TODAY: DIM. ANALYSIS & EFFECTIVE THEORY
LOOP DIAGRAMS

Recall: S, ACTION, ENCODES OUR DYNAMICS

in fact: contribution of a path to the amplitude is $\underbrace{\exp(iS)}$

$\Rightarrow S$ IS DIMENSIONLESS!
(in SI: $S/\hbar$ IS DIMENSIONLESS)

$$S = \underbrace{\int dt \int d^3x}_{\int d^4x} \; \underbrace{\mathcal{L}}_{\text{LAGRANGIAN DENSITY}}$$

$[d^4x] = -4 \quad \Rightarrow \quad [\mathcal{L}] = 4$

MASS DIM — eg $\mathcal{L}$ is in $[\text{GeV}]^4$

what are the dimensions of FIELDS/PARTICLES?

@ the bare minimum, the LAGRANGIAN CONTAINS THE KINETIC TERM

FACT: BOSONS: $(\partial\phi)^2$ OR $F_{\mu\nu}F^{\mu\nu} \sim (\partial A)^2$

FERMIONS: $\psi^\dagger i\bar{\sigma}^\mu \partial_\mu \psi$

So: $[\phi] = [A] = 1 \quad \leftarrow \quad [\partial_\mu] = \left[\frac{\partial}{\partial x}\right] = +1$

$\quad$ & $[(\partial\phi)^2] = 2[\partial] + 2[\phi] = 4$

AND $[\psi] = 3/2 \quad \leftarrow \quad \bar{\sigma}^\mu$ IS A MATRIX OF DIMENSIONLESS NUMBERS.

$[\psi^\dagger \partial \psi] = 2[\psi] + [\partial] = 4$

$[\psi^\dagger] = [\psi] \qquad [\partial] = 1$

this means the COUPLING constants of the GAUGE forces ARE DIMENSIONLESS

$$\partial_\mu \to D_\mu \equiv \partial_\mu + igA_\mu$$

SAME DIMENSION (+1)

$[g] = 0$, dimensionless

in physics: dimensionless numbers are usually: 0, ≈ 1, ∞

OR OTHERWISE THERE IS A REASON WHY THEY ARE NOT.

YUKAWAS: $y\, L^{\dagger} H e + \ldots$ } WHOLE TERM IS DIM 4

$\frac{3}{2}$ 1 $\frac{3}{2}$

$\Rightarrow [y] = 0$

nb: y is a matrix, naively expect all entries to be $\mathcal{O}(1)$

$\Rightarrow$ eigenvalues are $\propto$ MASS of fermions

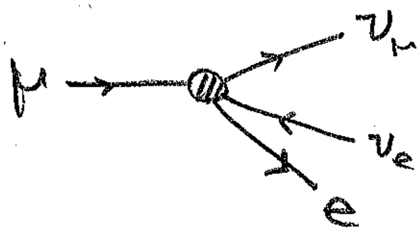
$\to$ why is $m_e \ll m_\tau$??

(eigenvalues of random matrices can be kind of odd...)

so far: all interactions are characterized by A DIMENSIONLESS coupling!

why not any other interaction?

<u>example</u> LONG AGO, WE OBSERVED



& WE THOUGHT: there SEEMS to BE A TERM IN the LAGRANGIAN,

$$\mathcal{L} \supset \# \underbrace{(e^\dagger \nu_e)(\nu_\mu^\dagger \mu)}_{\text{or maybe } (e^\dagger \mu)(\nu_\mu^\dagger \nu_e)\text{? etc.}}$$

BUT THIS # IS <u>DIMENSIONFUL</u>

$$\boxed{[\#] = -2}$$

there is a high power of fields
↳ many particles in vertex
→ LARGE MASS DIM

⇒ so "COUPLING" HAS A NEGATIVE MASS DIM s.t. $[\mathcal{L}] = 4$

↳ what mass scale?

well, now that we know the SM, we know where $\mu \to e \nu \bar{\nu}$ comes from:

$$\sim \frac{g^2}{p^2 - M_W^2} \approx \underbrace{\frac{g^2}{M_W^2}}$$

$\approx G_F$, FERMI COUPLING

(up to factors of $\sqrt{2}$ that I always mess up)

<u>OBSERVE</u>: the mass scale that shows up in our "effective coupling" is PHYSICALLY SIGNIFICANT

↳ IT IS THE SCALE @ WHICH NEW PHYSICS (W BOSON) "completes" our <u>effective</u> theory.

EFF THY only valid @ $E \ll M_W$

the "complete" theory only has couplings w/ zero mass dimension.

<u>REMARK</u>: @ $E \gg M_W$, you do <u>not</u> see the effective theory: you just produce the W on shell.

<u>REMARK</u>: GRAVITY HAS A COUPLING $G_N \sim 1/M_{Pl}^2$

↳ BUT $M_{Pl} \sim 10^{19}$ GeV

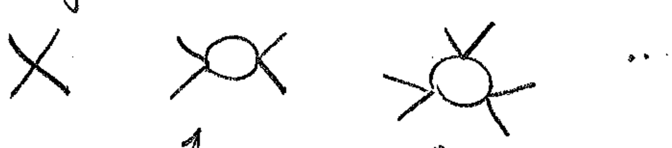
<u>RENORMALIZABILITY</u> ("deep idea" in physics)

for simplicity, imagine theory w/ one particle, ϕ that interacts w/ itself.

interactions w/ more than 4 powers of ϕ are suspicious → "NON RENORMALIZABLE"

↳ comes w/ negative mass DIM COUPLING

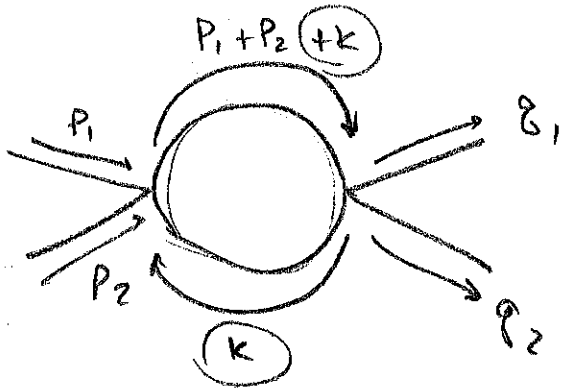
<u>why?</u> imagine $\lambda\phi^4$ interaction.



LOOP DIAGRAMS

<u>ASIDE:</u> LOOP DIAGRAMS : unconstrained momentum of internal particles

$\Rightarrow$ must sum (integrate) over all!



MUST SUM OVER ALL ALLOWED MOMENTA!

$$\int d^4k \; \lambda^2 \frac{1}{(P_1+P_2+k)^2 - m^2} \frac{1}{k^2 - m^2} \xrightarrow{\text{LARGE } k} \lambda^2 \underbrace{\int k^3 dk \frac{1}{k^4}}_{\ln \Lambda}$$

where Λ is a UV cutoff

($1/\Lambda$ is a small length scale)

SCALE @ which some new dynamics steps in to make physics sensible

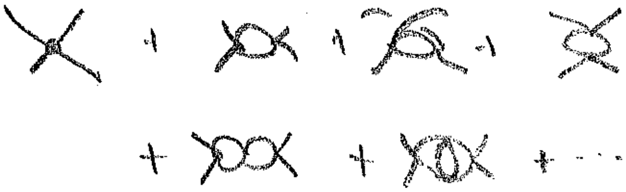
$\rightsquigarrow$ the NEW DYNAMICS WAS ALWAYS THERE, BUT THE EFFECT IS USUALLY SMALL.

(continuum limit: $\Lambda \to \infty$)

infinities in our physical theory?!

<u>RENORMALIZATION</u>: we never measure  $\leftarrow \lambda$

we measure

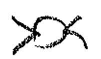


all these loop diagrams seem to have log <u>divergences</u> as $\Lambda \to \infty$

BUT WE <u>MEASURE</u> A FINITE CROSS SECTION that WE INTERPRET AS A MEASUREMENT of the COUPLING λ.

so if we calculate to $\mathcal{O}(\lambda^2)$ (ie 1-LOOP DIAGRAM)

then $\underbrace{\lambda_{meas}}_{\text{finite}} \sim \lambda_0 + \#\lambda_0^2 \underbrace{\log \Lambda}_{\to\infty}$

  etc.

$$\Rightarrow \underbrace{\lambda_0 \sim \lambda_{meas}}_{\text{LEADING ORDER}} - \#\lambda_{meas} \log \Lambda$$

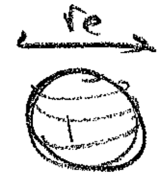
$\lambda_0 \to \lambda_{meas}$ @ L.O

DIVERGENT!

ie λ_0, the "BARE" COUPLING (UNCORRECTED) is FORMALLY DIVERGENT AS WELL in the continuum LIMIT!

→ this is <u>not</u> a problem. just keeping track of physical vs unphysical quantities eg. WHAT IS the MASS of the ELECTRON?

$m_e \sim 0.5$ MeV



some shell of net charge $-e$

$E \approx mc^2$ ← but E has contribution from POTENTIAL ENERGY

$$V \sim + \underbrace{\frac{e^2}{4\pi}}_{\alpha \approx 1/137} \frac{1}{r_e} = \Delta E_{\text{coulomb}}$$

$r_e < 10^{-14}$ cm

$10^{-2} \times 10^{14} \frac{1}{\text{cm}}$

"BARE MASS"

$$m = m_0 + \Delta E$$

USE: $\hbar c = 1 = 200$ MeV fm $= 200$ MeV $\times 10^{-13}$ cm

f $= 10^{-15}$

f -15
p -12
n -9
μ -6
m -3
c -2

$\Delta E = 10^{-2} \times 10^{14} \cdot 200$ MeV $\times 10^{-13}$
$= 2 \times 10^4$ MeV

$\frac{1}{r_e} \sim 2 \times 10^6$ MeV

$$0.5 \text{ MeV} = \left(\frac{m_0}{\text{MeV}} + 20{,}000\right) \text{MeV}$$

$$\boxed{m_0 = -19{,}999.5 \text{ MeV}}$$

"tuning" @ 4 ORDERS of MAGNITUDE

BARE MASS HAS NO PHYSICAL SIGNIFICANCE

BUT TUNING IS UNUSUAL.

(NB: PAIR PRODUCTION OCCURS @ LENGTH SCALES $\sim 1/2m_e$

compare to $r_e \sim \frac{1}{2 \times 10^6 \text{ MeV}}$ → tuning is much better!)

So: for $\lambda\phi^4$ interaction, we have to account for the log Λ divergence, but it just corrects the 4-point interaction.

What about  ?

 $\sim \underbrace{\int d^4k \, \frac{1}{k^6}}_{\text{finite!}}$ @ $\underbrace{\text{large } k}$

btw: small distance
we are probing UV micro-physics!

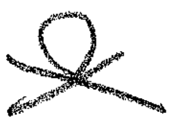
no need to redefine bare params.

WHAT ABOUT NONREN. INTERACTION?

e.g. $\frac{c}{M^2}\phi^6$ ⟵ 

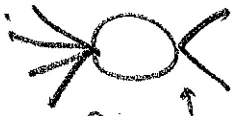
for some M^2 & dimless c

w/ this interaction, can write:

 $\sim \int d^4k \, \frac{1}{k^2} \sim \underbrace{\Lambda^2}_{\text{divergent}}$

so need to have × in theory w/ $\underline{\text{bare}}$ coupling λ_0 to absorb this divergence.

$\underline{\text{physical}}$ λ is then a not predictable quantity then can keep going!



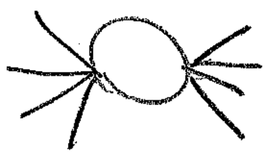
$\sim \int d^4k \frac{1}{k^4} \sim \underbrace{\log \Lambda}_{\text{divergent}}$

$\frac{c}{M^2}$ $\quad$ λ

so $\frac{c_0}{M^2}$ absorb to ABSORB divergence ... but now have to measure 6 point scattering.

(not predicted)

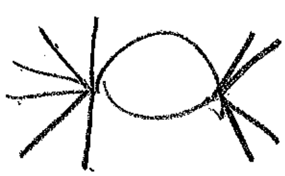
(test a theory: make at least N+1 measurements of a theory w/ N parameters)



$\sim \int d^4k \frac{1}{k^4} \sim \underbrace{\log \Lambda}_{\text{DIVERGENT}}$

$\Rightarrow$ NEED to ADD $\frac{d_0}{M^4}$ COEFF. 

to SOAK UP DIVERGENCE . . .
not predicted



$\sim \int d^4k \frac{1}{k^2} \sim \Lambda^2$

$\frac{d}{M^4}$ $\quad$ $\frac{c}{M^2}$

so now add $\frac{e_0}{M^6}$ COEFF 

to soak up divergence

... not predicted . . .

& ad infinitum!?!

$\Rightarrow$ so for A LONG TIME WE HAD ISSUES w/ non-renormalizable THEORIES

NOW WE KNOW BETTER

EFFECTIVE THEORY :

BREAKS down @ some scale

↳ usually scale where loop corrections are as large as leading diagrams

(TAYLOR EXP BREAKS)

⤳ implicit: a theory, not the theory.

next time : HIERARCHY PROBLEM

HW: 3rd IN W.C.

casa on HIERARCHY