14.01.2016

Q1. (a) For the statement $p \to \sim(q \wedge \sim r)$, write its contrapositive, converse and inverse. Then write your final answer without the connective '$\to$' and apply De Morgan's law where necessary. (6 marks)

Contrapositive: $\sim q \to \sim p$

$(q \wedge \sim r) \to \sim p$

$\equiv \sim(q \wedge \sim r) \vee \sim p$

$\equiv \sim q \vee r \vee \sim p$

Converse: $q \to p$

$\sim(q \wedge \sim r) \to p$

$\equiv (q \wedge \sim r) \vee \cancel{\sim p}\ p$

Inverse: $\sim p \to \sim q$

$\sim p \to (q \wedge \sim r)$

$\equiv \cancel{\sim\sim p}\ p \vee (q \wedge \sim r)$

(b) Let $A \equiv (p \leftrightarrow q) \veebar (\sim q \rightarrow r)$.

(i) Construct a truth table for the expression A. (3 marks)

(ii) Write the Principal Disjunctive Normal Form (PDNF) and Principal Conjunctive Normal Form (PCNF) of A. Hence deduce the PDNF and PCNF of $\sim A$. (6 marks)

$p\bar{q}r$, $\bar{p}qr$, $p\bar{q}r$, $pq\bar{r}$,

$\bar{p}\bar{q}r$, $\bar{p}q\bar{r}$, $p\bar{q}\bar{r}$, $\bar{p}\bar{q}\bar{r}$

Subject:

Date:

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BEYOND EDUCATION

(b)

same is 1

(i)

p	q	r	$\sim q$	$p \Leftrightarrow q$	$\sim q \rightarrow r$	A	
0	0	0	1	1	0	1	PDNF
0	0	1	1	1	1	0	
0	1	0	0	0	1	1	PDNF
0	1	1	0	0	1	1	PDNF
1	0	0	1	0	0	0	
1	0	1	1	0	1	1	PDNF
1	1	0	0	1	1	0	
1	1	1	0	1	1	0	

(ii)

PDNF of $A \equiv \bar{p}\bar{q}\bar{r} + \bar{p}q\bar{r} + \bar{p}qr + p\bar{q}r$

negate

PDNF of $\sim A \equiv \bar{p}\bar{q}r + p\bar{q}\bar{r} + pq\bar{r} + pqr$

PCNF of $A \equiv (p+q+\bar{r})(\bar{p}+q+r)(\bar{p}+\bar{q}+r)(\bar{p}+\bar{q}+\bar{r})$

PCNF of $\sim A = (p+q+r)(p+\bar{q}+r)(p+\bar{q}+\bar{r})(\bar{p}+q+\bar{r})$

negate

(c) Verify the following equivalence by using the laws of Logical Equivalence.
$(p \wedge q) \vee [p \wedge (\sim(\sim p \vee q))] \equiv p$ (5 marks)

1. (c) $(p \wedge q) \vee [p \wedge (\sim(\sim p \vee q))] \equiv p$

LHS: $(p \wedge q) \vee [p \wedge (p \wedge \sim q)] \equiv (p \wedge q) \vee [(p \wedge p) \wedge (p \wedge \sim q)]$

p ✓

$\equiv (p \wedge q) \vee [t \wedge (p \wedge \sim q)]$

$\equiv (p \wedge q) \vee (p \wedge \sim q)$ ✓

$\equiv p \wedge (q \vee \sim q)$

$\equiv p \wedge t$ ✓

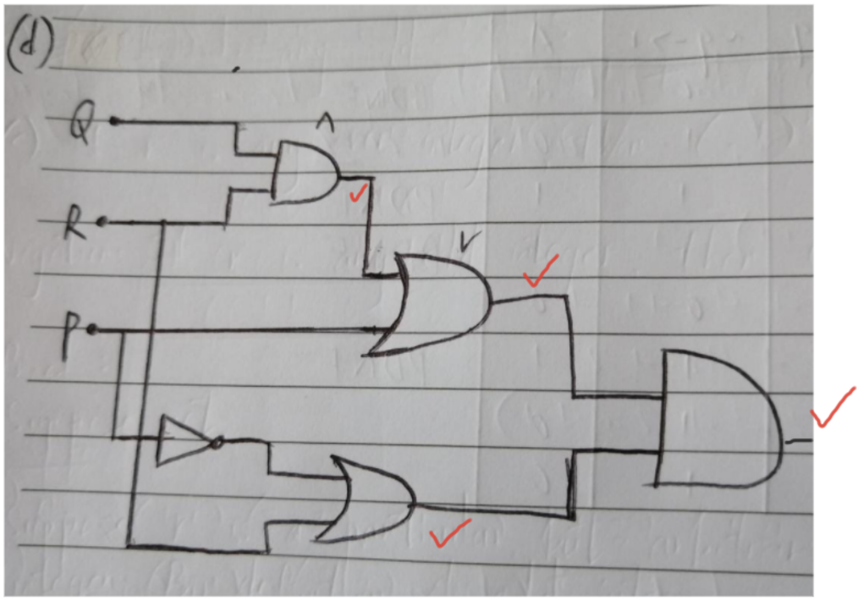
$\equiv p$ ✓ //

Logical Equivalences

Given any statement variables p, q, and r, a tautology t and a contradiction c, the following logical equivalences hold:

1. Commutative laws:	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
2. Associative laws:	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
3. Distributive laws:	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
4. Identity laws:	$p \wedge t \equiv p$	$p \vee c \equiv p$
5. Negation laws:	$p \vee \sim p \equiv t$	$p \wedge \sim p \equiv c$
6. Double negation laws:	$\sim(\sim p) \equiv p$	
7. Idempotent laws:	$p \wedge p \equiv p$	$p \vee p \equiv p$
8. De Morgan's laws:	$\sim(p \wedge q) \equiv \sim p \vee \sim q$	$\sim(p \vee q) \equiv \sim p \wedge \sim q$
9. Universal bound laws:	$p \vee t \equiv t$	$p \wedge c \equiv c$
10. Absorption laws:	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
11. Negation of t and c:	$\sim t \equiv c$	$\sim c \equiv t$

(d) Design a circuit for $[(Q \wedge R) \vee P] \wedge (\sim P \vee R)$. (5 marks)



Q2. (a) Determine the truth value of each of the following statements if the universe of each variable consists of all the integers. Give a reason to your answer if the statement is true and provide a counterexample for the false statement.

(i) $\forall x \forall y (x + y = y + x)$ (2 marks)

(ii) $\forall x \exists y (4xy = 12)$ (2 marks)

Subject : Q2

Date :

(a) (i) True ✓ because addition is commutative and for all the integers x, y, we have $x+y=y+x$.

(ii) False ✓

Let $x = 8$, $y = 3$

$4xy = 4(8)(3)$

$= 96 \neq 12$ ✓

1) Commutative laws:

- $p \wedge q \equiv q \wedge p$
- $p \vee q \equiv q \vee p$

Minor premise

(b) Use diagram to test the validity of the following argument.

Major premise (All numbers divisible by 4 are divisible by 2.) (24 is divisible by 4.) (Therefore, 24 is divisible by 2.) Conclusion

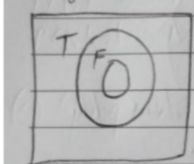
(5 marks)

(b) F: set of Numbers divisible by 4

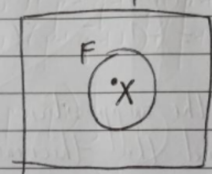
set of T: Numbers divisible by 2

X: 24

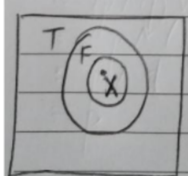
Major premise



Minor premise



Possible Conclusion:



24 is divisible by 2 since X falls inside the disc of T

∴ the argument is valid

(c) Prove the following statement.
"If r is any even integer, then $(-1)^r = 1$" (5 marks)

Proof:

Suppose r is a particular but arbitrarily chosen even integer. By definition of even integer, $r = 2m$, $m \in \mathbb{Z}$

Then $(-1)^r = (-1)^{2m}$ $\quad = [(-1)^2]^m$

$\cancel{= (-1)^{2m}}$ $\quad = (1)^m$

$= 1$ $\quad = 1$

Therefore, if r is any even integer, then $(-1)^r = +1$

1^{-3}

$= \frac{1}{1^3}$

$= 1$

(d) Find the greatest common divisor of 2020 and 888 by using Euclidean algorithm. Hence determine the least common multiple of 2020 and 888.

(8 marks)

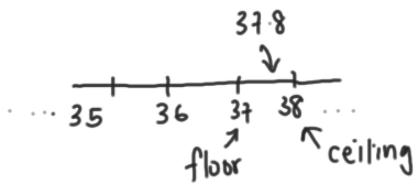
2. (d)

$$\begin{aligned}
&\gcd(2020, 888)\\
&= \gcd(888, 244)\\
&= \gcd(244, 156)\\
&= \gcd(156, 88)\\
&= \gcd(88, 68)\\
&= \gcd(68, 20)\\
&= \gcd(20, 8)\\
&= \gcd(8, 4)\\
&= \gcd(4, 0)\\
&= 4
\end{aligned}$$

$$\begin{aligned}
2020 &= 888(2) + 244\\
888 &= 244(3) + 156\\
244 &= 156(1) + 88\\
156 &= 88(1) + 68\\
88 &= 68(1) + 20\\
68 &= 20(3) + 8\\
20 &= 8(2) + 4\\
8 &= \underline{\underline{4}}(2) + 0
\end{aligned}$$

$$\begin{aligned}
\operatorname{lcm}(2020, 888) &= \frac{2020 \times 888}{\gcd(2020, 888)}\\
&= \frac{1793760}{4}\\
&= 448440
\end{aligned}$$

(e) Use binary representation to compute $\underset{}{\overset{37_{10}}{\lfloor 37.8 \rfloor}} + 15_{10}$. Leave your answer in binary digit. (3 marks)



$37_{10} = \overset{2^5}{1}00\overset{2^2}{1}0\overset{2^0}{1}_2$

$15_{10} = \dots_2$ ✓

$$
\begin{array}{r}
100101_2 \\
+ \quad 1111_2 \\
\hline
110100_2 \\
\hline
\end{array}
$$

$$
\begin{array}{r|l}
2 & 37 \\
2 & 18 - 1 \\
2 & 9 - 0 \\
2 & 4 - 1 \\
2 & 2 - 0 \\
 & 1 - 0
\end{array}
$$

$$
\begin{array}{r|l}
2 & 15 \\
2 & 7 - 1 \\
2 & 3 - 1 \\
 & 1 - 1
\end{array}
$$

$15_{10} = 1111_2$ ✓

$2^3 + 2^2 + 2^1 + 2^0$

$= 8 + 4 + 2 + 1$

Q3. (a) Let R be the relation on $\{1, 2, 3, 4\}$ given by $x\,R\,y$ if and only if $2x + 2y < 12$.

DM

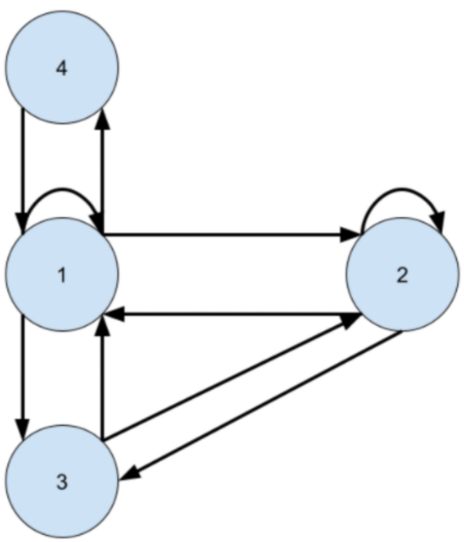
(i) List the ordered pairs belonging to the relation R. (2 marks)

(ii) Draw the digraph of R and write down the corresponding matrix M_R. (4 marks)

(iii) Find the in-degree and out-degree of each vertex. (2 marks)

(i) R = {(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (3,1), (3,2), (4,1)}

(ii)



Matrix (R):

[1 1 1 1]
| 1 1 1 0 |
| 1 1 0 0 |
[1 0 0 0]

(iii)

vertex	1	2	3	4
in degree	4	3	2	1
out degree	4	3	2	1

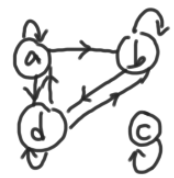
Alan

(b) Let $A = \{a, b, c, d\}$ and $R = \{(a, a), (a, b), (a, d), (b, b), (b, d), (c, c), (d, a), (d, b), (d, d)\}$.

Determine whether the relation R is reflexive, irreflexive, symmetric, asymmetric, antisymmetric or transitive. Give a counterexample if the answer is "No". (6 marks)

DATE:

$$M_R = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 1 & 1 & 0 & 1 \\ b & 0 & 1 & 0 & 1 \\ c & 0 & 0 & 1 & 0 \\ d & 1 & 1 & 0 & 1 \end{array}$$



Reflexive

$(a,a) \in R$, $(b,b) \in R$, $(c,c) \in R$, $(d,d) \in R$

Not irreflexive

$(a,a) \in R$

Not transitive

~~$(a,b) \in R$, $(b,d) \in R$, $(a,c) \notin R$~~

since $(b,d) \in R$, $(d,a) \in R$ but $(b,a) \notin R$

Not Symmetric

$(a,b) \in R$, $(b,a) \notin R$

Not asymmetric

Since $(a,a) \in R$ $(a,d) \in R$ and $(d,a) \in R$

Not Antisymmetric since $(a,d) \in R$ and $(d,a) \in R$, $a \neq d$

~~1-way $m_{ij} = 1$, $m_{ji} = 0$~~ not 1-way not loop

(c) Let $A = \{1, 2, 3, 4\}$ and R and T be the relations defined by the matrices

$$M_R = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \qquad M_T = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

HK

(i) Find the matrices of

Wo ✓ (1) $R \cap T^{-1}$ → inverse of T (2 marks)

Ricky → (2) $(\overline{T \circ R})$ → complement of T∘R (3 marks)

(ii) Use Warshall's algorithm to compute the transitive closure of $(R \cap T^{-1})$. (6 marks)

(c)(i)(1) $R \cap T^{-1}$

$$= \begin{bmatrix} 0&1&1&0 \\ 1&1&0&1 \\ 0&0&1&1 \\ 1&0&0&1 \end{bmatrix} \cap \begin{bmatrix} 1&0&0&1 \\ 0&1&0&0 \\ 1&0&1&1 \\ 0&0&1&0 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 0&1&1&0 \\ 1&1&0&1 \\ 0&0&1&1 \\ 1&0&0&1 \end{bmatrix} \cap \begin{bmatrix} 1&0&1&0 \\ 0&1&0&0 \\ 0&0&1&1 \\ 1&0&1&0 \end{bmatrix} ✓$$

$$= \begin{bmatrix} 0&0&1&0 \\ 0&1&0&0 \\ 0&0&1&1 \\ 1&0&0&0 \end{bmatrix} ✓$$

(2) $(\overline{T \circ R})$

$R = \{(1,2),(1,3),(2,1),(2,2),(2,4),(3,3),(3,4),(4,1),(4,4)\}$

$T = \{(1,1),(1,4),(2,2),(3,1),(3,3),(3,4),(4,3)\}$

$T \circ R = \{(1,2),(1,1),(1,3),(1,4),(2,1),(2,4),(2,2),(2,3),(3,1),(3,3),(3,4),(4,1),(4,4),(4,3)\}$

$(\overline{T \circ R}) = \{\cancel{(2,1),(1,1),(3,1),(4,1),(1,2)},(4,2), \cancel{(2,2)},(3,2), \cancel{(1,3),(3,3),(4,3),(1,4)}, \cancel{(4,4),(3,4)}\}$

$$M_{(\overline{T \circ R})} = \begin{bmatrix} 0&0&0&0 \\ 0&0&0&0 \\ 0&1&0&0 \\ 0&1&0&0 \end{bmatrix}$$

(i) (ii) $M_{T^{-1}} = \begin{bmatrix} 1&0&1&0 \\ 0&1&0&0 \\ 0&0&1&1 \\ 1&0&1&0 \end{bmatrix}$ ✓

$M_{R \cap T^{-1}} = \begin{bmatrix} 0&1&1&0 \\ 1&1&0&1 \\ 0&0&1&1 \\ 1&0&0&1 \end{bmatrix} \wedge \begin{bmatrix} 1&0&1&0 \\ 0&1&0&0 \\ 0&0&1&1 \\ 1&0&1&0 \end{bmatrix}$

$= \begin{bmatrix} 0&0&1&0 \\ 0&1&0&0 \\ 0&0&1&1 \\ 1&0&0&0 \end{bmatrix}$ ✓

$M_T \odot M_R$

$M_{T \circ R} = \begin{bmatrix} 0&1&1&0 \\ 1&1&0&1 \\ 0&0&1&1 \\ 1&0&0&1 \end{bmatrix} \odot \begin{bmatrix} 1&0&0&1 \\ 0&1&0&0 \\ 1&0&1&1 \\ 0&0&1&0 \end{bmatrix}$

$= \begin{bmatrix} 1&1&1&1 \\ 1&1&1&1 \\ 1&0&1&1 \\ 1&0&1&1 \end{bmatrix}$ ✓

$M_{(\overline{T \circ R})} = \begin{bmatrix} 0&0&0&0 \\ 0&0&0&0 \\ 0&1&0&0 \\ 0&1&0&0 \end{bmatrix}$ ✓

Let $W_0 = M_{R \cap T^{-1}} = \begin{bmatrix} 0&0&1&0 \\ 0&1&0&0 \\ 0&0&1&1 \\ 1&0&0&0 \end{bmatrix}$ $C_1 = 4$ ✓ $R_1 = 3$ ✓ Add $(4,3)$ ✓

$W_1 = \begin{bmatrix} 0&0&1&0 \\ 0&1&0&0 \\ 0&0&1&1 \\ 1&0&1&0 \end{bmatrix}$ $C_2 = 2$ ✓ $R_2 = 2$ ✓ Add $(2,2)$ ✓

$W_2 = \begin{bmatrix} 0&0&1&0 \\ 0&1&0&0 \\ 0&0&1&1 \\ 1&0&1&0 \end{bmatrix}$ $C_3 = 1, 3, 4$ ✓ $R_3 = 3, 4$ ✓ add $(1,3),(1,4),(3,3),(3,4),(4,3),(4,4)$ ✓

$W_3 = \begin{bmatrix} 0&0&1&1 \\ 0&1&0&0 \\ 0&0&1&1 \\ 1&0&1&1 \end{bmatrix}$ $C_4 = 1, 3, 4$ ✓ $R_4 = 1, 3, 4$ ✓ add $(1,1),(1,3),(1,4),(3,1),(3,3),(3,4),(4,1),(4,3),(4,4)$ ✓

$W_4 = \begin{bmatrix} 1&0&1&1 \\ 0&1&0&0 \\ 1&0&1&1 \\ 1&0&1&1 \end{bmatrix} = M_{R^\infty}$ ✓✓

$R^\infty = \{(1,1),(1,3),(1,4),(2,2),(3,1),(3,3),(3,4),(4,1),(4,3),(4,4)\}$ ✓✓

Q4. (a) Let $A = \{a, b, c, d, e, f, g\}$. Given $\rho_1 = (b, d, f, g)$ and $\rho_2 = (a, c, f, e)$ be two permutations on A.

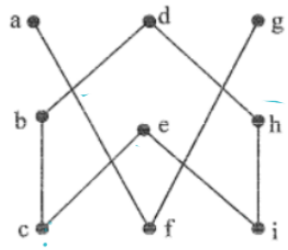
JM

(i) Compute $(\rho_1 \circ \rho_2)^{-1}$ and write the result as a product of disjoint cycles and as a product of transpositions. (6 marks)

(ii) Is $(\rho_1 \circ \rho_2)^{-1}$ an even or odd permutation? (1 mark)

(i) $\rho_1 \circ \rho_2 = \begin{pmatrix} a & b & c & d & e & f & g \\ a & d & c & f & e & g & b \end{pmatrix} \circ \begin{pmatrix} a & b & c & d & e & f & g \\ & & & & & & \end{pmatrix}$

(b) The Hasse diagram of a partially ordered set, P, is given below. Find, if exist(s):



K Weng (i) the maximal and minimal element(s) of P; (3 marks)

Jessie (ii) the greatest and least element(s) of P; (2 marks)

M Yi (iii) the upper bound and lower bound of {b, e, h}; (2 marks)

S Wai (iv) Least Upper Bound and Greatest Lower Bound of {b, h}. (2 marks)

b) i) maximal = {a, d, g, e} ✓

minimal = {c, f, i} ✓

Greatest = none

Least = none

b) (iii) upper bound of {b, e, h} = {~~a, d, g~~} none

lower bound of {b, e, h} = {~~c, f, i~~} none

(c) Simplify $(x \wedge y)' \wedge (x' \vee y) \wedge (y \vee y')$ to the simplest form by using the laws of Boolean algebra. (5 marks)

4 (c) $(x \wedge y)' \wedge (x' \vee y) \wedge (y \vee y') = (x' \vee y') \wedge (x' \vee y) \wedge 1$ ✓

$= x' \vee (y' \wedge y)$

$= x' \vee 0$ ✓

$= x'$ ✓

5

(d) Let $f(x, y, z) = (x' \wedge y' \wedge z') \vee (x' \wedge y \wedge z) \vee (x' \wedge y \wedge z') \vee (x \wedge y' \wedge z') \vee (x \wedge y \wedge z')$.
Construct a Karnaugh map in the form of

	y'	y'	y	y
x'				
x				
	z'	z	z	z'

and hence simplify $f(x, y, z)$ to the simplest form. (4 marks)

$f(x,y,z) = z' + x'y$

	y'	y'	y	y
x'	1	0	1	1
x	1	0	0	1
	z'	z	z	z'

Q3. (a) (i) Let $A = \{v, w, x, y, z\}$ and the relation on A is $R = \{(v, v), (w, w), (x, x), (y, y), (z, z)\}$. Determine whether the relation R is reflexive, irreflexive, symmetric, asymmetric, antisymmetric or transitive. Give a counterexample if the answer is "No". (7 marks)

(ii) Verify if R is an equivalence relation on A. (2 marks)

extra question

Q3(a)(i) R is reflexive since $(v,v), (w,w), (x,x), (y,y)$ and $(z,z) \in R$

R is not irreflexive since $(v,v) \in R$

R is ~~not~~ symmetric since $M_R = (M_R)^T$

R is not asymmetric since $(v,v) \in R$

R is antisymmetric

R is not transitive since $(a,b), (b,c)$ and $(a,c) \notin R$

I'm confused about symmetric, asymmetric, antisymmetric and transitive -ZY