Chi-Square Test - Qualitative variable (test the distribution based on freq distribution)

1. Goodness of fit test - 1 variable (m classes), 1 group

df/v = m - 1, Ei = np where p is/are from the claimed distribution (H0

E.g., Test whether the distribution of grade for DCS students is the same as students from FOAS which is 20%A, 30%B, 20%C, and 30%F. Claimed dist: 20%A. 30%B, 20%C, 30%F (must be stated in H0)

2. Independence test - 2 variables (R row categories, C column categories), 1 group

df/v = (R - 1)(C - 1), Eij = row total x column total / grand total

not related/not associated/ no r/s Ho

E.g., Test whether the gender and grade are independent among the students in TarUC.

3. Homogeneity test - 1 variable (R categories), 2 or more groups (C groups)

df/v = (R - 1)(C - 1), Eij = row total x column total / grand total

Ho

E.g. Test whether the distribution of gender is the same between students from FOCS and FOAS.

* If > 20% of the Ei/Eij (cells) are < 5, combine the groups or increase the sample size is necessary.

* If df/v = 1, Yate's correction has to be applied.

Q1. A die is thrown 132 times with the following results:

Number	1	2	3	4	5	6
Frequency	16	20	25	14	29	28

Test the hypothesis that the die is unbiased at the 5% significance level.

Shen Hoi

OR

fair

$\frac{1}{6} = P_1 = P_2 = P_3 = P_4 = P_5 = P_6$

Q1) H_0 : The die is unbiased (claim)

H_1 = The die is biased

At $\alpha = 0.05$, $v = 6 - 1 = 5$, critical value $= \chi^2_{0.05;5} = 11.070$

Critical region: $\chi^2 > 11.070$

Number	1	2	3	4	5	6	Total
O_i	16	20	25	14	29	28	132
E_i			$np = (132)(\frac{1}{6}) = 22$				132

$$\chi^2 = \sum_{i=1}^{6} \frac{(O_i - E_i)^2}{E_i} = \frac{(16-22)^2}{22} + \frac{(20-22)^2}{22} + \frac{(25-22)^2}{22} + \frac{(14-22)^2}{22} + \frac{(29-22)^2}{22} + \frac{(28-22)^2}{22}$$

$= 9$

Since $\chi^2 = 9 < 11.070$, failed to reject H_0 at $\alpha = 0.05$ and we can conclude that the die is unbiased.

Jun Dian

OK.

Q2. An engineer wishes to investigate whether the eight similar machines having the same performance or not. To test this, he runs an experiment and the actual number of items produced by these machines per hour period is given in the following table.

Machine	1	2	3	4	5	6	7	8
Total items produced / hour	8	7	6	9	10	8	6	10

Test the hypothesis that the eight machines have equal performance using $\alpha = 0.01$.

2. H_0 : The eight machines have equal performance ✓

H_1 : The eight machines do not equal performance

At $\alpha = 0.01$, $v = m-1 = 7$, critical value $= \chi^2_{0.01,7} = 18.475$ ✓

Critical region : $\chi^2 > 18.475$ ✓

$n = 64$, $p = \frac{1}{8}$

$E_i = 64 \times \frac{1}{8}$

$= 8$

$$\chi^2 = \sum_{i=1}^{8} \frac{(O_i - E_i)^2}{E_i} = \frac{(8-8)^2}{8} + \frac{(7-8)^2}{8} + \frac{(6-8)^2}{8} + \frac{(9-8)^2}{8} + \frac{(10-8)^2}{8} + \frac{(8-8)^2}{8} + \frac{(6-8)^2}{8} + \frac{(10-8)^2}{8}$$

$= 2.25$ ✓

Since $\chi^2 = 2.25 < 18.475$, failed to reject H_0 at $\alpha = 0.01$ ✓ and we can conclude that the eight machines have equal performance ✓

Shee Yeap

OK.

Q3. M&M's plain chocolate candies come in six different colours: brown, yellow, red, orange, green, and tan. According to the manufacturer (Mars, Inc.), the colour ratio in each large production batch is 30% brown, 20% yellow, 20% red, 10% orange, 10% green, and 10% tan. To test this claim, a researcher had count the colours of M&Ms found in "fun size" bags of the candy. The results are displayed in the table.

Colour	Brown	Yellow	Red	Orange	Green	Tan
No. of M&M's	84	79	75	49	36	47

Conduct a test to determine whether the true percentages of the colours produced differ from the manufacturer's stated percentages. Use $\alpha = 0.02$.

$m=6$

$n=370$

③ H_0: The ratio of each large production batch of brown, yellow, red, orange, green and tan are 3:2:2:1:1:1

H_1: The ratio of each large production batch is different from that specified in H_0

At $\alpha=0.02$, $v=m-1=5$, critical value $=\chi^2_{0.02;5}=13.388$

critical region $=\chi^2>13.388$

	Brown	Yellow	Red	Orange	Green	Tan	Total
O_i	84	79	75	49	36	47	370
Proportion, P_i	$\frac{3}{10}$	$\frac{2}{10}$	$\frac{2}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	1
$E_i=np$	111	74	74	37	37	37	370

$$\chi^2=\sum_{i=1}^{6}\frac{(O_i-E_i)^2}{E_i}=\frac{(84-111)^2}{111}+\frac{(79-74)^2}{74}+\frac{(75-74)^2}{74}+\frac{(49-37)^2}{37}+\frac{(36-37)^2}{37}+\frac{(47-37)^2}{37}=13.5405$$

$\therefore$ Since $\chi^2=13.5405>13.388$, reject H_0 at $\alpha=0.02$. We can conclude that the true percentage of the colours produced is significantly differ from the manufacturer's stated percentage.

Q4. In the past, 30% of the televisions sold by a store were small-screen, 40% were medium-screen, and 30% were large-screen. The manager wishes to determine whether the recent sales still following the past pattern of sales or not. To do this, he takes a random sample of 100 recent purchases and finds that 24 were small-screen, 37 were medium-screen, and 39 were large-screen. Test at the 2.5% level of significance, whether the recent sales follow the past pattern of sales.

Aaron

OK.

H_0 : The recent sales follow the past pattern of sales and the distribution as follows: 30% small screen, 40% medium screen, 30% large screen.

H_1 : The ~~survey~~ sample ~~that~~ does not follow the past pattern of sales

At $\alpha = 0.025$, $v = 3-1 = 2$, critical value $= \chi^2_{0.025;2} = 7.378$

critical region $= \chi^2 > 7.378$

	small	medium	large	total
O_i	24	37	39	100
E_i	$100 \times 30\% = 30$	$100 \times 40\% = 40$	$100 \times 30\% = 30$	100

$$\chi^2 = \sum_{i=1}^{3} \frac{(O_i - E_i)^2}{E_i}$$

$$= \frac{(24-30)^2}{30} + \frac{(37-40)^2}{40} + \frac{(39-30)^2}{30}$$

$$= 1.2 + 0.225 + 2.7$$

$$= 4.125$$

Since $\chi^2 = 4.125 < 7.378$, it is failed to reject H_0 at $\alpha = 0.025$.

Thus, the sample ~~is~~ follows the past pattern of sales.

Q5. Four coins are tossed 100 times. The numbers of heads obtained on each toss were recorded and are summarized below.

Y: no of heads when a fair coin is tossed 4 times

$Y \sim B(4, 0.5)$ fair ✓ not fair ✗

Li Yuet

Number of heads Y	0	1	2	3	4
Frequency	5	23	39	19	14

Using a 2.5% significance level, test the hypothesis that all four coins are fair.

Q5) H_0 : ~~The~~ the All ~~four~~ four coins are fair ✓

H_1 : ~~The~~ Not all four coins are ~~not~~ fair.

$\alpha = 0.025$, Critical value $\chi^2_{0.025;3} = 11.143$ ✓

$v = m-1$, Critical region : $\chi^2 > 11.143$ ✓

$= 4$

$Y \sim B(4, 0.5)$ 0 1 2 3 4

χ^2

O_i 5 23 39 19 14

E_i

P_i ${}^4C_0 0.5^0 0.5^4$ ${}^4C_1 0.5^1 0.5^3$ ${}^4C_2 0.5^2 0.5^2$ ${}^4C_3 0.5^3 0.5^1$ ${}^4C_4 0.5^4 0.5^0$

$$\chi^2 = \sum_{i=1}^{5} \frac{(O_i - E_i)^2}{E_i} = \frac{(5-20)^2}{20} + \frac{(23-20)^2}{20} + \frac{(39-20)^2}{20} + \frac{(19-20)^2}{20} + \frac{(14-20)^2}{20}$$

$= 11.25 + 0.45 + 18.05 + 0.05 + 1.8$

$= 31.6$

Since $\chi^2 = 31.6 > 11.143$, reject H_0 at $\alpha = 0.025$ and we can conclude that the all four coins are not fair.

Calon-calon mesti menggunakan kedua-dua belah muka.

H_0: A✓, B✓, C✓, D✓

opposite

H_1: not the case in H_0

- A✓ B✓ C✓ D✗
- A✓ B✓ C✗ D✓
- $\boxed{\text{A✗ B✗ C✗ D✗}}$
- A✗ B✓ C✗ D✓

H_1 ⋮

⇓

NOT all 4 coins are fair.

Correction Q5

...GAH KEBANGSAAN SERI INDAH
43300 SERI KEMBANGAN
SELANGOR DARUL EHSAN

Nama : ________ Angka Giliran : ________ Kp : ________

Tingkatan : ________ Perkara : ________ Tarikh : ________

Jangan tulis ap... di ruan...

8c) H_0 : All the four coins are fair

H_1 : Not all the four coins are fair

$\alpha = 0.025$, $V = 5-1 = 4$, critical value $\chi^2_{0.025;3} = 11.143$, critical region $= \chi^2 > 11.143$

Y	0	1	2	3	4
O_i	5	23	39	19	4
E_i	$0.0625 \times 100 = 6.25$	$0.25 \times 100 = 25$	$0.0375 \times 100 = 37.5$	$0.25 \times 100 = 25$	$0.0625 \times 100 = 6.25$
P_i	${}^4C_0(0.5)^0(0.5)^4 = 0.0625$	${}^4C_1(0.5)^1(0.5)^3$	${}^4C_2(0.5)^2(0.5)^2$	${}^4C_3(0.5)^3(0.5)^1$	${}^4C_4(0.5)^4(0.5)^0$
		$= 0.25$	$= 0.0375$	$= 0.25$	$= 0.0625$

$$\chi^2 = \sum_{i=1}^{5} \frac{(O_i - E_i)^2}{E_i} = \frac{(5-6.25)^2}{6.25} + \frac{(23-25)^2}{25} + \frac{(39-37.5)^2}{37.5} + \frac{(19-25)^2}{25} + \frac{(4-6.25)^2}{6.25}$$

$$= 0.25 + 0.16 + 0.06 + 1.44 + 9.61$$

$$= 11.52$$

Since $\chi^2 = 11.52 > 11.143$, reject H_0 at $\alpha = 0.025$ and we can conclude that Not all four coins are fair.

Q6. 1,000 flights from a major airport were classified according to intensity of bookings and type of flights. The results were summarized in the following table.

Intensity of bookings	Type of Flights		
	Internal	Regional	International
Fully booked	154	171	275
Not fully booked	96	79	225

Is there any evidence of significant association between the type of flights and the intensity of bookings? Use 1% significance level.

Cai Jie

OK.

H_1

H_0: There is no association....

H_1: There is an association...

Q6. Observed Result:

Intensity of booking	Type of Flights			
	Internal	Regional	International	total
Fully booked	154	171	275	600
Not fully booked	96	79	225	400
total	250	250	500	1000

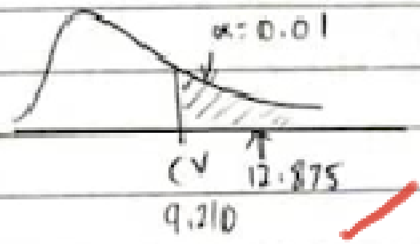
Expected Result:

Intensity of booking	Type of Flights			
	Internal	Regional	International	total
Fully booked	150	150	300	600
Not fully booked	100	100	200	400
total	250	250	500	1000

H_0: ~~Not associated~~ between the type of flight and the intensity of booking

H_1: associated between the type of flight and the intensity of booking.

$v = (R-1)(C-1)$
$= (2-1)(3-1)$
$= 1(2)$
$= 2$



critical value $= \chi^2_{0.01,2}$
$= 9.210$

critical region: $\chi^2 > 9.210$

Erc

$E_{1,1} = \frac{600(250)}{1000} = 150$ $E_{1,2} = \frac{600(250)}{1000} = 150$ $E_{1,3} = \frac{600(500)}{1000} = 300$

$E_{2,1} = \frac{250(400)}{1000} = 100$ $E_{2,2} = \frac{(400)(250)}{1000} = 100$ $E_{2,3} = \frac{400(500)}{1000} = 200$

$\chi^2_c = \sum \frac{(O_i - E_i)^2}{E} = \frac{(154-150)^2}{150} + \frac{(171-150)^2}{150} + \frac{(275-300)^2}{300} + \frac{(96-100)^2}{100} + \frac{(79-100)^2}{100} + \frac{(225-200)^2}{200}$
$= 0.1067 + 2.94 + 2.0833 + 0.16 + 4.41 + 3.125$
$= 12.825$

Since $\chi^2_c = 12.825 > 9.210$, H_0 is rejected at $\alpha = 0.05$.
There is a sufficient evidence of the association between the type of flight and the intensity of booking.

Mei Yi

OK

Q7. The machines in a factory were classified according to the observed degree of defectiveness over the previous year's operations, as in the following table:

% defective, d	Type of Machines		
	Cutting machine	Grinding machine	Milling machine
$d \le 1$	22	74	102
$1 < d < 2$	31	102	143
$d > 2$	7	64	55

Is there any evidence to show that the degree of defectiveness is independent from the types of machine? Use $\alpha = 0.02$.

Q7 H_0 : The degree of defectiveness is ~~not~~ independent from the types of machine.

H_1 : The degree of defectiveness is not independent from the types of machine.

At $\alpha = 0.02$, $v = (3-1)(3-1) = 4$, critical value : $\chi^2_{0.02;4} = 11.668$

critical region : $\chi^2 > 11.668$

O_{ij} (E_{ij})	Cutting machine	Grinding machine	Milling machine	Total
$d \le 1$	22 (19.8)	74 (79.2)	102 (99)	198
$1 < d < 2$	31 (27.6)	102 (110.4)	143 (138)	276
$d \ge 2$	7 (12.6)	64 (50.4)	55 (63)	126
	60	240	300	600

$E_{11} = \frac{198(60)}{600} = 19.8$ $\quad E_{12} = \frac{198(240)}{600} = 79.2$ $\quad E_{13} = \frac{198(300)}{600} = 99$

$E_{21} = \frac{276(60)}{600} = 27.6$ $\quad E_{22} = \frac{276(240)}{600} = 110.4$ $\quad E_{23} = \frac{276(300)}{600} = 138$

$E_{31} = \frac{126(60)}{600} = 12.6$ $\quad E_{32} = \frac{126(240)}{600} = 50.4$ $\quad E_{33} = \frac{126(300)}{600} = 63$

$$\chi^2 = \frac{(22-19.8)^2}{19.8} + \frac{(74-79.2)^2}{79.2} + \frac{(102-99)^2}{99} + \frac{(31-27.6)^2}{27.6} + \frac{(102-110.4)^2}{110.4} + \frac{(143-138)^2}{138} + \frac{(7-12.6)^2}{12.6} + \frac{(64-50.4)^2}{50.4} + \frac{(55-63)^2}{63}$$

$$= \frac{11}{45} + \frac{169}{495} + \frac{1}{11} + \frac{289}{690} + \frac{147}{230} + \frac{25}{138} + \frac{112}{45} + \frac{1156}{315} + \frac{64}{63}$$

$$= 9.0905$$

Since $\chi^2 = 9.0905 < 11.668$, it is failed to reject H_0 at $\alpha = 0.02$.

Hence, ~~there is insufficient evidence that~~ the degree of defectiveness is independent from types of machine.

Eason

Q8. The technical employees of a firm are either trained within the firm's training organization or, by outside training agencies. However, some are already trained on joining the firm. A survey is carried out to determine the productivity of technical employees, noting the training methods which have been used originally. The following table is available.

5/6

Training method	Productivity			
	High	Good	Average	Low
In-house	26	43	62	11
Outside agency	28	40	59	20
Previous employment	16	17	19	19

Test whether the level of productivity depends on the method of training. Use the 5% significance level.

H_0: The level of productivity is not dependent on the method of training

H_1: The level of productivity depends on the method of training (claim)

$\alpha = 0.05$, $v = (3-1)(4-1) = 6$, critical value $= \chi^2_{0.05;6} = 12.592$, rejection region $= \chi^2 > 12.592$

O_{ij} (E_{ij})	High	Good	Average	Low	Total
In-house	26 (27.6111)	43 (39.4444)	62	11	142
Outside agency	28	40	59	20	147
Previous employment	16	17	19	19	71
Total	70	100	140	50	360

$E_{11} = \frac{(142)(70)}{360} = 27.6111$ $E_{12} = \frac{142(100)}{360} = 39.4444$ $E_{13} = \frac{142(140)}{360} = 55.2222$ $E_{14} = \frac{142(50)}{360} = 19.7222$ $E_{21} = \frac{147(70)}{360} = 28.5833$

$E_{22} = \frac{147(100)}{360} = 40.8333$ $E_{23} = \frac{147(140)}{360} = 57.1667$ $E_{24} = \frac{(147)(50)}{360} = 20.4167$ $E_{31} = \frac{71(70)}{360} = 13.8056$ $E_{32} = \frac{71(100)}{360} = 19.7222$

$E_{33} = \frac{71(140)}{360} = 27.6111$ $E_{34} = \frac{71(50)}{360} = 9.8611$

$$\chi^2 = \sum_{i=1}^{3}\sum_{j=1}^{4}\frac{(O_{ij}-E_{ij})^2}{E_{ij}} = \frac{(26-27.6111)^2}{27.6111} + \frac{(43-39.4444)^2}{39.4444} + \frac{(62-55.2222)^2}{55.2222} + \frac{(11-19.7222)^2}{19.7222}$$
$$+ \frac{(28-28.5833)^2}{28.5833} + \frac{(40-40.8333)^2}{40.8333} + \frac{(59-57.1667)^2}{57.1667} + \frac{(20-20.4167)^2}{20.4167} +$$
$$\frac{(16-13.8056)^2}{13.8056} + \frac{(17-19.7222)^2}{19.7222} + \frac{(19-27.6111)^2}{27.6111} + \frac{(19-9.8611)^2}{9.8611}$$
$$= 0.09401 + 0.3205 + 0.8319 + 3.8574 + 0.01190 + 0.01701 + 0.05879$$
$$+ 0.0085 + 0.3488 + 0.3757 + 2.6856 + 8.4696$$
$$= 17.08$$

Since $\chi^2 = 17.08 > 12.592$, H_0 is rejected at $\alpha = 0.05$.

There is a sufficient evidence that the level of productivity depends on the method of training.

Q9. A coin is thrown 1000 times and 546 tails are obtained. Use Chi-square test to test whether the coin is fair at α − 0.02.

Yu Hong

OK

(Q9) ~~X~~ P = proportion of ~~a coin~~ getting tails ~~after thrown 1000 times~~ ~~X ~ B(1000, 0.5)~~

when a fair coin is tossed = 1/2

P in H0

H_0 : The coin is fair

H_1 : The coin is not fair

At the $\alpha = 0.02$, $v = 2 - 1 = 1$, critical value $\chi^2_{0.02} = 5.412$

critical region $\chi^2 > 5.412$

X	0	1	Total
O_i	454	546	1000
P	0.5	0.5	1
E_i	500	500	1000

np

χ^2

$$\chi^2 = \sum^{2} \frac{(|O_i - E_i| - 0.5)^2}{E_i} = \frac{(|454 - 500| - 0.5)^2}{500} + \frac{(|546 - 500| - 0.5)^2}{500}$$

$$= 8.281$$

$\therefore$ Since $\chi^2 = 8.281 > 5.142$, H_0 is rejected at $\alpha = 0.02$. There is insufficient evidence that the coin is fair.

Khai Jun

OK.

Q10. Two groups, A and B, consist of 100 people each who have a disease. A vaccine is given to group A but not to group B. it is found that in groups A and B, 75 and 65 people respectively, recover from the disease. Using Chi-square test at significance level of 0.05, test the hypothesis that the vaccine has relationship with recovery from the disease.

H_0: The vaccine has no significant relationship with recovery from the disease. ~~(claim)~~

H_1: The vaccine has a significant ~~no~~ relationship with recovery from the disease. (claim)

$\alpha = 0.05$, $v = (2-1)(2-1) = 1$, critical value $= \chi^2_{0.05;1} = 3.841$, rejection region: $\chi^2 > 3.841$

$O_{ij}(E_{ij})$	A	B	Total
Recover	75 (70)	65 (70)	140
Do not Recover	25 (30)	35 (30)	60
Total	100	100	200

$$\chi^2 = \underbrace{\sum_{i=1}^{2}\sum_{j=1}^{2} \frac{(|O_{ij}-E_{ij}|-0.5)^2}{E_{ij}}}_{} = \frac{(|75-70|-0.5)^2}{70} + \frac{(|65-70|-0.5)^2}{70} + \frac{(|25-30|-0.5)^2}{30} + \frac{(|35-30|-0.5)^2}{30}$$

$$= 0.2893 + 0.2893 + 0.675 + 0.675$$

$$= 1.9286$$

Since $\chi^2 = 1.9286 < 3.841$, H_0 is failed to reject at $\alpha = 0.05$.

Hence, there is ~~a~~ insufficient evidence that the vaccine has relationship with recovery from the disease.

Correlation

r_x	1	2	3	4	5
temp (°C)	25	26	27	29	30
r'_y	4	2	3	5	1
Sales of ice-cream	30	35	32	29	37
r_y	2	4	3	1	5
D^2	1	4	0	9	0
D'^2	9	0	0	1	16

$\Sigma D^2 = 14$

$r_s = 0.3$ (agreement rankings)

$r_x \uparrow$ $r_y \uparrow$

temp $\uparrow$ sales $\uparrow$

$\Sigma D^2 = 26$

$r_s = -0.3$ (disagreement)

$r_x \uparrow$ $r'_y \downarrow$

temp $\uparrow$ sales $\uparrow$

③ b)

Employee	A	B	C	D	E	F	G	H	
Weeks of experience, X	4 (1)	5 (2)	7 (3)	9 (4)	10 (5)	11 (6)	12 (7)	14 (8)	$\Sigma X = 72$
Number of rejects, Y	21 (7)	22 (8)	15 (5)	18 (6)	14 (3) 3.5	14 (4) 3.5	11 (1)	13 (2)	$\Sigma Y = 128$
X^2	16	25	49	81	100	121	144	196	$\Sigma X^2 = 732$
Y^2	441	484	225	324	196	196	121	169	$\Sigma Y^2 = 2\,156$
XY	84	110	105	162	140	154	132	182	$\Sigma XY = 1\,669$
r_x	1	2	3	4	5	6	7	8	
r_y	7	8	5	6	3.5	3.5	1	2	
$d = r_x - r_y$	-6	-6	-2	-2	1.5	2.5	6	6	
d^2	36	36	4	4	2.25	6.25	36	36	

$\Sigma d^2 = 160.5$

b)

$$r = \frac{n\Sigma XY - (\Sigma X)(\Sigma Y)}{\sqrt{[n\Sigma X^2 - (\Sigma X)^2][n\Sigma Y^2 - (\Sigma Y)^2]}} = \frac{8(1\,669) - (72)(128)}{\sqrt{[8(732) - (72)^2][8(2\,156) - (128)^2]}} = -0.8714$$

∴ There is a very strong negative linear correlation between weeks of experience and number of rejects. The higher the weeks of experience, the lesser the number of rejects.

c)

$$r_s = 1 - \frac{6\Sigma d^2}{n(n^2-1)} = 1 - \frac{6(160.5)}{8(8^2-1)} = -0.9107$$

∴ There is a very high degree of disagreement between the rankings of weeks of experience and number of rejects.

r_x ascending,

r_Y descending

r_s -ve

⇒ agreement in ranking

r_x ↑ r_Y ↑

exp ↑ no rejet ↓