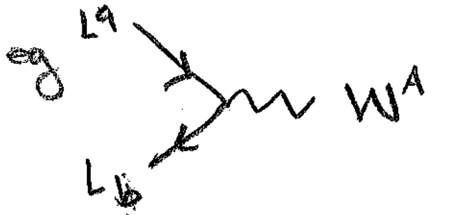
CREATING INVARIANTS

RECALL: INVARIANTS (WRT SYMMETRIES) CAN SHOW UP IN LAGRANGIAN

from which we READ off FEYNMAN RULES

eg


$= ig(T^A)^b{}_a$

<u>note</u>: we have updated our index convention

a, b, c : DOUBLET / FUNDAMENTAL INDEX
A, B, C : TRIPLET / ADJOINT INDEX

SU(2) NAME — GENERAL SYMM. NAME

ALWAYS UPPER INDEX. CONTRACT 2 REPEATED UPPER INDICES. (R REP, R REP HAS sense of charge conj → lower index)

UPPER : incoming particle / outgoing "anti-particle" → ARROW IN
LOWER : — " — anti / — " — PARTICLE → ARROW OUT

$(L^\dagger)_b$: $\dagger$ IS CHARGE CONJUGATION (<u>almost</u> antiparticle)

NOW LET US EXAMINE THIS for FROM the SYMMETRY GROUP PERSPECTIVE.

SU(2) : defined by unitary 2x2 mat w/ det = 1

$U^\dagger U = \mathbb{1}$ $\qquad$ $\det U = 1$

how many such matrices? $\boxed{\infty}$

JUST LIKE ∞ ROTATIONS IN 2D.

BUT: finite # of axes of rotation

eg x,y,z AXES for 3D ROT.

define a GENERATOR for EA AXIS

$$U(\theta^1, \theta^2, \theta^3) = e^{i(\theta^1 T^1 + \theta^2 T^2 + \theta^3 T^3)}$$

WHERE the T^A ARE TRACELESS, HERMITIAN

ensures $e^{i\theta T}$ is special $\qquad$ ensures $e^{iT\theta}$ is unitary

SU(2): $T^A = \frac{1}{2}\sigma^A$

IMPORTANT NORMALIZATION (CANNOT CHANGE THIS)

the KEY FEATURES of the SYMM. GROUP ARE ENCODED IN THE COMM. REL. OF THE GENERATORS.

$$U_1 U_2 - U_2 U_1 = (1 + i\theta_1^A T^A)(1 + i\theta_1^B T^B) - \dots$$
$$= i\theta_1^A \theta_2^B [T^A, T^B] + \mathcal{O}(\theta^3)$$

IN GEN: $\underbrace{[T^A, T^B]}_{\text{QUAD IN } T} = i f^{ABC} T^C$

LINEAR IN T

STRUCTURE CONST.

for SU(2): $f^{ABC} = \varepsilon^{ABC}$ ← Levi-Civita

$\varepsilon^{123} = 1$
$\varepsilon^{132} = -1$
$\varepsilon^{122} = 0$, etc

INDEX STRUCTURE: $(T^A)^a{}_b$

a ← fundamental (doublet)
b ← antifund. (antidoublet)
A ← ADJOINT (triplet)

The GENERATOR IS A $\boxed{\text{conversion factor}}$ between kinds of indices

eg LAST WEEK WE motivated COVARIANT DERIVATIVE.
if SU(2) is A LOCAL/GAUGE SYM, then WE INTRODUCE A GAUGE BOSON W:

$$\partial_\mu \to \partial_\mu + ig \underbrace{(T^A)^a{}_b W^A}$$

g ← SU(2) COUPLING strength

can think of this as $(W)^i{}_j = W^A T^A$

transforms as

$(W)^i{}_j \to U W U^\dagger$

eg

$(1+i\varepsilon T^1) W^2 T^2 (1-i\varepsilon T^1)$

$= W^2T^2 + i\varepsilon W^2 \underbrace{[T^1, T^2]}_{} + \mathcal{O}(\varepsilon^2)$

$- \varepsilon W^2 f^{12C} T^C$

(CAN CONVERT THIS INTO TRANSF ON A INDICES)

$W^2T^2 \to W^2T^2 - \varepsilon W^2 f^{12C} T^C$

$\boxed{Q}$: why only 3 W bosons?

<u>MATTER</u>: DOUBLET : $L^a = \begin{pmatrix} \nu \\ e \end{pmatrix}$

$$(L^\dagger)_b = (\nu^\dagger, e^\dagger)$$

can we write invariants?

$$L^a (L^\dagger)_a = L^a \longrightarrow\!\bullet\!\longrightarrow L_a$$

$$= \nu \longrightarrow\!\bullet\!\longrightarrow \nu + e \longrightarrow\!\bullet\!\longrightarrow e$$

<u>FACT</u>: SU(N) GIFTS US AN ADDITIONAL TENSOR, $\varepsilon^{a_1 \cdots a_N}$ & $\varepsilon_{a_1 \cdots a_N}$

So for SU(2), WE CAN ALSO USE ε^{ab} & ε_{ab}.

eg $L^a L^b \varepsilon_{ab} = L^a \longrightarrow\!\bullet\!\longleftarrow L^b \sim \varepsilon_{ab}$

$$\nu \longrightarrow\!\bullet\!\longleftarrow \nu \quad = 0$$

$$\nu \longrightarrow\!\bullet\!\longleftarrow e \quad = 1$$

$$e \longrightarrow\!\bullet\!\longleftarrow \nu \quad = 0$$

$$e \longrightarrow\!\bullet\!\longleftarrow e \quad = -1$$

that's WEIRD!

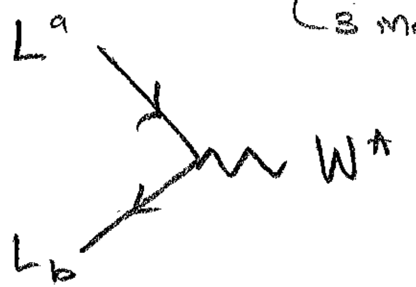
looks like ν can turn into e. clearly not SM

(but why should it be like SM?)

↳ what ingredients make this not possible in the SM?

3-point vertices

$$L^a (L^\dagger)_b \underbrace{(T^A)^b{}_a}_{\text{3 indices}} W^A$$



$\sim (T^A)^b{}_a$

we've already done this one.

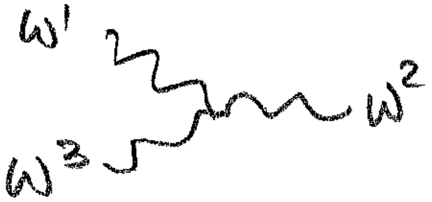
Are there other objects to use to get a 3-point?

$f^{ABC} = \underline{\varepsilon^{ABC}}$

like ε^{ab}, but came from a totally different place!

Note: ADJOINT indices

$$\varepsilon^{ABC} W^A W^B W^C$$



$\sim \varepsilon^{123}$

Q: why not $\varepsilon^{abc} (L^\dagger)_a (L^\dagger)_b (L^\dagger)_c$?

<u>excercise</u> : same thing for SU(3).

<u>GAUGED SU(3)</u> : color charge

$$q = \begin{pmatrix} q^1 \\ q^2 \\ q^3 \end{pmatrix} = \begin{pmatrix} r \\ g \\ b \end{pmatrix} \leftrightarrow q^a$$

$$q^+ = (\bar{r}, \bar{g}, \bar{b}) \leftrightarrow (q^+)_a$$

$$\partial_\mu \to D_\mu = \partial_\mu + i g_s T^A g^A$$

strong coupling GLUON

T^A : traceless, Hermitian 3x3 matrices
turns out there are 8 of them

could write as color/anticolor pair:

$$\underbrace{(g)^a{}_b}_{} = g^A T^A$$

3 colors x 3 anticolors = 9 gluons?

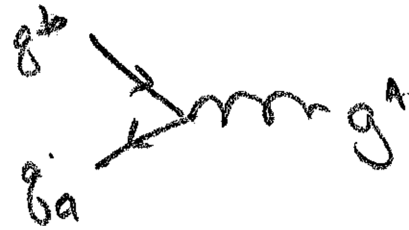
but structure of T^A prevents the 9th
→ eg tracelessness,

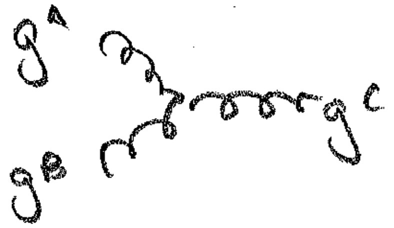
so the linear combo
w/ equal parts of r, g, b
("white") does not exist

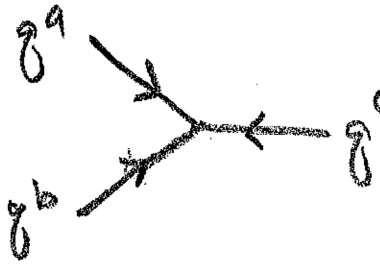
same structure constants f^{ABC}.

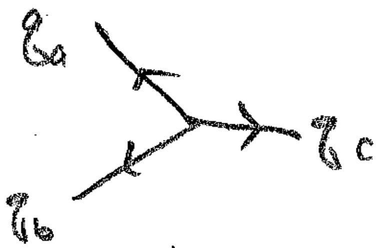
levi-civita: ε^{abc}, ε_{abc} ← SU(3)

<u>INVARIANTS</u>

 $= ig_s(T^A)^b{}_a$

 $= ig f^{ABC}$

 $= \varepsilon_{abc}$

 $= \varepsilon^{abc}$

kind of weird ... turns out not allowed once we include <u>SPIN</u> sym

<u>but</u>: this explains how 3 quarks become a bound state that is color-neutral.

$\boxed{\text{baryons}}$ (vs mesons ← $q\bar{q}$)