Question Paper (Matrices and Calculus) 2018-19

Matrices And Calculus (Babu Banarasi Das University)

Sl. No. 483 **BAS 2101**

No. of Printed Pages : 6

Following Paper ID and Roll No. to be filled in your Answer Book.	
PAPER ID : 29901	Roll No.

B. Tech. Examination 2018-2019

(Frist Semester)

MATRICES AND CALCULUS

Time : Three Hours] *[Maximum Marks : 60*

Note :– Attempt all questions.

Section-A

Note:- Attempt all parts. 1×8=8

1. (a) Find the rank of the matrix

$$A=\begin{bmatrix}1 & 2 & 3 & -4\\ -2 & 3 & 7 & -1\\ 1 & 9 & 16 & -13\end{bmatrix}$$

(b) If $A=\begin{bmatrix}2 & 3\\ 0 & 1\end{bmatrix}$ then find the eigen values of A^2 + A + I.

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(c) If $u = 2x + 3y$, $v = x - 3y$, then find $\frac{\partial v}{\partial x}$ and $\frac{\partial x}{\partial v}$.

(d) If $y = \frac{1}{1-x}$ then find y_{10}.

(e) Evaluate $\int_0^1 \int_0^{x^2} e^{y/x} dy dx$.

(f) Show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

(g) Find the unit normal to the surface $xy^3z^2 = 4$ at point $(-1, -1, 2)$.

(h) Determine a and b such that the vector field $\vec{A} = (2xy + 3yz)\hat{i} + (x^2 + axz - 4z^2)\hat{j} - (3xy + byz)\hat{k}$ is irrotational.

Section-B

Note:– Attempt any two parts 2×6=12

2. (a) Find the characteristic equation of matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

and verify that it is satisfied by A and hence obtain A^{-1}.

(b) Use the method of the Lagrange's multipliers to find the volume of the largest rectengular parallelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.

(c) Change the order of integration in

$$I=\int_0^1\int_{x^2}^{2-x} xy\,dy\,dx$$

and hence evaluate the same.

(d) Show that $\iint_S \vec{F}.\hat{n}ds=\frac{3}{2}$, where

$\vec{F}=4xzi-y^2\hat{j}+yz\hat{k}$ and S is the surface of the cube bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$.

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Section-C

Attempt any two parts from each question 5×8=40

3. (a) Find the eigen values and eigen vectors of the matrix

$$\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}.$$

(b) Test the consistency of following system of linear equations and hence find the solution $4x - y = 12$, $-x + 5y - 2z = 0$ and $-2y + 4z = -8$.

(c) Find the rank of the matrix

$$A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 4 & 1 & 2 & 1 \\ 3 & -1 & 1 & 2 \\ 1 & 2 & 0 & 1 \end{bmatrix}$$

by reducing it into Normal form,

4. (a) If $y = (\sin^{-1} x)^2$, find $(y_n)_0$.

(b) Verify Euler's theorem for $Z = \dfrac{x^{1/3} + y^{1/3}}{x^{1/2} + y^{1/2}}$.

(c) If $x + y + z = 4$, $y + z = 4v$, $z = 4\ vw$ then show that $\frac{\partial(x,y,z)}{\partial(4,v,w)} = u^2 v$.

5. (a) Show that $\overline{|n}\,\overline{|1-n} = \frac{\pi}{\sin n\pi}$ $(0 < n < 1)$.

(b) Change into polar coordinates and evaluate

$$\int_0^a \int_0^{\sqrt{a^2-x^2}} e^{-(x^2+y^2)} dy\, dx$$

(c) Find the volume of the cylindrical column standing on the erea common to the parabolas $y^2 = x$, $x^2 = y$ and cut off the surface $z = 12 + y - x^2$.

6. (a) Find the directional derivative of $\phi = 5x^2y - 5y^2z + \frac{5}{2}z^2x$ at the point p (1,1,1) in the direction of the line $\frac{x-1}{2} = \frac{y-3}{-2} = \frac{z}{1}$.

(b) Show that gradient field describing a motion is irrotational.

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(c) Using Green's theorem evaluate $\int_c (x^2 y dx + x^2 dy)$, where c is the boundary described counter clockwise of the triangle with vertices (0,0) (1,0), (1,1).

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