Short HW 8: **The exponential basis for Fourier Series**

Course: Physics 017, *Linear Algebra for Physics* (S22)
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Due by: **Thursday**, May 19

Note that this short assignment is due by class on Thursday. You have only *two days* to do it. This should be quick, I recommend doing it right after class on Tuesday.

In class we presented a basis of complex functions defined from $-L \leq x \leq L$:

$$|n\rangle = \sqrt{\frac{1}{2L}} e^{-\frac{in\pi}{L}x} \ . \tag{0.1}$$

A general function $f = |f\rangle$ is expanded as follows:

$$|f\rangle = \sum_{n=-\infty}^{\infty} c_n |n\rangle \ , \tag{0.2}$$

where we note that the sum extends over all integers from minus to plus infinity. We have not yet specified boundary conditions at $x = \pm L$. This imposes relationships between the coefficients c_n.

1 Reproducing the sine series

Suppose our function space has Dirichlet boundary conditions:

$$f(L) = 0 \qquad\qquad f(-L) = 0 \ . \tag{1.1}$$

What condition does this impose on the c_n? Hint: you should remember that these boundary conditions are satisfied by a Fourier sine series. What relations do c_n and c_{-n} satisfy for the expansion to be a sum of sine functions?

2 Orthonormality

2.1 Orthogonality

Show that $\langle n|m\rangle = 0$ for $n \neq m$.

2.2 Normality

Show that $\langle n|n\rangle = 1$. Explain why this integral is non-zero, even though the previous integral for $\langle n|m\rangle$ is zero for $n \neq m$.

3 But my textbook says something different...

If you look up the complex Fourier series in some physics textbooks[1], you may find a different expansion:

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{-\frac{in\pi}{L}x} \qquad\qquad c_n = \frac{1}{2L}\int_{-L}^{L} f(x)^* \, e^{-\frac{in\pi}{L}x} \; . \tag{3.1}$$

This looks like the normalization is totally different from our conventions! Show that the textbook rules above give the same Fourier series as our conventions.

COMMENT: our convention is better because our basis functions are normalized. The textbook definition uses not-normalized basis functions, and as a result the coefficients c_n swallow part of the normalization factor.

[1] e.g. Felder & Felder, *Mathematical Methods in Engineering and Physics*, (9.5.2).