SOLUTION OF BURGERS EQUTION USING CONSERVATIVE AND NON-CONSERVATIVE FORMS

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1. INTRODUCTION

This project focuses on the numerical simulation of Burgers' equation, a fundamental nonlinear partial differential equation that arises in fluid dynamics, acoustics, and other areas of physics. The primary objective is to implement numerical methods to solve Burgers' equation, compare results in both conservative and non-conservative forms, and create visualizations to gain insights into the behavior of the solutions.

2. METHODOLOGY

There is more than one method for solving a numerical problem. The chosen method is the Forward-Time Centered-Space (FTCS) scheme, a finite difference scheme known for its simplicity and ease of implementation. The FTCS scheme discretizes the temporal and spatial derivatives of Burgers' equation, allowing for a step-by-step solution. Solutions were implemented via MATLAB.

2.1 Mathematics

The mathematical foundation of the project is based on the Burgers' equation in its conservative and non-conservative forms. The conservative form,

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial x}\left(\frac{f^2}{2}\right) = \frac{\nu\,\partial^2 f}{\partial x^2} \quad (1)$$

represents the conservation of mass and the effects of viscosity. The non-conservative form,

$$\frac{\partial f}{dt} + \frac{f\,\partial f}{dx} = \frac{\nu\,\partial^2 f}{\partial x^2} \quad (2)$$

highlights the convective nature of the equation.

Using FTCS for those 2 equations creates factors which are used to simplify coding processes which are:

$$fac1 = 0.5 * \frac{\partial t}{\partial x} \quad (3)$$

$$fac2 = \frac{\nu\,\partial t}{\partial x^2} \quad (4)$$

$$fac3 = 0.25 * \frac{\partial t}{\partial x} \quad (5)$$

Equations 3 and 4 are used for non-conservative form, and 3 and 5 are used for conservative form.

2.2 Implementation

FTCS scheme was implemented on equations 1 and 2 using the equations 3,4 and 5. After all the new equations obtained Burger's equation was solved using the initial condition $f(x, t = 0) = \sin(2\pi x) + 1.0$ and the periodic boundary conditions.

Initially simulations were conducted with a space grid of 20 points and time grid of dt= 0.01. After that time grid was furthermore decreased due to the oscillations on the simulation and all the other space grids were conducted with dt = 0.001. After 20-point space grids 50,100 and lastly 200 grid points were used to simulate solution. When changing the number of grid points, convergence and stability of the solution were also checked with numerical experimentations and the integrated difference method. For integrated difference a grid between 20 and 200 was used with an increment of 10.

3. RESULTS AND DISCUSSION

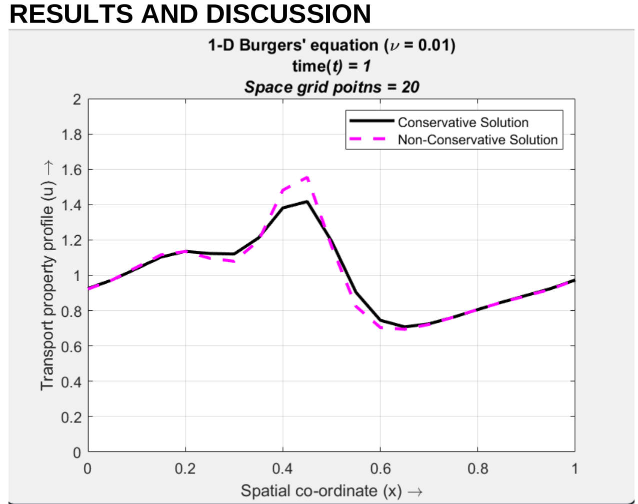

Figure 1. Burger's equation solution for 20 grid points.

In the 20 grid points we see that the solution is not very stable and has extra oscillations as seen on Fig.1 .

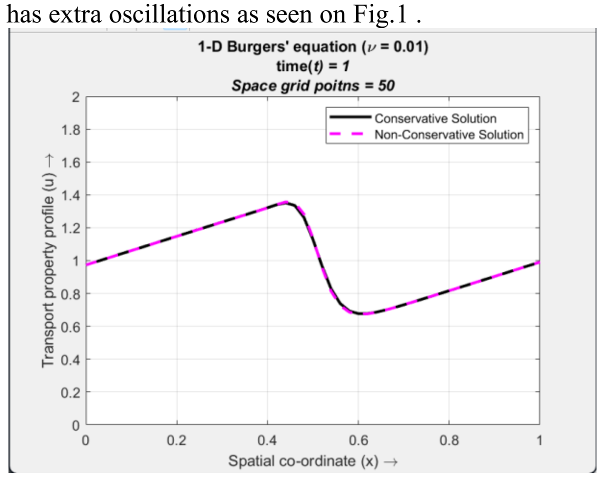

Figure 2. Burger's equation solution for 50 grid points.

When the gird is further increased to 50 points, we see that solution is much more stable but the non-conservative and the conservative solutions have some differences as demonstrated on Fig. 2.

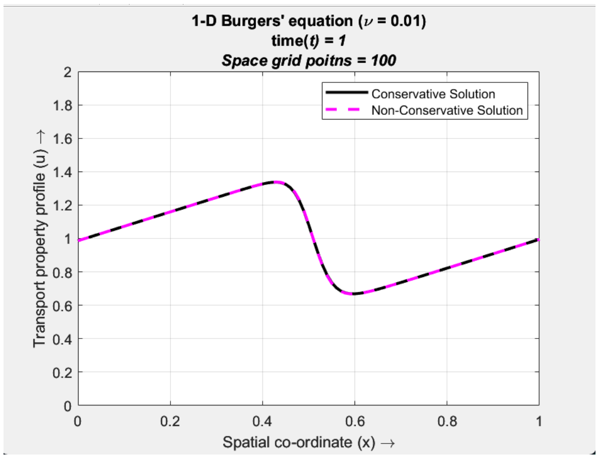

Figure 3. Burger's equation solution for 100 grid points.

A further increased solution which has 100 points has the same solutions for non-conservative and the conservative solutions.

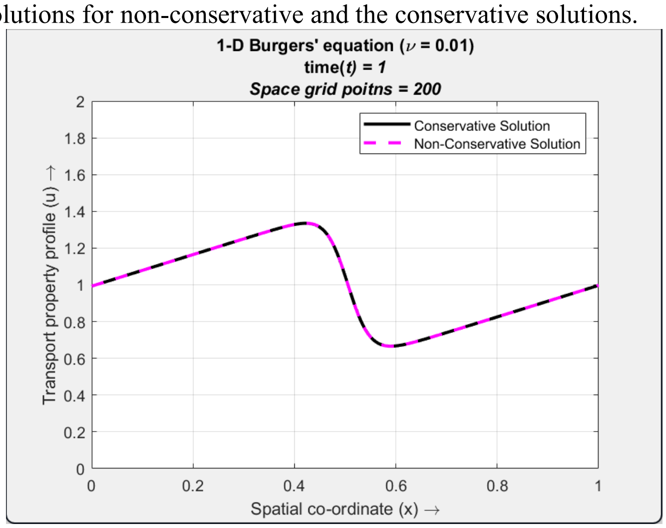

Figure 4. Burger's equation solution for 200 grid points.

To check the convergence lastly 200 grid points was used and the grid convergence for those two equations was satisfied.

In the implementation of those grid points also a stability analysis was done which resulted in the solution is only stable if the space grid points are maximum 220 and the solution is reasonable only if the space grid is higher than 20. Also, by checking the CFL number which is

$$CFL = \frac{\nu \Delta t}{\Delta x^2} \leq \frac{1}{2} \tag{6}$$

Is obtained when $\Delta x = 0.0045$ when grid is equal to 220 that makes CFL number equal to 0.48 which is smaller than 0.5. When this number is exceeded, the solution diverges into the infinity, or a bad grid is used solution becomes unreasonable.

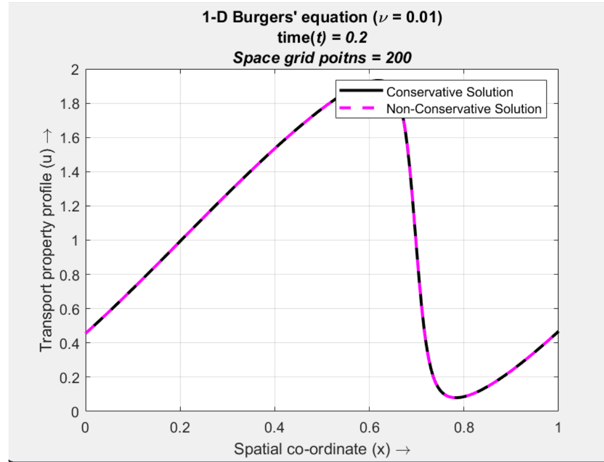

Figure 5. 200 space grids at t =0.2

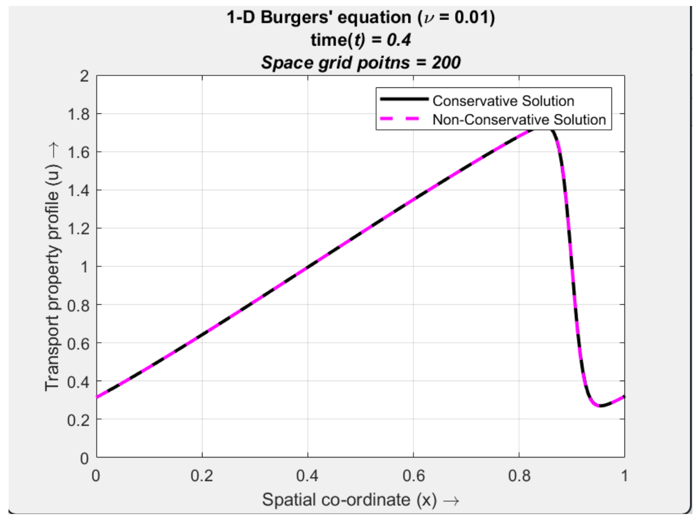

Figure 6. 200 space grids at t =0.4

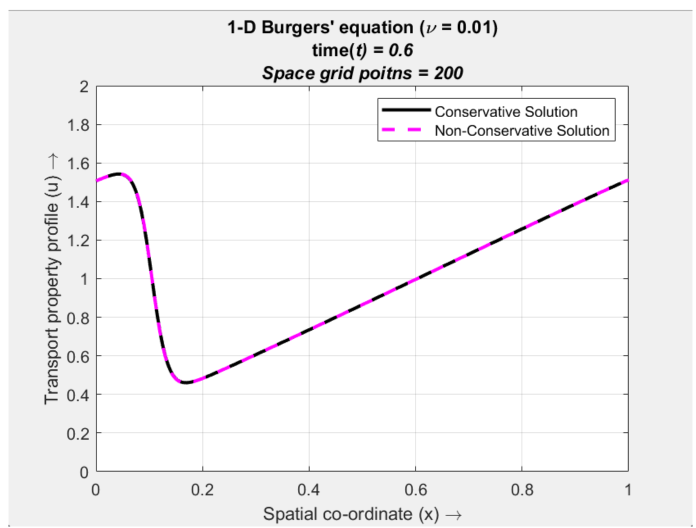

Figure 7. 200 space grids at t =0.6

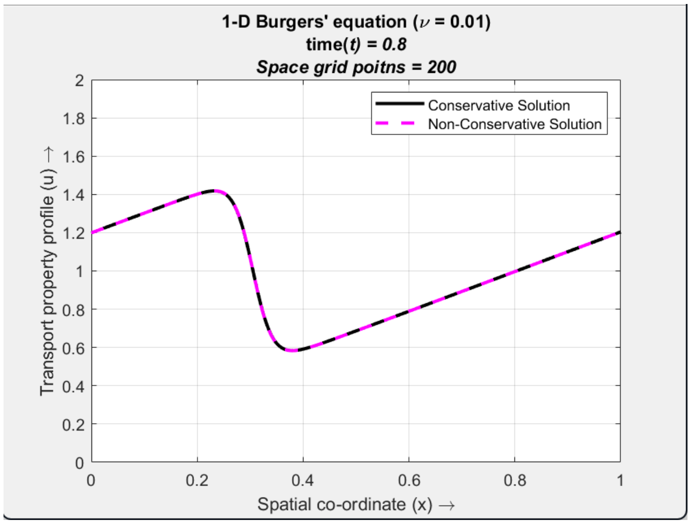

Figure 8. 200 space grids at t =0.8

From Fig.4-8 we see that solution converges into the final value with near zero oscillations due to solution being stable.

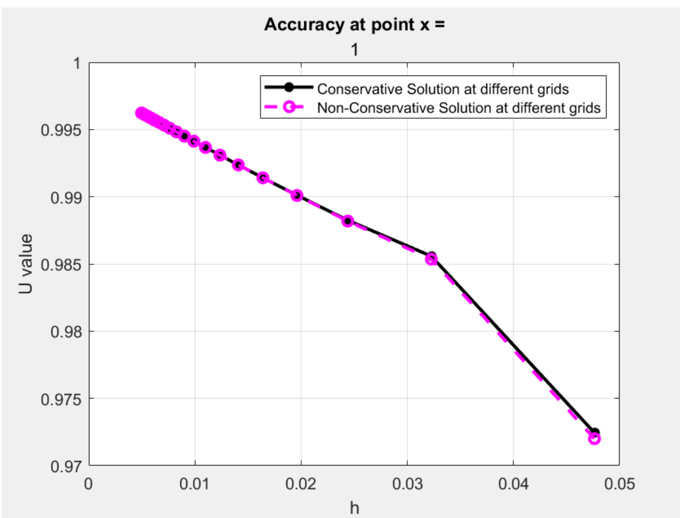

Figure 9. Integrated difference at x=1

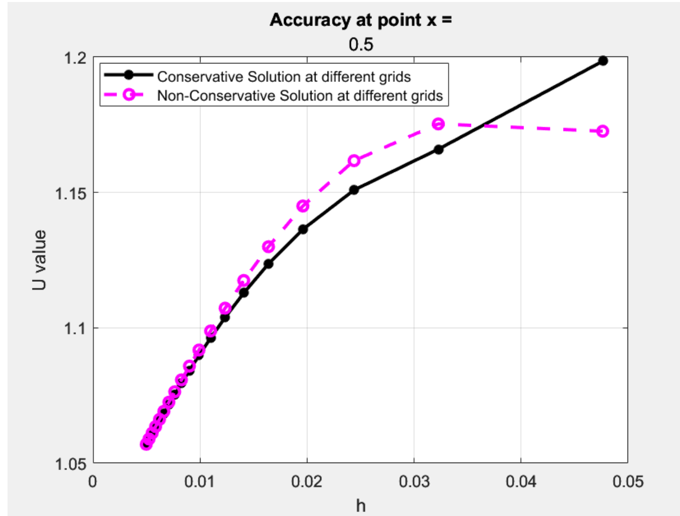

Figure 10. Integrated difference at x=0.5

For space point x=1 and x=0.5 integrated difference was applied for different grids. As Fig. 9 and 10 shows error at different space points are different. This might be due to the boundary conditions and implementation issues. But the common part of those two integrated difference graphs is that they show us grid convergence is obtained before the 200-space grid which corresponds to the 190-space grid.

4. CONCLUSION

In conclusion, .numerical solution of Burger's equation require a good mathematical model implementation and a fair amount of numerical solutions method to obtain reasonable solution. Otherwise, non-adjusted grid would cause oscillations or unexpected behaviors so that the final solution would diverge into infinity.