

## FOML – Linear Algebra

### Revision

Q1. Consider performing *K-Means Clustering* on a one-dimensional dataset containing four data points:  $\{5, 7, 10, 12\}$  using  $k = 2$ , Euclidean distance, and the initial cluster centres are  $c_1 = 3.0$  and  $c_2 = 13.0$ .

- (i) [3] What are the initial cluster assignments? (That is, which examples are in cluster  $c_1$  and which examples are in cluster  $c_2$ ?)
- (ii) What are the new cluster centers after making the assignments in (i)?

Q2.

Let  $\mathbf{u}$  and  $\mathbf{v}$  be the vectors

$$\mathbf{u} = \frac{1}{3} \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix} \quad \text{and} \quad \mathbf{v} = \frac{1}{3} \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}.$$

Check if the vectors are orthogonal.

Find REF

$$(a) \begin{bmatrix} 3 & -2 & 4 & 7 \\ 2 & 1 & 0 & -3 \\ 2 & 8 & -8 & 2 \end{bmatrix}$$

Q3.

Use Gaussian Elimination to find all solutions to the following systems of linear equations.

$$(a) \begin{cases} x + 2y + 3z = 9 \\ 2x - 2z = -2 \\ 3x + 2y + z = 7 \end{cases}$$

Q4.

Find the closest point to  $\mathbf{y}$  in the subspace  $W$  spanned by the vectors  $\mathbf{v}_1$  and  $\mathbf{v}_2$ .

$$\mathbf{y} = \begin{bmatrix} 3 \\ -1 \\ 1 \\ 13 \end{bmatrix} \quad \text{and} \quad \mathbf{v}_1 = \begin{bmatrix} 1 \\ -2 \\ -1 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -4 \\ 1 \\ 0 \\ 3 \end{bmatrix}.$$