Short HW 3: **Gram–Schmidt**

Course: Physics 017, *Linear Algebra for Physics* (S2022)
Instructor: Prof. Flip Tanedo (flip.tanedo@ucr.edu)
Due by: **Thursday**, April 14 (yes, *this* Thursday)

Note that this short assignment is due by class on Thursday. You have only *two days* to do it. This should be quick, I recommend doing it right after class on Tuesday.

1 Gram–Schmidt for a vectors in 3D Euclidean Space

You are given three vectors that are *linearly independent*[1]:

$$\mathbf{v} = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} \qquad \mathbf{w} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \qquad \mathbf{z} = \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} \tag{1.1}$$

Perform the Gram–Schmidt procedure to derive an orthonormal basis from these vectors. The first basis vector $\mathbf{e}_{(1)}$ should be parallel to $\mathbf{v}$. The second basis vector $\mathbf{e}_{(2)}$ should be on the $\mathbf{v}$–$\mathbf{w}$ plane.

2 Professor Tanedo doesn't understand the chain rule

In class last week we talked about partial derivatives being basis vectors for first-order differential operators:

$$D = d^1 \frac{\partial}{\partial x} + d^2 \frac{\partial}{\partial y} \ . \tag{2.1}$$

We claimed that it was 'obvious' that under a change of coordinages $x \to x'$, there is a natural change of basis

$$\frac{\partial}{\partial x} = \left(\frac{\partial x'}{\partial x}\right) \frac{\partial}{\partial x'} \ . \tag{2.2}$$

In class Professor Tanedo simply *cancelled out the $\partial x'$* factors. This is mathematically dubious at best! Show how to do this properly.

The new coordinates x' are a function of the old coordinates: $x'(x)$. This is simply the statement that every point in the old coordinates maps onto a single point in the new coordinates. Now take a test function $f(x')$ that is a function of the new coordinate x'. Take the derivative of $f(x')$ with respect to the old coordinate x by using the chain rule:

$$\frac{\partial}{\partial x} f\left(x'(x)\right) \ . \tag{2.3}$$

Since the result is true for any test function f, argue that (2.2) must be true in general.[2]

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[1] This means that you cannot write any vector as a linear combination of the other vectors. That is: each vector has at least some component that points in a 'new' direction relative to the plane *spanned* by the other vectors.

[2] There are some assumptions here about f being a sufficiently differentiable function... let us not concern ourselves with the deviant cases.