SET-B

Name: Student University Roll No.:	Printed Pages:

School of Engineering
Second Sessional Examination, Even Semester (AS: 2022-23)
B. Tech: All Branch [Year: First] [Semester: II]

Course Title: Differential Equations and Fourier Analysis Course Code: BAS3201	Max Marks: 60 Time: 3*hrs*

Instructions if any: Read the question Carefully.

	SECTION 'A' **Q.N.1. Attempt all parts of the following:**	CO	Marks
a)	Find the order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 = \left[1+\left(\frac{dy}{dx}\right)^2\right]^3$	1	1
b)	Find the particular integral of $\frac{d^2y}{dx^2} + y = sinx$	3	1
c)	Find the values of α and β for which $3x^2 = \alpha P_2(x) + \beta P_0(x)$.	7	1
d)	Show that $x = 1$ is a singular point of $(x-1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} + 2y = 0.$	7	1
e)	Write Dirichlet's conditions for a Fourier series.	8	1
f)	Find the constant term if the function $f(x) = x$ is expanded in Fourier series defined in $(-1, 1)$.	8	1
g)	Form the partial differential equation from $z = (x+a)(y+b)$ by eliminating the arbitrary constants a and b.	11	1
h)	Classify the partial differential equation $\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$	12	1
	SECTION 'B' **Q.N.2. Attempt any two parts of the following:**	CO	Marks
a)	Apply method of variation of parameters to solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \frac{e^x}{1+e^x}$	6	6
b)	Find the power series solution of $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$ about $x = 0$.	7	6
c)	Given that $f(x) = x + x^2$ for $-\pi < x < \pi$, Find the Fourier series expansion of $f(x)$. Deduce that $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots\ldots\ldots$	8	6
d)	Find the temperature in a bar of length 2 whose ends kept at zero and		

	lateral surface is insulated if the initial temperature is $\left[\sin(\frac{\pi x}{2}) + 3\sin(\frac{5\pi x}{2})\right]$	10	5
	SECTION 'C'	11	Marks
Q.N.3. Attempt any two parts of the following:			
a)	Solve $(3x+2)^2\frac{d^2y}{dx^2} - (3x+2)\frac{dy}{dx} - 12y = 6x$	5	5
b)	Solve $(D^2 - 4D + 3)y = 2xe^{3x}$	3	5
c)	Solve the simultaneous differential equations $\frac{dx}{dt} = 3x + 8y$ $\frac{dy}{dt} = -x - 3y$ with $x(0) = 6$ and $y(0) = -2$	4	5
Q.N.4. Attempt any two parts of the following:			
a)	Prove that $xJ_n' = nJ_n - xJ_{n+1}$	7	5
b)	Prove that $\int_{-1}^{1}[P_n(x)]^2\,dx = \frac{2}{2n+1}$	7	5
c)	Prove that $J_{1/2}(x) = \sqrt{\left(\frac{2}{\pi x}\right)}\sin x$	7	5
Q.N.5. Attempt any two parts of the following:			
a)	Find the half Range Fourier sine series of $f(x) = \begin{cases} x, & 0 < x < 2 \\ (4-x), & 2 < x < 4 \end{cases}$	8	5
b)	Expand the function $f(x) = x\cos x$, as a Fourier series in the interval $-\pi < x < \pi$.	9	5
c)	Find the half Range Fourier cosine series of $f(x) = \begin{cases} x, & 0 < x < \frac{\pi}{2} \\ (\pi - x), & \frac{\pi}{2} < x < \pi \end{cases}$	8	5
Q.N.6. Attempt any two parts of the following:			
a)	Solve $\left(D^2 + 2DD' + D'^2\right)z = \sin(2x+3y)$	3	5
b)	Solve $\frac{\partial u}{\partial x} = 3\frac{\partial u}{\partial t}$ using method of separation of variables.	10	5
c)	Solve $(D + D' - 1)(D + D' - 2)z = e^{2x-y}$	11	5