Q1. (a) For the statement $p \to \sim(q \wedge \sim r)$, write its contrapositive, converse and inverse. Then write your final answer without the connective '$\to$' and apply De Morgan's law where necessary. (6 marks)

14.01.2016

Contrapositive: $\sim q \to \sim p$

$(q \wedge \sim r) \to \sim p$

$\equiv \sim(q \wedge \sim r) \vee \sim p$

$\equiv \sim q \vee r \vee \sim p$

Converse: $q \to p$

$\sim(q \wedge \sim r) \to p$

$\equiv (q \wedge \sim r) \vee p$

Inverse: $\sim p \to \sim q$

$\sim p \to (q \wedge \sim r)$

$\equiv p \vee (q \wedge \sim r)$

(b) Let $A \equiv (p \leftrightarrow q) \veebar (\sim q \rightarrow r)$.

(i) Construct a truth table for the expression A. (3 marks)

(ii) Write the Principal Disjunctive Normal Form (PDNF) and Principal Conjunctive Normal Form (PCNF) of A. Hence deduce the PDNF and PCNF of $\sim A$. (6 marks)

Subject :

TUNKU ABDUL RAHMAN UNIVERSITY COLLEGE

BEYOND EDUCATION

Date :

same is 1

pqr, $\bar{p}qr$, $p\bar{q}r$, $pq\bar{r}$,

$\bar{p}\bar{q}r$, $\bar{p}q\bar{r}$, $p\bar{q}\bar{r}$, $\bar{p}\bar{q}\bar{r}$

(b)
(i)

p	q	r	$\sim q$	$p \leftrightarrow q$	$\sim q \rightarrow r$	A	
0	0	0	1	1	0	1	PDNF
0	0	1	1	1	1	0	
0	1	0	0	0	1	1	PDNF
0	1	1	0	0	1	1	PDNF
1	0	0	1	0	0	0	
1	0	1	1	0	1	1	PDNF
1	1	0	0	1	1	0	
1	1	1	0	1	1	0	

(ii)

PDNF of $A \equiv \bar{p}\bar{q}\bar{r} + \bar{p}q\bar{r} + \bar{p}qr + p\bar{q}r$

negate

PDNF of $\sim A \equiv \bar{p}\bar{q}r + p\bar{q}\bar{r} + pq\bar{r} + pqr$

PCNF of $A \equiv (p+q+\bar{r})(\bar{p}+q+r)(\bar{p}+\bar{q}+r)(\bar{p}+\bar{q}+\bar{r})$

PCNF of $\sim A = (p+q+r)(p+\bar{q}+r)(p+\bar{q}+\bar{r})(\bar{p}+q+\bar{r})$

negate

(c) Verify the following equivalence by using the laws of Logical Equivalence.
$(p \wedge q) \vee [p \wedge (\sim(\sim p \vee q))] \equiv p$ (5 marks)

1. (c) $(p \wedge q) \vee [p \wedge (\sim(\sim p \vee q))] \equiv p$

LHS : $(p \wedge q) \vee [p \wedge (p \wedge \sim q)] \equiv (p \wedge q) \vee [(p \wedge p) \wedge (p \wedge \sim q)]$

$\equiv (p \wedge q) \vee [t \wedge (p \wedge \sim q)]$

$\equiv (p \wedge q) \vee (p \wedge \sim q)$

$\equiv p \wedge (q \vee \sim q)$

$\equiv p \wedge t$

$\equiv p$ //

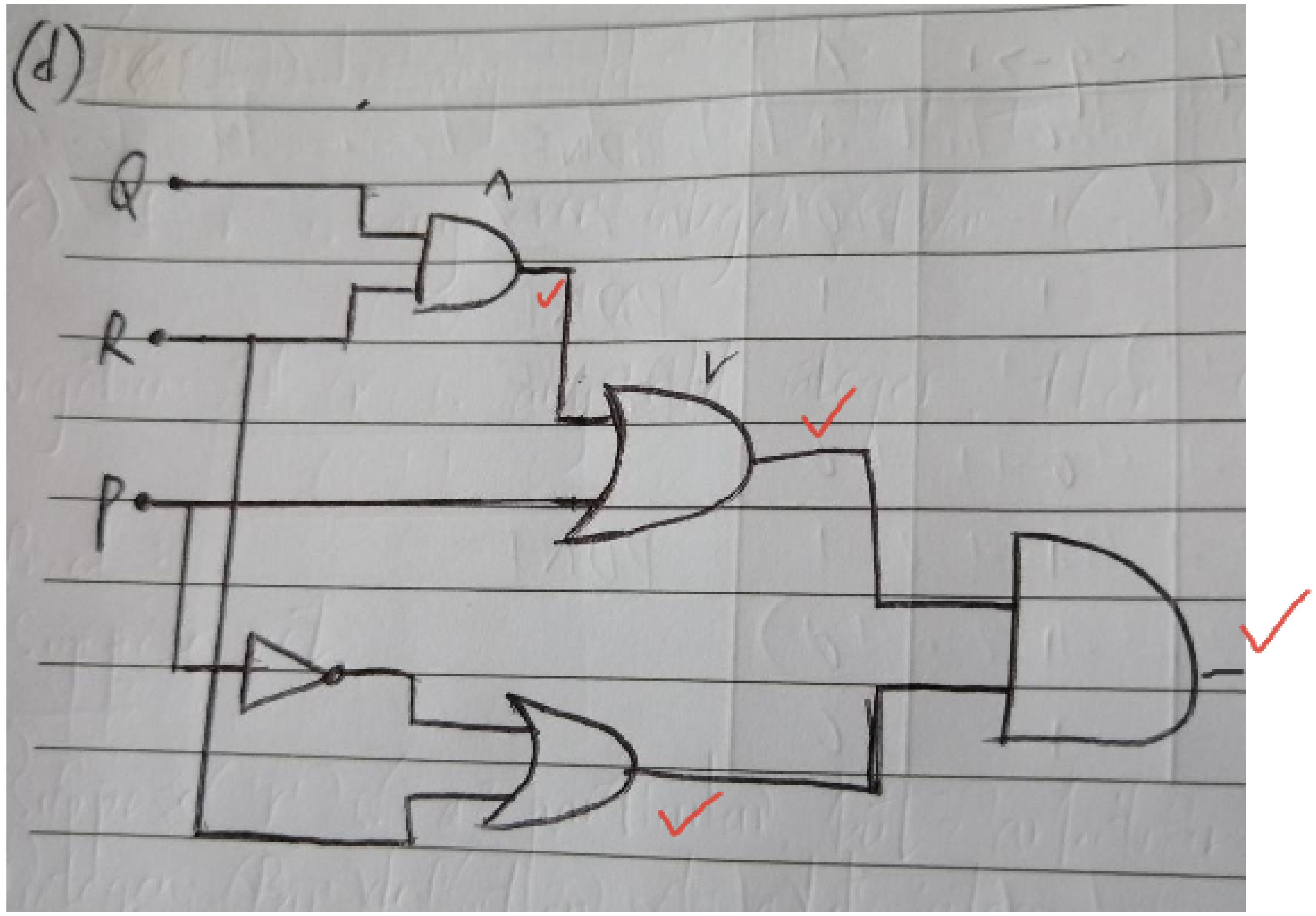
p

Logical Equivalences

Given any statement variables p, q, and r, a tautology t and a contradiction c, the following logical equivalences hold:

1. Commutative laws:	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
2. Associative laws:	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
3. Distributive laws:	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
4. Identity laws:	$p \wedge t \equiv p$	$p \vee c \equiv p$
5. Negation laws:	$p \vee \sim p \equiv t$	$p \wedge \sim p \equiv c$
6. Double negation laws:	$\sim(\sim p) \equiv p$	
7. Idempotent laws:	$p \wedge p \equiv p$	$p \vee p \equiv p$
8. De Morgan's laws:	$\sim(p \wedge q) \equiv \sim p \vee \sim q$	$\sim(p \vee q) \equiv \sim p \wedge \sim q$
9. Universal bound laws:	$p \vee t \equiv t$	$p \wedge c \equiv c$
10. Absorption laws:	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
11. Negation of t and c:	$\sim t \equiv c$	$\sim c \equiv t$

(d) Design a circuit for $[(Q \wedge R) \vee P] \wedge (\sim P \vee R)$. (5 marks)



Q2. (a) Determine the truth value of each of the following statements if the universe of each variable consists of all the integers. Give a reason to your answer if the statement is true and provide a counterexample for the false statement.

(i) $\forall x \forall y (x + y = y + x)$ (2 marks)

(ii) $\forall x \exists y (4xy = 12)$ (2 marks)

Subject : Q2

Date :

(a) (i) True ✓ because addition is commutative and for all the integers x, y, we have $x+y=y+x$.

(ii) False ✓

Let $x = 8$, $y = 3$

$4xy = 4(8)(3)$

$= 96 \neq 12$ ✓

1) Commutative laws:

- $p \wedge q \equiv q \wedge p$
- $p \vee q \equiv q \vee p$

(b) Use diagram to test the validity of the following argument.

Minor premise

Major premise (All numbers divisible by 4 are divisible by 2.) (24 is divisible by 4.) (Therefore, 24 is divisible by 2.) Conclusion

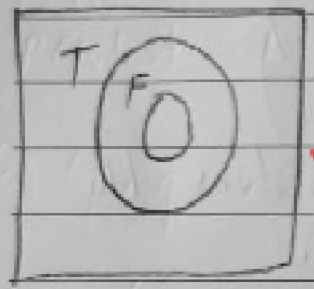
(5 marks)

(b) F: set of Numbers divisible by 4

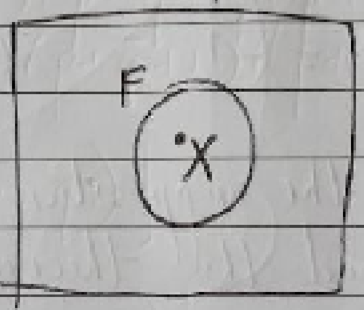
set of T: Numbers divisible by 2

X: 24

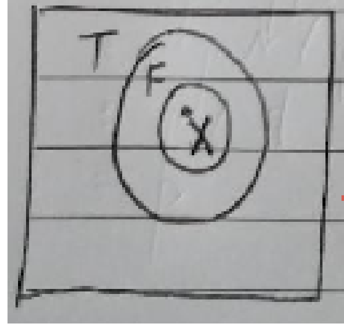
Major premise



Minor premise



Possible Conclusion:



24 is divisible by 2 since X falls inside the disc of T

∴ the argument is valid

(c) Prove the following statement.
"If r is any even integer, then $(-1)^r = 1$" (5 marks)

1^{-3}

$= \frac{1}{1^3}$

$= 1$

Proof:

Suppose r is a particular but arbitrarily chosen even integer. By definition of even integer, $r = 2m$, $m \in \mathbb{Z}$

Then $(-1)^r = (-1)^{2m}$

$\cancel{= (-1)^{2m}}$

$= 1$

$= [(-1)^2]^m$

$= (1)^m$

$= 1$

Therefore, if r is any even integer, then $(-1)^r = +1$

(d) Find the greatest common divisor of 2020 and 888 by using Euclidean algorithm. Hence determine the least common multiple of 2020 and 888.

(8 marks)

2. (d)

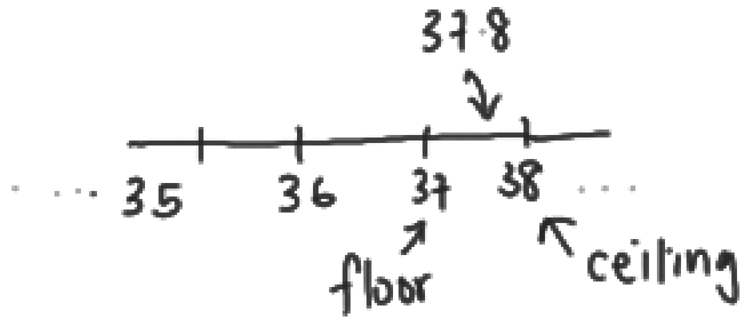
$$
\begin{aligned}
&\gcd(2020, 888)\\
&= \gcd(888, 244)\\
&= \gcd(244, 156)\\
&= \gcd(156, 88)\\
&= \gcd(88, 68)\\
&= \gcd(68, 20)\\
&= \gcd(20, 8)\\
&= \gcd(8, 4)\\
&= \gcd(4, 0)\\
&= 4
\end{aligned}
$$

$$
\begin{aligned}
2020 &= 888(2) + 244\\
888 &= 244(3) + 156\\
244 &= 156(1) + 88\\
156 &= 88(1) + 68\\
88 &= 68(1) + 20\\
68 &= 20(3) + 8\\
20 &= 8(2) + 4\\
8 &= \underline{\underline{4}}(2) + 0
\end{aligned}
$$

$$
\begin{aligned}
\operatorname{lcm}(2020, 888) &= \frac{2020 \times 888}{\gcd(2020, 888)}\\
&= \frac{1793760}{4}\\
&= 448440
\end{aligned}
$$

(e) Use binary representation to compute $\lfloor 37.8 \rfloor + 15_{10}$ Leave your answer in binary digit. (3 marks)

37_{10}



$2^5 \quad 2^2 \quad 2^0$

$37_{10} = 100101_2$

$15_{10} = \dots_2$ ✓

$$
\begin{array}{r}
100101_2 \\
+ \quad 1111_2 \\
\hline
110100_2 \\
\hline
\end{array}
$$

$$
\begin{array}{r|l}
2 & 37 \\
2 & 18 - 1 \\
2 & 9 - 0 \\
2 & 4 - 1 \\
2 & 2 - 0 \\
 & 1 - 0
\end{array}
$$

$$
\begin{array}{r|l}
2 & 15 \\
2 & 7 - 1 \\
2 & 3 - 1 \\
 & 1 - 1
\end{array}
$$

$15_{10} = 1111_2$ ✓

$2^3 + 2^2 + 2^1 + 2^0$

$= 8 + 4 + 2 + 1$

Q3. (a) Let R be the relation on $\{1, 2, 3, 4\}$ given by $x\,R\,y$ if and only if $2x + 2y < 12$.

DM

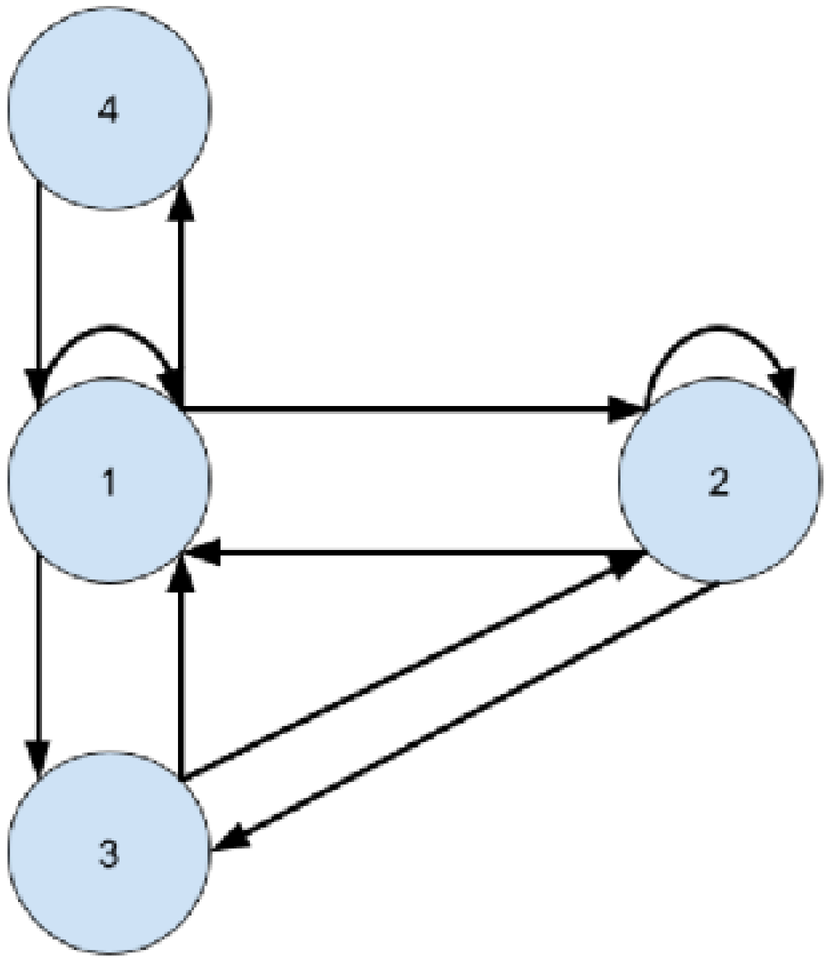
(i) List the ordered pairs belonging to the relation R. (2 marks)

(ii) Draw the digraph of R and write down the corresponding matrix M_R. (4 marks)

(iii) Find the in-degree and out-degree of each vertex. (2 marks)

(i) R = {(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (3,1), (3,2), (4,1)}

(ii)



Matrix (R):

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

(iii)

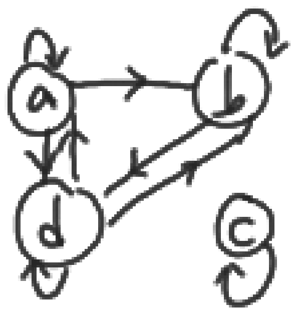
vertex	1	2	3	4
in degree	4	3	2	1
out degree	4	3	2	1

Alan

(b) Let $A = \{a, b, c, d\}$ and $R = \{(a, a), (a, b), (a, d), (b, b), (b, d), (c, c), (d, a), (d, b), (d, d)\}$.
Determine whether the relation R is reflexive, irreflexive, symmetric, asymmetric, antisymmetric or transitive. Give a counterexample if the answer is "No". (6 marks)

DATE:

$$M_R = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 1 & 1 & 0 & 1 \\ b & 0 & 1 & 0 & 1 \\ c & 0 & 0 & 1 & 0 \\ d & 1 & 1 & 0 & 1 \end{array}$$



Reflexive ✓
$(a,a) \in R$, $(b,b) \in R$, $(c,c) \in R$, $(d,d) \in R$

Not irreflexive
$(a,a) \in R$ ✓

Not transitive ✓
~~$(a,b) \in R$, $(b,d) \in R$, $(a,c) \in R$~~
since $(b,d) \in R$, $(d,a) \in R$ but $(b,a) \notin R$

Not symmetric ✓
$(a,b) \in R$, $(b,a) \notin R$ ✓

Not asymmetric ✓
since $(a,a) \in R$ $(a,d) \in R$ and $(d,a) \in R$

Not Antisymmetric since $(a,d) \in R$ and $(d,a) \in R$, $a \neq d$
~~1-way mirrored~~ not 1-way not loop

(c) Let $A = \{1, 2, 3, 4\}$ and R and T be the relations defined by the matrices

$$M_R = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \qquad M_T = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

HK

(i) Find the matrices of

Wo ✓ (1) $R \cap T^{-1}$ → inverse of T (2 marks)

(2) $(\overline{T \circ R})$ → complement of T∘R (3 marks)

Ricky

(ii) Use Warshall's algorithm to compute the transitive closure of $(R \cap T^{-1})$. (6 marks)

(c)(i)(1) $R \cap T^{-1}$

$$= \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \cap \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \cap \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \checkmark$$

(2) $(\overline{T \circ R})$

$R = \{(1,2), (1,3), (2,1), (2,2), (2,4), (3,3), (3,4), (4,1), (4,4)\}$

$T = \{(1,1), (1,4), (2,2), (3,1), (3,3), (3,4), (4,3)\}$

$T \circ R = \{(1,2), (1,1), (1,3), (1,4), (2,1), (2,4), (2,2), (2,3), (3,1), (3,3), (3,4), (4,1), (4,4), (4,3)\}$

$(\overline{T \circ R}) = \{$ ~~(2,1), (1,1), (3,1), (4,1), (1,2)~~, (4,2), ~~(2,2)~~, (3,2), ~~(1,3), (3,3), (4,3), (1,4), (4,4), (3,4)~~ $\}$

$$M_{(\overline{T \circ R})} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

(i) (ii) $M_{T^{-1}} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \checkmark$

$$M_{R \cap T^{-1}} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \wedge \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \checkmark$$

$M_T \odot M_R$

$$M_{T \circ R} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \odot \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \checkmark$$

$$M_{(\overline{T \circ R})} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \checkmark$$

Let $W_0 = M_{R \cap T^{-1}} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ $C_1 = 4$ ✓, $R_1 = 3$ ✓, Add $(4,3)$ ✓

$W_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$ $C_2 = 2$ ✓, $R_2 = 2$ ✓, Add $(2,2)$ ✓

$W_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$ $C_3 = 1, 3, 4$ ✓, $R_3 = 3, 4$ ✓, add $(1,3), (1,4), (3,3), (3,4), (4,3), (4,4)$ ✓

$W_3 = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$ $C_4 = 1, 3, 4$ ✓, $R_4 = 1, 3, 4$ ✓, add $(1,1), (1,3), (1,4), (3,1), (3,3), (3,4), (4,1), (4,3), (4,4)$ ✓

$W_4 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} = M_{R^\infty}$ ✓✓

$R^\infty = \{(1,1), (1,3), (1,4), (2,2), (3,1), (3,3), (3,4), (4,1), (4,3), (4,4)\}$ ✓✓

JM

Q4. (a) Let $A = \{a, b, c, d, e, f, g\}$. Given $\rho_1 = (b, d, f, g)$ and $\rho_2 = (a, c, f, e)$ be two permutations on A.

(i) Compute $(\rho_1 \circ \rho_2)^{-1}$ and write the result as a product of disjoint cycles and as a product of transpositions. (6 marks)

(ii) Is $(\rho_1 \circ \rho_2)^{-1}$ an even or odd permutation? (1 mark)

4) i) $p_1 \circ p_2 = \begin{pmatrix} a & b & c & d & e & f & g \\ a & d & c & f & e & g & b \end{pmatrix} \circ \begin{pmatrix} a & b & c & d & e & f & g \\ c & b & f & d & a & e & g \end{pmatrix}$

$= \begin{pmatrix} a & b & c & d & e & f & g \\ c & d & g & f & a & e & b \end{pmatrix}$ ✓

$(p_1 \circ p_2)^{-1} = \begin{pmatrix} a & b & c & d & e & f & g \\ e & g & a & b & f & d & c \end{pmatrix}$ ✓

(a, c)(e, e)
(b, b)(b, d)
(c, f)(f, g)
(d, d)(d, f)
(e, a)(a, a)
(f, e)(e, e)
(g, g)(g, b)

product of disjoint cycles

$= (a, e, f, d, b, g, c)$ ✓

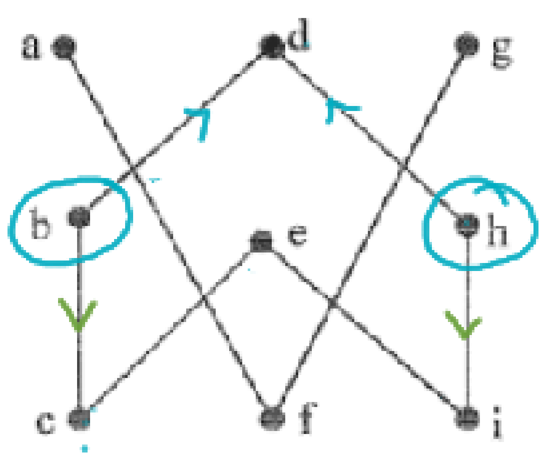
product of transpositions

$= (a, c) \circ (a, g) \circ (a, b) \circ (a, d) \circ (a, f) \circ (a, e)$

1 2 3 4 5 6

ii) $(p_1 \circ p_2)^{-1}$ is even ✓ permutation.

(b) The Hasse diagram of a partially ordered set, P, is given below. Find, if exist(s):



b)i) maximal = {a, d, g, e}
minimal = {c, f, i}

K Weng (i) the maximal and minimal element(s) of P; (3 marks)

none none

Jessie (ii) the greatest and least element(s) of P; (2 marks)

M Yi (iii) the upper bound and lower bound of {b, e, h}; (2 marks)

S Wai (iv) Least Upper Bound and Greatest Lower Bound of {b, h}. (2 marks)

LUB of {b, h} = d
GLB of {b, h} = none

Greatest = none
Least = none

b) (iii) upper bound of {b, e, h} = ~~{a, d, g}~~ none

lower bound of {b, e, h} = ~~{c, f, i}~~ none

(c) Simplify $(x \wedge y)' \wedge (x' \vee y) \wedge (y \vee y')$ to the simplest form by using the laws of Boolean algebra. (5 marks)

4 (c) $(x \wedge y)' \wedge (x' \vee y) \wedge (y \vee y') = (x' \vee y') \wedge (x' \vee y) \wedge 1$ ✓

$= x' \vee (y' \wedge y)$

$= x' \vee 0$ ✓

$= x'$ ✓

5

(d) Let $f(x, y, z) = (x' \wedge y' \wedge z') \vee (x' \wedge y \wedge z) \vee (x' \wedge y \wedge z') \vee (x \wedge y' \wedge z') \vee (x \wedge y \wedge z')$.

Construct a Karnaugh map in the form of

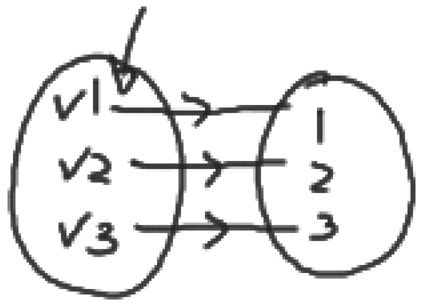
	y'	y'	y	y
x'				
x				
	z'	z	z	z'

and hence simplify $f(x, y, z)$ to the simplest form. (4 marks)

	y'	y'	y	y
x'	1	0	1	1
x	1	0	0	1
	z'	z	z	z'

$f(x,y,z) = z' + x'y$

①

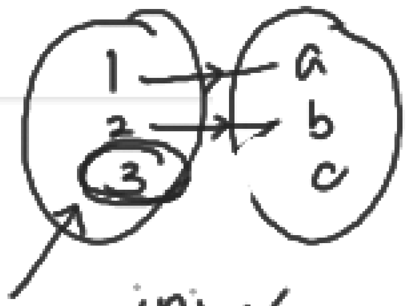


inj. ✓

every where def. ✓

surj ✓

②



inj. ✓

not everywhere def.

~~not~~ surjective

Surjective:

codomain = range

Q3. (a) (i) Let $A = \{v, w, x, y, z\}$ and the relation on A is $R = \{(v, v), (w, w), (x, x), (y, y), (z, z)\}$. Determine whether the relation R is reflexive, irreflexive, symmetric, asymmetric, antisymmetric or transitive. Give a counterexample if the answer is "No". (7 marks)

(ii) Verify if R is an equivalence relation on A. (2 marks)

extra question

Q3(a)(i) R is reflexive since $(v,v), (w,w), (x,x), (y,y)$ and $(z,z) \in R$ ✓

R is not irreflexive since $(v,v) \in R$ ✓

R is ~~not~~ symmetric since $M_R = (M_R)^T$ ✓

R is not asymmetric since $(v,v) \in R$ ✓

R is antisymmetric ✓

R is ~~not~~ transitive ~~since $(a,b), (b,c)$ and $(a,c) \notin R$~~

I'm confused about symmetric, asymmetric, antisymmetric and transitive -ZY

$M_{ij} = 1$

$M_{ji} = 0$

$M_{12} = 1$

$M_{21} = 0$

$$M_R = \begin{array}{c c} & \begin{array}{ccccc} v & w & x & y & z \end{array} \\ \begin{array}{c} v \\ w \\ x \\ y \\ z \end{array} & \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array} = M_R^T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M_R \odot M_R = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} = M_R$$

(ii) R is an equivalence relation on A, since R is reflexive, symmetric and transitive.

(c) Use the Laws of Logical Equivalence to show that

$$[p \vee (q \vee \sim r)] \rightarrow (q \wedge r) \vee (p \wedge r) \equiv r$$

(5 marks)

Q2. (a) Let $D = \{-9, -6, -3, 0, 2, 4, 8\}$. Determine which of the following statements are true and which are false. Prove those true statements and provide counterexamples for those false statements.

YH (i) $\exists x \in D$, if x is odd, then $3|x$.

XY (ii) $\forall x \in D$, if x is even, then x is positive.

WJ (iii) $\exists x \in D$, $x > -10$ and $x \bmod 3 = 1$.

Tzer K (iv) $\forall x \in D$, $x^2 > 0$. (4 marks)

(b) Use diagrams to determine the validity of the following argument.

Janet

Everyone in the class is IT major.
Sally is an IT major.
Therefore, Sally is in the class. (5 marks)

(c) Prove or disprove the below statement.

For all integers a, b and c, if $a|b$ and $a|c$, then $a|(b-c)$. (5 marks)

(a)(i)True

(ii)False, counter example -6 != positive

(iii)

(iv)

(b) Let $A=\{ a, b, c, d, e \}$ and $R=\{(a,a), (a,b), (a,d), (a,e), (b,a), (b,b), (b,d), (c,c), (d,a), (e,a)\}$. Determine whether the relation is reflexive, irreflexive, symmetric, asymmetric, antisymmetric or transitive. Give a simple explanation if the answer is "no". (6 marks)

(c) Let $A = \{1, 2, 3, 4\}$ and R and S be the relation on A described by the matrices

$$M_R = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \text{ and } M_S = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(i) Compute $M_{R^{-1}}$, $M_{\overline{R}}$, $M_{R \cup S}$ and $M_{S^{-1} \circ R}$. (5 marks)

(ii) Use Warshall's algorithm to compute the transitive closure matrix of R. (6 marks)

Let $A = \{1, 2, 3, 6, 18, 24\}$ and R be the relation on A whose matrix is

$$M_R = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(i) Draw the Hasse diagram of R. (3 marks)

(ii) Is $[A, R]$ a linearly ordered set? (1 mark)

(iii) Determine all the minimal and all the maximal elements of the poset. (2 marks)

(iv) Find the least and greatest elements of the poset. (2 marks)

(v) Find the least upper bound of $B = \{1,2,3\}$. (1 mark)

(vi) Find the greatest lower bound of $B = \{1,2,3\}$. (1 mark)