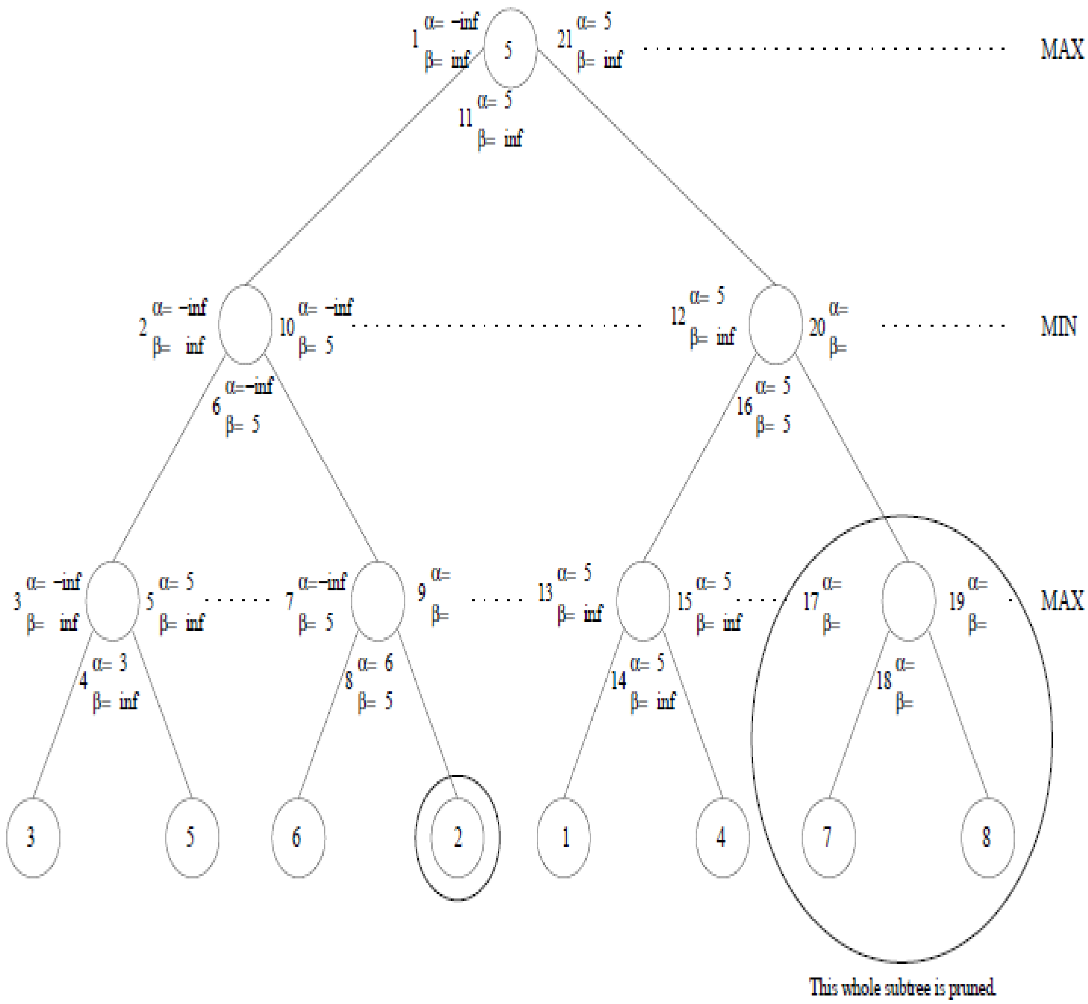
1.



1 α= −inf β= inf
5
21 α= 5 β= inf
11 α= 5 β= inf
MAX
2 α= −inf β= inf
10 α= −inf β= 5
6 α=−inf β= 5
12 α= 5 β= inf
20 α= β=
16 α= 5 β= 5
MIN
3 α= −inf β= inf
4 α= 3 β= inf
5 α= 5 β= inf
7 α=−inf β= 5
8 α= 6 β= 5
9 α= β=
13 α= 5 β= inf
14 α= 5 β= inf
15 α= 5 β= inf
17 α= β=
18 α= β=
19 α= β=
MAX
3
5
6
2
1
4
7
8
This whole subtree is pruned.

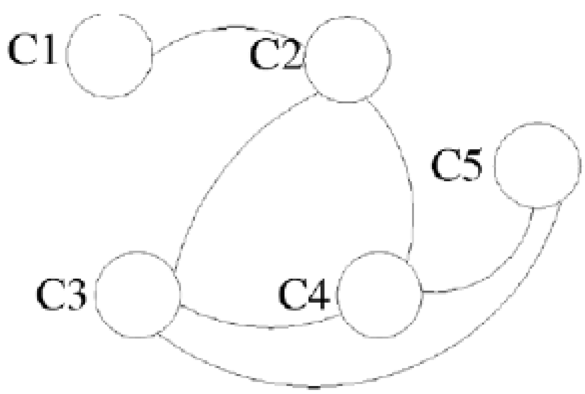
2. A

$Variables$	$Domains (or unary constraints)$
$C1$	C
$C2$	B, C
$C3$	A, B, C
$C4$	A, B, C
$C5$	B, C

Constraints:

$C1 \neq C2$
$C2 \neq C3$
$C3 \neq C4$
$C4 \neq C5$
$C2 \neq C4$
$C3 \neq C5$

B.



C.

$Variable$	$Domain$
$C1$	C
$C2$	B
$C3$	A, C
$C4$	A, C
$C5$	B, C

Note that C5 cannot possibly be C, but arc consistency does not rule it out.

D.

C1 = C, C2 = B, C3 = C, C4 = A, C5 = B. One other solution is possible (where C3 and C4 are switched).

(a) Translate the following sentences in first-order logic.

i. Star Trek, Star Wars and The Matrix are science fiction movies.
Answer: $SciFi(StarTrek) \wedge SciFi(StartWars) \wedge SciFi(Matrix)$

ii. Every AI student loves Star Trek or Star Wars.
Answer: $\forall x AIStudent(x) \rightarrow Loves(x, StarTrek) \vee Loves(x, StartWars)$

iii. Some AI students do not love Star Trek.
Answer: $\exists x AIStudent(x) \wedge \neg Loves(x, StarTrek)$

iv. All AI students who love Star Trek also love The Matrix.
Answer: $\forall x AIStudent(x) \wedge Loves(x, StarTrek) \rightarrow Loves(x, Matrix)$

v. Every AI student loves some science fiction movie.
Answer: $\forall x AIStudent(x) \rightarrow (\exists y SciFi(y) \wedge Loves(x, y)$

vi. No science fiction movie is loved by all AI students.
Answer: $\neg(\exists y SciFi(y) \wedge (\forall x AIStudent(x) \rightarrow Loves(x, y)))$

vii. There is an AI student who loves all science fiction movies.
Answer: $\exists x AIStudent(x) \wedge (\forall y SciFi(y) \rightarrow Loves(x, y))$

(b) Based on the knowledge base above, prove formally that there exists some AI student who loves Star Wars.

Answer: We can re-write the first statement as:

$$\neg AIStudent(x) \vee Loves(x, StarTrek) \vee Loves(x, StartWars)$$

The second statement can be re-written through skolemization as:

$$AIStudent(X0) \wedge \neg Loves(X0, StarTrek)$$

By unification and resolution between these two statements, we get:

$$Loves(X0, StartWars)$$

which proves the conclusion

1. (50 pts total, 5 pts each) **Try it Yourselves.** Prove that the unicorn is magical.

If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

Use these propositional variables:

Y = unicorn is m**Y**thical	**R** = unicorn is mo**R**tal	**M** = unicorn is a ma**M**mal
H = unicorn is **H**orned	**G** = unicorn is ma**G**ical	

1.a. Convert the English into propositional logic implicative form and conjunctive normal form (CNF). The first one is done for you as an example. (Note: "immortal" means "not mortal.")

1.a.1. If the unicorn is mYthical, then it is not moRtal.

S1: Implicative $Y \Rightarrow \neg R$. CNF $(\neg Y \vee \neg R)$

It is OK not to put the parentheses around the CNF clauses.

1.a.2. If the unicorn is not mYthical, then it is moRtal.

S2: Implicative $\neg Y \Rightarrow R$. CNF $(Y \vee R)$

1.a.3. If the unicorn is not mYthical, then it is a maMmal.

S3: Implicative $\neg Y \Rightarrow M$. CNF $(Y \vee M)$.

1.a.4. If the unicorn is not moRtal, then it is Horned.

S4: Implicative $\neg R \Rightarrow H$. CNF $(R \vee H)$.

1.a.5. If the unicorn is a maMmal, then it is Horned.

S5: Implicative $M \Rightarrow H$. CNF $(\neg M \vee H)$.

1.a.6. The unicorn is maGical if it is Horned.

This sentence means the same as, "If the unicorn is Horned, then it is maGical."

S6: Implicative $H \Rightarrow G$. CNF $(\neg H \vee G)$.

1.b. (25 pts total, 5 pts each) **Resolution Theorem Proving.** Use the conjunctive normal form (CNF) expressions from **1.a** above to prove that the unicorn is magical. The first and last steps are done for you.

Here you will get full credit if you do the resolution steps correctly based on your answers for the two resolved sentences (even if those sentences were not correct). I.e., this question asks only that you do the resolution step correctly, regardless of content.

1.b.1. The negated goal is S7.

S7: (¬ G) .

1.b.2. Resolve S6 and S7 to give S8.

S8: (¬ H) .

1.b.3. Resolve S5 and S8 to give S9.

S9: (¬ M) .

1.b.4. Resolve S4 and S8 to give S10.

S10: (R) .

1.b.5. Resolve S3 and S9 to give S11.

S11: (Y) .

1.b.6. Resolve S1 and S11 to give S12.

S12: (¬ R) .

1.b.7. Resolve S10 and S12 to give the empty clause, thus proving the goal sentence is true.

S13: ( ) .