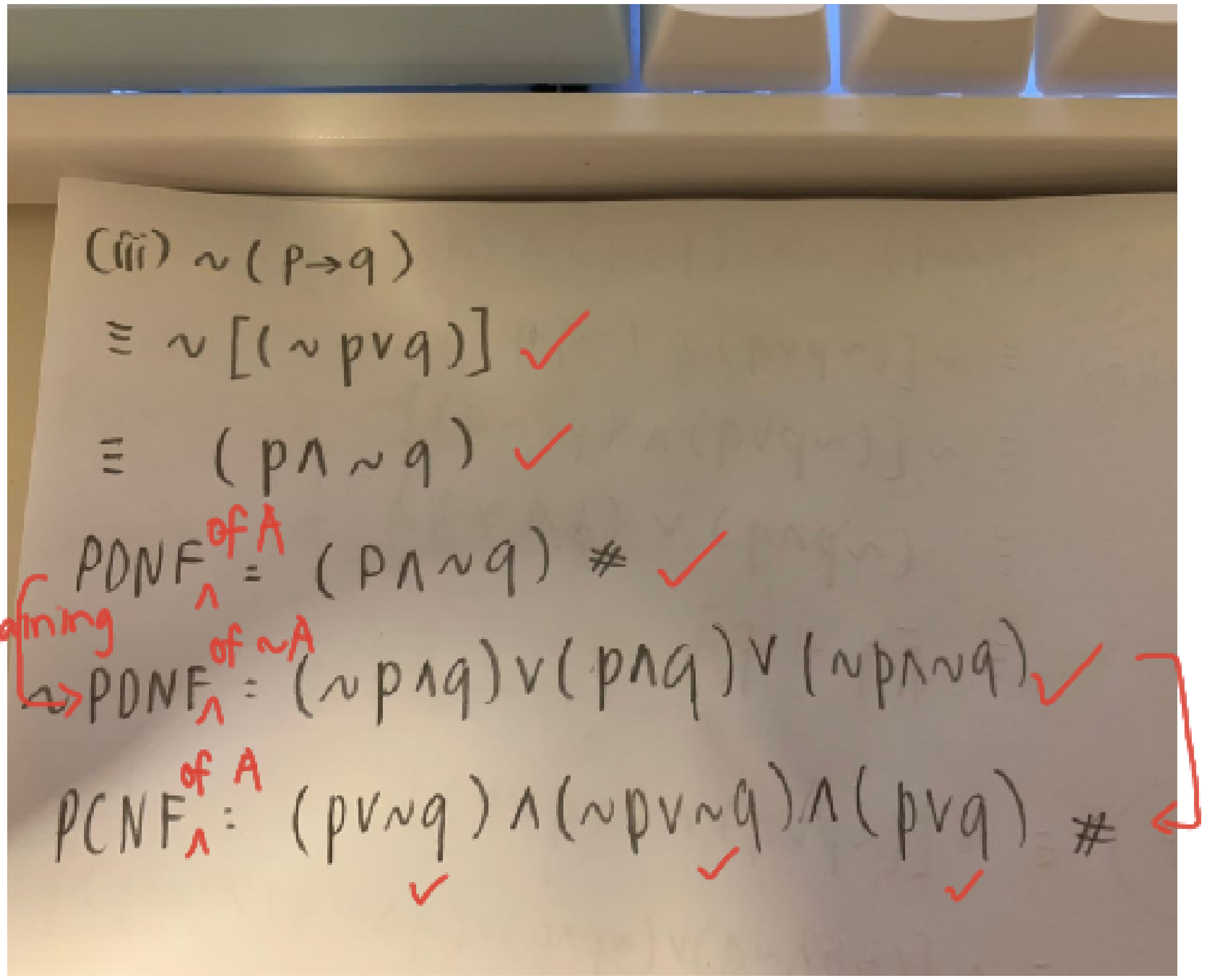
PDNF of $\sim A \equiv p \times q \equiv pq$ ✓ or $(p \wedge q)$ ✓

remaining

PDNF of A $\equiv \bar{p}q + p\bar{q} + \bar{p}\bar{q}$ or $(\sim p \wedge q) \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q)$

7. Obtain the PDNF and PCNF of each of the following expressions:

iii) $\sim(p \rightarrow q)$ iv) $\sim(p \leftrightarrow q)$



(iii) $\sim(p \rightarrow q)$

$\equiv \sim[(\sim p \vee q)]$

$\equiv (p \wedge \sim q)$

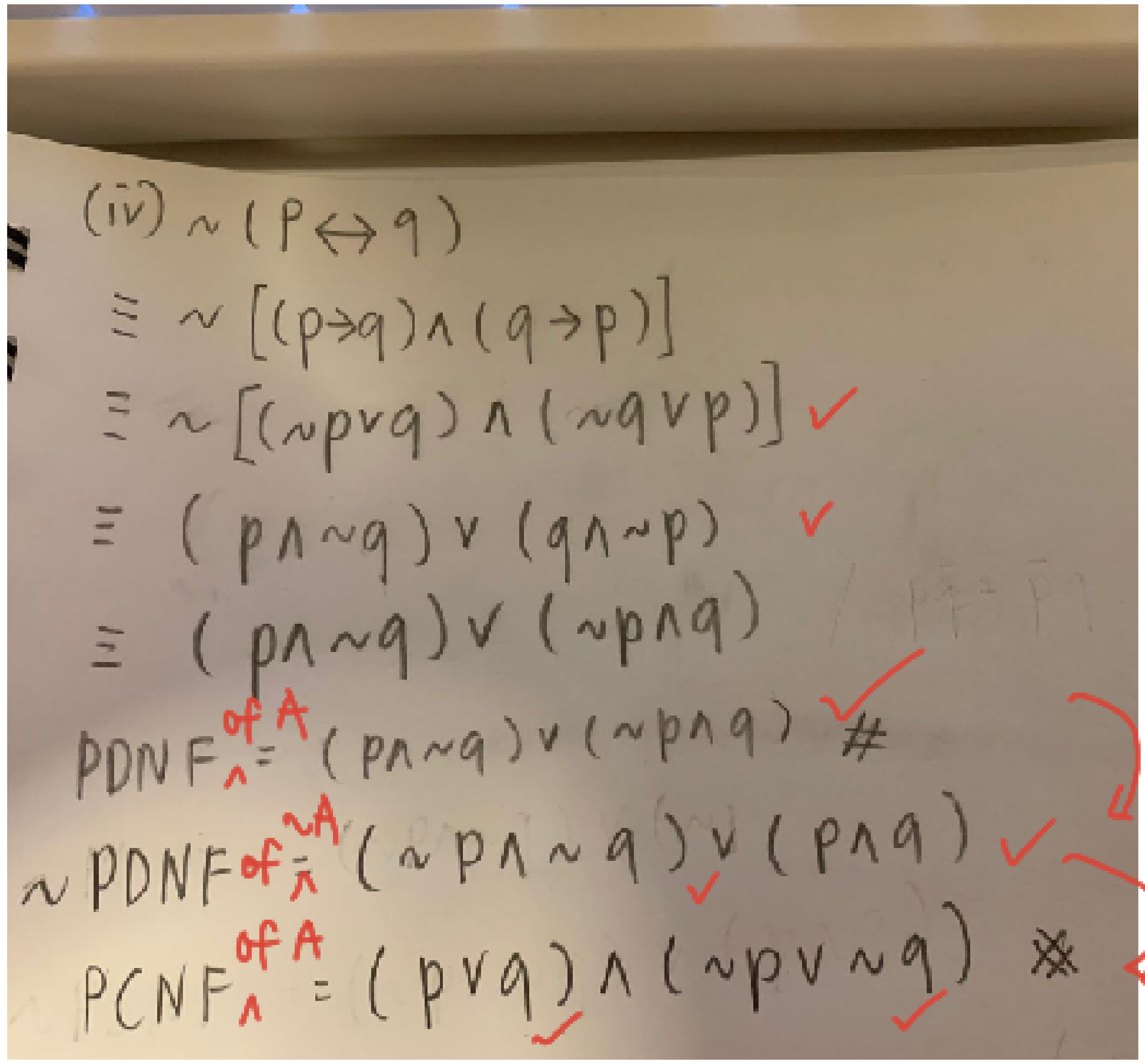
PDNF of A $= (p \wedge \sim q)$ #

remaining

$\sim$PDNF of $\sim$A $: (\sim p \wedge q) \vee (p \wedge q) \vee (\sim p \wedge \sim q)$

negate

PCNF of A $: (p \vee \sim q) \wedge (\sim p \vee \sim q) \wedge (p \vee q)$ #



(iv) $\sim(p \leftrightarrow q)$

$\equiv \sim[(p \rightarrow q) \wedge (q \rightarrow p)]$

$\equiv \sim[(\sim p \vee q) \wedge (\sim q \vee p)]$

$\equiv (p \wedge \sim q) \vee (q \wedge \sim p)$

$\equiv (p \wedge \sim q) \vee (\sim p \wedge q)$

PDNF of A $= (p \wedge \sim q) \vee (\sim p \wedge q)$ #

remaining

$\sim$PDNF of $\sim$A $= (\sim p \wedge \sim q) \vee (p \wedge q)$

negate

PCNF of A $= (p \vee q) \wedge (\sim p \vee \sim q)$ #

8. Construct a truth table for the expression $A \equiv (p \rightarrow q) \wedge (\sim p \vee r)$. Based on the truth table, write the PDNF of A, the PDNF of $\sim A$, the PCNF of A, and the PCNF of $\sim A$.

8.

p	q	r	$p \rightarrow q$	$\sim p$	$\sim p \vee r$	$(p \rightarrow q) \wedge (\sim p \vee r)$		
0	0	0	1	1	1	1	minterm	$\bar{p}\bar{q}\bar{r}$
0	0	1	1	1	1	1		$\bar{p}\bar{q}r$
0	1	0	1	1	1	1		$\bar{p}q\bar{r}$
0	1	1	1	1	1	1		$\bar{p}qr$
1	0	0	0	0	0	0	maxterm	$\bar{p}+q+r$
1	0	1	0	0	1	0		$\bar{p}+q+\bar{r}$
1	1	0	1	0	0	0		$\bar{p}+\bar{q}+r$
1	1	1	1	0	1	1		pqr

or

$\sim p \wedge \sim q \wedge \sim r$

$\sim p \wedge \sim q \wedge r$

$\vdots$

negate

PDNF of A

$= (\sim p \wedge \sim q \wedge \sim r) \vee (\sim p \wedge \sim q \wedge r) \vee (\sim p \wedge q \wedge \sim r)$
$\vee (\sim p \wedge q \wedge r) \vee (p \wedge q \wedge r)$

negate

PCNF of A

$= (p \wedge \sim q \wedge \sim r) \wedge (p \wedge \sim q \wedge r) \wedge (p \wedge q \wedge \sim r)$

PCNF of A

$\equiv (\sim p \vee q \vee r) \wedge (\sim p \vee q \vee \sim r) \wedge (\sim p \vee \sim q \vee r)$ or $(\bar{p}+q+r)(\bar{p}+q+\bar{r})(\bar{p}+\bar{q}+r)$

PDNF of ~A

$= (p \wedge q \wedge r) \vee (p \wedge q \wedge \sim r) \vee (p \wedge \sim q \wedge r) \vee$
$(p \wedge \sim q \wedge \sim r) \vee (\sim p \wedge \sim q \wedge \sim r)$

PDNF of ~A

$\equiv (p \wedge \sim q \wedge \sim r) \vee (p \wedge \sim q \wedge r) \vee (p \wedge q \wedge \sim r)$

PCNF of ~A

$= (\sim p \wedge q \wedge r) \wedge (\sim p \wedge q \wedge \sim r) \wedge (\sim p \wedge \sim q \wedge r)$

PCNF of ~A

$\equiv (p \vee q \vee r) \wedge (p \vee q \vee \sim r) \wedge (p \vee \sim q \vee r)$
$\wedge (p \vee \sim q \vee \sim r) \wedge (\sim p \vee \sim q \vee \sim r)$

$p q,\ \bar{p} q,\ p\bar{q},\ \bar{p}\bar{q}$

9. Without constructing truth tables, obtain the PDNF of A, the PDNF of $\sim A$, the PCNF of A, and the PCNF of $\sim A$, (in any order), if the normal forms exist.

i) $A \equiv q \wedge (p \vee \sim q)$ ii) $A \equiv (\sim p \vee \sim q) \rightarrow (p \leftrightarrow \sim q)$

iii) $A \equiv p \rightarrow [p \wedge (q \rightarrow p)]$ iv) $A \equiv (q \rightarrow p) \wedge (\sim p \wedge q)$

9 i)

A q /\ (p V ~q)

= (q /\ p) V (q /\ ~q)

(q /\ p) V c

= pq ✓

PDNF of A = pq ✓ negate

negate PDNF of ~A = (~p)(~q) + p(~q) + (~p)q ✓

PCNF of A = (p + q) x (~p + q) x (p + ~q) ✓

PCNF of ~A = ~p+~q ✓

p	q	p V ~q	q /\ (p V ~q)
0	0	1	0
0	1	0	0
1	0	1	0
1	1	1	1

missing factor

missing factor

$[p \vee \underbrace{(q \vee \sim q)} \wedge (p \vee q)] \vee [(p \vee q) \wedge q \vee (p \vee \sim p)]$

$\equiv [(p \vee q) \vee (p \vee \sim q) \wedge (p \vee q)] \vee [(p \vee q) \wedge (q \vee p) \vee (q \vee \sim p)]$

$\equiv p \vee q$

$a \vee \sim a \equiv t$

9 ii)

A = (~p V ~q) -> (p <-> ~q)

= (~p V ~q) -> [(~p V ~q) /\ (q V p)] ✓

= ~(~p V ~q) V [(~p V ~q) /\ (q V p)] ✓

= (p /\ q) V [(~p V ~q) /\ (q V p)] ✓ $: pq + [(\bar{p}+\bar{q})(p+q)]$

$t \wedge [(p \wedge q) \vee (q \vee p)]$ = [(p /\ q) V (~p V ~q)] /\ [(p /\ q) V (q V p)] ✓

$\bar{p}p \equiv c \equiv 0$

$\bar{q}q \equiv c \equiv 0$

= (p /\ q) V (p V q) ✓ "$\sim(p \wedge q)$"

= [(p /\ q) V p] V q $[(p \wedge q) \vee q]$

= ~~p V q~~ $[(p \vee p) \wedge (p \vee q)] \vee (p \vee q) \wedge (q \vee q)$

$[p \wedge (p \vee q)] \vee [(p \vee q) \wedge q]$

PDNF of A = p q + ~p q + p ~q

PDNFof ~A = ~p~q

PCNF of A = p + q

PCNF of ~A = (~p + ~q) x (~p + q) x (p + ~q)

9(ii)

ii) $A \equiv (\sim p \vee \sim q) \rightarrow (p \leftrightarrow \sim q)$

$A \equiv \sim(\sim p \vee \sim q) \vee (p \leftrightarrow \sim q)$

$\equiv (p \wedge q) \vee [(p \rightarrow \sim q) \wedge (\sim q \rightarrow p)]$

$\equiv (p \wedge q) \vee [(\sim p \vee \sim q) \wedge (q \vee p)]$ ✓

$\equiv pq + [(\bar{p} + \bar{q})(p + q)]$

$\equiv pq + \underbrace{\bar{p}p}_{\equiv C} + \bar{p}q + p\bar{q} + \underbrace{\bar{q}q}_{\equiv C}$

$\equiv pq + \bar{p}q + p\bar{q}$ (PDNF of A)

PDNF of $\sim A \equiv \bar{p}\bar{q}$

PCNF of $A \equiv p + q$ negate

PCNF of $\sim A \equiv (\bar{p} + \bar{q})(p + q)(\bar{p} + q)$

9 ii)

A = (~p V ~q) -> (p <-> ~q)

= (~p V ~q) -> [(~p V ~q) /\ (q V p)]

= ~(~p V ~q) V [(~p V ~q) /\ (q V p)]

= (p /\ q) V [(~p V ~q) /\ (q V p)] ✓

= [(p /\ q) V (~p V ~q)] /\ [(p /\ q) V (q V p)] ✓

= (p /\ q) V (p V q) ✓

= ((p /\ q) V p) V [(p /\ q) V q] ✓

= [(p V p) /\ (p V q)] V [(p V q) /\ (q V q)] ✓

= (p V q) V (p V q)

= p V q

PDNF of A = p q + ~p q + p ~q ✓

PDNFof ~A = ~p~q ✓

PCNF of A = p + q ✓

PCNF of ~A = (~p + ~q) x (~p + q) x (p + ~q) ✓

9. iii) $A \equiv p \to [p \wedge (q \to p)]$

$\equiv \sim p \vee [p \wedge (\sim q \vee p)]$

$\equiv \sim p \vee [(p \wedge \sim q) \vee (p \wedge p)]$ $\equiv p$

$\equiv \sim p \vee p \vee (p \wedge \sim q)$

$\equiv t \vee (p \wedge \sim q)$

$\equiv t$ ✓

PCNF of $A \equiv t$ (no possible maxterms) ✓

PCNF of $\sim A \equiv (p \vee q) \wedge (\sim p \vee q) \wedge (p \vee \sim q) \wedge (\sim p \vee \sim q)$ ✓

PDNF of $\sim A \equiv c$ (no possible ~~maxterms~~ minterms)

PDNF of $A \equiv (\sim p \wedge \sim q) \vee (p \wedge \sim q) \vee (\sim p \wedge q) \vee (p \wedge q)$ ✓

mint 1 1 1 1

maxt. 0 X

iv) $A \equiv (q \to p) \wedge (\sim p \wedge q)$

$\equiv (\sim q \vee p) \wedge (\sim p \wedge q)$

$\equiv (p \vee \sim q) \wedge (\sim p \wedge q)$ ✓

$\equiv \sim(\sim p \wedge q) \wedge (\sim p \wedge q)$

$\equiv c$ ✓

PDNF of $A \equiv c$ ✓ (no possible ~~max~~ min terms)

PDNF of $\sim A \equiv (\sim p \wedge \sim q) \vee (p \wedge \sim q) \vee (\sim p \wedge q) \vee (p \wedge q)$ ✓

negate

PCNF of $A \equiv (p \vee q) \wedge (\sim p \vee q) \wedge (p \vee \sim q) \wedge (\sim p \vee \sim q)$ ✓

PCNF of $\sim A \equiv t$ ✓ (no possible maxterms) ✓

0 0 0 0

minterms 1 X