BABU BANARASI DAS UNIVERSITY (Set-B)

Department of Mathematics & Computer Science

B. Tech. [Ist Year], 2nd Sessional Test (AS: 2021-22)

Subject Code: BAS3201 (Differential Equations and Fourier Analysis)

Time: 3:00 Hrs Maximum Marks: 60

Section-A

Q.1 Attempt all parts of the following: $8 \times 1 = 8$ 5

a. Find the general solution of $(D^2 - 3D + 4)y = 0$.

b. Find the particular integral of $\frac{d^2y}{dx^2} - y = x^2$.

c. Evaluate: $\int_{-1}^{1} P_3^2(x)dx$.

d. Express J_4 in terms of J_2 and J_1.

e. If $f(x) = k$ is expanded in Fourier sine series in $(0, \pi)$ then find the value of b_n.

f. Find the constant term if the function $f(x) = x^3$ is expanded in Fourier series defined in $(-1, 1)$.

g. Form the partial differential equation from $z = x + y + f(xy)$, by eliminating the arbitrary function.

h. Classify the partial differential equation $x^2\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = u$, where $x > 0$.

Section-B

Q.2 Attempt any two parts of the following: $2 \times 6 = 12$

a. Solve by method of variation of parameters: $\frac{d^2y}{dx^2} + y = \sec x$.

b. Find the power series solution of $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = 0$ about $x = 0$.

c. Obtain the Fourier series to represent $f(x) = x^2$ in the interval $0 \le x \le 2\pi$. Hence, show that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots = \frac{\pi^2}{12}$.

d. Find the temperature in a bar of length 2 whose ends kept at zero and lateral surface insulated if the initial temperature is $\sin\frac{\pi x}{2} + 3\sin\frac{5\pi x}{2}$.

Section-C

Q.3 Attempt any two parts of the following: $2 \times 5 = 10$

a. Solve the differential equation $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 3y = x^2 \log x$.

b. Solve the differential equation $x^2y'' - (x^2 + 2x)y' + (x + 2)y = x^3e^x$ given that $y = x$ is a solution.

c. Obtain the general solution of the differential equation $(D^2 - 2D + 2)y = x + e^x \cos x$.

Q.4 Attempt any two parts of the following: $2 \times 5 = 10$

a. Prove that: $xJ_n' = -nJ_n + xJ_{n-1}$.

b. Prove that: $\int_{-1}^{1}[P_n(x)]^2\,dx = \frac{2}{2n+1}$.

c. Show that: $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\ \cos x$.

Q.5 Attempt any two parts of the following: $2 \times 5 = 10$

a. Find the Fourier series for $f(x) = \begin{cases} x, & -\pi < x < 0 \\ -x, & 0 < x < \pi \end{cases}$

b. Expand the function $f(x) = x\cos x$ as a Fourier series in interval $-\pi \le x \le \pi$.

c. Find half-range Fourier cosine series of $f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 2(2 - x), & 1 < x < 2 \end{cases}$

Q.6 Attempt any two parts of the following: $2 \times 5 = 10$

a. Solve: $(D^2 + 2DD' + D'^2)z = \sin(2x + 3y)$

b. Solve: $(D + D' - 2)(D + D' - 3)z = e^{3x-2y}$

c. Using method of separation of variables, solve: $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, where $u(x, 0) = 6e^{-3x}$