

SHORT HW 4: Momentum Flow

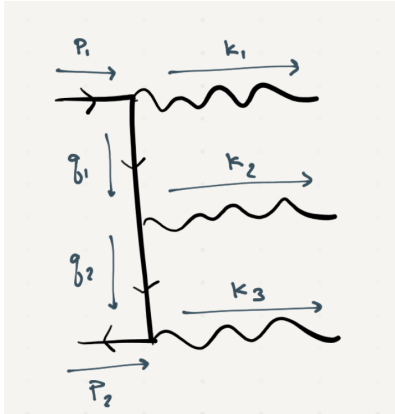
COURSE: Physics 165, *Introduction to Particle Physics* (2022)

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DUE BY: Thursday, April 21

1 Total Momentum Conservation for a $2 \rightarrow 3$ process

The following diagram is one contribution to the process $e^+e^- \rightarrow 3\gamma$:



Using the conservation of four-momentum at each vertex, show that the total four-momentum is conserved. That is, prove:

$$(p_1 + p_2)^\mu = (k_1 + k_2 + k_3)^\mu. \quad (1.1)$$

HINT: Start by writing the conservation of four-momentum at each vertex. Those are three equations with unspecified q_1^μ and q_2^μ . Use two equations to determine what these virtual momenta are, then plug them into the last equation to prove the above relation.

2 QED+ μ

Here are the Feynman rules for “QED+ μ ,” a theory of electrons, photons, and a second type of matter: muons, denoted μ .

$$\begin{array}{c} e \\ \swarrow \\ \text{---} \\ \nwarrow \\ e \end{array} \text{---} \text{wavy line} = ie$$

$$\begin{array}{c} \mu \\ \swarrow \\ \text{---} \\ \nwarrow \\ \mu \end{array} \text{---} \text{wavy line} = ie$$

$$\text{wavy line} \text{ with } \vec{p} \text{ above it} = \frac{i}{p^2}$$

$$\begin{array}{c} e \\ \text{---} \\ \vec{p} \end{array} = \frac{i}{p^2 - m_e^2}$$

$$\begin{array}{c} \mu \\ \text{---} \\ \vec{p} \end{array} = \frac{i}{p^2 - m_\mu^2}$$

[only approximate!]

Muons have the same charge as electrons and so the strength of their vertex with the photon is exactly the same. What are the conserved charges in this theory?

HINT: as you can guess, electric charge is conserved. However, there's a bit more to it than that. For example, $\mu^- \rightarrow e^- \gamma$ conserves electric charge. When the muon mass m_μ is larger than the photon mass, then $\mu^- \rightarrow e^- \gamma$ is even kinematically allowed. However, this process cannot happen dynamically in the theory QED+ μ .

COMMENT: Professor Tanedo's early Ph.D research focused on this process for theories with extra dimensions.¹

¹<https://arxiv.org/abs/1004.2037>