Tutorial 10

Q1 i)

1. Let $A = \{0, 2, 4, 6\}$ and $B = \{1, 3, 5, 7\}$. Determine which of the following relations between A and B forms a function with domain A and codomains B. For those whose are functions, determine whether they are injective ①, surjective ② or bijective ①,②.

1-to-1
many-to-1

i) $\{(6, 3), (2, 1), (0, 3), (4, 5)\}$
ii) $\{(2, 3), (4, 7), (0, 1), (6, 5)\}$
iii) $\{(2, 1), (4, 5), (6, 3)\}$
iv) $\{(6, 1), (0, 3), (4, 1), (0, 7), (2, 5)\}$

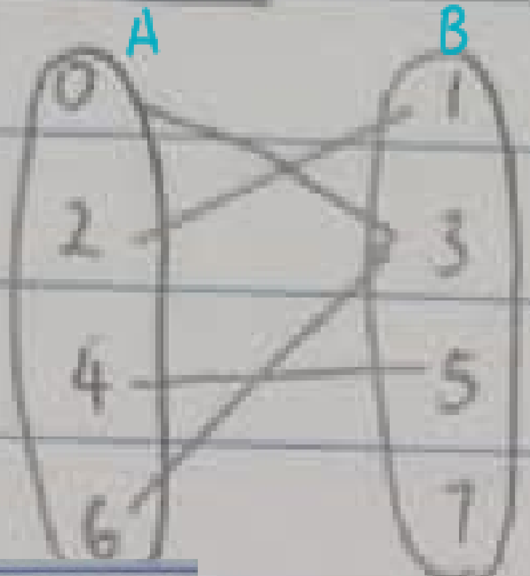
1-to-1

codomain = range

Angka Giliran : .... Perkara : ....

Tutorial 10

i)



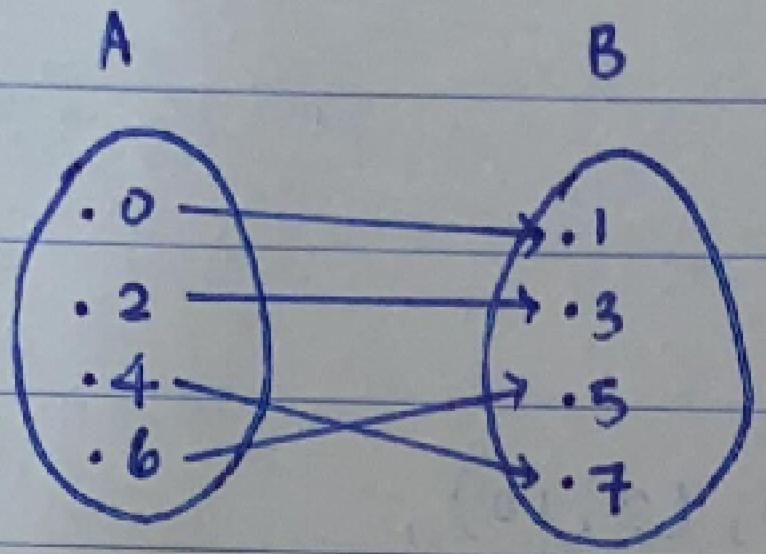
∴ This is a function

It is not injective ✓ since $f(0) = f(6) = 3$

It is not surjective ✓ since $7 \notin R(f)$ or codomain $\neq$ Range $= \{1, 3, 5\}$

∴ It is not bijective ✓

ii)

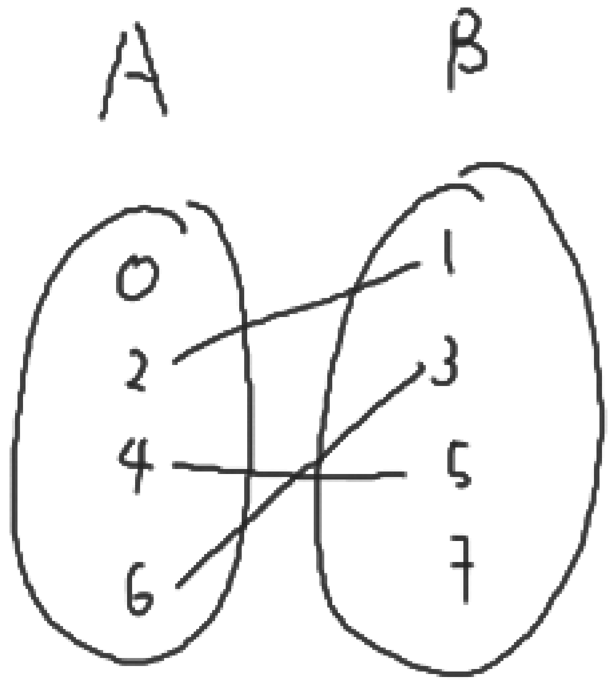


∴ The relation is a function, ✓ f is bijective ✓

f is injective ✓

f is surjective ✓

Q1 (iii)



{(2,1),(4,5),(6,3)}

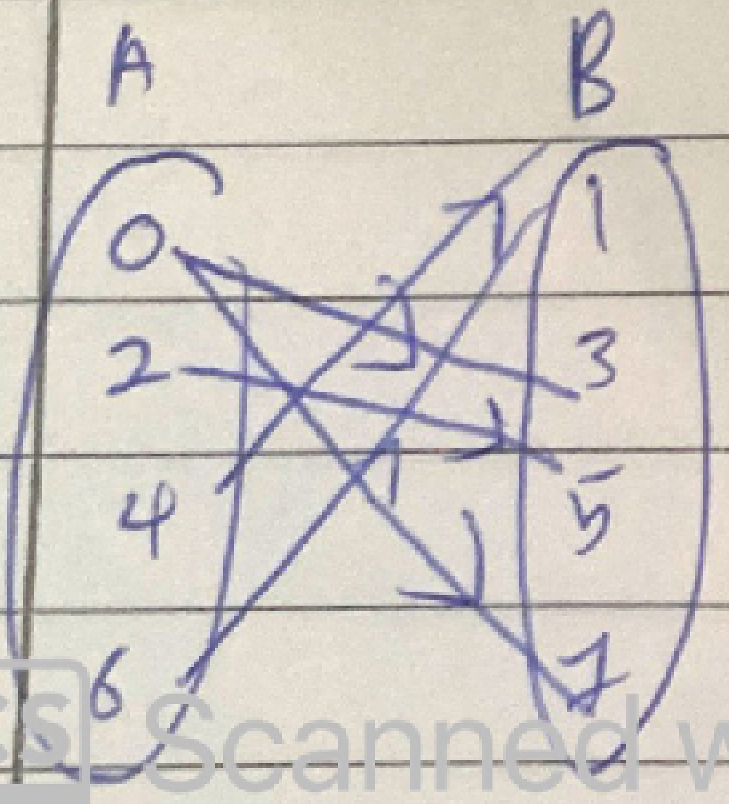
it is not injective
it is not surjective (codomain not equal to range)
it is not bijective

It is not function since no element is associated to 0

or since f(0) is undefined

1) $A = \{0, 2, 4, 6\}$ $B = \{1, 3, 5, 7\}$

iv) $\{(6,1), (0,3), (4,1), (0,7), (2,5)\}$



it is not injective since many-to-one.

It is surjective since onto.

$\therefore$ it is not bijective function

This is not a function since $f(0) = 3$ and $f(0) = 7$

domain ✓ codomain ✓

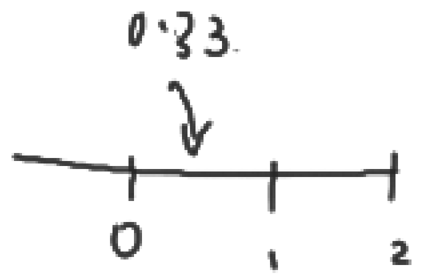
2. Let $A = \{-1, 0, 1, 2\}$ and $f: A \rightarrow \mathbb{Z}$ be given by $f(x) = \left\lfloor \frac{x^2+1}{3} \right\rfloor$.

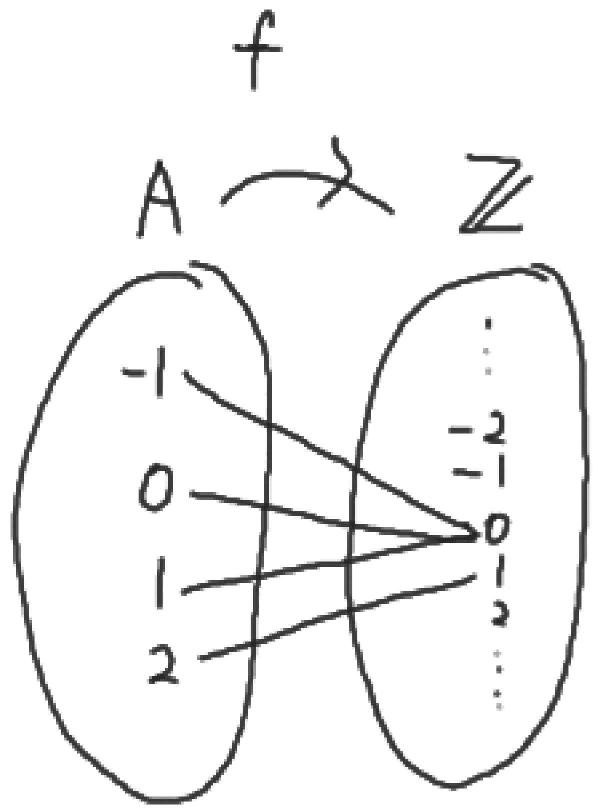
integers

i) Find the range of f.

ii) Determine whether the function f is injective, surjective or bijective. Justify your answer.

$$f(-1) = \left\lfloor \frac{(-1)^2+1}{3} \right\rfloor = \left\lfloor \frac{2}{3} \right\rfloor = \lfloor 0.66\ldots \rfloor = 0 ✓$$





$$f(0) = \left\lfloor \frac{0^2+1}{3} \right\rfloor = \left\lfloor \frac{1}{3} \right\rfloor = \lfloor 0.33\ldots \rfloor = 0$$

$f(1) =$ (1+1)/3=$\lfloor 0.66 \rfloor$=0 ✓

$f(2) =$ (4+1)/3=$\lfloor 1.67 \rfloor$=1 ✓ $\therefore$ Range = $\{0, 1\}$

ii) It is not ✓ injective ✓ because its many ✓ to one and not surjective

It is not ✓ bijective because it is not injective.

it is not ✓ surjective becuase not every Z has atleast one A.

since $2 \notin R(f)$ /

since Range $\neq$ Codomain

3. Given $f(x) = 2x - 1$, a function from $X = \{1, 2, 3\}$ to $Y = \{1, 2, 3, 4, 5\}$. Find the domain and range of the function f. Hence determine whether the function is a bijective function and explain your answer.

When x=1
f(x) = 2(1) - 1 = 1 ✓

When x=2
f(x) = 2(2) - 1 = 3 ✓

When x=3
f(x) = 2(3) - 1 = 5 ✓

Domain = {1,2,3} ✓ Range = {1,3,5} ✓

Injective : Yes, because it is a one-to-one function ✓

Surjective : No , because not every element in the codomain is a value of the f(x).Missing {2,4}. ✓

or since $2 \notin R(f)$

Bijective : Not bijective ✓ because the function is not a surjective ✓ function.

4. Let $A = \{1, 2, 3, 4, 5, 6\}$ and $p_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 1 & 2 & 6 & 5 \end{pmatrix}$, $p_2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 1 & 5 & 4 & 6 \end{pmatrix}$, $p_3 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 3 & 2 & 5 & 4 & 1 \end{pmatrix}$. Compute the following.

i) p_1^{-1} ii) $p_3 \circ p_1$

iii) p_3^{-1} iv) $p_1^{-1} \circ p_2^{-1}$

Q4 i) $P_1^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 1 & 2 & 6 & 5 \end{pmatrix}$

start

ii) $P_3 \circ P_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 3 & 2 & 5 & 4 & 1 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 1 & 2 & 6 & 5 \end{pmatrix}$

$= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 5 & 6 & 3 & 1 & 4 \end{pmatrix}$

$1 \to 3,\ 3 \to 2$
$2 \to 4,\ 4 \to 5$
$3 \to 1,\ 1 \to 6$
$4 \to 2,\ 2 \to 3$
$5 \to 6,\ 6 \to 1$
$6 \to 5,\ 5 \to 4$

iii) $P^{-3} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 3 & 2 & 5 & 4 & 1 \end{pmatrix}$

iv) $P1^{-1} \circ P2^{-1}$

$P2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 1 & 5 & 4 & 6 \end{pmatrix}$

$P2^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 2 & 5 & 4 & 6 \end{pmatrix}$

Start

$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 1 & 2 & 6 & 5 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 2 & 5 & 4 & 6 \end{pmatrix}$

$= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 3 & 4 & 6 & 2 & 5 \end{pmatrix}$

nxt wk

$(1,1)(1,1)(1,1)$

$(2,5)(5,7)(7,7)$

$(3,4)(4,4)(4,4)$

5. Let $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$. Compute the following products.

i) $(3, 5, 7, 8) \circ (1, 3, 2)$ ii) $(2, 6) \circ (3, 5, 7, 8) \circ (2, 5, 3, 4)$

start

i) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 2 & 5 & 4 & 7 & 6 & 8 & 3 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 1 & 2 & 4 & 5 & 6 & 7 & 8 \end{pmatrix}$

$= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 5 & 1 & 2 & 4 & 7 & 6 & 8 & 3 \end{pmatrix}$

start

ii) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \underline{\underline{1}} & 6 & 3 & \underline{\underline{4}} & 5 & 2 & \underline{\underline{7}} & 8 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 2 & 5 & 4 & 7 & 6 & 8 & 3 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 5 & 4 & 2 & 3 & 6 & 7 & 8 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 7 & 4 & 6 & 5 & 2 & 8 & 3 \end{pmatrix}$

Q6. Let $A = \{1,2,3,4,5,6,7,8\}$. Write each permutation as a product of transpositions

i) $(2,1,4,5,8,6) = \underbrace{(2,6)}_{\text{transposition}} \circ (2,8) \circ (2,5) \circ (2,4) \circ (2,1)$ ✓

cycle

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 1 & 3 & 5 & 8 & 2 & 7 & 6 \end{pmatrix}$$

ii) $(3,1,6) \circ (4,8,2,5) = \underbrace{(3,6) \circ (3,1) \circ (4,5) \circ (4,2) \circ (4,8)}_{\text{product of transpositions}}$ ✓

product of disjoint cycles

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 6 & 2 & 1 & 4 & 5 & 3 & 7 & 8 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 5 & 3 & 8 & 4 & 6 & 7 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 6 & 5 & 1 & 8 & 4 & 3 & 7 & 2 \end{pmatrix}$$

7. Let $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$. Determine the given permutation is even or odd.

i) $(6, 4, 2, 1, 5)$ ii) $(4, 8) \circ (3, 5, 2, 1) \circ (2, 4, 7, 1)$

(i) $(6, 5) \circ (6, 1) \circ (6, 2) \circ (6, 4)$ ✓

Since there are 4 transpositions, therefore this is an even permutations ✓

(ii) $(4, 8) \circ (3, 1) \circ (3, 2) \circ (3, 5) \circ (2, 1) \circ (2, 7) \circ (2, 4)$

Since there are 7 transpositions, therefore this is an odd permutations ✓

8. Let $A = \{1, 2, 3, 4, 5\}$. Let $f = (5, 3, 2)$ and $g = (3, 4, 1)$ be permutations of A. Compute each of the following and write the results as the (product of disjoint cycles.)

i) $f \circ g$ ii) $f^{-1} \circ g^{-1}$

Start

a) $f \circ g$ $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 5 & 2 & 4 & 3 \end{pmatrix}$ $g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 2 & 4 & 1 & 5 \end{pmatrix}$

$f \circ g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 2 & 5 & 4 & 1 & 3 \end{pmatrix} = (1, 2, 5, 3, 4)$: product of disjoint cycles

Start

b) $f^{-1} \circ g^{-1}$ $f^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 5 & 4 & 2 \end{pmatrix}$ $g^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 2 & 1 & 3 & 5 \end{pmatrix}$

$f^{-1} \circ g^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 4 & 3 & 1 & 5 & 2 \end{pmatrix} = (1, 4, 5, 2, 3)$

9. Let $A = \{1, 2, 3, 4, 5, 6\}$ and $p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 3 & 1 & 5 & 6 \end{pmatrix}$ be a permutation of A.

i) Write p as a product of disjoint cycles.
ii) Compute p^{-1}.
iii) Compute p^2.

Q9. $A = \{1, 2, 3, 4, 5, 6\}$

$$p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 3 & 1 & 5 & 6 \end{pmatrix}$$

i. $p = (1, 2, 4)$

ii. $$p^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 1 & 3 & 2 & 5 & 6 \end{pmatrix}$$

iii. $$p^2 = p \circ p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 3 & 1 & 5 & 6 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 3 & 1 & 5 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 1 & 3 & 2 & 5 & 6 \end{pmatrix}$$