Chapter 1

Group

1 1 1 1

1 1 1 1

1 1 . - - -

. - - - -

(individual)

prob dist

X

prob?

$\Downarrow$

estimate no. of items in this $\boxed{\text{group}}$ = prob × total items

rounded to the nearest integer

$n \uparrow$

CLT

$\bar{X} \sim N$

$\hat{p} \sim N$

$\bar{X}_1 - \bar{X}_2 \sim N$

$\hat{p}_1 - \hat{p}_2 \sim N$

Chapter 2.

Group

n n n

S1 S2 S3

n - - - -

S4

sampling dist

prob? $\bar{X}$, $\hat{p}$, $\bar{X}_1 - \bar{X}_2$, $\hat{p}_1 - \hat{p}_2$

estimate no. of samples in this $\boxed{\text{group}}$

= prob × total samples

rounded to the nearest int

* estimate sample size, n

→ rounded up!

Ze Xuan

Q1. If $X \sim N(200, 80)$ and a random sample of size 5 is taken from the distribution, find the probability that the sample mean

OK (a) is greater than 207, (b) lies between 201 and 209.

1.

$X \sim N(200, 80)$, $n = 5$

$\bar{X} \sim N(200, \frac{80}{5})$

a) $P(\bar{X} > 207) = P\left(Z > \frac{207 - 200}{\sqrt{\frac{80}{5}}}\right)$

$= P(Z > 1.75)$

$= 0.0401$

b) $P(201 < \bar{X} < 209) = P(0.25 < Z < 2.25)$

$= P(Z > 0.25) - P(Z > 2.25)$

$= 0.4013 - 0.01222$

$= 0.38908$

Sean

Q2. The heights of a new variety of sunflower are normally distributed with mean 2m and standard deviation 40cm. 100 samples of 50 flowers each are measured. How many samples with the sample mean

(a) between 195cm and 205cm? (b) less than 197cm?

or

round to the nearest integer

(b) $P(\bar{x} < 197) = P\left(Z < \frac{197-200}{40/\sqrt{50}}\right)$

$= P(Z < -0.53)$

$= P(0.53 < Z)$

$= 0.2981$

$= \cancel{0.30}$

no. of samples

$\cancel{0.30} \times 100$ $= 0.2981 \times 100$

$\therefore \cancel{x} = 30$

$= 29.81$

≈ 30 //

$\mu = 2m$ $\sigma = 40c$ 100 50

(a) $195 < x < 205$

let X be the height of the variety of sunflower.

$x \sim N(200, 40^2) \Rightarrow \bar{x} \sim N\left(200, \frac{40^2}{50}\right)$

$P(195 < \bar{x} < 205) = P\left(\frac{195-200}{40/\sqrt{50}} < Z < \frac{205-200}{40/\sqrt{50}}\right)$

$= P\left(-\frac{5\sqrt{2}}{8} < Z < \frac{5\sqrt{2}}{8}\right)$

$= P(-0.88\cancel{39} < Z < 0.88\cancel{39})$

$= P(-0.88 < Z < 0.88)$

$= 1 - 2P(Z > 0.88)$

$= 1 - 2(0.1894)$

right-tail area

$= 1 - 0.3788$

$= 1 - 0.3788$

0.6212

no. of samples

$n \Rightarrow 0.6212 \times 100$

$n = 62.12$

$\therefore n = 62$

Scanned with CamScanner

Aaron

Q3. If large number of samples size n are taken from a population which follows a normal distribution with mean 74 and standard deviation 6, find n if the probability that the sample mean
(a) exceeds 75 is 0.281, (b) is less than 70.4 is 0.00135.

OK.

sample size $\Rightarrow$ round up

3a) $\bar{x} \sim \left(74, \frac{6^2}{n}\right)$

$P(\bar{x} > 75) = 0.281$

$P\left(Z > \frac{75-74}{\frac{6}{\sqrt{n}}}\right) = 0.281$ <0.5

+ve

$\frac{\sqrt{n}}{6} = 0.58$

$n = 12.11$

$n \approx 13$

b) $P(\bar{x} < 70.4) = 0.00135$

$P\left(Z < \frac{70.4-74}{\frac{6}{\sqrt{n}}}\right) = 0.00135$ <0.5

$\frac{(70.4-74)}{\frac{6}{\sqrt{n}}} = -3.0$ -ve

$\frac{-3.6\sqrt{n}}{6} = -3$

$\sqrt{n} = 5$

$n = 25$

Cecilia

Q4. A random sample of size 100 is taken from Bin (20, 0.6). Find the probability that the sample mean is
(a) is greater than 12.4. (b) is less than 12.2.

OK.

$\mu = 20 \times 0.6 = 12$

$\sigma^2 = 20 \times 0.6 \times 0.4 = 4.8$

4) n = 100

$\boxed{X \sim B(20, 0.6)}$

a) Since np > 5, n > 30 and nq > 5, thus normal approximation is be used $X \sim N(12, 4.8)$.

$P(\bar{X} > 12.4) = P(Z > \frac{12.4-12}{\sqrt{4.8}/\sqrt{100}})$

$= P(Z > 1.83)$

$= 0.0336$

b) $P(\bar{X} < 12.2) = P(Z < \frac{12.2-12}{\sqrt{4.8}/\sqrt{100}})$

$= P(Z < 0.91)$

$= 1 - P(Z > 0.91)$

$= 1 - 0.1814$

$= 0.8186$

sample size

According to Central Limit Theorem, when n > 30,

$\bar{X} \sim N(12, \frac{4.8}{100})$

$\frac{\sigma^2}{n}$ ← sample size

Mei Yi

Q5. If a large number of samples of size n is taken from Bin (20, 0.2) and approximately 90% of the sample means are less than 4.354, estimate n.

5K

sample size ⇒ round up

CLT

Q5 $X \sim B(20, 0.2)$

$\mu = np = 4$

$\sigma^2 = npq = 3.2$

Since n is large, $\bar{X} \sim N(4, \frac{3.2}{n})$

$P(\bar{X} < 4.354) = 0.9$

Same $P(\bar{X} > 4.354) = 1 - 0.9 = 0.1$

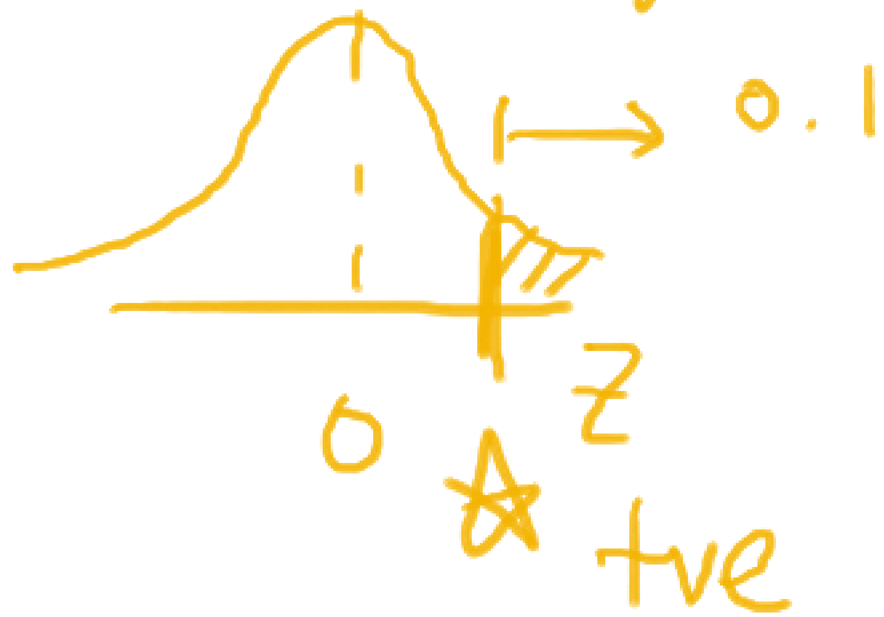
$P\left(Z > \frac{4.354 - 4}{\sqrt{3.2/n}}\right) = \underline{0.1} < 0.5$

$\frac{0.354\sqrt{n}}{\sqrt{3.2}} = 1.2816$

Appendix 21 (stat table)

$n \approx 41.8573$ 94

$n \approx 42$



Cai Jie

Q6. If a large number of samples of size n is taken from $P_o(2.9)$ and approximately 1% of the sample means are greater than 3.41, estimate n.

sample size → round up

applies to sample statistics $\bar{x}, \hat{p}$

No.: T4 Date:

$X \sim P_o(2.9)$ individual $\Rightarrow$ n is large, CLT

$\Rightarrow \bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$

Q6 $\mu = 2.9$, $\sigma = 2.9$

$\mu_{\bar{x}} = 2.9$ $\sigma_{\bar{x}} = \sqrt{\frac{2.9}{n}}$

Sampling dist,

$\bar{X} \sim N\left(2.9, \frac{2.9}{n}\right)$

$P(\bar{x} > 3.41) \approx 0.01$

$P\left(Z > \frac{3.41-2.9}{\sqrt{\frac{2.9}{n}}}\right) \approx 0.01$ < 0.5

Appendix 2 in statistical table

$\Rightarrow \frac{0.51}{\frac{1.7029}{\sqrt{n}}} = 2.3263$ +ve

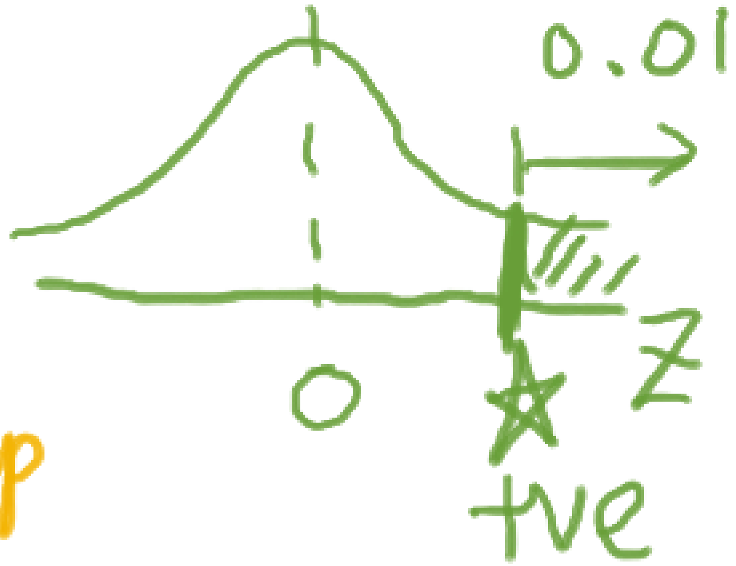
$0.2995\sqrt{n} = 2.3263$

$\sqrt{n} = 7.7796$

$n = (7.7796)^2$

$n = 60.52$

$n \approx 61$ round up



Jun Yan

Q7. Given the discrete uniform population $f(x) = 0.25$ for $x = 0, 1, 2, 3$. Find the probability that a random sample of size 36, selected with replacement, will yield a sample mean greater than 1.4 but less than 1.8.

independent samples

$n = 36$ (large sample size)
from Central Limit Theorem,

Discrete uniform

X	$f(x) = P(X=x)$
0	0.25
1	0.25
2	0.25
3	0.25

$\mu = \sum x P(X=x)$
$=$

$\sigma^2 = \sum x^2 P(X=x) - \mu^2$
$=$

Q 7

mean, $\mu = \sum x P(X=x)$
$= 0(0.25) + 1(0.25) + 2(0.25) + 3(0.25)$
$= 1.5$

variance, $\sigma^2 = \sum x^2 P(X=x) - \mu^2$
$= [0^2(0.25) + 1^2(0.25) + 2^2(0.25) + 3^2(0.25)] - 1.5^2$
$= 1.25$

$\bar{x} \sim N(1.5, \frac{1.25}{36})$

$P(1.4 < \bar{x} < 1.8) = P\left(\frac{1.4 - 1.5}{\sqrt{1.25}/\sqrt{36}} < Z < \frac{1.8 - 1.5}{\sqrt{1.25}/\sqrt{36}}\right)$
$= P(-0.54 < Z < 1.61)$
$= 1 - P(Z > 0.54) - P(Z > 1.61)$
$= 1 - 0.2946 - 0.0537$
$= 0.6517$

Pui Mun

Q8. A fair coin is tossed 150 times. Find the probability that heads will occur
(a) less than 40% of the tosses, (b) between 44% and 56% of the tosses.

ac

heads when for

Let $\hat{p}$ = sample proportion of heads when a coin is tossed for 150 times

$\hat{p} \sim N\left(0.5, \frac{0.5(0.5)}{150}\right) = \hat{p} \sim N\left(0.5, \frac{1}{600}\right)$

(a) $P(\hat{p} < 0.40) = P\left(Z < \frac{0.40 - 0.50}{\sqrt{\frac{1}{600}}}\right)$

$= P(Z < -2.45)$

$= P(Z > 2.45)$

$= 0.00714$

(b) $P(0.44 < \hat{p} < 0.56) = P\left(\frac{0.44 - 0.50}{\sqrt{\frac{1}{600}}} < Z < \frac{0.56 - 0.50}{\sqrt{\frac{1}{600}}}\right)$

$= P(-1.47 < Z < 1.47)$

$= 1 - 2P(Z > 1.47)$

$= 1 - 2(0.0708)$

$= 0.8584$

Shee Yeap

Q9. It has been found that 2% of the tools produced by a certain machine are defective. Find the probability that in a shipment of 400 such tools (a) 3% or more will be defective, (b) 2% or less will be defective.

ore

(9) Let $\hat{p}$ = sample proportion of tools produced by a certain machines that are defective

$$\hat{P} \sim N\left(0.02, \frac{0.02(1-0.02)}{400}\right) \Rightarrow \hat{P} \sim N(0.02, 0.000049)$$

(a) $P(\hat{P} \geq 0.03) = P\left(Z \geq \frac{0.03-0.02}{\sqrt{0.000049}}\right)$

$= P(Z \geq 1.43)$

$= 0.764$

(b) $P(\hat{P} \leq 0.02) = P\left(Z \leq \frac{0.02-0.02}{\sqrt{0.000049}}\right)$

$= P(Z \leq 0)$

$= 0.5$

Jing Jet

Q10. A certain candidate standing for election is known to have the support of proportion 0.46 of the electors. Find the probability that the candidate will have a majority in a random sample of

(a) 200 electors, (b) 1000 electors.

or

2 possible outcomes
- support
- not support

mean $\mu_{\hat{p}}$ 0,46, variance $\sigma^2_{\hat{p}}$

$\hat{p} \sim N(p, \frac{pq}{n})$

population proportion

$\hat{p} \sim$ N(0.46(1-0.46)/n)

Let $\hat{p}$ = sample proportion of a certain candidate standing for election who have the support of the electors

a) $P[\hat{p} > 0.5]$

$= P\left[z > \frac{0.5 - 0.46}{\sqrt{\frac{0.46(0.54)}{200}}}\right]$

$= P[z > 1.14]$

$= 0.1271$

b) $P[\hat{p} > 0.5]$

$= P\left[z > \frac{0.5 - 0.46}{\sqrt{\frac{0.46(0.54)}{1000}}}\right]$

$= P[z > 2.54]$

$= 0.00554$

Sampling dist for sample proportion, $\hat{p} \sim N\left(0{,}46, \frac{0.46(0.54)}{n}\right)$

Jia Jie

Q11. Two populations of measurements are normally distributed with $\mu_1 = 57$ and $\mu_2 = 25$. The two populations' standard deviations are $\sigma_1 = 12$ and $\sigma_2 = 6$. Two independent samples of $n_1 = n_2 = 36$ are taken from the populations.

ok

(a) What is the expected value of the difference in the sample means $\bar{X}_1 - \bar{X}_2$?

(b) What is the standard deviation of the distribution of $\bar{X}_1 - \bar{X}_2$?

(c) What is the distribution of $\bar{X}_1 - \bar{X}_2$?

(d) Find $P(29 \le \bar{X}_1 - \bar{X}_2 \le 31)$.

(e) What proportion of the time will the means of the samples differs by 34 or more?

a) $\mu_{\bar{x}_1 - \bar{x}_2} = \mu_1 - \mu_2$

b) $\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = \sqrt{5} = 2.2361$

c) $\bar{X}_1 - \bar{X}_2 \sim$ N(32, 5)

d) $P(\bar{X}_1 - \bar{X}_2 \ge 34)$

Q11

~~let Pi be measurements of 1 and 2. i = 1, 2~~

(a) $57 - 25 = 32$

(b) $\frac{12^2}{36} + \frac{6^2}{36} = 5$

$\sqrt{5} = 2.2361$?

c) Normal Distribution

d) $P(29 \le \bar{x}_1 - \bar{x}_2 \le 31)$, ~~$P(32, 2.2361)$~~

$= P\left[\frac{29-32}{2.2361} \le Z \le \frac{31-32}{2.2361}\right]$

$= P[-1.34 \le Z \le -0.45]$ flip

$= P[0.45 \le Z \le 1.34]$

$= P[Z \ge 0.45] - P[Z \ge 1.34]$

$= 0.3264 - 0.0901$

$= 0.2363$

e) $P(\bar{x} \ge 34)$

$P\left[Z \ge \frac{34-32}{2.2361}\right]$

$= P[Z \ge 0.89]$

$= 0.1867$

Khai Jun

Q12. Yields for plantation bonds (X_1) over a period have had a mean of 0.12 and a standard deviation of 0.02, whereas yields for industrial bonds (X_2) have had a mean of 0.13 and a standard deviation of 0.03. Independent random samples, of size 49 and 64, are obtained from the two very large populations of bonds.

(a) What is the expected value of the difference in the sample means yields, $\bar{X}_1 - \bar{X}_2$?

(b) What is the standard deviation of the difference in the sample mean yields?

(c) What is the distribution of the difference in the sample mean yields?

(d) Find the probability that the sample mean yield of plantation bonds is less than the sample mean yield of industrial bonds.

(e) Find the probability that the sample mean yield for industrial bonds will be at least 0.005 more than the sample mean yield for plantation bonds.

$$\underbrace{\bar{X}_2 \geq 0.005 + \bar{X}_1}$$

Q12 $\mu_{x_1} = 0.12$ $\mu_{x_2} = 0.13$

$\sigma_{x_1} = 0.02$ $\sigma_{x_2} = 0.03$

Let $\bar{x}_1$ be the sample mean number of yields for plantation bonds, a

let $\bar{x}_2$ be the sample mean number of yields for industrial bonds

$\mu_{\bar{x}_1 - \bar{x}_2} = 0.12 - 0.13 = \mu_1 - \mu_2$

$= -0.01$ ✓

b) $(\bar{x}_1 - \bar{x}_2 \sim N(-0.01, 0.0000222^2)) \rightarrow (c)$

$\sqrt{0.0000222} = 0.00471$

c) Approximate normal

d) $P(\bar{x}_1 < \bar{x}_2)$ ✓

$= P(\bar{x}_1 - \bar{x}_2 < 0)$ ✓

$= P\left(Z < \frac{0-(-0.01)}{0.00471}\right)$

$= P(Z < 2.12)$ ✓ $= 1 - P(Z > 2.12)$

$= 0.983$ ✓

e) $P(\bar{x}_1 - \bar{x}_2 \leq 0.005)$

$Z \leq \frac{0.005 - (-0.01)}{0.00471}$

$Z \leq 3.1$ ✗

$1 - 0.000736$

$= 0.9993$

$P(\bar{x}_1 - \bar{x}_2 \leq -0.005)$

e) $\bar{x}_2 \geq 0.005 + \bar{x}_1$

$= P(\bar{x}_1 - \bar{x}_2 \leq -0.005)$

$= P\left(Z \leq \frac{-0.005 - (-0.01)}{0.00471}\right)$

$= P(Z \leq 1.0616)$

$= 1 - 0.1446$

$= 0.8554$

$\mu_{\bar{x}_1 - \bar{x}_2} \neq \bar{x}_1 - \bar{x}_2$

b) $\sigma_{\mu_1 - \mu_2} = \sqrt{(0.02^2)/49 + (0.03^2)/64}$

$= 0.0000222$ ✓

Jun Dian

OK

Q13. A weight reduction clinic has offices in Kuala Lumpur and Ipoh. It has found that the proportion of people signed up for its weight-reducing classes who actually complete their entire program is $p_1 = 0.80$ in Kuala Lumpur and $p_2 = 0.72$ in Ipoh. A class of $n_1 = 80$ participants just started the program in Kuala Lumpur and a class of $n_2 = 60$ just started the program in Ipoh. Consider these two groups to be independent random samples.

(a) What is the expected value of the difference in the completion rates, $\hat{p}_1 - \hat{p}_2$, between these Kuala Lumpur and Ipoh groups?

(b) What is the standard deviation of this difference over many pairs of classes of this size?

(c) What is the probability that the difference $\hat{p}_1 - \hat{p}_2$ will be 10% or larger?

13. Let $\hat{P}_i$ be the sample rate of those complete weight-reducing classes for i-group, i = 1 (Kuala Lumpur), 2 (Ipoh).

a. $P_1 - P_2 = 0.80 - 0.72 = \mu_{\hat{P}_1 - \hat{P}_2}$

$= 0.08$

b. $\hat{P}_1 - \hat{P}_2 \sim N(0.08, 0.00536)$

$\sigma^2_{\hat{P}_1 - \hat{P}_2} = 0.00536 = \frac{P_1 q_1}{n_1} + \frac{P_2 q_2}{n_2}$

refer chat box

$\sigma_{\hat{P}_1 - \hat{P}_2} = \sqrt{0.00536}$

$= 0.0732$

c. $P(\hat{P}_1 - \hat{P}_2 \geq 0.10) = P\left(Z \geq \frac{0.10 - 0.08}{0.0732}\right)$

$= P(Z \geq 0.27)$

$= 0.3936$

Jia Yu

OK

Q14. Two larger national companies in the same industry differ in the proportions of women in their production labor forces. The first company has a proportion of women that is $p_1 = 0.30$. The second company's proportion of women is $p_2 = 0.18$. Randomly selected groups of $n_1 = 80$ and $n_2 = 70$ production workers are being sent by their companies to an industry-sponsored training program in Miami. What is the probability that the proportion of women in the first company's groups will exceed the proportion of women in the second company's group between 10% and 20%?

14) $P_1 = 0.30,\ P_2 = 0.18,\ n_1 = 80,\ n_2 = 70,\ q_1 = 0.70,\ q_2 = 0.82$

∴ Let P_i be the ~~no. of~~ sample proportion of women in company i in the production labor forces, $i = 1, 2$

$$\hat{P}_1 \sim \hat{P}_2 \sim N\left(P_1 - P_2,\ \frac{P_1 q_1}{n_1} + \frac{P_2 q_2}{n_2}\right)$$

$$\Rightarrow \hat{P}_1 - \hat{P}_2 \sim N\left(0.30 - 0.18,\ \frac{0.3(0.7)}{80} + \frac{0.18(0.82)}{70}\right)$$

$$\equiv \hat{P}_1 - \hat{P}_2 \sim N(0.12,\ 0.00473)$$

$$P(0.1 < \hat{P}_1 - \hat{P}_2 < 0.2)$$ ← between 10% and 20%

$10\% = 0.1$

$20\% = 0.2$

$$= P\left(\frac{0.1 - 0.12}{\sqrt{0.00473}} < Z < \frac{0.2 - 0.12}{\sqrt{0.00473}}\right)$$

$$= P(-0.29 < Z < 1.16)$$

$$= 1 - P(Z > 0.29) - P(Z > 1.16)$$

$$= 1 - 0.3859 - 0.1230$$

$$= 0.4911$$