

2.

Consider the crypto system below and compute $H(K-C)$

$P = \{a, b, c\}$ with $P_P(a) = 1/3$ $P_P(b) = 1/6$ $P_P(c) = 1/2$

$K = (k_1, k_2, k_3)$ with $P_K(k_1) = \frac{1}{2}$ $P_K(k_2) = \frac{1}{4}$ $P_K(k_3) = \frac{1}{4}$

$C = \{1, 2, 3, 4\}$

$e_{k_1}(a) = 1$ $e_{k_1}(b) = 2$ $e_{k_1}(c) = 2$

$e_{k_2}(a) = 2$ $e_{k_2}(b) = 3$ $e_{k_2}(c) = 1$

$e_{k_3}(a) = 3$ $e_{k_3}(b) = 4$ $e_{k_3}(c) = 4$

Compute:

$$P_c(1) = \frac{7}{24}$$

$$P_c(2) = \frac{5}{12}$$

$$P_c(3) = \frac{1}{8}$$

$$P_c(4) = \frac{1}{6}$$

Since $H(X) = - \sum_{i=1}^n p_i \log_2 p_i$, find $H(p)$, $H(k)$, and $H(c)$

$$H(K) = -(\frac{1}{2} \log_2 \frac{1}{2} + \frac{1}{4} \log_2 \frac{1}{4} + \frac{1}{4} \log_2 \frac{1}{4}) = 1.5$$

$$H(P) = -(\frac{1}{3} \log_2 \frac{1}{3} + \frac{1}{6} \log_2 \frac{1}{6} + \frac{1}{2} \log_2 \frac{1}{2}) = 1.459$$

$$H(C) = -(\frac{7}{24} \log_2 \frac{7}{24} + \frac{5}{12} \log_2 \frac{5}{12} + \frac{1}{8} \log_2 \frac{1}{8} + \frac{1}{6} \log_2 \frac{1}{6}) = 1.851$$

$$H(K) + H(P) - H(C) = 1.5 + 1.459 - 1.851 = 1.108$$