Tutorial 1

1. Determine whether each of the following sentences is a $\boxed{\text{statement.}}$

   i) $y + 3$ is a positive integer. Not a statement

   ii) $128 = 2^6$. statement

   iii) $x = 2^6$. Not a statement

   iv) Is 2 a positive number? Not a statement

2. Give the negation of the following statement.

i) $2 + 7 \leq 11$ $2+7 > 11$

ii) $2 + 1 = 3$ $2+1 \neq 3$

iii) p: (2 is an even integer) and (8 is an odd integer.) q

iv) Today is Wednesday.

and : $\wedge$

or : $\vee$

iii) $p \wedge q$

$\sim (p \wedge q) \equiv \sim p \vee \sim q$

2 is <u>not</u> an even integer <u>or</u> 8 is <u>not</u> an odd integer.

iv) Today is not Wednesday.

$\vee$: or
$\wedge$: and
$\sim p$: not p

3. Let p, q, r be the propositions
p : You have a flu.
q : You miss the final exam.
r : You pass the course.
Express each of these propositions as in English sentences.
i) $p \vee q \vee r$ ii) $(p \wedge q) \vee (\sim q \wedge r)$ iii) $\sim p \wedge \sim q \wedge r$

i) You have a flu or you miss the final exam or you pass the course.

ii) Either you have a flu and miss the final exam or you didn't miss the final exam and pass the course.

iii) You did not have a flu and you did not miss the final exam and you pass the course.

but $\equiv$ and

4. Let h : " John is healthy."

w : "John is wealthy."

s : "John is wise."

$\wedge$, $\vee$, ...

Use the indicated letters and logical connectors to represent the following compound statements.

i) John is healthy and wealthy. $h \wedge w$

ii) John is healthy and not wise. $h \wedge \sim s$

iii) John is healthy and wealthy but not wise. $h \wedge w \wedge \sim s$

iv) John is not wealthy but he is healthy and wise. $\sim w \wedge (h \wedge s)$

v) John is either wealthy or healthy, or both. $w \vee h$

vi) John is wealthy or he is healthy but not wealthy and healthy. $w \underline{\vee} h$

vii) John is neither healthy nor wealthy. $\sim (h \wedge w)$ or $h | w$

viii) John is neither healthy, wealthy, nor wise.

$\sim (h \wedge w \wedge s)$

T: True

F: False

5. Determine the truth or falsity of each of the following statement.

i) $2 \geq 3$ and 3 is a positive integer. False

ii) $2 < 3$ or 3 is not a positive integer. True

iii) 2 is a prime but 3 is not a prime. False

iv) It is not true that 2 is not a prime or 3 is prime. False

v) It is false that 2 is prime or multiple of 4. False

prime: 2, 3, 5, 7, 11, 13, ...

multiple of 4: 4, 8, 12, 16, 20, ...

(i) $F \wedge T \equiv F$ $\quad (T \wedge T \equiv T)$

(ii) $T \vee F \equiv T$ $\quad (F \vee F \equiv F)$

(iii) $T \wedge F \equiv F$

(iv) $\sim (F \vee T) \equiv \sim T \equiv F$

(v) $\sim (T \vee F) \equiv \sim T \equiv F$

$p: T$, $q: F$, $r: T$

6. Find the truth value of each proposition if p and r are true and q is false.

i) $\sim p \wedge (q \vee r)$ False

ii) $p \wedge (\sim(q \vee \sim r))$ True

iii) $(r \wedge \sim q) \vee (p \wedge r)$ True

iv) $(q \wedge r) \wedge (p \vee \sim r)$ False

i) $F \wedge (F \vee T) \equiv F \wedge T \equiv F$

ii) $T \wedge (\sim(F \vee F)) \equiv T \wedge (\sim F) \equiv T \wedge T \equiv T$

iii) $(T \wedge T) \vee (T \wedge T) \equiv T \vee T \equiv T$

iv) $(F \wedge T) \wedge (T \vee F) \equiv F \wedge T \equiv F$

7. Construct a truth table for the following compound statements.

i) $(p \underline{\vee} q) \underline{\vee} (q \underline{\vee} r)$

$\underline{\vee}$: not both

p	q	r	$p \underline{\vee} q$	$q \underline{\vee} r$	$(p \underline{\vee} q) \underline{\vee} (q \underline{\vee} r)$
0	0	0	0	0	0
0	0	1	0	1	1
0	1	0	1	1	0
0	1	1	1	0	1
1	0	0	1	0	1
1	0	1	1	1	0
1	1	0	0	1	1
1	1	1	0	0	0

ii) $(p \downarrow q) \wedge (q \downarrow r)$

$p \downarrow q \equiv \sim(p \vee q)$

$0, 0 \Rightarrow 1$

p	q	r	$p \downarrow q$	$q \downarrow r$	$(p \downarrow q) \wedge (q \downarrow r)$
0	0	0	1	1	1
0	0	1	1	0	0
0	1	0	0	0	0
0	1	1	0	0	0
1	0	0	0	1	0
1	0	1	0	0	0
1	1	0	0	0	0
1	1	1	0	0	0

iii) $(p \mid q) \land r$

$p \mid q \equiv \sim (p \land q)$

$1, 1 \Rightarrow 0$

p ✓	q ✓	r ✓	$p \mid q$ ✓	$(p \mid q) \land r$
0	0	0	1	0
0	0	1	1	1
0	1	0	1	0
0	1	1	1	1
1	0	0	1	0
1	0	1	1	1
1	1	0	0	0
1	1	1	0	0

iv) $(p \mid q) \vee (p \mid r)$

$p \mid q \equiv \sim(p \wedge q)$

$1, 1 \Rightarrow 0$

p	q	r	$p \mid q$	$p \mid r$	$(p \mid q) \vee (p \mid r)$
0	0	0	1	1	1
0	0	1	1	1	1
0	1	0	1	1	1
0	1	1	1	1	1
1	0	0	1	1	1
1	0	1	1	0	1
1	1	0	0	1	1
1	1	1	0	0	0

8. Determine which of the pairs of statement forms are logically equivalent. Justify your answers using truth tables.

i) $\boxed{\sim(p \wedge (\sim(p \wedge q)))}$ and $\boxed{p \vee (\sim q \wedge \sim p)}$

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim(p \wedge q)$	$p \wedge \sim(p \wedge q)$	$\sim(p \wedge (\sim(p \wedge q)))$	$\sim q \wedge \sim p$	$p \vee (\sim q \wedge \sim p)$
0	0	1	1	0	1	0	1	1	1
0	1	1	0	0	1	0	1	0	0
1	0	0	1	0	1	1	0	0	1
1	1	0	0	1	0	0	1	0	1

not equal

$\therefore$ not logically equivalent

ii) $\underbrace{(p \downarrow p) \downarrow (q \downarrow q)}_{\text{LHS}}$ and $\underbrace{p \wedge q}_{\text{RHS}}$.

$p \downarrow q \equiv \sim(p \vee q)$

$0, 0 : 1$

p	q	$p \downarrow p$	$q \downarrow q$	$(p \downarrow p) \downarrow (q \downarrow q)$	$p \wedge q$
0	0	1	1	0	0
0	1	1	0	0	0
1	0	0	1	0	0
1	1	0	0	1	1

equal

$\therefore$ logically equivalent

iii) $(p \veebar q) \wedge r$ and $(p \wedge r) \veebar (q \wedge r)$

p	q	r	$p \veebar q$	$(p \veebar q) \wedge r$	$p \wedge r$	$q \wedge r$	$(p \wedge r) \veebar (q \wedge r)$
0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	1	0	1	1
1	0	0	1	0	0	0	0
1	0	1	1	1	1	0	1
1	1	0	0	0	0	0	0
1	1	1	0	0	1	1	0

equal

$\therefore$ logically equivalent

9. Use truth table to determine each of the statement forms below is a tautology, contradiction or contingency.

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i) $((\sim p \wedge q) \wedge (q \wedge r)) \wedge \sim q$

p	q	r	$\sim p$	$\sim q$	$\sim p \wedge q$	$q \wedge r$	$(\sim p \wedge q) \wedge (q \wedge r)$	$((\sim p \wedge q) \wedge (q \wedge r)) \wedge \sim q$
0	0	0	1	1	0	0	0	0
0	0	1	1	1	0	0	0	0
0	1	0	1	0	1	0	0	0
0	1	1	1	0	1	1	1	0
1	0	0	0	1	0	0	0	0
1	0	1	0	1	0	0	0	0
1	1	0	0	0	0	0	0	0
1	1	1	0	0	0	1	0	0

✓

$\therefore$ contradiction

ii) $(\sim p \vee q) \vee (p \wedge \sim q)$

p	q	$\sim p$	$\sim q$	$\sim p \vee q$	$p \wedge \sim q$	$(\sim p \vee q) \vee (p \wedge \sim q)$
0	0	1	1	1	0	1
0	1	1	0	1	0	1
1	0	0	1	0	1	1
1	1	0	0	1	0	1

✓

$\therefore$ tautology

10. Simplify the following statements using law of logical equivalences.

i) $(p \vee q) \wedge \sim(\sim p \wedge q)$

ii) $((p \vee q) \wedge (p \vee \sim q)) \vee q$

$(2 \times 3) + (2 \times 4)$

$= 2(3+4)$

i) $(p \vee q) \wedge (p \vee \sim q) \equiv p \vee \underbrace{(q \wedge \sim q)}_{\equiv c} \equiv p \vee c \equiv p$ #

ii) $\Big(p \vee \overbrace{(q \wedge \sim q)}^{\equiv c}\Big) \vee q \equiv p \vee q$ #

4) Identity laws:

- $p \wedge t \equiv p$
- $p \vee c \equiv p$

5) Negation/Inverse laws:

- $p \vee \sim p \equiv t$
- $p \wedge \sim p \equiv c$

11. Use the law of logical equivalences to verify the following logical equivalences.

i) LHS : $(p \vee q) \vee (\sim p \wedge \sim q \wedge r) \equiv p \vee q \vee r$ RHS

ii) $\sim(p \downarrow q) \equiv (\sim p \mid \sim q)$

9) Universal Bound / Domination laws:

- $p \vee t \equiv t$
- $p \wedge c \equiv c$

i) LHS : $$\big((p \vee q) \vee \sim p\big) \wedge \big((p \vee q) \vee \sim q\big) \wedge \big((p \vee q) \vee r\big)$$

$$\equiv \big(\overset{\equiv t}{(p \vee \sim p)} \vee q\big) \wedge \big(p \vee \overset{\equiv t}{(q \vee \sim q)}\big) \wedge (p \vee q \vee r)$$

$$\equiv (t \vee q) \wedge (p \vee t) \wedge (p \vee q \vee r)$$

$$\equiv t \wedge t \wedge (p \vee q \vee r) \equiv p \vee q \vee r_{\#} \text{ (RHS)}$$

ii) LHS : $$\sim(p \downarrow q) \equiv \sim(\sim(p \vee q))$$

$$\equiv p \vee q \checkmark$$

RHS: $$\sim p \mid \sim q \equiv \sim(\sim p \wedge \sim q) \equiv p \vee q_{\#} \text{ (LHS)}$$