Tutorial 4

prime 2 : 1, 2
3 : 1, 3
5 : 1, 5
11 : 1, 11

not prime : 6 : 1, 2, 3, 6
10 : 1, 2, 5, 10
1 : 1, ~~x~~

even: $\boxed{2\times n}$
$2 = 2\times 1$ ✓
$10 = 2\times 5$ ✓
$100 = 2\times 50$ ✓
$0 = 2\times 0$ ✓

real numbers : 0.53, −0.2, 3, $\sqrt{2}$, π, $\frac{3}{7}$, ...

not real numbers : $\sqrt{-2}$, $-3+i$, ...

1. Determine the truth value of the following universal statements. If a statement is false, suggest a counterexample for the statement.

integers

..., −2, −1, 0, 1, 2, ...

real numbers

i) $\forall x \in \{1, 2, 3, 5, 11\}$, x is prime.
ii) $\forall x \in \{0, 2, 6, 12, 36, 48, 52\}$, x is nonnegative and even.
iii) $\forall x \in Z$, the square of x is positive.
iv) $\forall a \in Z$, $(a-1)/a$ is not an integer.
v) $\forall x, y \in R$, $\sqrt{x+y} = \sqrt{x} + \sqrt{y}$.
vi) $\forall$ prime x, x^3 is odd.

i) False. Counterexample: 1 is not a prime number.

ii)True ✓

iii)False.Counterexample: 0^2 is not positive ✓

$0^2 = 0$

1.732

LHS: $\sqrt{1+2} = \sqrt{3}$

RHS: $\sqrt{1} + \sqrt{2} = 2.414$

Let a = 1,

Let x = 1, y = 2

iv)False. Counterexample : let a =1 , (a-1)/a=(1-1)/1=0 (integer)
v)False. Counterexample : Let x=1, y=2 ,the LHS is not equal to RHS
vi)False. Counterexample : Let X =2, x^3 = x^3 =8 (even number)

Let $x = 2$, $x^3 = 2^3 = 8$ (even)

2. Let D = {-48, -14, -8, 0, 1, 3, 16, 23, $\boxed{26}$, $\boxed{32}$, $\boxed{36}$}. Determine which of the following statements are true and which are false. Provide counterexamples for those statements that are false.

i) $\forall\, x \in D$, if x is odd, then $x > 0$.

ii) $\forall\, x \in D$, if x is less than 0 then x is even.

iii) $\forall\, x \in D$, if x is even then $x \le 0$.

iv) $\forall\, x \in D$, if the ones digit of x is 2, then the tens digit is 3 or 4.

v) $\forall\, x \in D$, if the ones digit of x is 6, then the tens digit is 1 or 2.

even = 2n

-8 : 2(-4)

3 2
tens ones

4 2
tens ones

i. true

ii. true

Let x = 16,

iii. false, counterexample : zero is neutral number / 16 > 0

iv. True

Let x = 36,

v. False, Counterexample: ones digit of 6, the tens digit is 3.

iii) $p \to q$ true

3) i) The sum of any even integer is even

- $\forall$ x and y $\in \mathbb{Z}$, if x and y are even, then the sum of x and y are even. ✓
- $\forall$ even integers x and y, sum of x and y are even ✓

ii) All polynomial functions are continuous

- $\forall$ x, if x are polynomial functions, then x are continuous ✓
- $\forall$ ~~even x are~~ polynomial functions x, x are continuous ✓

iii) No integer is a factor of 4.

→ (All integer is not a factor of 4) ✓

- $\forall$ x ~~$\in \mathbb{Z}$~~, if x is an integer ~~4 is not divisible of x~~, then x is not a factor of 4
- $\forall$ ~~even~~ integer x ~~is integer~~, x is not a factor of 4 ✓

4. Determine the truth value of the following existential statements. Prove or disprove the statements.

i) $\exists x \in \{1, 2, 3, 5, 11\}$ such that x is prime and even.

ii) $\exists x \in \{2, 4, 8, 16, 32\}$ such that x is not divisible by 2.

iii) $\exists x \in Z^-$, such that x equals its square.

iv) $\exists x \in Z^+$ such that $4x^2 - 1 = 0$.

negative integers

positive integers

(i)

$P(1)$: 1 is not prime, 1 is not even (F)

$P(2)$: 2 is prime, 2 is even (T)

$P(3)$: 3 is prime, 3 is not even (F)

$P(5)$: 5 is prime, 5 is not even (F)

$P(11)$: 11 is prime, 11 is not even (F)

$\therefore \exists x \in \{1, 2, 3, 5, 11\}$ is true.

(ii)

$P(2)$: $\frac{2}{2} = 1$ (divisible by 2; F)

$P(4)$: $\frac{4}{2} = 2$ (divisible by 2; F)

$P(8)$: $\frac{8}{2} = 4$ (divisible by 2; F)

$P(16)$: $\frac{16}{2} = 8$ (divisible by 2; F)

$P(32)$: $\frac{32}{2} = 16$ (divisible by 2; F)

$\therefore \exists x \in \{2, 4, 8, 16, 32\}$ is false.

iii)

Z^-: negative integers: -1, -2, -3, …

P (-1) :$(-1)^2 = 1$ (F)

P (-2) :$(-2)^2 = 4$ (F)

P (-3) :$(-3)^2 = 9$ (F)

$\therefore \exists \chi \in Z^-$ is false

iv)

Z^+: positive integers: 1, 2, 3, …

$4x^2 - 1 = 0$

$x = \pm \frac{1}{2}$

P (1):$4(1)^2-1=3 \neq 0$ (F)

P (2):$4(2)^2-1=15 \neq 0$ (F)

P (3):$4(3)^2-1=35 \neq 0$ (F)

$\therefore \exists \chi \in Z^+$ is false real numbers: $-2, 0.53, \frac{2}{7}, \ldots, \sqrt{2}$

5. Consider the following statement:

$\exists x \in R$ such that $x^2 = 2$.

Which of the following are equivalent ways of expressing this statement?

i) If x is a real number, then $x^2 = 2$.

ii) Some real number has square 2.

iii) Some real numbers have square 2.

iv) The number x has square 2, for some real number x.

v) The square of each real number is 2. all

vi) There is at least one real number whose square is 2.

i. not equilvalent

ii. equivalent e.g. $\sqrt{2}^2 = 2$

iii. Equilvalent $\sqrt{2}^2 = 2$, $(-\sqrt{2})^2 = 2$

iv) Equivalent

v) Not Equivalent ✓

vi) Equivalent ✓

explain x

6. Rewrite the following statements in the two forms "∃ _x such that _" and "∃ x such that __ and __".

i) Some exercises have answers.
ii) Some questions are easy.
iii) There exists an even integer divisible by 4.
iv) Some people are rich but unhappy.

6. i. ∃ exercises x such that x have answers.
∃x such that x are exercises and x have answers

ii. ∃ questions x such that x are easy.
∃x such that x are questions and x are easy.

iii) There exists an even integer divisible by 4

1 : ∃ even integer x such that x is divisible by 4.

2 : ∃ x such that x is an even integer and x is divisible by 4

iv) Some people are rich but unhappy

1 : ∃ rich people x such that x is unhappy

2 : ∃ x such that x is rich people and x is unhappy