Set-1: GDP and Trade Modeling

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CS302, Modeling and Simulation

In this lab, we modeled the logistic equation to observe the GDP and trade dynamics of six countries with the highest GDP in the world. These countries are the USA, China, Japan, Germany, India and the UK. We set arbitrary constants to minimize the mean and standard deviation for the relative error. In addition, we tried to find the correlation between GDP and annual trade of a country.

I. DIFFERENTIAL EQUATIONS

The logistic equation

$$\dot{x} = \frac{dx}{dt} = f(x) = ax - bx^2 \tag{1}$$

represents a basic model of a nonlinear function, where a and b are fixed parameters. By solving this equation with the initial condition $x(0) = x_0$ and $k = \frac{a}{b}$, we obtain the following solution:

$$x(t) = \frac{kx_0e^{at}}{k + x_0(e^{at} - 1)} \tag{2}$$

The nonlinear time scale is given by:

$$t_{\mathrm{nl}} = \frac{1}{a}\ln\left(\frac{k}{x_0} - 1\right) \tag{3}$$

Next, applying the logistic equation to model GDP of a country, we have:

$$\dot{G} = \frac{dG}{dt} = G(G) = \gamma_1 G - \gamma_2 G^2 \tag{4}$$

where $G = G(t)$ is the GDP measured in US dollars and t is time in years. The parameters γ_1 and γ_2 correspond to a and b, respectively, with $k_G = \frac{\gamma_1}{\gamma_2}$.

For modeling annual trade of a country, we have:

$$\dot{T} = \frac{dT}{dt} = T(T) = \tau_1 T - \tau_2 T^2 \tag{5}$$

where $T = T(t)$ represents the annual trade of a country, and t is time in years. The parameters τ_1 and τ_2 correspond to a and b, respectively, with $k_T = \frac{\tau_1}{\tau_2}$.

To explore the correlated GDP (G) and annual trade (T) of a country, we establish a linked autonomous dynamical system denoted by:

$$\dot{G} = G(T, G), \quad \dot{T} = T(T, G)$$

Introducing the parameter $\alpha = \frac{\gamma_1}{\tau_1}$, the solutions for the relationship between annual trade and GDP are simplified into a power-law expression:

$$G(T) \sim T^{\alpha} \tag{6}$$

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II. GRAPHS

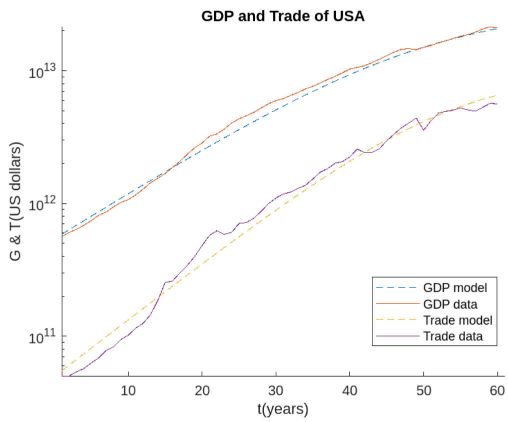

FIG. 1: Modelling the dynamics of GDP (upper plot) and trade (lower plot) using World Bank data for USA [2, 8]. The dotted curves follow the logistic equation with the parameter values in Table I. The zero year of both plots is 1960. The GDP plot ends in 2020, but the trade plot ends in 2019

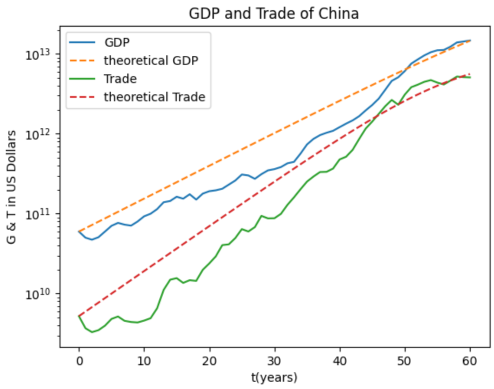

FIG. 2: Modelling the dynamics of GDP (upper plot) and trade (lower plot) using World Bank data for China [3, 9]. The dotted curves follow the logistic equation with the parameter values in Table I. The zero year of both plots is 1960, and both end in 2020.

FIG. 4: Modelling the dynamics of GDP (upper plot) and trade (lower plot) using World Bank data for Germany [5, 11]. The dotted curves follow the logistic equation with the parameter values in Table I. The zero year of both plots is 1970, and both end in 2020.

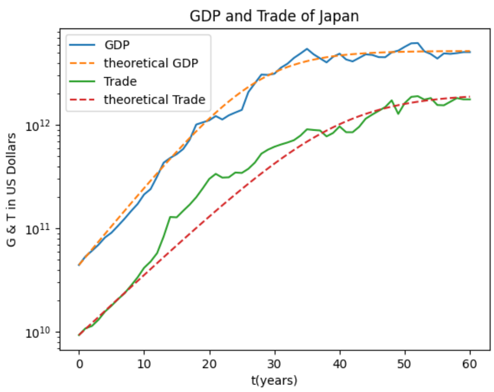

FIG. 3: Modelling the dynamics of GDP (upper plot) and trade (lower plot) using World Bank data for Japan [4, 10]. The dotted curves follow the logistic equation with the parameter values in Table I. The zero year of both plots is 1960, and both end in 2019.

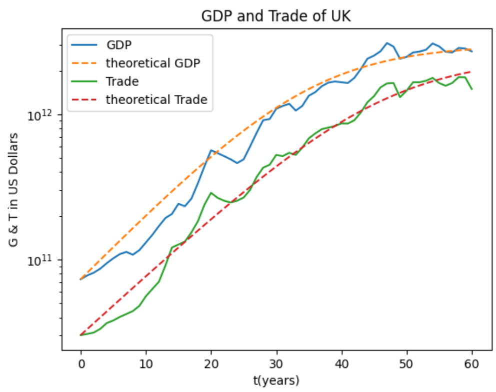

FIG. 5: Modelling the dynamics of GDP (upper plot) and trade (lower plot) using World Bank data for UK [6, 12]. The dotted curves follow the logistic equation with the parameter values in Table I. The zero year of both plots is 1960, and both end in 2020.

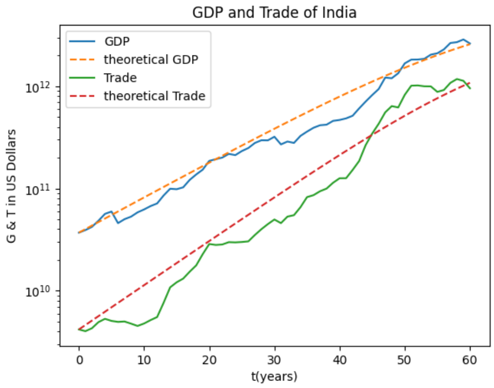

FIG. 6: Modelling the dynamics of GDP (upper plot) and trade (lower plot) using World Bank data for India [7, 13]. The dotted curves follow the logistic equation with the parameter values in Table I. The zero year of both plots is 1960, and both end in 2020.

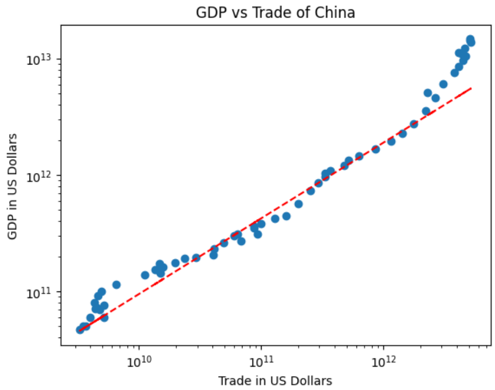

FIG. 8: Plotting GDP against trade using World Bank data for China [3, 9]. The dotted line follows Eq. (6) with $\alpha = 0.65$ (see Table III). The plot begins in 1960 and ends in 2020.

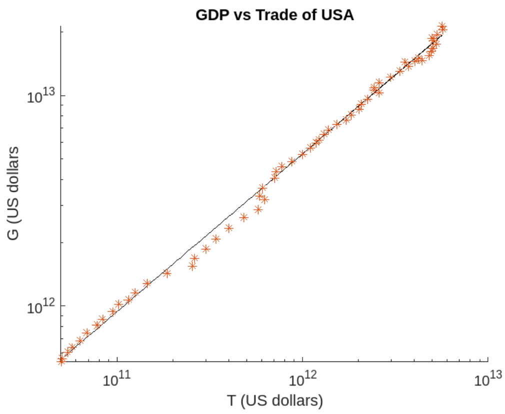

FIG. 7: Plotting GDP against trade using World Bank data forUSA [2, 8]. The dotted line follows Eq. (6) with $\alpha = 0.75$ (see Table III). The plot begins in 1960 and ends in 2019.

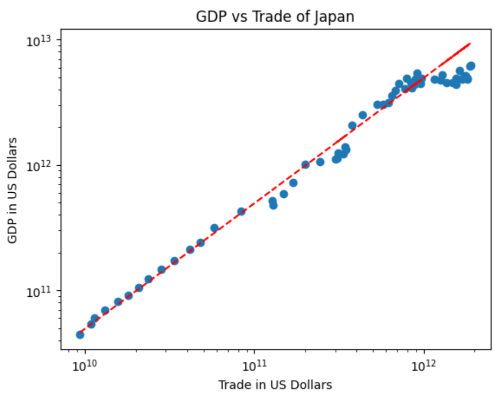

FIG. 9: Plotting GDP against trade using World Bank data for Japan [4, 10]. The dotted line follows Eq. (6) with $\alpha = 1.00$ (see Table III). The plot begins in 1960 and ends in 2019.

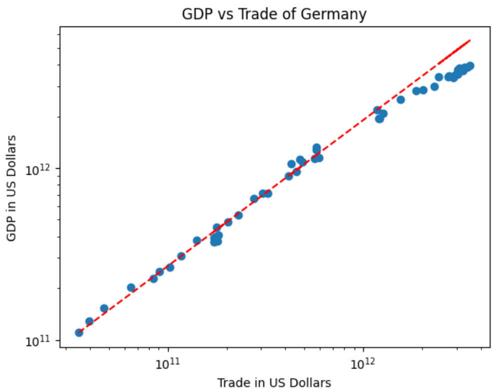

FIG. 10: Plotting GDP against trade using World Bank data for Germany [5, 11]. The dotted line follows Eq. (6) with α = 0.85 (see Table III). The plot begins in 1970 and ends in 2020.

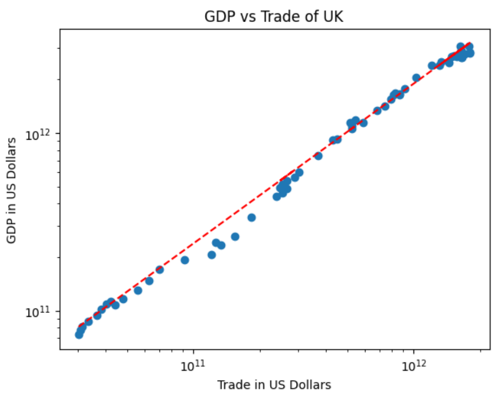

FIG. 11: Plotting GDP against trade using World Bank data for UK [6, 12]. The dotted line follows Eq. (6) with $\alpha = 0.90$ (see Table III). The plot begins in 1960 and ends in 2020.

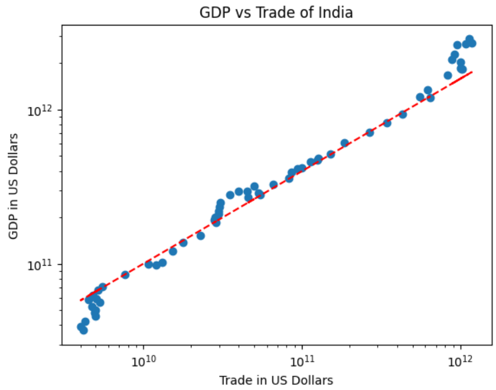

FIG. 12: Plotting GDP against trade using World Bank data for India [7, 13]. The dotted line follows Eq. (6) with α = 0.60 (see Table III). The plot begins in 1960 and ends in 2020.

III. COMPARISION BETWEEN COUNTRIES

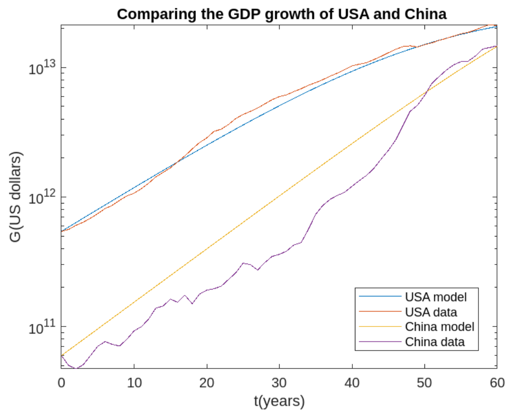

FIG. 13: Comparing the GDP growth of the USA and China, which are, respectively, the countries with the highest and the second highest GDPs in the world. The smooth dotted curves model the GDP growth of both countries according to Eq. (2), with the values of γ and k in Table 1. The World Bank data of the annual GDP from 1960 (t = 0) to 2020 (USA GDP data n.d.; China GDP data n.d.) show a much more ordered progression for the USA than for China. Consequently, the logistic modelling of the GDP growth of the USA shows a greater closeness with the actual data than what it does in the case of China.

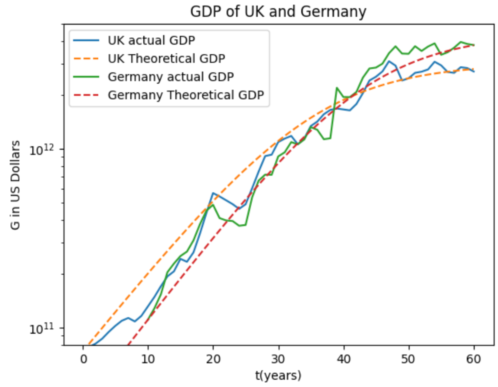

FIG. 15: Comparing the GDP growth of Germany and the UK, which, after the USA, are, respectively, the countries with the second and the third highest GDPs in the North-Atlantic region. The World Bank data of the annual GDP of the UK start from 1960 (t = 0) and end in 2020 (UK GDP data n.d.). For Germany, however, the GDP data (Germany GDP data n.d.) start from 1970 (t = 10 years). Till 1999-2000, both countries ran each other very close in terms of their GDP growth. Thereafter, the GDP of Germany has led the GDP of the UK. The beginning of the lead for Germany is theoretically captured by the intersection of the smooth dotted curves around the year 2000 (shown in Fig. 4). These two theoretical logistic curves model the GDP growth of both countries according to Eq. (2), with the values of γ and k in Table 1. In the case of Germany the theoretical logistic curve has been extrapolated backward before 1970.

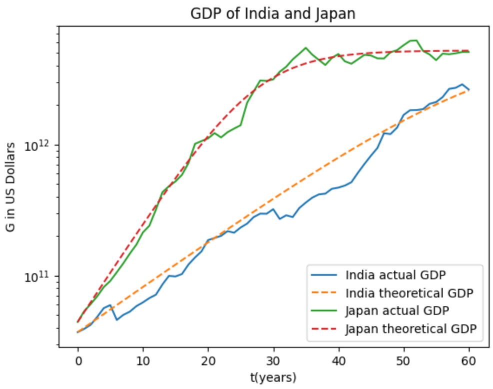

FIG. 14: Comparing the GDP growth of Japan and India, which, after China, are, respectively, the countries with the second and the third highest GDPs in the Indo-Pacific region. The World Bank data of the annual GDPs of both countries start from 1960 (t = 0) (Japan GDP data n.d.; India GDP data n.d.). The GDP data end in the year 2019 for Japan, and the year 2020 for India. The GDP growth of Japan has a steep gradient in the early years, but by the year 2000, the growth has visibly stagnated. Both of these features are modelled closely by the logistic function — the smooth dotted curve. In contrast, the GDP growth of India has been slow but on the whole steady, and by the year 2020, the GDP of India grows with a higher gradient than the GDP of Japan. At this rate, the GDP of India will eventually overtake the GDP of Japan. This forecast is theoretically modelled in Fig. 4 by extrapolating the logistic curves of Japan and India beyond 2020. These two theoretical logistic curves model the GDP growth of both countries according to Eq. (2), with the values of γ and k in Table 1.

IV. IMPORTANT VALUES TABLE:

Country	**Parameters to fit G (GDP)**		
	γ_1 (per annum)	k_G (trillion US dollars)	t_{nl} (years)
USA	0.080	30.0	50
China	0.095	80.0	76
Japan	0.175	5.2	26
Germany	0.110	4.4	32
UK	0.105	3.0	35
India	0.080	6.0	64

TABLE I: Parameters to fit G (GDP)

Country	Parameters to fit T (Trade)		
	τ_1 (per annum)	k_T (trillion US $)	t_{nl} (years)
USA	0.099	10.0	53
China	0.130	10.0	58
Japan	0.135	2.0	39
Germany	0.130	3.9	36
UK	0.095	2.5	46
India	0.100	3.0	66

TABLE II: Parameter values of the logistic equation for dynamically modelling the World Bank trade data [8–13] of the six countries that are listed in the first column. The data have been plotted and modelled in Figs. 1 to 6.

Country	G-T Correlation	α
USA	0.992	0.75
China	0.983	0.65
Japan	0.919	1.00
Germany	0.987	0.85
UK	0.993	0.90
India	0.982	0.60

TABLE III: This table lists the values of the power-law exponent (α) in the correlated growth of GDP and trade, as plotted and modelled in Figs. 7 to 12.

Country	Statistical analysis of G(t) (GDP)	
	μ_G	σ_G
USA	0.0492	0.0873
China	-0.3568	0.2504
Japan	-0.0833	0.1395
Germany	0.0489	0.1744
UK	-0.1089	0.1651
India	-0.1359	0.1743

TABLE IV: Statistical analyses of the relative difference between the actual data and the model functions. This lists the mean μ_G and the standard deviation σ_G of the yearly relative variations of the GDP data [12–17] with respect to the logistic model.

Country	Statistical analysis of T(t) (Trade)	
	μ_T	σ_T
USA	0.1160	0.2040
China	-0.3570	0.3393
Japan	-0.3570	0.3393
Germany	0.0736	0.2411
UK	0.0053	0.1679
India	-0.1630	0.3534

TABLE V: Statistical analyses of the relative difference between the actual data and the model functions. This lists the mean μ_G and the standard deviation σ_G of the yearly relative variations of the GDP data [12–17] with respect to the logistic model.

Country	Statistical analysis of α	
	μ_α	σ_α
USA	-0.0012	0.0024
China	0.0075	0.0113
Japan	-0.0045	0.0082
Germany	-0.0034	0.0051
UK	-0.0019	0.0033
India	0.0015	0.0079

TABLE VI: Statistical analyses of the relative difference between the actual data and the model functions. This lists the mean μ_G and the standard deviation σ_G of the yearly relative variations of the GDP data [12–17] with respect to the logistic model.

V. PREDICTING WHEN INDIA WILL BECOME A $4 AND $5 TRILLION ECONOMY

Using Equation (5), we can predict when India will become a $ 4 and $ 5 economy respectively.

According to our predictions using the given model and stated equation, India will become a **4 trillion dollar economy in 2032** and a **5 trillion dollar economy in 2043**.

VI. SALIENT FEATURES:

1. Japan GDP has reached its saturation i.e. its GDP growth has became constant over time.

2. India will overtake Germany and Japan's GDP to become 3(rd) largest economy in the world by around 2044.

3. GDP consists of many components i.e. consumption, investment, trade and government spendings. GDP is almost proportional to a specific power of trade. This specific power is different for different countries.

[1] Arnab K. Ray *Logistic modeling of economic dynamics*, Dhirubhai Ambani University.

[2] "USA GDP data", https://data.worldbank.org/ indicator/NY.GDP.MKTP.CD?locations=US.

[3] "China GDP data", https://data.worldbank.org/ indicator/NY.GDP.MKTP.CD?locations=CN.

[4] "Japan GDP data", https://data.worldbank.org/ indicator/NY.GDP.MKTP.CD?locations=JP.

[5] "Germany GDP data", https://data.worldbank.org/ indicator/NY.GDP.MKTP.CD?locations=DE.

[6] "UK GDP data", https://data.worldbank.org/ indicator/NY.GDP.MKTP.CD?locations=GB.

[7] "India GDP data", https://data.worldbank.org/ indicator/NY.GDP.MKTP.CD?locations=IN.

[8] "USA trade data", https://data.worldbank.org/ indicator/NE.TRD.GNFS.ZS?locations=US.

[9] "China trade data", https://data.worldbank.org/ indicator/NE.TRD.GNFS.ZS?locations=CN.

[10] "Japan trade data", https://data.worldbank.org/ indicator/NE.TRD.GNFS.ZS?locations=JP.

[11] "Germany trade data", https://data.worldbank.org/ indicator/NE.TRD.GNFS.ZS?locations=DE.

[12] "UK trade data", https://data.worldbank.org/ indicator/NE.TRD.GNFS.ZS?locations=GB.

[13] "India trade data", https://data.worldbank.org/ indicator/NE.TRD.GNFS.ZS?locations=US.