Following Paper ID and Roll No. to be filled in your Answer Book.	
PAPER ID : 39912	Roll No.

B. Tech. Examination 2023-24

(Odd Semester)

COMPLEX ANALYSIS AND INTEGRAL TRANSFORMS

Time : Three Hours]   *[Maximum Marks : 60*

Note :– Attempt all questions.

SECTION–A

1. Attempt all parts of the following : $8 \times 1 = 8$

   (a) Define an analytic function.

   (b) Find the value of m, if the function

   $$2x - x^2 + m y^2$$

   be harmonic.

   (c) State Liouville theorem.

(d) Define an isolated singularity of the function with example.

(e) Find the Laplace transform of $(t-1) \cup (t-1)$.

(f) Find the inverse Laplace transform of $\dfrac{1}{s^{1/2}}$.

(g) State the Fourier integral theorem.

(h) Write the Z-transform of a sequence $\{f(k)\}$.

SECTION – B

2. Attempt any two parts of the following : $2\times6=12$

(a) If $u - v = (x - y)(x^2 + 4xy + y^2)$ and $f(z) = u + iv$ is an analytic function of $z = x + iy$, find $f(z)$ in terms of z by Milne-Thomson method.

(b) Evaluate :
$$\int_0^{2\pi} \frac{1}{5 - 4\sin\theta}\, d\theta$$

by contour integration in the complex plane.

(c) Solve :
$$y'' + 4y' + 4y = 6e^{-t}$$
$$y(0) = -2,\ y'(0) = 8$$

using Laplace transform method.

(d) Find the inverse Z-transform of :

$$\frac{1}{(z-3)(z-2)}$$

(i) $2 < |z| < 3$

(ii) $|z| > 3$

SECTION – C

Note :– Attempt all questions. Attempt any two parts from each question. $\qquad 5 \times 8 = 40$

3. (a) Using the Cauchy-Riemann equations, show that $f(z) = z^3$ is analytic in the entire plane.

(b) Find the value of the integral :

$$\int_0^{1+i} \left(x - y + i x^2\right) dz$$

from $z = 0$ to $z = 1$ and then along a line parallel to the imaginary axis from $z = 1$ to $z = 1 + i$.

(c) Use Cauchy integral formula to evaluate :

$$\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$$

Where C is the circle $|z| = 3$.

[P. T. O.