Set-9: Competitive exclusion and predator-prey dynamics

Shail Patel (202201297)* and Divyakumar Tandel (202201469)†
Dhirubhai Ambani Institute of Information & Communication Technology, Gandhinagar, Gujarat 382007, India
CS302, Modeling and Simulation

This report studies Competitive Exclusion and Predator-Prey dynamics using the Euler method. Simulate species interactions over time, highlighting extinction, co-existence, and the impact of human interference. Key behaviors are illustrated through plots.

I. MODELS :

Competitive exclusion

In nature, it is common for two similar species to compete for resources and space within the same ecological habitat.

Let $x(t)$ and $y(t)$ represent the population densities (individuals per unit area) of species X and species Y, respectively. The dynamics of these populations are described by coupled differential equations:

$$\dot{x} = Ax - Bx^2 - \alpha xy \tag{1}$$

$$\dot{y} = Cy - Dy^2 - \beta xy \tag{2}$$

Euler's method approximates the solutions for $x(t)$ and $y(t)$ using the iterative formulas:

$$x(i) = x(i-1) + dx(x(i-1), y(i-1)) \cdot \Delta t \tag{3}$$

$$y(i) = y(i-1) + dy(x(i-1), y(i-1)) \cdot \Delta t \tag{4}$$

where $dx(x, y) = \dot{x}$ and $dy(x, y) = \dot{y}$.

Predator-prey dynamics

In the interaction between a prey species X and a predator species Y, let $x(t)$ represent the population density (number per unit of area) of the prey and $y(t)$ represent that of the predator. The dynamics of their populations are described by the following coupled differential equation system:

$$\dot{x} = Ax - Bxy - \epsilon x \tag{5}$$

$$\dot{y} = -Cy + Dxy - \epsilon y \tag{6}$$

Euler's method can be used to approximate the solutions for $x(t)$ and $y(t)$ as:

$$x(i) = x(i-1) + dx(x(i-1), y(i-1)) \cdot \Delta t \tag{7}$$

$$y(i) = y(i-1) + dy(x(i-1), y(i-1)) \cdot \Delta t \tag{8}$$

where $dx(x, y) = \dot{x}$ and $dy(x, y) = \dot{y}$.

II. GRAPHS

A. Competitive exclusion

No competition within the species

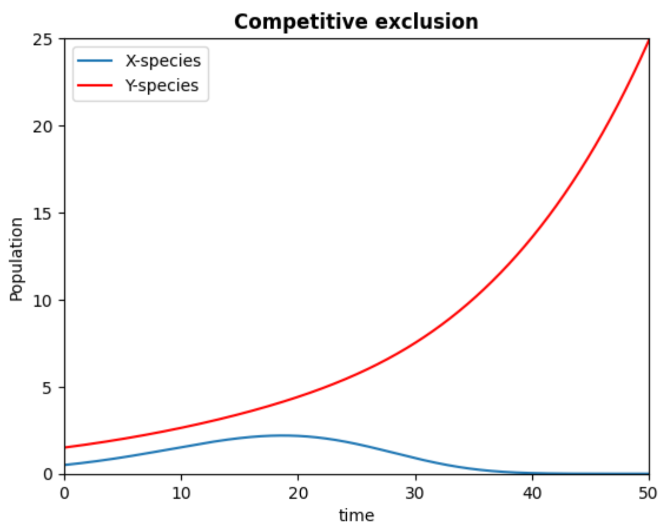

FIG. 1: The graph shows population density of a species vs time.

Here x(0) = 0.5, y(0) = 1.5, A = 0.21827, B = 0 , C = 0.06069, D = 0, α = 0.05289, β = 0.00459, Δt = 0.0001.

*Electronic address: 202201297@daiict.ac.in
†Electronic address: 202201469@daiict.ac.in

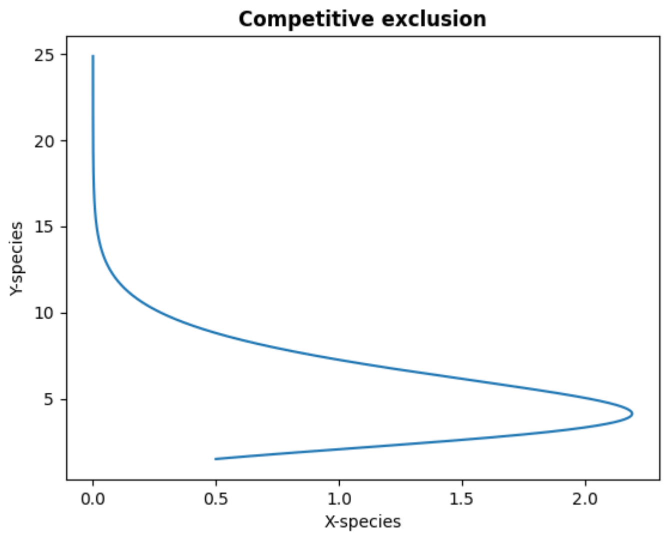

FIG. 2: The graph shows population density of a Y-species vs population density of a X-species.

Here x(0) = 0.5, y(0) = 1.5, A = 0.21827, B = 0 , C = 0.06069, D = 0, α = 0.05289, β = 0.00459, Δt = 0.0001.

Competition within the species

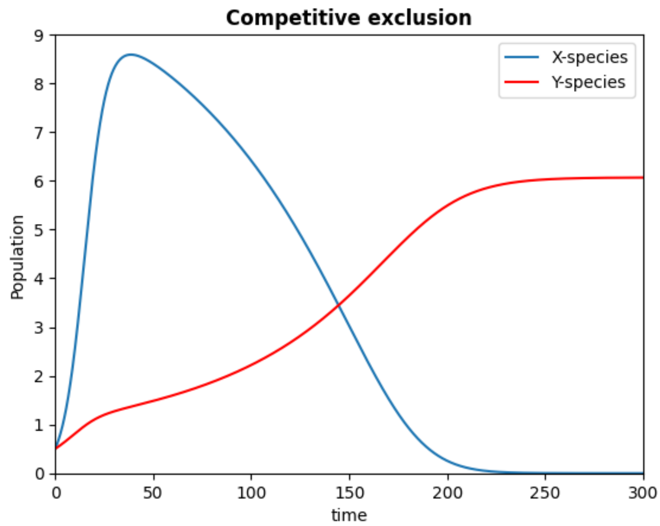

FIG. 3: The graph shows population density of a species vs time.

Here x(0) = 0.5, y(0) = 0.5, A = 0.21827, B = 0.017 , C = 0.06069, D = 0.010, α = 0.05289, β = 0.00459, Δt = 0.0001.

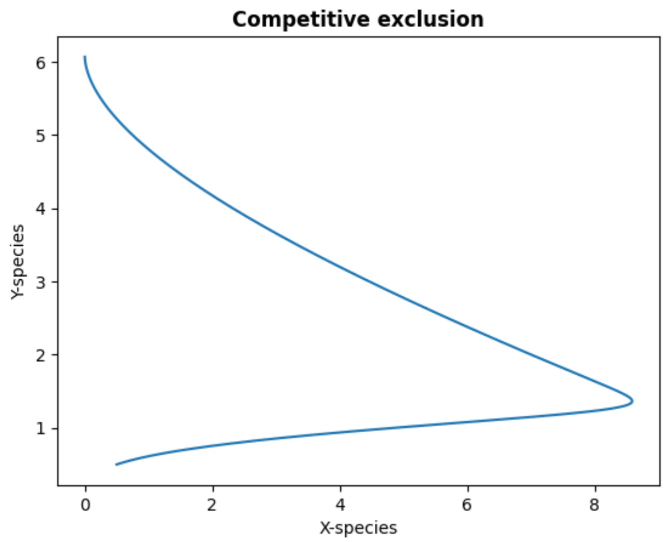

FIG. 4: The graph shows population density of a Y-species vs population density of a X-species.

Here x(0) = 0.5, y(0) = 0.5, A = 0.21827, B = 0.017, C = 0.06069, D = 0.010,α = 0.05289, β = 0.00459, Δt = 0.0001.

The time at which x reaches its maximum value is : 50 years.
The maximum value of x : 8.5897.
The maximum value of y : 6.0669.

No inter species competition

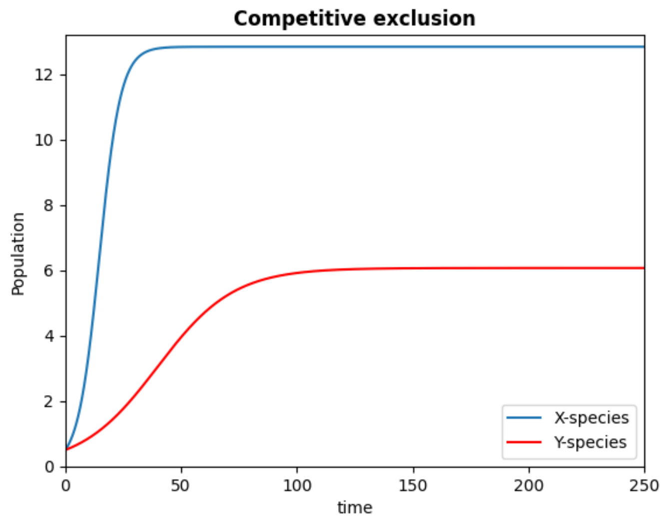

FIG. 5: The graph shows population density of a species vs time.

Here x(0) = 0.5, y(0) = 0.5, A = 0.21827, B = 0.017 , C = 0.06069, D = 0.010, α = 0, β = 0, Δt = 0.0001.

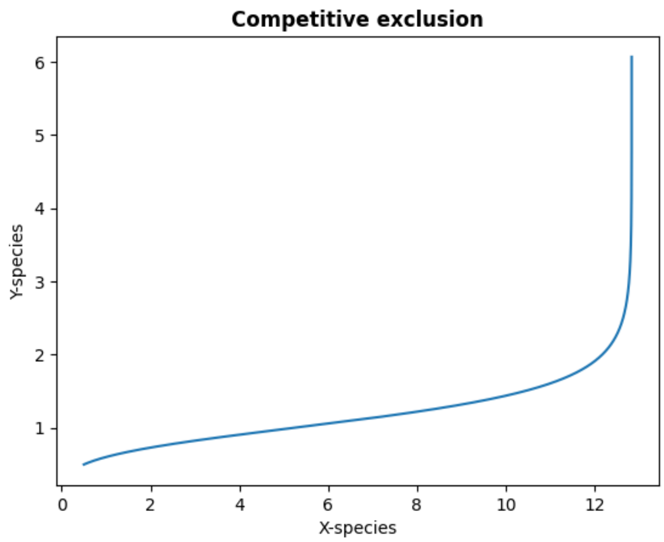

FIG. 6: The graph shows population density of a Y-species vs population density of a X-species.

Here x(0) = 0.5, y(0) = 0.5, A = 0.21827, B = 0.017 , C = 0.06069, D = 0.010, $\alpha = 0$,$\beta = 0$,$\Delta t = 0.0001$.

Peak value of X-species: 12.8394
Peak value of Y-species: 6.0689

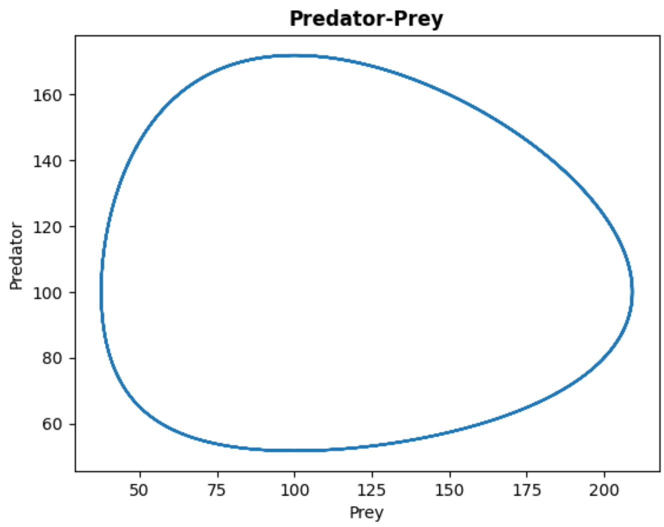

FIG. 8: The graph shows population density of a Y-species vs population density of a X-species.

Here x(0) = 200, y(0) = 80, A = 1.0, B = 0.01, C = 0.5, D = 0.005, $\epsilon = 0$, Δ t = 0.0001.

The maximum value of y : 171.8382.

B. Predator-prey dynamics

Without fishing(Without human interference)

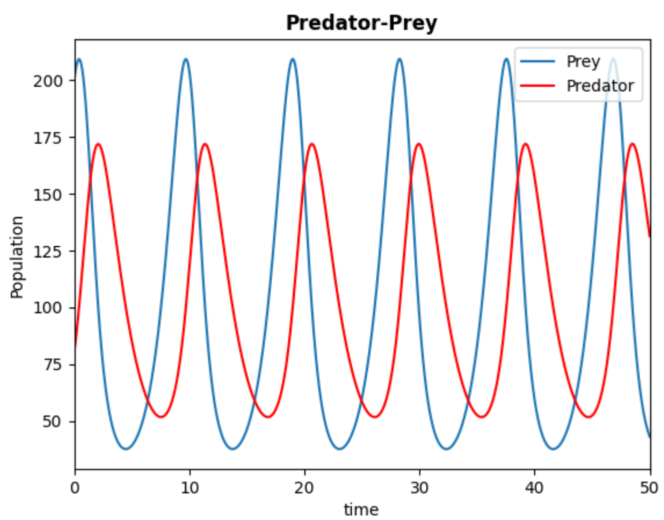

FIG. 7: The graph shows population density of a species vs time.

Here x(0) = 200, y(0) = 80, A = 1.0, B = 0.01, C = 0.5, D = 0.005, $\epsilon = 0$, $\Delta t = 0.0001$.

With fishing(Human interference)

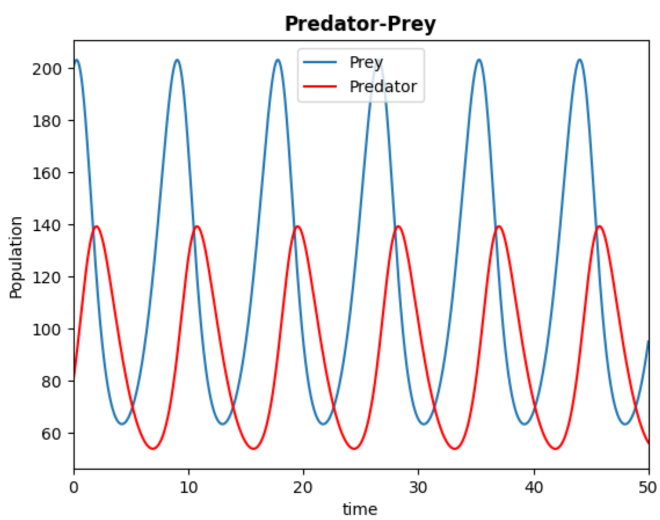

FIG. 9: The graph shows population density of a species vs time.

Here x(0) = 200, y(0) = 80, A = 1.0, B = 0.01, C = 0.5, D = 0.005, $\epsilon = 0.1$, $\Delta t = 0.0001$.

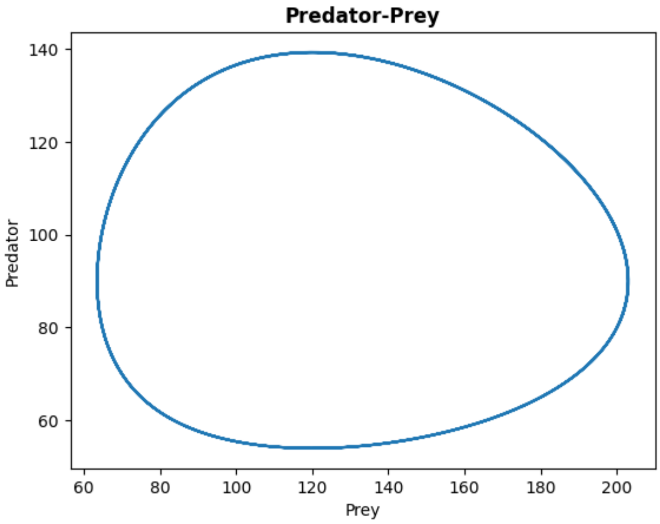

FIG. 10: The graph shows population density of a Y-species vs population density of a X-species.

Here x(0) = 200, y(0) = 80, A = 1.0, B = 0.01, C = 0.5, D = 0.005, $\epsilon = 0.1$, $\Delta t = 0.0001$.

The maximum value of y : 139.2702.
The maximum value of Y-species is higher when there is no human interference(without fishing).

No predator

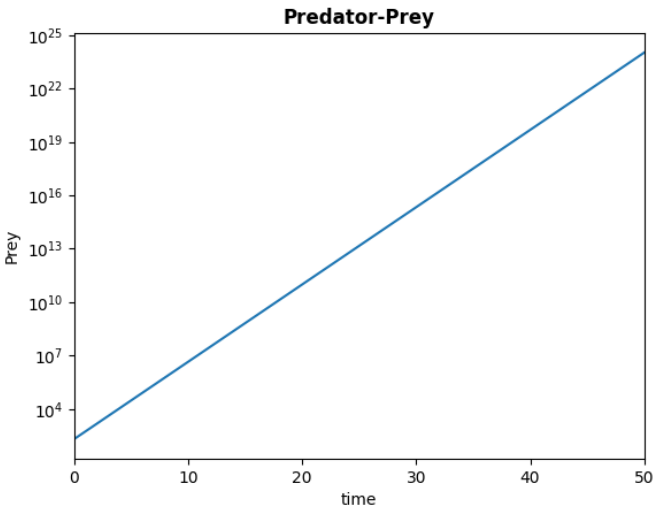

FIG. 11: The graph shows population density of a X-species(logarithmic) vs time.

Here x(0) = 200, y(0) = 0, A = 1.0, B = 0.01, C = 0.5, D = 0.005, $\epsilon = 0$, $\Delta t = 0.0001$.

Here the X-species(logarithmic scale) is linearly increased when there is no predator(Y-species).

No prey

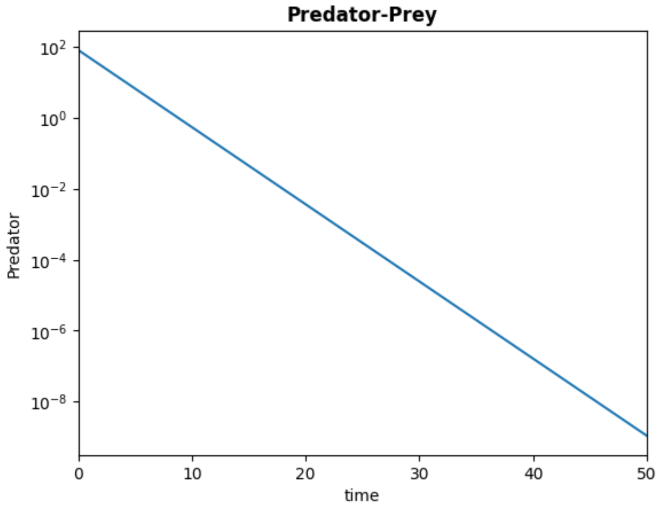

FIG. 12: The shows population density of a Y-species(logarithmic) vs time.

Here x(0) = 0, y(0) = 80, A = 1.0, B = 0.01, C = 0.5, D = 0.005, $\epsilon = 0$, $\Delta t = 0.0001$.

Here, the Y-species(logarithmic scale) is linearly decrease when there is no prey(X-species).
ch