1) [10 Marks] Capital-centered Internet plan

A country has *n* cities. The government has decided to establish communication network that provides the Internet to all these cities. At first, the main server is built at city 0, the capital. The other cities should be connected to this main server via high-speed lines.

For any cities *i*, *j*, it is possible to build a high-speed line between them in c_{ij} million dollars. Because of the difficulty to build high-speed line in some area, only *m* candidate lines are being considered. These *m* potential lines are more than adequate to cover the entire country, but building all of them is redundant. The country is in crisis after a civil war, so the government decided to build only a few high-speed lines. Of course from every city there must be a path along the lines to the capital.

Find the minimum possible cost to build all necessary high-speed lines.

INPUT:

1st line : the number of cities *n*, $2 \le n \le 100$, and the number of high-speed lines between pairs of cities, *m*, $n \le m \le 2{,}500$

Each of the following *m* lines consists of city *i*, city *j*, and cost c_{ij}, $0 \le i, j \le n$-1

OUTPUT: The minimum possible cost to build all necessary highs-speed lines

EXAMPLE

INPUT	OUTPUT

4 4
0 3 15
1 2 20
1 3 24
2 3 17
	52

2) [10 Marks] Square-Matrix Transformation

In this question, you are given a solution program for Square-Matrix Transformation; matrixTrans.py. However, the program utilizes a recursive bruteforce technique and thus can only handle a small matrix. The time limit of 1 second, excluding the input and output time as shown in the code, will be exceeded when the matrix dimension size, n, is more than 5.

INPUT:

1st line : the square matrix dimension size, n, $3 \leq n \leq 140$

Each of the following n lines list rows of the matrix from top to bottom. Each row consists of n *positive* integers representing elements in the row, ordered by column.

Every $A[i][i]$ of matrix A is 0

OUTPUT: The transformed matrix

Your task for this question is to transform the matrix into a more efficient version so that it can handle a square matrix of size up to 140.

Use the recursive program to verify the correctness of your code.

3) [10 Marks] Sinking Ship

Lupin is training himself with simulation of a crisis scenario. This time, the crisis he simulated is sinking cruise ship.

A gigantic cruise ship is hit an underwater iceburg and is sinking. Water is flooding the bilge and passengers are evacuating to life boats. Lupin the thief, however, finds this to be the time that he can take valuable things of others without being caught. He has the map of all the valuable things he has targeted in his laptop computer. The captain announces that the ship is sinking at the rate of one floor per 2 minutes. During the trip, Lupin has installed a special climbing machine in a vertical duct that allow him to get to any of the passenger floor very fast. However, provided that the floor is still dry *when he reaches the floor*, it will take him 2 minutes to go from any position in the duct into any floor, run to the targeted valuable thing, grab it and return to the duct, even while water is filling up the floor.

The water will just reach the bottom floor right when Lupin starts his stealing. Lupin has to plan on which valuable things he can take. Lupin is keen on computer programming and he has a good idea. He quickly creates a program that helps him decide which valuable thing he has to go for at each two minutes, in such a way that the sum of values he takes will be the maximum possible.

Write a program that determine such the maximum possible total value that Lupin can take.

INPUT:

1st line : the number of floors n, $10 \le n \le 1000$
Each of the following n lines represents each floor from the top down to the bottom. Each line contains a list of integers representing values of things in the respective floor. There are at most 1000 valuable things in each floor. The value of each thing is at most $10000.

OUTPUT: The maximum total value that Lupin can take.

EXAMPLE

INPUT	OUTPUT
5 4 46 **56** 44 **52** 29 29 25 **54** 2 **55** 30 11 20 46 33 11 5 29 5 18 51 15 **68**	285

Note: the bold values are taken by Lupin

4) [10 Marks] Typing jobs

Somchai needs to finish typing a lot of documents during this weekend. Contents of all the documents are well prepared and drafted but need to be typed beautifully in a word processor. There are exactly N number of documents to be submitted to his boss on Saturday night and another N number of documents to be submitted on Sunday night. Realizing he cannot himself type all the documents on time, he decides to hire N number of professional typists to type the documents for him.

So each typist needs to type one document on Saturday and another one on Sunday. Of course, each document has different number of pages to be typed. Somchai will pay each typist minimum of 2,000 Baht for the two days of typing work. However, if the total number of pages a typist needs to type during the two days exceeds T pages, he will pay the typist extra R Baht per page for each extra page after the first T pages. Somchai wishes to minimize the extra payments he may have to make for the extra pages of typing.

Somchai needs to find a way of assigning each typist one Saturday typing job and one Sunday typing job so that total extra payments he may have to make is minimized.

INPUT:

1st line: Three integers 1 <= N <= 100, 1 <= T <= 10000, and 1 <= R <= 10, as described above, in the order given, separated by white space.

2nd line: N positive integers separated by a white space, each integer (guaranteed to be <= 10000) denoting the number of pages to be typed for each typing job to be done on Saturday. (The integers are NOT in any particular order)

3rd line: N positive integers separated by a white space, each integer (guaranteed to be <= 10000) denoting the number of pages to be typed for each typing job to be done on Sunday. (The integers are NOT in any particular order)

OUTPUT:

A single line of output containing an integer denoting the minimum possible extra payments Somchai must pay.

EXAMPLE

INPUT	**OUTPUT**

2 30 5
20 15
15 20
	50

3 400 10
40 20 30
50 80 30
	0

3 5 1
1 4 7
3 1 2
	4

5) [10 Marks] Largest Cloud

As a part of environmental management and local weather forecast, an array of cameras is placed around the city to keep track of the sky image.

At each fixed interval, a snapshot of the camera will be processed by the information extraction program. One of the required features is to detect the size of the largest cloud in the snapshot image. Given that the image is already processed into a matrix of black/white pixels. The program is required to report the number of pixels occupied by the largest cloud in the image.

A pixel is considered connected to an adjacent pixel only in one of the 4 directions, which are up, down, left, and right.

Write the program that report the size of the largest cloud in the given image.

INPUT:
1st line: The number of rows, $M \leq 500$, and the number of columns, $N \leq 1000$, of the image.

Each of the following M lines list rows of the image from top to bottom. Each row consists of N pixels ordered by column. Each pixel is either 0 (sky) or 1 (cloud).

OUTPUT: The size, in number of pixels, of the largest cloud in the image

EXAMPLE

INPUT	OUTPUT
4 6 0 **1** 0 0 0 0 **1 1 1** 0 0 0 0 0 **1** 0 1 1 0 0 **1** 0 1 0	6

Note: The largest cloud consists of the bold pixels.

6) [10 Marks] M3Tile version II (huge length)

The original M3Tile problem is described at https://www.spoj.com/problems/M3TILE/.

A concept for solving M3Tile can be a simple one, which is in fact a refined reduction of the method listed in worksheet 7.

$$f(L) = \begin{cases} 4f(L-2) - f(L-4) & ; L \text{ } is \text{ } even \\ 0 & ; \text{ } L \text{ } is \text{ } odd \end{cases}$$

$$f(0) = 1$$

$$f(2) = 3$$

INPUT: An integer, the length of path way, L ; $0 \leq L \leq 10^{20}$

OUTPUT: The number of possible tiling patterns for length L, modulo by 44711.

EXAMPLE

INPUT	OUTPUT
4999998	28306
4999999998	36159

NOTE: Only one test case will have $L \leq 5{,}000{,}000$

HINT: According to the following fact, f_L can be obtained by computing M^k.

$$\begin{bmatrix} f_L \\ f_{L-2} \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} f_{L-2} \\ f_{L-4} \end{bmatrix}$$

$$\begin{bmatrix} f_L \\ f_{L-2} \end{bmatrix} = M \begin{bmatrix} f_{L-2} \\ f_{L-4} \end{bmatrix}$$

$$\begin{bmatrix} f_L \\ f_{L-2} \end{bmatrix} = M^2 \begin{bmatrix} f_{L-4} \\ f_{L-6} \end{bmatrix}$$

$$\begin{bmatrix} f_L \\ f_{L-2} \end{bmatrix} = M^k \begin{bmatrix} f_2 \\ f_0 \end{bmatrix}$$

k is the number of time for f_{L-2} to reduce to f_2

7) [20 Marks] Minimal damage

Ironman is fighting the Chitauri forces (the villain army in The Avengers 2012). N attacking aircrafts of the Chitauri form a circle around Ironman. In each aircraft, there are a number of Chitauri soldiers. Ironman's repulsor can destroy three adjacent aircrafts per one shoot, which immediately kills all the soldiers in those aircrafts. However, after the shoot, the survival Chitauri soldiers also fire back at Ironman causing some damage to him — one unit per soldier. Ironman will further follow with new shoot and so on until all aircrafts are destroyed. It is required to define the minimum amount of damage, which can be dangerous to Tony Stark.

INPUT: N integers, amount of soliders on each consecutive aircraft (not less than 1 and no more than 100 on each). 3 ≤ N ≤ 20.

OUTPUT: The minimum amount of damage.

EXAMPLE

INPUT	OUTPUT
3 4 2 2 1 4 1	9

Elaboration:

1) Shoot the 2nd aircraft destroys 3+4+2, damage = 2+1+4+1 = 8
2) Shoot the 5th aircraft destroys 2+1+4, damage += 1 = 9

8) [10 Marks] Pairings For The Best Chance to Win

A university tennis team is pairing up for traditional inter-collegiate mixed-double event. A mixed-double match features a pair of tennis players competing against another pair, in which each pair comprises one male and one female tennis player.

In this traditional event, n matches will be played, simultaneously in n tennis courts. Therefore, each team must line up n mixed double pairs.

There is no information about how the line-up of the opposing team will be, therefore the coach can only try to maximize the chance of winning. He has the records of every pairing of his selected n male players and n female players. So he graded every pairing with a rating system; the lower rating value indicates the pair the won more, while the higher rating indicates the pair that lost more.

With the complete table of every pairing, the coach would like to determine the best possible pairings of the entire team. He defines such pairings as the one that has the minimum sum of the ratings of all the selected n pairs.

Write a program to determine the minimum possible sum of ratings for this university team.

Input:

1st line : the number of pairings required, n, $3 \leq n \leq 17$

Each of the following n lines consists of positive n integers in fixed order. Thus, these n lines form an $n \times n$ matrix, A, which describes pairings of n male players with n female players. The element in column j of row i, denoted A_{ij}, is the rating of pairing female player i with male player j.

Output: Minimum possible sum of ratings for n mixed-double pairings.

EXAMPLE

INPUT	OUTPUT
4 4 3 2 **1** 5 5 **1** 4 2 **1** 3 4 **2** 3 1 4	5

Note: the selected 4 pairings are indicated with the bold numbers.