AAMS3244 (30 Apr 2021)

Q1 (a) continuous, pdf

"prob = $\int$ (area)"

$$f(x) = \begin{cases} kx^3, & 0 < x \le 1 \\ 0, & \text{otherwise} \end{cases}$$ defined range

(i) Total prob = 1

$\int_{-\infty}^{\infty}$ $\quad \int_0^1 kx^3\,dx = 1 \Rightarrow \left[\frac{1}{4}kx^4\right]_0^1 = 1$

$\frac{k}{4}(1^4 - 0^4) = 1 \Rightarrow k = 4$ (shown)

(ii) $\mu = \int_{-\infty}^{\infty} x f(x)\,dx$

$= \int_0^1 4x^4\,dx = \left[\frac{4}{5}x^5\right]_0^1 = \frac{4}{5}$

(iii) $\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2 = \int_0^1 4x^5\,dx - \left(\frac{4}{5}\right)^2$

$= \left[\frac{4}{6}x^6\right]_0^1 - \frac{16}{25} = \frac{2}{3} - \frac{16}{25} = \frac{2}{75} \Rightarrow \sigma = \sqrt{\frac{2}{75}} = 0.1633$

b) normally dist., mean. std dev

take note on the unit of measurement

X : driving time from home to office

$X \sim N(40, 5^2)$ use the stat table attached during FOA

i) prob

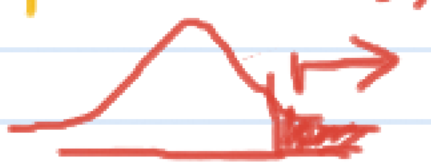
App 1

(1) $P(X > 50) = P(Z > \frac{50-40}{5}) = P(Z > 2) = 0.02275$

(2) $P(35 < X < 45) = P(-1 < Z < 1) = 1 - 2P(Z > 1) = 1 - 2(0.1587) = 0.6826$

ii) Let k = Min. time of 10% of the longest trip.

$P(X > k) = 0.1 \Rightarrow P(Z > \underbrace{\frac{k-40}{5}}_{+ve}) = 0.1$ ← App 2



$\frac{k-40}{5} = 1.2816 \Rightarrow k = 46.41$ minutes

c) Poisson, average 12 per hr, find prob in 10 min.

X : no of public cabs that stop at the cabstand per 10 minute

$X \sim Po(12/6 = 2)$

$P(X \geq 2) = 1 - P(X < 2) = 1 - \overbrace{P(X=0) - P(X=1)} = 1 - e^{-2}(1+2) = 0.594$

Q2 a) survey → sample : $n=15$, $\bar{x}=80$, $s=15$

CI for all → population.

Let μ : true population mean score of job fit for all project managers

99% confidence interval for μ is

$$\bar{x} \pm t_{0.005,14} \frac{s}{\sqrt{n}}$$

$$\Rightarrow 80 \pm 2.977 \frac{15}{\sqrt{15}} = [68.4701, 91.5299]$$

Note: If the var. is specified in any unit, pls state the unit.

b) $n_A = 400$, $\bar{x}_A = 7.2$, $S_A = 0.5$; $n_B = 400$, $\bar{x}_B = 8$, $S_B = 0.4$

Test mean, different (2-tailed)

Let μ_i : true population mean satisfaction index for Campus i, $i = A, B$

$H_0: \mu_A = \mu_B \Rightarrow H_0: \mu_A - \mu_B = 0$

$H_1: \mu_A \neq \mu_B \qquad H_1: \mu_A - \mu_B \neq 0$

2-tailed test

question

conclude

σ_A, σ_B ×

$n_A, n_B > 30$

$\alpha = 0.02$. Critical value $= \pm Z_{0.01} = \pm 2.3263 \Rightarrow$ critical region:

$Z > 2.3263$ or

$Z < -2.3263$

cont Q2(b)

Test statistic, $Z = \dfrac{\bar{x}_A - \bar{x}_B}{\sqrt{\dfrac{s_A^2}{n_A} + \dfrac{s_B^2}{n_B}}} = \dfrac{7.2 - 8}{\sqrt{\dfrac{0.5^2}{400} + \dfrac{0.4^2}{400}}} = -24.9878$ (use the same no of dp as c.v)

Since $Z = -24.9878 < -2.3263$, H_0 is rejected at $\alpha = 0.02$.

H_1

Hence, the mean satisfaction indexes for all students for the two campuses are different.

(c) left-handed (qualitative) → $x_M = 20$, $x_F = 15$, $n_M = 50$, $n_F = 60$

Test rate → proportion : $p_M > p_F$ claim 1-tailed test

Let p_i = true population proportion of left-handed among i-group, i = M(male), F(female)

conclude → $H_0: p_M \le p_F \Rightarrow H_0: p_M - p_F \le 0$

$H_1: p_M > p_F \qquad H_1: p_M - p_F > 0$ right-tailed

Cont Q1(c)

$\alpha = 0.01$, critical value $= +Z_{0.01} = 2.3263$

critical region : $Z > 2.3263$

"pooled"

$$\hat{p} = \frac{20+15}{50+60} = \frac{35}{110} \quad , \hat{q} = \frac{75}{110}$$

Test statistic, $$Z = \frac{\hat{p}_M - \hat{p}_F}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_M}+\frac{1}{n_F}\right)}} = \frac{\frac{20}{50}-\frac{15}{60}}{\sqrt{\frac{35}{110}\left(\frac{75}{110}\right)\left(\frac{1}{50}+\frac{1}{60}\right)}} = 1.6818$$

Since $Z = 1.6818 < 2.3263$, H_0 is failed to reject at $\alpha = 0.01$.

H_0

Hence, the rate of left-handedness among males is not more than females, i.e. the claim is not true.

Q3 a) $n=600$, χ^2 → association / r/s btw 2 qualitative var.
freq given

indep / no r/s

H_0: There is no association between the gender and marital status.

H_1: There is a significant association between gender and marital status.

$\alpha = 0.05$, $df = (2-1)(3-1) = 2$, critical value $= 5.991$.

critical region: $\chi^2 > 5.991$

O_{ij} (E_{ij})	S	MA	Ot	Total
M	82 (79.1667)	239 (228.6333)	59 (72.2)	380
F	43 (45.8333)	122 (132.3667)	55 (41.8)	220
T	125	361	114	600

$$\chi^2 = \sum\sum \frac{(O_{ij}-E_{ij})^2}{E_{ij}} = \frac{(82-79.1667)^2}{79.1667} + \cdots + \frac{(55-41.8)^2}{41.8} = 8.1402$$

Since $\chi^2 = 8.140 > 5.991$, H_0 is rejected at $\alpha = 0.05$. and we can conclude that there is a significant association between gender and marital status for those who hold more than one job.

b) mass, normally. $\mu = 800$, $\sigma = 25$

X : mass of oranges, $X \sim N(800, 25^2)$

n = 50 (sample). prob/prop of {sample mean}

$\bar{X} \sim N\left(800, \frac{25^2}{50}\right)$

std error

$P(\bar{X} < 790) = P\left(Z < \frac{790-800}{25/\sqrt{50}}\right) = P(Z < -2.83) = P(Z > 2.83)$

$= 0.00233$

c)

	P_0	q_0	P_1	q_1	P_0q_1	P_0q_0	P_1q_1
(i) X	100		120				
Y							

Simple → "one"

compared to 2019.

i) $I_X = \frac{120}{100} \times 100 = 120$

ii) L QI $= \frac{\sum P_0 q_1}{\sum P_0 q_0} \times 100 = \frac{8400}{18000} \times 100 = 46.67$

base

<100

Interpret

The quantity of the 2 types of toy cars had been decreased about 53.33% on average In 2020 as

iii) PPI $= \dfrac{\sum q_1 \quad p_1}{\sum q_1 \quad p_0} \times 100 = \dfrac{10280}{8400} \times 100 = 122.38$

current

...price ...... increased about 22.38% on average 2020 ... 2019.

Q4 a) X - IV , Y - DV

If ask for interpretation
strength, direct, form
how the change of IV/DV

(i) $r = \underline{\qquad} = 0.9376$

(ii)

	J	F	M	A	M	J	J
r_x	2.5	4	1	5.5	5.5	2.5	7
r_y	2	4	1	5	6	3	7
d							
d^2							

$\Sigma d^2 = 1$

$r_s = \underline{\qquad} = 0.9821$

iii) electricity charges (Y) ON mthly output (X)

$b = \underline{\qquad} = 0.9367$

$a = \underline{\qquad} = 3.8825$

$Y' = 3.8825 + 0.9367X$

iv) $X = 6$

$Y' = 3.8825 + 0.9367(6)$

$= 9.5027$ (RM'000)

b) 3 weeks, each wk has 5 days

season = 5 day

Wk	Day	Sales (RM'00)	5-Day Moving Total	(i) 5-day moving average, T	(ii) Y-T
					additive
1	S		–	–	–
	M		–	–	–
	T	9	57	11.4	-2.4
	W		58	11.6	2.4
	T		59	11.8	4.2
2	S		57	11.4	-2.4
	M		58	11.6	-0.6
	T		62	12.4	-5.4
	W		63	12.6	2.4
	T		65	13	7
3	S		72	14.4	-4.4
	M		73	14.6	-4.6
	T		71	14.2	-0.2
	W		–	–	–
	T		–	–	–

(ii)

Wk	S	M	T	W	T
1	–	–	Y	Y	Y
2	Y	Y	Y	Y	Y
3	Y	Y	Y	–	–
Total	−6.8	−2.2	−8	4.8	11.2
Average	−3.4	−1.1	−2.6667	2.4	5.6
Adj	+				
S	−3.5667	−1.2667	−2.8334	2.2333	5.4333

1.3t

Y − T

total = 0.8333 ≠ 0

Adj: $-0.8333/5 = -0.1667$

total −0.0002

adj = total / 5 ≈ 0.0000

(iii) mean increment in Trend = $\frac{14.2 - 11.4}{11-1} = 0.28$

M, W4 $T_{est} = 14.2 + 4(0.28) =$ ____

$Y_{est} =$ ____ $+ (-1.2667) = 14.0533$ (RM'00)