Lecture-20

Probability and Random Processes (15B11MA301)

CO3

Module: Probability Distribution

Content:

Additive Property of Normal Distribution with Proof.

Examples.

Normal Distribution

Additive Property of Normal Distribution

Assume that $X_1, X_2, \ldots X_n$ are independent random variables and

$$X_i \sim N(\mu_i, \sigma_i^2);$$

then

$$a_i X_i \sim N(a_i \mu_i, a_i^2 \sigma_i^2),$$

and

$$m_{a_i X_i}(t) = \exp\left(a_i \mu_i t + \tfrac{1}{2} a_i^2 \sigma_i^2 t^2\right).$$

Hence

$$m_{\sum a_i X_i}(t) = \prod_{i=1}^{n} m_{a_i X_i}(t) = \exp\left[\left(\sum a_i \mu_i\right)t + \tfrac{1}{2}\left(\sum a_i^2 \sigma_i^2\right)t^2\right],$$

which is the moment generating function of a normal random variable; so

$$\sum_1^n a_i X_i \sim N\left(\sum_1^n a_i \mu_i, \sum_1^n a_i^2 \sigma_i^2\right).$$

Normal Distribution

Example

If $X_1, \ldots, X_n$ are independent and identically distributed random variables distributed $N(\mu, \sigma^2)$, then

$$\overline{X}_n = \frac{1}{n}\sum X_i \sim N\left(\mu, \frac{\sigma^2}{n}\right);$$

Normal Distribution

Example : IQ examination scores for sixth-graders are normally distributed with mean value 100 and standard deviation 14.2.
(a) What is the probability a randomly chosen sixth-grader has a score greater than 130?
(b) What is the probability a randomly chosen sixth-grader has a score between 90 and 115?

Normal Distribution

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Solution (a)

$X \sim N(100, (14.2)^2)$ (given)

so, $\frac{X-100}{14.2} = Z \sim N(0,1)$

$P(X>130) = P\left[\frac{X-100}{14.2} > \frac{130-100}{14.2}\right]$

$= P[Z > 2.113] = 1 - \Phi(2.113)$

$= 0.017$.

Normal Distribution

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(b) $P[90 < X < 115]$

$$= P\left[\frac{90-100}{14.2} < \frac{X-100}{14.2} < \frac{115-100}{14.2}\right]$$

$$= P[-0.704 < Z < 1.056]$$

$$= \Phi(1.056) - \Phi(-0.704)$$

$$= 0.854 - 0.242$$

$$= 0.612$$

Normal Distribution

Example : Suppose the amount of time a light bulb works before burning out is a normal random variable with mean 400 hours and standard deviation 40 hours. If an individual purchases two such bulbs, one of which will be used as a spare to replace the other when it burns out, what is the probability that the total life of the bulbs will exceed 750 hours?

Normal Distribution

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<u>Solution</u> : Let X : Life time of first bulb

Y : Life time of second bulb.

Find $P(X+Y>750)$.

$X+Y \sim N(800, 40^2+40^2)$.

so, $$\frac{X+Y-800}{\sqrt{40^2+40^2}} = Z \sim N(0,1).$$

Normal Distribution

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$$P(X+Y>750)$$

$$= P\left(\frac{X+Y-800}{\sqrt{40^2+40^2}} > \frac{750-800}{\sqrt{40^2+40^2}}\right)$$

$$= P(Z > -0.884)$$

$$= 1-\Phi(-0.884)$$

$$= \Phi(0.884)$$

$$= 0.81$$

References

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Thank You