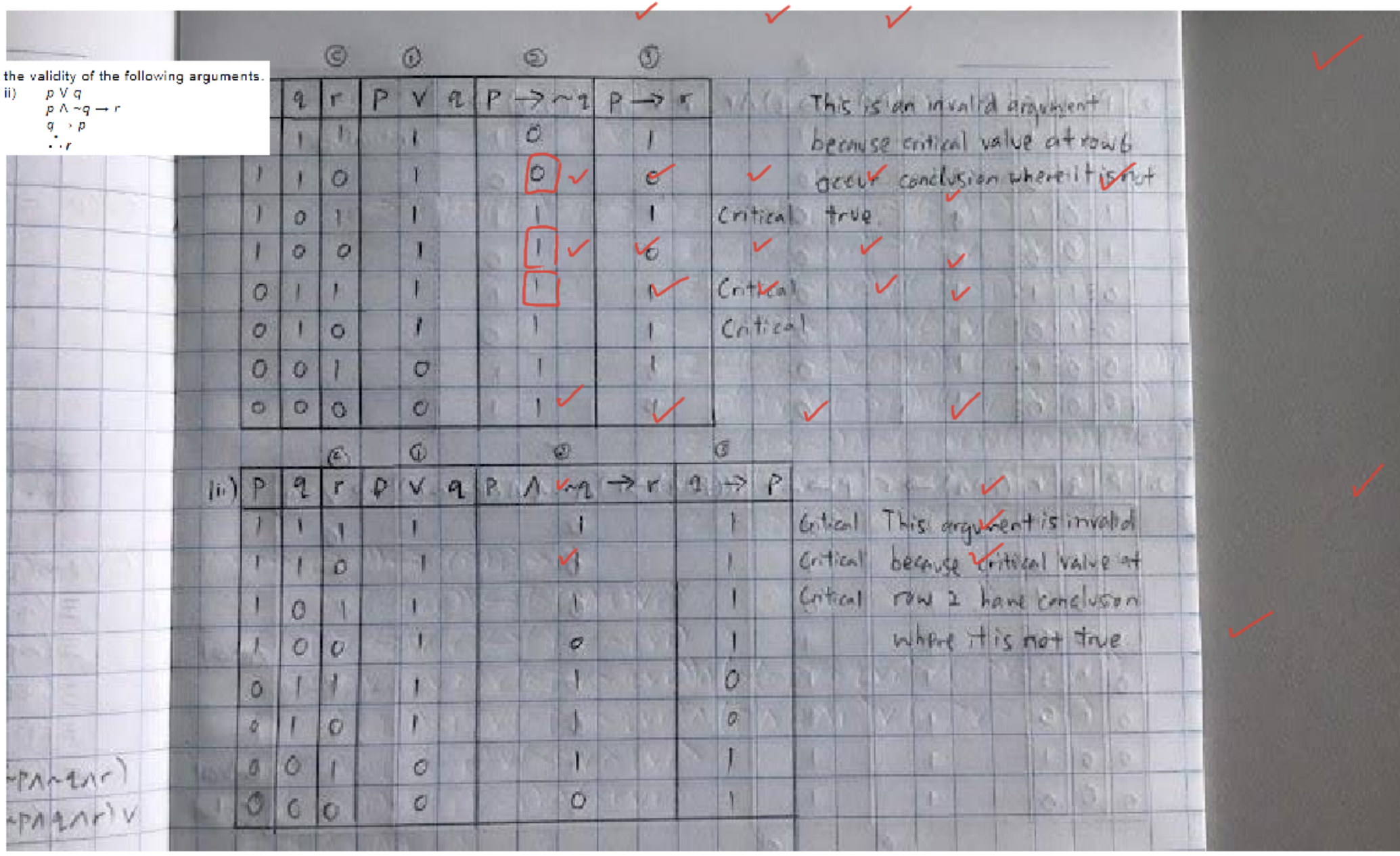
Tutorial 3

1. Use truth table to determine whether the validity of the following arguments.

i) 1 $p \vee q$
2 $p \rightarrow \sim q$
3 $p \therefore r$
$\therefore r$ conclusion

ii) $p \vee q$
$p \wedge \sim q \rightarrow r$
$q \therefore p$
$\therefore r$



	q	r	p ∨ q	p → ~q	p → r	
	1	1	1	0	1	
1	1	0	1	0	0	
1	0	1	1	1	1	Critical
1	0	0	1	1	0	
0	1	1	1	1	1	Critical
0	1	0	1	1	1	Critical
0	0	1	0	1	1	
0	0	0	0	1	1	

This is an invalid argument because critical value at row 6 occur conclusion where it is not true

ii)

p	q	r	p ∨ q	p ∧ ~q → r	q → p	
1	1	1	1	1	1	Critical
1	1	0	1	1	1	Critical
1	0	1	1	1	1	Critical
1	0	0	1	0	1	
0	1	1	1	1	0	
0	1	0	1	1	0	
0	0	1	0	1	1	
0	0	0	0	0	1	

This argument is invalid because critical value at row 2 have conclusion where it is not true

~p∧~q∧r)
~p∧q∧r) ∨

2. The following are valid arguments. Establish the validity of each by means of the truth table. In each case, determine the critical rows for assessing the validity of the argument and which rows can be ignored.

i) $(p \wedge (p \rightarrow q) \wedge r) \rightarrow ((p \vee q) \rightarrow r)$

ii) $((p \wedge q) \rightarrow r) \wedge \neg q \wedge (p \rightarrow \neg r) \rightarrow (\neg p \vee \neg q)$

iii) $(p \vee (q \vee r)) \wedge \neg q \rightarrow p \vee r$

(i)

p	q	r	$(p \rightarrow q)$	$(p \vee q)$	$((p \vee q) \rightarrow r)$
0	0	0	1	0	1
0	0	1	1	0	1
0	1	0	1	1	0
0	1	1	1	1	1
1	0	0	0	1	0
1	0	1	0	1	1
1	1	0	1	1	0
1	1	1	1	1	1

critical value row

The argument is valid.

The validity of the argument follows the result from row 8.

(ii)

p	q	r	$p \wedge q$	$(p \wedge q) \rightarrow r$	$\neg q$	$p \rightarrow \neg r$	$\neg p$	$\neg p \vee \neg q$
0	0	0	0	1	1	1	1	1 C·row
0	0	1	0	1	1	1	1	1 C·row
0	1	0	0	1	0	1	1	1
0	1	1	0	1	0	0	1	1 C·row
1	0	0	0	1	1	1	0	1
1	0	1	0	1	1	0	0	1
1	1	0	1	0	0	1	0	0
1	1	1	1	1	0	0	0	0

critical value

The argument is valid.

The validity of the argument follows the result at row 1, 2, 5.

(iii)

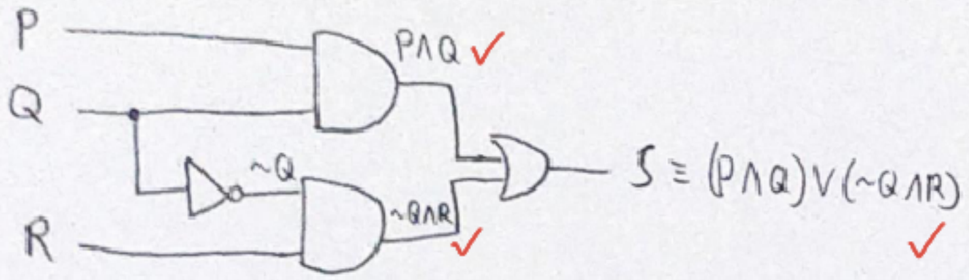
p	q	r	$p \vee (q \vee r)$	$\neg q$	$p \vee r$
0	0	0	0	1	0
0	0	1	1	1	1 critical value row
0	1	0	1	0	0
0	1	1	1	0	1
1	0	0	1	1	1 critical value row
1	0	1	1	1	1 critical value row
1	1	0	1	0	1
1	1	1	1	0	1

The argument is valid.

The validity of the argument follows the result from row 2, 5 and 6.

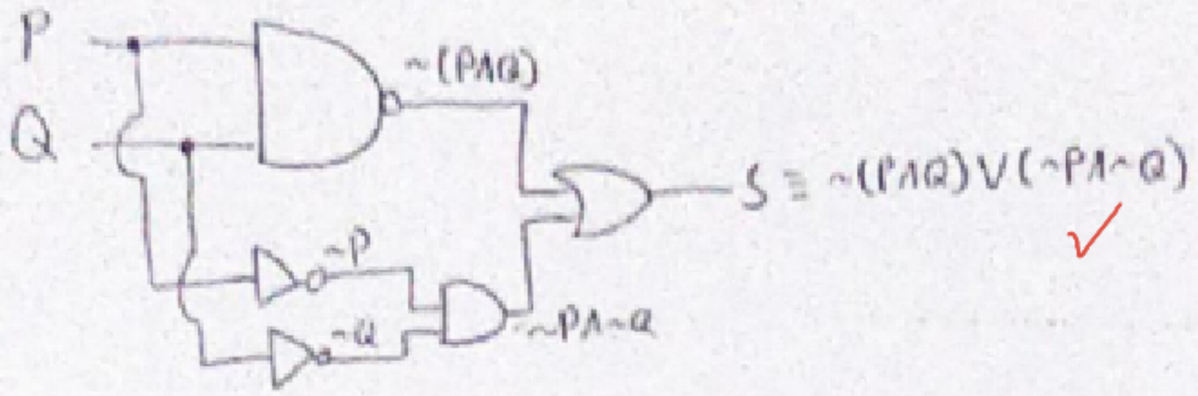
③ Write an input / output table and then find the Boolean expression that corresponds to the circuit below.

i)



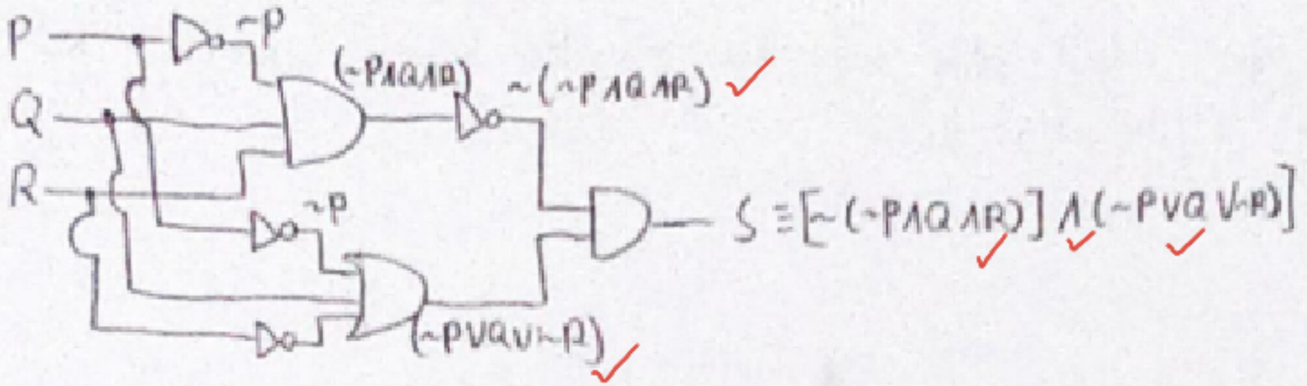
Input			Output
P	Q	R	S
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

③ ii)



Input		Output
P	Q	S
0	0	1
0	1	1
1	0	1
1	1	0

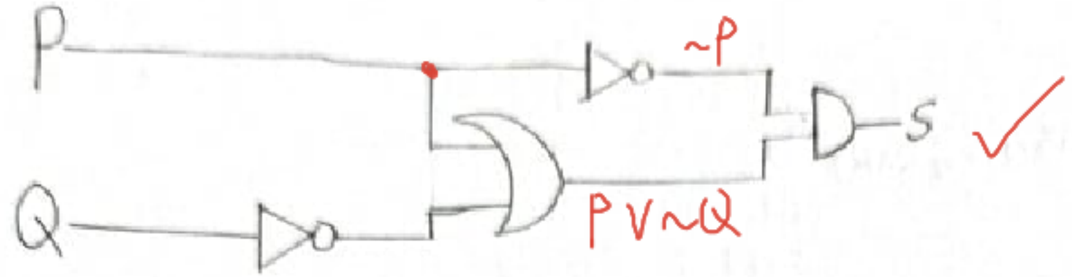
iii)



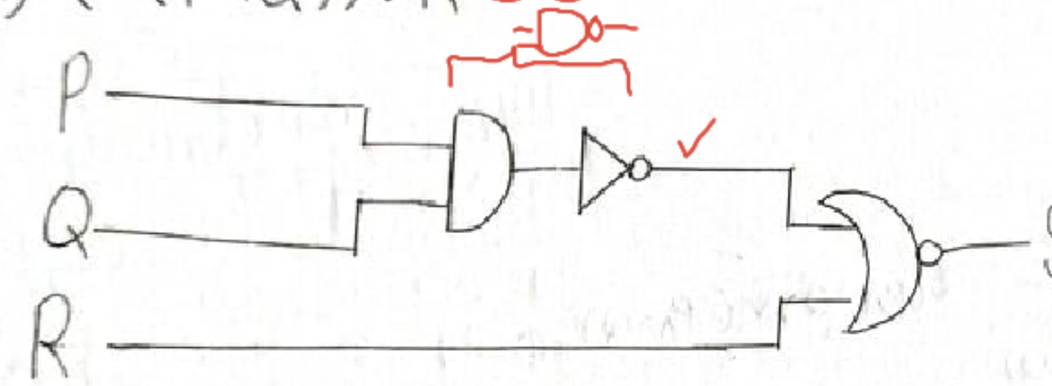
Input			Output
P	Q	R	S
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

$A \downarrow B \equiv \sim(A \vee B)$

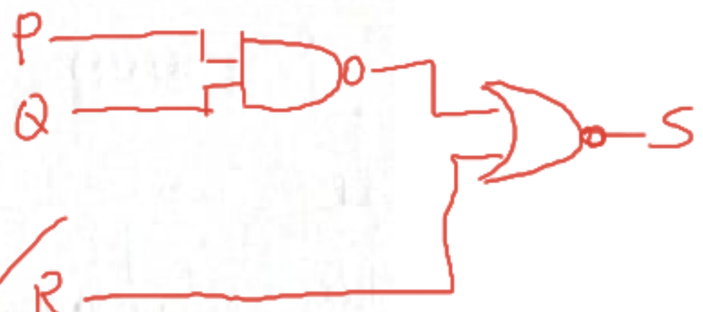
4. i) $S \equiv (P \vee \sim Q) \wedge \sim P$



ii) $(\sim(P \wedge Q)) \downarrow R \equiv S$

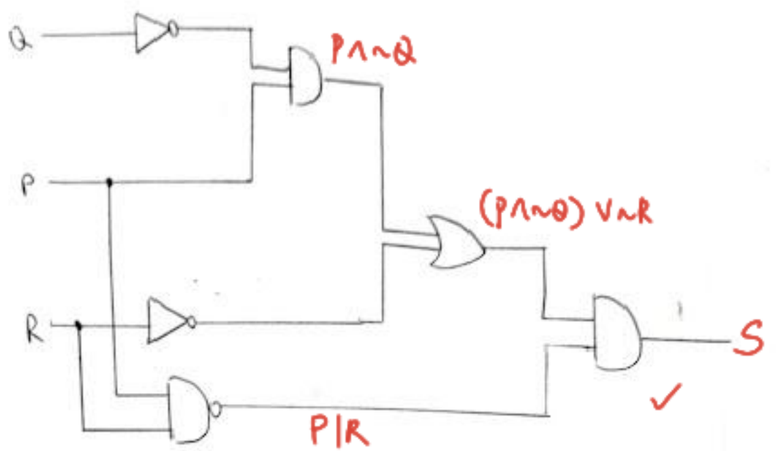


optional :



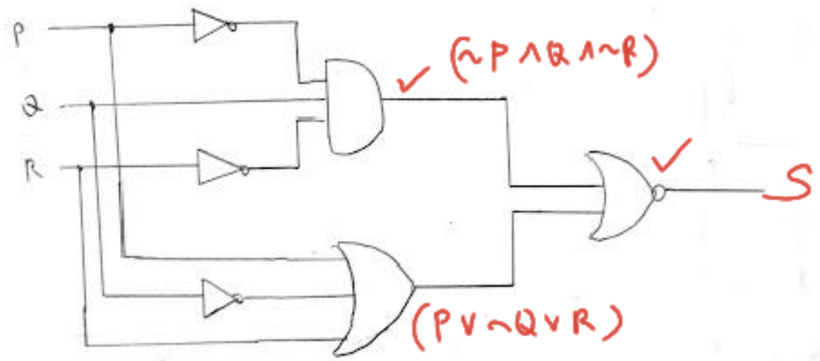
iii) $((P \wedge \sim Q) \vee \sim R) \wedge (P \mid R) \equiv S$ ✓

$A \mid B \equiv \sim (A \wedge B)$



iv) $(\sim P \wedge Q \wedge \sim R) \downarrow (R \vee \sim Q \vee P) \equiv S$

$A \downarrow B \equiv \sim (A \vee B)$

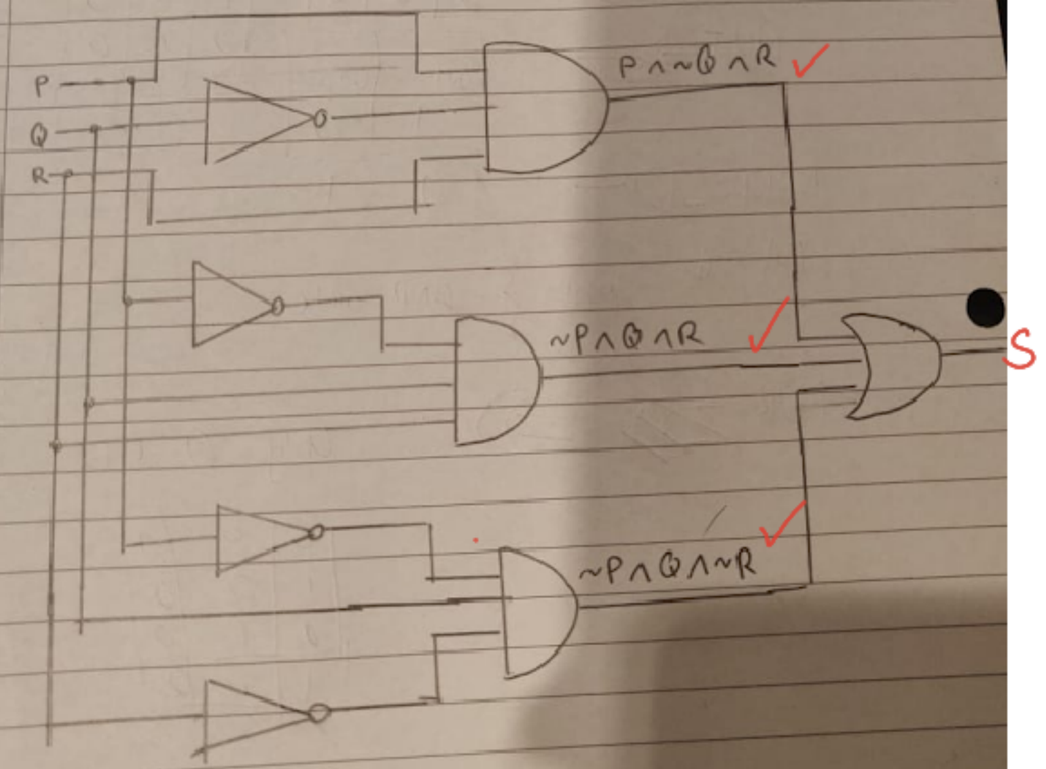


Tutorial 3 5 i)

P	Q	R	S (Output)
0	0	0	0
0	0	1	0
0	1	0	1 ✓
0	1	1	1 ✓
1	0	0	0
1	0	1	1 ✓
1	1	0	0
1	1	1	0

$\bar{P}Q\bar{R}$ ✓ or $\bar{P}QR$ ✓ or $P\bar{Q}R$ ✓

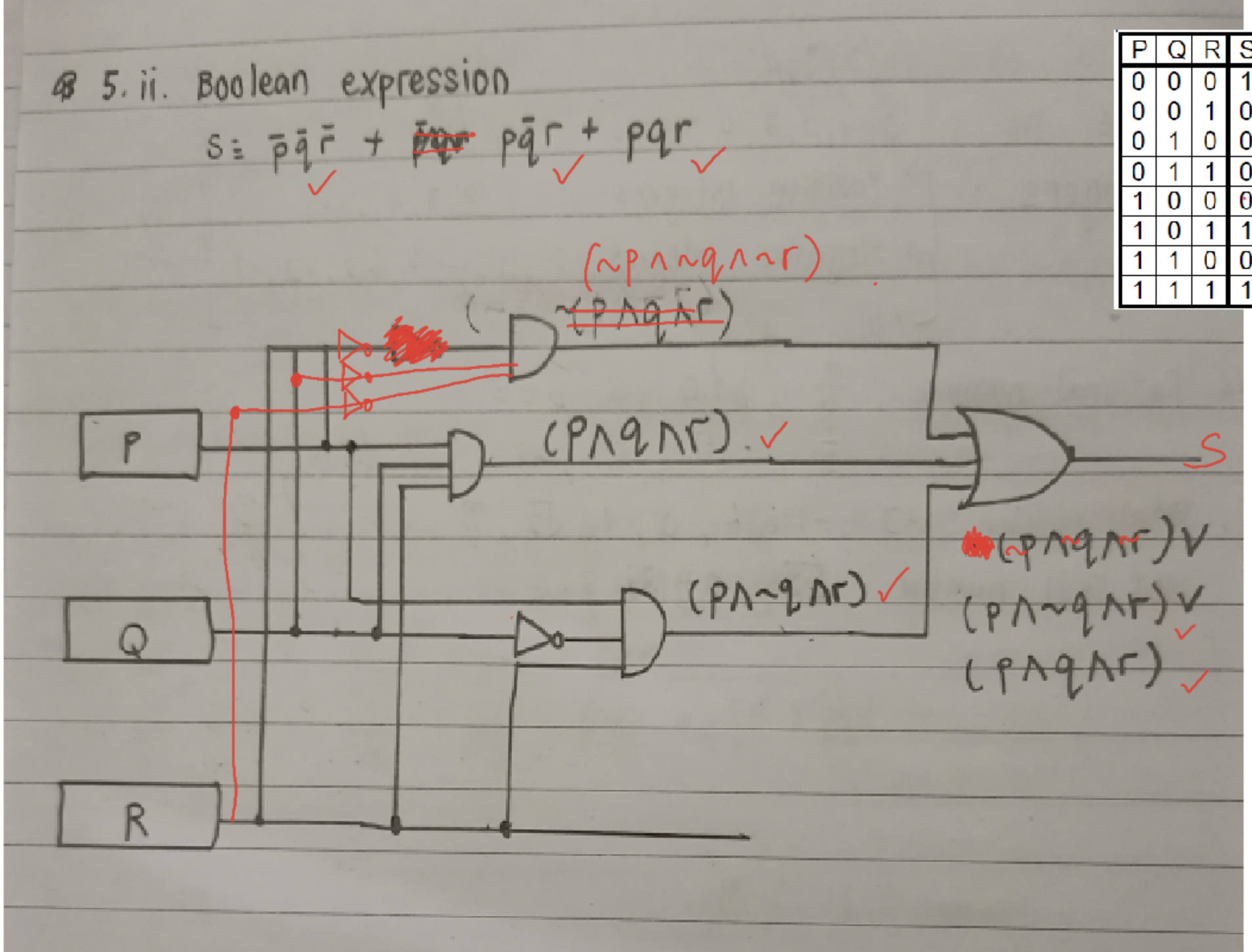
$S \equiv (\sim P \wedge Q \wedge \sim R) \vee (\sim P \wedge Q \wedge R) \vee (P \wedge \sim Q \wedge R)$ ✓



(B) 5. ii. Boolean expression

$S = \bar{p}\bar{q}\bar{r} + p\bar{q}r + pqr$ ✓

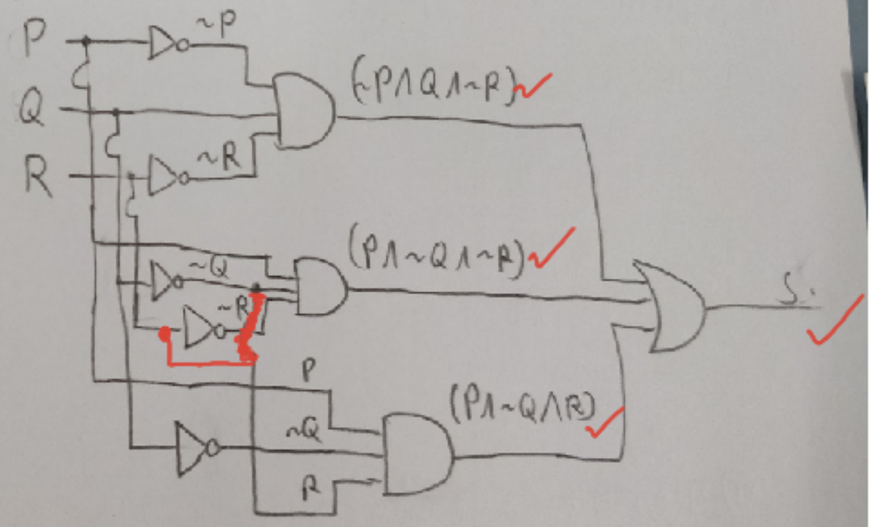
P	Q	R	S (output)
0	0	0	1 ✓
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1 ✓
1	1	0	0
1	1	1	1 ✓



⑤ iv)

P	Q	R	S (output)
0	0	0	0
0	0	1	0
0	1	0	✓ 1 ✓ $\bar{P}Q\bar{R}$ ✓
0	1	1	0
1	0	0	✓ 1 ✓ $P\bar{Q}\bar{R}$ ✓
1	0	1	✓ 1 ✓ $P\bar{Q}R$ ✓
1	1	0	0
1	1	1	0

$S \equiv (\sim P \wedge Q \wedge \sim R) \vee (P \wedge \sim Q \wedge \sim R) \vee (P \wedge \sim Q \wedge R)$ ✓ ✓ ✓



⑥ Find the binary notation for the following decimal integers.

i) (a

ii) 139

iii) 186

iv) 390_{10}

6) i)

2	69
2	34 -1
2	17 -0
2	8 -1
2	4 -0
2	2 -0
	1 -0

$69_{10} = 1000101_2$ ✓

ii)

2	139
2	69 -1
2	34 -1
2	17 -0
2	8 -1
2	4 -0
2	2 -0
	1 -0

$139_{10} = 10001011_2$ ✓

iii)

2	186
2	93 -0
2	46 -1
2	23 -0
2	11 -1
2	5 -1
2	2 -1
	1 -0

$186_{10} = 10111010_2$ ✓

iv)

2	390
2	195 -0
2	97 -1
2	48 -1
2	24 -0
2	12 -0
2	6 -0
2	3 -0
	1 -1

$390_{10} = 110000110_2$ ✓

7. i) 11001_2

$= 2^4 + 2^3 + 2^0$

$= 16 + 8 + 1$

$= 25_{10}$

ii) 101000_2

$= 2^5 + 2^3$

$= 32 + 8$

$= 40_{10}$

iii) 11001100_2

$= 2^7 + 2^6 + 2^3 + 2^2$

$= 128 + 64 + 8 + 4$

$= 204_{10}$

(Q8) (i) $11001_2 + 1001_2$

$= 101010_2$ ✓

$$\begin{array}{r} 11001 \\ +\ 1001_2 \\ \hline 101010 \end{array}$$ ✓

(ii) $1100111_2 + 11_2$

$= 1101010_2$ ✓

$$\begin{array}{r} 1100111 \\ +\ \ \ \ 11_2 \\ \hline 1101010 \end{array}$$ ✓

$1+1 = 2_{10} = 10_2$

$3_{10} = 11_2$

8 iii) $11\overset{\sim}{0}011_2$

$-\ 1001_2$

101010_2 ✓

8 iv) $\overset{\rightarrow}{1}\overset{\rightarrow}{0}\overset{\rightarrow}{0}\overset{\rightarrow}{0}\overset{\rightarrow}{0}0_2$

$-\ 1_2$

1111_2 ✓