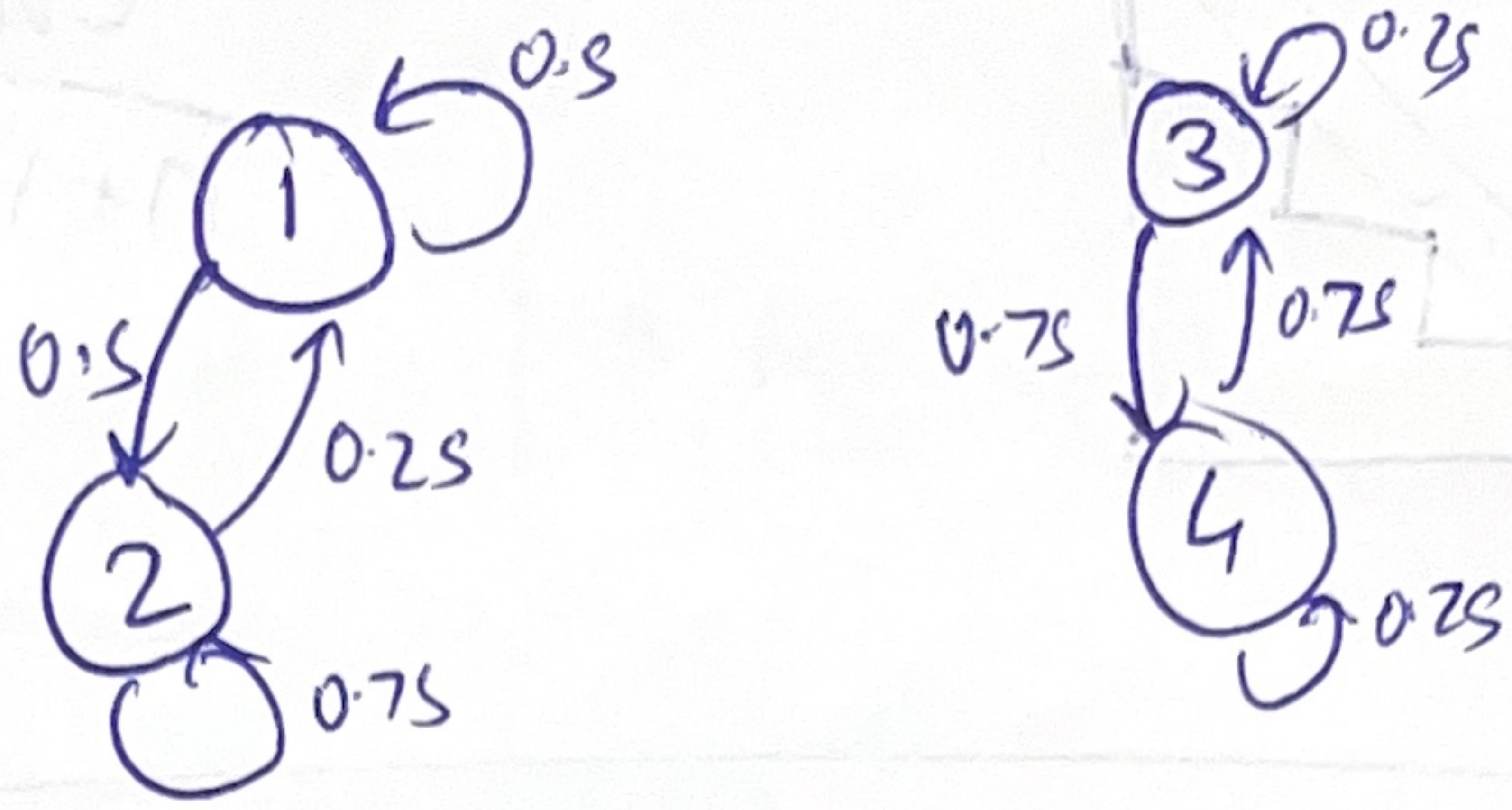
# SMFD Assignment 2.1

1. Two State Loop

$\{1\,2\,3\,4\}$




a) $$Q = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0.5 & 0.5 & 0 & 0 \\ 2 & 0.25 & 0.75 & 0 & 0 \\ 3 & 0 & 0 & 0.25 & 0.75 \\ 4 & 0 & 0 & 0.75 & 0.25 \end{array}$$

b) States 1 & 2 }
States 3 & 4. } Two independent closed communicating class.

Recurrent state
↳ once entered;
$P=1$ of returning to it.

Both (1,2) & (3,4) are closed classes.
⇒ all 4 states are recurrent

no transient states.
cannot leave its class & never return

c) Stationary distribution $\pi$ satisfies $\pi Q = \pi$.

→ $\pi$ (states 1 & 2) [3 & 4 set to 0]

Let $\pi = [a, b, 0, 0]$ & $a+b=1$;

$$\begin{bmatrix} a = 0.5a + 0.25b \\ b = 0.5a + 0.75b \end{bmatrix} \quad \begin{array}{l} 0.5a = 0.25b \\ 2 \\ 3a = 1 \\ a = \frac{1}{3};\ b = \frac{2}{3} \end{array}$$

$$\pi^{(1)} = \left[\frac{1}{3}, \frac{2}{3}, 0, 0\right]$$

→ $\pi$ (states 3 & 4) [1 & 2 = 0]

$\pi(0, 0, c, d)$ & $c+d=1$

$c = 0.25c + 0.75d$
$d = 0.75c + 0.25d$
$c = d = 0.5$

$$\pi^{(2)} = \left[0, 0, \frac{1}{2}, \frac{1}{2}\right]$$

their convex combination also stationary

$$\left(\alpha\pi^{(1)} + (1-\alpha)\pi^{(2)},\ 0 \leq \alpha \leq 1\right)$$

2. Winning Streak

W — 0.8W
W — 0.2L
L — 0.3W
L — 0.7L

W — 0.7D
L — 0.2D

a) $P(\text{win}) = \underline{(0.8)\,P(\text{win}) + (0.3)\,(1 - P(\text{win}))}$

$x = 0.8x + 0.3 - 0.3x$

$0.5x = 0.3$

$x = \frac{3}{5} = 0.6$

$\underline{P(\text{win}) = 0.6}$ ✓

b) $P(\text{dinner}) = P(\text{dn}/\text{lost})P(\text{lost}) + P(\text{dn}/\text{win})P(\text{win})$ $\cancel{\frac{P(\text{dn} \cap \text{lost})}{P(\text{lost})} + \frac{P(\text{dn} \cap \text{win})}{P(\text{win})}}$

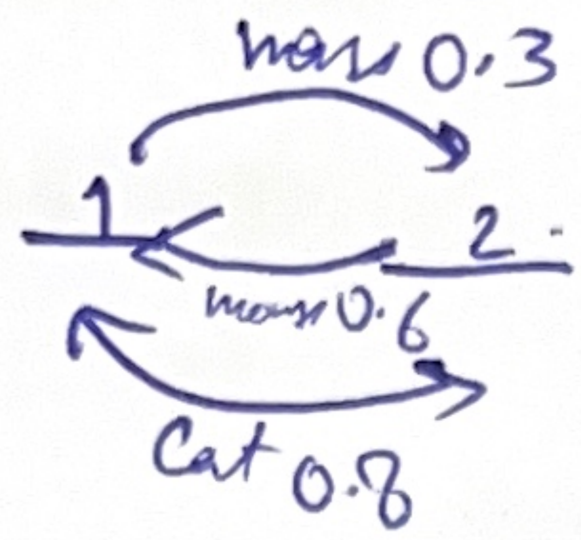
$= \cancel{\frac{(0.2)}{0.4} + \frac{0.7}{0.6} = \frac{1}{2}}$

$= (0.2)(0.4) + (0.7)(0.6) = \begin{array}{r} 0.08 \\ +0.42 \end{array} = \underline{\underline{0.5}}$

c) #games = ?

$P(\text{dinner}) \to 1 \Rightarrow \frac{1}{P(\text{dinner})} = 2$ games

3. Cat & Mouse Game




moving 0.1 game
Cat 0.2 game.

a)

$$C = \begin{array}{c|cc} \text{room} & 1 & 2 \\ \hline 1 & 0.2 & 0.8 \\ 2 & 0.8 & 0.2 \end{array} \qquad M = \begin{array}{c|cc} \text{from} \backslash \text{to} & 1 & 2 \\ \hline 1 & 0.7 & 0.3 \\ 2 & 0.6 & 0.4 \end{array}$$

Cat chain

$\pi = [p_1, p_2]$

$\pi C = \pi$

$p_1 = 0.2p_1 + 0.8p_2$

$p_2 = 0.8p_1 + 0.2p_2$.

$p_1 + p_2 = 1 \quad p_1 = p_2 = 0.5$. ✓

$\pi_C = [0.5, 0.5]$

mouse chain

$\pi [q_1, q_2]$

$\pi C = \pi$

$q_1 = 0.7q_1 + 0.6q_2$

$q_2 = 0.3q_1 + 0.4q_2$

$q_1 + q_2 = 1 \quad \cancel{0.3}q_1 = 0.6q_2$

$q_1 = \frac{2}{3} \quad q_2 = \frac{1}{3}$ ✓

$\pi_M = \left[\frac{2}{3}, \frac{1}{3}\right]$

b)

4 possible (M, C) states: (1,1), (1,2), (2,1), (2,2)

$Z_n \Rightarrow$ no of current state at time n $\Rightarrow$ 4 states.

$\Rightarrow$ M, C move independently

$\Rightarrow$ (M, C) joint process is Markov chain

P(next state) @ current state

NOT @ past.

## 4. Wandering King

Classification: 4 corner sqs.
24 edges.
36 inner.

| | | #moves. | | |
|---|---|---|---|---|
| from | C | 3 | ×4 | → 12 |
| | E | 5 | ×24 | → 120 |
| | I | 8 | ×36 | → 288 |
| | | | | 420 (moves) |

stationary probabilities by type.

$$\pi_{corner} = \frac{3}{420} = \frac{1}{140} \quad \times 4$$

$$\pi_{edge} = \frac{5}{420} = \frac{1}{84} \quad \times 24$$

$$\pi_{interior} = \frac{8}{420} = \frac{2}{105} \quad \times 36$$

$$\Sigma = 1$$

→ portfolio rebalancing.
→ credit risk modeling
→ market regime analysis

Detailed balancing principle:
→ risk-neutral valuation for derivative pricing
→ Monte Carlo simulations; option pricing models.
→ eqlbm models in asset pricing theory.

## 5. Stock Price Model

tick size | NSE $\Delta$ = Rs 0.01 increments only.

10am — 3pm. (5hrs)
↓
₹120

t = 5s
↑ P(0.1) +0.01
~ P(0.85)
↓ P(0.05) −0.01

a) $E(\text{change}) = 0.1(+1^{tick}) + 0.85(0) + 0.05(-1)$
$= \underline{0.05} > 0.$
expected drift.

→ transient random walk

↗ does not return to starting 100% ⇒ not recurrent.

⇒ growth stock (bull market)

5. a) No, the stock price is not recurrent

$\therefore P(\text{moving up}) = 0.1$

$P(\text{moving down}) = 0.05$   $0.1 > 0.05$

So chain has a upward drift.

Probability to return back at starting price decrease over time

So, Process is transient

b) NO, a stationary distribution does not exist for this process.

c) Time window $= 180\text{min} = 10{,}800\,\text{sec}$

No. of steps $= \frac{10{,}800}{5} = 2160$ steps

$P_{up} = 0.1$   $P_{stay} = 0.85$   $P_{down} = 0.05$

To achieve 130 Rs from Rs 120
1000 ticks up within 2160 steps

Expected change per step:

$\Delta = 0.1 \times 0.01 + 0.85 \times 0 + 0.05 \times -0.01$
$= 0.0005$ per step

expected total change $= 2160 \times 0.0005 = 1.08$

expected price at 1:00pm $= 121.08$

Probability to achieve 130 is $\simeq 0$