# Lecture-19

# Probability and Random Processes (15B11MA301)

## CO3

## Module: Probability Distributions

# Content

- Normal Distribution
- Standard Normal Distribution
- Curve of Normal Distribution
- Use of Standard Normal Distribution Table
- Mean, Median and Mode of the normal distribution

# Normal Distribution

**Definition :** A random variable $X$ is defined to be *normally* distributed if its probability density function is given by

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

where the parameters μ and $\sigma$ satisfy -∞< μ < ∞, < and $\sigma$ >0.

**Moments :**

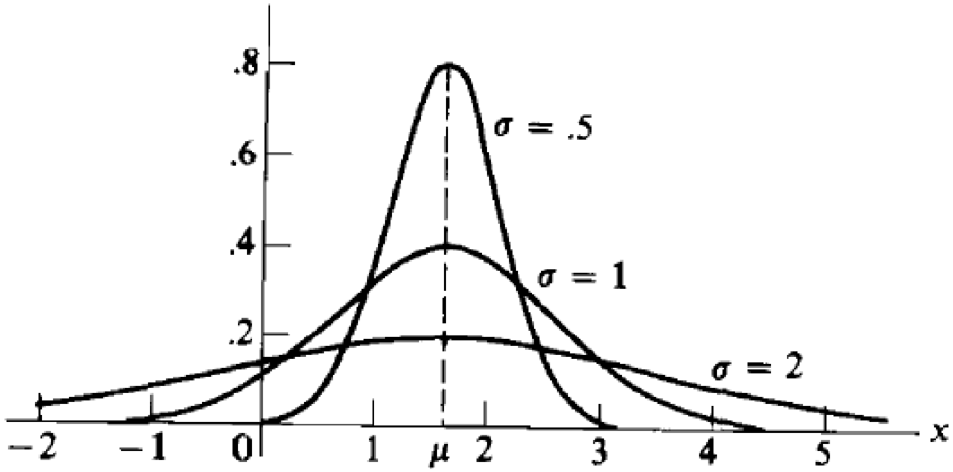
$$E[X] = \mu \text{ (mean)}$$

$$\text{var}(X) = \sigma^2 \text{ variance}$$

$$m_X(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2} \text{ (moment generating function)}$$

# Normal Distribution

Graph of probability density function of Normal Distribution



# Normal Distribution

## Standard Normal Distribution

If the normal random variable has mean 0 and variance 1, it is called a *standard* or *normalized* normal random variable. The pdf and cumulative distribution function (cdf) are given as follows:

$$\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} \quad \text{and} \quad \Phi(x) = \int_{-\infty}^{x} \phi(u)\, du.$$

# Normal Distribution

## Properties of Normal Distribution

**1.** If $X \sim N(\mu, \sigma^2)$, then $\dfrac{X-\mu}{\sigma} \sim N(0,1)$ and

$$P[a < X < b] = \Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right).$$

**2.**

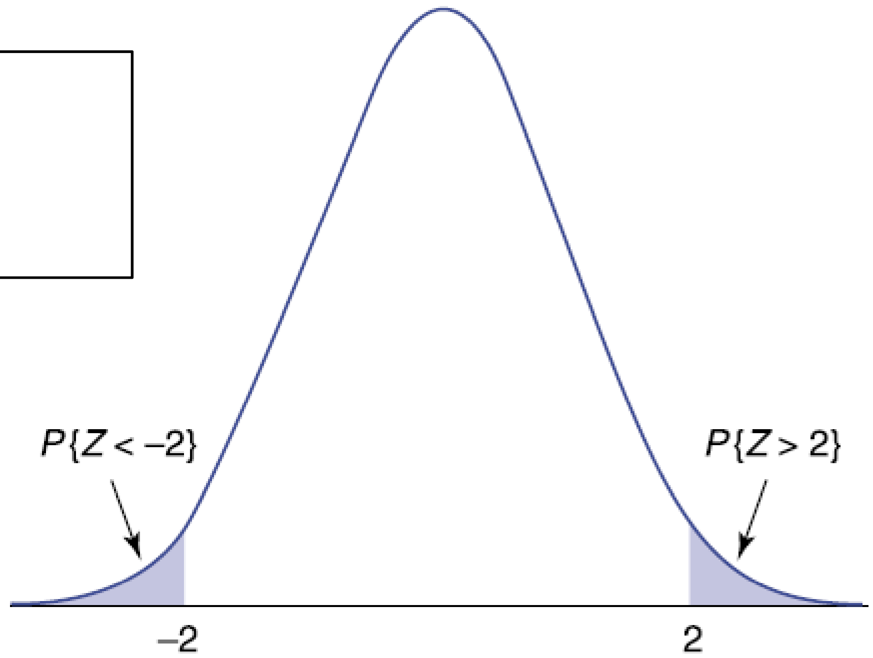
$$\Phi(x) = 1 - \Phi(-x).$$

Note: $\Phi(x) = P(Z<x)$, where Z is Standard normal random variable

# Normal Distribution

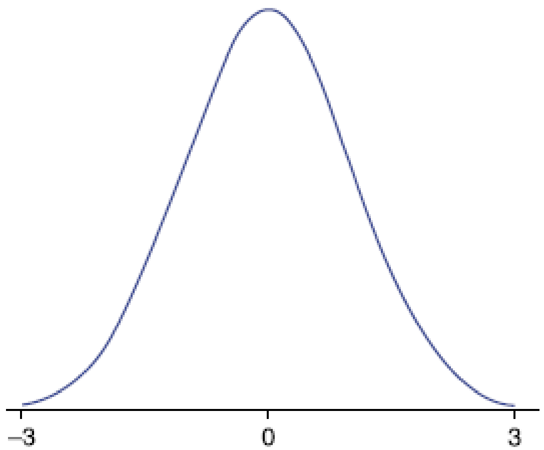
Properties of Normal Distribution

**2.** $$\Phi(x) = 1 - \Phi(-x).$$




Note: For normal distribution,
mean = median = mode

# Normal Distribution




**Approximation Rule**

A normal, random variable with mean μ and standard deviation σ will be
Between μ−σ and , μ+σ with approximate probability 0.68
Between , μ−2σ and μ+2σ with approximate probability 0.95
Between μ−3σ and μ+3σ with approximate probability 0.997.
Verify using the standard normal table.

# Normal Distribution. Verify using standard normal table.

$z_u$: 1-$u$ percentile of variable Z

$$\boxed{P(Z \le z_u) = 1 - u}$$

Remark:

$z_u = -z_{1-u}$

$z_{0.025} = 1.96$, $z_{0.975} = -1.96$

$z_{0.05} = 1.64$, $z_{0.95} = -1.64$,

# References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|---|---|
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Thank You