Problem Reduction Search: AND/OR Graphs & Game Trees

Problem Reduction Search

- Planning how best to solve a problem that can be recursively decomposed into sub-problems in multiple ways
  - Matrix multiplication problem
  - Tower of Hanoi
  - Blocks World problems
  - Theorem proving

Formulations

- **AND/OR Graphs**
  - An OR node represents a choice between possible decompositions
  - An AND node represents a given decomposition
- **Game Trees**
  - Max nodes represent the choice of my opponent
  - Min nodes represent my choice

The AND/OR graph search problem

- **Problem definition:**
  - **Given: [G, s, T] where**
    - G: implicitly specified AND/OR graph
    - S: start node of the AND/OR graph
    - T: set of terminal nodes
    - h(n) heuristic function estimating the cost of solving the sub-problem at n
  - **To find:**
    - A minimum cost solution tree

Algorithm AO*

1. Initialize: **Set $G^* = \{s\}$, $f(s) = h(s)$**

   If $s \in T$, label s as SOLVED
2. Terminate: **If s is SOLVED, then Terminate**
3. Select: **Select a non-terminal leaf node n from the marked sub-tree**
4. Expand: **Make explicit the successors of n**

   For each new successor, m:

   Set $f(m) = h(m)$

   If m is terminal, label m SOLVED
5. Cost Revision: **Call cost-revise(n)**
6. Loop: **Go To Step 2.**

Cost Revision in AO*: cost-revise(n)

```
1. Create Z = {n}
2. If Z = { } return
3. Select a node m from Z such that m has no
   descendants in Z
4. If m is an AND node with successors
   $r_1, r_2, \ldots r_k$:
      Set $f(m) = \Sigma\ [\, f(r_i) + c(m, r_i)\, ]$
      Mark the edge to each successor of m
      If each successor is labeled SOLVED,
         then label m as SOLVED
```

Cost Revision in AO*: cost-revise(n)

5. If m is an OR node with successors $r_1, r_2, \ldots r_k$:
   - Set $f(m) = \min \{ f(r_i) + c(m, r_i) \}$
   - Mark the edge to the best successor of m
   - If the marked successor is labeled SOLVED, label m as SOLVED
6. If the cost or label of m has changed, then insert those parents of m into Z for which m is a marked successor
7. Go to Step 2.

Searching OR Graphs

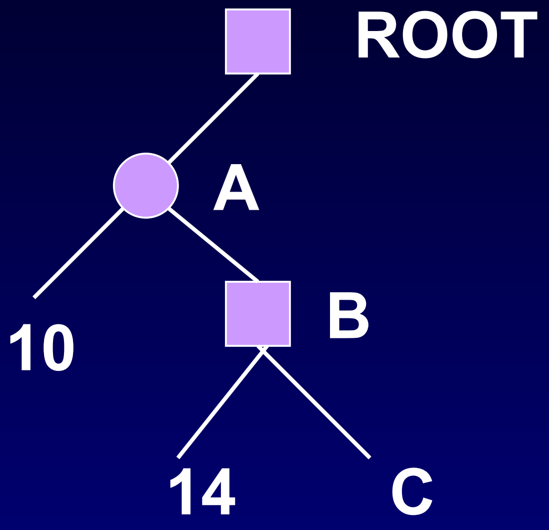
- How does AO* fare when the graph has only OR nodes?

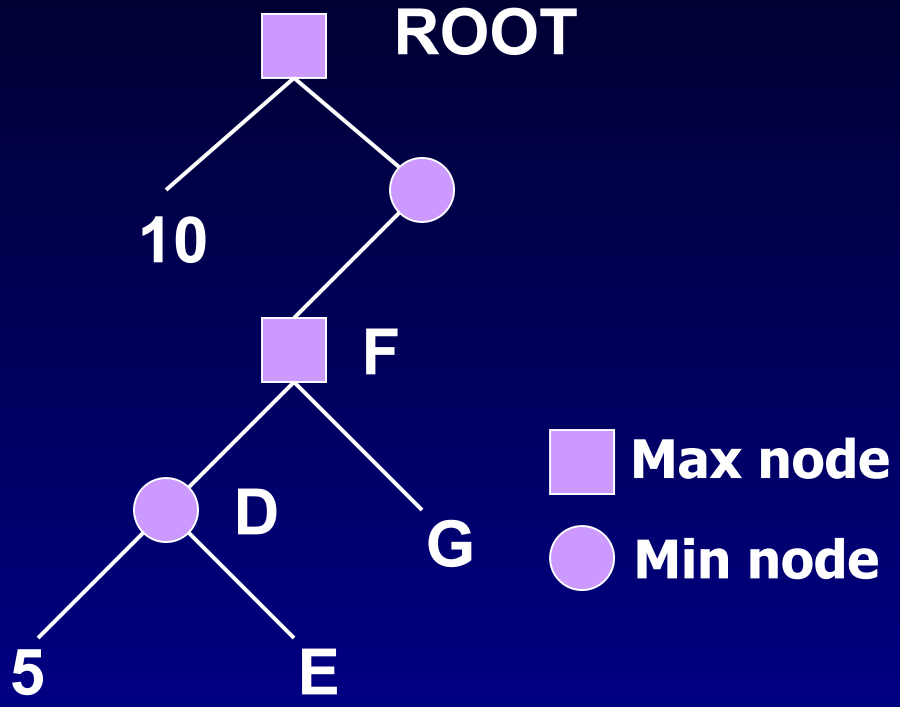
Searching Game Trees

- Consider an OR tree with two types of OR nodes, namely Min nodes and Max nodes
- In Min nodes, select the min cost successor
- In Max nodes, select the max cost successor

- Terminal nodes are winning or loosing states
  - It is often infeasible to search up to the terminal nodes
  - We use heuristic costs to compare non-terminal nodes

Shallow and Deep Pruning





Shallow Cut-off

Deep Cut-off

Alpha-Beta Pruning

- Alpha Bound of J:
  - **The max current val of all MAX ancestors of J**
  - **Exploration of a min node, J, is stopped when its value equals or falls below alpha.**
  - **In a min node, we update beta**
- Beta Bound of J:
  - **The min current val of all MIN ancestors of J**
  - **Exploration of a max node, J, is stopped when its value equals or exceeds beta**
  - **In a max node, we update alpha**
- In both min and max nodes, we return when $\alpha \geq \beta$

Alpha-Beta Procedure: $V(J;\alpha,\beta)$

```
1. If J is a terminal, return V(J) = h(J).
2. If J is a max node:
       For each successor $J_k$ of J in succession:
           Set $\alpha = \max \{ \alpha, V(J_k; \alpha, \beta) \}$
           If $\alpha \geq \beta$ then return $\beta$, else continue
       Return $\alpha$
3. If J is a min node:
       For each successor $J_k$ of J in succession:
           Set $\beta = \min \{ \beta, V(J_k; \alpha, \beta) \}$
           If $\alpha \geq \beta$ then return $\alpha$, else continue
       Return $\beta$
```