

Set-8 : Modelling Battle Strategies

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In this lab, we used Lanchester's model to analyse the battle of Iwo Jima and the battle of Trafalgar.

I. BATTLE OF IWO JIMA

In 1945, a fierce battle was fought on the Japanese island of Iwo Jima between the Imperial Japanese Army and the US Army. At a time t , the number of Japanese troops is $J(t)$ and the number of American troops is $A(t)$. Here, $a = 0.0106$ and $j = 0.0544$ are the combat effectiveness parameters of the American and the Japanese troops, respectively. The initial number of Japanese soldiers is $J(0) = J_0 = 18274$ and American soldiers is $A(0) = A_0 = 66454$.

A. Model

The system of differential equations governing the battle is given by:

$$\frac{dA}{dt} = -jJ \quad (1)$$

$$\frac{dJ}{dt} = -aA \quad (2)$$

Additionally, the combat effectiveness relation is:

$$aA^2 - jJ^2 = K \quad (3)$$

where K is given by:

$$K = aA_0^2 - jJ_0^2 \quad (4)$$

B. Results

By Lanchester's Square Law, the US Army will win the battle. The battle will end in 31 days. The number of active troops in the US Army is 51,527 and the number of casualties in the US Army is 14,927. Here the parameters are:

- $a = 0.0106$
- $j = 0.0544$
- $J(0) = J_0 = 18274$
- $A(0) = A_0 = 66454$

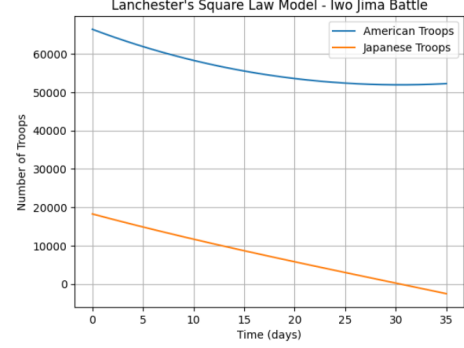


FIG. 1. Number of Japanese and American troops vs Time.

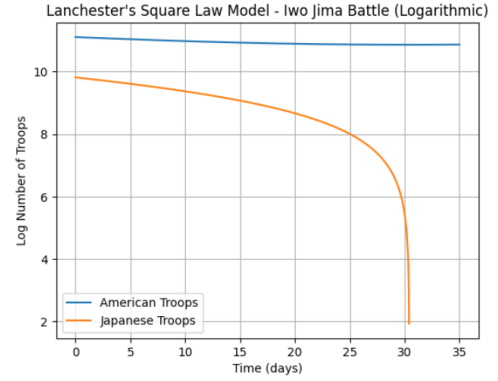


FIG. 2. Logarithmic plot of Number of Japanese and American troops vs Time.

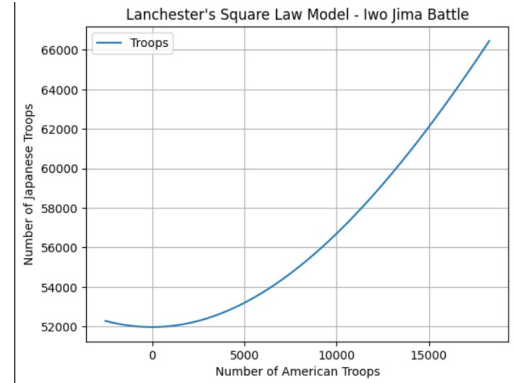


FIG. 3. Plot of American troops vs. Japanese troops.

FIG. 4. Visualization of troop dynamics in the Battle of Iwo Jima.

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II. BATTLE OF TRAFALGAR

In 1805, the Battle of Trafalgar was fought between Napoleon's French Navy and the British Royal Navy, which was under the command of Lord Nelson. The British emerged victorious in this naval battle. If at a time t , the number of French ships is $F(t)$ and the number of British ships is $B(t)$, the combat effectiveness parameters are $f = b = 0.05$. The initial number of ships in the French fleet is $F(0) = F_0 = 33$ and in the British fleet is $B(0) = B_0 = 27$.

Noting the numerical superiority of the French fleet, Lord Nelson divided his overall battle plan into three stages.

A. Model

The system of differential equations governing the battle is given by:

$$\frac{dB}{dt} = -fF \quad (5)$$

$$\frac{dF}{dt} = -bB \quad (6)$$

Additionally, the combat effectiveness relation is:

$$bB^2 - fF^2 = K \quad (7)$$

where K is given by:

$$K = bB_0^2 - fF_0^2 \quad (8)$$

B. Results

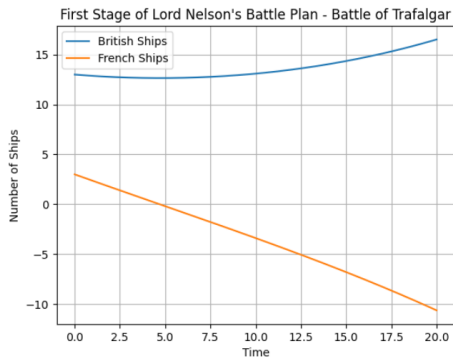


FIG. 5. Number of British and French ships vs Time for the First Stage. $B_0 = 13$, $F_0 = 3$.

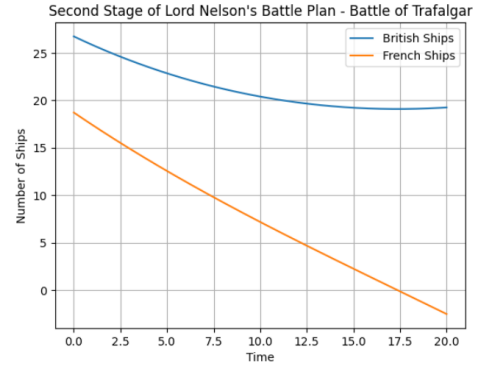


FIG. 6. Number of British and French ships vs Time for the Second Stage. $B_0 = 26.73$, $F_0 = 18.71$.

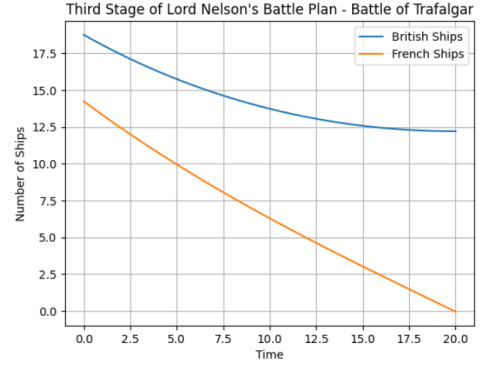


FIG. 7. Number of British and French ships vs Time for the First Stage. $B_0 = 18.76$, $F_0 = 14.24$.

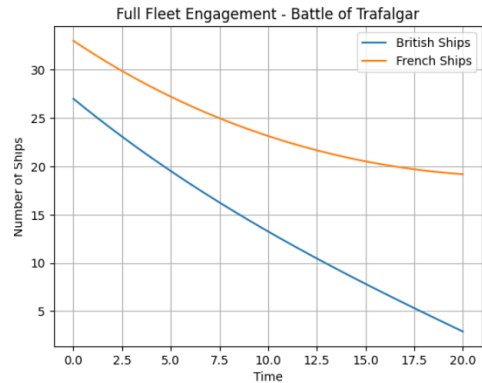


FIG. 8. Number of British and French ships vs Time for the First Stage. $B_0 = 27$, $F_0 = 33$.