2. Part A

a) $S_0 = 100$, strike $k = 105$, 10 days

S_T {Terminal price} $= 100 + 2K - 10$

K = number of up move

Solve $100 + 2K - 10 > 105$

$K > 7.5 \Rightarrow \boxed{K \geq 8}$

Probability.

$$P(K \geq 8) = \sum_{K=8}^{10} {}^{10}C_K (0.5)^{10}$$

$$(0.5)^{10}\left[{}^{10}C_8 + {}^{10}C_9 + {}^{10}C_{10}\right]$$

$$= \boxed{0.05468}$$

b) Expected payoff

Payoff $= \max(S_T - 105, 0)$

$$\text{Expected payoff} = \sum_{K=0}^{10} \max(100 + 2K - 10 - 105, 0) \cdot {}^{10}C_K (0.5)^{10}$$

$$= 0.078125$$

c) Fair value (No discounting)

= Expected payoff = \$0.078125

Part B:

a) $E[|x|] = \sigma\sqrt{2/\pi} = 1$

$$\sigma_{daily} = \frac{1}{\sqrt{2/\pi}} = \sqrt{\frac{\pi}{2}} \simeq 1.2533$$

Scaled to 10 days

$$\sigma_{10} = \sigma_{daily}\sqrt{10} \approx 3.9633$$

b) $E[\max(S_T - K, 0)] = \int_K^{\infty} (S-K) f_{S_T}(S)\, dS$

$f_{S_T}(S) = \frac{1}{\sigma_{10}\sqrt{2\pi}} \exp\left(-\frac{(S-100)^2}{2\sigma_{10}^2}\right)$

Probability density function

$\sigma_{10} = 3.2633$

Part C

a) $E[|x|] = \int_a^b |x| f(x)\, dx = \int_{-b}^{b} |x| \cdot \frac{1}{2b}\, dx$

$a = -b$

~~b-a=2c~~

$= \frac{b}{2}$ $\quad f(x) = \frac{1}{b-a}$

$E[|x|] = 1 \quad b = 2$

$x \sim \text{Uniform}[-2, 2]$

b) Binomial: Discrete, symmetric, variance $= 10 \times 1 = 10$

Normal: continuous, symmetric, variance ≈ 15.71

Uniform sum:

Daily variance $= \frac{(2-(-2))^2}{12} = \frac{4}{3}$

10-day variance $= 10 \times \frac{4}{3} \approx 13.33$

c) Simulate daily moves:

for each day $i = 1$ to 10, Sample $x_i \sim \text{Uniform}[-2, 2]$

Compute terminal price: $S_T = 100 + \sum_{i=1}^{10} x_i$

Compute payoff: $\max(S_T - 105, 0)$

Average over trials:

Run $N = 10{,}000$ iteration

Fair value $\approx$ mean of payoffs