16 DECEMBER 2020

Question 1

a)
$$f(x) = \begin{cases} k(x^2-2x+2), & 0<x\leq 3 \\ 3k & 3<x\leq 4 \\ 0 & \end{cases}$$

(i) $\int_0^3 k(x^2-2x+2)\,dx + \int_3^4 3k\,dx = 1$

$k\left[\frac{x^3}{3} - \frac{2x^2}{2} + 2x\right]_0^3 + 3k[x]_3^4 = 1$

$k\left[\left(\frac{3^3}{3} - 3^2 + 2(3)\right) - 0\right] + 3k(4-3) = 1$

$\rightarrow k(9-9+6) + 3k = 1$

$6k + 3k = 1$

$9k = 1$

$k = \frac{1}{9}$ #

(ii) $\mu = \int_0^3 x\left[\frac{1}{9}(x^2-2x+2)\right]dx + \int_3^4 x\left[3\left(\frac{1}{9}\right)\right]dx$

$= \frac{1}{9}\int_0^3 x^3 - 2x^2 + 2x\,dx + \frac{1}{3}\int_3^4 x\,dx$

$= \frac{1}{9}\left[\frac{x^4}{4} - \frac{2x^3}{3} + \frac{2x^2}{2}\right]_0^3 + \frac{1}{3}\left[\frac{x^2}{2}\right]_3^4$

$= \frac{1}{9}\left(\frac{3^4}{4} - \frac{2(3)^3}{3} + 9\right) + \frac{1}{3}\left(\frac{4^2}{2} - \frac{3^2}{2}\right)$

$= \frac{5}{4} + \frac{7}{6}$

$= \frac{29}{12}$

(iii) $\sigma^2 = \int_0^3 x^2\left[\frac{1}{9}(x^2-2x+2)\right]dx + \int_3^4 x^2\left[3\left(\frac{1}{9}\right)\right]dx - \mu^2$

$= \frac{1}{9}\int_0^3 x^4 - 2x^3 + 2x^2\,dx + \frac{1}{3}\int_3^4 x^2\,dx - \left(\frac{29}{12}\right)^2$

$= \frac{1}{9}\left[\frac{x^5}{5} - \frac{2x^4}{4} + \frac{2x^3}{3}\right]_0^3 + \frac{1}{3}\left[\frac{x^3}{3}\right]_3^4 - \frac{841}{144}$

$= \frac{1}{9}\left(\frac{3^5}{5} - \frac{3^4}{2} + \frac{2(3)^3}{3}\right) + \frac{1}{3}\left(\frac{4^3}{3} - \frac{3^3}{3}\right) - \frac{841}{144}$

$= \frac{281}{240}$

$\sigma = \sqrt{\frac{281}{240}} = 1.0821$

Question 1 (continued)

b) $X \equiv$ lifetime of a certain brand of scientific calculator

$X \sim N(5, 0.5^2)$

(i) 1) $P[X<4] = P[Z < \frac{4-5}{0.5}]$

$= P[Z<-2]$

$= P[Z>2]$

$= 0.02275$

2) $P[5.5 < X < 7] = P[\frac{5.5-5}{0.5} < Z < \frac{7-5}{0.5}]$

$= P[1<Z<4]$

$= P[Z>1] - P[Z>4]$

$= 0.1587 - 0.00003$

$= 0.15867$

(ii) $P[X>k] = 0.02$

$P[X > \frac{k-5}{0.5}] = 0.02$

$\frac{k-5}{0.5} = 2.0537$

$k-5 = 1.02685$

$k = 6.02685.$

≈ 6.03 years

c) $X \equiv$ no. of car batteries are defective.

$X \sim B(10, 0.07)$

$P[X \geq 1] = 1 - P[X=0]$

$= 1 - {}^{10}C_0 (0.07)^0 (0.93)^{10}$

$= 1 - 0.4840$

$= 0.5160$

Question 2

a) $n = 20 \quad \bar{x} = 70 \quad s = 10$

95% C.I. $\Rightarrow \alpha = 0.05 \Rightarrow \alpha/2 = 0.025$

Since σ^2 is unknown, $n = 20 < 30$, t-dist is used

$$t_{0.025,19} = 2.093$$

The 95% confidence interval for the mean score job satisfaction of all the system analysis is

$$\bar{x} \pm t_{0.025,19} \frac{s}{\sqrt{n}} = 70 \pm 2.093\left(\frac{10}{\sqrt{20}}\right)$$

$$= [65.3199, 74.6801]$$

b) Let μ_i be the true pop. mean satisfaction index for all customers for supermarket i where $i = 1(\text{I}), 2(\text{II})$

$H_0: \mu_1 = \mu_2$ $\Rightarrow$ $H_0: \mu_1 - \mu_2 = 0$

$H_1: \mu_1 \neq \mu_2$ $\quad$ $H_1: \mu_1 - \mu_2 \neq 0$

$n_1 = 300 \quad n_2 = 350$

$\bar{x}_1 = 7.6 \quad \bar{x}_2 = 8.1$

$s_1 = 0.75 \quad s_2 = 0.59$

Since σ_1^2 and σ_2^2 are unknown, but $n_1 = 300 > 30$ and $n_2 = 350 > 30$, z-test is used.

At $\alpha = 0.01$, critical value $= \pm Z_{0.005} = \pm 2.5758$

critical region : $-2.5758 < Z < 2.5758$

$$Z = \frac{7.6 - 8.1}{\sqrt{\frac{0.75^2}{300} + \frac{0.59^2}{350}}} = -9.3339$$

Since $Z = -9.3339 < -2.5758$, H_0 is rejected at $\alpha = 0.01$.

There is sufficient evidence that the mean satisfaction indexes for all customers for the two supermarkets are different.

Question 2 (continued)

c) Let $x_i \equiv$ no. of item returned at Store i where $i = A$ (Store A), B (Store B)

Let $P_i \equiv$ true pop. proportion of item that returned at Store i where $i = A, B$

$H_0: P_A \leq P_B$ $\Rightarrow$ $H_0: P_A - P_B \leq 0$ $n_A = 800$ $n_B = 900$

$H_1: P_A > P_B$ $\quad$ $H_1: P_A - P_B > 0$ $x_A = 280$ $x_B = 279$

At $\alpha = 0.01$, critical value $= Z_{0.01} \doteq 2.3263$

critical region : $Z > 2.3263$

$$\hat{p} = \frac{280 + 279}{800 + 900} = \frac{559}{1700} \quad , \quad \hat{q} = 1 - \frac{559}{1700} = \frac{1141}{1700}$$

$$Z = \frac{\frac{280}{800} - \frac{279}{900}}{\sqrt{\frac{559}{1700}\left(\frac{1141}{1700}\right)\left(\frac{1}{800} + \frac{1}{900}\right)}} = 1.7523$$

Since $Z = 1.7523 < 2.3263$, H_0 is fail to reject at $\alpha = 0.01$.

We can conclude that the proportion of all sales for which at least one item is returned at Store A is higher than Store B.

Question 3

a)

$O_{ij}(E_{ij})$	Yes	No	Uncertain	Total
Women	125(115.3125)	59(70.725)	21(18.9625)	205
Men	100(109.6875)	79(67.275)	16(18.0375)	195
Total	225	138	37	400

H_0: There is no association between the gender and response
H_1: There is a significant association between the gender and response.

At $\alpha = 0.01$, $v = (2-1)(3-1) = 2$, critical value $= \chi^2_{0.01,2} = 9.210$

critical region: $\chi^2 > 9.210$

$$\chi^2 = \sum_{i=1}^{2}\sum_{j=1}^{3} \frac{(O_{ij}-E_{ij})^2}{E_{ij}} = \frac{(125-115.3125)^2}{115.3125} + \dots + \frac{(16-18.0375)^2}{18.0375}$$

$$= 6.1058$$

Since $\chi^2 = 6.106 < 9.210$, H_0 is rejected at $\alpha = 0.01$.
We can conclude that there is no association between the gender and response for those who consider to ~~mary~~ marry someone who was RM 25 000 or more in debt.

b) $X \equiv$ mass of potato $\quad X \sim N(650, 30^2) \Rightarrow \bar{X} \sim N(650, \frac{30^2}{40})$

$$P[\bar{X} > 640] = P\left[Z > \frac{640-650}{30/\sqrt{40}}\right]$$
$$= P[Z > -2.11]$$
$$= 1 - P[Z > 2.11]$$
$$= 1 - 0.01743$$
$$= 0.98257$$

5

Question 3 (continued)

c)

Item	2018 = 100 Price (RM)	Quantity	2019 Price (RM)	Quantity	p_0q_0	p_0q_1	p_1q_1	q_0p_1
A	30	100	38	108	3000	3240	4104	3800
B	34	55	46	46	1870	1564	2116	2530
	p_0	q_0	p_1	q_1	$\Sigma p_0q_0 = 4870$	$\Sigma p_0q_1 = 4804$	$\Sigma p_1q_1 = 6220$	$\Sigma q_0p_1 = 6330$

(i) Simple price index of item B for the year 2019

$= \frac{P_1}{P_0} \times 100$

$= \frac{46}{34} \times 100$

$= 135.29\%$

(ii) ~~Da~~ Laspeyres price index for the year 2019

$= \frac{\Sigma q_0 p_1}{\Sigma q_0 p_0} \times 100$

$= \frac{6330}{4870} \times 100$

$= 129.98\%$

∴ The price of the 2 types of electrical items had been increased about 29.98% on average in 2019 as compared to 2018.

(iii) Paasche quantity index for the year 2019

$= \frac{\Sigma \cancel{q_1} p_1 q_1}{\Sigma p_1 q_0} \times 100$

$= \frac{6220}{6330} \times 100$

$= 98.26\%$

∴ The quantity of the 2 types of electrical items had been decreased about 1.74% on average in 2019 as compared to 2018.

Question 4

a)

CGPA, X	2.25	2.05	3.94	2.42	3.20	3.81	2.90
Starting Salary, Y (RM '00)	22	19	32	21	23	28	23
r_x	2	1	7	3	5	6	4
r_y	3	1	7	2	4.5	6	4.5
$d = r_x - r_y$	-1	0	0	1	0.5	0	-0.5
d^2	1	0	0	1	0.25	0	0.25

$$\sum d^2 = 2.5$$

(i)

$$r = \frac{n\sum XY - \sum X(\sum Y)}{\sqrt{[n\sum X^2 - (\sum X)^2][n\sum Y^2 - (\sum Y)^2]}} = \frac{7(512.33) - 20.57(168)}{\sqrt{[7(63.8111) - (20.57)^2][7(4152) - (168)^2]}}$$

$$= 0.9281$$

(ii)

$$r_s = 1 - \frac{6\sum d^2}{n(n^2-1)} = 1 - \frac{6(2.5)}{7(7^2-1)} = 0.9554$$

(iii)

$$b = \frac{n(\sum XY) - \sum X(\sum Y)}{n\sum X^2 - (\sum X)^2} = \frac{7(512.33) - 20.57(168)}{7(63.8111) - 20.57^2} = 5.5429$$

$$a = \frac{\sum Y}{n} - b\frac{\sum X}{n} = \frac{168}{7} - 5.5429\left(\frac{20.57}{7}\right) = 7.7118$$

$$Y' = 7.7118 + 5.5429X$$

Question 4 (continued)

a) (iv) CGPA, $X = 3.5$

$$Y' = 7.7118 + 5.5429(3.5)$$

$$= 27.1120 \text{ (RM '00)}$$

(i) b) Let ~~S ≡ Sunday~~, W ≡ Wednesday, T ≡ Thursday, F ≡ Friday, Sat ≡ Saturday, Sun ≡ Sunday

Week	Day	Sales	5-day moving total	5-day moving average, T	Y - T
1	W	20	-	-	-
	T	45	-	-	-
	F	20	153	30.6	-10.6
	Sat	28	161	32.2	-4.2
	Sun	40	166	33.2	6.8
2	W	28	171	34.2	-6.2
	T	50	173	34.6	15.4
	F	25	178	35.6	-10.6
	Sat	30	182	36.4	-6.4
	Sun	45	192	38.4	6.6
3	W	32	201	40.2	-8.2
	T	60	208	41.6	18.4
	F	34	-	-	-
	Sat	37	-	-	-
	Sun	-	-	-	-

Question 4 (continued)

b) (ii)

Week \ Day	W	T	F	Sat	Sun
1	–	–	-10.6	-4.2	6.8
2	-6.2	15.4	-10.6	-6.4	6.6
3	-8.2	18.4	–	–	–
Total	-14.4	33.8	-21.2	-10.6	13.4
Average	-7.2	16.9	-10.6	-5.3	6.7
Adjustment	$-\frac{0.5}{5} = -0.1$				
Average daily variation, S	-7.3	16.8	-10.7	-5.4	6.6

Σ avg. $= 0.5 \neq 0$

(iii) Average change per time period $= \frac{41.6 - 30.6}{10 - 1} = \frac{11}{9}$

Week 3, Sunday:

$T_{est} = 41.6 + 3\left(\frac{11}{9}\right) = 45.2667$

$S = 6.6$

$Y_{est} = 45.2667 + 6.6$

$= 51.87$ (RM)

9