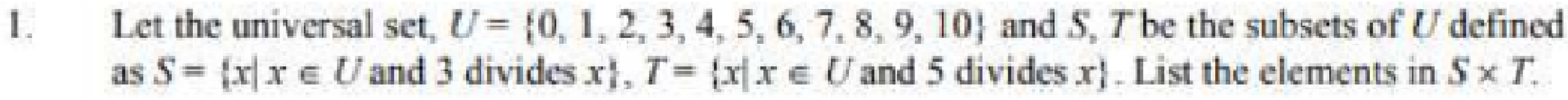
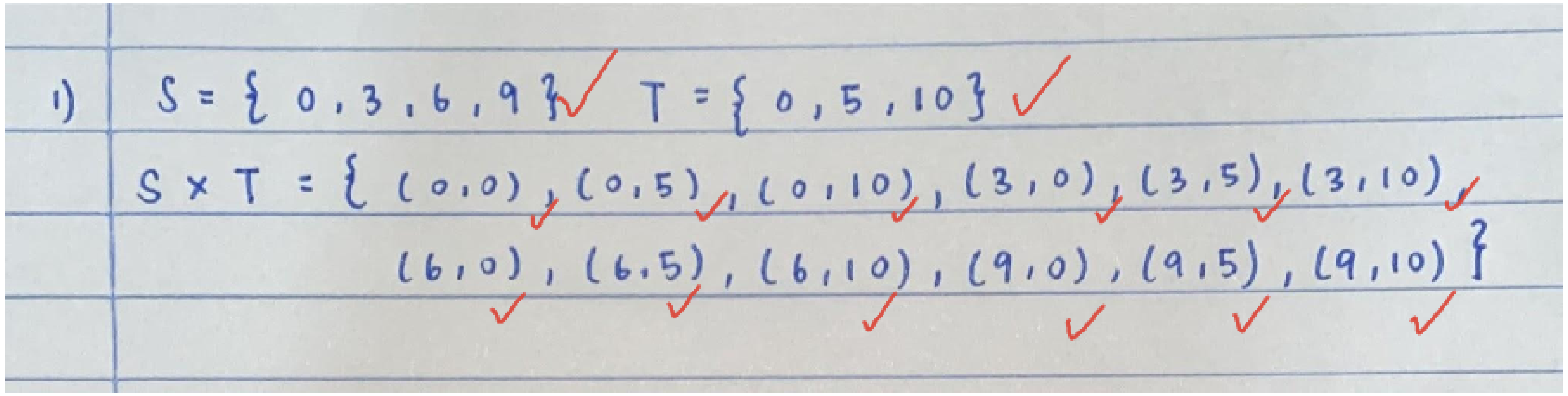
Tutorial 7



1. Let the universal set, $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and S, T be the subsets of U defined as $S = \{x \mid x \in U \text{ and 3 divides } x\}$, $T = \{x \mid x \in U \text{ and 5 divides } x\}$. List the elements in $S \times T$.



1) $S = \{0, 3, 6, 9\}$ ✓ $T = \{0, 5, 10\}$ ✓

$S \times T = \{(0,0)$, ✓ $(0,5)$, ✓ $(0,10)$, ✓ $(3,0)$, ✓ $(3,5)$, ✓ $(3,10)$, ✓
$(6,0)$, ✓ (6.5), ✓ $(6,10)$, ✓ $(9,0)$, ✓ $(9,5)$, ✓ $(9,10)\}$ ✓

2. Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $A_1 = \{1, 2, 3, 4\}$, $A_2 = \{5, 6, 7\}$, $A_3 = \{4, 5, 7, 9\}$, $A_4 = \{4, 8, 10\}$, $A_5 = \{8, 9, 10\}$, $A_6 = \{1, 2, 3, 6, 8, 10\}$.
List the possible partitions of A.

1) $A_1 \cap A_2 \cap A_5 = \emptyset$

2) $A_1 \cup A_2 \cup A_5 = A$

1) $A_3 \cap A_6 = \emptyset$ ✓

2) $A_3 \cup A_6 = A$ ✓

$\{A_1, A_2, A_5\}$ and $\{A_3, A_6\}$ are partitions of A ✓ ✓

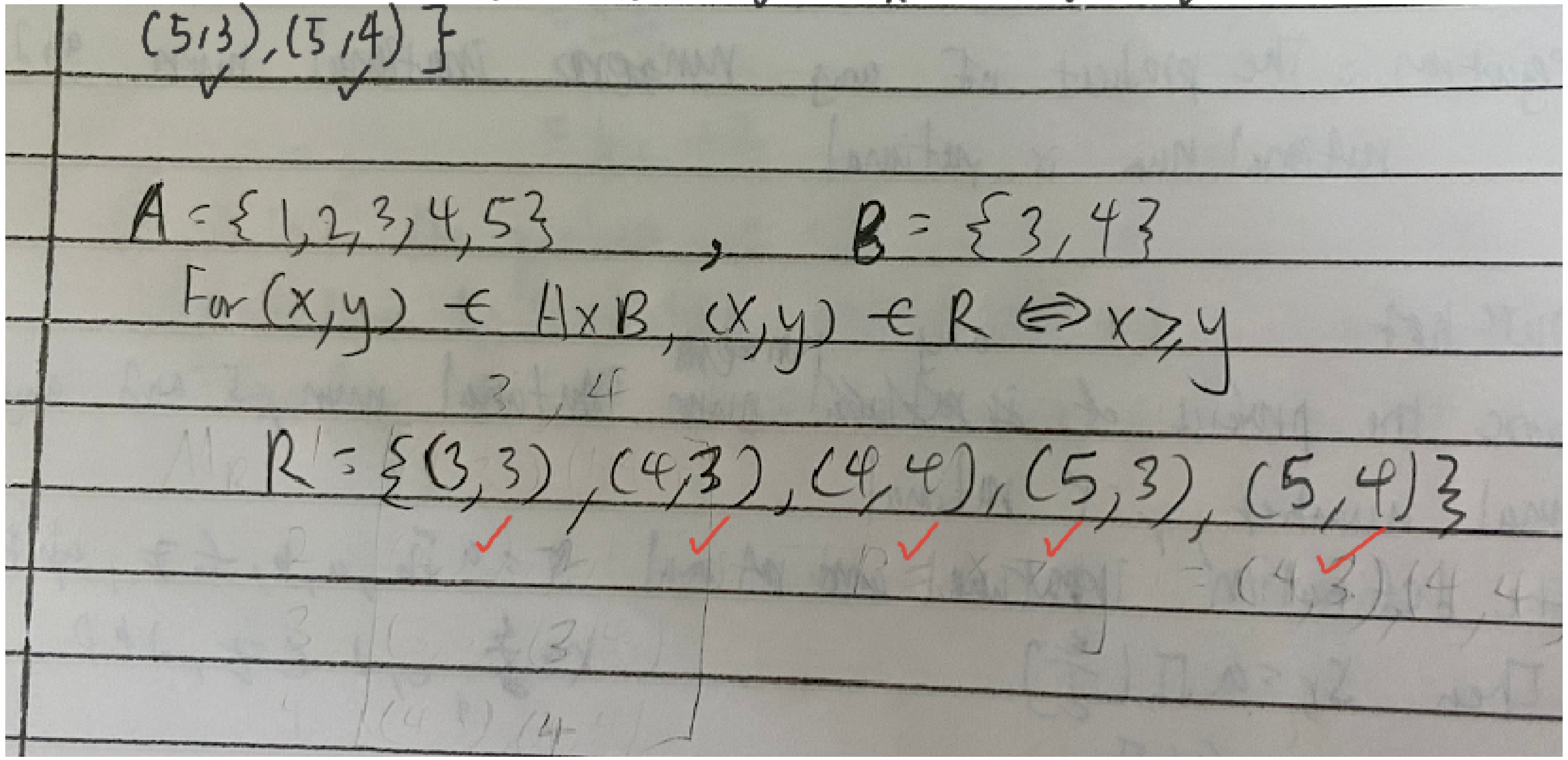
List the possible partitions of A.

3. Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{3, 4\}$ and define a binary relation R from A to B as follows:

For $(x, y) \in A \times B$, $(x, y) \in R \Leftrightarrow x \geq y$.

Write R as a set of ordered pairs.

optional: $A \times B = \{\underset{\times}{(1,3)}, \underset{\times}{(1,4)}, \underset{\times}{(2,3)}, \underset{\times}{(2,4)}, \underset{\checkmark}{(3,3)}, \underset{\times}{(3,4)}, \underset{\checkmark}{(4,3)}, \underset{\checkmark}{(4,4)}, \underset{\checkmark}{(5,3)}, \underset{\checkmark}{(5,4)}\}$



$A = \{1,2,3,4,5\}$, $B = \{3,4\}$

For $(x,y) \in A \times B$, $(x,y) \in R \Leftrightarrow x \geq y$

3, 4

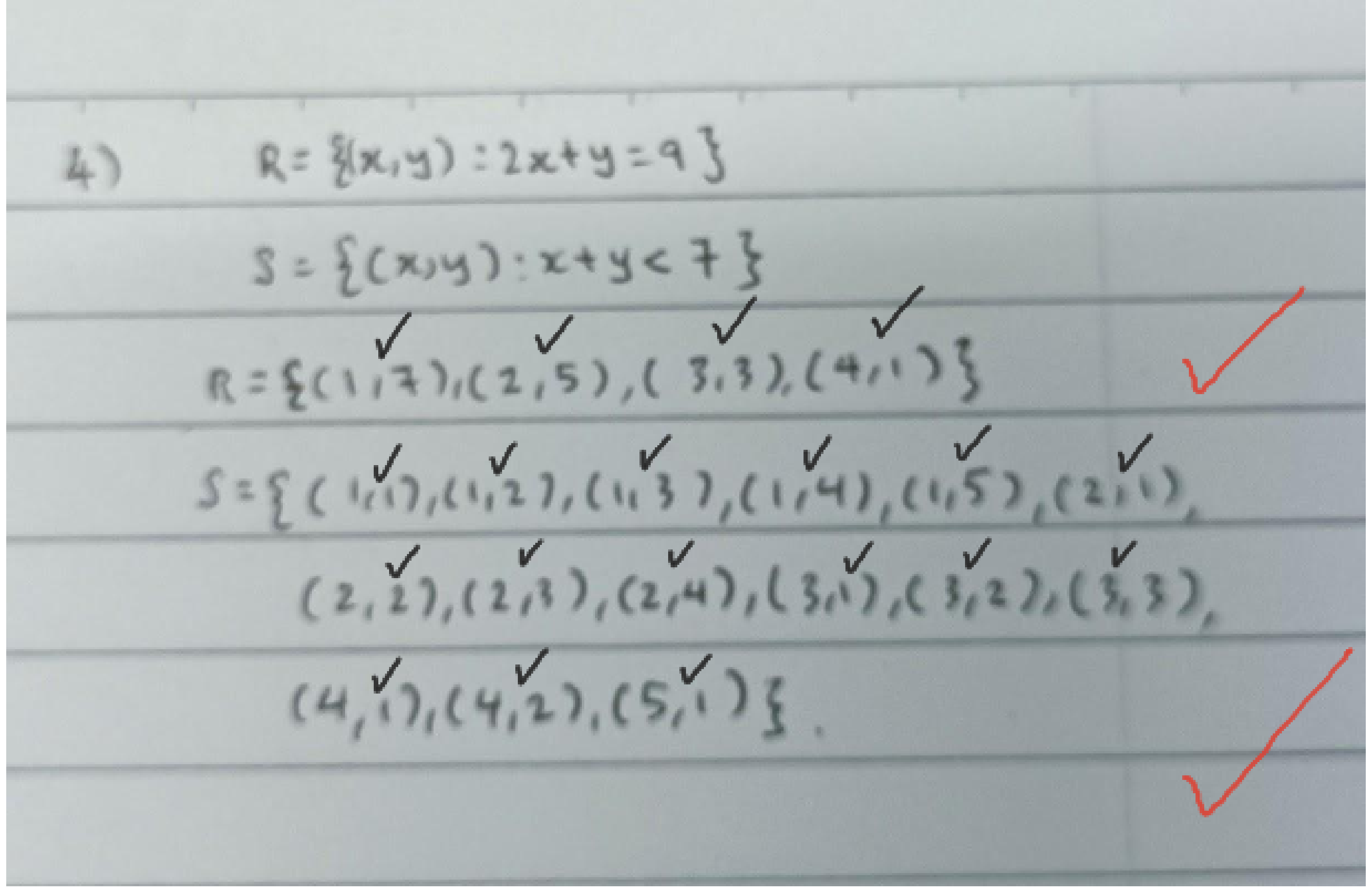
$R = \{\underset{\checkmark}{(3,3)}, \underset{\checkmark}{(4,3)}, \underset{\checkmark}{(4,4)}, \underset{\checkmark}{(5,3)}, \underset{\checkmark}{(5,4)}\}$

Natural number, $N = \{1, 2, 3, 4, 5, \dots\}$

4. For each of the following relation on N, list the ordered pairs that belong to the relation.

$R = \{(x, y): 2x + y = 9\}$

$S = \{(x, y): x + y < 7\}$



4) $R = \{(x,y) : 2x+y=9\}$

$S = \{(x,y) : x+y<7\}$

$R = \{(1,7),(2,5),(3,3),(4,1)\}$ ✓

$S = \{(1,1),(1,2),(1,3),(1,4),(1,5),(2,1),$

$(2,2),(2,3),(2,4),(3,1),(3,2),(3,3),$

$(4,1),(4,2),(5,1)\}.$ ✓

5. Let $A = \{1, 3, 5, 7\}$ and R be the relation on A whose matrix is given below.

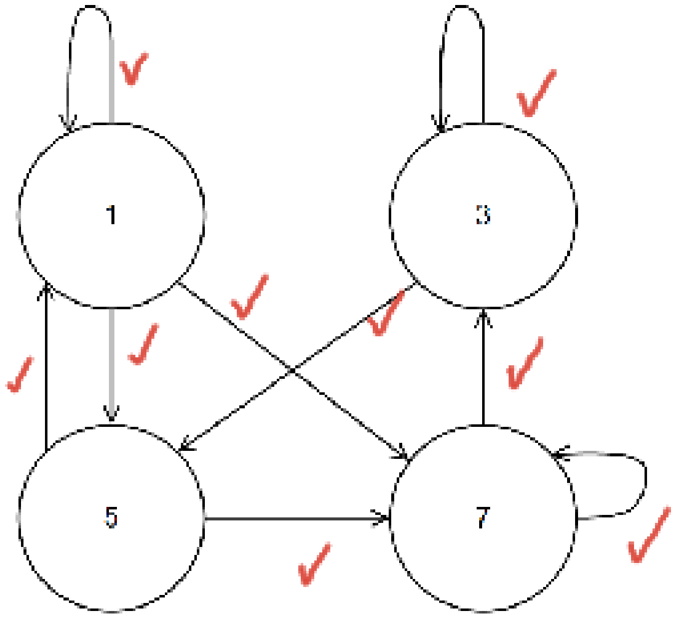
$$\mathbf{M}_R = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

- i) Write R as a set of ordered pairs.
- ii) Draw the digraph of R.
- iii) Find the domain and range of R.
- iv) Give the in-degree and out degree of each vertex.

$$\begin{array}{c|cccc} & 1 & 3 & 5 & 7 \\ \hline 1 & 1 & 0 & 1 & 1 \\ 3 & 0 & 1 & 1 & 0 \\ 5 & 1 & 0 & 0 & 1 \\ 7 & 0 & 1 & 0 & 1 \end{array}$$

i)R= {(1,1),(1,5),(1,7),(3,3),(3,5),(5,1),(5,7),(7,3),(7,7)}

ii)



iii)Domain(R)=Range(R)= {1,3,~~4~~ 7,5}

iv)

	Vertex	1	3	5	7	
column	In-degree	2 ✓	2 ✓	2 ✓	3 ✓	9
row	Out-degree	3 ✓	2 ✓	2 ✓	2 ✓	9

6. Let R be the relation on {1, 2, 3, 4} given by $u \, R \, v$ if and only if $u + 2v$ is odd. Represent R in each of the following ways:

i) as a set of ordered pairs; ii) in graphical form;

iii) in matrix form.

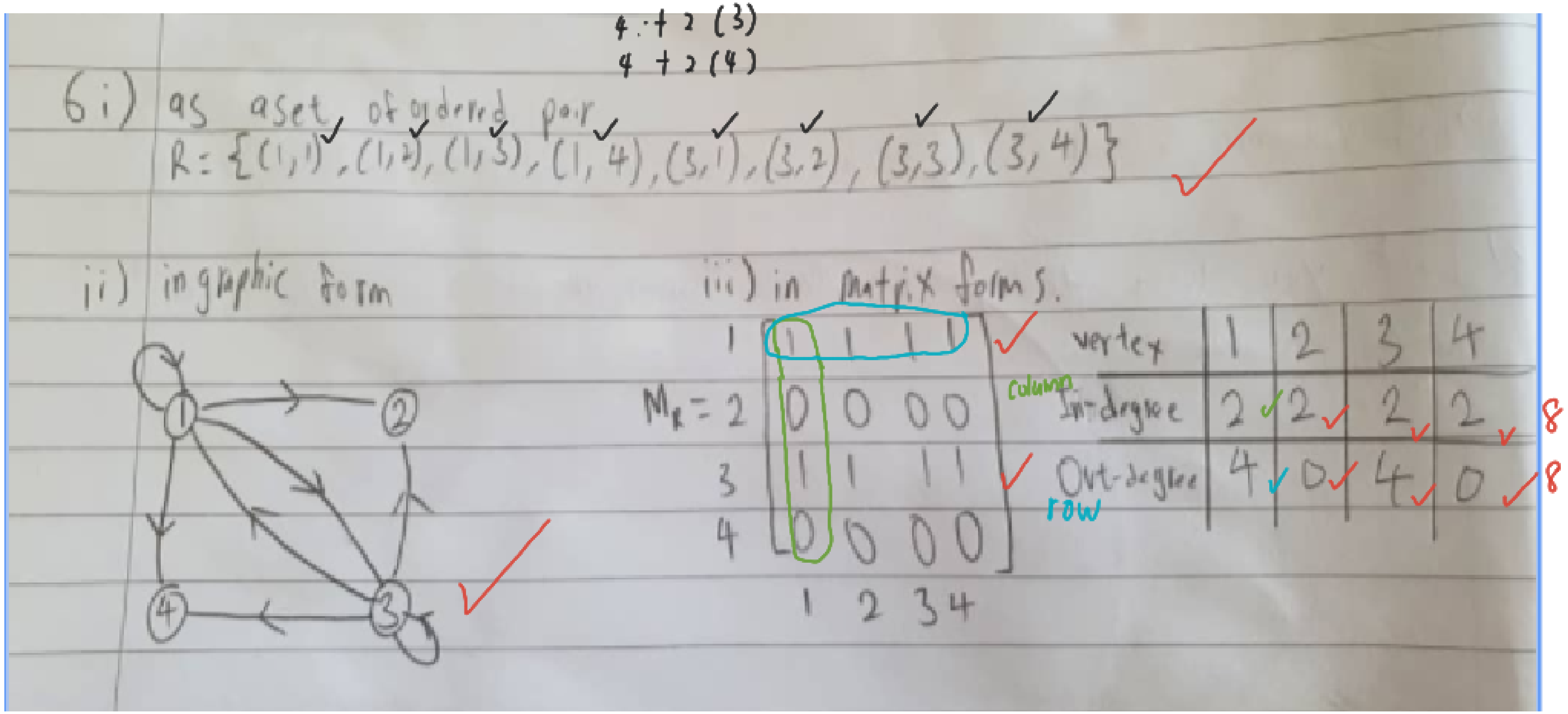
Give the in-degree and out-degree of each vertex.

$4 + 2(1)$

$4 + 2(2)$

$4 + 2(3)$

$4 + 2(4)$



6i) as aset of ordered pair

$R = \{(1,1),(1,2),(1,3),(1,4),(3,1),(3,2),(3,3),(3,4)\}$

ii) in graphic form

iii) in matrix forms.

$$M_R = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ \\ \end{array} \begin{array}{cccc} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 2 & 3 & 4 \end{array}$$

column

row

vertex	1	2	3	4	
In-degree	2	2	2	2	8
Out-degree	4	0	4	0	8

7. Find the domain ✓, range ✓, matrix ✓, and, when $A = B$, the digraph of the relation R.

i) $A = \{1, 2, 3, 4, 8\} = B$; $a\,R\,b$ if and only if $a = b$.

ii) $A = \{1, 2, 3, 4, 6\} = B$; $a\,R\,b$ if and only if a is a multiple of b.

iii) $A = \{1, 3, 5, 7, 9\}$, $B = \{2, 4, 6, 8\}$; $a\,R\,b$ if and only if $b < a$.

(x, y)

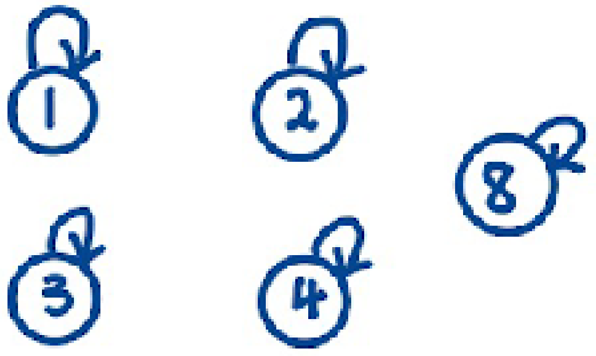
Domain Range

domain (R) : $\{1.2.3.4.8\}$ ✓

range (R) : $\{1.2.3.4.8\}$ ✓

$$\text{matrix}(R) = \begin{array}{cc|ccccc|} & & & & & B & \\ & & 1 & 2 & 3 & 4 & 8 \\ \hline & 1 & 1 & 0 & 0 & 0 & 0 \\ & 2 & 0 & 1 & 0 & 0 & 0 \\ A & 3 & 0 & 0 & 1 & 0 & 0 \\ & 4 & 0 & 0 & 0 & 1 & 0 \\ & 8 & 0 & 0 & 0 & 0 & 1 \\ \hline \end{array}$$ ✓

$R = \{(1,1).(2.2).(3.3).(4.4).(8.8)\}$ ✓



✓

multiple of 2 : 2x1, 2x2, 2x3, ... = 2, 4, 6, ...

multiple of 1 : 1x1, 1x2, 1x3, ... = 1, 2, 3, ...

1 is a multiple of 1

2 is a multiple of 1

3 is a multiple of 1

⋮

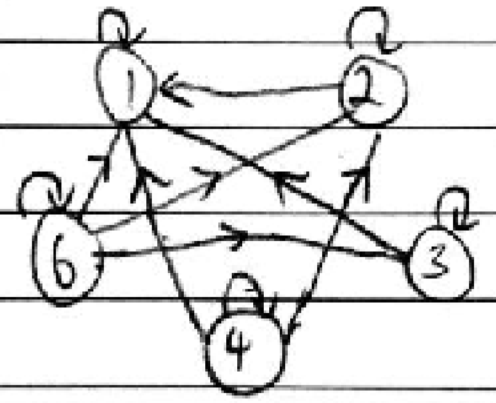
⑦ ii) $A = \{1,2,3,4,6\}$ $B = \{1,2,3,4,6\}$

Domain(R) = $\{1,2,3,4,6\}$ ✓

Range (R) = $\{1,2,3,4,6\}$ ✓

$$M(R) = \begin{array}{cc} & b \\ a & \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 6 \\ \hline 1 & 1 & 0 & 0 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 & 0 \\ 3 & 1 & 0 & 1 & 0 & 0 \\ 4 & 1 & 1 & 0 & 1 & 0 \\ 6 & 1 & 1 & 1 & 0 & 1 \end{array} \end{array}$$ ✓

$R = \{(1,1), (2,1), (2,2), (3,1), (3,3), (4,1), (4,2), (4,4), (6,1), (6,2), (6,3), (6,6)\}$ ✓



✓

7.

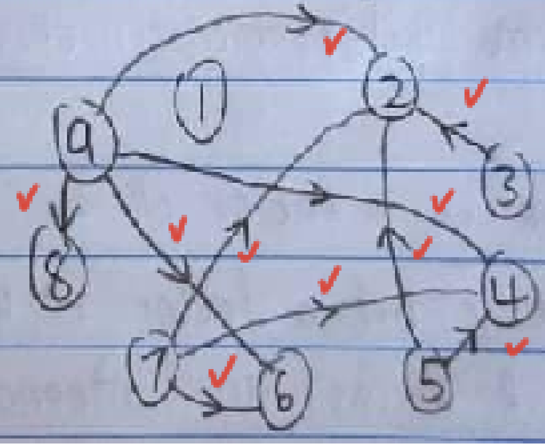
iii) $A = \{1,3,5,7,9\}$, $B = \{2,4,6,8\}$; aRb if and only if $b < a$

$R = \{(3,2), (5,2), (5,4), (7,2), (7,4), (7,6), (9,2),$ ✓ $---(9,8)\}$ ✓ ✓ ✓ ✓ ✓ ✓ ✓

(9,4), (9,6),

Dom (R) = $\{3,5,7,9\}$ ✓ Ran (R) = $\{2,4,6,8\}$ ✓

$$M_R = \begin{array}{c|cccc} 1 & 0 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 & 0 \\ 5 & 1 & 1 & 0 & 0 \\ 7 & 1 & 1 & 1 & 0 \\ 9 & 1 & 1 & 1 & 1 \\ & 2 & 4 & 6 & 8 \end{array}$$ ✓

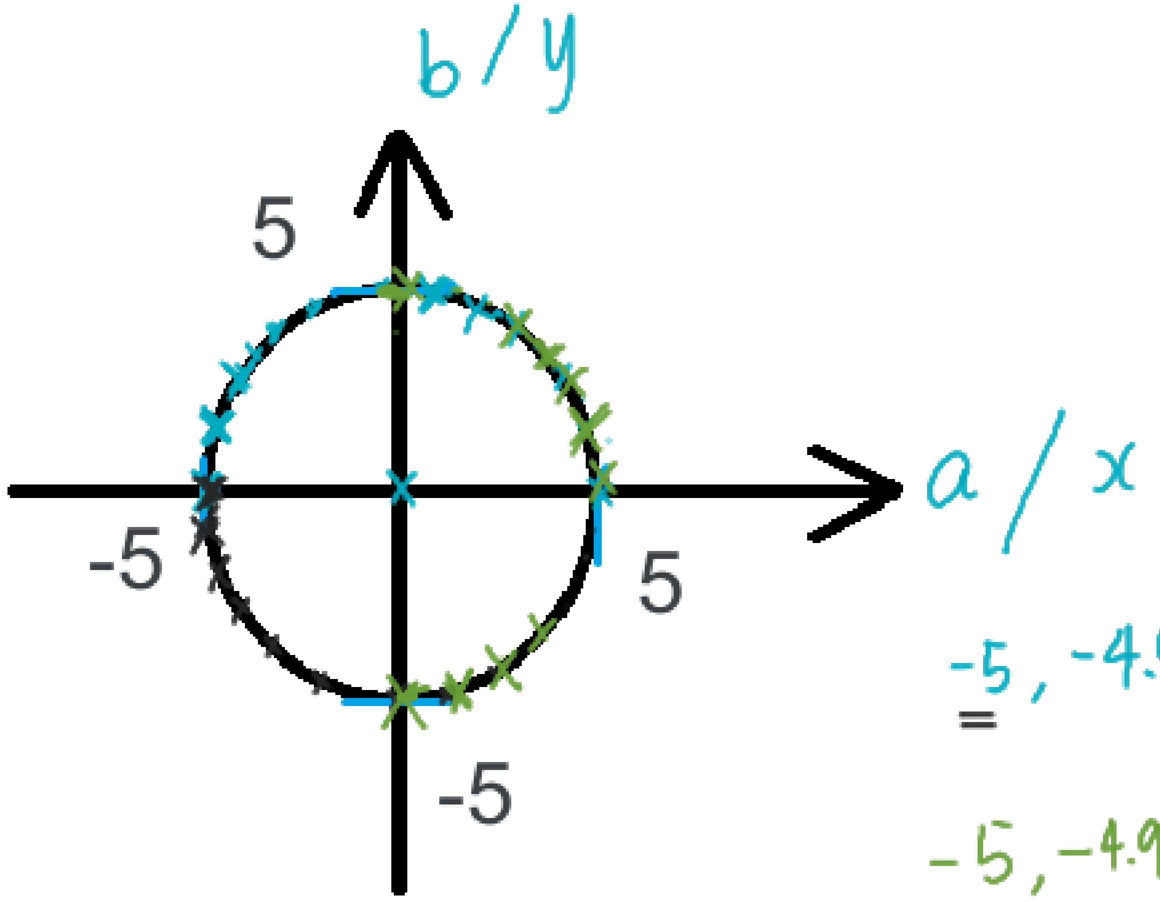


8. Let $A = \mathbb{R}$, set of real numbers. Consider the following relation R on A: $a\ R\ b$ if and only if $\boxed{a^2 + b^2 = 25}$. Find Dom (R) and Ran (R).

$(a-0)^2 + (b-0)^2 = 25$

↳ center: $(0,0)$

radius: $\sqrt{25} = 5$



Dom(R): $-5 \leq x \leq 5$

Ran(R): $-5 \leq y \leq 5$ ✓

$\underline{\underline{-5}}, -4.92, -4.6, -3, 0, 0.5, \ldots \underline{\underline{5}}$: $-5 \leq a \leq 5$

$\underline{\underline{-5}}, -4.96, \ldots 0, \ldots \underline{\underline{5}}$: $-5 \leq b \leq 5$

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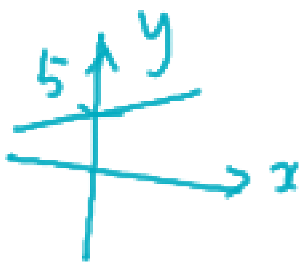
① straight line

$y = mx + c$ → slope, m ; y-intercep, c

$y = 2x + 5$

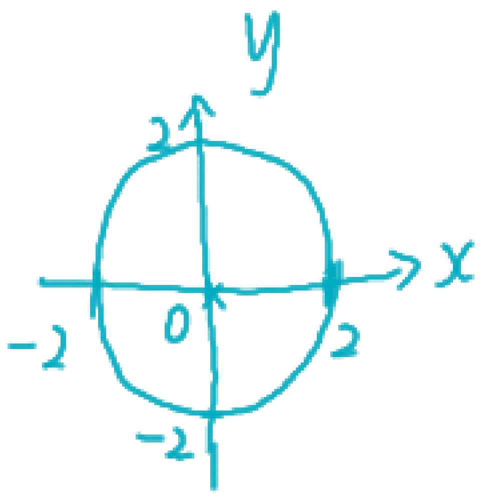


② circle equation

$(\underbrace{x-p}_{})^2 + (\underbrace{y-q}_{})^2 = r^2$ → center $= (p, q)$; radius $= \sqrt{r^2} = r$

e.g. center: $(0,0)$, radius $= \sqrt{4} = 2$

$(x-0)^2 + (y-0)^2 = 4$



9. Let $A = \{1, 2, 3, 4, 6\}$ and R be the relation defined as $a\,R\,b$ if and only if a is a multiple of b. Find each of the following.

i) $R(3)$ ii) $R(6)$ iii) $R(\{2, 4, 6\})$

,(2,1), (3,1), (4,1), (6,1), ,(4,2), (6,2) (6,3),

R= {(1,1) ✓ ~~(1,2) (1,3) (1,4) (1,6)~~ (2,2) ✓ ~~(2,4) (2,6)~~ (3,3) ✓ ~~(3,6)~~ (4,4) ✓ (6,6) ✓}

i) R (3) = {1,3}

ii) R (6) = {1,2,3,6}

iii) R ({2,4,6}) = {1,2,3,4,6} ✓

$= R(2) \cup R(4) \cup R(6)$

$= \{1,2\} \cup \{1,2,4\} \cup \{1,2,3,6\}$

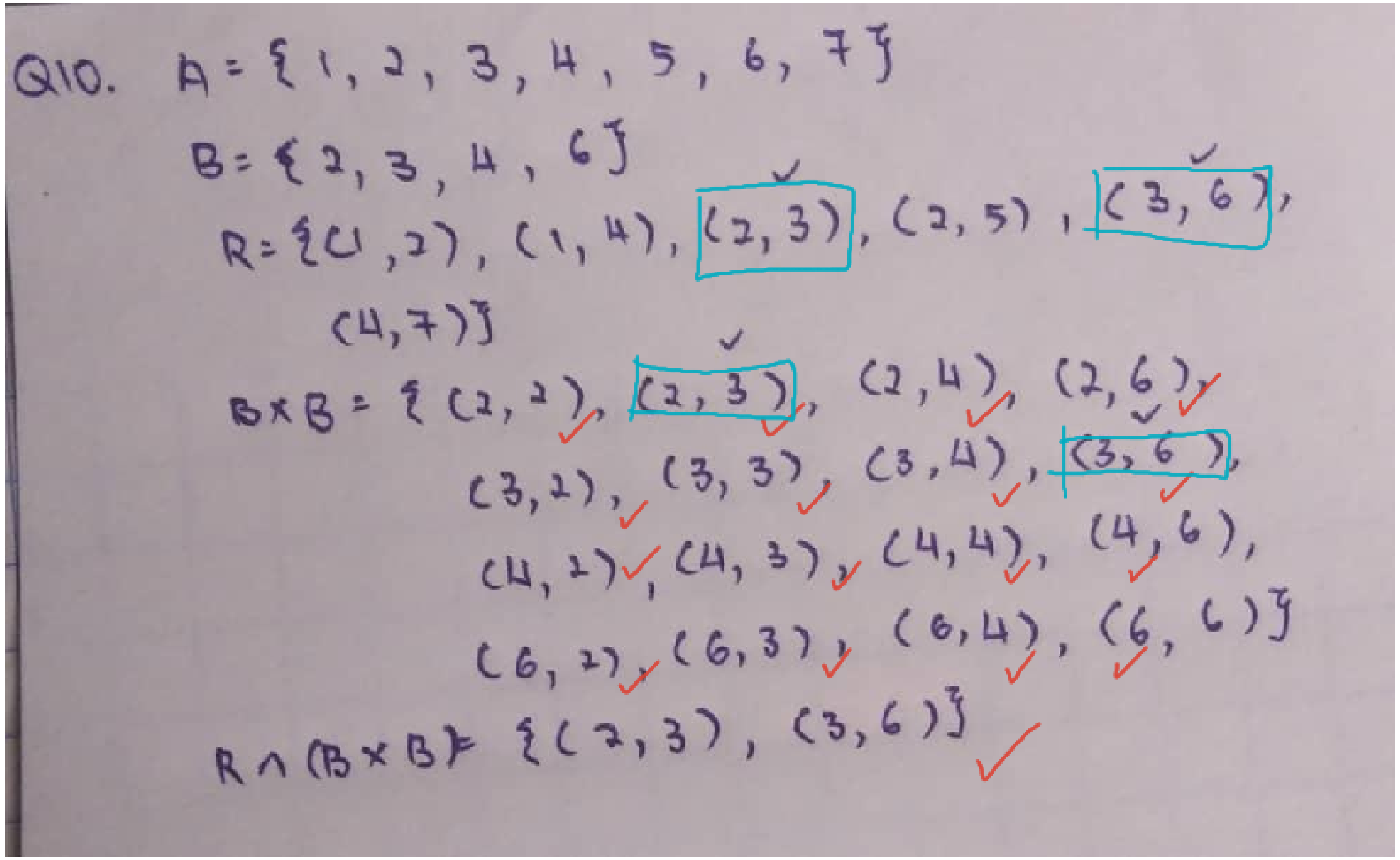
multiple of 1 : 1×1, 1×2, 1×3, 1×4, ... = 1,2,3,4, ...

multiple of 3 : 3×1, 3×2, 3×3, 3×4, .. = 3,6,9,12, ...

3 is a multiple of 3
6 is a multiple of 3
9 is a multiple of 3
⋮

1 is a multiple of 1
✓2 is a multiple of 1
✓3 is a multiple of 1
⋮

10. Let $A = \{1, 2, 3, 4, 5, 6, 7\}$, $B = \{2, 3, 4, 6\}$, and $R = \{(1, 2), (1, 4), (2, 3), (2, 5), (3, 6), (4, 7)\}$. Compute the restriction of R to B.



Q10. $A = \{1, 2, 3, 4, 5, 6, 7\}$

$B = \{2, 3, 4, 6\}$

$R = \{(1, 2), (1, 4), \boxed{(2, 3)}, (2, 5), \boxed{(3, 6)},$
$(4, 7)\}$

$B \times B = \{(2, 2), \boxed{(2, 3)}, (2, 4), (2, 6),$
$(3, 2), (3, 3), (3, 4), \boxed{(3, 6)},$
$(4, 2), (4, 3), (4, 4), (4, 6),$
$(6, 2), (6, 3), (6, 4), (6, 6)\}$

$R \cap (B \times B) = \{(2, 3), (3, 6)\}$