Probability and Random Processes (15B11MA301)

Lecture- 35

(Content covered: Correlation Ergodic, Examples)

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Correlation Ergodic Random Process

A random processs $\{X(t)\}$ is said to be correlation ergodic if

$$\langle R_{XX}(\tau)\rangle = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} X(t)\, X(t+\tau)\,dt = R_{XX}(\tau)$$

Example

If the WSS process $\{X(t)\}$ is given by $X(t) = 10 \cos(100t + \theta)$, where θ is uniformly distributed over $(-\pi, \pi)$, prove that $\{X(t)\}$ is correlation ergodic.

Solution $\quad R(\tau) = E[10 \cos(100t + 100\tau + \theta) \times 10 \cos(100t + \theta)]$

$$= 50 \cos(100\tau)$$

Consider
$$\bar{Z}_T = \frac{1}{2T} \int_{-T}^{T} X(t+\tau)\, X(t)\, dt$$

$$= \frac{1}{2T} \int_{-T}^{T} 100 \cos(100\, t + 100\, \tau + \theta) \cos(100\, t + \theta)\, dt$$

$$= \frac{25}{T} \int_{-T}^{T} \cos(100\, \tau) dt + \frac{25}{T} \int_{-T}^{T} \cos(200\, t + 100\, \tau + 2\theta) dt$$

$$= 50 \cos(100\, \tau) + \frac{25}{T} \int_{-T}^{T} \cos(200\, t + 100\, \tau + 2\theta)\, dt$$

Now $\quad \lim_{T \to \infty} (Z_T) = 50 \cos(100\, \tau)$

$$= R(\tau)$$

Therefore, $\{X(t)\}$ is correlation-ergodic

Distribution Ergodic Random Process

The random process $\{X(t)\}$ is said to be distribution erogodic random process if there is another random process $\{Y(t)\}$ such that $Y(t) = \begin{cases} 1, & X(t) \le x \\ 0, & X(t) > x \end{cases}$ and also the process $\{Y(t)\}$ is mean ergodic random process.

We note that

$$\begin{aligned} E\{Y(t)\} &= 1 \times P\{X(t) \le x\} + 0 \times P\{X(t) > x\} \\ &= F_X(x) \end{aligned}$$

$\{X(t)\}$ is distribution-ergodic,

if $\frac{1}{2T} \int_{-T}^{T} Y(t)\, dt \to F_X(x)$ as $T \to \infty$

Practice Problem

Consider a random process $X(t) = \cos(\omega t + \theta)$ where ω is a constant and θ is a random variable with a probability density,

$$p(\theta) = \begin{cases} \dfrac{1}{2}, & 0 \le \theta \le 2\pi \\ 0, & \text{otherwise} \end{cases}.$$

Is $\{X(t)\}$ correlation ergodic process?

Ans: Yes.

THANK YOU