

No. of Printed Pages: 06

Following Paper ID and Roll No. to be filled in your Answer Book.

PAPER ID: 39901 Roll No.

B. Tech. Examination 2022-23

(Special Carry Over Paper)
MATRICES AND CALCULUS

Time: Three Hours | [N

[Maximum Marks: 60

Note :- Attempt all questions.

SECTION-A

1. Attempt all parts of the following:

 $8 \times 1 = 8$

(a) If the matrix

$$A = \begin{bmatrix} 1+i & 3-5i \\ 2i & 5 \end{bmatrix}$$

find (Au).

(b) Find latent roots of matrix:

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$$

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5. (a) Prove that:

(i)
$$\sqrt{\frac{1}{2}} = \sqrt{\pi}$$

(ii)
$$\int_0^1 \left(\log \frac{1}{y} \right)^{n-1} dy = \ln \left($$

(b) Evaluate:

$$\int_0^\infty \int_0^\infty e^{-\left(x^2+y^2\right)} dx dy$$

by changing polar co-ordinates. Hence, show that:

 $\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$

- (c) Calculate the volume of the solid bounded by the surface x = 0, y = 0, x + y + z = 1 and z = 0.
- 6. (a) Find the directional derivative of the function $\phi = x^2 y^2 + 2z^2$ at the point P (1, 2, 3) in the direction of the line PQ where Q is the point (5, 0, 4).
 - (b) A fluid motion is given:

$$\overline{V} = (y + z) i + (z + x) j + (x + y) k$$

show that the motion is irrotational and hence find the velocity potential.

order.

Order.

What is switch case statement? wny

What is switch suitable example.

Explain with suitable example.

(c) If
$$u = \sin^{-1} \frac{x}{y} + \tan^{-1} \frac{x}{y}$$

then find the value of:

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$$

- (d) Find nth derivative of log x2.
- (e) Prove that:

$$B(m+1, n) = \frac{m}{m+n} B(m, n)$$

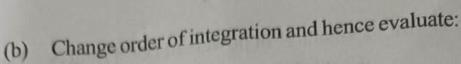
(f) Evaluate:

$$\int_0^1 \int_0^x e^{y/x} dx dy$$

- (g) Find a unit normal vector to the surface $x^2 + 3y^2 + 2z^2 = 6$ at point (2, 0, 1).
- (h) If $f = 2 x^2 3 y^2 + 4 z^2$, find the value of curl (grad f).

SECTION-B

- 2. Attempt any two parts of the following: $2 \times 6 = 12$
 - (a) State Stoke's theorem and evaluate $\oint_C \overline{F} \cdot d\overline{r}$ by Stoke's theorem, where $\overline{F} = y^2 i + x^2 j (x + z)$ K and C is the boundary of triangle with vertices (0, 0, 0), (1, 0, 0) and (1, 1, 0).



$$\int_0^a \int_{\sqrt{ax}}^a \frac{y^2}{\sqrt{y^4 - a^2 x^2}} dx dy$$

(c) Use the method of the Lagrange's multipliers to find the volume of the largest rectangular parallelopiped that can be inscribed in the ellipsoid:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

(d) If $y = \tan -1 x$, prove that:

$$(1 + x^2) y_{n+2} + (2 n + 1) \times y_{n+1} + n (n + 1) y_n = 0$$

hence, determine the values of all the derivatives of y w.r.t. x using x = 0.

SECTION-C

Note: Attempt all questions. Attempt any two parts from each question. 5×8=40

3., (a) If $x^x y^y z^z = c$. Show that at:

$$x = y = z$$
, $\frac{\partial^2 z}{\partial x \partial y} = -(x \log e x)^{-1}$

- (b) Expand x^y in powers of (x-1) and (y-1) upto the third degree terms.
- (c) Examine $f(x, y) = x^3 + y^3 3$ a xy for maximum and minimum values.
- 4. (a) Reduce the matrix A to its normal form, when:

$$A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ -1 & -2 & 6 & -7 \end{bmatrix}$$

0)

hence, find the rank of A.

Tests for consistency and solve the following system of equations:

$$5x+3y+7z=4$$

 $3x+26y+2z=9$
 $7x+2y+11z=5$

(c) Find the eigen values and the corresponding eigen vectors for the following matrix:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 3 & 2 & 3 \end{bmatrix}$$

- 5. (a) Prov
 - (i)
 - (ii)
 - (b) Eva

by

- (c) Ca
- 6. (a) Fi
 - d
 - (b)