Tutorial 5

1. Negate and simplify each of the following.

   i) $\exists\, x \ni [p(x) \vee q(x)]$ ii) $\forall\, x, [p(x) \wedge \sim q(x)]$

i) $\exists x \ni [p(x) \vee q(x)]$

$\sim (\exists x \ni [p(x) \vee q(x)]] = \forall x, \sim [p(x) \vee q(x)]$

$= \forall x\ [\sim p(x) \wedge \sim q(x)]$

ii) $\forall x, [p(x) \wedge \sim q(x)]$

$\sim (\forall x, [p(x) \wedge \sim q(x)]) = \exists x \ni \sim [p(x) \wedge \sim q(x)]$

$= \exists x \ni [\sim p(x) \vee q(x)]$

Q1. iii) $\forall x, [p(x) \rightarrow \sim q(x)]$ iv) $\exists x \ni [p(x) \vee q(x) \rightarrow r(x)]$

iii) $\forall x, [p(x) \rightarrow \sim q(x)] : \forall x, [\sim p(x) \vee \sim q(x)]$ ✓

negate $\longrightarrow$ $\exists x \ni [p(x) \wedge q(x)]$ ✓

such that

iv) $\exists x \in [p(x) \vee q(x) \rightarrow r(x)] \xrightarrow{\text{negate}} \forall x, [\sim p(x) \wedge \sim q(x) \rightarrow \sim r(x)]$

$= \forall x, \cancel{[\sim p(x) \wedge q(x) \wedge r(x)]}$

negation $= \forall x, [(p(x) \vee q(x)) \wedge \sim r(x)]$

$(p \vee q) \rightarrow r \equiv \sim (p \vee q) \vee r$

negate: $(p \vee q) \wedge \sim r$

(i) $\exists\, x \in \mathbb{R}^{\text{nonneg}} \ni x > 3$ and x is even but $x^2 \leq 9$ ✓

(ii) $\exists\, n \in \mathbb{Z}$ such that n is ✓ prime and n is not odd but $n \neq 2$ ✓

(iii) $\exists\, a, b, c \in$ integers $\ni (a-b)$ is even and $(b-c)$ is even but $(a-c)$ is not even. ✓

(iv) $\forall\, x \in \mathbb{R},\ x^2 \neq 2$ ✓

ii) $\forall n \in \mathbb{Z}$, if $\overset{p}{(n \text{ is prime})}$, then $\overset{q}{(n \text{ is odd})}$ or $\overset{r}{(n = 2)}$

$\forall n \in \mathbb{Z},\ p \to (q \vee r)$

$\therefore\ p \to (q \vee r) \equiv \sim p \vee (q \vee r)$

$\downarrow$ negate

$p \wedge \sim(q \vee r) \equiv p \wedge (\sim q \wedge \sim r)$

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v) $\exists x \in \mathbb{Z}^+$ such that $(x \text{ is } \overset{p}{\text{even}})$ and $(\overset{q}{\text{prime}})$ $\quad p \wedge q$

$\forall x \in \mathbb{Z}^+$, x is not even or not prime $\quad \sim(p \wedge q)$

vi) All even integers have even squares

Some even integers do not have even squares

vii) No irrational numbers are integers

$\Rightarrow$ All irrational numbers are not integers.

Some irrational numbers are integers.

viii) If $(\text{an integer is divisible}^{\,p} \text{ by } 2)$, then $(\text{it is } \overset{q}{\text{even}})$ $\quad p \rightarrow q \equiv \sim p \vee q$

negat

An integer is divisible by 2 and an integer is not even.

Quantifiers : $\forall$, $\exists$

variables : $x, y, \ldots$

3. i) Everybody trusts somebody

$\forall x \in$ people, $\exists y \in$ people $\ni$ x trusts y ✓

• $\exists$ exercise x such that x have answers.

ii) Somebody trusts everybody

$\exists x \in$ people $\ni$ $\forall y \in$ people, x trusts y ✓

iii) The number of rows in any truth table equals 2^n for some integer n.

$\forall x \in$ number of rows in any truth table, $\exists n \in \mathbb{Z} \ni x = 2^n$ ✓

$\forall x \in$ number of rows in any truth table, $\exists y \in$ integer n such that x equals 2^n for y.

Q3) iv) v)

iv) $\forall x \in$ action, $\exists y \in$ reaction $\ni x$ has an equal & opposite reaction.

y

v) $\exists x \in$ prime number, $\forall y \in \mathbb{R} \ni x$ between y and $2x$.

y

$\equiv \mathbb{Z}$

4) i) $\exists x \ni \exists y \ni [xy=1]$ True ✓

ii) $\exists x \ni \forall y, [xy=1]$ False ✓ e.g. $1(1)=1$

$1(2) \neq 1$

$1(3) \neq 1$

iii) $\forall x, \exists y \ni [xy=1]$ False ✓

e.g. $(2)1 \neq 1$

$(3)1 \neq 1$

iv. $\exists x \ni \exists y \ni [(2x+y=5) \wedge (x-3y=-8)]$

$2x+y=5$, $x-3y=-8$

$y=5-2x$ $\quad x+6x=7$

$y=3$ ✓ $\in \mathbb{Z}^{\text{nonzero}}$ $\quad x=1$ ✓ $\in \mathbb{Z}^{\text{nonzero}}$

$\therefore$ True ✓

v. $\exists x \ni \exists y \ni [(3x-y=7) \wedge (2x+4y=3)]$

$3x-y=7$, $2x+4y=3$

$y=3x-7$ $\quad 2x+12x=31$

$y=3\left(\frac{31}{14}\right)-7$ $\quad 14x=31$

$=-\frac{5}{14} \notin \mathbb{Z}^{\text{nonzero}}$ $\quad x=\frac{31}{14}$ / $2.2142 \notin \mathbb{Z}^{\text{nonzero}}$

$\therefore$ False ✓

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(5) Rewrite each of the following quantifier statements formally using quantifier and variables, then write a negation for the statement.

i) For every odd integer n, there is an integer k such that $n = 2k+1$

$\forall\, n \in$ odd integer, $\exists\, k \in \mathbb{Z} \ni n = 2k+1$

Negation : $\exists\, n \in$ odd integer $\ni \forall\, k \in \mathbb{Z},\ n \neq 2k+1$

ii) Any even integers equals twice some other integer.

$\forall\, x \in$ even integers, $\exists\, y \in \mathbb{Z} \ni x$ equals twice y // $x = 2y$

Negation : $\exists\, x \in$ even integers $\ni \forall\, y \in \mathbb{Z}$, x not equals twice y

//

$x \neq 2y$

6) i) No college cafeteria food is good. Major premise

No good food is wasted. Minor premise

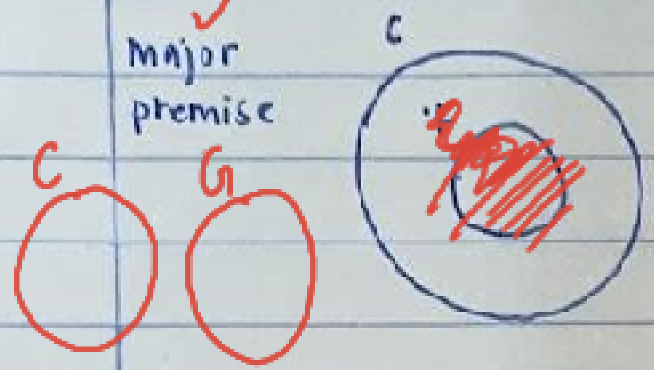
∴ No college cafeteria food is wasted. Conclusion

Let C : set of college cafeteria food

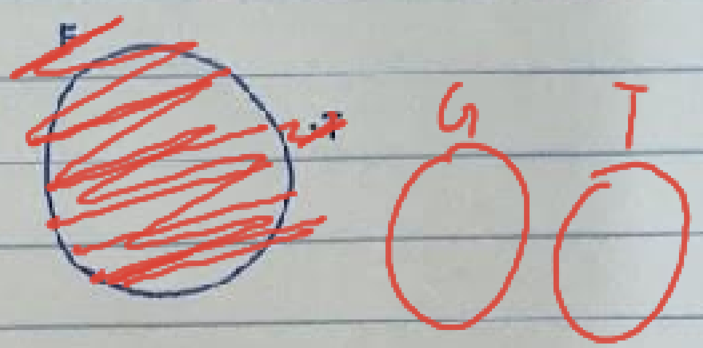
F : set of good food

T : ~~This particular~~ set of wasted food

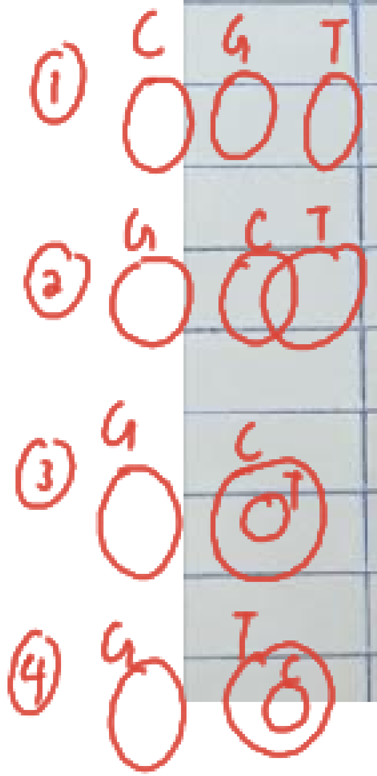
major premise



minor premise



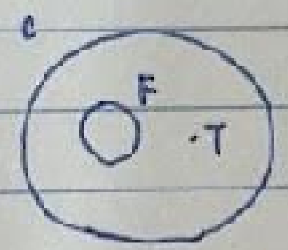
Possible conclusions :



2 outcomes :

This particular food is not good as T falls outside the disc F

This particular food is good as T falls inside the disc F.



∴ There is a contradiction between the conclusions, therefore argument is invalid

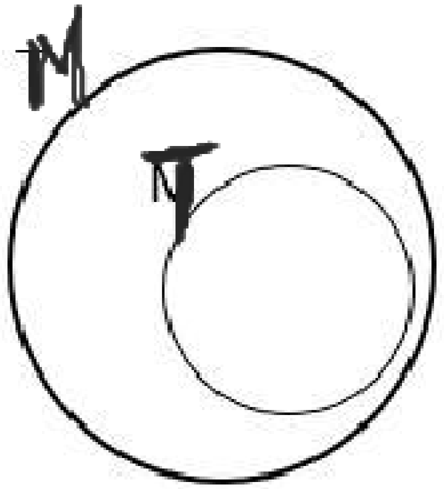
Q6 (ii)

Let T: set of teachers

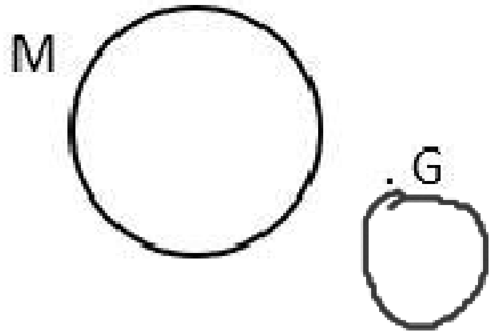
M: set of ~~teachers~~ people who make mistakes

G: set of gods

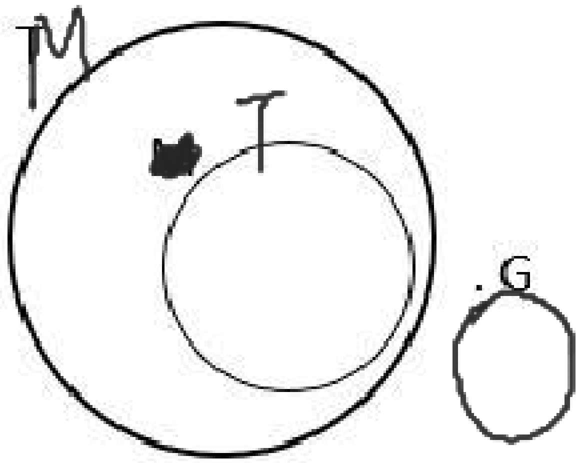
Major premise



Minor premise



The possible conclusion is



Therefore, no teachers are gods as G falls outside the disc of ~~M~~ T.

Hence, the argument is valid

6(iii)

All mathematics lectures have studied calculus.

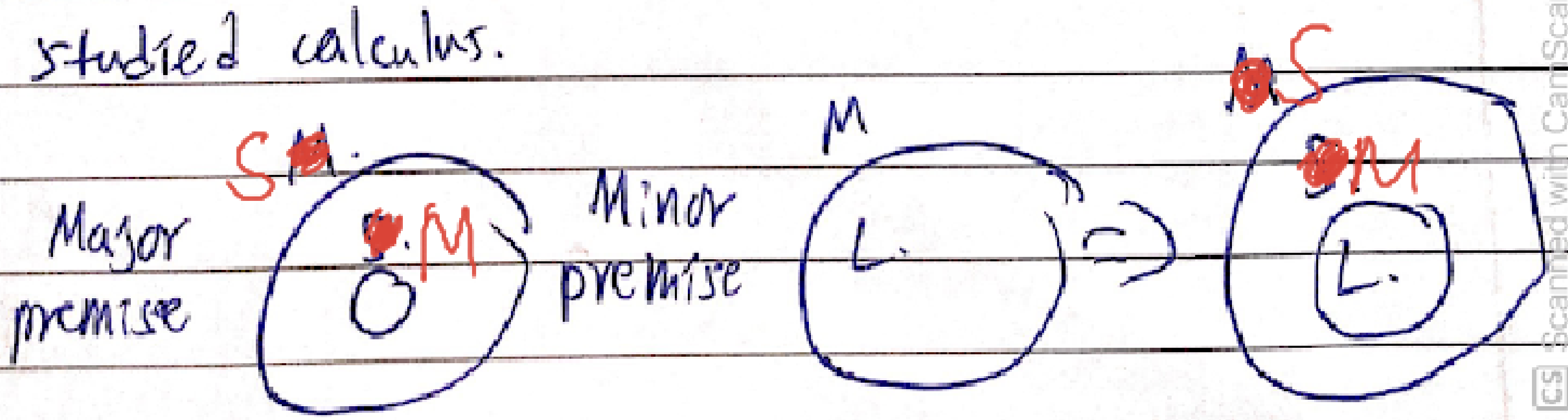
Lena is a mathematics lecture.

∴ Lena had studied calculus.

M= set of mathematics lectures.

S= Set of studied calculus.

L= Lena



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The Arguments is valid ✓

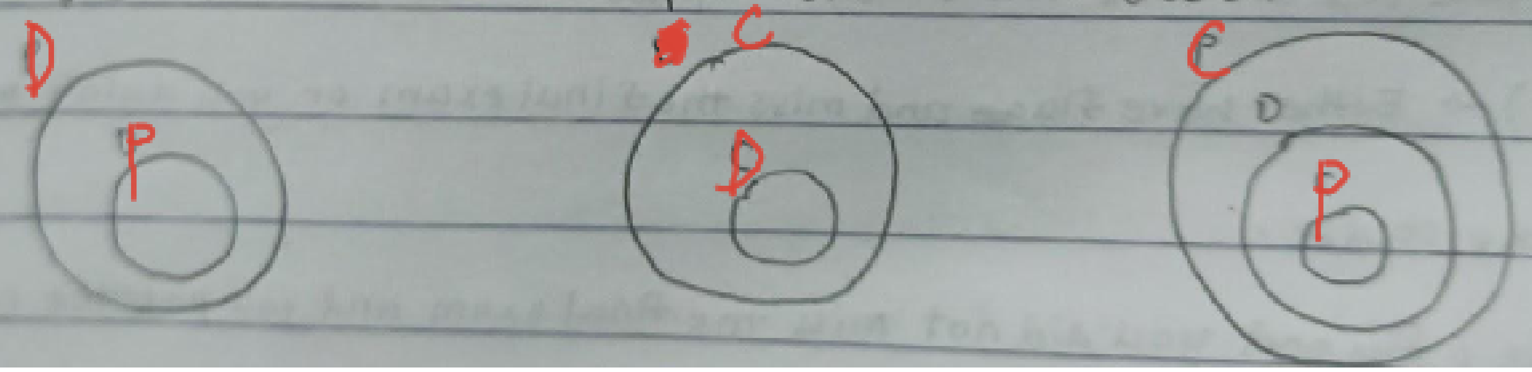
iii) All polynomial functions are differentiable.
All differentiable functions are continuous.
∴ All polynomial functions are continuous.

6.iii) let P : Sets of all polynomial functions ✓

D : Sets of all differentiable functions ✓

C : sets of Continuous functions

major premise   minor premise   conclusion



∴ The argument is valid.