For each question, please define the random variable used and state its probability distribution together with the relevant parameters.

X: number of ...... (success)

1. $X \sim U(a, b)$
2. $X \sim g(x; p)$
3. $X \sim B(n, p)$

Q1. A roulette wheel is divided into 25 sectors of equal area numbered from 1 to 25.

Aaron

(a) Find a formula for the probability distribution of X, the number that occurs when the wheel is spun.

(b) Find the mean and variance of X.

(c) Find the probability that the number is a prime number.

OK

$X \sim U(1, 25)$

1a) $f(x;k) = \frac{1}{k}$

$f(x;25) = \frac{1}{25}$ ✓ , $1 \le x \le 25$

b) $\mu = \frac{k+1}{2} = \sum x P(X=x)$

$= \frac{25+1}{2}$

$= 13$ ✓

$\sigma^2 = \frac{k^2-1}{12} = \sum x^2 P(X=x)$

$= \frac{25^2-1}{12} = 52$ ✓

c) Prime numbers → 2, 3, 5, 7, 11, 13, 17, 19, 23

total = 9 ✓

Prob. $= \frac{9}{25} = 0.36$ ✓

Q2. Suppose that 20% of the applicants for a certain industrial job possess advanced training in computer programming. Applicants are interviewed sequentially and are selected at random from the pool.

(a) Find the probability that the first applicant with advanced training in computer programming

(i) is found on the fifth interview,

(ii) is found before the fifth interview.

(b) Find the mean and variance of the number of applicants who need to be interviewed in order to find the first one with advanced training in computer programming.

Cai Jie

X: number of applicants for a certain industrial job possess advanced training in computer programming.

$X \sim g(x, 0.2)$

required to obtain the first applicant who

2) a) i) $P(X=5) = \cancel{g(5; 0.2)} = (0.8)^4(0.2)$

$= 0.08192$

ii) $P(X<5) = P(X=1) + P(X=2) + P(X=3) + P(X=4)$

$= 0.2 + (0.8 \times 0.2) + ((0.8)^2(0.2)) + ((0.8)^3(0.2))$

$= 0.5904$

b) $\mu = \frac{1}{0.2}$

$= 5$

$\sigma^2 = \frac{1-p}{p^2} = \frac{1-0.2}{(0.2)^2} = 20$

Q3. Over a long period of time a drug has been effective in 40% of cases in which it has been prescribed. If 4 patients are treated by this drug, find the probability that it will be effective for

Shee Yeap

ok.

(a) at least 3 patients, (b) none of the patients, (c) 1 or 2 patients.

X: number of patients ~~are~~ who is treated by the drug and find it is effective out of 4 patients

③ $P(X=r) = {}^4C_r\, 0.4^r\, 0.6^{4-r}$, $r = 0,1,2,3,4$ $\Rightarrow X \sim B(4, 0.4)$

(a) $P(X \geq 3) = P(X=3) + P(X=4)$

$= {}^4C_3\, 0.4^3\, 0.6^1 + {}^4C_4\, 0.4^4\, 0.6^0$

$= 0.1792$

(b) $P(X=0) = {}^4C_0\, 0.4^0\, 0.6^4 = 0.1296$

(c) $P(X=1) + P(X=2) = {}^4C_1\, 0.4^1\, 0.6^3 + {}^4C_2\, 0.4^2\, 0.6^2$

$= 0.6912$

Q3. Give the probability distribution for the number of patients that the drug is effective for them.

Chun Wai

ok

x	0	1	2	3	4
$P(X = x)$	0.1296	0.3456	0.3456	0.1536	0.0256

ok.
Q4. A fair coin is tossed 10 times. Find the probability that the head appears

Janet (a) at most 2 times, (b) more than 3 times, (c) between 4 and 8 times.

Q4. X = number of head appears at a fair coin out of 10 times of tosses

$X \sim B(10, 0.5)$, $\Rightarrow P(X=r) = {}^{10}C_r\, 0.5^r\, 0.5^{10-r}$, $r = 1, 2, 3, \cdots, 10$

A. $P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$

$= {}^{10}C_0\, 0.5^0 0.5^{10} + {}^{10}C_1\, 0.5^1 0.5^9 + {}^{10}C_2\, 0.5^2 0.5^8$

$= 0.0009766 + 0.009766 + 0.04395$

$= 0.0547$

B. $P(X > 3) = 1 - P(X \leq 2) - P(X = 3)$

$= 1 - 0.0547 - {}^{10}C_3\, 0.5^3 0.5^7$

$= 1 - 0.0547 - 0.1172$

$= 0.8281$

C. $P(4 < X < 8) = P(X=5) + P(X=6) + P(X=7)$

$= {}^{10}C_5\, 0.5^5 0.5^5 + {}^{10}C_6\, 0.5^6 0.5^4 + {}^{10}C_7\, 0.5^7 0.5^3$

$= 0.2461 + 0.2051 + 0.1172$

$= 0.5684$

Q4. Give the probability distribution for the number of heads appeared.

Jun Dian

ok

x	$P(X=x) = f(x)$
0	${}^{10}C_0\, 0.5^0\, 0.5^{10} = \frac{1}{1024}$
1	${}^{10}C_1\, 0.5^1\, 0.5^9 = \frac{5}{512}$
2	${}^{10}C_2\, 0.5^2\, 0.5^8 = 0.0439$
3	${}^{10}C_3\, 0.5^3\, 0.5^7 = 0.1172$
4	${}^{10}C_4\, 0.5^4\, 0.5^6 = 0.2051$
5	${}^{10}C_5\, 0.5^5\, 0.5^5 = 0.2461$
6	${}^{10}C_6\, 0.5^6\, 0.5^4 = 0.2051$
7	${}^{10}C_7\, 0.5^7\, 0.5^3 = 0.1172$
8	${}^{10}C_8\, 0.5^8\, 0.5^2 = 0.0439$
9	${}^{10}C_9\, 0.5^9\, 0.5^1 = \frac{5}{512}$
10	${}^{10}C_{10}\, 0.5^{10}\, 0.5^0 = \frac{1}{1024}$

Jing Jet

Q5. A shoe store's records show that 30% of customers making a purchase use credit cards to make payment. This morning, 20 customers purchased shoes from the store.

(a) What is the probability that at least 3 customers, but not more than 6, used credit cards?

(b) What is the mean and standard deviation of the number of customers who used credit cards?

(c) Find the probability that exactly 14 customers did not used credit cards.

X: No.of customer make payment by credit cards out of 20 customers

X~B (20,0.3)

5) a) $P(3 \le X \le 6) = {}^{20}C_3\ 0.3^3\ 0.7^{17} + {}^{20}C_4\ 0.3^4\ 0.7^{16} + {}^{20}C_5\ 0.3^5\ 0.7^{15} + {}^{20}C_6\ 0.3^6\ 0.7^{14}$

$= 0.0716 + 0.1304 + 0.1789 + 0.1916$

$= 0.5725$

b) $\mu = 20(0.3)$

$= 6$

$\sigma = \sqrt{20(0.3)(0.7)}$

$= 2.0494$

c) $P[X=6] = 0.1916$

Q5. Give the probability distribution for the number of customers use credit card to make payment.

$0 <= X <= 10$ — Mei Yi

OK

Q5 $0 \le X \le 10$

X	$P(X = x)$
0	${}^{20}C_0\ 0.3^0\ 0.7^{20} = 0.00080$
1	${}^{20}C_1\ 0.3\ 0.7^{19} = 0.00684$
2	${}^{20}C_2\ 0.3^2\ 0.7^{18} = 0.02785$
3	0.0716
4	0.1304
5	0.1789
6	0.1916
7	${}^{20}C_7\ 0.3^7\ 0.7^{13} = 0.1643$
8	${}^{20}C_8\ 0.3^8\ 0.7^{12} = 0.1144$
9	${}^{20}C_9\ 0.3^9\ 0.7^{11} = 0.0654$
10 ✓	${}^{20}C_{10}\ 0.3^{10}\ 0.7^{10} = 0.0308$ ✓

$11 <= X <= 20$ — Eason

OK

X	$P(X = x)$
11	${}^{20}C_{11} \left(\frac{3}{10}\right)^{11} \left(\frac{7}{10}\right)^9 = 0.012$
12	${}^{20}C_{12} \left(\frac{3}{10}\right)^{12} \left(\frac{7}{10}\right)^8 = 0.039$ 0.0039 3.73×10^{-3}
13	${}^{20}C_{13} \left(\frac{3}{10}\right)^{13} \left(\frac{7}{10}\right)^7 = 0.010$ 0.0010
14	${}^{20}C_{14} \left(\frac{3}{10}\right)^{14} \left(\frac{7}{10}\right)^6 = 0.022$ 0.0002
15	${}^{20}C_{15} \left(\frac{3}{10}\right)^{15} \left(\frac{7}{10}\right)^5 = 0.0037$ 0.000037
16	${}^{20}C_{16} \left(\frac{3}{10}\right)^{16} \left(\frac{7}{10}\right)^4 = 0.0005$
17	${}^{20}C_{17} \left(\frac{3}{10}\right)^{17} \left(\frac{7}{10}\right)^3 = 0.00001$
18	${}^{20}C_{18} \left(\frac{3}{10}\right)^{18} \left(\frac{7}{10}\right)^2 = 0.0000036$
19	${}^{20}C_{19} \left(\frac{3}{10}\right)^{19} \left(\frac{7}{10}\right)^1 = 0.00000016$
20	${}^{20}C_{20} \left(\frac{3}{10}\right)^{20} \left(\frac{7}{10}\right)^0 = 3.49 \times 10^{-11}$

$X = 16$ to 20: $P(X = x) \approx 0$

Q6. In a group of people, the expected number who wears glasses is 2 and the va[riance] is 1.6. Find the probability that
(a) a person chosen at random from the group wears glasses,
(b) 6 people in the group wear glasses.

Sean

ok -

Given

$\sigma^2 = 1.6$ — ①

$\mu = 2$ — ②

$\mu = np$

$\sigma^2 = npq = np(1-p)$

Substitute ② into ①

$1.6 = 2q$

$\frac{1.6}{2} = q$

$0.8 = q$

$(1-q) = p$

$1 - 0.8 = 0.2$

$p = 0.2$

Substitute p=0.2 into ①

$1.6 = n(0.2)(0.8)$

$10 = n$

(a) $E(x) = 2$

x is the no. of people who wear glasses out of 10.

people

$X \sim B(10, 0.2)$

$P(X=1) =$

$P(x=1) = {}^{10}C_1 (0.2)^1 (0.8)^9$

$= 0.2684$

(b) $P(x=x) = {}^nC_x p^x q^{n-x}$;

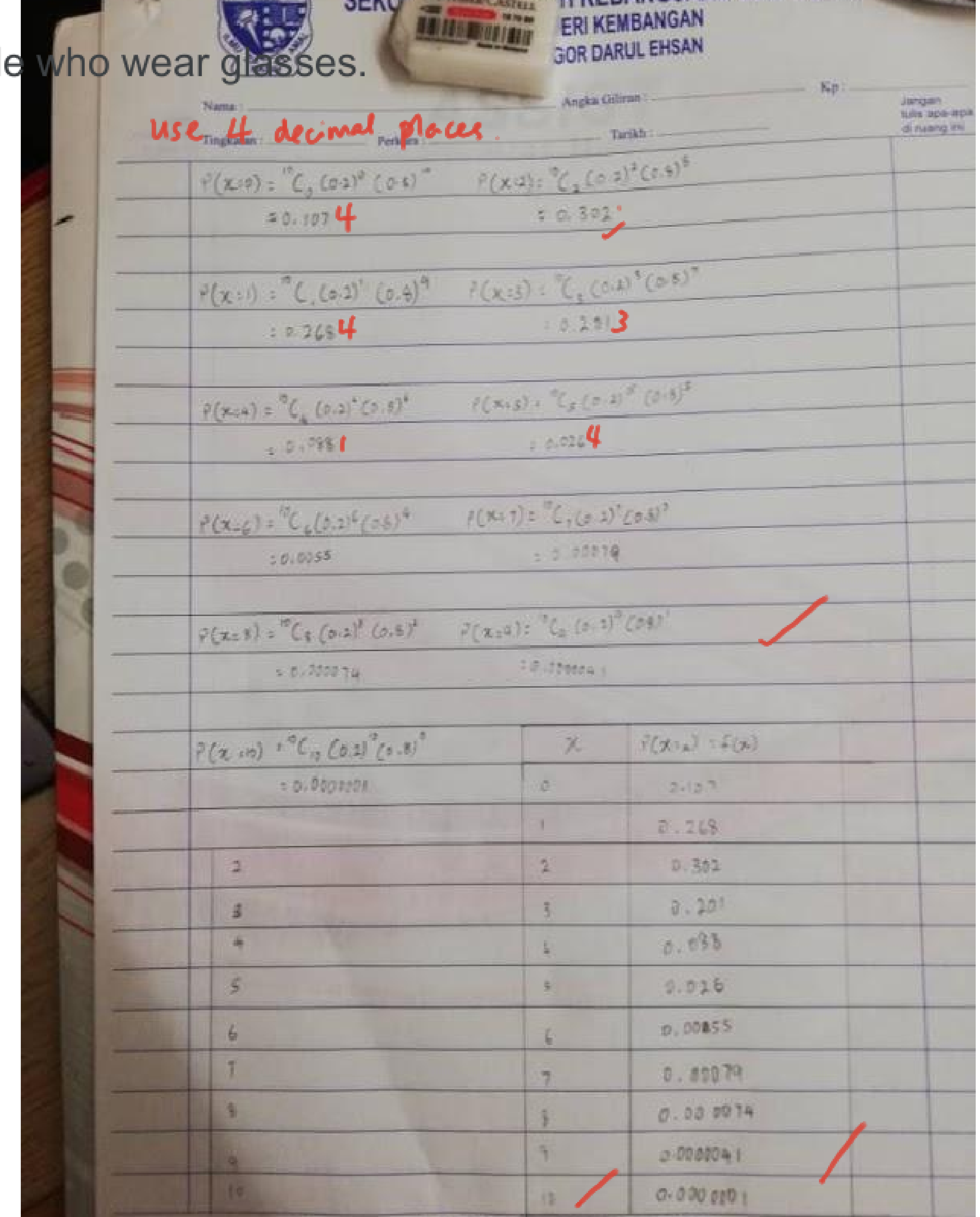
$P(x=6) = {}^{10}C_6 (0.2)^6 (0.8)^4$

$= 0.0055$

Q6. Give the probability distribution for the number of people who wear glasses.

ok

Li Yuet



Poisson Distribution

X: Number of success/occurrences in a particular INTERVAL (space)
eg, in 1kg, in a day, in a room, etc
x = 0, 1, 2, ..., infinity -> P(X > r) = complement rule = 1 - P(X<=r)

X ~ Po (λ), lambda = mean/average number = variance
λ : mean/average number of success/occurences in a particular interval
* Lambda will change proportionately according to the change of the interval
Eg if the mean for 1kg is 2, then the mean for 3 kg will be 3x2 = 6.

$$P(X=r) = \frac{e^{-\lambda}\lambda^r}{r!}, \quad r = 0, 1, 2, \ldots, \infty$$

Binomial: in a sample size/trials of n, p (prob of success) remains the same no matter how difference is the n.
Poission: in an interval, lambda (mean number) changes proportionately based on the change of the interval

Poisson Approximation to Binomial

When

1. $n \to$ infinity (large);
2. $p \to 0$ (small);
3. $np < 5$.

np ~= npq

$X \sim B(n, p) \dashrightarrow X \sim Po(\lambda = np)$

For Poisson distribution, since the mean number may change according to the interval, thus we always re-define the random variable (preferable using different alphabets) for the distribution of different interval.

X: no of success in 1 kg, $X \sim Po(2)$

Y: no the success in 3 kg, $Y \sim Po(6)$

Q7. During the summer months (June to August inclusive), an average of 5 marriages per month take place in a small city. Assuming that these marriages occur randomly and independently of one another, find the probability that

1 summer month

(a) fewer than 3 marriages will occur in June, Ze Xuan ok

(b) exactly 10 marriages occur during the 2 months of July and August, Cecilia ok

(c) at least 14 but not more than 18 marriages occur during the entire 3 months of summer. Pui Mun ok

X= Number of marriages in ~~June~~ in a small city,
X~P0(5)

a) $P(X<3)$

$= P(X=0) + P(X=1) + P(X=2)$

$= \frac{e^{-5}(5^0)}{0!} + \frac{e^{-5}(5^1)}{1!} + \frac{e^{-5}(5^2)}{2!}$

$= 0.006738 + 0.03369 + 0.08422$

$= 0.1246$

2-summer month

b) X :Number of marriages in ~~July and August~~ in a small city
X~Po(10)

b) $P(x=10) = \frac{10^{10}e^{-10}}{10!}$

$= 0.1251$

X = number of marriage in 3-summer month in a small city

$X \sim P_0(15)$

$P(X=r) = \frac{e^{-\lambda}\lambda^r}{r!}$

$P(14 \le X \le 18) = P(X=14) + P(X=15) + P(X=16) + P(X=17) + P(X=18)$

$= \frac{e^{-15}(15^{14})}{14!} + \frac{e^{-15}(15^{15})}{15!} + \frac{e^{-15}(15^{16})}{16!} + \frac{e^{-15}(15^{17})}{17!} + \frac{e^{-15}(15^{18})}{18!}$

$= 0.1024 + 0.1024 + 0.0960 + 0.0847 + 0.0706$

$= 0.4561$

13/7

Q8. An insurance company receives on average 2 claims per week from a certain factory. Assuming that the number of claims follows a Poisson distribution, find the probability that

(a) it receives more than 3 claims in a given week, Khai Jun ok

(b) it receives 2 or more claims in a given fortnight, Jing Xian ok

(c) it receives no claims on a given day, assuming that the factory operates on a 5-day week. Jia Yu ok

a) X=Number of claims per week from a certain factory

X~Po(2)

Q8. a) $P(X>3)$

$= 1 - P(X=0) - P(X=1) - P(X=2) - P(X=3)$

$= 1 - \frac{e^{-2}2^0}{0!} - \frac{e^{-2}2^1}{1!} - \frac{e^{-2}2^2}{2!} - \frac{e^{-2}2^3}{3!}$

$= 1 - 0.1353 - 0.2707 - 0.2707 - 0.1804$

$= 0.1429$

b

X = Number of claims in 2 weeks from a certain factory

X ~ Po(4)

$P(X \geq 2) = 1 - P(X=0) - P(X=1)$

$= 1 - \frac{e^{-4}4^0}{0!} - \frac{e^{-4}4^1}{1!}$

$= 0.9084$

(c) X = Number of claims in a ~~week~~ day ~~week~~ from a certain factory. $X \sim Po(\frac{2}{5})$ / $X \sim Po(0.4)$

$P(X=0) = \frac{e^{-0.4}0.4^0}{0!}$

$= 0.6703$

13/7

Q9. Cars arrive at a petrol station at an average rate of 30 per hour. Assume that the number of cars arriving at the petrol station follows a Poisson distribution.

(a) Find the probability that

(i) no cars arrive during a particular 5 minute interval, Jia Jie ok.

(ii) less than 3 cars arrive during a 10 minute interval.

(b) Find the mean and standard deviation of the number of cars arrive at the petrol station in 15 minutes. Jun Yan ok.

interval ~ Poisson

b)

~~mean = λ~~

= 30 per hour

= 30 per 60 minutes

↓ ÷4 ↓ ÷4

7.5 per 15 minutes

$\lambda = 7.5 = \frac{30}{4}$

Variance, $\sigma^2 = 7.5$

Standard deviation, $\sigma = \sqrt{7.5}$

$= 2.7386$

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30 per hour = 30 per 60 minutes

↓ ÷4 ↓ ÷4

7.5 per 15 minutes

$\lambda = \frac{30}{4}$

$= 7.5$

variance, $\sigma^2 = 7.5$

standard deviation, $\sigma = \sqrt{7.5}$

$= 2.7386$

(a) $X \sim Po(2.5)$ per 5-min interval

(i) X = no of cars arriving at the petrol station

i) $\frac{60}{5} = \frac{30}{x}$

$x = 2.5$

$P(x=0) = (e^{-2.5}\, 2.5^0)/0!$

$= 0.0821$

ii) $P(Y<3) = \frac{e^{-5}5^0}{0!} + \frac{e^{-5}5^1}{1!} + \frac{e^{-5}5^2}{2!}$

$= 0.1246$

Y: no of cars arrive at the petrol station per 10-minute interval

$Y \sim Po(5)$

Q10. A restaurant prepares a tossed salad containing on the average 5 vegetables. Find the probability that the salad contains more than 5 vegetables

(a) on a given day, Yee Hao ok.

(b) on 3 of the next 4 days, Shen Hoi ok

(c) for the first time in April on April 5. Kang Hong ok.

"success"

(a) X: number of vegetables contain in the tossed salad, X ~ P0(5)

$P[X>5] = 1 - P[X \leq 5]$

$= 1 - \left(\frac{e^{-5}5^5}{5!} + \frac{e^{-5}5^4}{4!} + \frac{e^{-5}5^3}{3!} + \frac{e^{-5}5^2}{2!} + \frac{e^{-5}5^1}{1!} + \frac{e^{-5}5^0}{0!}\right)$

$= 1 - (0.1755 + 0.1755 + 0.1404 + 0.0842 + 0.0337 + 0.0067)$

$= 1 - 0.6160$

$= 0.3840$

b) X: number of day that more than 5 vegetables are contained in ~~the~~ a tossed salad, out of 4 days X~B(4,0.384)

(b) $X \sim B(4, 0.384)$, $P(X=r) = {}^4C_r (0.384)^r (0.616)^{4-r}$, $r = 0, 1, \ldots$

$P(X=3) = {}^4C_3 (0.384)^3 (0.616)^1$

$= 0.1395$

X: number of ~~time until~~ days for the first time with a tossed salad that contains more than 5 vegetables.

$X \sim g(x; 0.3840)$

10.(c) $P(X=5) = (1-0.3840)^4 (0.3840)^1$

$= 0.0553$

Q11. The probability that a particular make of light bulb is faulty is 0.01. The light bulbs are packed in boxes of 100. Find the probability that in a certain box there are

Mavis (a) no faulty light bulbs,

ok (b) 2 faulty light bulbs,

(c) more than 3 faulty light bulbs.

11) X : The ~~probability~~ no. of ~~that a particular make~~ of light bulb that is faulty in a ~~certain~~ box. of 100 bulbs

(a) $X \sim B(100, 0.01)$

$n = 100$ (large), $p = 0.01$ (small) $\Rightarrow np < 5$

$\therefore X \sim Po(1)$ approximately n is used

$$P(X=0) = \frac{e^{-1}\,1^0}{0!}$$

$$= 0.3679$$

(b) $$P(X=2) = \frac{e^{-1}\,1^2}{2!}$$

$$= 0.1839$$

(c) $$P(X>3) = 1 - P(X \leq 3)$$

$$= 1 - [P(X=0) + P(X=1) + P(X=2) + P(X=3)]$$

$$= 1 - e^{-1}\left[\frac{1^0}{0!} + \frac{1^1}{1!} + \frac{1^2}{2!} + \frac{1^3}{3!}\right]$$

$$= 0.0190$$

Q12. A sample of 200 items is taken from a large batch, which is known to produce defective with probability 0.005. Find the probability that there are at most 2 defectives in the sample using a Poisson approximation to the Binomial distribution.

n p $P(X \le 2)$

Yu Hong

$n = 200$ $p = 0.005$ $q = 0.995$

$\mu = 200 \times 0.005 = \lambda = 1$

③ Since $n \to \infty$, $p \to 0$ and $np = 1 < 5$, Poisson approximation can be used.

$\Rightarrow X \sim Po(1)$

② $X \sim B(200, 0.005)$

① X = number of defective in 200 items ~~however~~ ③

success group size

④ $p(X \le 2) = \frac{(e^{-1}(1^2))}{2!} + \frac{(e^{-1}(1^1))}{1!} + \frac{(e^{-1}(1^0))}{0!} = 0.9197$

$P(X=2) + P(X=1) + P(X=0)$