Jing Xian

OK

Q1. An electrical firm manufactures light bulbs that have a length of life that is approximately normally distributed with a standard deviation of 40 hours. If a random sample of 30 bulbs has an average life of 780 hours, find a 96% confidence interval for the population mean of all bulbs produced by this firm.

1)

$\sigma = 40$

$n = 30$

$\bar{x} = 780$

96% CI $\Rightarrow \alpha = 4\% = 0.04 \Rightarrow \alpha/2 = 0.02$

Since σ is given, we will use Z dist $\Rightarrow Z_{0.02} = 2.0537$

The 96% confidence interval for the population mean of all bulbs produced by this firm is

$$\bar{x} \pm Z_{0.02} \frac{\sigma}{\sqrt{n}} = 780 \pm 2.0537 \frac{40}{\sqrt{30}}$$

$$= [765.0019, 794.9981] \text{ (hours)}$$

Eason

15K

Q2. A sample of 50 college students showed mean height of 167.16 cm and standard deviation of 6.86 cm.

(a) Estimate the mean height of all college students.

(b) Construct a 98% confidence interval for the mean height of all college students.

2. $n=50$, $\bar{x}=167.16$, $\cancel{\sigma}\ s=6.86$

a) estiminated mean, $\bar{x} = 167.16$ cm $= \hat{\mu}$

b) 98% CI $\Rightarrow \alpha = 0.02 \Rightarrow \alpha/2 = 0.01$

Since σ is not given but $n > 30$, Z-distribution (Z) is used, $Z_{0.01} = 2.3263$

The 98% confidence interval for the mean height of all college students

$$\bar{x} \pm Z_{0.01}\frac{s}{\sqrt{n}} = 167.16 \pm 2.3263\frac{6.86}{\sqrt{50}}$$

$$= [164.9031,\ 169.4169]\ \text{(cm)}$$

a) Question asks for $\hat{\mu}$,

$\hat{\mu} = \bar{x} = \ldots$ point estimate

Kang Hong Q3. A random sample of eight cigarettes of a certain brand has average nicotine content of 18.6 milligrams and a standard deviation of 2.4 milligrams. Construct a 99% confidence interval for the true average nicotine content of this particular brand of cigarettes, assuming an approximately normal distribution.

Sample (specific) info: sample statistics

OK. 9/8/2021

3. $s = 2.4$ $df = 8-1 = 7$ $\bar{x} = 18.6$

Since σ is unknown, $n < 30$, t-dist is used.

99% CI $\Rightarrow \alpha = 0.01 \Rightarrow \alpha/2 = 0.005 \Rightarrow t_{0.005,7} = 3.499$

The 99% confidence interval for the ~~mean~~ true average nicotine content of this particular brand of cigrattes

$$\bar{x} \pm t_{0.005,7} \frac{s}{\sqrt{n}} = 18.6 \pm 3.499 \frac{2.4}{\sqrt{8}}$$

$$= 18.6 \pm 2.9690$$

$$= [15.6310, 21.5690] \text{mg}$$

Chun Wai

Q4. A machine is producing metal pieces that are cylindrical in shape. A sample of pieces is taken, and the diameters are 1.01, 0.97, 1.03, 1.04, 0.99, 0.98, 0.99, 1.01 and 1.03 inches. Find a 99% confidence interval for the mean diameter of all pieces from this machine, assuming a normal distribution.

ok

$\Sigma x^2 = 9.1051$

$\Sigma x = 9.05$

[Small, big]

$\bar{x} - E$ $\quad$ $\bar{x} + E$

Walao ei
Hardworking

Tarikh:

4. 1.01, 0.97, 1.03, 1.04, 0.99, 0.98, 0.99, 1.01, 1.03

σ^2 is unknown → $n = 9 < 30$ → t distribution → need to compute s

$df = v = n-1 = 8$

$$s = \sqrt{\frac{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}{n-1}}$$

$$= \sqrt{\frac{9.1051 - \frac{(9.05)^2}{9}}{9-1}}$$

$$= 0.0246$$

99% confidence interval → $\alpha = 0.01$ → $\frac{\alpha}{2} = 0.005$ → t

$\alpha = 0.01$

$\frac{\alpha}{2} = 0.005$

$t_{0.005,8} = 3.355$

The 99% confidence interval for the mean diameter of all pieces from this machine is

$$\bar{x} \pm t_{0.005,8} \frac{s}{\sqrt{n}} = \frac{9.05}{9} \pm 3.355 \frac{0.0246}{\sqrt{9}}$$

$$= \left[\frac{9.05}{9} + 3.355 \frac{0.0246}{\sqrt{9}}, \frac{9.05}{9} - 3.355 \frac{0.0246}{\sqrt{9}}\right]$$

$$= \boxed{[1.0331, 0.9780]}$$ inches

Janet

OK.

Q5. According to a survey of 1500 adults conducted by the National Sleep Foundation, 1125 of them said that they have symptoms of sleep problems such as frequent walking during the night or snoring.

(a) What is the point estimate of the population proportion?

(b) Find a 97% confidence interval for the percentage of all adults who have such symptoms.

sample mean

$\hat{p}$

Q5. A $\cancel{\bar{x}} = \frac{\Sigma x}{n} = \frac{1125}{1500} \doteq 0.75$

B 97% CI $\Rightarrow \alpha = 0.03 \Rightarrow \frac{\alpha}{2} = 0.015 \Rightarrow Z_{0.015} = 2.1701$

$$\hat{p} \pm Z_{0.015}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.75 \pm (2.1701)\sqrt{\frac{0.75(1-0.75)}{1500}}$$

$$= [0.7257, 0.7743] \times 100$$

$$= [72.57, 77.43] \ (\%)$$

$\therefore$ 97% confidence interval for the percentage of adults who have such symptoms is between 72.57% to 77.43%

Yee Hao Q6. A large corporation is concerned about the declining quality of medical services provided by their group health insurance. A random sample of 100 office visits by employees of this corporation to primary care physicians last year found that the doctors spent an average of 19 minutes with each patient. This year a random sample of 108 such visits showed that doctors spent an average of 15.5 minutes with each patient. Assume that the population standard deviations are 2.7 and 2.1 minutes respectively. Construct a 94% confidence interval for the difference between the two population means for these two years.

9L

⑥ Let μ_1 and μ_2 be the pop. mean time spent by doctor with patients of the employees of the large corporation to primary care physicians in last year(1) and this year (2)

$n_1 = 100 \quad \bar{x}_1 = 19 \quad \sigma_1 = 2.7$

$n_2 = 108 \quad \bar{x}_2 = 15.5 \quad \sigma_2 = 2.1$

94% C.I. $\Rightarrow \alpha = 0.06 \Rightarrow \alpha/2 = 0.03$

Since σ_1^2 and σ_2^2 are known, Z dist. is to be used $\Rightarrow Z_{0.03} = 1.8808$

The 94% C.I. for the diff. between the two population means for these two years is

can just write $\mu_1 - \mu_2$ as μ_i are defined earlier

$$(\bar{x}_1 - \bar{x}_2) \pm Z_{0.03}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = 3.5 \pm 1.8808\sqrt{\frac{2.7^2}{100} + \frac{2.1^2}{108}} = [2.8657, 4.1343] \text{ (minutes)}$$

Shen Hoi

9/C

Q7. To compare the starting salaries of graduates majoring in education and accounting, random samples of 50 recent graduates in each major were selected and the following information was obtained:

Major	Mean	Standard Deviation
Education	RM 2,555.40	RM 222.50
Accounting	RM 2,334.80	RM 237.50

(a) Find a point estimate for the difference in the average starting salaries of all graduates majoring in education and accounting.

(b) Construct a 95% confidence interval for $\mu_1 - \mu_2$, the difference in the average starting salaries.

Let μ_1 and μ_2 be the population means for the starting salaries of all graduates majoring in education (1) and accounting (2).

$n_1 = n_2 = 50$; $\bar{x}_1 = 2555.40$, $s_1 = 222.50$; $\bar{x}_2 = 2334.80$, $s_2 = 237.50$

a) point estimate of $\mu_1 - \mu_2 = \bar{x}_1 - \bar{x}_2$

$= 2555.40 - 2334.80$

$=$ RM220.60

b) Since σ_1 and σ_2 are unknown, $n_1, n_2 \geq 30$, we use Z distribution ①

95% CI $\Rightarrow \alpha = 0.05 \Rightarrow \alpha/2 = 0.025 \Rightarrow z_{0.025} = 1.960$

The 95% confidence interval $\mu_1 - \mu_2$ is

$$(\bar{x}_1 - \bar{x}_2) \pm z_{0.025}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = 220.10 \pm 1.960\sqrt{\frac{222.50^2}{50} + \frac{237.50^2}{50}}$$

$= [130.392, 310.808]$ (RM)

Li Yuet

OK.

Q8. A sample of 14 cans of Brand A diet soda gave the mean number of calories of 23 per can with a standard deviation of 3 calories. Another sample of 16 cans of Brand B diet soda gave the mean number of calories of 25 per can with a standard deviation of 4 calories. Assume that the calories per can of diet soda are normally distributed for both brands with equal standard deviations. Find the 98% confidence interval for the difference in the mean number of calories for these two brands of diet soda.

CI for $\mu_2 - \mu_1$

$(\bar{x}_2 - \bar{x}_1) \pm E$

Let μ_1 and μ_2 be the population mean number of calories of diet soda Brand A and Brand B

Q8) $\bar{x}_1 = 25 \quad s_1 = 3 \quad n_1 = 14$

Since σ_1 and σ_2 are unknown, $\bar{x}_2 = 25 \quad s_2 = 4 \quad n_2 = 16$

$n_1 < 30, n_2 < 30$, assume $\sigma_1^2 = \sigma_2^2$

t-distribution with S_p is used

98% CI $\Rightarrow \alpha = 0.02$

$\frac{\alpha}{2} = 0.01$

$t_{0.01, 28} = 2.467$

$$S_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

$$= \sqrt{\frac{(14-1)(3)^2 + (16-1)(4)^2}{14+16-2}}$$

$$= 3.5707$$

$\mu_1 - \mu_2$

The 98% confidence interval for μ_1 and μ_2 is

$$(\bar{x}_1 - \bar{x}_2) \pm t_{\frac{\alpha}{2}, n_1+n_2-2} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$= (23-25) \pm (2.467)(3.5707)\sqrt{\frac{1}{14} + \frac{1}{16}}$$

$$\rightarrow [-5.2237, 1.2237]$$

calories

Cai Jie

OK 9/8/2021

Q9. Ten soldiers were selected at random from each of two companies to participate in a rifle-shooting competition. Their scores are listed in the table below.

Company 1	72	29	62	60	68	59	61	73	38	48
Company 2	75	43	63	63	61	72	73	82	47	43

Assume that the scores for both companies having equal variances. Construct a 90% confidence interval for the difference between the mean scores for the two companies.

S: sample std dev

σ = population std dev

from sample data

Since σ_1, σ_2 unknown, n_1 & $n_2 < 30$, t-dist is used

No.:　　　　Date:

Q9. $\bar{x}_1 = \frac{72+29+62+60+68+59+61+73+38+48}{10} = 57$

$\bar{x}_2 = \frac{75+43+63+63+61+72+73+82+47+43}{10} = 62.2$

$S_1 = \sqrt{\frac{(72-57)^2+(29-57)^2+(62-57)^2+(60-57)^2+(68-57)^2+(59-57)^2+(61-57)^2+(73-57)^2+(38-57)^2+(48-57)^2}{10-1}}$

$= \sqrt{\frac{1882}{9}} = 14.4607$

$S_2 = \sqrt{\frac{(75-62.2)^2+(43-62.2)^2+(63-62.2)^2+(63-62.2)^2+(61-62.2)^2+(72-62.2)^2+(73-62.2)^2+(82-62.2)^2+(47-62.2)^2+(43-62.2)^2}{10-1}}$

$= \sqrt{\frac{1739.6}{9}} = 13.9028$

CI = 0.9

$\alpha = \frac{1-0.9}{2} = 0.05$

$n_1+n_2-2 = 10+10-2 = 18$

$t_{0.05,18} = 1.734$

$\sqrt{\frac{1}{10}+\frac{1}{10}} = 0.4472$

$S_p = \sqrt{\frac{(n_1-1)S_1^2+(n_2-1)S_2^2}{n_1+n_2-2}}$

$= \sqrt{\frac{(10-1)(14.4607)^2+(10-1)(13.9028)^2}{10+10-2}}$

$= \sqrt{\frac{1882.0066+1739.5906}{18}}$

$= \sqrt{201.1998}$

$= 14.18449$

$(\bar{x}_1-\bar{x}_2) \pm t_{\frac{\alpha}{2}, n_1+n_2-2} \cdot S_p \cdot \sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$

$= (57-62.2) \pm (1.734)(0.4472)(14.18449)$

$= -5.2 \pm 10.9996$

$(-16.1996, 5.7996)$

Ze Xuan

OK

Q10. A researcher wanted to find the effect of a special diet on systolic blood pressure. He selected a sample of seven adults and put them on this dietary plan for three months. The following table gives the systolic blood pressures of these seven adults before and after the completion of this plan.

Before	210	180	195	220	231	199	224
After	193	186	186	223	220	183	233

Construct a 95% confidence interval for the mean reduction in the systolic blood pressures due to this special dietary plan for all adults. Assume that the population of paired differences is approximately normally distributed.

Let $D = x_{before} - x_{after}$

Q10

	D_i
1	17
2	-6
3	9
4	-3
5	11
6	16
7	-9

$\Sigma D_i = 35$

$\Sigma D_i^2 = 873$

$$\bar{D} = \frac{\Sigma D_i}{n} = \frac{35}{7} = 5$$

$$S_D = \sqrt{\frac{873 - \frac{(35)^2}{7}}{7-1}} = 10.7858$$

The 95% confidence interval for the mean reduction in the systolic blood pressures due to this special plan for all adults is

$$\bar{D} \pm t_{0.025,6} \frac{S_D}{\sqrt{n}}$$

$$= 5 \pm 2.447 \left(\frac{10.7858}{\sqrt{7}}\right)$$

$$= [-4.9756, 14.9756]$$

Yu Hong

5/

Q11. A certain geneticist is interested in the proportion of males and females in the population that have a certain minor blood disorder. In a random sample of 1000 males 250 are found to be afflicted, whereas 275 of 1000 females tested appear to have the disorder.

(a) Estimate the difference in the true proportion of males and females that have the blood disorder.

(b) Compute a 95% confidence interval for the difference between the proportion of males and females that have the blood disorder.

P_i = population proportion ... (a parameter)

$\hat{P}_i$ = sample proportion ... (a statistic)

Q11) P_1 and P_2 be the population proportion of males (1) and female (2) that have a certain minor blood disorder.

$\hat{P}_1 = \frac{250}{1000} = 0.25 \quad \hat{P}_2 = \frac{275}{1000} = 0.275$

a) point estimate of p_1 ~~and~~ p_2 ~~estimate~~ $= 0.25 - 0.275 = -0.025$

$\hat{P}_1 - \hat{P}_2$ point estimate of $P_1 - P_2$

$\sigma_{\hat{p}_1 - \hat{p}_2} = \sqrt{\frac{0.25(0.75)}{1000} + \frac{0.275(0.725)}{1000}}$

$= 0.0197$

b) The 95% C.I for the diff between proportion of males and females that have blood disorder is.

$(\hat{P}_1 - \hat{P}_2) \pm Z_{0.025}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} = -0.025 \pm (1.96)(0.0197)$

$= [-0.0636, 0.0136]$

Aaron

OK.

Q12. A marketing researcher wants to find a 95% confidence interval for the mean amount that visitors to a theme park spend per person per day. He knows that the standard deviation of the amounts spent per person per day by all visitors to this park is RM11. How large a sample should the researcher select so that the estimate will be within RM2 of the population mean?

Sample size → Round UP

No. of samples → Round to the nearest integer

$\alpha = 100\% - 95\% = 5\% = 0.05$

$\alpha/2 = 0.05/2 = 0.025$

$Z_{\alpha/2} = 1.96$

$\sigma = 11$, ~~popu mean = 2~~

$E = 2$

interval estimate = point est $\pm E$

$\underbrace{\pm E}_{\text{within } E}$

$n = \left(\frac{1.96 \times 11}{2}\right)^2 = 116.208$

≈ 117

up

Shee Yeap

Q13. Johnny's Pizza guarantees all pizza deliveries within 30 minutes of the placement of orders. A researcher wants to estimate the proportion of all pizzas delivered within 30 minutes by Johnny's. The 99% confidence interval for the population proportion has a margin of error to within 0.02.

(a) What is the most conservative estimate of the sample size?

(b) Assume that a preliminary study has shown that 93% of all Johnny's pizzas are delivered within 30 minutes. How large should the sample size?

a) conservative, $p = q = \frac{1}{2}$

13 (a) $n = \left(\frac{z_{0.005}}{0.02}\right)^2 \underbrace{(0.5)(1-0.5)}_{p \quad 1-p}$

$= \left(\frac{z_{0.005}}{0.02}\right)^2 (0.5)(0.5)$

$= \left(\frac{2.5758}{0.02}\right)^2 (0.25)$

$= 4146.7160$

≈ 4147

OR

$\left(\frac{z_{\alpha/2}}{2E}\right)^2$

$= \left(\frac{2.5758}{2(0.02)}\right)^2$

$= 4147$

same formula

(b) $n = \left(\frac{z_{0.005}}{0.02}\right)^2 (0.93)(1-0.93)$

$= \left(\frac{2.5758}{0.02}\right)^2 (0.0651)$

$= 1079.8049$

≈ 1080

Jun Dian

Q14. A small insurance company has 1861 life insurance policyholders. A random sample of 100 life insurance policyholders showed that the mean premium they pay on their life insurance policies is RM685 per year with a standard deviation of RM71. Assuming that the life insurance policy premiums for all life insurance policyholders have a normal distribution, construct a 92% confidence interval for the population mean.

ok.

$N = 1861$ (population size)

$\Rightarrow$ apply finite population adjustment

$* \sqrt{\frac{N-n}{N-1}}$ in E

TS No: .... Date:

14.

$n = 100 \quad \bar{x} = 685 \quad s = 71$

92% CI $\Rightarrow \alpha = 8\% = 0.08 \Rightarrow \alpha/2 = 0.04$

(σ^2 is unknown $\Rightarrow n > 30 \Rightarrow z$ dist ⊙)

$z_{0.04} = 1.7507$

The 92% confidence interval for the average premium ~~they pay~~ paid by all the policyholders on their life insurance policies is

$$\bar{x} - z_{0.04}\frac{s}{\sqrt{n}} * \le \mu \le \bar{x} + z_{0.04}\frac{s}{\sqrt{n}} *$$

$$= 685 - 1.7507\frac{71}{\sqrt{100}} \le \mu \le 685 + 1.7507\frac{71}{\sqrt{100}}$$

$$= 672.5700 \le \mu \le 697.42997 \text{ (RM)}$$

$$\sqrt{\frac{N-n}{N-1}} = 0.9730$$

$$= 685 \pm \left(1.7507\frac{71}{\sqrt{100}} \times 0.9730\right)$$

$$= 672.9056 \le \mu \le 697.0944 \text{ (RM)}$$

Mei Yi

OIC

Q15. In a large metropolitan area in which a total of 800 gasoline service stations are located, a random sample of 36 gasoline service stations, 20 of the stations carry a particular nationally advertised brand of oil. Find a 95% confidence interval estimate for
(a) the proportion of all stations in the area which carry the oil.
(b) the total number of service stations in the area which carry the oil.

$N = 800$ (population size)

$n = 36$ (sample size)

$\frac{20}{36} = \hat{p}$ = sample proportion of the stations that

Q15 $N = 800$ (population size) $n = 36$ (sample size)

$\frac{20}{36} = \hat{p}$ = sample proportion of 95% CI $\Rightarrow \alpha = 5\% = 0.05 \Rightarrow \alpha/2 = 0.025$

(a) 95 % confidence interval for p:

$\hat{p} \pm Z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \sqrt{\frac{N-n}{N-1}}$

$= \frac{20}{36} \pm 1.96 \sqrt{\frac{\frac{20}{36}(1-\frac{20}{36})}{36}} \sqrt{\frac{800-36}{800-1}}$

$= \frac{20}{36} \pm 0.1623(0.97785)$

$= \frac{20}{36} \pm 0.1587$

$= [0.3969, 0.7143]$

(b) total number of stations

= prob × total number

95% confidence interval ~~for total~~ number that carry the oil

$= [0.3969(800), 0.7143(800)]$

$= [317.48, 571.44]$

Additional Questions

Cecilia

1. A recent survey of 15 project managers showed that the mean score of job fit is 80 with standard deviation of 15. Compute a 99% confidence interval for the mean score of job fit of all the project managers. ([68.4701, 91.5299])

Variable : Score of job fit
Since population variance is not given,n = 15 < 30, t distribution is used
99% CI => a = 0.01 => a/2 = 0.005
t0.005;14=2.977 mean=80 s.d=15 n=15
99% confidence interval of mean score of jobfit of all the project manager ~~is~~
X - t0.005,14(s/sqrt(n)) <+ u <=X + t0.005,14(s/sqrt(n))
80+-2.977(15/sqrt(15))=[68.4701,91.5299]

Dui Mun

2. The following data show the height (in cm) of a sample of 9 kindergarten children:
122, 112, 118, 127, 111, 113, 110, 124, 107
Assume that the heights of all kindergarten children have an approximately normal distribution with standard deviation of 6.6.

i) Find a 90% confidence interval for the population mean height. ([112.3812, 119.6188])

ii) If the sample is taken from a population of 1520 kindergarten children, construct a 95% confidence interval for the population mean height. ([111.6994, 120.3006])

a) 90% CI $\Rightarrow \alpha = 0.1 \Rightarrow \frac{\alpha}{2} = 0.05 \Rightarrow z_{0.05} = 1.6449$

The 90% confidence interval for the population mean height of kindergarten children is

$$\bar{X} \pm z_{0.05}\frac{\sigma}{\sqrt{n}} = 116 \pm 1.6449\frac{6.6}{\sqrt{9}}$$

$$= [112.3812, 119.6188]$$ cm

b) 95% CI $\Rightarrow \alpha = 0.05 \Rightarrow \frac{\alpha}{2} = 0.025 = z_{0.025} = 1.9600$

The 95% confidence interval for the population mean height of kindergarten children is

$$\bar{X} \pm z_{0.025}\frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}} = 116 \pm 1.96\frac{7}{\sqrt{9}}\sqrt{\frac{1520-9}{1520-1}}$$

$$= \cancel{[111.1570, 120.2598]}$$

6.6

$$\cancel{s = \sqrt{\frac{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}{n-1}}}$$

$$\cancel{s = \sqrt{\frac{121496 - \frac{1089936}{9}}{8}}}$$

$$\cancel{s = \sqrt{\frac{121496 - 121104}{8}}}$$

$$\cancel{= 7}$$

3. XYZ University has two campuses in Malaysia. The university's quality assurance department wanted to check if the students are equally satisfied with the service provided at these two campuses. A sample of 14 students selected from Campus A produced a mean satisfaction index of 7.2 with a standard deviation of 0.5. Another sample of 18 students selected from Campus B produced a mean satisfaction index of 8.0 with a standard deviation of 0.4. Assume the distributions of the satisfaction index for both campuses have equal variances. Find a 98% confidence interval for the difference in the mean satisfaction indexes for all students for the two campuses. What can we infer from the confidence interval obtained? ([-1.1906, -0.4094])

Mavis

3) Let μ_1 and μ_2 be the population mean satisfaction indexes for all students for the Campus A (1) and Campus B (2).

$n_1 = 14$ $\quad \bar{x}_1 = 7.2$ $\quad s_1 = 0.5$

$n_2 = 18$ $\quad \bar{x}_2 = 8.0$ $\quad s_2 = 0.4$

Since σ_1 and σ_2 are unknown, $n_1, n_2 < 30$, assume $\sigma_1^2 = \sigma_2^2$, t-distribution with S_p is used.

98% C.I. $\Rightarrow \alpha = 0.02 \Rightarrow \frac{\alpha}{2} = 0.01 \Rightarrow t_{0.01,30} = 2.457$

$$S_p = \sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}}$$

$$= \sqrt{\frac{(14-1)(0.5)^2 + (18-1)(0.4)^2}{14+18-2}}$$

$$= 0.4461$$

The 98% confidence interval for $\mu_1 - \mu_2$ is

$$(\bar{x}_1 - \bar{x}_2) \pm t_{0.01,30}\, S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} = (7.2-8.0) \pm 2.457(0.4461)\sqrt{\frac{1}{14}+\frac{1}{18}}$$

$$= [-1.1906, -0.4094]$$

98% confident

Based on the confidence interval obtained, we are ~~not confident~~ that the students are not equally satisfied with the service provided at the two campuses.

Li Yvet

4. In a random sample of 50 males selected from a city, it was found that 20 of them write with their left hands. In a random sample of 60 females from the city, it was found that 15 of them write with their left hands.
Construct a 99% confidence interval for the difference in the rates of left-handedness between males and females. Interpret the results. ([-0.0793, 0.3793])

Sern

(4)

$n_1 = 50 \quad \hat{p}_1 = \frac{20}{50} = 0.4 \quad q_1 = 0.6$

$n_2 = 60 \quad \hat{p}_2 = \frac{15}{60} = 0.25 \quad q_2 = 0.75$

p_i be the population proportion of males (i=1) & females (i=2) who were found writing with their left hand

Con. I = 99 for $p_1 - p_2$,

$\alpha = 0.01$

$\alpha/2 = 0.005$

$$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$$

$$= (0.4 - 0.25) \pm 2.5758\sqrt{\frac{0.4(0.6)}{50} + \frac{0.25(0.75)}{60}}$$

$$= [-0.0793, 0.3793] = [-7.93\%, 37.93\%]$$

Interpret

We are 99% confidence that the difference in the rates of left handedness between males and females are -0.0793 and 0.3793

5. Determine the sample size that is needed to estimate the population mean to within half a standard deviation with 95% confidence. (16)

$E = \frac{1}{2}\sigma$

5) $n = \left(\frac{1.96\sigma}{\frac{1}{2}\sigma}\right)^2 = 15.3664$

≈ 16

95% CI => $\alpha = 0.05$ => $\frac{\alpha}{2} = 0.025$: $z_{0.025} = 1.96$ $E = \frac{1}{2}$

Yee How

6. A researcher investigates whether single people who own pets are generally happier than singles without pets. A group of non-pet owners is compared to pet owners using a mood inventory. The pet owners are matched one to one with the non-pet owners for income, number of close friendships, and general health. The data are as follow:

Matched pair	Non-pet owner ~~Before~~ After	Pet owner ~~After~~ Before	Diff, D_i	D_i^2
A	10	15	-5	25
B	9	9	0	0
C	11	10	1	1
D	11	19	-8	64
E	5	17	-12	144
F	9	15	-6	36
			$\Sigma D_i = -30$	$\Sigma D_i^2 = 270$

Calculate a 95% confidence interval for the mean difference of the mood scores for non-pet owners versus pet owners.

$D = x_{pet} - x_{non}$

$\Downarrow$

$[-0.142, 10.142]$

Let $D = x_{\cancel{before}\ non} - x_{\cancel{After}\ pet}$

~~Let μ_1 and μ_2 be the true pop. mean of the mood scores for non-pet owners (1) and pet owners (2)~~

~~$n_1 = n_2 = 6$ $\bar{x}_1 = \frac{55}{6} = 9.1667$ $\bar{x}_2 = \frac{85}{6} = 14.1667$ $s_1 =$~~

$\bar{D} = \frac{-30}{6} = -5$ $\quad s_D = \sqrt{\dfrac{270 - \frac{30^2}{6}}{5}} = 4.8990$

let μ_D be

The 95% C.I. for μ_D is

$$\bar{D} \pm t_{0.025,5} \frac{s_D}{\sqrt{n}} = -5 \pm 2.571 \left(\frac{4.8990}{\sqrt{6}}\right)$$

$$= [-10.142, 0.142]$$

([-10.142, 0.142])