AACS3064
Computer Systems Architecture

Chapter 1: Numbering Systems

Chapter Overview

1) The basic of numbering system

- How numbers work, the nature of counting, how calculations are performed.

2) Conversion between numbering systems

- base 2(binary), 10(decimal), or 16(hexadecimal)

3) Performing Arithmetic in different numbering systems

- Addition, subtraction and multiplication

1. The Basic of Numbering Systems

1. Numbering Systems

Introduction

performed arithmetic using ***decimal number system***.

(base 10)

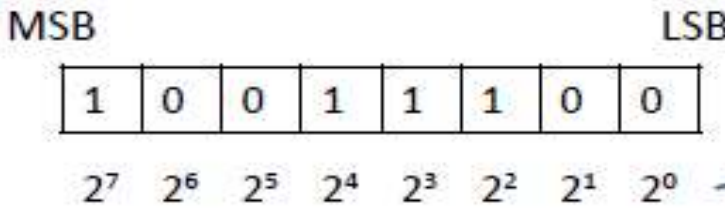
perform arithmetic using ***binary number system***.

(base 2)

1. Numbering Systems (Continued)

Introduction – Binary Numbers

- Bit :
  - The fundamental building block of computer storage.
- Digits are 1 and 0 (1 = true / on, 0 = false / off)
  - MSB – most significant bit
  - LSB – least significant bit



Each bit represents a power of 2

1. Numbering Systems (Continued)

Introduction

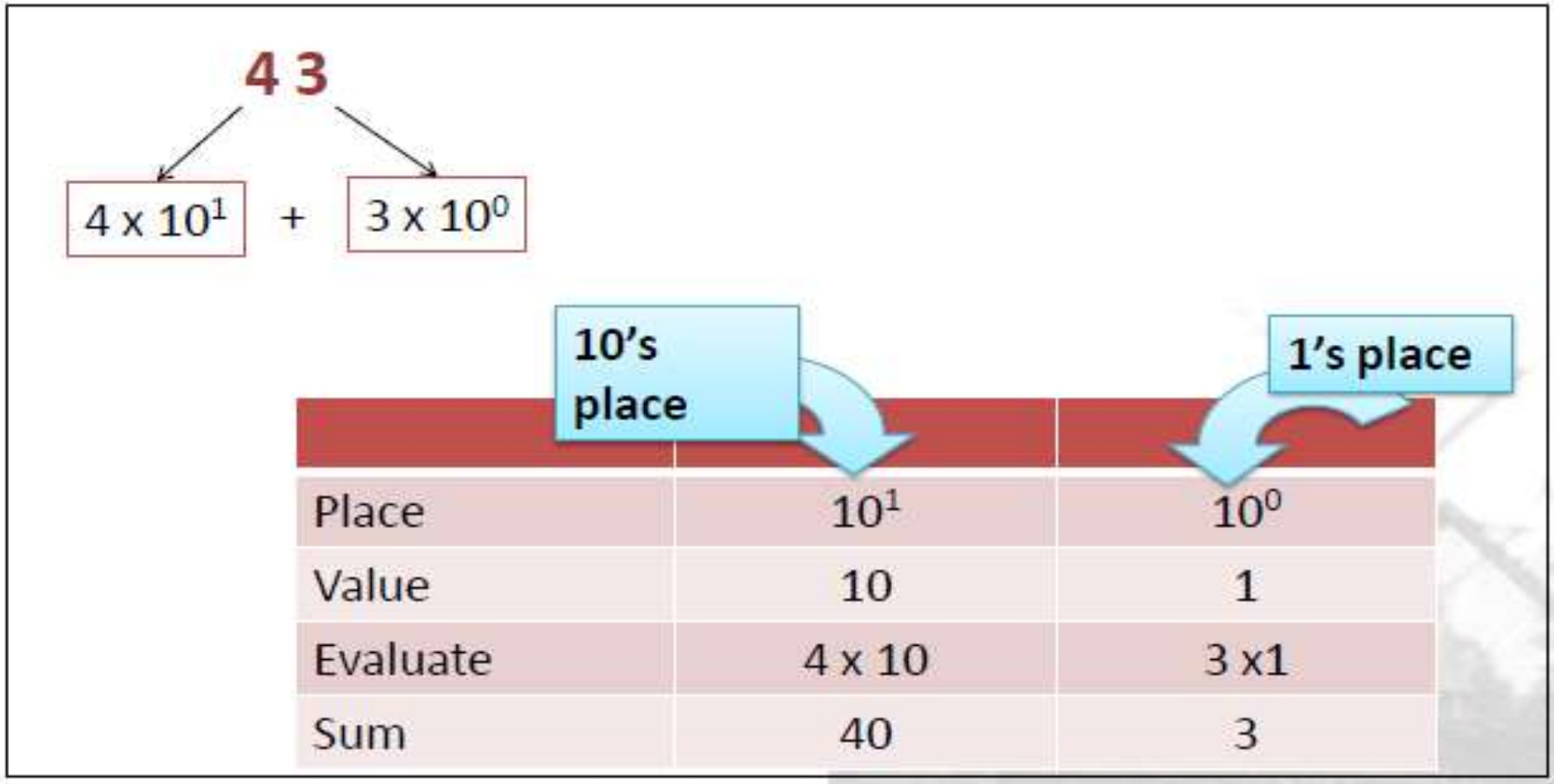
- Base means the total number of digits including zero.

Base 2 (Binary)	Base 10 (Decimal)
2 digits (0,1)	10 digits (0,1,2,3,4,5,6,7,8,9)

Base 8 (Octal)	Base 16 (Hexadecimal)

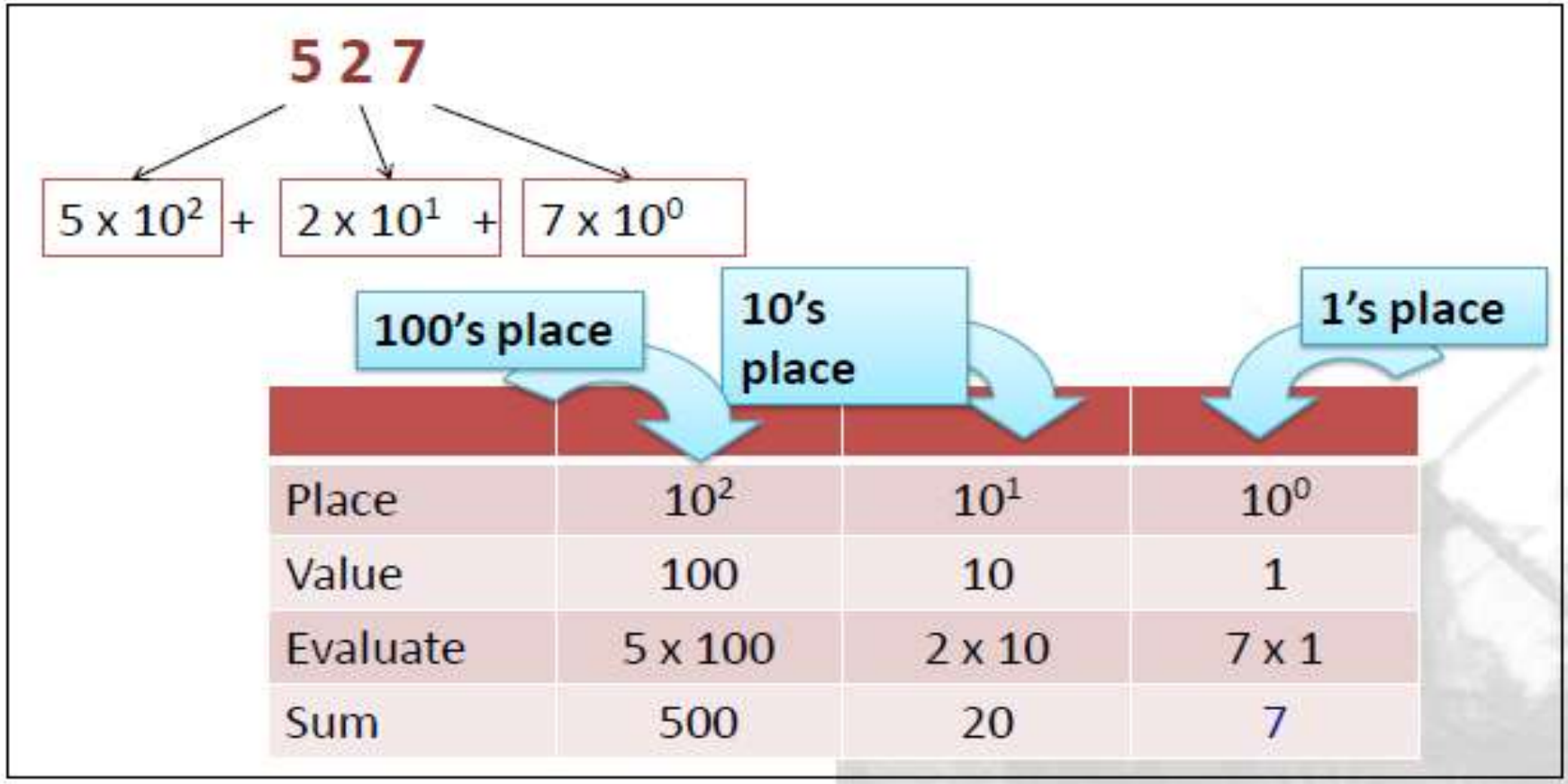
1. Numbering Systems (Continued)

Decimal Systems : Position Notation : Base 10



1. Numbering Systems (Continued)

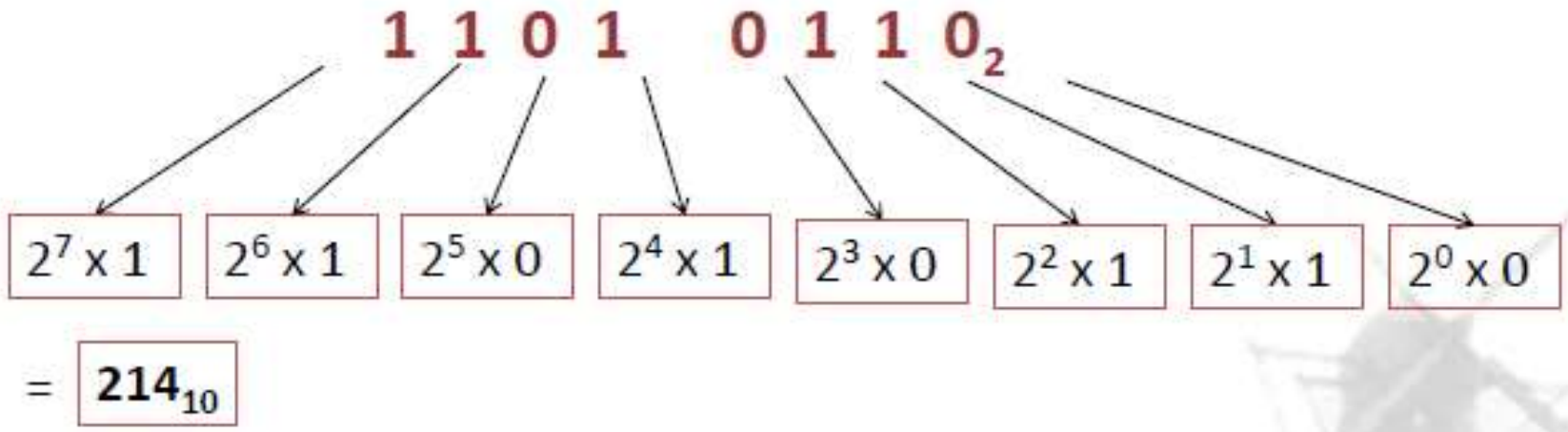
Decimal Systems : Position Notation : Base 10



Place	10^2	10^1	10^0
Value	100	10	1
Evaluate	5 x 100	2 x 10	7 x 1
Sum	500	20	7

1. Numbering Systems (Continued)

Binary Numbering Systems



= 214_{10}

1. Numbering Systems (Continued)

Hexadecimal Numbering Systems

computer works on the binary system

E.g. :

111010001100

111010001100111101110001

101001011001000000100110100 1010

1. Numbering Systems (Continued)

Hexadecimal Numbering Systems

Binary	Hexadecimal	Binary	Hexadecimal
0000	0	1000	8
0001	1	1001	9
0010	2	1010	A
0011	3	1011	B
0100	4	1100	C
0101	5	1101	D
0110	6	1110	E
0111	7	1111	F

1. Numbering Systems (Continued)

Hexadecimal Numbering Systems

Purpose :

- To solve the problems of numbers written in binary tend to be long and difficult to express.
- Conversion between binary and hex is easy.

Hex: 000000	Hex: 000033	Hex: 000066
R: 000 G: 000 B: 000	R: 000 G: 000 B: 051	R: 000 G: 000 B: 102

Hex: 000099	Hex: 0000CC	Hex: 0000FF
R: 000 G: 000 B: 153	R: 000 G: 000 B: 204	R: 000 G: 000 B: 255

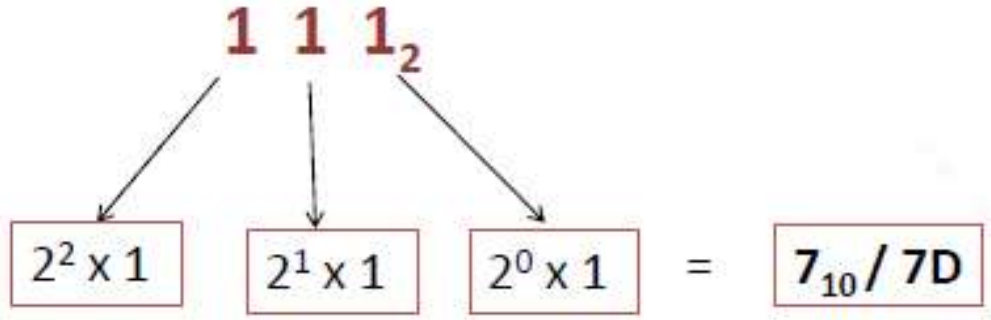
2. Conversions Between Numbering Systems

2. Conversion

Conversion between number systems

Example 1 : base 2 -> base 10

(i) 111_2

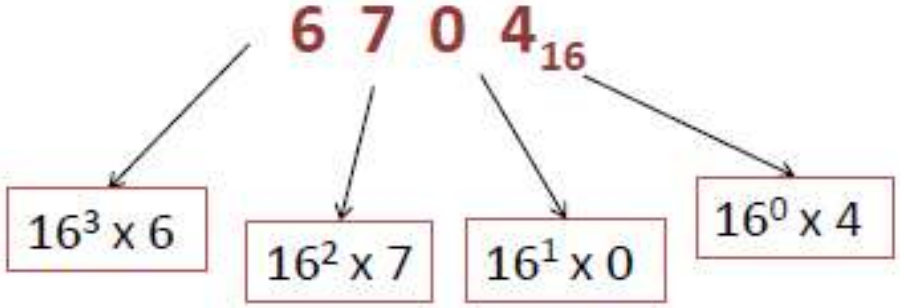


2. Conversion (Continued)

Conversion between number systems

Example 2 : base 16 -> base 10

(i) 6704H / 6704_{16}



= **$26{,}372_{10}$ / 26,372D**

2. Conversion (Continued)

Conversion between number systems

Example 3 : base 2 -> base 16

(i) $111_2 = 7H$

(ii) $1001\ 0100_2 = 148_{10} / 16 = 94H$

$$\underset{\downarrow}{1\ 0\ 0\ 1}\quad \underset{\downarrow}{0\ 1\ 0\ 0_2}$$

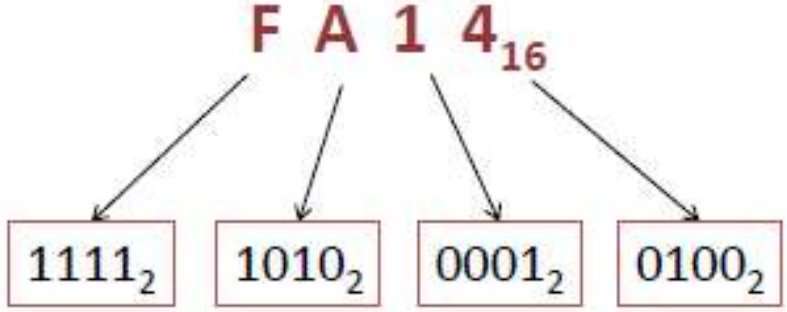
$$9 \qquad 4 \qquad = \qquad 94_{16} \text{ / } 94H$$

2. Conversion (Continued)

Conversion between number systems

Example 4 : base 16 -> base 2

(i) FA14H / $FA14_{16}$

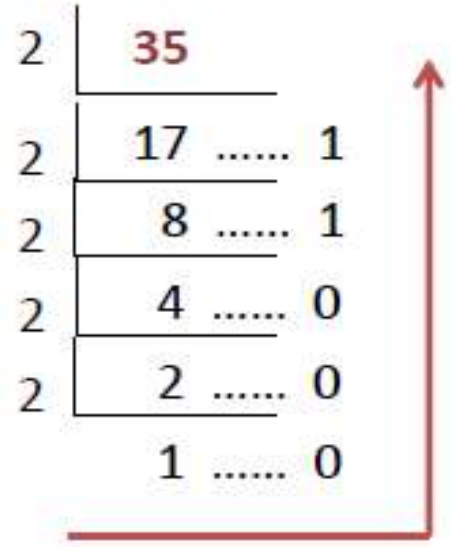


2. Conversion (Continued)

Conversion between number systems

Example 5 : base 10 -> base 2

(i) 35



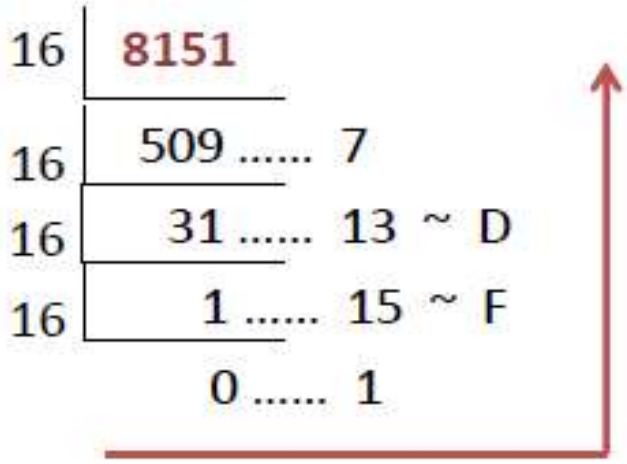
$10\,0011_2$

2. Conversion (Continued)

Conversion between number systems

Example 6 : base 10 -> base 16

(i) 8151



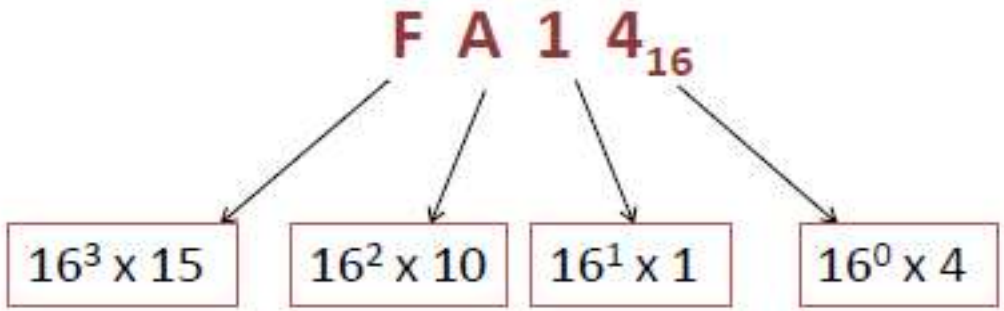
$1FD7_{16}$

2. Conversion (Continued)

Conversion between number systems

Example 7 : base 16 -> base 10

(i) FA14H / $FA14_{16}$



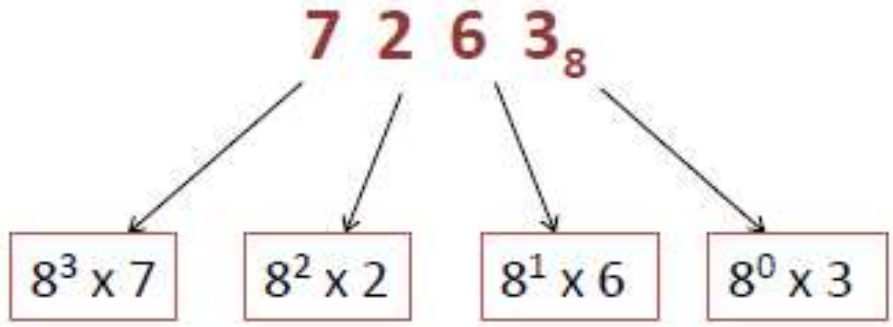
$64{,}020_{10}$ / 64,020D

2. Conversion (Continued)

Conversion between number systems

Example 8 : base 8 -> base 10

(i) 7263_8



3763_{10}

2. Conversion (Continued)

Conversion between number systems

Example 8 : base 8 -> base 10

Alternative conversion multiplication method

(i) 7263_8

7

x 8

56 + 2 = 58

x 8

464 + 6 = 470

x 8

3760 + 3 = 3763_{10}

3. Performing Arithmetic in Different Numbering Systems

3. Arithmetic

Binary Addition

Base 2 addition table

+	0	1
0	0	1
1	1	10

Example :

0	0	1	1	1
+0	+1	+0	+1	1
0	1	1	10	+1
				11

3. Arithmetic (Continued)

Binary Addition

Example :

Decimal	Binary
109 + 22 ------ 131 ------	110 1101 + 1 0110 ---------- 1000 0011 ----------

NOTE: After addition, please check to ensure that the binary sum equals to that decimal values you added.

3. Arithmetic (Continued)

Binary Subtraction

Example **:**

Decimal	Binary
12	0 1 1 ~~1~~ ~~1~~0 ¹0
- 9	- 1 0 0 1
-----	----------
3	0 0 1 1
-----	----------

3. Arithmetic (Continued)

Binary Multiplication

Base 2 Multiplication table

x	0	1
0	0	0
1	0	1

Example 1 :

Binary

```
      1101
  x    101
----------
      1101
     0000
    1101
----------
   1000001
----------
```

3. Arithmetic (Continued)

Binary Multiplication

Example 2 :

Binary

```
          1101101
      x     10110
-----------------------------
        1101101
       1101101
 1101101
-----------------------------
 1000000101110
-----------------------------
```

3. Arithmetic (Continued)

Hexadecimal Addition

Example :

(i) 13B4H + 0033H

Hexadecimal
1 3 B 4 H
+ 0 0 3 3 H
1 3 E 7 H

HINT : perform calculation from right to left

- ***Step 1 :*** Think of the decimal equivalent of each digit.
- ***Step 2 :*** Add or subtract the decimal equivalents.
- ***Step 3 :*** Reconvert the decimal result to its hexadecimal

3. Arithmetic (Continued)

Hexadecimal Addition

Example **:**

(ii) 53CDH + 0004H

Hexadecimal
5 3 C D H
+ 0 0 0 4 H
5 3 D 1 H

- D+4 = 13+4 = 17. Binary representation for 17 = 0001 0001 = 11 (resulted in a carry of one bit)
- C+0+**1** (carry bit from the previous sum) = 12+1=13=D
- 3+0=3 ; 5+0=5

3. Arithmetic (Continued)

Hexadecimal Addition

Example **:**

(iii) 8797H + 0777H

Hexadecimal	
	8 7 9 7 H
+	0 7 7 7 H
	8 F 0 E H

- 7+7 = 14 = E
- 9+7=16 .Binary representation for 16 = 0001 0000 = 10 (resulted in a carry of one bit)
- 7+7+**1** (carry bit from the previous sum) = 15 = F
- 8+0 =8

3. Arithmetic (Continued)

Hexadecimal Subtraction

Example **:**

(i) 53CBH - 0004H

Hexadecimal
53CBH
- 0004H

53C7H
----

3. Arithmetic (Continued)

Hexadecimal Subtraction

Example **:**

(ii) 13E7H – 13B4H

Hexadecimal
13E7H
- 13B4H

0033H
----

3. Arithmetic (Continued)

Hexadecimal Multiplication

Example **: method 1**

(i) E2H x 4AH

Hexadecimal

STEP 1:

$E2 = 224 + 2 = 226_{10}$

$4A = 64 + 10 = 74_{10}$

$226 \times 74 = 16724_{10}$

Hexadecimal

STEP 2:

16	16724	
16	1045	 4
16	65	 5
16	4	 1
	0	 4

3. Arithmetic (Continued)

Hexadecimal Multiplication

Example **: method 2**

(i) E2H x 4AH

```
        E2H
  x     4AH
-------------
       8D4H
     388  H
-------------
      4154H
-------------
```

Chapter Review

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