Hypothesis Testing

A) Test on Population Mean (Quantitative Variable)

1. One group (z or t test)
2. Two independent groups - each subject is assigned in one of the two groups and measured once, n = total subjects (z or t test)
3. Paired/dependent groups - each subject (pair) is measured repeatedly, n = total pair of observations (t test)

B) Test on Population Proportion (Qualitative)

1. One group
2. Two independent groups

~ z test ~

Steps:

Define the test variable.

1. Hypotheses (null, alternative)
2. Critical value and critical/rejection region (rule)
3. Test statistic
4. Result (reject or fail to reject null hypothesis)
5. Conclusion (answer the claim/question)

Identify: test variable, values of parameter (if given) and statistics, test value (claim), level of significance

Q1. The mean weekly sale of the Chocolate Bar in candy stores is 146.3 bars per store with standard deviation of 17.2 bars. After an advertising campaign, the mean weekly sale in 22 stores for a typical week is 153.7 bars with a standard deviation of 16.7 bars. Was the advertising campaign successful? Use $\alpha = 0.05$.

Khai Jun

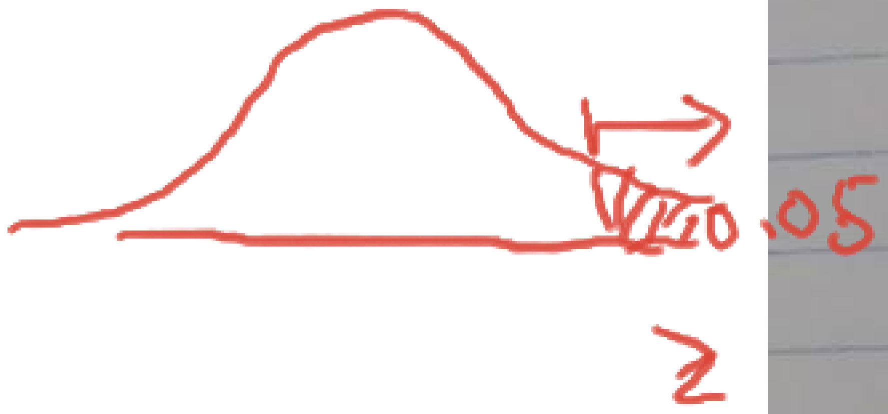
OK.

Q1) $\bar{x} = 153.7$, $\sigma = 17.2$, $n = 22$, $\alpha = 0.05$

let μ = the population mean weekly sale in candy stores

$H_0: \mu \le 146.3$

right $H_1: \mu > 146.3$ claim (right) Successful campaign



$\boxed{T/m}$ α

At $\alpha = 0.05$, since σ is known, z-test is used

critical value = $\cancel{\pm}$ $z_{0.05}$ = $\cancel{\pm 1.9600}$ 1.6449

rejection region = $z >$ $\cancel{1.96}$ 1.6449

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

$$= \frac{153.7 - 146.3}{17.2 / \sqrt{22}}$$

$= 2.018$

≈ 2.02

Since $z = 2.018 >$ ~~1.96~~ 1.6449, we reject Ho at $\alpha = 0.05$ and we can conclude that the ~~mean weekly sale in candy stores is not more than 146.3 bars.~~ advertising campaign was successful.

Q2. The manufacturer of a certain brand of car batteries claims that the mean life of these batteries is 45 months. A consumer protection agency that wants to check this claim took a random sample of 36 such batteries and found that the mean life for this sample is 43.15 months. The lives of all such batteries have a normal distribution with standard deviation of 6.1 months. Test the hypothesis that the mean life of these batteries is less than 45 months at the 2.5% significance level.

OK.

population

Jia Yu

Q2) $\bar{x} = 43.15$, $\sigma = 6.1$, $n = 36$

Let μ = true population mean life of ~~the manufacturer of~~ a certain brand of car batteries

$H_0 : \mu \geq 45$

$H_1 : \mu < 45$ claim, $\}$ one tail

left

At $\alpha = 0.025$, since σ is known, z test is used

-ve

critical value $= -z_\alpha = -z_{0.025} = -1.96$

rejection region : $z \leq -1.96$

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{43.15 - 45}{6.1/\sqrt{36}} = -1.819 \approx -1.82$$

Since $z = -1.819 > -1.96$, we failed to reject H_0 at $\alpha = 0.025$. Hence, we can conclude that the mean number of life of batteries is not less than 45 months

Q3. The manufacturer of a certain oil-additive claims that the mean net weight of jars of his product is 1 kg. A random sample of size 49 of a large consignment supplied to your company is found to have a mean net weight of 0.99 kg with a standard deviation of 0.02 kg. Test the manufacturer's claim at $\alpha = 0.05$.

Jia Jie

ok.

μ $=1$

$s = 0.02$

$\bar{x} = 0.99$

sample

(b) $\mu = 1$, $n = 49$, $\sigma = 0.02$

① Let μ = true population mean of the net weight of jars ~~manufacturer of a certain oil-additive claimed~~.

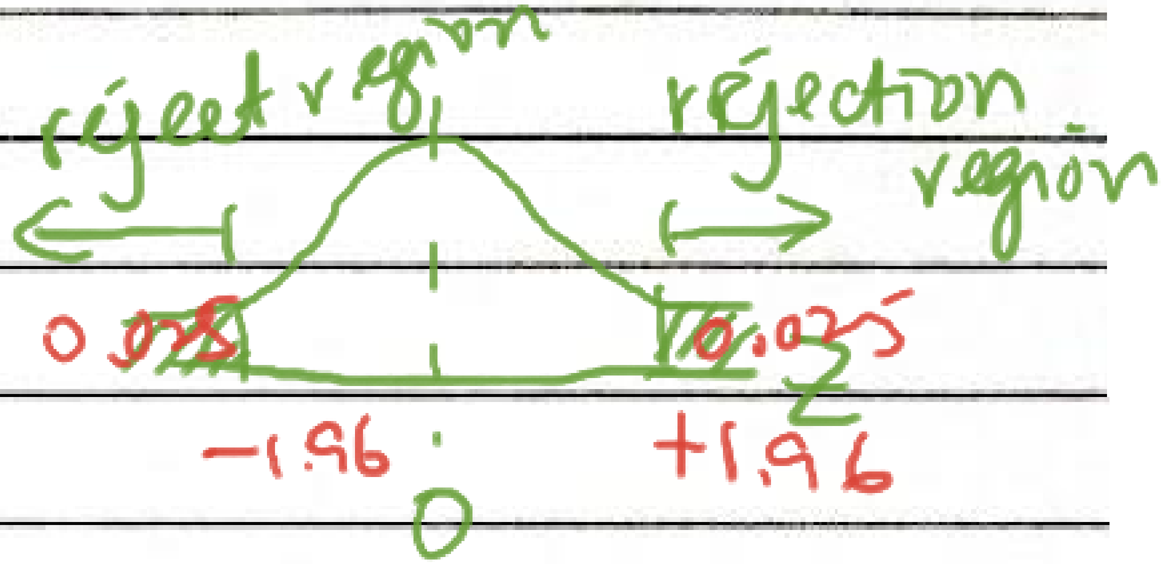
② $H_0 : \mu = 1$ (claim)

$H_1 : \mu \neq 1$ └ 2-tailed test

③ Since $\alpha = 0.05$, σ is unknown, $n = 49 > 30$, Z ~~test~~ is used

critical value $= \pm z_{\alpha/2} = -z_{0.025}$ ~~$= -1.64$~~ ± 1.96

rejection region $= z < -1.96$ or $z > 1.96$



④ $Z = \dfrac{\bar{x} - \mu}{s/\sqrt{n}} = \dfrac{0.99 - 1}{0.02/\sqrt{49}}$

$= -3.5$ ✓

⑤ Since $z = -3.5 < -1.96$, we reject H_0 at $\alpha = 0.05$. ✓ Hence, we can conclude that the ~~mean number~~ net weight of the jar of ~~the manufacturer~~ a certain oil-additive ~~claimed~~ is not ~~less than~~ 1 kg, ie the claim is not true.

Test stat $\leftrightarrow$ critical/rejection region

Jing Jet

Q4. The mean weight of all babies born at a hospital last year was 3.45 kg. A random sample of 35 babies born at this hospital this year produced the following data in kg.

3.72	4.13	3.13	2.63	2.90	4.67	5.49	4.13	2.68	3.31	5.08	3.76
2.95	3.22	3.63	4.17	2.59	4.31	3.76	2.86	2.22	3.45	4.58	4.17
3.81	3.40	3.27	3.76	3.27	4.40	2.72	3.67	2.77	3.76	3.04	

σ

Test at the 3% significance level whether the mean weight of babies born at this hospital this year is exceed last year.

4. Let μ be the true population mean weight of all babies born at a hospital ~~last year~~.

$\mu = 3.45$ $\quad$ $\bar{x} = \frac{125.41}{35} = 3.5832$ $\quad$ $H_0 : \mu \leq 3.45$ $\quad$ $H_1 : \mu > 3.45$ claim $\quad$ exceed

$n = 35$

$$s = \sqrt{\frac{468.4307 - \frac{(125.41)^2}{35}}{35 - 1}} = 0.7487$$

$\alpha = 0.03$, since σ is unknown, $n > 30$, z-test is used

Critical value $= Z_{0.03} = 1.8808$

Critical region : $z > 1.8808$

$$Z = \frac{3.58 - 3.45}{0.7487/\sqrt{35}} = 1.0272$$

Since $Z = 1.0272 < 1.8808$, we are failed to reject H_0 at $\alpha = 0.03$.

Hence, there is insufficient evidence that the mean weight of babies born at this hospital this year is exceeding last year.

Q5. A soft-drink vending machine is set to dispense 250 ml per cup. If the machine is tested 25 times, yielding a mean cup fill of 241.80 ml with a standard deviation of 6.67 ml, is this evidence at the 1% significance level that the machine is underfilling? → $<$ α

Jun Yan OK

Let μ be the true population mean ~~of ml~~ ~~that~~ volume of fill dispensed from a soft drink vending machine ~~is set to dispense~~

$n = 25$

$\bar{x} = 241.80$

$s = 6.67$

$H_0 : \mu \geq 250$ ~~(claim)~~

$H_1 : \mu < 250$ underfilling (Claim)

$\alpha = 0.01$, since σ is unknown and $n < 30$, t-statistic is used

critical value $= \cancel{\pm} t_{0.01,24} = \cancel{\pm} 2.492$

critical region $= t < -2.492$

$$T = \frac{241.80 - 250}{6.67/\sqrt{25}} = -6.147 \checkmark$$

Since $t = -6.147 < -2.492$, we reject H_0 at $\alpha = 0.01$. ✓

Hence, there is ~~in~~sufficient evidence that the soft-drink vending machine is ~~set to dispense 250 ml~~ underfilling.

test stat

$t = -6.147$

p-value

$= P(t < -6.147)$

$= P(t > 6.147)$

From t-table

$P(t > 3.745)$

$= 0.0005$

≈ 0 estimate

pvalue < 1%

→ reject H_0

Q6. A computer company that recently introduced a new software product claims t
the mean time it takes to learn how to use this software is not more than 2 ho
for people who are somewhat familiar with computers. A random sample of
such persons was selected. The following data give the times taken (in hours)
these persons to learn how to use this software.

1.75 2.25 2.40 1.90 1.50 2.75 2.15 2.25 1.80 2.20 3.25 2.6

Test at the 1% significance level whether the company's claim is true. Assume t
the times taken by all persons who are somewhat familiar with computers to le
how to use this software are approximately normally distributed.

ok

Kang Hong

$\mu > 2$ more than 2 ×

Claim $\mu \le 2$ not more than 2

6. Let μ the true population mean time it takes to learn how to use this software ~~is not more than 2 hours~~

$\mu = 2$

$H_0 : \mu \le 2$ Claim (not more than 2)

$H_1 : \mu > 2$ ~~claim~~

$n = 12$, t-test is used

At $\alpha = 0.01$, since σ is unknown, ~~$n \ge 30$~~, ~~z-test is used~~

~~critizal value~~

critizal value $= +t_{0.01,11} = +2.718$

Rejection region: $t > +2.718$

~~Non-Rejection region:~~

$$T = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{2.2333 - 2}{\frac{0.4807}{\sqrt{12}}} = 1.681$$

consistent in term of dp!

Since $t = 1.681 < 2.718$, it is failed to reject H_0 at $\alpha = 0.01$. Hence, we can conclude that the mean time it takes to learn how to use this software is not more than 2 hours for people. i.e the claim is true.

Q7. A machine assesses the life of a ball-point pen, by measu[...] continuous line drawn using the pen. A random sample of 8[...] a mean writing length of 1.21 km. A random sample of 75 p[...] mean writing length of 1.25 km. Assuming that the stand[...] writing length of a single pen is 0.15 km for both brands, t[...] significance, whether the mean writing lengths of the [...] significantly.

general

ok. Jing Xian

Note

If we just write $Z = -1.6591 > -1.96$,

it is possible that $Z > 1.96$

⇒ misleading the result then

⇒ Do not write like this!

$\bar{x}_X = 1.21$ $\quad n_X = 80$ $\quad \sigma_X = 0.15$

$\bar{x}_Y = 1.25$ $\quad n_Y = 75$ $\quad \sigma_Y = 0.15$

Let μ_i be the true population mean writing lengths of a ball-point pen brand i, where $i = 1(X)$, $i = 2(Y)$

$H_0: \mu_1 = \mu_2$ $\Rightarrow$ $H_0: \mu_1 - \mu_2 = 0$

$H_1: \mu_1 \neq \mu_2$ $\quad$ $H_1: \mu_1 - \mu_2 \neq 0$ claim

At $\alpha = 0.05$, since σ_1 and σ_2 are given, z-test is used

critical value $= \pm z_{0.025} = \pm 1.96$

critical region: $z < -1.96$ or $z > 1.96$

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$= \frac{(1.21 - 1.25) - 0}{\sqrt{\frac{0.15^2}{80} + \frac{0.15^2}{75}}}$$

$$= \frac{-0.04}{0.0241}$$

$$= -1.6591$$

∴ Since $-1.96 < z = -1.6591 < 1.96$, H_0 is failed to reject at $\alpha = 0.05$ and conclude that the mean writing length of the two brands ~~are difference~~ do not differ significantly.

Q8. The management of a supermarket wanted to investigate whether the male customers spend less money on average than the female customers. A sample of 35 male customers who shopped at this supermarket showed that they spent an average of RM80 with a standard deviation of RM17.50. Another sample of 40 female customers who shopped at the same supermarket showed that they spent an average of RM96 with a standard deviation of RM14.40. Using the 2% significance level, can you conclude that the mean amount spent by all male customers at this supermarket is less than that spent by all female customers?

Cecilia

$\mu_2 - \mu_1 > 0$

$\mu_1 < \mu_2 \Rightarrow \mu_1 - \mu_2 < 0$

Let μi:true population mean amount of spent by all i customers at this supermarket,i=male,female (1, 2)

n1=35 X1=80 S1=17.50 ($\bar{x}_1$)

n2=40 X2=96 S2=14.40 ($\bar{x}_2$)

$\boxed{\mu 1<\mu 2 \quad \mu 1-\mu 2 < 0}$

H_0:μ1-μ2 >= 0

H_1:μ1-μ2 < 0 claim

Left-tailed test

σ_1, σ_2 unknown

a=0.02, $n_1 > 30$, $n_2 > 30$, n>30,use Z-dist $Z_{0.02}$=2.0537

Critical value: -2.0537

Critical region: Z<-2.0537

X1-X2=80-96=-16

Z=(-16-0)/(sqrt(17.50²/35+14.40²/40))=-4.2863

Since Z<-2.0537, H_0 is rejected at α=0.02. Hence, we can conclude that the mean amount spent by all male customers at this supermarket is less than that spent by all female customers.

-4.2863

alternatively,

$H_0: \mu_2 - \mu_1 \leq 0$

$H_1: \mu_2 - \mu_1 > 0$ right

c.v = 2.0537

c.r : Z > 2.0537

Z = 4.2863

Q9. The manager of a package courier service believes that packages shipped at the end of the month are heavier than those shipped early in the month. To test this, he weighed a random sample of 20 packages at the beginning of the month. He found that the mean weight was 20.25 kg with standard deviation was 5.84 kg. Ten packages randomly selected at the end of the month had a mean weight of 24.80 kg with standard deviation of 5.67 kg. At the 0.05 significance level, can we conclude that the packages shipped at the end of the month weigh more? Assume equal variances between the packages shipped at the early of the month and at the end of the month.

OK

$\mu_2 > \mu_1$

Sean

$n_1 = 20$ $\quad \bar{x}_1 = 20.25$ $\quad s_1 = 5.84$

$n_2 = 10$ $\quad \bar{x}_2 = 24.80$ $\quad s_2 = 5.67$ $\quad \sigma_1^2 = \sigma_2^2$

Let μ_1 & μ_2 be the true population mean weight of the packages shipped at the beginning of the month and at the end of month, respectively.

$H_0: \mu_1 \geq \mu_2 \quad \Rightarrow \quad H_0: \mu_1 - \mu_2 \geq 0$

$H_1: \mu_1 < \mu_2 \quad \quad \quad H_1: \mu_1 - \mu_2 < 0$

or $H_0: \mu_2 - \mu_1 \leq 0$

$H_1: \mu_2 - \mu_1 > 0$

At $\alpha = 0.05$, σ_1 & σ_2 are unknown, $\sigma_1^2 = \sigma_2^2$, $n_1 < 30$ & $n_2 < 30$, t-statistic is used.

Critical value: $-T_{0.05, 28} = -1.701$

Critical region: $T < -1.701$

C.V $= T.701$

C.r: $T > 1.701$

$t = 2.030$

$$S_p = \sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}}$$

$$= \sqrt{\frac{(19)(5.84)^2 + (9)(5.67)^2}{28}}$$

$$= 5.7859$$

$$T = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$= \frac{(20.25 - 24.8) - (0)}{5.7859\sqrt{\frac{1}{20} + \frac{1}{10}}}$$

$$= -2.030$$

Since $t_1 = -2.030 < -1.701$, we reject H_0 at $\alpha = 0.05$.

Hence, we can conclude that the package shipped at the end of the month weight more

or

There is sufficient evidence that the packages shipped at the end of the month weight more

Q10. Computer response time is defined as the length of time a user has to wait for the computer to access information on the disk. Suppose a data center wants to compare the average response times of its two computer disk drives. To test this, independent random samples of 13 response times for disk 1 and 15 response times for disk 2 were selected. The data (recorded in milliseconds) are as follows.

Disk 1	59	73	74	61	92	60	84	33	54	73	47	102	75		
Disk 2	71	63	40	34	38	48	60	75	47	41	44	86	53	68	39

Assume that the response times for both disks having same variances. Test at the 1% significance whether the mean response times of the two disks differ significantly.

Mavis

ok.

$\Sigma x_1 = 887,\ \Sigma x_1^2 = 64699$

$\Sigma x_2 = 807,\ \Sigma x_2^2 = 46915$

10) Let μ_i be the true population mean response times for computer disk ~~1~~ i, where ~~and disk 2~~, i=1 (1), i=2 (2)

$H_0: \mu_1 = \mu_2 \Rightarrow H_0: \mu_1 - \mu_2 = 0$ $\qquad n_1 = 13,\ \bar{x}_1 = 68.2308,\ s_1 = 18.6599$

$H_1: \mu_1 \neq \mu_2$ claim $\qquad H_1: \mu_1 - \mu_2 \neq 0$ claim $\qquad n_2 = 15,\ \bar{x}_2 = 53.8,\ s_2 = 15.8078$

At $\alpha = 0.01$, since σ_1 and σ_2 are unknown, n_1 and $n_2 < 30$, assume $\sigma_1^2 = \sigma_2^2$, t-dist with S_p is used.

Critical value $= \pm t_{0.005,26} = \pm 2.779$

critical region: $T < -2.779$ or $T > 2.779$

$$S_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2} = \frac{12(18.6599)^2 + 14(15.8078)^2}{26} = 295.2582$$

$$T = \frac{(\bar{x}_1-\bar{x}_2)-(\mu_1-\mu_2)}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{(68.2308-53.8)-0}{\sqrt{295.2582\left(\frac{1}{13}+\frac{1}{15}\right)}} = 2.2163$$

$-2.779 < t = 2.2163 < 2.779$,

Since t = 2.2163 < 2.779, it is failed to reject H_0 at $\alpha = 0.01$. Hence, we can conclude that the mean response time of the two disks are not significantly different.

claim

Test xxxx : ← let say H_0.

If result : reject H_0.

claim is true

⇒ xxx is not in conclusion.

claim is true

⇒ insufficient evidence that xxxx.

If result : fail to reject H_0.

claim is true

⇒ xxx is in conclusion.

claim is true

⇒ sufficient evidence that xxxx.

claim
Test xxxx ← let say H_1.

If result : reject H_0 ⇒ conclude H_1

claim is true
⇒ xxx is in conclusion.

claim is true
⇒ sufficient evidence that xxxx.

If result : fail to reject H_0 ⇒ cannot conclude H_1

claim is true
⇒ xxx is not in the conclusion.

claim is true
⇒ insufficient evidence that xxxx.

1. Classical: critical value vs test statistic
2. p-value: prob of tail area beyond test statistic vs alpha

If p-value =< alpha, H0 is rejected. Otherwise, it is failed to reject H0.

3. Confidence interval: CI vs test value (in H0)

If the CI includes the test value, H0 is failed to reject. Otherwise, we reject H0.

1. one/two tailed test, z/t test
2. one/two tailed test, z test
   (* can estimate for t test, eg17)
3. two tailed test, z/t test

Q1. Classical; p-value
Q2. Classical; p-value
Q3. Classical; p-value; CI
Q4. Classical; p-value
Q5. Classical; p-value by estimation
Q6. Classical; p-value by estimation
Q7. Classicalp-value; CI
Q8. Classical; p-value
Q9. Classical; p-value by estimation
Q10. Classical; p-value by estimation, CI

Q11. Classical; p-value by estimation
Q12. Classical; p-value; CI
Q13. Classical; p-value
Q14. Classical; p-value; CI
Q15. Classical; p-value

Q11. A company claims that its 12-week special exercise program significantly reduces weight. A random sample of six persons was selected and these persons were put on this exercise program for 12 weeks. The following table gives the weights (in kg) of those six persons before and after the program.

Before the program	81.6	88.5	80.3	100.2	94.3	90.3
After the program	83.0	84.8	73.0	92.5	89.4	85.7

Test the hypothesis that the 12-week special exercise program significantly reduces weight at $\alpha = 0.01$.

ok

paired / dependent sample

Janet

reduce weight

(2) after < (1) before

before − after > 0

estimate p-value

P-value = $P(t > 3.3388)$ *

t-table, df = 5

0.1	0.05	0.025	0.01	...
1.476	2.015	2.571	3.365	...

$\underbrace{2.571 \quad 3.365}_{3.3388}$

$0.01 < \text{p-value} < 0.025$

$P(t > 3.3388) \rightarrow$ p-value $> \alpha$ *

Q11.

Before program (x_1)	After program (x_2)	Difference, D ($D = x_1 - x_2$)	D^2
81.6	83.0	-1.4	1.96
88.5	84.8	3.7	13.69
80.3	73.0	7.3	53.29
100.2	92.5	7.7	59.29
94.3	89.4	4.9	24.01
90.3	85.7	4.6	21.16
Total		26.8	173.4

$\bar{D} = \frac{26.8}{6} = 4.4667$, $S_D = \sqrt{\frac{\Sigma D^2 - \frac{(\Sigma D)^2}{n}}{n-1}} = \sqrt{\frac{173.4 - \frac{(26.8)^2}{6}}{6-1}} = 3.277$

Let μ_D : true population mean of the difference in the weight before and after the program

$\boxed{H_0 : \mu_D \leq 0}$

$H_1 : \mu_D > 0$ ← claim, significant reduce weight

At $\alpha = 0.01$, critical value $= t_{0.01, 5} = 3.365$

rejection region : $T > 3.365$

$T = \frac{\bar{D} - \mu_D}{S_D/\sqrt{n}} = \frac{4.4667 - 0}{3.277/\sqrt{6}} = 3.3388$

Since $T = 3.3388 < 3.365$, it is $\boxed{\text{failed to reject } H_0}$ at $\alpha = 0.01$. Hence, we can conclude that the 12-week special exercise program ~~significantly~~ does not significantly reduce~~s~~ weight.

OK. Yee Hao

Q12. A coin is thrown 1000 times and 546 tails are obtained. Test whether the coin is $\boxed{\text{fair}}$ at $\alpha = 0.04$.

→ equal chance of getting any outcome (2 outcomes)

→ $P_t = P_h = 0.5$

$$\hat{p} = \frac{546}{1000} \leftarrow \text{no of tails}$$

Classical approach

$n = 1000 \quad \hat{p} = 0.546$

Let p be the true pop. proportion of ~~a coin~~ getting a tail when a coin is thrown

fair

$H_0: p = 0.5$ (claim)

$\boxed{H_1: p \neq 0.5}$

critical value:

At $\alpha = 0.04$ $\pm z_{0.02} = \pm 2.0537$

critical region: $z < -2.0537$ or $z > 2.0537$

$$z = \frac{\hat{p}\downarrow\; 0.546 - 0.5}{\sqrt{\frac{0.5 \times 0.5}{1000}}} = 2.9093$$

Since $z = 2.9093 > 2.0537$, $\boxed{H_0 \text{ is rejected}}$ at $\alpha = 0.04$.

There is insufficient evidence that the coin is fair.

2 tailed

$$p\text{-value} = 2P(Z > 2.91) = 2(0.00181) = 0.00362 < \alpha = 0.04 \Rightarrow \text{reject } H_0$$

C.I. Approach:

$H_0: p = 0.5$

$H_1: p \neq 0.5$

$\alpha = 0.04 \Rightarrow 96\%$ C.I. for p is

$$\hat{p} \pm z_{0.02}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$= 0.546 \pm 2.0537\sqrt{\frac{0.546(0.454)}{1000}}$$

$$= [0.5137, 0.5783]$$

Since the C.I. does not include 0.5, H_0 is rejected at $\alpha = 0.04$.

Q13. A manufacturer claimed that at least 95% of the equipment which he supplied to a factory conformed to specifications. An examination of a sample of 200 pieces of equipment revealed that 18 were faulty. Test the manufacturer's claim at a significance level of 0.05.

Ze Xuan

ok.

Q13 Let p : true population proportion of equipment supplied conformed to specifications

$H_0: p \geq 0.95$ claim, at least 95% $\hat{p} = \frac{182}{200} = \frac{200-18}{200}$

$H_1: p < 0.95$

At $\alpha = 0.05$, critical value $= -z_{0.05} = -1.6449$

rejection region : $z < -1.6449$

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.91 - 0.95}{\sqrt{\frac{0.95(0.05)}{200}}} = -2.60$$

Since $z = -2.60 < -1.6449$, H_0 is rejected at $\alpha = 0.05$

Thus, we can conclude that not at least 95% of equipment the manufacturer supplied conformed to specifications, i.e. the claim is not true.

Q14. The manager of a supermarket chain wants to determine the difference between the proportion of morning shopper who are men and the proportion of afternoon shoppers who are men. Over a period of 2 weeks, the chain's researchers conduct a systematic random sample survey of 400 morning shoppers, which reveals that 352 are women and 48 are men. During this same period, a systematic random sample of 480 afternoon shoppers reveals that 312 are women and 168 are men. At the 1% significance level, is there a difference between the proportion of men shopper during morning and afternoon period?

Pui Mun

9

Let p_i be the true population proportion of men shoppers ~~i~~ during ~~morning and afternoon~~ period, i = morning (1) and afternoon (2)

x_i = no. of men shopper during morning and afternoon period

$H_0 : p_1 - p_2 = 0$

$\boxed{H_1 : p_1 - p_2 \neq 0}$ (claim, difference)

At $\alpha = 0.01$

Critical value : $\pm z_{\frac{\alpha}{2}} = \pm z_{\frac{0.01}{2}} = \pm z_{0.005} = \pm 2.5758$

rejection region : $z < -z_{\frac{\alpha}{2}}$ OR $z > z_{\frac{\alpha}{2}}$

$= z < -2.5758$ OR $z > 2.5758$

$\hat{p} = \dfrac{x_1 + x_2}{n_1 + n_2} = \dfrac{48 + 168}{400 + 480} = \dfrac{216}{880} = \dfrac{27}{110}$

$\hat{q} = 1 - \hat{p} = 1 - \dfrac{27}{110} = \dfrac{83}{110}$

$z = \dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \dfrac{\frac{48}{400} - \frac{168}{480}}{\sqrt{\frac{27}{110}\left(\frac{83}{110}\right)\left(\frac{1}{400} + \frac{1}{480}\right)}}$

$= -7.90$

∴ Since $z = -7.90 < -2.5758$, $\boxed{\text{reject } H_0}$ at $\alpha = 0.01$

We can conclude that there is a different between proportions of men shopper during morning and afternoon period

Q15. One thousand items from factory B are examined and found to contain 3% defectives. One thousand five hundred similar items from factory C are found to contain only 2% defectives. Can you conclude that the items of factory C are superior to those of factory B? Use $\alpha = 0.05$.

OK

Chun Wai

1,000 items => factory B => 3% defectives
1,500 items => factory C => 2% defectives
$\alpha = 0.05$

C superior than B

$\Downarrow$

C is better

$\Downarrow$

$P_C < P_B$
$\;\;\; 2 \;\;\;\;\; 1$

$\Downarrow$

$P_2 - P_1 < 0$

Let P_i: true population proportion of defective items from factory i, i = B (1), C (2)

$\boxed{H_0: P_2 < P_1 \Rightarrow P_2 - P_1 \geq 0}$

$H_1: P_2 > P_1 \Rightarrow P_2 - P_1 < 0$ ← claim, C superior than B.

At $\alpha = 0.05$, $Z_{0.05} = -1.6449$

Rejection region: $Z < -1.6449$

$$\hat{P} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{(1000 \times 0.03) + (1500 \times 0.02)}{1000 + 1500} = \frac{60}{2500}$$

$$\hat{q} = 1 - \frac{60}{2500} = \frac{2440}{2500}$$

$$Z = \frac{\cancel{\hat{P}_1 - \hat{P}_2}}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$\hat{P}_2 - \hat{P}_1$

$$= \frac{\frac{30}{1000} - \frac{30}{1500}}{\sqrt{\frac{60}{2500}\left(\frac{2440}{2500}\right)\left(\frac{1}{1000} + \frac{1}{1500}\right)}}$$

$= -1.6005$

Alternatively,
$H_1: P_1 - P_2 > 0$
C.V = 1.6449
C.R: $Z > 1.6449$
$Z = 1.6005$

Since $Z = -1.6005 > -1.6449$, it is $\boxed{\text{failed to reject } H_0}$ at $\alpha = 0.05$. Thus, we can conclude that the items of factory C are not superior to those of factory B.