Lecture-21

Probability and Random Processes (15B11MA301)

CO3

Module: Probability Distribution

Content:

Miscellaneous Examples on Normal Distribution

Examples

For a certain period normal distribution, the first moment about 10 is 40 and that the 4th moment about 50 is 48, what are the parameters of the distribution?

Solution Let μ be the mean and σ^2 the variance, then $\mu_1' = 40$ about $A = 10$.

$$\text{Mean} = A + \mu_1' = 10 + 40 \quad \Rightarrow \quad \mu = 50$$

$\mu_4 = 48 \Rightarrow 3\sigma^4 = 48 \Rightarrow \sigma^4 = 16 \Rightarrow \sigma = 2.$

Thus parameters are, mean $\mu = 50$, SD $= \sigma = 2$.

Examples

X is a normal variate with mean 2 and variance 4. Y is another normal variate independent of X with mean 2 and variance 3. What is the distribution of $X + 2Y$.

Solution Given X and Y are independent normal variates and $E(X) = 2$, $\text{Var}(X) = 4$, $E(Y) = 2$, $\text{Var}(Y) = 3$.

$$\therefore \quad \text{Mean of } (X + 2Y) = E(X + 2Y) = E(X) + E(2Y)$$

$$= E(X) + 2E(Y) = 1 + (2)\ (2) = 5$$

$$\text{Var}(X + 2Y) = \text{Var}(X) + 4\ \text{Var}(Y)$$

$$= (1 \times 4) + (4 \times 3) = 16$$

$\therefore$ $X + 2Y$ follows normal distribution with mean 5 and variance 16.

(In view of additive property of normal distribution)

Examples: Practice Problem

In a distribution exactly normal, 7% of the items are under 35 and 89% are under 63. What are the mean and standard deviation of the distribution?

mean of the distribution is 50.3 and $\sigma = 10.33$.

Examples

If X is a normal variate with mean 50 and SD 10, find $P(Y \leq 3137)$, where $Y = X^2 + 1$.

Solution Given: $P(Y \leq 3137) = P(X^2 + 1 \leq 3137)$

$$P(X^2 \leq 3136) = P(-56 \leq X \leq 56)$$

$$= P(|X| \leq 56)$$

But, $$Z = \frac{X - \mu}{\sigma}$$

When $X = -56$ $$Z = \frac{-56 - 50}{10} = -10.6$$

When $X = 56$ $$Z = \frac{56 - 50}{10} = 0.6$$

$$\begin{aligned} \therefore \quad P(Y \leq 3137) &= P(-56 \leq X \leq 56) \\ &= P(-10.6 \leq Z \leq 0.6) \\ &= P(-10.6 \leq Z \leq 0) + P(0 \leq Z \leq 0.6) \\ &= 0.5 + 0.2257 = 0.7257 \end{aligned}$$

Examples

In a test on 2000 electric bulbs, it was found that the life of a particular make was normally distributed with an average life of 2040 hours and SD of 60 hours. Estimate the number of bulbs likely to burn for

(i) more than 2150 hours,
(ii) less than 1950 hours, and
(iii) more than 1920 hours but less than 2160 hours.

Examples

Solution Given: $\mu = 2040$ hours, $\sigma = 60$ hours

(i) To find P(more than 2150 hours) = $P(X > 2150$ hours):
We know that

$$Z = \frac{X - \mu}{\sigma} = \frac{X - 2040}{60}$$

When $X = 2150$, $Z = \dfrac{2150 - 2040}{60} = 1.833$

$$\therefore \quad P(X > 2150) = P(Z_1 > 1.833)$$
$$= 0.5 - P(0 < Z < 1.833)$$
$$= 0.5 - 0.4664 = 0.0336$$

$\therefore$ The number of bulbs expected to burn for more than 2150 hours $= 2000 \times 0.0336 = 67$ (nearly)

Examples

(ii) To find $P(\text{less than 1950 hours}) = P(X < 1950 \text{ hours})$:

When $X = 1950$, $Z = \dfrac{1950 - 2040}{60} = -1.5$

$$\begin{aligned} \therefore \quad P(X < 1950) &= P(Z < -1.5) \\ &= 0.5 - P(-1.5 < Z < 0) \\ &= 0.5 - P(0 < Z < 1.5) \\ &= 0.5 - 0.4332 = 0.0668 \end{aligned}$$

$\therefore$ The number of bulbs expected to burn for less than 1950 hours $= 2000 \times 0.0668 = 134$ (nearly)

Examples

(iii) To find P(more than 1920 hours but less than 2160 hours):

When $X = 1920$, $Z = \dfrac{1920 - 2040}{60} = -2$

When $X = 2160$, $Z = \dfrac{2160 - 2040}{60} = 2$

$$\therefore \quad P(1920 < X < 2160) = P(-2 < Z < 2)$$
$$= 2P(0 < Z < 2)$$
$$= 2 \times 0.4773 = 0.9546$$

$\therefore$ The number of bulbs expected to burn for more than 1920 hours but less than 2160 hours $= 2000 \times 0.9546 = 1909$ (nearly)

References

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Thank You