

Jun Yan

X

- Q1. The thickness of photoresist applied to wafers in semiconductor manufacturing at a particular location on the wafer is uniformly distributed between 0.2050 and 0.2150 micrometers.

b

- (a) Find the proportion of wafers that exceeds 0.2125 micrometers.
(b) What thickness is exceeded by 10% of the wafers?
(c) Find the mean and standard deviation of photoresist thickness.

a

x = the thickness of photoresist applied to wafers

$$f(x) = \begin{cases} \frac{1}{0.01}, & 0.2050 < x < 0.2150 \\ 0, & \text{otherwise} \end{cases}$$
$$\frac{1}{b-a} = \frac{1}{0.2150 - 0.2050}$$
$$= \frac{1}{0.01}$$

$$x \sim U(0.2050, 0.2150)$$

$$\text{(a)} \quad P(x > 0.2125) = \int_{0.2125}^{0.2150} \frac{1}{0.01} dx \quad \text{or } 1 - \int_0^{0.2150}$$
$$= \frac{1}{0.01} [x]_{0.2125}^{0.2150}$$
$$= \frac{1}{0.01} [0.2150 - 0.2125]$$
$$= 0.25$$

$$\text{(b)} \quad P(x > y) = \int_y^{0.2150} \frac{1}{0.01} dx$$
$$\frac{1}{10} = \frac{1}{0.01} [x]_y^{0.2150}$$
$$\frac{1}{1000} = 0.2150 - y$$
$$0.2140 = y$$
$$\text{(c)} \quad E(x) = \mu = \frac{a+b}{2}$$
$$= \frac{0.2050 + 0.2150}{2}$$
$$= 0.21 \mu\text{m}$$

$$0.2140 = y$$

$$y = 0.2140 \mu\text{m}$$

$$\text{Var}(x) = \sigma^2 = \frac{(0.2150 - 0.2050)^2}{12}$$

$$\sigma^2 = \frac{1}{12 \cdot 0.0001}$$
$$\sigma = \sqrt{\frac{1}{12 \cdot 0.0001}}$$
$$= 0.0029 \mu\text{m}$$

Jun Dian

- Q2. The lifetime of a mechanical assembly in a vibration test is exponentially distributed with a mean of 400 hours.

- ok.
- (a) What is the probability that an assembly on test fails in less than 100 hours?
 - (b) Find the probability that an assembly operates for more than 500 hours before failure.
 - (c) Find the standard deviation of the lifetimes of mechanical assemblies in vibration tests.

x:lifetime of a mechanical assembly in a
vibration test

$$X \sim \text{Exp}(400), x > 0$$

$$\begin{aligned} a. P(x < 100) &= \int_0^{100} \frac{1}{400} e^{-\frac{x}{400}} dx \\ &= \frac{1}{400} \left[-e^{-\frac{x}{400}} \right]_0^{100} \\ &= -e^{-\frac{100}{400}} - (-e^0) \rightarrow 1 \\ &= 0.2212 \end{aligned}$$

P(x > 500)

$$\begin{aligned} b. 1 - P(x \leq 500) &= 1 - \int_0^{500} \frac{1}{400} e^{-\frac{x}{400}} dx \\ &= 1 - \left[-e^{-\frac{500}{400}} - (-e^0) \right] \\ &= 0.2865 \end{aligned}$$

No:

$$\mu = 6$$

$$\begin{aligned} 2. c. \text{Var}(x) &= 400^2 = \mu^2 \\ &= 160000 \end{aligned}$$

| $\sigma = 400$ hrs.

$$3. a. P(z < -0.6) = 0.2743$$

$$b. P(z > -1.28)$$

Standard normal dist , $P(Z < a) = \infty$ ← prob
 std normal \downarrow
 Z score $\cdot a$ tve or -ve
 $|Z| \geq 0$

$$P(Z \geq a) = 0.1711$$



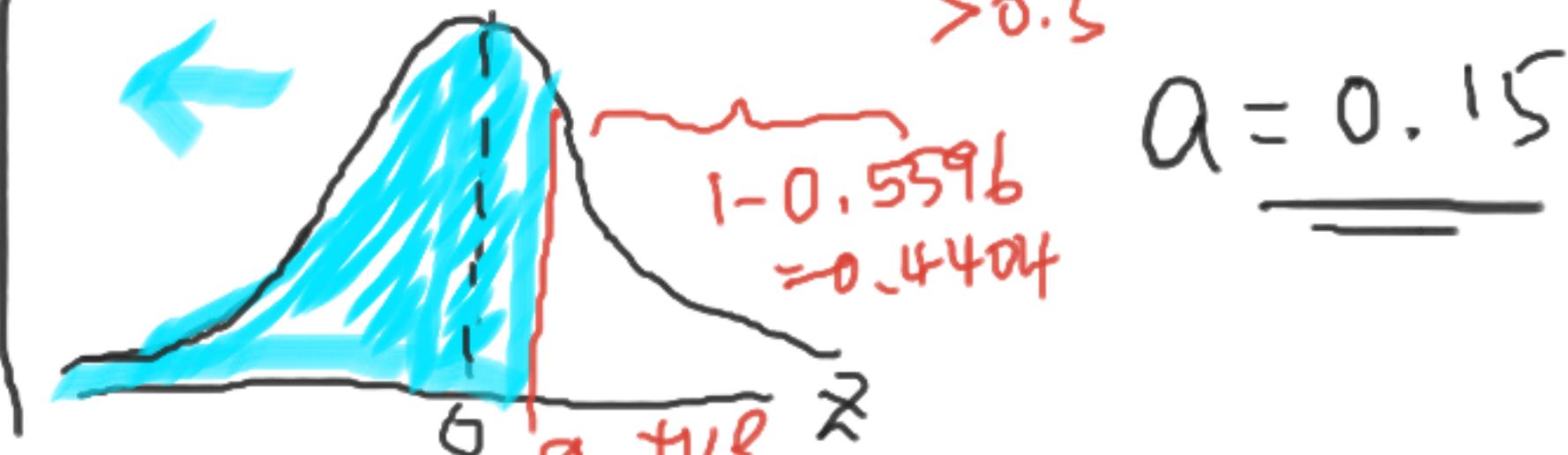
$$P(Z < a) = 0.1711$$



$$P(Z > a) = 0.5596$$



$$P(Z < a) = 0.5596$$



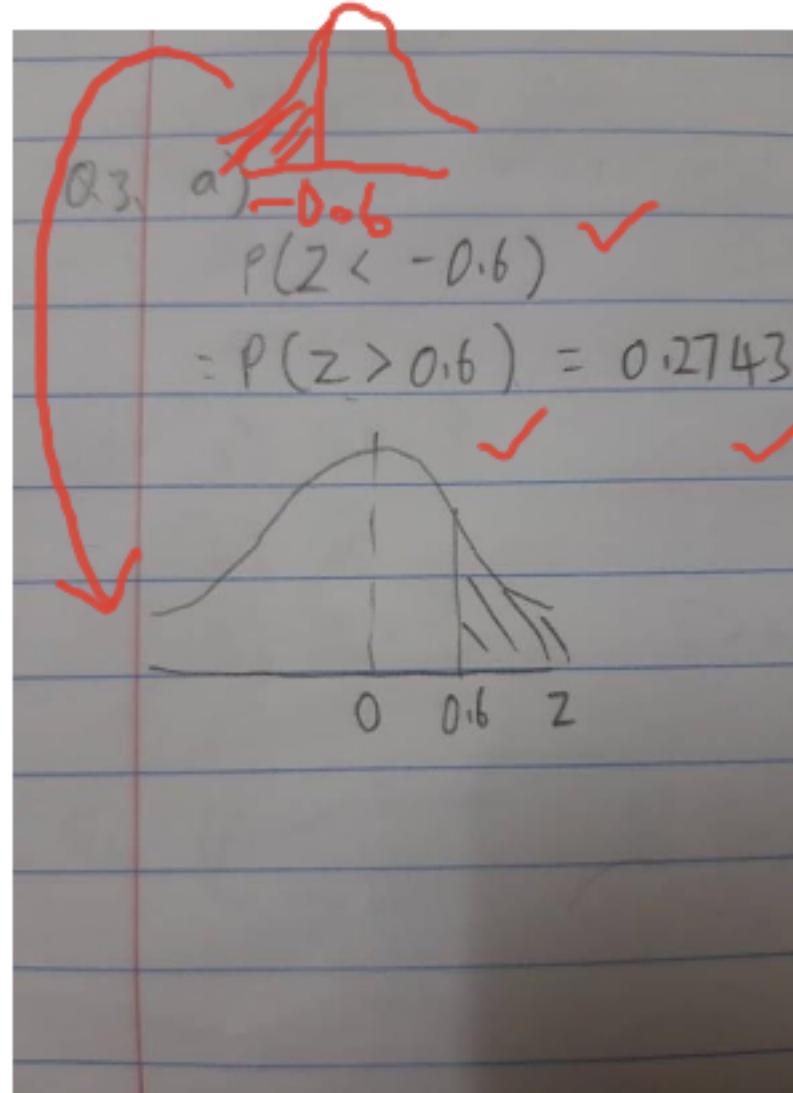
5L

- Q3. Let Z be the standardized normal variable, i.e. Z follows normal distribution with the mean $\mu = 0$, and variance $\sigma^2 = 1$. Find the value of the following probabilities.

In each case, sketch a curve and shade the area representing the probability.

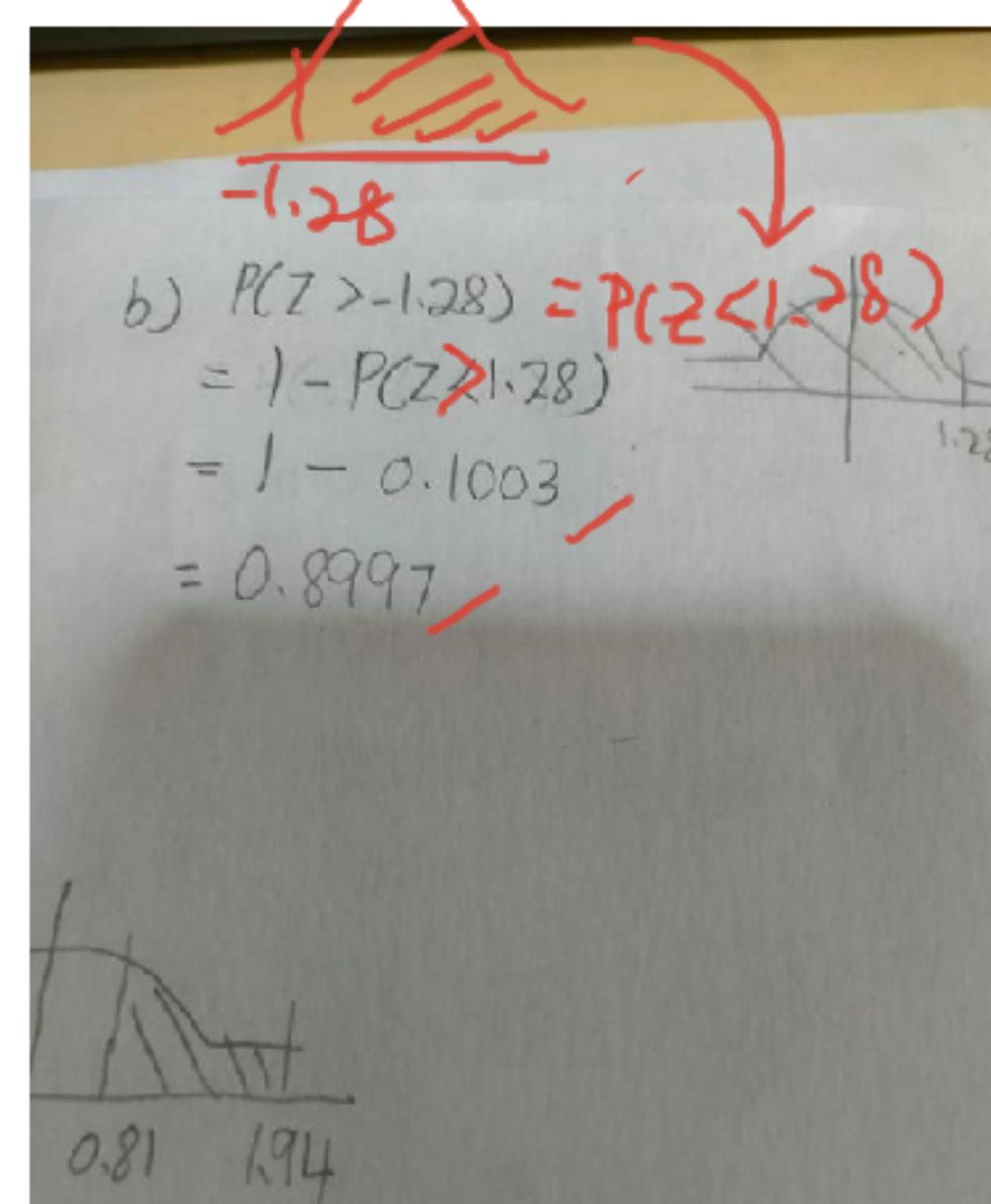
(a) $P(Z < -0.6)$

Khai Jun



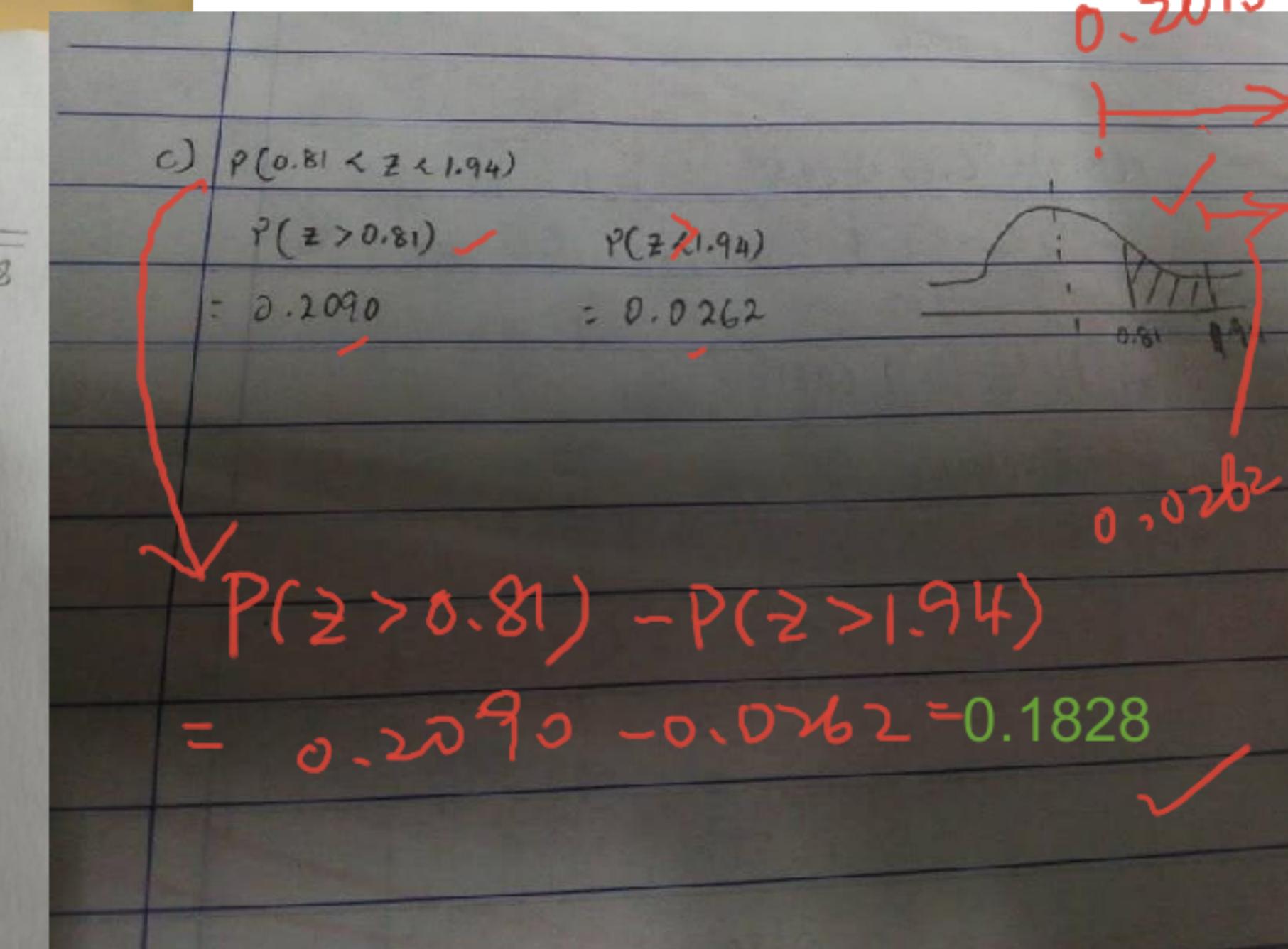
(b) $P(Z > -1.28)$

Eason



(c) $P(0.81 < Z < 1.94)$

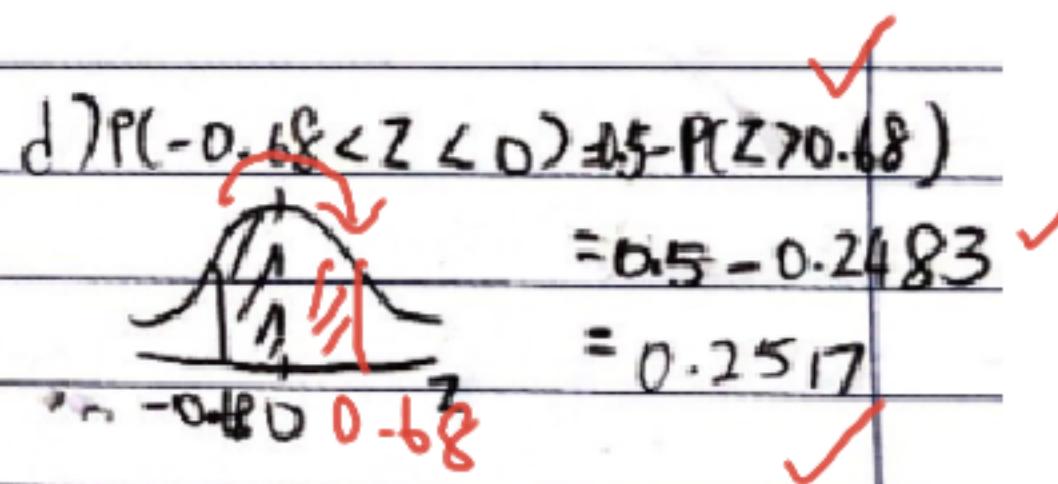
Li Yuet



Q3. Let Z be the standardized normal variable, i.e. Z follows normal distribution with the mean $\mu = 0$, and variance $\sigma^2 = 1$. Find the value of the following probabilities. In each case, sketch a curve and shade the area representing the probability.

(d) $P(-0.68 < Z < 0)$

Cecilia



(e) $P(-0.46 < Z < 2.21)$

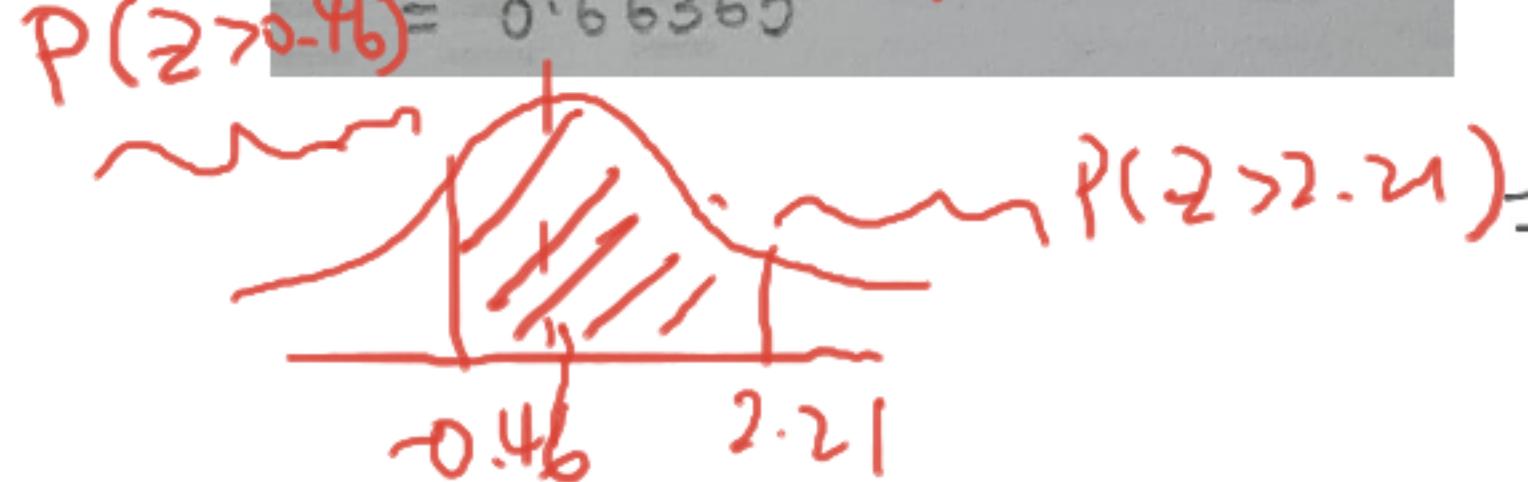
Ze Xuan

e) $P(-0.46 < Z < 2.21)$

$$= 1 - P(Z > 0.46) - P(Z > 2.21)$$

$$= 1 - 0.3228 - 0.01355$$

$P(Z > 0.46) = 0.66365$

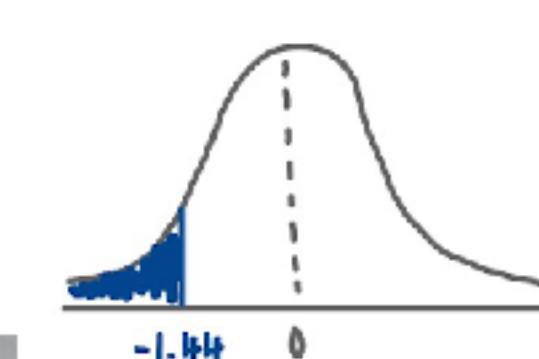


(f) $P(Z < -1.44 \text{ or } Z > 2.05)$

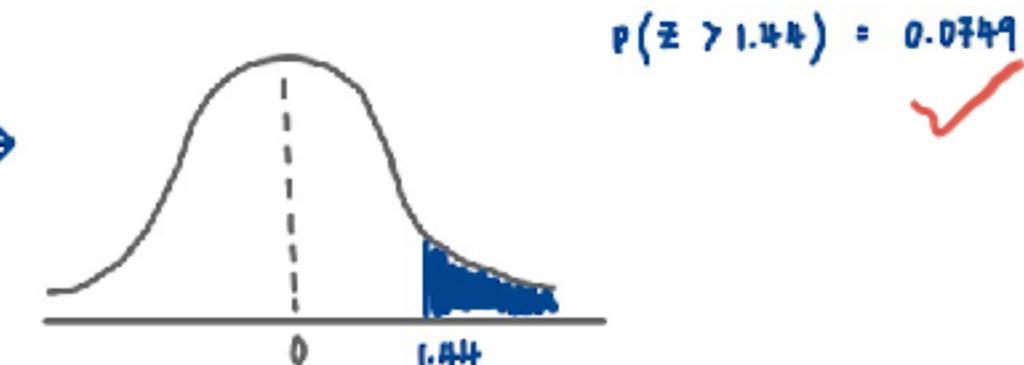
Pui Mun

(f) $P(Z < -1.44 \text{ or } Z > 2.05) = 0.09508$

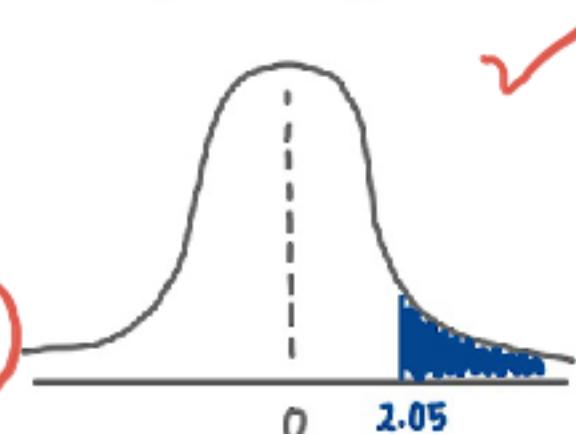
$P(Z < -1.44)$



"flip"



$P(Z > 2.05) = 0.02018$



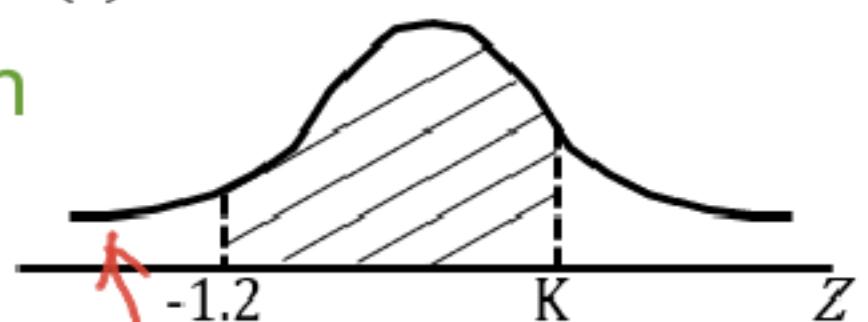
$$\begin{aligned} & P(Z < -1.44) + P(Z > 2.05) \\ & = P(Z > 1.44) + P(Z > 2.05) \\ & = \underline{\hspace{2cm}} + \underline{\hspace{2cm}} \end{aligned}$$



Q4. Find the value of K in each of the following cases. Given your answer correct to 2 decimal places.

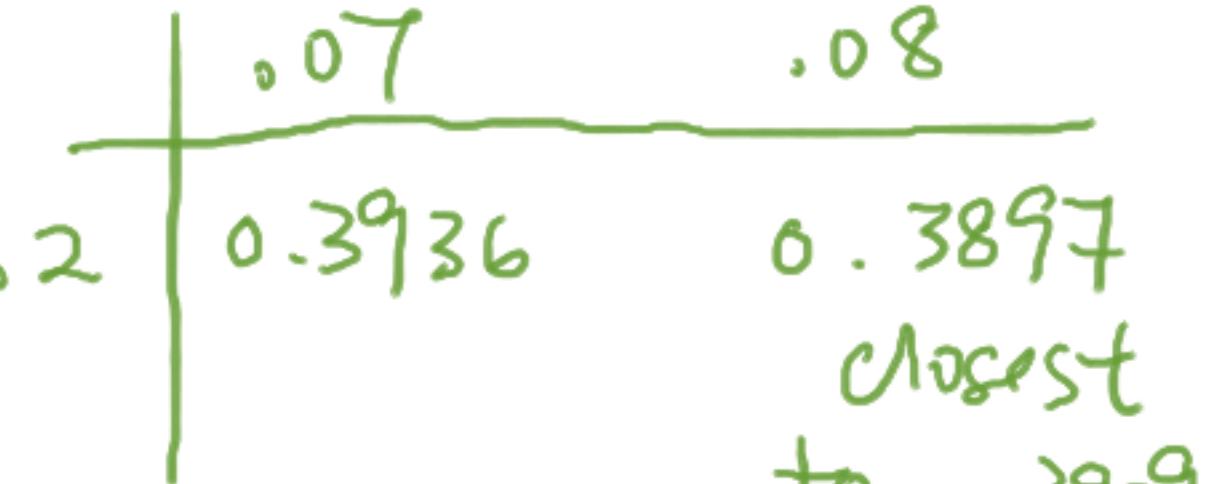
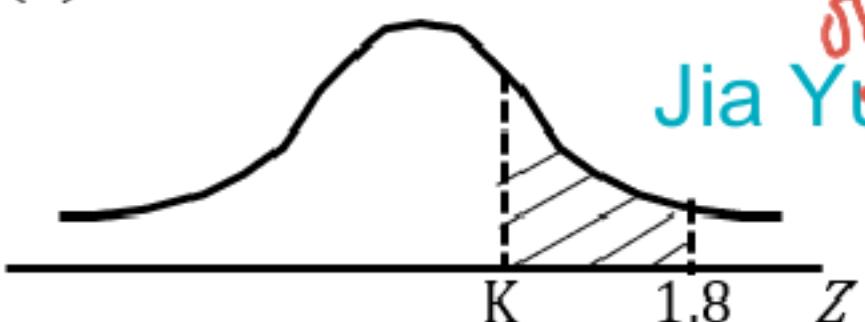
(a) Shaded area = 0.523

Aaron

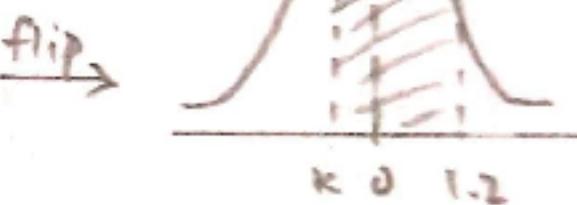


(b) Shaded area = 0.355

Jia Yu ✓



$$\begin{aligned} P(Z < -1.2) &= P(Z > 1.2) = \underline{0.1151} \\ 1 - P(Z > 1.2) - 0.523 &= 0.3619 \\ K &\approx 0.35 \quad \checkmark \\ &\downarrow \\ &= P(Z > K) \end{aligned}$$



(b)

Shaded = 0.355

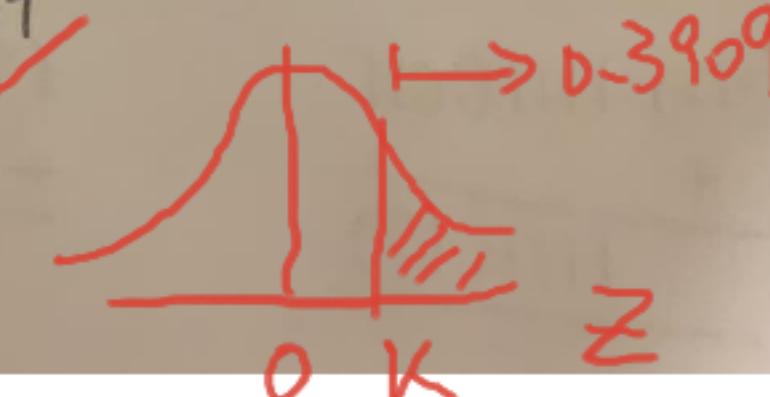
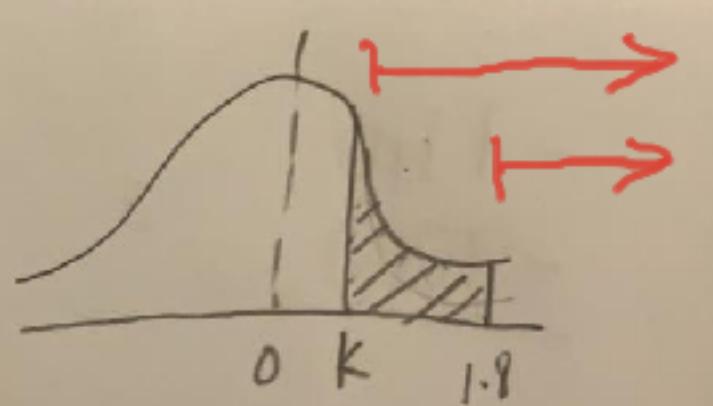
$$P(Z > 1.8) = \underline{0.0359} \quad \checkmark$$

$$0.355 = P(Z > K) - P(Z > 1.8)$$

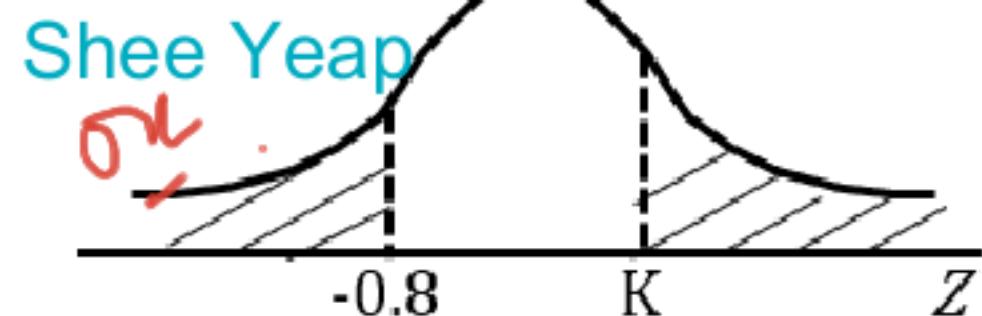
$$P(Z > K) = \underline{0.3909} \quad \checkmark$$

$$\therefore K \approx \underline{0.28} \quad \checkmark$$

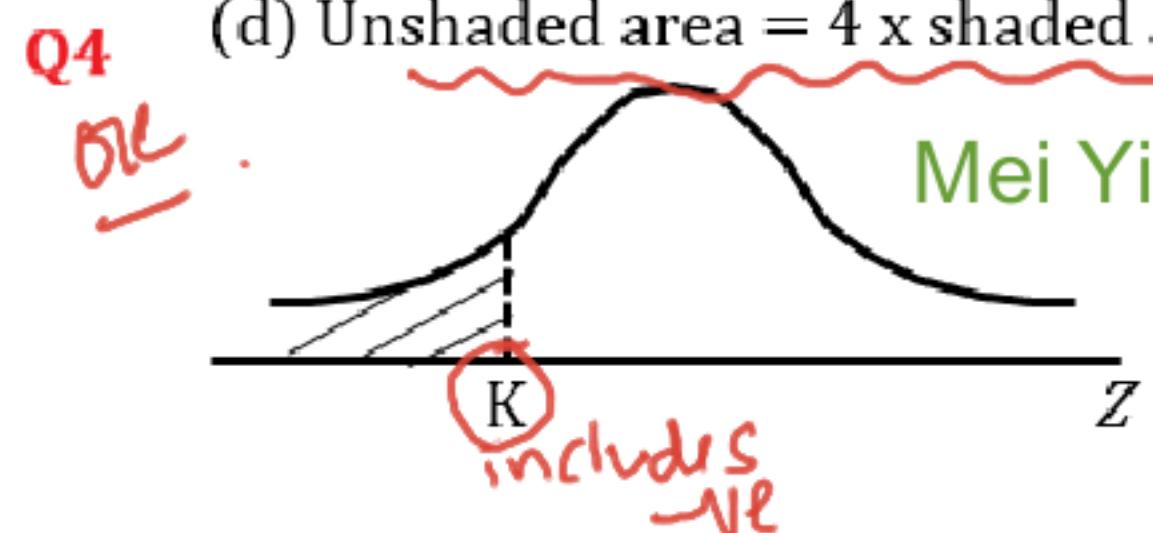
closest



(c) Sum of shaded area = 0.616



(d) Unshaded area = 4 x shaded area

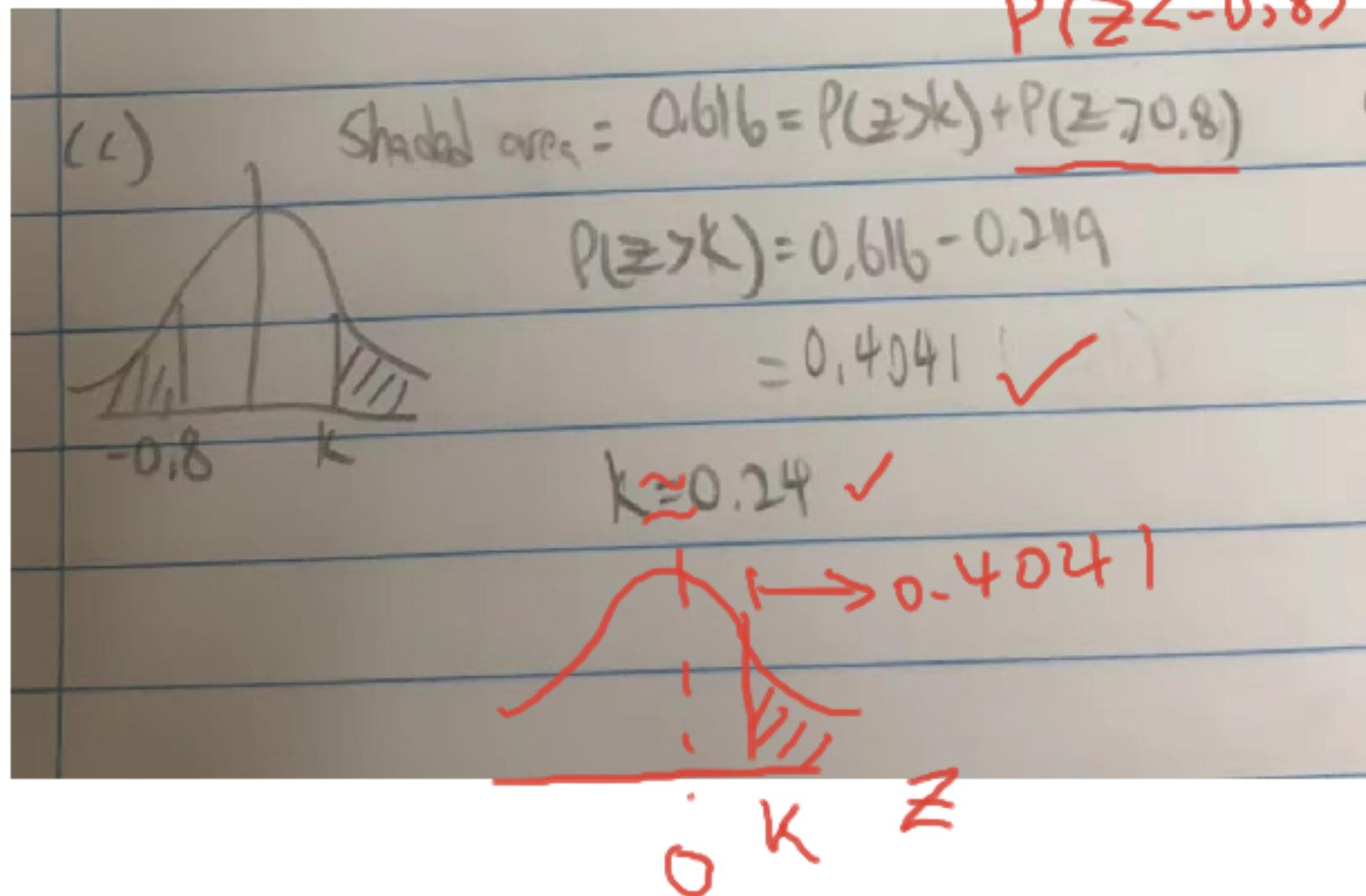


Let $P(z < K) = x$

$$P(z > K) = 4 \times P(z < K)$$

$$1 - P(z < K) = 4 \times P(z > K)$$

$$5 \times P(z > K) = 1$$

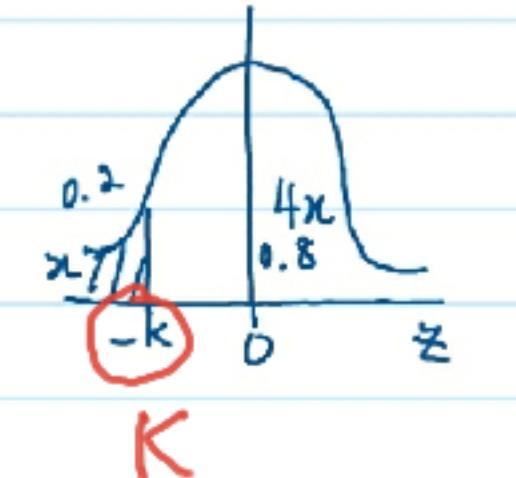


4d) $P(z < K) = 0.2$

$$P(z > K) = 0.2$$

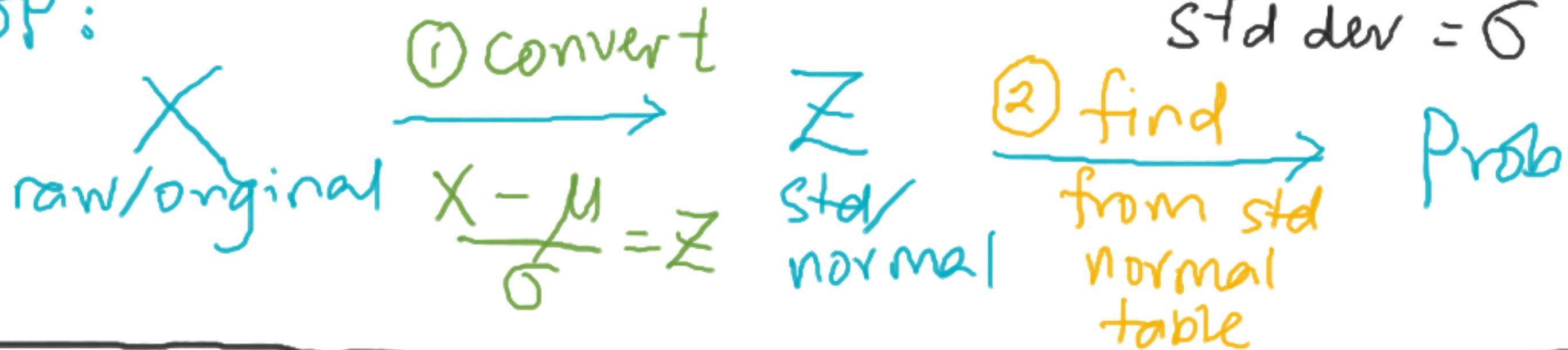
$$-k = 0.84$$

$$k = -0.84$$



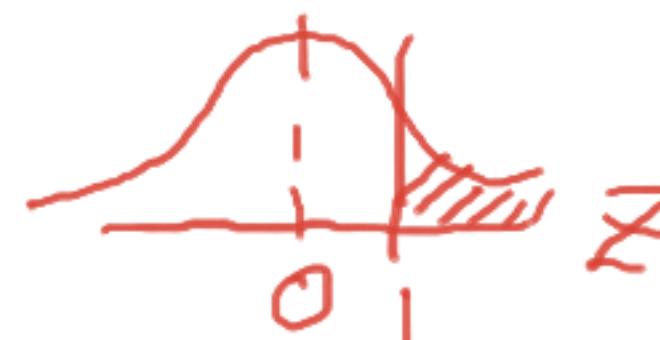
X : Continuous random variable, $X \sim N(\mu, \sigma^2)$
mean = μ
std dev = σ

SOP:



X : weight (kg), $X \sim N(10, 2^2)$ mean = 10kg, std dev = 2kg

$$P(X > 12) = P(Z > \frac{12 - 10}{2}) = P(Z > 1) = \underline{\underline{0.1587}}$$



or

$$P(X < 12) = P(Z < 1) = 1 - P(Z > 1) = 1 - 0.1587 = \underline{\underline{0.8413}}$$

Q5. Steel rods are manufactured to a specification of 20cm length and are accepted only if they are within the limits of 19.9cm and 20.1cm. The lengths are normally distributed with the mean 20.02cm and standard deviation 0.05cm. Find percentage of rods which will be rejected as
(a) undersized. (b) oversized.

Sean

X: length of steel rod

$$X \sim N(20.02, 0.05^2)$$

$$\text{Mean} = 20.02$$

$$G = 0.05$$

$$P(19.7 < \alpha < 20.1)$$

$$Z = \frac{x-1}{\theta}$$

(a) undersi

$$P(x < 19.9) = P(z < -2.4)$$

$$\bar{z} = \frac{19,9 - 20,0}{0,05}$$

- 2 -

$$8\text{kg} = P \quad (\pi > 19.9)$$

@ table

0,0082 x 100%

≈ 0,82%

(iv) Outcome

$$P(Z > 20.1) = \overline{P(Z > 1.6)}$$

$$z = \frac{90.1 - 20.02}{11.05}$$

1

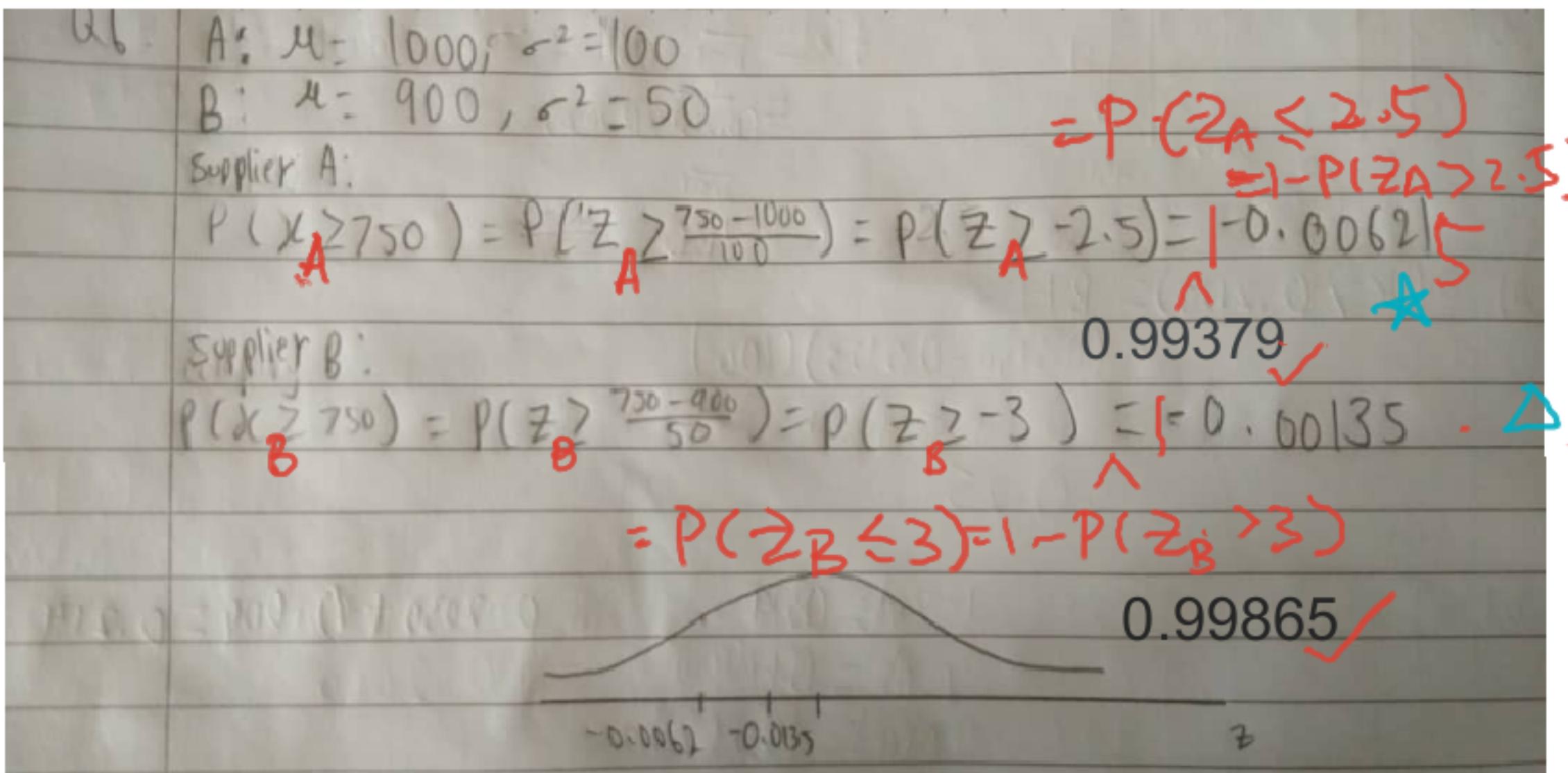
$$= \underline{5,48\%} \quad (2)$$

$$24 = 0.054$$

Q6.

Cai Jie

- Your company requires a special type of inelastic rope, which is available from only two suppliers. Supplier A's ropes have a mean breaking strength of 1000kg, with a standard deviation of 100kg. Supplier B's ropes have a mean breaking strength of 900kg, with a standard deviation of 50kg. The distribution of the breaking strengths of each type of rope is normal. Your company requires that the breaking strength of a rope be at least 750kg. All other things being equal, which supplier should the company order the rope? Show your workings.



Ans: Supplier B

Compare \star and Δ , (Δ higher prob)
 as its prob is higher.

X_A : breaking strength of supplier A's rope
 $X_A \sim N(1000, 100^2)$

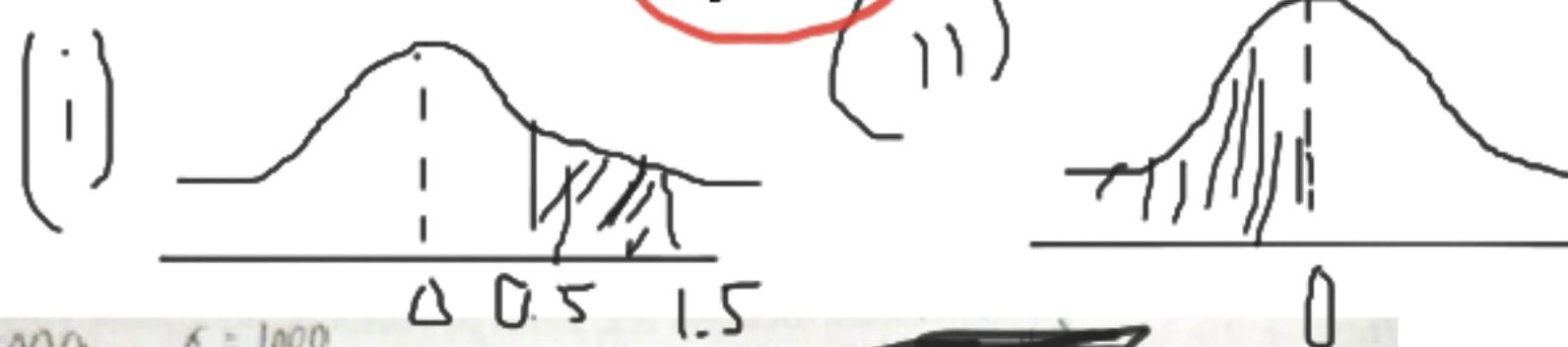
X_B : breaking strength of supplier B's rope
 $X_B \sim N(900, 50^2)$



Q7. The life of a television tube measured in hours of use, is normally distributed with a mean of 5000 hours and a standard deviation of 1000 hours.

- b1**
- (a) Find the probability that a TV tube will last for
 - (i) between 5500 and 6500 hours, (ii) less than the expected life of a tube.
 - (b) Find the probability that for someone using a TV for 1500 hours per year, the tube will last for less than 5 years.

Yee Hao

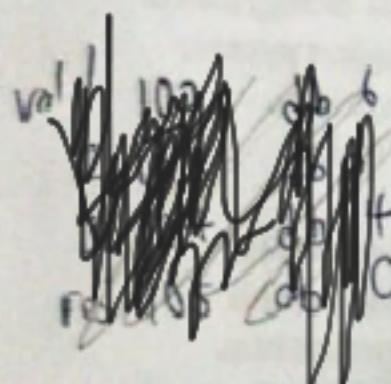


$$\mu = 5000 \quad \sigma = 1000$$

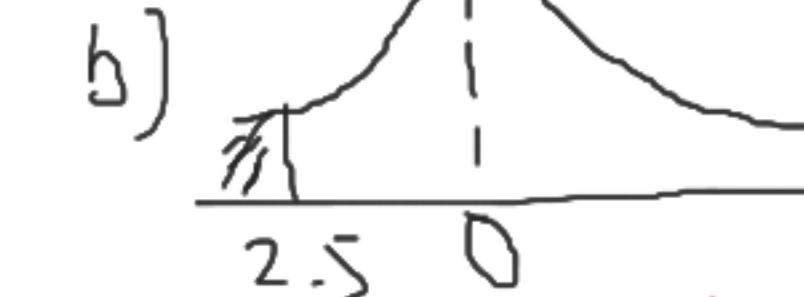
X = no. of hours that a TV tube will last for,

$$\checkmark X \sim N(5000, 1000)$$

$$\begin{aligned} \text{a) (i)} \quad P[5500 < X < 6500] &= P[0.5 < Z < 1.5] = 0.2417 \quad \checkmark \\ &= P[X > 5500] - P[X > 6500] \\ &= P[Z > \frac{5500 - 5000}{1000}] - P[Z > \frac{6500 - 5000}{1000}] \\ &= P[Z > 0.5] - P[Z > 1.5] \end{aligned}$$



$$\begin{aligned} \text{(ii)} \quad P[X < 5000] &= P[Z < \frac{5000 - 5000}{1000}] \\ &= P[Z < 0] \\ &= 0.5 \quad \checkmark \end{aligned}$$



$$SX (1500)$$

$$\begin{aligned} \text{b) } &P[X < 7500] = P[Z < \frac{7500 - 5000}{1000}] \\ &= P[Z < 2.5] \quad \checkmark \\ &\approx 1 - P[Z > 2.5] \quad \checkmark \\ &\approx 1 - 0.00621 \\ &\approx 0.99379 \quad \checkmark \end{aligned}$$

Chun Wai

- Q8. The average weight of a packet of sugar packed by a certain machine is 500g and the standard deviation is 20g. Assume that the weights are normally distributed.

(a) In a batch of 2000 packets, how many packets weight

(i) more than 520g? (ii) less than 470g? (iii) between 520g and 530g?

(b) Find the minimum weight of the 5% heaviest packets.

$$X = \text{no. of packets that weight } X \sim N(500, 20^2)$$

(a) (i)

$$P(X > 520)$$

$$= P\left(Z > \frac{520 - 500}{20}\right)$$

$$= P(Z > 1)$$

$$= 0.1587$$

$$2000 \times 0.1587 = 317.4 \approx 317$$

no. of packets =

(a) (ii)

$$P(X < 470)$$

$$= P\left(Z < \frac{470 - 500}{20}\right)$$

$$= P(Z < -1.5)$$

$$= P(Z > 1.5)$$

$$= 0.0668$$

$$2000 \times 0.0668 = 133.6 \approx 134$$

$$Z = \frac{X - \mu}{\sigma}$$

(a) (iii)

$$P(520 < X < 530)$$

$$= P\left(\frac{520 - 500}{20} < Z < \frac{530 - 500}{20}\right)$$

$$= P(1 < Z < 1.5)$$

$$= P(Z > 1) - P(Z > 1.5)$$

$$= 0.1587 - 0.0668$$

$$= 0.0919$$

$$2000 \times 0.0919 = 183.8 \approx 184$$

(b)

$$5\% = 0.05$$

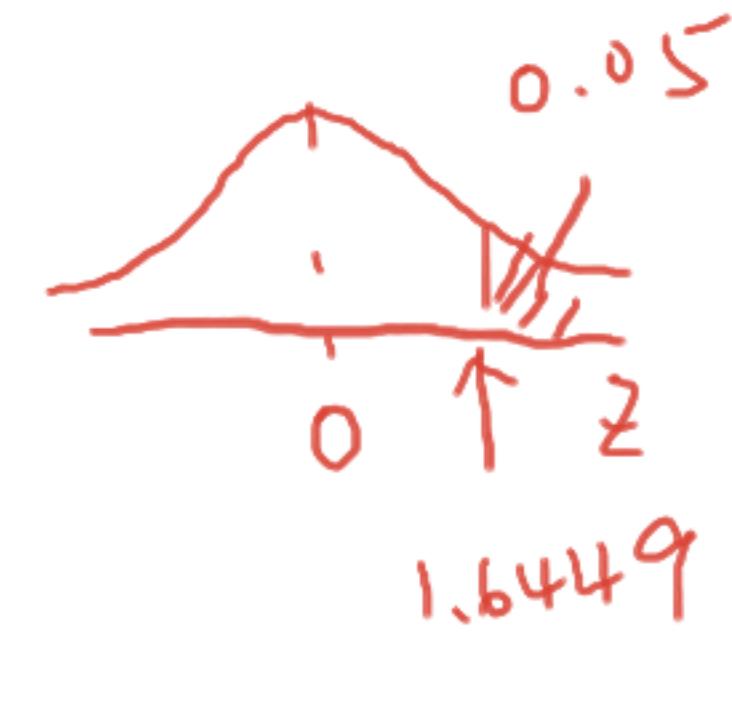
$$P(X > k) = 0.05$$

$$P\left(Z > \frac{k - 500}{20}\right) = 0.05$$

$$\frac{k - 500}{20} = 1.6449$$

$$k - 500 = 32.898$$

$$k = 532.898 \text{ g}$$



- Q9. The amount of time required for a certain type of transmission repair at a service garage is normally distributed with a mean of 45 minutes and a standard deviation of 8 minutes. The service manager plans to charge a customer 52 minutes for the transmission of a customer's car. He tells the customer that the car will be ready within 50 minutes.
- What is the probability that he will be wrong?
 - What is the probability that the working time will exceed 52 minutes?
 - What is the required working time allotment such that the transmission repair will be completed within 50 minutes?
 - What is the working time allotment such that the transmission repair can be completed within 52 minutes?

9) X = amount of time required for a certain type of automobile transmission repair at a service garage

$$X \sim N(45, 8^2)$$

$$(a) P(X > 50) = P(Z > \frac{50-45}{8})$$

$$= P(Z > 0.625)$$

$$= P(Z > 0.63)$$

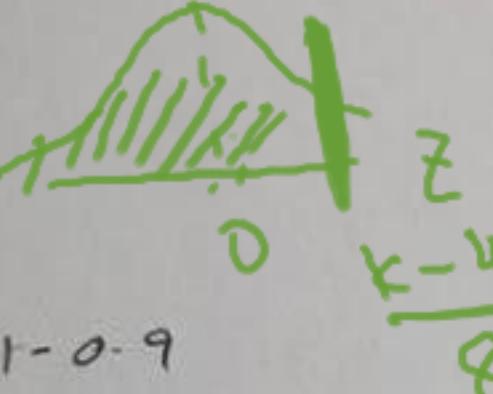
Exclude 10 minutes before repair from 1 hour

$$(b) P(X < k) = 0.9$$

$$P(Z < \frac{k-45}{8}) = 0.9$$

$$\therefore P(Z > \frac{k-45}{8}) = 1 - 0.9$$

$$= 0.1$$



$$(c) P(40 < X < 52) = P(\frac{40-45}{8} < Z < \frac{52-45}{8})$$

$$= P(-0.625 < Z < 0.875)$$

$$= 1 - P(Z > 0.63) - P(Z > 0.88)$$

$$= 1 - 0.2643 - 0.1894$$

$$= 0.5463$$



$$(d) P(X < k) = 0.3$$

$$P(Z < \frac{k-45}{8}) = 0.3$$

$$\frac{k-45}{8} = -0.5244$$

$$k = 40.8048 \text{ mins.}$$



$$\frac{k-45}{8} = +1.2816$$

$$k = 55.2528 \text{ mins.}$$

Janet

- Q10. 10% of males suffer from a certain disease. Find the probability that more than 60 men in a randomly selected group of 500 males will suffer from the disease.

50.

in a group of 500 males

number of
males suffer from the disease, $X \sim B(500, 0.1)$

Since $n = 500 > 30$, $p = 0.1$, $np = 50 > 5$ and $nq = 450 > 5$
thus, normal approximation is be used, $X \sim N(\mu = np = 50, \sigma^2 = npq = 4.5)$

$P(X > 60) \approx P(X > 60.5)$
 $= P(Z > \frac{60.5 - 50}{\sqrt{4.5}})$
 $= P(Z > 1.54) \quad 1.57$
 $= 0.0582$

Jia Jie ~~OK~~

- Q11. 10% of the chocolates produced in a factory are mis-shapes. In a sample of 1000 chocolates, find the probability that the number of mis-shapes is
 (a) less than 80, (b) from 90 to 115, (c) 120 or more.

⑪ $X = \text{no. of mis-shape chocolates}$ ~~out of 1000~~, $X \sim B(1000, 0.1)$

Since $n = 1000 > 30$, $np = 100 > 5$, $nq = 900 > 5$, thus normal approx is to be used, $X \sim N(100, \underline{\sigma^2})$

$$\begin{aligned} a) P[X < 80] &\approx P[X < 79.5] = P\left[Z < \frac{79.5 - 100}{3\sqrt{10}}\right] \\ &= P[Z < -2.16] \\ &= P[Z > 2.16] \end{aligned}$$

$$\begin{aligned} b) P[90 \leq X \leq 115] &\approx P[89.5 \leq X \leq 115.5] = P\left[\frac{89.5 - 100}{3\sqrt{10}} \leq Z \leq \frac{115.5 - 100}{3\sqrt{10}}\right] \\ &= P[-1.11 \leq Z \leq 1.65] \\ &= 1 - P[Z > 1.11] - P[Z > 1.65] \end{aligned}$$

$$\begin{aligned} c) P[X \geq 120] &\approx P[X \geq 119.5] = P\left[Z > \frac{119.5 - 100}{3\sqrt{10}}\right] \\ &= P[Z > 2.06] \end{aligned}$$

- 0.01970 ✓

$$\begin{aligned} &3(\sqrt{10})^2 a \\ &(3\sqrt{10})^2 90 b \\ &\downarrow \\ &\neq 3\sqrt{10}^2 30 \end{aligned}$$

Jing Jet

- Q12. The number of bacteria on a plate viewed under a microscope follows a Poisson distribution with parameter 60.

- (a) Find the probability that there are between 55 and 75 bacteria on a plate.
(b) A plate is rejected if less than 38 bacteria are found. If 2000 such plates are viewed, how many will be rejected?

60

$$X \sim N(\mu, \sigma^2)$$

12.	$X \equiv$ number of bacteria on a plate, $X \sim P_6(60)$ ✓ Since $\lambda = 60 > 20$, normal approx to be used, $X \sim N(60, 2\sqrt{6})$ ✓
a)	$P[55 < X < 75] \approx P[55.5 < Z < 74.5] = P\left[\frac{55.5 - 60}{2\sqrt{6}} < Z < \frac{74.5 - 60}{2\sqrt{6}}\right]$ $= P[-0.58 < Z < 1.81]$ $= 1 - P[Z > 0.58] - P[Z > 1.81]$ $= 1 - 0.2810 - 0.0301$ $= 0.6883$ ✓
b)	$P[X < 38] \approx P[X < 37.5] = P\left[Z < \frac{37.5 - 60}{2\sqrt{6}}\right]$ 0.00187 $\times 2000$ $= P[Z < -2.9]$ ✓ $= 3.74$ $= P[Z > 2.9]$ ∵ 4 plates $= 0.00187$ ✓

Shen Hoi

- Q13. Customers arrive randomly at a department store at an average rate of 18.6 per minute. Assuming the customers arrivals form a Poisson distribution, calculate the probability that at most 25 customers arrive in any particular minute.

OK

λ
lambda

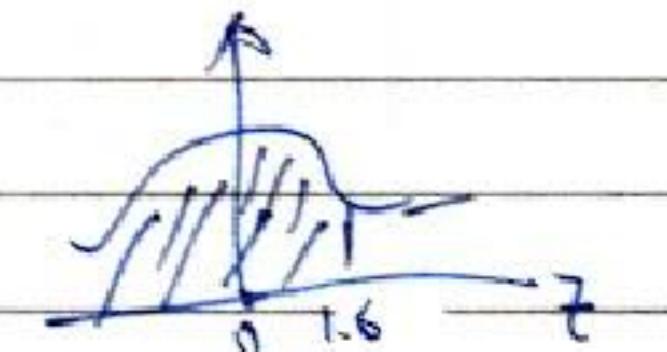
arrive

per minute

213) \checkmark customer arrives randomly at a department store $x \sim Po(18.6)$ ✓
 $x \sim N(18.6, 18.6)$

Since $\lambda = 18.6 \approx 20$, Normal approximation
is to be used.

$$\begin{aligned} P(x \leq 25) &\approx P(x < 25.5) \\ &= P\left(z < \frac{25.5 - 18.6}{\sqrt{18.6}}\right) \\ &= P(z < 1.6) \quad \checkmark \\ &= 1 - \Phi(1.6) \quad \checkmark \\ &= 1 - 0.9441 \\ &= 0.9452 \quad \checkmark \end{aligned}$$



\approx

Probability Distributions (Additional questions)

- Q1. Suppose in a city, 85% of all households have cable TV. **Kang Hong**
- i) Find the probability that you would have to ask three households before finding one who has cable TV. **(0.0191)**
 - ii) Find the probability that among the four randomly selected households, at least one household have cable TV. **(0.9995)**
 - iii) Give the mean and standard deviation for the distribution in ii). **(3.4, 0.7141)**
- the 1st one* *geometric*
binomial
- Q2. 0.5% of the households in City A spend more than RM2500 per month for grocery items. A group of 500 households are selected.
- i) Calculate the probability that 2 to 4 households spend more than RM2500 per month for grocery. **(0.6039)**
 - ii) Calculate the probability that at least one household spent more than RM2500 per month for grocery items. **(0.9179)**
- Q3. Suppose the weights of all packages of corn flour are normally distributed with a mean of 1000 grams and a standard deviation of 20 grams. A package is considered to meet the weight standard if the weight is between 960.6 grams and 1039 grams. **Mavis**
- i) Find the percentage of the package that meet the weight standard. **(0.95)**
 - ii) 2.5% of the packages are more than k grams. Find the value of k . **(1039.2)**
 - iii) A random sample of 10 packages is selected. Find the probability that the sample contains more than 8 packages that meet the weight standard. **(0.9138)**
- Q4. The distribution of the number of public buses stop at a particular bus stand has an average of 12 buses per hour. Find the probability that at least 2 buses stop in an interval of 10 minutes. **(0.5940)** **Li Yuet**
- Q5. A traffic survey shows that 60% of the cars turn left at a T-junction. Find the probability that out of 100 cars, more than 70 cars will turn left at the T-junction. **(0.01618)** **Eason**

(i) x = No of households that would have to ask before finding one
who has cable tv, $X \sim g(x; 0.85)$, $x = 1, 2, 3, \dots$

Q1. (i) $P(X=3) = 0.85^3 \cdot 0.15 = 0.1913 = 0.0191$

(ii) x = No of households that have cable tv
out of 4, $X \sim B(4, 0.85)$, $x = 0, 1, 2, 3, 4$

$$= 1 - [4C_0 \cdot 0.85^0 \cdot 0.15^4] = 1 - 0.0005 = 0.9995$$

(iii) $E(X) = np = 4(0.85) = 3.4$ households

$$\sigma^2 = npq = 4(0.85)(0.15) = 0.51$$

$$\sigma = \sqrt{0.51}$$

$$= 0.7141$$
 households

DL

DL

that
(2) x = No of households in City A spent more than RM2500 per month
for grocery items out of 500 households. $\rightarrow X \sim B(500, 0.0015)$

Since $n = 500$, $p = 0.0015$, $np = 2.5 < 5$ and $nq = 2.5 < 5$

i) $P(2 \leq x \leq 4) \approx e^{-2.5} \left(\frac{2.5^2}{2!} + \frac{2.5^3}{3!} + \frac{2.5^4}{4!} \right) = 0.0821(3.125 + 2.603 + 1.628) = 0.6039$

Poisson approximation
is used.
 $X \sim Po(2.5)$

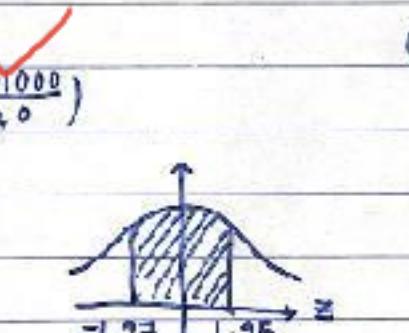
ii) $P(X \geq 1) = 1 - P(X=0) = 1 - \frac{e^{-2.5} 2.5^0}{0!} = 1 - 0.0821 = 0.9179$

Q3

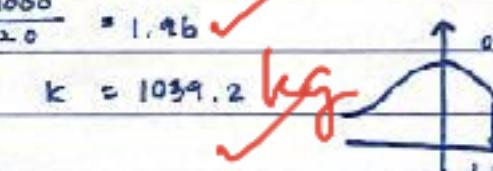
3) X : Weights of the packages of corn flour

$$X \sim N(1000, 20^2)$$

$$\begin{aligned} \text{(i)} \quad & P(960.6 < X < 1039) \\ & = P\left(\frac{960.6-1000}{20} < Z < \frac{1039-1000}{20}\right) \\ & = P(-1.97 < Z < 1.95) \\ & = (1 - 0.0256 - 0.0244) \times 100 \\ & = 95\% \end{aligned}$$



$$\begin{aligned} \text{(ii)} \quad & P(X > k) = 0.025 \\ & P\left(Z > \frac{k-1000}{20}\right) = 0.025 \\ & \frac{k-1000}{20} = 1.96 \\ & k = 1039.2 \end{aligned}$$



(iii) $X \sim B(10, 0.95)$ X : Weights of the packages of corn flour out of 10 packages

$$\begin{aligned} P(X > 8) &= {}^{10}C_9 (0.95)^9 (0.05)^1 + {}^{10}C_{10} (0.95)^9 (0.05)^1 \\ &= 0.9139 \end{aligned}$$

that meet the weight standard

Q4. Let x be the number of cars turn left at a T-junction. $X \sim B(100, 0.6)$

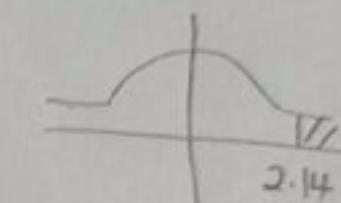
Since $n = 100 > 30$, $P = 0.6 \approx 0.5$, $np = 60$, $npq = 24$, normal distribution is used. $X \sim N(60, 24)$

$$P(X > 70) \approx P(X > 70.5)$$

$$= P(Z > \frac{70.5-60}{\sqrt{24}})$$

$$= P(Z > 2.14)$$

$$= 0.01618$$



Q4.

(a)

X : no of public buses stop at a particular bus stand in an interval of 10 mins

$$X \sim Po(2)$$

$$P(X \geq 2) = 1 - P(X < 2)$$

$$\begin{aligned} &= 1 - \left(\frac{e^{-2} 2^0}{0!} + \frac{e^{-2} 2^1}{1!} \right) \\ &= 1 - 0.4060 \end{aligned}$$

$$= 0.5940$$

interval	mean
60	12
10	$\frac{12}{6} = 2$
2	$\lambda = 2$