Machine Learning I: Foundations

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Due: 10/05/2019 - 10 am

Basic intructions

- Groups from 1 to 3 students. From the Sheet 3 you **CAN'T** change your group.
- Each student needs to submit the answers on moodle (same file for all members of the group).
- You must submit a ZIP file with, at least:
  - PDF generated from your LaTeX(dont sent the LaTeXcode!);
  - Source files from coding exercises (e.g. File.py);
  - A "README.txt" file with the execution instructions.
- All plots and/or figures generated in coding exercises must be added in the *LaTeX* PDF. We strongly recommend that you add your source code as well. It can help you during the oral examination.

Exercises Sheet 2

1. In this question we will be exploring eigenvectors and eigenvalues. Let $A \in \mathbb{R}^{n\times n}$. Recall that a vector $\mathbf{v} \in \mathbb{R}^n$ is an *eigenvector* of A if there exists $\lambda \in \mathbb{R}$ such that $\lambda\mathbf{v} = A\mathbf{v}$, and we call λ an *eigenvalue* of A, associated with the vector $\mathbf{v}$. If we have some eigenvector $\mathbf{v}$ of A note that $a\mathbf{v}$ is also trivially an eigenvector for any $a \in \mathbb{R}$. Because of this and a are not considered different eigenvectors: i.e. colinear eigenvectors are not considered distinct. Prove the following:

   Proposition 1. If A has a finite number of distinct eigenvectors then each eigenvector must have a distinct eigenvalue.

2. Let A a symmetric matrix in $\mathbb{R}^{n\times n}$. A is called positive definite if for all $\mathbf{x} \in \mathbb{R}^n$

   $$\mathbf{x}^\top A\mathbf{x} > 0\,.$$

   (a) Prove that the eingenvalues of A are positive if A is positive definite.

   (b) Prove that if the eingenvalues of A are positive then A is positive definite.
   Consider the function $F : \mathbb{R}^2 \longrightarrow \mathbb{R} : (x, y) \mapsto F(x, y) = x^2 + 2y^2 + 4.97$.

   (c) Find the gradient vector ∇F.

   (d) Find the critical point of F.

   (e) Let $\mathbf{H}$ be the Hessian matrix of F. Find $\mathbf{H}$.

   (f) Note that $\mathbf{H}$ is symmetric. If $\mathbf{H}$ is positive definite in a critical point then it is a local minimum. Show that the critical point of item (d) is a local minimum.

3. In this exercises we will implement the Nearest Centroid Classifier (NCC) (Slide 13 - Lecture 2) for the Disneyland dataset (see Exercises Sheet 1).

   (a) Make a function to calculate the centroids.

   (b) Then use these centroids to calculate the linear classifier $f(\mathbf{x}) = (\mathbf{w}^\top\mathbf{x} + b)$.

   (c) Make a scatter plot where the x-axis is the height of the citizens and the y-axis is the weight of the citizens. The color of the points need to be different for males and females. In the same figure, plot the linear classifier calculated in item (b)

   (d) Use the dataset "DWH_Training.csv" to train your model and the dataset "DWH_Test.csv' to check the accuracy. Compare the NCC accuracy with the accuracy of the models of Exercises Sheet 1. **Attention:** you need to code the algorithm. Don't use any library that implements the Nearest Centroid Classifier.