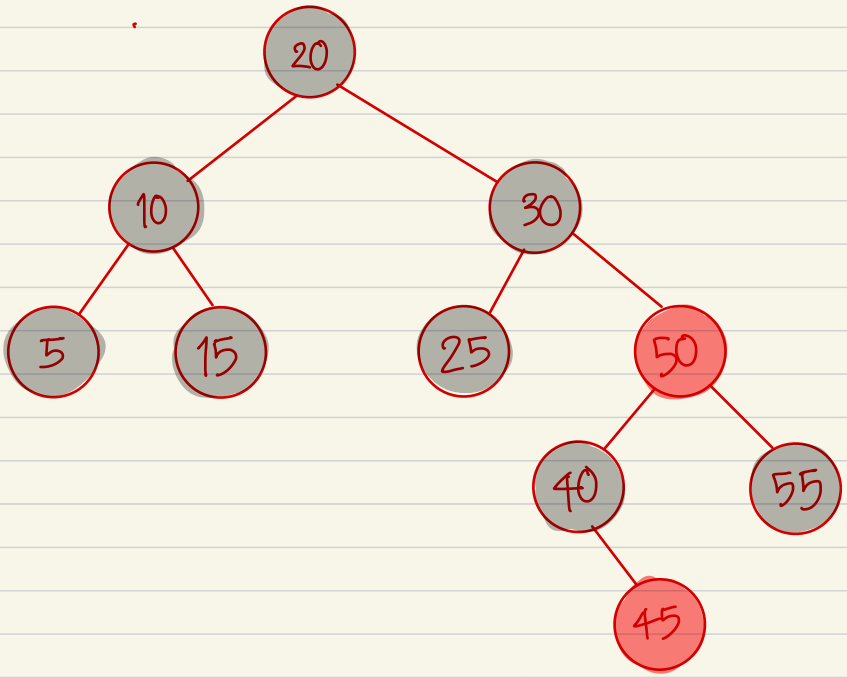
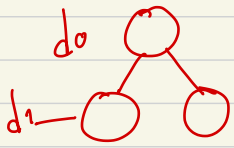
1

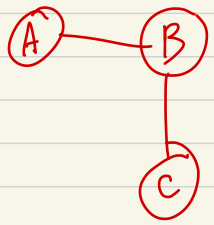




depth 0
0 indent

```
R---- 20(BLACK)
     L---- 10(BLACK)
     |     L---- 5(BLACK)
     |     R---- 15(BLACK)
      R---- 30(BLACK)
           L---- 25(BLACK)
           R---- 50(RED)
                 L---- 40(BLACK)
                 |     R---- 45(RED)
                 R---- 55(BLACK)
```

a)

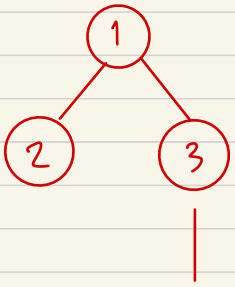


$|E| = 2$
$|V| = 3$

$|E| < |V|$

∴ Despite $|E| < |V|$, the graph is connected. False

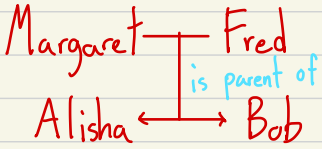
b)



← True, if add an edge, not a tree.

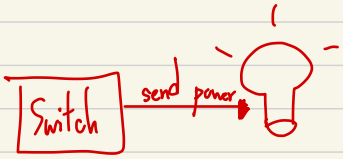
(Theory: a tree with n vertices must have exactly n-1 edges)

c)



True, Margaret is parent of Bob, but Bob is not parent of Margaret. Thus, directed Graph.

d) True

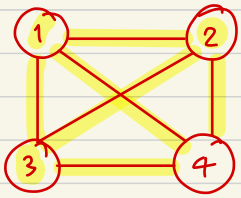


switch can send power to light bulb.
light bulb don't send power to switch.

$|V| = 4$

$3+2+1+0$

e)

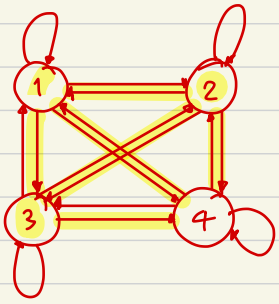


False. Figure has $|E| = 6$, $|V| = 4$, while $|V|^2 - |V| = 12$.

The $|V|^2 - |V|$ counts for duplicated edges

(1,2) (1,3) (1,4)
(2,1) (2,3) (2,4)
(3,1) (3,2) (3,4)
(4,1) (4,2) (4,3)

Directed Graph: If loop: $|V|^2$, Else: $|V|^2 - |V|$



In order to find the Max $|E|$, we have to count distinct edges, thus yeild the formula

$$|E|_{max} = \overbrace{(|V|-1) + (|V|-2) + \dots + 2 + 1 + 0}^{|V| \text{ terms}}$$

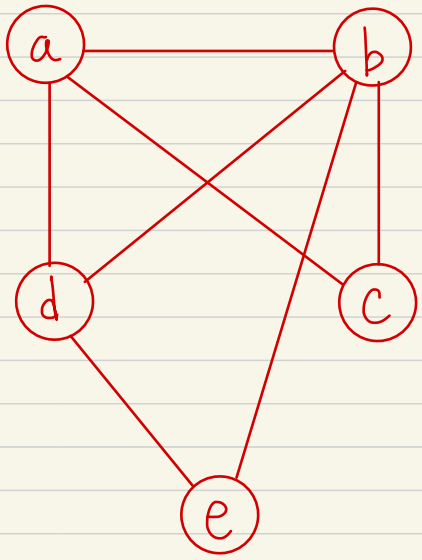
$$= \frac{|V|(|V|-1+0)}{2}$$

$$= \frac{(|V|^2 - |V|)}{2}$$

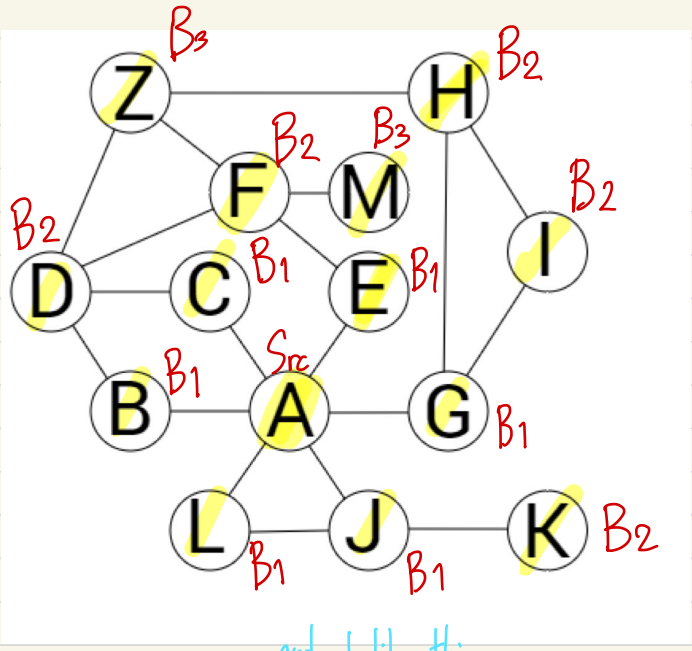
3.

```
----Adjacency List----
a -->  b c d
b -->  c d a e
c -->  a b
d -->  a e b
e -->  b d

----Adjacency Matrix----
   a  b  c  d  e
a  0  1  1  1  0
b  1  0  1  1  1
c  1  1  0  0  0
d  1  1  0  0  1
e  0  1  0  1  0
```



4. A



Vertices Reached order = [start] + B_1 + B_2 + B_3 (ordered like this)

start = A

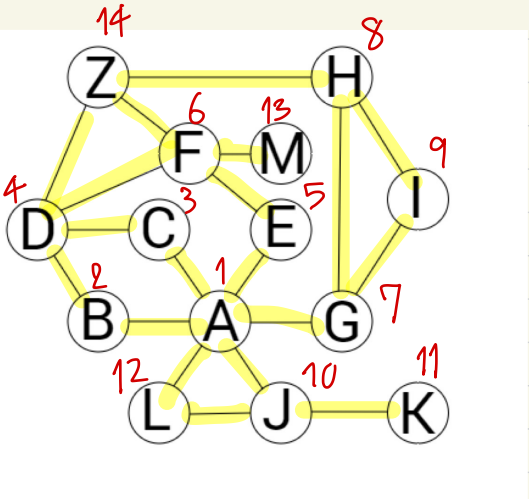
B_1 = [B, C, E, G, J, L]

B_2 = [K, D, F, I, H]

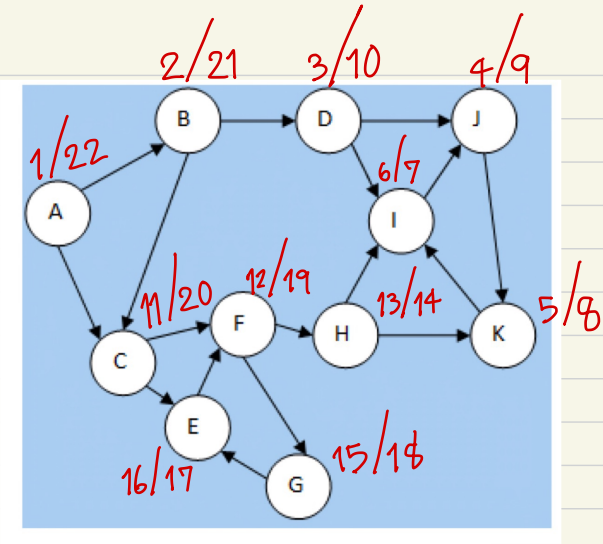
B_3 = [Z, M]

} order within lists does not matter

∴ One ex of Vertices Reached order is [A, B, C, E, G, J, L, K, D, F, I, H, Z, M]



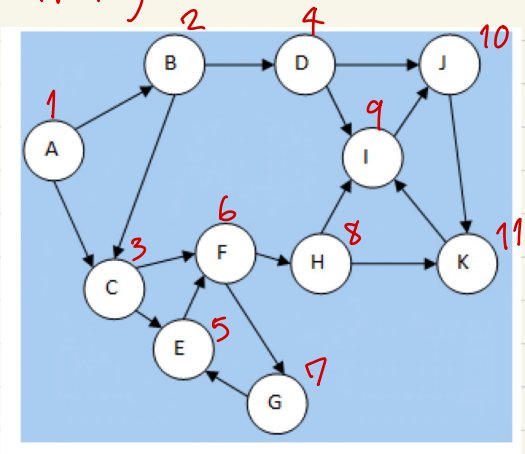
5



V

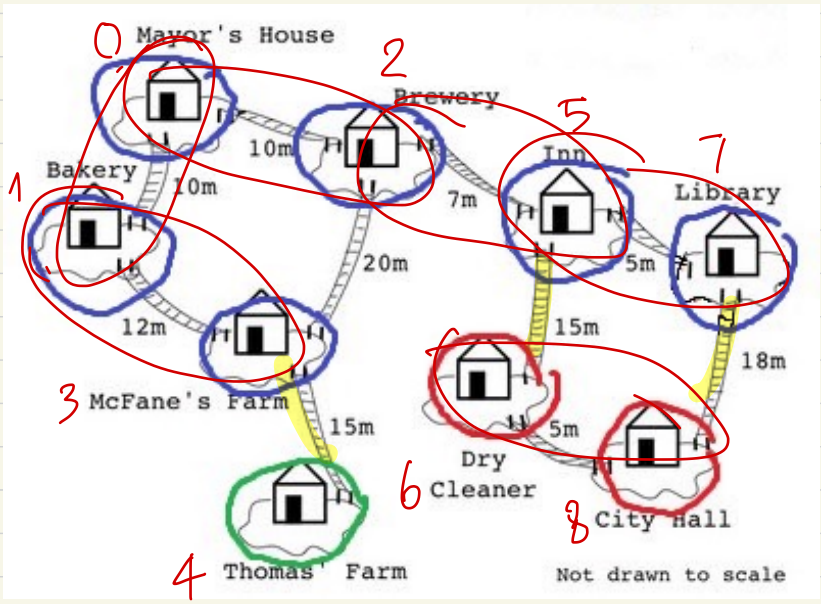
A	1	22
B	2	21
C	11	20
D	3	10
E	16	17
F	12	19
G	15	18
H	13	14
I	6	7
J	4	9
K	5	8

Hashing Vertices



setCount: 9

6



Edgelist | setCount

5 7 5	8
6 8 5	7
2 5 7	6
0 1 10	5
0 2 10	4
1 3 12	3
3 4 15	
5 6 15	
7 8 18	
2 3 20	

village.in

```
9 10
0 1 10
0 2 10
1 3 12
2 3 20
2 5 7
3 4 15
5 6 15
5 7 5
6 8 5
7 8 18
3
```

V | E | EdgeList | k

```
s = DisjointSets(V)
edgeList.sort(key = lambda a:a[2])
W = 0
edgeCount = 0
for u,v,w in edgeList:
    if s.findset(u) != s.findset(v):
        W += w
        s.union(u,v)
        edgeCount += 1
```

setCount = V

setCount -= 1

if setCount == k:
    break

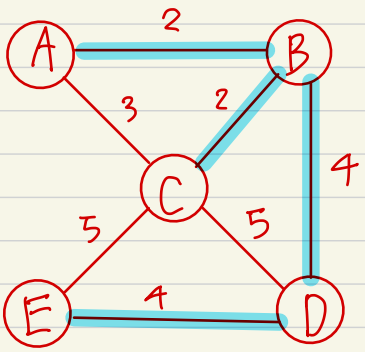
Smallest = 10^{100000}

for u,v,w in EdgeList:
if findset(u) != findset(v) and w < Smallest:
    Smallest = w

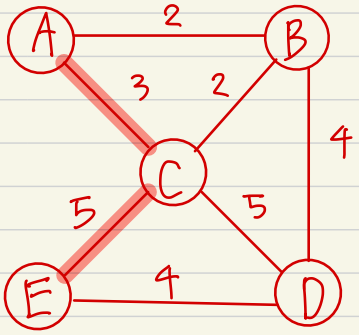
Answer = 15

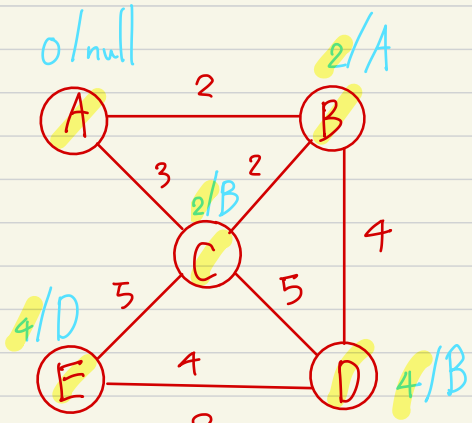
```
edgeList = []
V, E = map(int, input().split())
for i in range(E):
    edgeList.append(tuple(map(int, input().split())))
k = int(input())
```

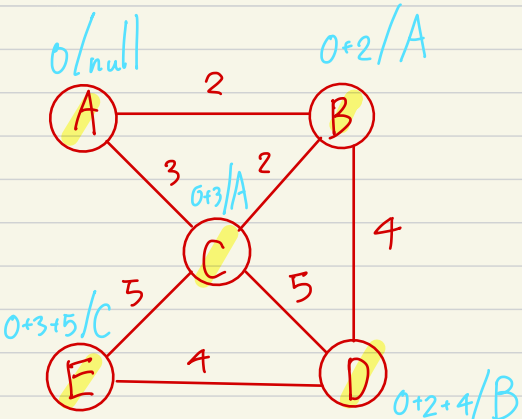
7. Prim's MST



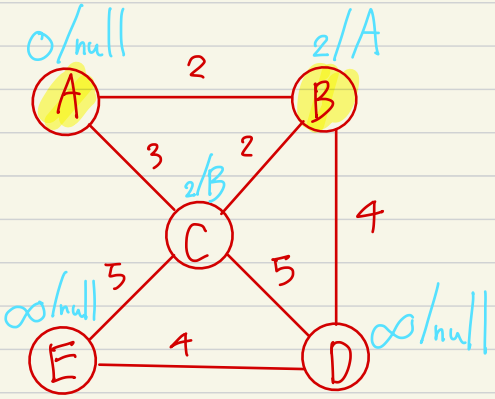
Dijkstra



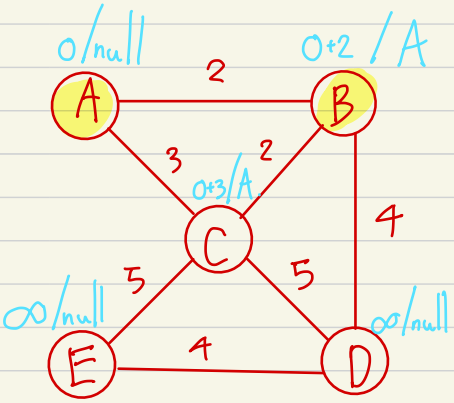


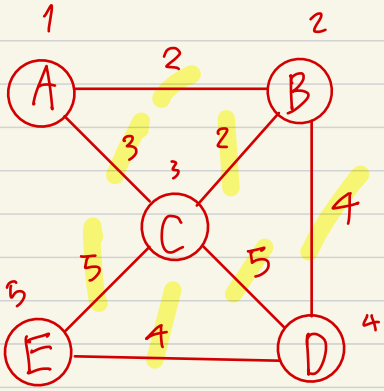


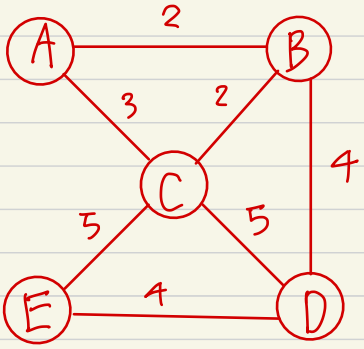
Prim



D.







ASSUMPTION UNIVERSITY

Vincent Mary School of Science and Technology
Department of Computer Science
Department of Information Technology

Final Examination
Semester 2/2021

Subject Code :	CSX3003/IT2230/ITX2010, BIS4787
Subject Title :	Data Structure and Algorithms, Data Structure
Date :	March 25, 2022
Time :	**13.00 – 16.00 (3 hours)**
Instructors :	Asst. Prof. Dr. Thitipong Tanprasert (Full-Time)

Instructions:

1. Read the questions carefully and answer each question completely, legibly, and concisely.
2. An answer to a question is either a text file, or docx file, or a Python 3 program.
3. To submit your answers, compress (zip) all of your answers into one file and upload the zipped file to the Final Examination created as an assignment for the course no later than 16:00 the latest. Any late submission received after 16:01 will not be graded.
4. This is an opened book examination; you can use any materials as references, including online search. However, any form of communication with anyone regarding the exam, directly or indirectly, will be considered "CHEATING".
5. You MUST turn on your camera and microphone, share your working screen (the entire screen of your PC), and record the video in MS Team for the whole examination period. The answered file will NOT BE GRADED IF THERE IS NO COMPLETELY RECORDED VIDEO CLIP.
6. This examination paper and recorded video are an intellectual property of Assumption University; you are NOT allowed to duplicate, share, or publicize it.
7. If you cheat or contribute to cheating at the exam, you will get zero score and will be considered to get the grade 'F' for this course.

Marking Scale:

Essay and/or Programming	7 questions	70 marks

Total 70 marks

Total 3 pages (including this page)

1) [10 marks] Show the Red-Black Tree resulted from the following sequence operations. At the beginning, the Red-Black Tree is empty.
   - Insert 5, 10,15, 20, 25, 30, 35, 40, 45, 50, 55
   - Delete 35

   You can illustrate the resulted Red-Black Tree in any format that is unambiguous. However, because the answer must be either a text file or docx file, if you choose to sketch in the paper and take a photo of it, you must insert the photo into a docx file for submission.

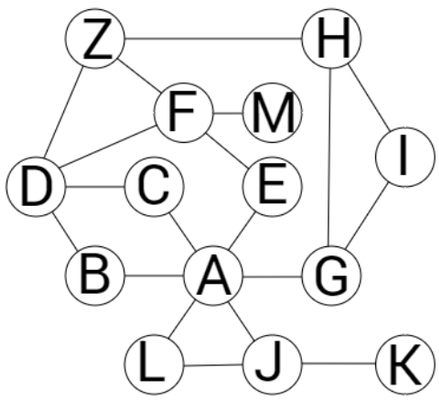
2) [10 marks] Given that a graph is defined by (V, E) where V is the vertex set and E is the edge set, indicate whether each of the following sentence is TRUE or FALSE (2 marks each).
   $|V|$ is the size of vertex set and $|E|$ is the size of edge set.

   a) If $|E| < |V|$, the graph is disconnected.
   b) Adding an edge to a tree makes the graph no longer be a tree.
   c) A family tree is a directed graph
   d) An electrical wiring of a house in a directed graph.
   e) For an undirected graph, the maximum possible value of $|E|$ is $|V|^2 - |V|$

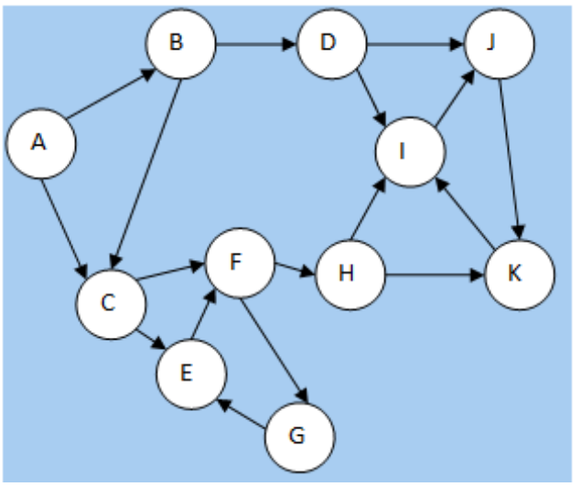
3) [10 marks] Given the following edge list, provide the corresponding adjacency list and adjacency matrix. The graph is undirected.

$$(a, b), (b, c), (c, a), (b, d), (d, a), (d, e), (e, b)$$

4) [10 marks] The Breadth-First Search is run on the graph below with vertex A being the source vertex. List out the vertices according to order that they are reached by the search. If a vertex is not reachable by the search, it must not be included in the list. There can be more than one correct order. List only one of them for the answer.



5) [10 marks] The Depth-First Search is run on the graph below. Give the discovery time and the finish time of each vertex. There are more than one correct searching orders. Answer only one of them.



6) [10 marks] Given a *connected* undirected graph $G = (V, E)$ where V is the vertex set and E is the edge set. A "k-clustering" is the grouping of vertices into k clusters such that only the vertices in the same cluster are connected (by a subset of E).

A *gap* between a pair of clusters is the weight of any edge that does not connect vertices of the same cluster.

A *maximally separated k-clustering* is the k-clustering whose smallest gap is the largest possible.

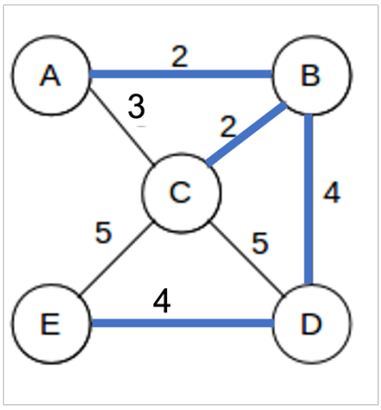
Write a Python 3 program that reads an edge list and k as input and prints the smallest gap' weight of maximally separated k-clustering as output.

Hint Utilize the Kruskal's algorithm. Stop the algorithm when the number of sets is k.

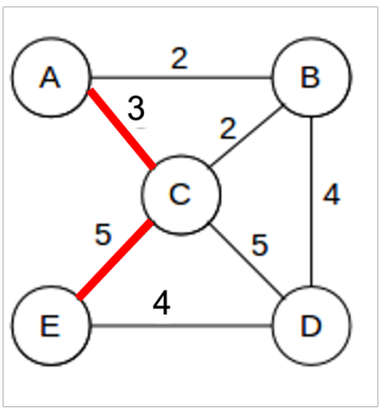
An example input file and the corresponding output is provided with this exam.

7) [10 marks] Prim's algorithm and Dijkstra's algorithm have the same algorithmic structure. Although a minimum spanning tree is an undirected graph, Prim's can be run on a directed graph. Similarly, Dijkstra's single-source shortest path algorithm can be run on an undirected graph as well.

However, if both algorithms are run on the same graph, using the same source vertex, the shortest path from the source vertex to another vertex, according to Dijkstra's algorithm, is not necessarily a path on the minimum spanning tree that is generated by Prim's algorithm.
The figure below illustrates such a scenario.



Prim's Minimum Spanning Tree



Shortest path from A to E

Identify what makes the selected set of edges of the two algorithms different.