AAMS 3244 24 April 2020

Question 1

a)

$$f(x) = \begin{cases} kx^3(1+x) & , 0 \le x < 2 \\ 0 & , \text{elsewhere} \end{cases}$$

(i) $\int_0^2 kx^3(1+x)\,dx = 1$

$k\int_0^2 x^3 + x^4\,dx = 1$

$k\left[\frac{x^4}{4} + \frac{x^5}{5}\right]_0^2 = 1$

$k\left(\frac{2^4}{4} + \frac{2^5}{5}\right) = 1$

$\rightarrow k\left(\frac{16}{4} + \frac{32}{5}\right) = 1$

$k = \frac{5}{52}$ #

(ii) $\mu = \int_0^2 x\left[\frac{5}{52}x^3(1+x)\right]dx = \frac{5}{52}\int_0^2 x^4 + x^5\,dx$

$= \frac{5}{52}\left[\frac{x^5}{5} + \frac{x^6}{6}\right]_0^2$

$= \frac{5}{52}\left(\frac{32}{5} + \frac{64}{6}\right)$

$\rightarrow = \frac{64}{39}$

(iii) $\sigma^2 = \int_0^2 x^2\left[\frac{5}{52}x^3(1+x)\right]dx - \mu^2 = \frac{5}{52}\int_0^2 x^5 + x^6\,dx - \left(\frac{64}{39}\right)^2$

$= \frac{5}{52}\left[\frac{x^6}{6} + \frac{x^7}{7}\right]_0^2 - \left(\frac{64}{39}\right)^2$

$= \frac{5}{52}\left(\frac{2^6}{6} + \frac{2^7}{7}\right) - \left(\frac{64}{39}\right)^2$

$\rightarrow = \frac{5}{52}\left(\frac{608}{21}\right) - \left(\frac{64}{39}\right)^2$

$= \frac{968}{10647}$

$\sigma = \sqrt{\frac{968}{10647}}$

$\doteq 0.3015$

Question 1 (continued)

b) Let X be the no. of people prefer brand A hand phone than brand B hand phone

$X \sim B(10, 0.65)$

$P[X \geq 2] = 1 - P[X<2]$

$= 1 - {}^{10}C_0 (0.65)^0 (0.35)^{10} - {}^{10}C_1 (0.65)^1 (0.35)^9$

$= 0.9995$

c) Let X be the weights of canned sardine

$X \sim N(250, 12^2)$

(i) 1) $P[X < 270] = P[Z < \frac{270-250}{12}]$

$= P[Z < 1.67]$

$= 0.0475$

2) $P[260 < X < 280]$

$= P[\frac{260-250}{12} < Z < \frac{280-250}{12}]$

$= P[0.83 < Z < 2.5]$

$= P[Z > 0.83] - P[Z > 2.5]$

$= 0.2033 - 0.00621$

$= 0.19709$

(ii) $P[X < x] = 0.05$

$P[Z < \frac{x-250}{12}] = 0.05$

$\frac{x-250}{12} = 1.6449$

$x = 269.7388\text{g}$

Question 2

a) $n = 12 \quad \bar{x} = 20 \quad s = 4$

98% C.I. $\Rightarrow \alpha = 0.02 \Rightarrow \alpha/2 = 0.01$

Since σ is unknown and $n = 12 < 30$, t-distribution is used.

$t_{0.01,11} = 2.718$

The 98% confidence interval for the true mean time taken by all the computer science students to solve this computer assignment is

$$\bar{x} \pm t_{0.01,11} \frac{s}{\sqrt{n}} = 20 \pm 2.718 \left(\frac{4}{\sqrt{12}}\right)$$

$$= [16.8615, \ 23.1385] \text{ (minutes)}$$

b) Let x_i be the no. of people who read newspaper i, where $i = 1(X), 2(Y)$

P_i be the true pop. proportion of people who read newspaper i, where $i = 1(X), 2(Y)$

$n_1 = 240 \quad n_2 = 250$

$H_0: P_1 \leq P_2 \qquad H_0: P_1 - P_2 \leq 0$

$H_1: P_1 > P_2 \Rightarrow H_1: P_1 - P_2 > 0$

At $\alpha = 0.01$, critical value $= Z_{0.01} = 2.3263$

critical region : $Z > 2.3263$

$$\hat{p} = \frac{180 + 150}{240 + 250} = \frac{33}{49}, \quad \hat{q} = 1 - \frac{33}{49} = \frac{16}{49}$$

$$Z = \frac{\frac{180}{240} - \frac{150}{250}}{\sqrt{\frac{33}{49}\left(\frac{16}{49}\right)\left(\frac{1}{240} + \frac{1}{250}\right)}} = 3.5395$$

Since $Z = 3.5395 > 2.3263$, H_0 is rejected at $\alpha = 0.01$.

There is sufficient evidence that the proportion of people who read newspaper X was higher than the proportion of people who read newspaper Y.

Question 2 (continued)

c) Let μ_i be the true pop. mean scores of students from group i, where $i = 1\,(A), 2\,(B)$

$H_0: \mu_1 = \mu_2$ $\Rightarrow$ $H_0: \mu_1 - \mu_2 = 0$

$H_1: \mu_1 \neq \mu_2$ $\quad$ $H_1: \mu_1 - \mu_2 \neq 0$

$n_1 = 40$ $\quad$ $n_2 = 42$

$\bar{x}_1 = 82$ $\quad$ $\bar{x}_2 = 75$

$s_1 = 4$ $\quad$ $s_2 = 6$

Since σ_1 and σ_2 are unknown, but $n_1 = 40 > 30$ and $n_2 = 42 > 30$, z-test is used

At $\alpha = 0.05$, critical value $= \pm z_{0.025} = \pm 1.9600$

critical region: $-1.9600 < z < 1.9600$

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{82 - 75}{\sqrt{\frac{4^2}{40} + \frac{6^2}{42}}}$$

$$= 6.2432$$

Since $z = 6.2432 > 1.9600$, H_0 is rejected at $\alpha = 0.05$.

There is sufficient evidence that the mean scores for the students from group A was different from the mean scores of students from group B after implementing two different teaching methods.

Question 3

a)

$O_{ij}(E_{ij})$ Sex	Brand of sport shoes: Adida	Niky	Old Balance	Total
Boys	65 (46.8)	45 (52)	20 (31.2)	130
Girls	25 (43.2)	55 (48)	40 (28.8)	120
Total	90	100	60	250

H_0: There is no association between the choices of brand of sport shoes and sex

H_1: There is a significant association between the choices of brand of sport shoes and sex

At $\alpha = 0.02$, $v = (2-1)(3-1) = 2$, critical value $= \chi^2_{0.02, 2} = 7.378$

critical region : $\chi^2 > 7.378$

$$\chi^2 = \sum_{i=1}^{2}\sum_{j=1}^{3} \frac{(O_{ij} - E_{ij})^2}{E_{ij}} = \frac{(65 - 46.8)^2}{46.8} + \dots + \frac{(40 - 28.8)^2}{28.8}$$

$$= 25.0846$$

Since $\chi^2 = 25.085 > 7.378$, H_0 is rejected at $\alpha = 0.02$.

We can conclude that there is a significant association between the choices of brand of sport shoes and sex

Question 3 (continued)

b) Let X be the monthly phone bill.

$$X \sim N(55, 12^2) \Rightarrow \bar{X} \sim N\left(55, \frac{12^2}{15}\right)$$

$$P[\bar{X} > 50] = P\left[Z > \frac{50-55}{12/\sqrt{15}}\right]$$
$$= P[Z > -1.61]$$
$$= 1 - P[Z > 1.61]$$
$$= 1 - 0.0537$$
$$= 0.9463$$

c)

Item	Year 2016 = 100		Year 2017		P_1q_1	P_1q_0	P_0q_0
	Price (RM)	Quantity (units)	Price (RM)	Quantity (units)			
A	23	350	25	420	10 500	8 750	8050
B	18	460	20	530	10 600	9 200	8 280
C	8	280	10	340	3 400	2800	2240
	P_0	q_0	P_1	q_1	$\Sigma P_1q_1 = 24500$	$\Sigma P_1q_0 = 20750$	$\Sigma P_0q_0 = 18570$

(i) Simple price index for item A

$$= \frac{25}{23} \times 100$$
$$= 108.70\%$$

(ii) Simple aggregate quantity index

$$= \frac{\Sigma q_1}{\Sigma q_0} \times 100$$
$$= \frac{1290}{1090} \times 100$$
$$= 118.35\%$$

(iii) Paasche quantity index

$$= \frac{\Sigma P_1q_1}{\Sigma P_1q_0} \times 100$$
$$= \frac{24500}{20750} \times 100$$
$$= 118.07\%$$

(iv) Laspeyres price index

$$= \frac{\Sigma q_0P_1}{\Sigma q_0P_0} \times 100$$
$$= \frac{20750}{18570} \times 100$$
$$= 111.74\%$$

Question 4

Let ~~indepent~~ independent variable, X = Grade Point Average (GPA)
dependent variable, Y = Salary (RM 00)

(i) $$b = \frac{n\sum XY - \sum X(\sum Y)}{n\sum X^2 - (\sum X)^2} = \frac{7(516.35) - 21.2(169)}{7(64.79) - (21.2)^2} = 7.7384$$

$$a = \frac{\sum Y}{n} - b\frac{\sum X}{n} = \frac{169}{7} - 7.7384\frac{(21.2)}{7} = 0.7066$$

$$Y' = 0.7066 + 7.7384X$$

(ii) GPA, $X = 3.80$

$$Y' = 0.7066 + 7.7384(3.8)$$
$$= 30.1125 \text{ (RM 00)}$$

(iii)

$$r = \frac{n\sum XY - \sum X(\sum Y)}{\sqrt{[n\sum X^2 - (\sum X)^2][n\sum Y^2 - (\sum Y)^2]}} = \frac{7(516.35) - 21.2(169)}{\sqrt{[7(64.79) - (21.2)^2][7(4127) - (169)^2]}}$$

$$= 0.8641$$

$\therefore$ There is a very strong positive linear correlation between GPA and salary. As the GPA higher, the salary will be higher as well.

Question 4 (continued)

a) (iv)

GPA, X	2.85	3.50	3.25	2.75	3.10	2.60	3.15
Salary (RM 00), Y	24	28	27	23	22	20	25
r_X	3	7	6	2	4	1	5
r_Y	4	7	6	3	2	1	5
$D = r_X - r_Y$	-1	0	0	-1	2	0	0
$\cancel{D^2}$ d^2	1	0	0	1	4	0	0

$\Sigma d^2 = 6$

$$r_s = 1 - \frac{6\Sigma d^2}{n(n^2-1)} = 1 - \frac{6(6)}{7(7^2-1)} = 0.8929$$

Question 4 (continued)

b) (i)

Week	Day	Income (RM)	5-day moving Total	5-day moving Average, T	(ii) Y - T
1	Mon	–	–	–	–
	Tue	230	–	–	–
	Wed	200	–	–	–
	Thu	220	1110	222	-2
	Fri	280	1090	218	62
2	Mon	180	1115	223	-43
	Tue	210	1110	222	-12
	Wed	225	1090	218	7
	Thu	215	1115	223	-8
	Fri	260	1130	226	34
3	Mon	205	1135	227	-22
	Tue	225	1130	226	-1
	Wed	230	1145	229	1
	Thu	210	–	–	–
	Fri	270	–	–	–

(ii)

Week \ Day	Mon	Tue	Wed	Thu	Fri
1	–	–		-2	62
2	-43	-12	7	-8	34
3	-22	-1	1	–	–
Total	-65	-13	7	-10	96
Average	-32.5	-6.5	7	-5	48
Adjustment			$-\frac{11}{5} = -2.2$		
Seasonal Variation, S	-34.7	-8.7	4.8	-7.2	45.8

b) (iii) Average change per time period $= \frac{229 - 222}{10 - 1} = \frac{7}{9}$

Week 4, Tuesday :

$T_{est} = 229 + \frac{7}{9}(4)$

$= 232.1111$

$S = -8.7$

$Y_{est} = 232.1111 + (-8.7)$

$= 223.41$ (RM)