extra question

Q3. (a) (i) Let $A = \{v, w, x, y, z\}$ and the relation on A is $R = \{(v, v), (w, w), (x, x), (y, y), (z, z)\}$. Determine whether the relation R is reflexive, irreflexive, symmetric, asymmetric, antisymmetric or transitive. Give a counterexample if the answer is "No". (7 marks)

(ii) Verify if R is an equivalence relation on A. (2 marks)

(Q3)(a)(i) R is reflexive since $(v,v), (w,w), (x,x), (y,y)$ and $(z,z) \in R$ ✓

R is not irreflexive since $(v,v) \in R$ ✓

R is ~~not~~ symmetric since $M_R = (M_R)^T$ ✓

R is not asymmetric since $(v,v) \in R$ ✓

R is antisymmetric ✓

R is ~~not~~ transitive ~~since (a,b), (b,c) and (a,c) ∉ R~~

I'm confused about symmetric, asymmetric, antisymmetric and transitive -ZY

$M_{ij} = 1$

$M_{ji} = 0$

$M_{12} = 1$

$M_{21} = 0$

$$M_R = \begin{array}{c} \\ v \\ w \\ x \\ y \\ z \end{array} \begin{array}{c} \begin{array}{ccccc} v & w & x & y & z \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array} = M_R^T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M_R \odot M_R = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} = M_R$$

(ii) R is an equivalence relation on A, since R is reflexive, symmetric and transitive.

(c) Use the Laws of Logical Equivalence to show that

$$[p \vee (q \vee \sim r)] \to (q \wedge r) \vee (p \wedge r) \equiv r$$ (5 marks)

$[p \vee (q \vee \sim r)] \to (q \wedge r) \vee (p \wedge r) \equiv r$

LHS $= \sim p \wedge (\sim q \wedge r) \vee (q \wedge r) \vee (p \wedge r)$

$= (\sim p \wedge \sim q \wedge r) \vee (q \wedge r) \vee (p \wedge r)$

$= r \wedge [(\sim p \wedge \sim q) \vee (q \vee p)]$

$= r \wedge [\underbrace{(\sim p \wedge \sim q)} \vee \sim \underbrace{(\sim q \wedge \sim p)}]$

$= r \wedge t$

$= r$

$\Leftarrow$ let $k = \sim p \wedge \sim q$

$(\sim p \wedge \sim q) \vee \sim (\sim q \wedge \sim p) = k \vee \sim k$

$= t$

Q2. (a) Let D = {−9 , −6 , −3 , 0 , 2 , 4 , 8 }. Determine which of the following statements are true and which are false. Prove those true statements and provide counterexamples for those false statements.

YH (i) $\exists x \in D$, if x is odd, then $3|x$. $\frac{-9}{-3} = -3 \in \mathbb{Z}$, $\frac{-3}{3} = -1 \in \mathbb{Z}$

XY (ii) $\forall x \in D$, if x is even, then x is positive.

WJ (iii) $\exists x \in D$, $x > -10$ and x mod 3=1.

Tzer K (iv) $\forall x \in D$, $x^2 > 0$. (4 marks)

(b) Use diagrams to determine the validity of the following argument.

Janet

Everyone in the class is IT major.
Sally is an IT major.
Therefore, Sally is in the class. (5 marks)

(c) Prove or disprove the below statement.
For all integers a, b and c, if a|b and a|c, then a|(b − c). (5 marks)

(a)(i)True

(ii)False, counter example ~~-6~~ $\neq$ positive

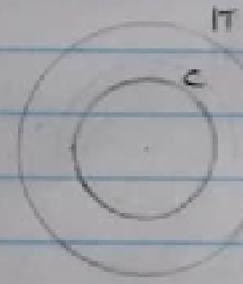
(iii)True, 4 > -10 and 4 mod 3 = 1

(iv)False, counter example 0^2 = 0, 0=0

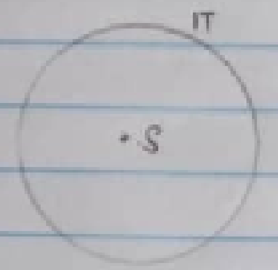
B. Everyone in the class is IT major - major premiums
Sally is an IT major - minor premiums
Therefore, Sally is in the class - Conclusion

Let C = set of everyone in the class
IT = set of IT major
S = Sally

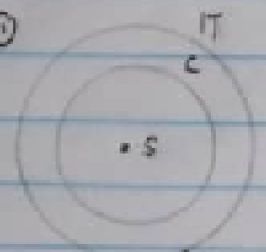
Major premiums :

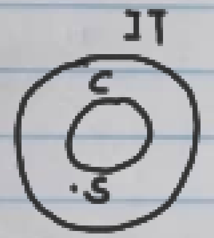


Minor premiums :



Possible conclusion :

① 

② 

S falls inside of C S falls outside of C

∴ The argument is invalid since the possible conclusions contradict with each other.

Therefore, a|(b-c) is proved.

(b) Let $A=\{a, b, c, d, e\}$ and $R=\{(a,a), (a,b), (a,d), (a,e), (b,a), (b,b), (b,d), (c,c), (d,a), (e,a)\}$. Determine whether the relation is reflexive, irreflexive, symmetric, asymmetric, antisymmetric or transitive. Give a simple explanation if the answer is "no". (6 marks)

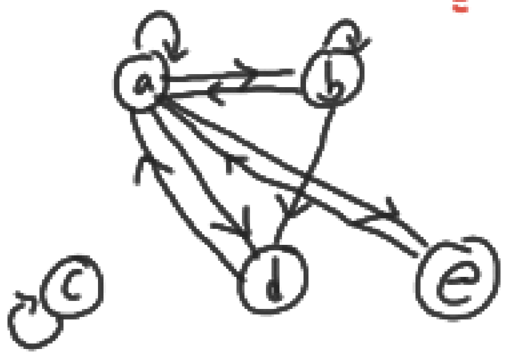
(c) Let $A = \{1, 2, 3, 4\}$ and R and S be the relation on A described by the matrices

$$M_R = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \text{ and } M_S = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(i) Compute $M_{R^{-1}}$, $M_{\overline{R}}$, $M_{R\cup S}$ and $M_{S^{-1}\circ R}$. (5 marks)

(ii) Use Warshall's algorithm to compute the transitive closure matrix of R. (6 marks)

b) R is not reflexive since $(e,e) \notin R$
not irreflexive since (a,a) E R.
not symmetric since (b,d) E R but (d,b) !E R
not asymmetric since (a,b) and (b,a) E R
not antisymmetric since (a,b) and (b,a) E R but a != b
not transitive since $(d,a) \in R$, $(a,b) \in R$ but $(d,b) \notin R$.



c)(ii)

$$W_0 = M_R = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$C_1 = 1, 4$
$R_1 = 1, 2, 4$
Add = (1,1), (1,2), (1,4), (4,1), (4,2), (4,4)

$$W_1 = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

$C_2 = 1, 2, 4$
$R_2 = 2, 3$
Add = (1,2), (1,3), (2,2), (2,3), (4,2), (4,3)

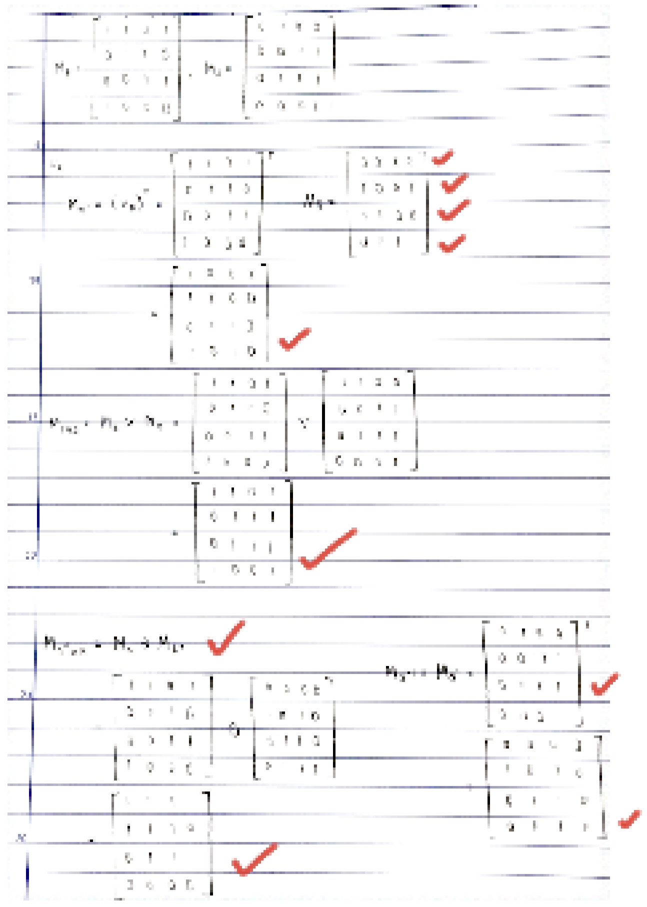
$$W_2 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$C_3 = 1, 2, 3, 4$
$R_3 = 3, 4$
Add = (1,3), (1,4), (2,3) (2,4), (3,3), (3,4), (4,3), (4,4)

$$W_3 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$C_4 = 1, 2, 3, 4$
$R_4 = 1, 2, 3, 4$
Add = (1,1) (1,2) (1,3) (1,4) (2,1) (2,2) (2,3) (2,4) (3,1) (3,2) (3,3) (3,4) (4,1) (4,2) (4,3) (4,4)

$$W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} = M_{R^\infty}$$



Let $A = \{1, 2, 3, 6, 18, 24\}$ and R be the relation on A whose matrix is

$$\begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 6 & 18 & 24 \end{matrix} \\ \hline \begin{matrix} 1 \\ 2 \\ 3 \\ 6 \\ 18 \\ 24 \end{matrix} & M_R = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

(i) Draw the Hasse diagram of R. (3 marks)

(ii) Is $[A, R]$ a linearly ordered set? (1 mark)

(iii) Determine all the minimal and all the maximal elements of the poset. (2 marks)

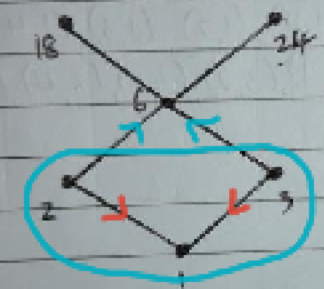
(iv) Find the least and greatest elements of the poset. (2 marks)

(v) Find the least upper bound of $B = \{1,2,3\}$. (1 mark)

(vi) Find the greatest lower bound of $B = \{1,2,3\}$. (1 mark)

b) R = {(1,1), (1,2), (1,3), (1,6), (1,18), (1,24), (2,2), (2,6), (2,18), (2,24), (3,3), (3,6), (3,18), (3,24), (6,6), (6,18), (6,24), (18,18), (24,24)}

(i)



(ii) [A, R] is not a linearly ordered set

(iii) maximal = {18, 24}
minimal = {1}

(iv) greatest = none
least = {1}

(v) LUB ({1,2,3}) = {6}

(vi) GLB ({1,2,3}) = {1}

$q \to p$ $\sim p \to \sim q$ $\sim q \to \sim p$

State and determine the truth value for the negation, converse, inverse and contrapositive of the following statement:

$\boxed{\text{For all rational numbers } x \text{ and } y}$ if $\overset{p}{x < y}$, then $\overset{q}{(x^2 < y^2.)}$ (12 marks)

I got it!
Thank You

$\sim(p \to q)$

$\equiv \sim(\sim p \vee q)$

$\equiv p \wedge \sim q$

negation : Exists some rational numbers x and y, $x < y$ and $x^2 \geq y^2$. (True)

converse : for all rational numbers x and y, if x^2 < y^2, then x < y false
-2^2 < -3^2 -2 > -3

inverse : for all rational numbers x and y, if x >= y, then x^2 >= y^2 (FALSE)
-2 > -3 ~~4~~ $\not>$ 9

contrapositive : for all rational numbers x and y, if x^2 >= y^2, then x >= y (false)


Thank You
WELL DONE