KOLEJ UNIVERSITI TUNKU ABDUL RAHMAN

FACULTY OF COMPUTING AND INFORMATION TECHNOLOGY

ACADEMIC YEAR 2020/2021

Assignment 2

MATHEMATICS AAMS3163
ALGEBRA

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Course Title: ALGEBRA
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Q1.

Let $A = \begin{bmatrix} 1 & 3 & 3 \\ 0 & 2 & 2 \\ 2 & 4 & 3 \end{bmatrix}$

Q1.(a) $[A|I] = \left[\begin{array}{ccc:ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ 0 & 2 & 2 & 0 & 1 & 0 \\ 2 & 4 & 3 & 0 & 0 & 1 \end{array}\right]$

$\xrightarrow{R_3 - R_1 \to R_3} \left[\begin{array}{ccc:ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ 0 & 2 & 2 & 0 & 1 & 0 \\ 1 & 1 & 0 & -1 & 0 & 1 \end{array}\right]$

$\xrightarrow{R_1 - R_2 \to R_1} \left[\begin{array}{ccc:ccc} 1 & 1 & 1 & 1 & -1 & 0 \\ 0 & 2 & 2 & 0 & 1 & 0 \\ 1 & 1 & 0 & -1 & 0 & 1 \end{array}\right]$

$\xrightarrow{R_1 - R_3 \to R_1} \left[\begin{array}{ccc:ccc} 0 & 0 & 1 & 2 & -1 & -1 \\ 0 & 2 & 2 & 0 & 1 & 0 \\ 1 & 1 & 0 & -1 & 0 & 1 \end{array}\right]$

$\xrightarrow{2R_1 - R_2 \to R_2} \left[\begin{array}{ccc:ccc|ccc} 0 & 0 & 1 & \cancel{2} & \cancel{-1} & \cancel{-1} & 2 & -1 & -1 \\ 0 & -2 & 0 & \cancel{2} & \cancel{-2} & \cancel{-1} & 4 & -3 & -2 \\ 1 & 1 & 0 & \cancel{-1} & \cancel{0} & \cancel{1} & -1 & 0 & 1 \end{array}\right]$

$\xrightarrow{-R_2 - R_3 \to R_2} \left[\begin{array}{ccc:ccc|ccc} 0 & 0 & 1 & \cancel{2} & \cancel{-1} & \cancel{-1} & 2 & -1 & -1 \\ -1 & 1 & 0 & \cancel{-1} & \cancel{2} & \cancel{0} & -3 & 3 & 1 \\ 1 & 1 & 0 & \cancel{-1} & \cancel{0} & \cancel{1} & -1 & 0 & 1 \end{array}\right]$

$\xrightarrow{R_3 - R_2 \to R_2} \left[\begin{array}{ccc:ccc|ccc} 0 & 0 & 1 & \cancel{2} & \cancel{-1} & \cancel{-1} & 2 & -1 & -1 \\ 2 & 0 & 0 & \cancel{0} & \cancel{-2} & \cancel{1} & 2 & -3 & 0 \\ 1 & 1 & 0 & \cancel{-1} & \cancel{0} & \cancel{1} & -1 & 0 & 1 \end{array}\right]$

$\xrightarrow{\frac{1}{2}R_2 \to R_2} \left[\begin{array}{ccc:ccc} 0 & 0 & 1 & 2 & -1 & -1 \\ 1 & 0 & 0 & 1 & -\frac{3}{2} & 0 \\ 1 & 1 & 0 & -1 & 0 & 1 \end{array}\right]$

$\xrightarrow{R_3 - R_2 \to R_3} \left[\begin{array}{ccc:ccc} 0 & 0 & 1 & 2 & -1 & -1 \\ 1 & 0 & 0 & 1 & -\frac{3}{2} & 0 \\ 0 & 1 & 0 & -2 & \frac{3}{2} & 1 \end{array}\right]$

$\xrightarrow[R_2 \leftrightarrow R_3]{R_2 \leftrightarrow R_1} \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 1 & -\frac{3}{2} & 0 \\ 0 & 1 & 0 & -2 & \frac{3}{2} & 1 \\ 0 & 0 & 1 & 2 & -1 & -1 \end{array}\right]$

$A^{-1} = \begin{bmatrix} 1 & -\frac{3}{2} & 0 \\ -2 & \frac{3}{2} & 1 \\ 2 & -1 & -1 \end{bmatrix}$

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Q1.(b)
$$\begin{bmatrix} 1 & 3 & 3 \\ 0 & 2 & 2 \\ 2 & 4 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \\ 9 \end{bmatrix}$$
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -\frac{3}{2} & 0 \\ -2 & \frac{3}{2} & 1 \\ 2 & -1 & -1 \end{bmatrix}\begin{bmatrix} 7 \\ 4 \\ 9 \end{bmatrix}$$
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$
$x=1, y=1, z=1$

Q2.
$$\begin{bmatrix} 1 & -1 & -2 \\ 2 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} +\begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} & -\begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix} & +\begin{vmatrix} 2 & 2 \\ 3 & 1 \end{vmatrix} \\ -\begin{vmatrix} -1 & -2 \\ 1 & 1 \end{vmatrix} & +\begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & -1 \\ 3 & 1 \end{vmatrix} \\ +\begin{vmatrix} -1 & -2 \\ 2 & 3 \end{vmatrix} & -\begin{vmatrix} 1 & -2 \\ 2 & 3 \end{vmatrix} & +\begin{vmatrix} 1 & -1 \\ 2 & 2 \end{vmatrix} \end{bmatrix}$$
$$= \begin{bmatrix} \cancel{-1} & \cancel{7} & \cancel{-4} \\ \cancel{-1} & \cancel{7} & \cancel{-4} \\ \cancel{1} & \cancel{-7} & \cancel{4} \end{bmatrix} \begin{bmatrix} -1 & 7 & -4 \\ -1 & 7 & -4 \\ 1 & -7 & 4 \end{bmatrix}$$
$$= \begin{bmatrix} -1 & -1 & 1 \\ 7 & 7 & -7 \\ -4 & -4 & 4 \end{bmatrix}$$

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Q2. $\left[\begin{array}{ccc|c} 1 & -1 & -2 & a \\ 2 & 2 & 3 & b \\ 3 & 1 & 1 & c \end{array}\right]$

$\xrightarrow{R_3 - R_2 \rightarrow R_2} \left[\begin{array}{ccc|c} 1 & -1 & -2 & a \\ 1 & -1 & -2 & c-b \\ 3 & 1 & 1 & c \end{array}\right]$

$\xrightarrow{R_2 - R_1 \rightarrow R_1} \left[\begin{array}{ccc|c} 0 & 0 & 0 & c-b-a \\ 1 & -1 & -2 & c-b \\ 3 & 1 & 1 & c \end{array}\right]$

$\cancel{c-b-a=0}$
$\cancel{a=c-b}$

$\cancel{x-y-2z=c-b}$
$\cancel{3x+y+z=c}$
$\cancel{0=c-b-a}$
$\cancel{a=c-b}$
$\cancel{a=c-b}$

$x-y-2z=c-b$
$3x+y+z=c$
$0=c-b-a$
$c=b+a \quad , \quad b,a \in R$

$\therefore c=b+a \quad b,a \in R$

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Q3.

$$\text{Let } A = \begin{bmatrix} 1 & 4 & 2 & 3 \\ 2 & 2 & 6 & 1 \\ 3 & 8 & 4 & 3 \\ 2 & 1 & 2 & 1 \end{bmatrix}$$

$$\xrightarrow[R_2 - R_4 \to R_2]{R_3 - 2R_1 \to R_3} \begin{bmatrix} 1 & 4 & 2 & 3 \\ 0 & 1 & 4 & 0 \\ 1 & 0 & 0 & -3 \\ 2 & 1 & 2 & 1 \end{bmatrix}$$

$$\xrightarrow{R_3 \leftrightarrow R_4} \begin{bmatrix} 1 & 4 & 2 & 3 \\ 0 & 1 & 4 & 0 \\ 2 & 1 & 2 & 1 \\ 1 & 0 & 0 & -3 \end{bmatrix}$$

$$\xrightarrow{R_1 - R_3 \to R_1} \begin{bmatrix} -1 & 3 & 0 & 2 \\ 0 & 1 & 4 & 0 \\ 2 & 1 & 2 & 1 \\ 1 & 0 & 0 & -3 \end{bmatrix}$$

$$\xrightarrow{R_1 - R_2 \to R_1} \begin{bmatrix} -1 & 2 & -4 & 2 \\ 0 & 1 & 4 & 0 \\ 2 & 1 & 2 & 1 \\ 1 & 0 & 0 & -3 \end{bmatrix}$$

$$\xrightarrow{R_3 - 2R_4 \to R_3} \begin{bmatrix} -1 & 2 & -4 & 2 \\ 0 & 1 & 4 & 0 \\ 0 & 1 & 2 & 7 \\ 1 & 0 & 0 & -3 \end{bmatrix}$$

$$\xrightarrow{R_2 - R_3 \to R_3} \begin{bmatrix} -1 & 2 & -4 & 2 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 2 & -7 \\ 1 & 0 & 0 & -3 \end{bmatrix}$$

$$\xrightarrow{R_1 + R_4 \to R_1} \begin{bmatrix} 0 & 2 & -4 & -1 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 2 & -7 \\ 1 & 0 & 0 & -3 \end{bmatrix}$$

$$\xrightarrow{-2R_3 + R_2 \to R_2} \begin{bmatrix} 0 & 2 & -4 & -1 \\ 0 & 1 & 0 & 14 \\ 0 & 0 & 2 & -7 \\ 1 & 0 & 0 & -3 \end{bmatrix}$$

$$\xrightarrow{2R_2 - R_1 \to R_1} \begin{bmatrix} 0 & 0 & 4 & 29 \\ 0 & 1 & 0 & 14 \\ 0 & 0 & 2 & -7 \\ 1 & 0 & 0 & -3 \end{bmatrix}$$

$$\xrightarrow{2R_3 - R_1 \to R_1} \begin{bmatrix} 0 & 0 & 0 & -43 \\ 0 & 1 & 0 & 14 \\ 0 & 0 & 2 & -7 \\ 1 & 0 & 0 & -3 \end{bmatrix}$$

$$\xrightarrow{R_1 \leftrightarrow R_4} \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 14 \\ 0 & 0 & 2 & -7 \\ 0 & 0 & 0 & -43 \end{bmatrix}$$

$$\begin{aligned} \operatorname{Det} A &= |(1 \times 1 \times 2 \times -43)| \\ &= |-86| \\ &= 86 \end{aligned}$$

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Q4. $\begin{vmatrix} 3 & x & 5 \\ 1 & 2 & x+2 \\ x & 2 & -3 \end{vmatrix} = 0$

$0 = 3\begin{vmatrix} 2 & x+2 \\ 2 & -3 \end{vmatrix} - x\begin{vmatrix} 1 & x+2 \\ x & -3 \end{vmatrix} + 5\begin{vmatrix} 1 & 2 \\ x & 2 \end{vmatrix}$

$= 3(-6-(2x+4)) - x(-3-(x^2+2x)) + 5(2-2x)$

$= -30-6x+3x+x^3+2x^2+10-10x$

$= x^3+2x^2-13x-20$

~~$x = 3.45, x = -4, x = -1.45$~~

$= (x+4)(x^2-2x-5)$

~~$x = -4, x = 3.45, x = -1.45$~~

$x = -4$, $a = 1, b = -2, c = -5$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \dfrac{-(-2) \pm \sqrt{(-2)^2-4(1)(-5)}}{2(1)}$

$= \dfrac{2 \pm \sqrt{24}}{2}$

$x = \dfrac{2+\sqrt{24}}{2}$ $\qquad$ $x = \dfrac{2-\sqrt{24}}{2}$

$= 3.45$ $\qquad$ $= -1.45$

$\therefore x = -4, x = 3.45, x = -1.45$

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Q5. ~~Let A~~ $A = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$ $\det(A) = 7$

Q5.(a) $\det(11A) = 11^3 \det A$
$= 1331(7)$
$= 9317$

Q5.(b) $\det((A^{-1})^T) = 1^3 \det(A^{-1})^T$
$= \dfrac{1}{1^3 \det(A)^T}$
$= \dfrac{1}{7}$

$A^T = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ $\det(A)^T = 7$

Q5.(c) $\cancel{\det\left(\begin{bmatrix} d & e & f \\ g & h & i \\ a & b & c \end{bmatrix}\right) = \begin{vmatrix} a & d & g \\ b & e & h \\ c & f & i \end{vmatrix}}$
$\cancel{= (-1)(-1)(7)}$
$\cancel{= 7}$

Q5.(c) $\det\left(\begin{bmatrix} d & e & f \\ g & h & i \\ a & b & c \end{bmatrix}\right) = \underset{R_1 \leftrightarrow R_2}{(-1)}\ \underset{R_2 \leftrightarrow R_3}{(-1)} \begin{vmatrix} a & d & g \\ b & e & h \\ c & f & i \end{vmatrix}$

$= (-1)(-1)(7)$
$= 7$

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Q6. $A = \begin{pmatrix} 1 & 0 \\ 5 & 2 \end{pmatrix}$

$\lambda I - A = \begin{bmatrix} \lambda-1 & 0 \\ -5 & \lambda-2 \end{bmatrix}$

$|\lambda I - A| = 0$

$(\lambda-1)(\lambda-2) + 0 = 0$

$\lambda_1 = 1,\ \lambda_2 = 2$ $\therefore$ eigen value : 1, 2

$\lambda_1 = 1$, $\left[\begin{array}{cc|c} 1-1 & 0 & 0 \\ -5 & 1-2 & 0 \end{array}\right]$

$= \left[\begin{array}{cc|c} 0 & 0 & 0 \\ -5 & -1 & 0 \end{array}\right]$

$-5x_1 - x_2 = 0$

Let $x_2 = 5$, $x_1 = -1$

$\therefore$ eigenvector $V_1 = \begin{pmatrix} -1 \\ 5 \end{pmatrix}$, $V_1 = (-1, 5)$

$\lambda_2 = 2$, $\left[\begin{array}{cc|c} 2-1 & 0 & 0 \\ -5 & 2-2 & 0 \end{array}\right]$

$\xrightarrow{5R_1 + R_2 \to R_2} \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$

$1x_1 - 0x_2 = 0$

Let $x_2 = 1$, $x_1 = 0$

$\therefore V_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, $V_2 = (0, 1)$

$\therefore$ eigenvector $V_1 = (-1, 5)$, $V_2\ (0, 1)$

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Q7.

$(AB)^{-1} = (A)^{-1}(B)^{-1}$

$A^T = A$
$(A^{-1})^T = (A^T)^{-1}$
$= A^{-1}$ $\therefore A^{-1}$ is symmetric

$B^T = B$
$(B^{-1})^T = (B^T)^{-1}$
$= B^{-1}$ $\therefore B^{-1}$ is symmetric

$(AB)^T = A^T B^T$
$= AB$

$[(AB)^{-1}]^T = [(A^{-1})(B^{-1})]^T$
$= [(A^T)(B^T)]^{-1}$
$= (AB)^{-1}$

$\therefore (AB)^{-1}$ is symmetric

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