Assignment 3

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Kruskal Proof

Proof:

Let the minimum spanning tree of a graph G be T*

Suppose that Kruskal's algorithm produces T.

Assume first, that the two trees are different, and the lightest edge in T* which is not in T is e.

Adding e to T will create a **cycle** (let's call it C)

- Why? Because T is a spanning tree, and adding any edge to it will create a cycle.

There must be an edge in C that is not in T*. Why? (let's call this edge f)

This is because T* is a spanning tree and does not contain any cycles. When you create a cycle by adding edge e to T, at least one edge in that cycle must not be in T* which is edge f.

Removing f from T will create a new spanning tree. Why? (let's call this new spanning tree T1)

It is because when we added e to T, it created a cycle and since f is one of the path in the cycle, removing f will break the cycle and form the new Minimum Spanning Tree.

T1 will be more similar to T* than T. But is the total weight of T1 less than T ?

Suppose that it is, this means that e < f. (a comparing operation) (e is less than f)

If e was really so, would it be picked by Kruskal's algorithm before f ?

Yes, if it was, but it is impossible, if e was really less than f then why would Kruskal picked f in the first place and not e. Kruskal's algorithm always selects edges in ascending order of weight.

Therefore, the fact must be that e >= f. b (e is more than or equal to f)

Therefore, we can conclude that weight of T1 >= T. (Weight of T1 is more than or equal to weight of T)

Continually transforming T1 further in the same fashion until the resulted spanning tree becomes T*.

Will the transformed spanning tree has less total weight than T? Why?

No, the transformed spanning tree(T1) will not have less weight than T even though we keep transforming T1 further. With every e and f, it is impossible that e is < f, because it that is true then e would have been picked by Kruskal the first time, it would not have pick f instead. The only possible explanation is that T1 is greater than or equal to T.

But the total weight of T* is the minimum possible (by assumption). Therefore, the only possible conclusion is the total weight of T **is equal to** the total weight of T*. PROVED!