Machine Learning I: Foundations

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Placement math test

1. Let S be the vector space of polynomials of degree less than or equal to 2. Given the polynomials a $= x^2 + 5$, b $= 3x^2 + 4x + 27$ and c $= x + 3$. Are a, b and c linearly independent?

2. Consider the following matrix $\mathbf{A}$:

$$\mathbf{A} = \begin{bmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{bmatrix}.$$

   (a) Find the characteristic polynomial of $\mathbf{A}$.

   (b) Diagonalize the matrix $\mathbf{A}$.

3. Consider the following matrix $\mathbf{A}$:

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{bmatrix}.$$

   (a) Is the matrix $\mathbf{A}$ singular? Why?

   (b) What is the rank of $\mathbf{A}$?

   (c) If it is possible, find the inverse of $\mathbf{A}$.

4. Consider the function $F : \mathbb{R}^2 \longrightarrow \mathbb{R} : (x, y) \mapsto F(x, y) = x^2 + 2y^2 + 4.97$.

   (a) Find the gradient vector ∇F.

   (b) Find the critical point of F.

   (c) Let $\mathbf{H}$ be the Hessian matrix of F. Find $\mathbf{H}$.

   (d) Show if the critical point of item (b) corresponds to a local maximum, a local minimum or a saddle point.

5. (a) Find A_{23}.

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \times \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

   (b) Find B^{10}.

$$\mathbf{B} = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$