Tommy Olney October 9, 2018

1.

a. Prove that a ≡ b(mod n) implies b ≡ a(mod n)

$a \equiv b \; mod \; n \rightarrow n|(b - a) \rightarrow n|(-1)(b - a) \; or \; n|(a - b) \therefore b \equiv a \; mod \; n.$

b. Prove that a ≡ b(mod n) and b ≡ c(mod n) imply a ≡ c (mod n)

$a \equiv b \; mod \; n \; and \; b \equiv c \; mod \; n \rightarrow n|(b-a) \; and \; n|(c-b) \rightarrow n|(b - a + c - b) \rightarrow n|(c - a) \therefore a \equiv c \; mod \; n.$

2.

a. 1234 mod 4321

x	Remainder
0	4321
1	1234
-3	619
4	615
-7	4
1075	3
-1082	1
3239	0

b. 24140 mod 40902

x	Remainder
0	40902
1	24140
-1	16762
2	7378
-5	2006
17	1360
-22	646
61	68
-571	34
No inverse	0

c. 550 mod 1769

x	Remainder
0	1769
1	550
-3	119
13	74
-16	45
29	29
-45	16
74	13
-119	3
550	1
550	0

3.

a. Reducible: $x^3 + 1 = (x + 1)(x^2 + x + 1)$
b. Irreducible
c. Reducible: $x^4 + 1 = (x + 1)^4$

4.
a. $1\ (mod\ 2)$
b. $x + 1 (mod\ 3)$

5. Shown in pictures below.

$$\sum_{k=0 \to k=4} Pr(1/k) \cdot Pr(k)$$

for k_1, $e_{k_1}(?) = 1$ $? = a$, $a = 1/3$

$Pr(k_1) = 1/2$, $1/2 \cdot 1/3 + k_2 ...$

$Pr(1) = \sum_{k=1 \to 4} Pr(1|k) \cdot Pr(k)$

k_1 a or c $((1/4 + \frac{1}{2}) \cdot \frac{1}{2}) = 1/2$

k_2 $Pr(1|k_2)$ $(1/2 \cdot 1/4)$

k_3 $Pr(1|k_3)$ $0 \cdot$ $3/8$

k_4 $Pr(1|k_4)$ 0

$Pr(2|k) \cdot Pr(k)$

$Pr(2)$ k_1 b $1/4 \cdot 1/2$ $2/16$ $= 1/4$

k_2 a $1/4 \cdot 1/4$ $1/16$

k_3 b $1/4 \cdot 1/4$ $1/16$

k_4 0

$Pr(3|k) \cdot Pr(k)$

$Pr(3)$ k_1 0 $= 1/8$

k_2 b $1/4 \cdot 1/4$

k_3 a $1/4 \cdot 1/4$

k_4 a $1/4 \cdot 0$

$Pr(4|k) \cdot Pr(k)$

$Pr(4)$ k_1 0 $= 1/8$

k_2 0

k_3 c $\frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$

k_4 0

$$Pr(h|c) = Pr\frac{(c|h) \cdot Pr(h)}{Pr(c)}$$

$(1\|h_1)$	a and c	3/4
$(1\|h_2)$	c	1/2
$(1\|h_3)$	0	0
$(1\|h_4)$	0	0
$(2\|h_1)$	b	1/4
$(2\|h_2)$	a	1/4
$(2\|h_3)$	b	1/4
$(2\|h_4)$	0	0
$(3\|h_1)$	0	0
$(3\|h_2)$	b	1/4
$(3\|h_3)$	a	1/4
$(3\|h_4)$	a	1/4
$(4\|h_1)$	0	0
$(4\|h_2)$	0	0
$(4\|h_3)$	c	1/2
$(4\|h_4)$	b and c	3/4

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$Pr(h_1|1) = Pr(1|h_1)\,Pr(h_1)/Pr(1)$

$1/3 \cdot 1/2 \cdot 7/21$

$Pr(h_1|1) \quad (3/4 \cdot 1/2)/1/2 = 3/4$

$Pr(h_1|2) \quad (1/4 \cdot 1/2)/1/4 = 1/2$

$Pr(h_1|3) \quad 0$

$Pr(h_1|4) \quad 0$

$Pr(h_2|1) \quad (1/2 \cdot 1/4)/1/2 = 1/4$

$Pr(h_2|2) \quad (1/4 \cdot 1/4)/1/4 = 1/4$

$Pr(h_2|3) \quad (1/4 \cdot 1/4)/1/8 = 1/2$

$Pr(h_2|4) \quad 0$

$Pr(h_3|1) \quad 0$

$Pr(h_3|2) \quad (1/4 \cdot 1/4)/1/4 = 1/4$

$Pr(h_3|3) \quad (1/4 \cdot 1/4)/1/8 = 1/2$

$Pr(h_3|4) \quad (1/2 \cdot 1/4)/1/8 = 1$

$Pr(h_4|1) \quad 0$

$Pr(h_4|2) \quad 0$

$Pr(h_4|3) \quad (1/4 \cdot 0) \quad 0$

$Pr(h_4|4) \quad 0$

$$-\Big(1/2\big(3/4\log_2(3/4) + 1/4\log_2(1/4) + 0\log(0)\big) + \rightarrow \frac{3\log(3)}{8\log(2)} - 1$$

$$+\,1/4\big(1/2\log_2(1/2) + 1/4\log_2(1/4) + 1/4\log_2(1/4)\big) \rightarrow -3/8$$

$$+\,1/8\big(0\log 0 + 1/2\log_2(1/2) + 1/2\log_2(1/2)\big) \rightarrow -1/8$$

$$\boxed{= .9056}$$