Debbie

25.04.2020

pqr, $\bar{p}qr$, $p\bar{q}r$, $pq\bar{r}$,
$\bar{p}\bar{q}r$, $\bar{p}q\bar{r}$, $p\bar{q}\bar{r}$, $\bar{p}\bar{q}\bar{r}$

p	q	r	~p	~p∧q	q→r	A
						0
						0
						0
						1 ✓ minterm +
						0
						1 ✓ minterm

Question 1

a) Let A= (~p ∧ q) ↔ (q → r)

(i) Construct a truth table for A and determine whether A is a tautology, contradiction or contingency. (6 marks)

(ii) Write the Principal Disjunctive Normal Form and the Principal Conjunctive Normal Form of the statement A and ~A. (8 marks)

PDNF of A = minterm 1 + minterm 2 + ⋮

negate

PCNF of ~A

PDNF of ~A

negate

PCNF of A

p	q	r	~p	~p^q	q-->r	(~p^q) <--> (q^r)
0	0	0	1	0	1	0
0	0	1	1	0	1	0
0	1	0	1	1	0	0
0	1	1	1	1	1	1
1	0	0	0	0	1	0
1	0	1	0	0	1	0
1	1	0	0	0	0	1
1	1	1	0	0	1	0

PDNF A = p'qr + pqr' ✓

PDNF ~A = p'q'r' + p'q'r + p'qr' + pq'r' + pq'r + pqr

PCNF A = (p+q+r) * (p+q+r') * (p+q'+r) * (p'+q+r) * (p'+q+r') * (p'+q'+r') ✓

negate

PCNF ~A = (p+q'+r') * (p'+q'+r) ✓

negate

b) Using the Laws of Logical Equivalence, simplify the following statement.

$\sim(p \rightarrow (q \vee r)) \rightarrow ((\sim r \wedge q) \rightarrow \sim p)$ (6 marks)

$\sim[p \rightarrow (q \vee r)] \rightarrow [(\sim r \wedge q) \rightarrow \sim p]$

$= [\sim p \vee (q \vee r)] \vee [\sim(\sim r \wedge q) \vee \sim p]$

$= [\sim p \vee q \vee r] \vee [r \vee \sim q \vee \sim p]$

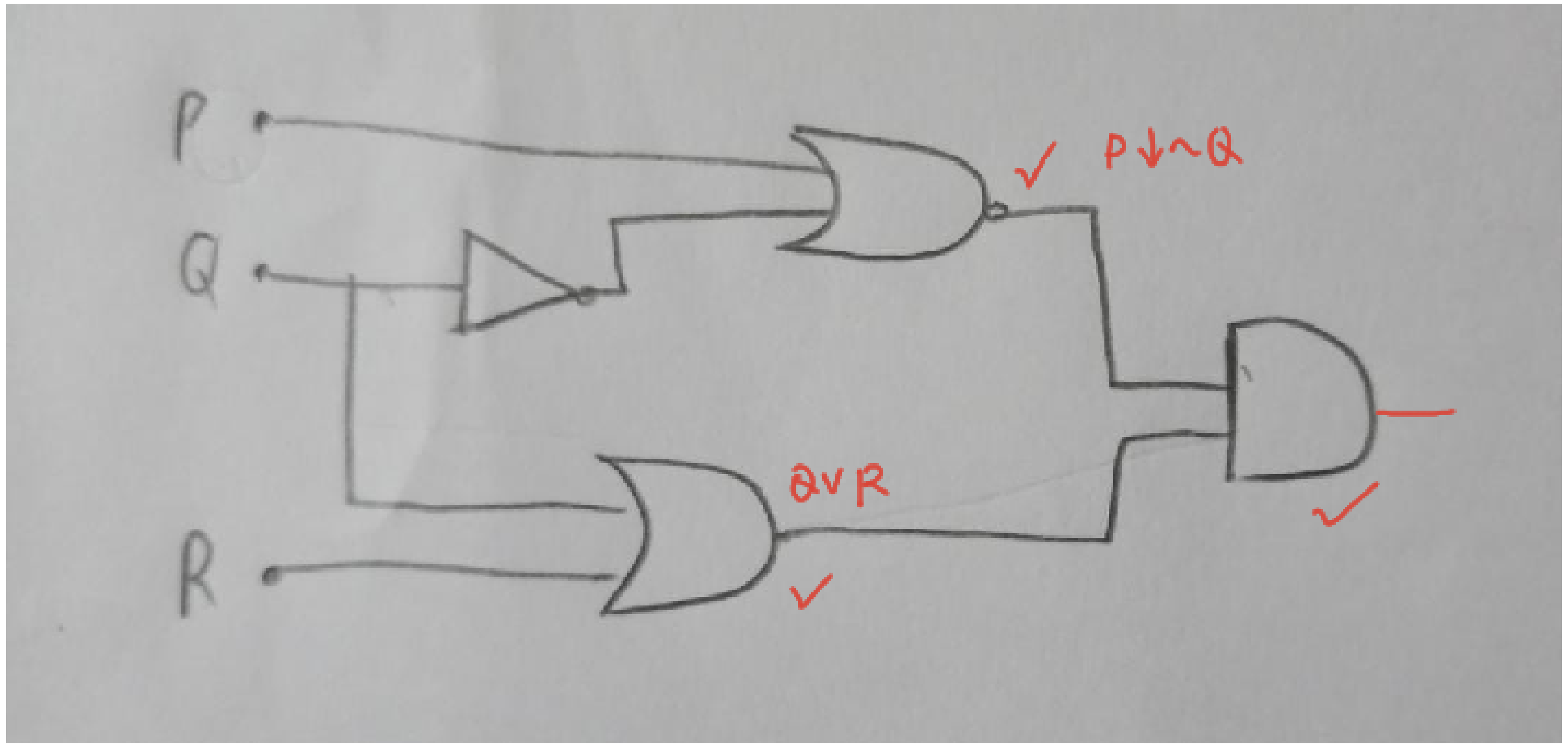
$= (\sim p \vee \sim p) \vee (q \vee \sim q) \vee (r \vee r)$

$= \sim p \vee t \vee r = t$

NOR

c) Design a circuit for $(P \downarrow \sim Q) \wedge (Q \vee R)$ (5 marks)

and



KL

Question 2

a) Let the set of all integers be the universe of discourse and let

A(x): $x < 0$,
✓ B(x): x is even,
✓ C(x): x is divisible by 5.

$\forall, \exists$ x $\rightarrow$

Rewrite the following statements formally using quantifiers, variables and connectives. Then determine their truth values. For each false statement, provide a counterexample.

(i) There exists a non-negative integer that is even. (2 marks)

(ii) If x is even, then it is divisible by 5. (3 marks)

$x = 2 \rightarrow$ non-negative ✓
$\rightarrow$ even ✓

$\exists \sim A(x) \ni \sim A(x) = B(x)$

(i) $\exists x \ni [\sim A(x) \wedge B(x)]$
True

(ii) $\forall x$ ~~B(x)~~, $B(x) \rightarrow C(x)$

false ✓, ✓ ~~2 ≠ 5~~ $2 \div 5 = 0.4 \notin \mathbb{Z}$ ✓

Let $x=2$ (even)

SSiew SHua WChong ZTeng

b) State and determine the truth value for the negation, converse, inverse and contrapositive of the following statement.

For all real numbers x and y, if $x^2 < y^2$, then $x < y$. (12 marks)

p q

$\sim(p \to q)$

$= \sim(\sim p \vee q)$

$= p \wedge \sim q$

Negation : Some of the real numbers x and y, such that x^2 < y^2, and x ≥ y. (~~False~~) True

Inverse : For all real numbers x and y, if x^2 ≥ y^2, then x ≥ y. ~~(T)~~ F

$\sim p \to \sim q$

$x^2 \geq y^2$, $x \geq y$

$(-4)^2 \geq (-2)^2$, $-4 \geq -2$ X

$16 \geq 4$

Converse: For all real numbers x and y, if x < y, then x^2 < y^2 (False)

$q \to p$

$x < y$, $x^2 < y^2$

$-4 < -2$, $(-4)^2 < (-2)^2$

$16 < 4$ X

Contrapositive: For all real numbers x and y, if x >= y , then x^2 >= y^2. ~~(True)~~ False

$\sim q \to \sim p$

$x \geq y$, $x^2 \geq y^2$

$-1 \geq -2$, $(-1)^2 \geq (-2)^2$

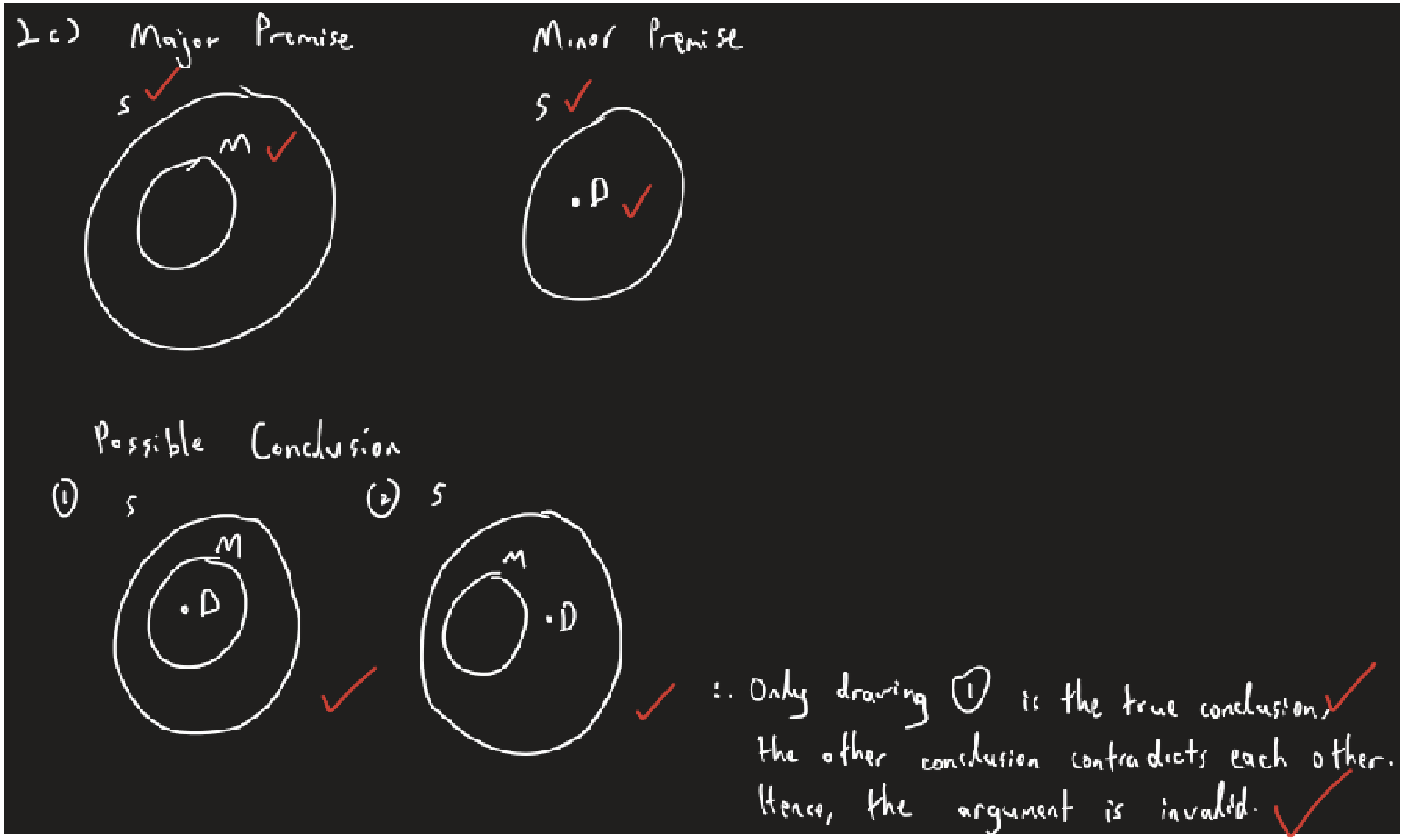
$1 \geq 4$ X

Hx c) Determine the validity of the following argument using diagrams.

All mathematicians are smart. Major premise
Danny is smart. Minor premise
Therefore, Danny is a mathematicians. Conclusion

Notes: Students may use the following notations.

Let S: Set of people who are smart.
M: Set of people who are mathematicians.
D: Danny



ZY

Question 3

a) Show that if n is an odd integer, then $n^2 + 1$ is even. (5 marks)

Proof:

Suppose n is a particular but arbitrarily chosen odd integer.

By definition of odd, $n = 2m+1$, $m \in \mathbb{Z}$

$$\begin{aligned} \text{Then } n^2+1 &= (2m+1)^2+1 \\ &= 4m^2+4m+1+1 \\ &= 4m^2+4m+2 \\ &= 2(2m^2+2m+1) \\ &= 2k \end{aligned}$$

Since the sum of products of integers is an integer, $2m^2+2m+1$ is an integer. By definition of even integer, $2(2m^2+2m+1)$ is an even integer.

Therefore, if ~~n~~ n is an odd integer, then n^2+1 is even.

b) Use the Euclidean algorithm to find the greatest common divisor of 120 and 32. Write the greatest common divisor in the form of $s120 + t32$, $s,t \in Z$. Hence, find the least common multiple of 120 and 32. (6 marks)

$a = 120 \quad b = 32$

$120 = 32(3) + 24$ ✓

$32 = 24(1) + 8$ ✓

$24 = 8(3) + 0$

$\gcd(120, 32) = \gcd(32, 24)$ ✓

$= \gcd(24, 8)$ ✓

$= \gcd(8, 0)$ ✓

$= 8$ ✓

$8 = 32 - 24$

$= 32 - [120 - 32(3)]$ ✓

$= 32(4) - 120$

$= -120 + 4(32)$

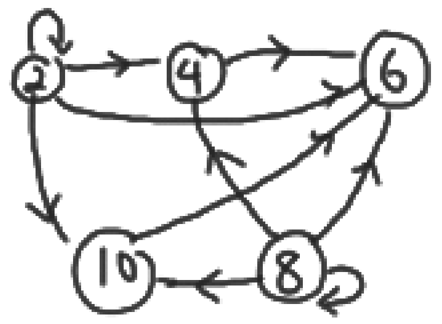
$s = -1$ ✓ $t = 4$ ✓

$\text{lcm}(120, 32) = \dfrac{120 \times 32}{8}$ ✓

$= 480$ ✓

Jia Yi Q3c) Let A = {2, 4, 6, 8, 10} and R be the relation on A whose matrix is given below.

$$M_R = \begin{array}{c} \\ 2 \\ 4 \\ 6 \\ 8 \\ 10 \end{array} \begin{array}{c} \begin{array}{ccccc} 2 & 4 & 6 & 8 & 10 \end{array} \\ \begin{pmatrix} 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \end{array}$$



Determine whether the relation is reflexive, irreflexive, symmetric, asymmetric, antisymmetric or transitive. Give a counterexample if the answer is "No". (5 marks)

optional

R is not reflexive since (4,4), (6,6), (10,10) $\notin$ R ... since $4 \not R 4$

R is not irreflexive since (2,2) $\in$ R ... since $2 R 2$

R is not symmetric since (2,4) $\in$ R but (4,2) $\notin$ R ... since $2 R 4$ but $4 \not R 2$

R is not asymmetric since (2,2) $\in$ R ... since $2 R 2$

R is antisymmetric

R is transitive

Q3
kae lun

d) Let A = {a,b,c,d} and R and S be the relation on A described by the matrices.

$$M_R = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 1 & 0 & 0 & 1 \\ b & 1 & 0 & 0 & 0 \\ c & 0 & 1 & 1 & 1 \\ d & 1 & 1 & 1 & 0 \end{array} \quad \text{and} \quad M_S = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 1 & 0 & 0 \\ b & 0 & 1 & 0 & 0 \\ c & 0 & 0 & 0 & 1 \\ d & 1 & 1 & 0 & 1 \end{array}$$

(i) Compute $M_{\bar{R}}$, $M_{S \circ R}$ and $M_{(S \circ R)^{-1}}$. (4 marks)

(ii) Use Warshall's algorithm to compute the transitive closure matrix of R. (5 marks)

Complement of R, interchanging of "0" and "1".

i) $M_{\bar{R}} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ ✓

$M_R \circ M_S$

$M_{S \circ R} = \cancel{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}} \circ \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \circ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$ ✓

Inverse of (S∘R), transpose the original matrix

$M_{(S \circ R)^{-1}} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix}$ ✓

ii) Let $M_R = W_0 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$

C_1: a, b, d ✓
R_1: a, d ✓
Add: (a,a), (a,d), (b,a), (b,d), (d,a), (d,d) ✓

$W_1 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

C_2: c, d ✓
R_2: a, d ✓
Add: (c,a), (d,a), (c,d), (d,d) ✓

$W_2 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

C_3: c, d ✓
R_3: a, b, c, d ✓
Add: (c,a), (c,b), (c,c), (c,d), (d,a), (d,b), (d,c), (d,d) ✓

$W_3 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

C_4: a, b, c, d ✓
R_4: a, b, c, d ✓
Add: (a,a), (a,b), (a,c), (a,d), (b,a), (b,b), (b,c), (b,d), (c,a), (c,b), (c,c), (c,d), (d,a), (d,b), (d,c), (d,d) ✓

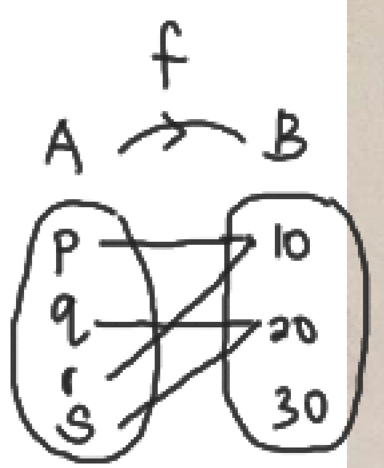
✓ $W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} = M_{R^\infty}$

∴ Transitive closure, R^∞ =
{(a,a), (a,b), (a,c), (a,d),
(b,a), (b,b), (b,c), (b,d),
(c,a), (c,b), (c,c), (c,d),
(d,a), (d,b), (d,c), (d,d)} ✓

Question 4 Joel

(x, y) Domain range

a) Let A = {p , q , r , s}, B = {10 ,20 ,30} and let f={(p, 10) , (q, 20) , (r, 10) , (s, 20)} be a function from A to B. Find the domain and range of the function f. Hence, determine whether the function is everywhere defined and onto. Explain your answers. (5 marks)



4) $f: A \to B = \{(p,10), (q,20), (r,10), (s,20)\}$

$dom(f) = A$ ✓ $range(f) = \cancel{B}\ \{10, 20\}$

:- ~~Hence~~ Function is everywhere defined as ~~each~~ every element is being mapped from the domain. ✓ / since $Dom(f) = A$

Function is not onto as ✓ ~~no~~ there is an element that is not mapped to. / since $30 \notin Range(f)$.

f is not onto since $Range(f) \neq B$ (codomain).

1, 2, 5, 1 3, 4, 6, 3

MLeong

b) Let $\rho = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 5 & 4 & 6 & 1 & 3 \end{pmatrix}$ be a permutation of the set A={1, 2, 3, 4, 5, 6}.

(i) Write ρ as a product of disjoint cycles. (2 marks)

(ii) Write ρ as a product of transpositions. (2 marks)

(iii) Determine whether ρ is even or odd. (2 marks)

b) $\rho = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 5 & 4 & 6 & 1 & 3 \end{pmatrix}$

i) disjoint cycles

$= (1, 2, 5) \circ (3, 4, 6)$ ✓ ✓

ii) transposition

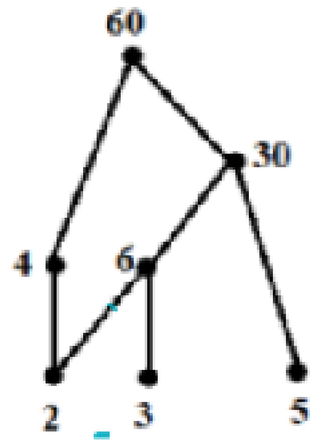
$= (1,2) \circ (1,5) \circ (3,4) \circ (3,6)$

$(1,5) \circ (1,2) \circ (3,6) \circ (3,4)$

iii) It is even since there are 4 transpositions. ✓

Nicholas

c) The Hasse diagram for a poset, P is given below. Find, if exist(s):



(i) the maximal element(s) of P; **(1 mark)**

(ii) the minimal element(s) of P; **(3 marks)**

(iii) the greatest and the least element(s); **(2 marks)**

(iv) the least upper bound of {2, 3}; **(1 mark)**

(v) the greatest lower bound of {2, 3}. **(1 mark)**

i) Maximal = {60} ✓

ii) Minimal = {2,3,5} ✓

iii) Greatest = {60} ✓ least = none, more than one minimal ✓

iv) Least upper bound of {2,3} = {6} ✓ | upper bound = {6,30,60} ✓

v) Greatest lower bound of {2,3} = none ~~exists~~ ✓ | lower bound = ∅ ✓

d) Let $f(x,y,z) = (x'\wedge y\wedge z')\vee(x'\wedge y\wedge z)\vee(x\wedge y\wedge z')\vee(x\wedge y\wedge z)\vee(x\wedge y'\wedge z)$. Construct a Karnaugh map in the form of

	y'	y'	y	y
x'				
x				
	z'	z	z	z'

and hence simplify $f(x,y,z)$ to the simplest form. (6 marks)

	y'	y'	y	y
x'	0	0	1	1
x	0	1	1	1
	z'	z	z	z'

$f(x,y,z) = y + xz$ ✓ ✓

14.01.2016

Q1. (a) For the statement $p \rightarrow \sim(q \wedge \sim r)$, write its contrapositive, converse and inverse. Then write your final answer without the connective '$\rightarrow$' and apply De Morgan's law where necessary. (6 marks)

Contrapositive: $\sim q \rightarrow \sim p$

$(q \wedge \sim r) \rightarrow \sim p$

$\equiv \sim(q \wedge \sim r) \vee \sim p$

$\equiv \sim q \vee r \vee \sim p$

Converse: $q \rightarrow p$

$\sim(q \wedge \sim r) \rightarrow p$

$\equiv (q \wedge \sim r) \vee p$

Inverse: $\sim p \rightarrow \sim q$

$\sim p \rightarrow (q \wedge \sim r)$

$\equiv p \vee (q \wedge \sim r)$

(b) Let $A \equiv (p \leftrightarrow q) \veebar (\sim q \rightarrow r)$.

(i) Construct a truth table for the expression A. (3 marks)

(ii) Write the Principal Disjunctive Normal Form (PDNF) and Principal Conjunctive Normal Form (PCNF) of A. Hence deduce the PDNF and PCNF of $\sim A$. (6 marks)

(b)

same is 1

(i)

p	q	r	$\sim q$	$p \leftrightarrow q$	$\sim q \rightarrow r$	A	
0	0	0	1	1	0	1	PDNF
0	0	1	1	1	1	0	
0	1	0	0	0	1	1	PDNF
0	1	1	0	0	1	1	PDNF
1	0	0	1	0	0	0	
1	0	1	1	0	1	1	PDNF
1	1	0	0	1	1	0	
1	1	1	0	1	1	0	

(ii)

PDNF of $A \equiv \bar{p}\bar{q}\bar{r} + \bar{p}q\bar{r} + \bar{p}qr + p\bar{q}r$

PDNF of $\sim A \equiv \bar{p}\bar{q}r + p\bar{q}\bar{r} + pq\bar{r} + pqr$

PCNF of $A \equiv (p+q+\bar{r})(\bar{p}+q+r)(\bar{p}+\bar{q}+r)(\bar{p}+\bar{q}+\bar{r})$

PCNF of $\sim A = (p+q+r)(p+\bar{q}+r)(p+\bar{q}+\bar{r})(\bar{p}+q+\bar{r})$