

Set-1:

Shail Patel (202201297)* and Divyakumar Tandel (202201469)[†]
*Dhirubhai Ambani Institute of Information & Communication Technology,
 Gandhinagar, Gujarat 382007, India
 CS302, Modeling and Simulation*

In this lab, we modeled the logistic equation to observe the growth of Revenue and Human Resource for IBM. We set arbitrary constants to minimize the mean and standard deviation for the relative error. In addition, we tried to find the correlation between Annual Revenue and IBM Human Resources count.

I. DIFFERENTIAL EQUATIONS

The logistic equation

$$\dot{x} = \frac{dx}{dt} = f(x) = ax - bx^2 \quad (1)$$

represents a basic model of a nonlinear function, where a and b are fixed parameters. By solving this equation with the initial condition $x(0) = x_0$ and $k = \frac{a}{b}$, we obtain the following solution:

$$x(t) = \frac{kx_0e^{at}}{k + x_0(e^{at} - 1)} \quad (2)$$

The nonlinear time scale is given by:

$$t_{nl} = \frac{1}{a} \ln \left(\frac{k}{x_0} - 1 \right) \quad (3)$$

Next, applying the logistic equation to model revenue, we have:

$$\dot{R} = \frac{dR}{dt} = R(R) = \rho_1 R - \rho_2 R^2 \quad (4)$$

where $R = R(t)$ is the revenue measured in US dollars and t is time in years. The parameters ρ_1 and ρ_2 correspond to a and b , respectively, with $k_R = \frac{\rho_1}{\rho_2}$.

For modeling human resources, we have:

$$\dot{H} = \frac{dH}{dt} = H(H) = \eta_1 H - \eta_2 H^2 \quad (5)$$

where $H = H(t)$ represents the human resources of IBM, and t is time in years. The parameters η_1 and η_2 correspond to a and b , respectively, with $k_H = \frac{\eta_1}{\eta_2}$.

To explore the correlated growth of revenue (R) and human resources (H), we establish a linked autonomous dynamical system denoted by:

$$\dot{R} = R(H, R), \quad \dot{H} = H(H, R)$$

Introducing the variables $V = \frac{1}{R} - \frac{1}{k_R}$ and $U = \frac{1}{H} - \frac{1}{k_H}$, along with the parameter $\beta = \frac{\rho_1}{\eta_1}$, the solutions for the relationship between human resources and revenue are simplified into a power-law expression:

$$V \sim U^\beta \quad (6)$$

II. GRAPHS

A. Annual revenue growth plot of IBM from 1914 to 2006

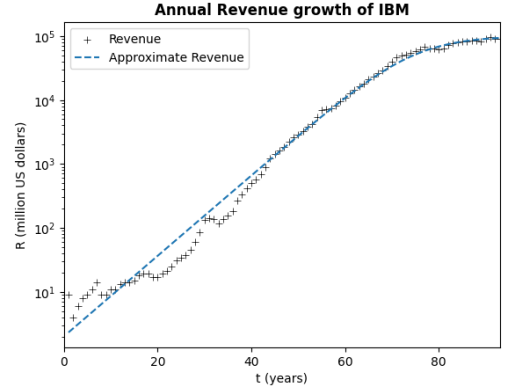


FIG. 1: The dotted graph is the annual revenue of IBM. The Line graph is the modeled revenue given by Eq(2) with parameter values $\rho_1 = 0.145 \text{ year}^{-1}$ and $k_R = \$ 100 \text{ billion}$.

B. Human resources growth of IBM from 1914 to 2006

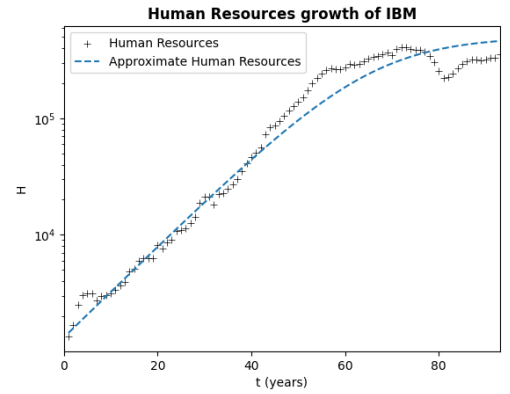


FIG. 2: The dotted graph is the Human Resource count of IBM. The Line graph is the modeled Human Resource count given by Eq(2) with parameter values $\eta_1 = 0.09 \text{ year}^{-1}$ and $k_H = 500000$.

*Electronic address: 202201297@daict.ac.in

[†]Electronic address: 202201469@daict.ac.in

C. Net annual earnings of IBM

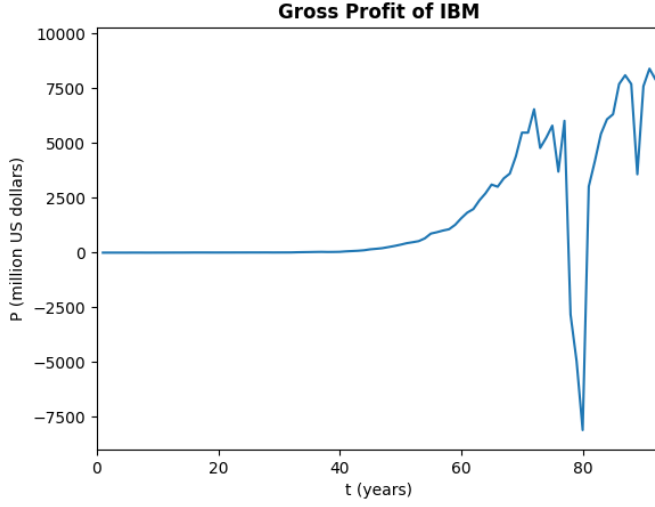


FIG. 3: Net annual earnings (the profit P) of IBM (in million US dollars) grow steadily till about 75-80 years (the early years of the 1990s). Around this time IBM suffered major losses in its net earnings (\$ 8 billion in 1993).

III. EVALUATION METRICS

Annual Revenue:

Mean: 0.0250, Standard Deviation: 0.4870

Human Resource:

Mean: 0.0901, Standard Deviation: 0.2980

IV. SALIENT FEATURES:

D. Correlated growth of annual revenue and human resource strength of IBM

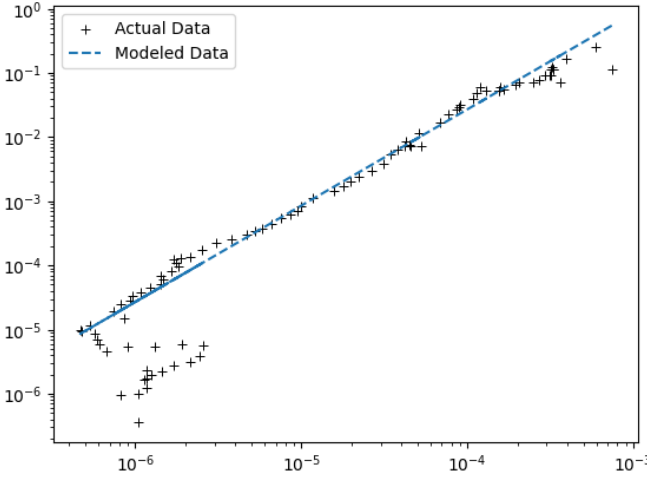


FIG. 4: For the dotted graph, $U = \frac{1}{H} - \frac{1}{k_H}$ and $V = \frac{1}{R} - \frac{1}{k_R}$. For the line graph, $U = \frac{1}{H} - \frac{1}{k_H}$ and $V = \alpha \cdot U^\beta$, where $\beta = \frac{\rho_1}{\eta_1} \approx 1.5$, $\alpha = 32000$, $k_H = 500000$, and $k_R = 100000$.

1. From the Fig 1. we can see that the IBM revenue is approaching the saturation the revenue will increase at slower rate. It will reach maximum revenue of \$ 100 billions

2. From the Fig 2. we can see the the gross profit of IBM dropped drastically in early 1990s which lead to layoff of some human resources as we can see in Fig 2. The reason for the decrease is due to poor strategic management decisions made by the company.

3. We have observed from given graphs that annual revenue is directly related to human resources. Our obsevation is due to the fact that whenever company's human resources increase, their revenue also increases and vice-versa.

[1] Arnab K. Ray *Logistic modeling of economic dynamics*, Dhirubhai Ambani University.