Set-10: Modelling epidemics and endemic breakouts of infectious diseases

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This report models the spread of epidemic and endemic diseases using the Euler method. It analyzes infection dynamics in fixed and changing populations, observing peak infection times and outbreak patterns. Results are shown through time series and phase-space plots.

I. MODELS :

EPIDEMIC DISEASES

Consider a population size of N, through which an infection spreads. The population is divided into three classes: the infected class $x(t)$, the susceptible class $y(t)$, and the recovered class $z(t)$, so that $x(t)+y(t)+z(t) = N$ (constant).

The coupled dynamics of these variables is given by

$$\dot{x} = Axy - bx \tag{1}$$

$$\dot{y} = -Axy \tag{2}$$

$$\dot{z} = Bx \tag{3}$$

The Euler solutions to the above equations are given as:

$$x_{n+1} = x_n + (Ax_ny_n - Bx_n)\,\Delta t \tag{4}$$

$$y_{n+1} = y_n + (-Ax_ny_n)\,\Delta t \tag{5}$$

$$z_{n+1} = z_n + (Bx_n)\,\Delta t \tag{6}$$

The total number of students in a boarding school is 763. Initially, a single student introduces an infectious disease into this population. The parameters used for the simulation are:

$$A = 2.18 \times 10^{-3}\ \text{day}^{-1}, \quad B = 0.44\ \text{day}^{-1}$$

These values are applied consistently across all plots.

ENDEMIC DISEASES

Endemic diseases persist in a population and develop from time to time. In this case N = N(t), that is, the total population size changes. If the per capita death rate is a and the per capita birth rate is b $(a, b > 0)$, then the relevant coupled system of equations is given by

$$\dot{x} = Axy - Bx - ax \tag{7}$$

$$\dot{y} = bN - Axy - ay \tag{8}$$

$$\dot{z} = Bx - az \tag{9}$$

$$\dot{N} = (b - a)N \tag{10}$$

The Euler solutions to the above equations are given as:

$$x_{n+1} = x_n + (Ax_ny_n - Bx_n - ax_n)\,\Delta t \tag{11}$$

$$y_{n+1} = y_n + (bN_n - Ax_ny_n - ay_n)\,\Delta t \tag{12}$$

$$z_{n+1} = z_n + (Bx_n - az_n)\,\Delta t \tag{13}$$

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II. GRAPHS

A. Epidemic Diseases

Plot of Infected and Susceptible Population

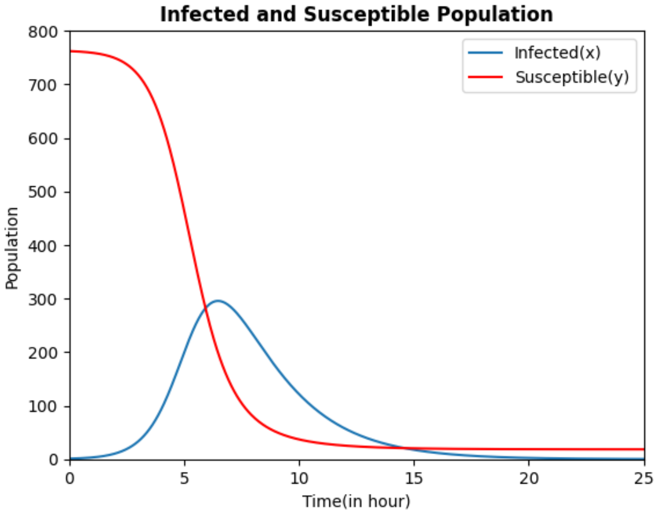

FIG. 1: The graph shows infected class $x(t)$ and susceptible class $y(t)$ versus number of days.

Here, $\Delta t = 1$ hour, $x_0 = 1$, $y_0 = 762$, $A = 2.18 \times 10^{-3}$ day^{-1}, $B = 0.44$ day^{-1}, $N = 763$, $t = 25$ days.
Time at which x is maximum: 6.4583 days.

Log Plot of Infected and Susceptible Population

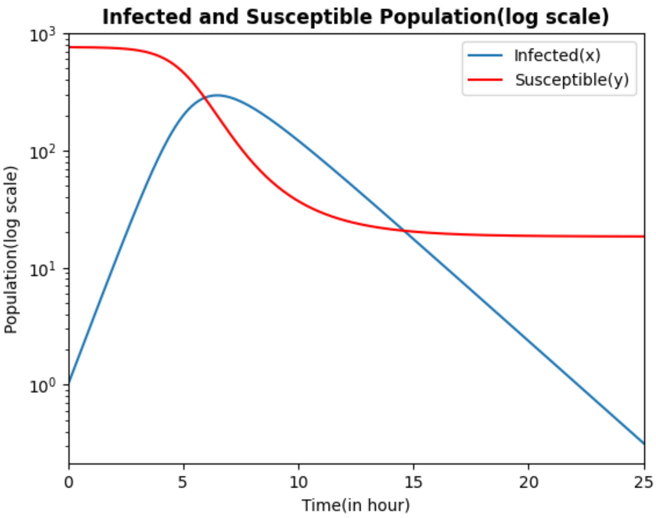

FIG. 2: The graph shows infected and susceptible classes (logarithmic scale) versus number of days.

Here, $\Delta t = 1$ hour, $x_0 = 1$, $y_0 = 762$, $A = 2.18 \times 10^{-3}$ day^{-1}, $B = 0.44$ day^{-1}, $N = 763$, $t = 25$ days.

Plot of Recovered Population

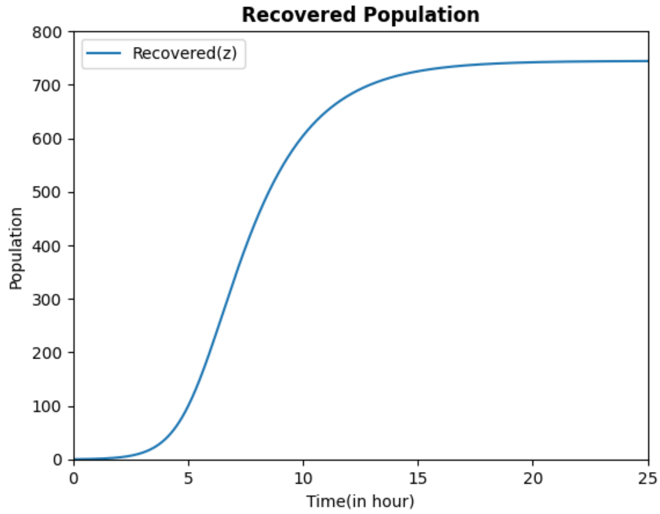

FIG. 3: The graph shows population density of a species versus time.

Here, $\Delta t = 1$ hour, $z_0 = 0$, $A = 2.18 \times 10^{-3}$ day^{-1}, $B = 0.44$ day^{-1}, $N = 763$, $t = 25$ days.

Log Plot of Recovered Population

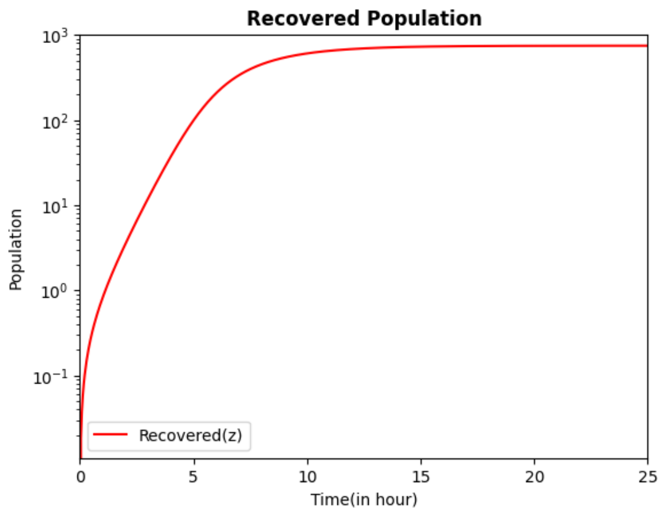

FIG. 4: The graph shows population density of Y-species versus X-species.

Here, $\Delta t = 1$ hour, $z_0 = 0$, $A = 2.18 \times 10^{-3}$ day^{-1}, $B = 0.44$ day^{-1}, $N = 763$, $t = 25$ days.

Plot of Infected Population vs Susceptible Population

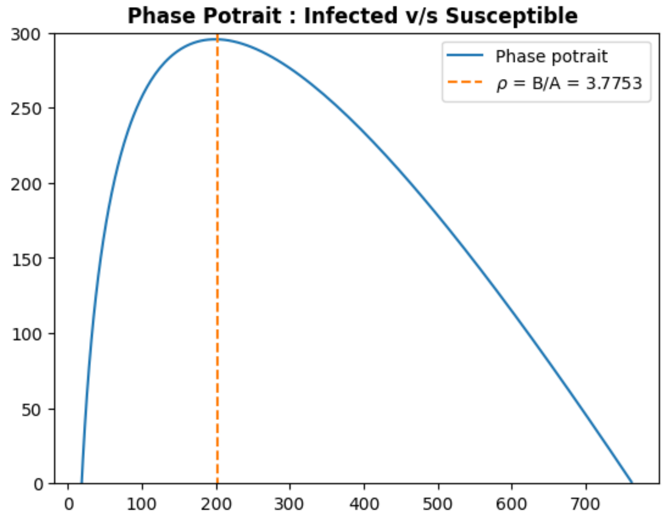

FIG. 5: The graph shows infected class $x(t)$ versus susceptible class $y(t)$.

Here, $\Delta t = 1$ hour, $x_0 = 1$, $y_0 = 762$, $A = 2.18 \times 10^{-3}$ day^{-1}, $B = 0.44$ day^{-1}, $N = 763$, $t = 25$ days.

Threshold value: 201.85
Value of R: 3.7753
Since $R > 1$, an epidemic breaks out.

B. Endemic Diseases

Plot of Infected Population

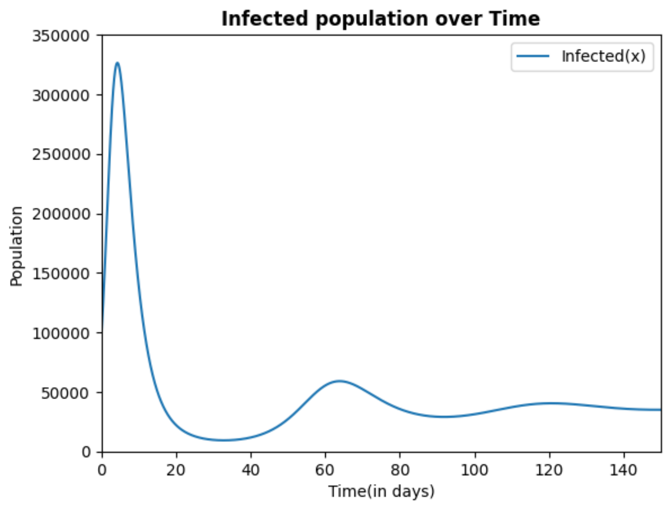

FIG. 6: The graph shows infected class $x(t)$ versus number of years.

Here, $\Delta t = 1$ day, $a = b = 0.02$ year^{-1} so that $\dot{N} = 0$, i.e., N is fixed. Take $A = 10^{-6}$ year^{-1}, $B = 0.333$ year^{-1}, $N = 10^6$, $x_0 = 10^5$.

- Time when x reaches peak 1: 4.2411 years.
- Time when x reaches peak 2: 63.8603 years.
- Time when x reaches peak 3: 120.7288 years.

Plot of Susceptible Population

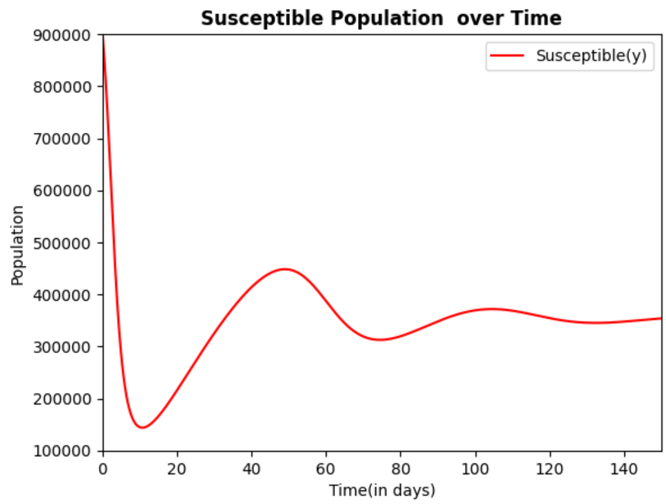

FIG. 7: The graph shows susceptible class $y(t)$ versus number of days.

Here, $\Delta t = 1$ day, $a = b = 0.02$ year^{-1} so that $\dot{N} = 0$, i.e., N is fixed. Take $A = 10^{-6}$ year^{-1}, $B = 0.333$ year^{-1}, $N = 10^6$, $y_0 = 9 \times 10^5$.

Plot of Recovered Population

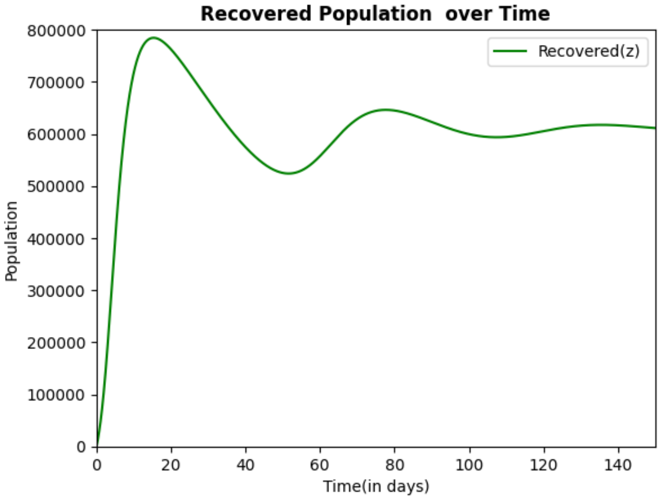

FIG. 8: The graph shows recovered class $z(t)$ versus number of days.

Here, $\Delta t = 1$ day, $a = b = 0.02$ year^{-1} so that $\dot{N} = 0$, i.e., N is fixed. Take $A = 10^{-6}$ year^{-1}, $B = 0.333$ year^{-1}, $N = 10^6$, $z_0 = 0$.