STUDENT'S DECLARATION OF ORIGINALITY

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	Marks /50	Mark /100
Section 1 (40%)		
Section 2 (10%)		
Total		

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AAMS3184 Discrete Mathematics

SECTION 1

Q1. Using the laws of Algebra of propositions, simplify $(p \to q) \to ((p \to r) \to (p \to (q \wedge r)))$.

$$
\begin{aligned}
& (p \to q) \to ((p \to r) \to (p \to (q \wedge r))) \\
\equiv\ & \sim(p \to q) \vee ((p \to r) \to (p \to (q \wedge r))) \\
\equiv\ & \sim(p \to q) \vee (\sim(p \to r) \vee (p \to (q \wedge r))) \\
\equiv\ & \sim(\sim p \vee q) \vee \sim(p \to r) \vee (p \to (q \wedge r)) \\
\equiv\ & \sim(\sim p \vee q) \vee \sim(\sim p \vee r) \vee (\sim p \vee (q \wedge r)) \\
\equiv\ & (p \wedge \sim q) \vee (p \wedge \sim r) \vee (\sim p \vee (q \wedge r)) \\
\equiv\ & [p \wedge (\sim q \vee \sim r)] \vee [\sim p \vee (q \wedge r)] \\
\equiv\ & [p \wedge (\sim q \vee \sim r)] \vee \sim[p \wedge (\sim q \vee \sim r)] \\
\equiv\ & t
\end{aligned}
$$

$\therefore$ It is using negation law, the answer is t.

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Question 2

Q2. Let set $S = \{-15, -9, -6, 0, 3, 6, 10, 21, 36\}$. Determine whether each of the following statements is true or false. Prove those true statements and provide counterexample(s) for the false statements.

Q2.(a) $\forall x \in S$, x is divisble by 3

Counterexample: Let $x = 10$, $S(x) = \frac{x}{3}$

$S(10) = \frac{10}{3}$

$= 3.3333$ (false)

$\therefore$ ~~$\forall x \in S$ is false~~.

$\therefore$ The statement is false.

Q2.(b) $\forall x \in S$, all the ten digit of x is odd.

Counterexample: Let $x = 21$, the ~~the~~ ten digit of 21 is even.

$\therefore$ The statement is false.

Q2.(c) $\exists x \in S, \exists y \in S$ such that $[(x + 2y = 23) \wedge (11x - y = 23)]$.

~~Proof~~ $11x - y = 23$, $x + 2y = 23$

$\cancel{y = 11}$ $\quad$ $\cancel{x = 23 - 2y}$

$y = 11x - 23$ $\quad$ $x + 2(11x - 23) = 23$

$y = 11(3) - 23$ $\quad$ $x + 22x - 46 + 23$

$y = 10$ $\quad$ $23x = 69$

$x = 3$

Proof: Let $x = 3$, $y = 10$

$x + 2y = 3 + 2(10)$
$= 23$

~~$11x - y = 11(3) + 2(10)$~~ $\quad$ $11x - y = 11(3) - 10$
$= 23$

$\therefore$ since the value 3 and 10 are part of set S.

The statement is true.

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Question 3

Q3. Let $x \in \{0,1\}$ and $y \in \{3,4\}$. Consider the predicates $A(x): 2x^2 > x+1$ and $B(y)$: y is even. Rewrite the expression $\exists x \forall y [A(x) \rightarrow B(y)]$ by eliminating the symbol, $\rightarrow$ and quantifiers. Hence, determine ~~is~~ its truth value.

Let set $M = \{0,1\}$ $\quad$ $A(x) = 2x^2 > x+1$

set $N = \{3,4\}$ $\quad$ $B(y)$: y is even

$$\exists x \forall y [A(x) \rightarrow B(y)] \equiv \exists x \forall y [\sim A(x) \vee B(x)]$$

$\therefore$ There exists a $x \in M$ for all $y \in N$ such that $2x^2 > x+1$ or y is even.

~~$[\sim A(0) \vee B(3)] \wedge [$~~

$[\sim A(0) \vee B(3)] \wedge [\sim A(0) \vee B(4)] \;\vee\; [\sim A(1) \vee B(3)] \wedge [\sim A(1) \vee B(4)]$

$\equiv [(T \vee F) \wedge (T \vee T)] \;\vee\; [(T \vee F) \wedge (T \vee T)]$

$\equiv (T \wedge T) \vee (T \wedge T)$

$\equiv T$

$\therefore$ The expression is true.

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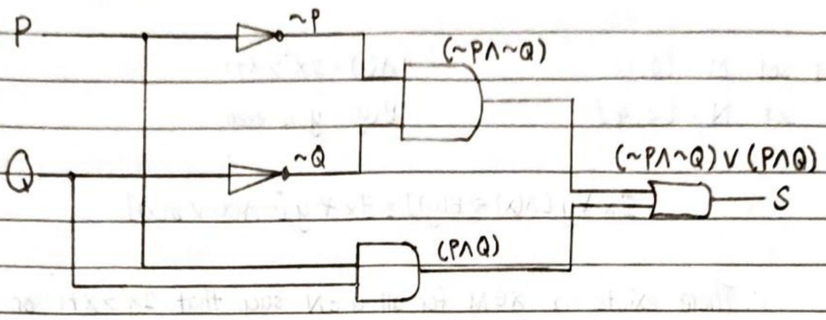
Question 4

Q4. Construct an input/output table and find the Boolean expression that corresponds to the circuit below.



~~S ≡ (~P∧~Q) ∨ (P∧~~

$S \equiv (\sim P \wedge \sim Q) \vee (P \wedge Q)$

Input		output
P	Q	S
0	0	1
0	1	0
1	0	0
1	1	1

Boolean expression

$S \equiv \bar{p}\bar{q} + pq$

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Question 5

Q5. Use the Euclidean algorithm to find the greatest common divisor and the least common multiple of ~~a~~ $a = 20220$ and $b = 238$. Find s and t such that $d = sa + tb$ where $s, t \in \mathbb{Z}$.

$a = 20220$, $b = 238$

$$20220 = 238(84) + 228 \qquad \gcd(20220, 238) = \gcd(238, 228)$$
$$238 = 228(1) + 10 \qquad = \gcd(228, 10)$$
$$228 = 10(22) + 8 \qquad = \gcd(10, 8)$$
$$10 = 8(1) + 2 \qquad = \gcd(8, 2)$$
$$8 = 2(4) + 0 \qquad = \gcd(2, 0)$$

$$\gcd(20220, 238) = 2$$

$$\cancel{\text{tt}}\ \text{LCM}(20220, 238) = \frac{20220 \times 238}{\gcd(20220, 238)}$$
$$= \frac{20220 \times 238}{2}$$
$$= 2{,}406{,}180$$

$$2 = 10 - 8(1)$$
$$= 10 - [228 - 10(22)]$$
$$= 23(10) - 228$$
$$= 23[238 - 228(1)] - 228$$
$$\cancel{= 23(238) - 23(224) - 228}$$
$$= 23(238) - 23(228) - 228$$
$$= 23(238) - 24(228)$$
$$= 23(238) - 24[20220 - 238(84)]$$
$$= 23(238) - 24(20220) + \cancel{2016(84)}\ 2016(238)$$
$$= -24(20220) + 2039(238)$$

$$\therefore s = -24, \ t = 2039$$

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Question 6

Q6. Prove that the sum of four consecutive integers is even.

Proof:

suppose $n, n+1, n+2,$ and $n+3$ are particular but arbitrarily chosen consecutive integer.

By the parity property, n is either odd or even.

Case 1: If n is even

By definition of even, $n = 2k$, $k \in \mathbb{Z}$

Then, $(2k)+(2k+1)+(2k+2)+(2k+3)$

$= 8k+6$

$= 2(4k+3)$

$= 2m$, $m = 4k+3$, $m \in \mathbb{Z}$

since the sum of integers is an integer, $4k+3$ is an integer and hence by definition of even, $n+(n+1)+(n+2)+(n+3) = 2m$ is an even integer.

Therefore, if n is even, then $n+(n+1)+(n+2)+(n+3)$ is even.

Case 2: If n is odd

By definition of odd, $n = 2k+1$, $k \in \mathbb{Z}$

Then, $(2k+1)+[(2k+1)+1]+[(2k+1)+2]+[(2k+1)+3]$

$= 8k+10$

$= 2(4k+5)$

$= 2l$, $l = 4k+5$, $l \in \mathbb{Z}$

since the sum of integers is an integer, $4k+5$ is an integer and hence by definition of even, $n+(n+1)+(n+2)+(n+3) = 2l$ is an even integer.

Therefore if n is odd, then $n+(n+1)+(n+2)+(n+3)$ is even.

$\therefore$ Regardless of which case actually occurs, either n is odd number or even number, the ~~other~~ sum of four consecutive integers is even.

Question 7

Q7. Let set $S = \{a,b,c,d,e,f\}$ and
$R = \{(a,a),(a,b),(a,c),(a,e),(b,a),(b,b),(b,c),(c,b),(c,c),(c,d),(d,e),(e,f)\}$ be a relation on the set S.

Q7.(a) Find the domain and range of the relation R.

$DOM(R) = \{a,b,c,d,e\}$
$Ran(R) = \{a,b,c,d,e,f\}$

Q7.(b) Find the in-degree and out-degree of each vertex:

vertex	a	b	c	d	e	f
In-degree	2	3	3	1	2	1
out-degree	4	3	3	1	1	0

$$\begin{array}{c|cccccc} & a & b & c & d & e & f \\ \hline a & 1 & 1 & 1 & 0 & 1 & 0 \\ b & 1 & 1 & 1 & 0 & 0 & 0 \\ c & 0 & 1 & 1 & 1 & 0 & 0 \\ d & 0 & 0 & 0 & 0 & 1 & 0 \\ e & 0 & 0 & 0 & 0 & 0 & 1 \\ f & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

Q7.(C) Determine whether the relation R is reflexive, irreflexive, symmetric, asymmetric, antisymmetric or transitive. Give a counterexample if the answer is "No".

R is not reflexive since $(d,d) \notin R$
R is not irreflexive since $(a,a) \in R$
R is not symmetric since $(a,c) \in R$ but $(c,a) \notin R$
R is not asymmetric since (a,b) and $(b,a) \in R$
R is not antisymmetric since (a,b) and $(b,a) \in R$, and $a \neq b$
R is not transitive since (b,c) and $(c,d) \in R$, but $(b,d) \notin R$