sorry, no notes from LAST TIME.

<u>PUNCHLINE</u>: IN ADDITION TO the QUdLe's, the TENSORS FOR $SU(3) \times SU(2) \times U(1)$

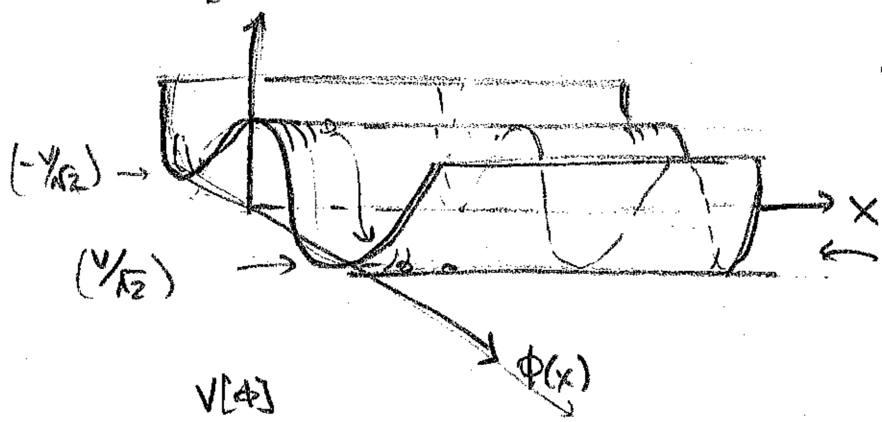
the HIGGS HAS A <u>VACUUM EXPECTATION VALUE</u> $\langle H^a \rangle = \frac{v}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix}$

"vev"

spontaneous sym breaking

→ this means @ every point in space & time the HIGGS has a constant value.

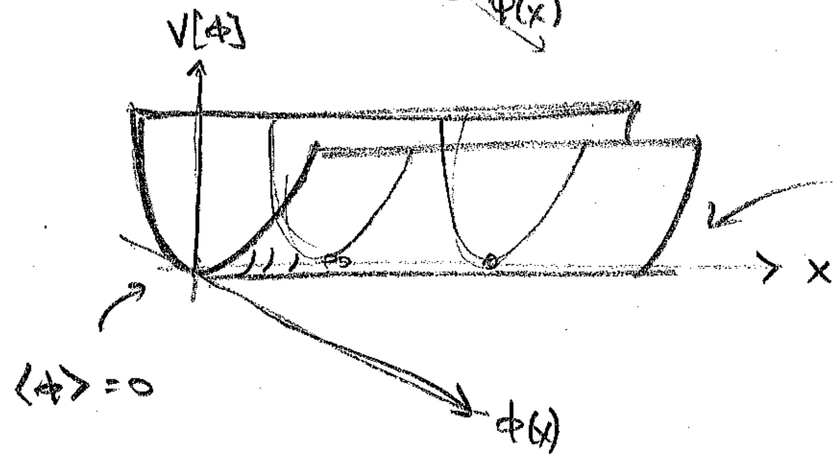
$V(\phi)$ ← @ EACH x, $\phi(x)$ is A "PARTICLE" w/ $\langle \phi(x) \rangle = v/\sqrt{2}$



HIGGS FIELD

vev

minimum



ORDINARY FIELD

no vev

ϕ is "off" by default until we excite it quantum mechanically.

So what? we still have HIGGS <u>PARTICLES</u> ← QUANTUM excitations of the HIGGS FIELD w.r.t the VACUUM.

→ BUT WE ALSO HAVE $\langle H \rangle$ AS AN <u>ORDER PARAMETER</u> of SYMMETRY BREAKING

↑ a new tensor to write LAGRANGIAN terms (just replace H w/ $\langle H \rangle$)

So what: can identify mass terms in LAGRANGIAN
two fields, no derivatives

eg YUKAWAS:

① $y_e L^\dagger H e$ ← we already checked invariance ... now BREAK IT!

$\langle H \rangle = \frac{v}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix}$

$$\frac{v}{\sqrt{2}} y_e (\nu_L^\dagger \;\; e_L^\dagger)\begin{pmatrix}0\\1\end{pmatrix} e_R = \underbrace{\left(\frac{v y_e}{\sqrt{2}}\right)}_{\text{MASS}} e_L^\dagger e_R$$

② $$y_d Q^\dagger H d_R \rightarrow \frac{v}{\sqrt{2}} y_d (u_L^\dagger \;\; d_L^\dagger)\begin{pmatrix}0\\1\end{pmatrix} d_R = \frac{v y_d}{\sqrt{2}} d_L^\dagger d_R$$

③ $$y_u Q^\dagger_a \, \varepsilon^{ab} (H^\dagger)_b \, u_R \rightarrow \frac{v}{\sqrt{2}} y_u (u_L^\dagger \;\; d_L^\dagger)\begin{pmatrix}1\\0\end{pmatrix} u_R = \frac{v y_u}{\sqrt{2}} u_L^\dagger u_R$$

q_Y: $-1/6 \quad -\frac{1}{2} \quad 2/3 \quad = 0$ ✓

$\varepsilon^{ab} H^\dagger{}_b = \begin{pmatrix} & 1\\ -1 & \end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}$

So: e_L & e_R PAIR UP → MASSIVE electron e

u_L & u_R → MASSIVE UP QUARK u

d_L & d_R → MASSIVE down quark d

COMBINATION of 2 WEYL FERMIONS = DIRAC FERMION

Weyl = CHIRAL

ALL HAVE WELL DEFINED ELECTRIC CHARGE

$$\underbrace{Q}_{\text{electric charge}} = \underbrace{T^3}_{\frac{1}{2}\begin{pmatrix}1 & \\ & -1\end{pmatrix}} + \underbrace{Y}_{\text{HYPERCHARGE}} \quad \leftarrow \text{eg } Q\begin{pmatrix}\nu_L\\ e_L\end{pmatrix}_{-1/2} = \begin{pmatrix}\frac{1}{2} - \frac{1}{2}\\ -\frac{1}{2} - \frac{1}{2}\end{pmatrix} = \begin{pmatrix}0\\-1\end{pmatrix}$$

where did ELECTRODYNAMICS come from?

① GAUGE BOSONS $\leftrightarrow$ force particles

↳ under GAUGE symmetry,

$$B_\mu \to B_\mu + \frac{i}{g}\partial_\mu \theta$$

this prohibits mass term: $\underbrace{M^2 B^2}_{\text{not invariant under shift.}}$

② therefore: when a force particle picks up a mass from the Higgs ... what happens?

eg $\underbrace{|DH|^2}_{\text{kinetic term}} = |(\partial_\mu + ig' B_\mu)\langle H\rangle|^2 \supset \frac{g'^2 v^2}{2} B^2$

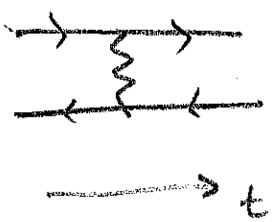
$\boxed{\text{massive force particle} \leftrightarrow \text{short range force}}$

"broken" force

intuition: $\Delta E \, \Delta t \sim \hbar$

MASS

$\Rightarrow$ only allowed for short times



IN GORY DETAIL?

$$D_\mu \langle H\rangle = \Big(\partial_\mu + ig\underbrace{\tfrac{1}{2}\begin{pmatrix} W^3 & \tilde{W}^+ \\ \tilde{W}^- & -W^3\end{pmatrix}_\mu}_{W^A \cdot T^A} + ig' \tfrac{1}{2} B_\mu\Big) \underbrace{\tfrac{v}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix}}_{\langle H\rangle}$$

$\partial_\mu(\text{const}) = 0$ $\quad$ $\tilde{W}^\pm = W^1 \mp iW^2$ $\quad$ $Y_H = \frac{1}{2}$

$$= \frac{i}{2}\begin{pmatrix} g'B + gW^3 & g\tilde{W}^+ \\ g\tilde{W}^- & g'B - gW^3 \end{pmatrix}\begin{pmatrix}0\\ v/\sqrt{2}\end{pmatrix}$$

$$= \frac{iv}{2\sqrt{2}}\begin{pmatrix} g\tilde{W}^+ \\ g'B - gW^3\end{pmatrix}$$

PROPER NORMALIZ

$W^\pm = \frac{1}{\sqrt{2}}\tilde{W}^\pm$

multiplied by -1 (SQUARED ANYWAY)

$$|D_\mu\langle H\rangle|^2 = \underbrace{\left(\frac{gv}{2}\right)^2}_{M_W^2}(W^+W^-) + \underbrace{\frac{(g^2 + (g')^2)v^2}{8}}_{\frac{1}{2}M_Z^2}\ \underbrace{\frac{(g'B + gW^3)^2}{(g')^2 + g^2}}_{= Z^2}$$

convention for mass of self-conjugate particle

normalized

$\boxed{Z \text{ boson}}$ $\Rightarrow$ has mass ("broken" symmetry)

$\hookrightarrow$ LINEAR COMBINATION of B & W^3

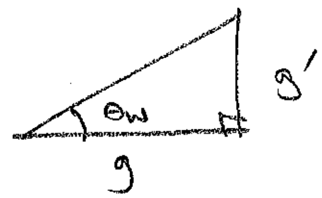
... so there is an orthogonal combination

$$Z = \underbrace{\frac{-g'}{\sqrt{g'^2 + g^2}}}_{-\sin\theta_W} B + \underbrace{\frac{g}{\sqrt{g'^2 + g^2}}}_{\cos\theta_W} W^3$$

$\leftarrow$ WEINBERG (MIXING) ANGLE : $s_W^2 \approx .23$

orthogonal combination:

$$A = c_W B + s_W W^3$$



no mass term → this linear combination of B & W^3 represents an unbroken (good) gauge sym.

$$\underbrace{\begin{pmatrix} A \\ Z \end{pmatrix}}_{\text{MASS EIGENSTATE}} = \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix} \underbrace{\begin{pmatrix} B \\ W^3 \end{pmatrix}}_{\text{"GAUGE" EIGENSTATE}} \rightarrow \begin{pmatrix} B \\ W^3 \end{pmatrix} = \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix} \begin{pmatrix} A \\ Z \end{pmatrix}$$

MASS EIGENSTATE — "GAUGE" EIGENSTATE ← $SU(2) \times U(1)$

COVARIANT DERIVATIVE ACTING ON A DOUBLET (eg $L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$)

$$D_\mu = \partial_\mu + \underbrace{\frac{ig}{2} \begin{pmatrix} W^3 & \tilde{W}^+ \\ \tilde{W}^- & -W^3 \end{pmatrix} + ig' q_Y \begin{pmatrix} B & \\ & B \end{pmatrix}}$$

IGNORE for now, only DOUBLETS talk to $W^\pm$ ... and $W^\pm$ connects the two components!

$$ig W^3 \underbrace{T^3}_{\pm 1/2 \text{ for upper/lower component of doublet}} + ig' B q_Y$$

±1/2 for UPPER/LOWER COMPONENT OF DOUBLET

$$= i\underbrace{g}_{\sqrt{\cdots}\, c_W}(s_W A + c_W Z) T^3 + i\underbrace{g'}_{\sqrt{\cdots}\, s_W}(c_W A - s_W Z) q_Y$$

$\underbrace{\sqrt{\cdots}}$ c_W — $\sqrt{\cdots}$ s_W

call this g_Z

$$\mathcal{L} = i g_Z c_W s_W A \left(T^3 + q_Y\right) + i g_Z c_W^2 Z T^3 - i g_Z s_W^2 Z q_Y$$

electric coupling

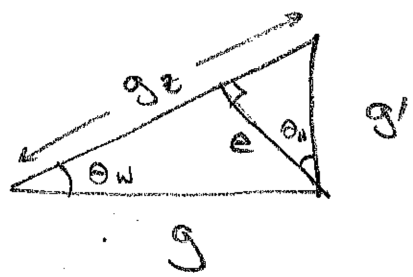
ELECTRIC CHARGE

$$= i \boxed{g_Z c_W s_W} A \boxed{\left(T^3 + q_Y\right)} + i g_Z Z \underbrace{\left(c_W^2 T^3 - s_W^2 q_Y\right)}$$

Z-charge (kinda complicated)

Z coupling

$$g_Z^2 = (g')^2 + g^2$$



$$e = g' c_W = (g_Z s_W) c_W$$

nb: Acting on singlets is easier

So that's where ELECTRICITY comes from: it is the unbroken symmetry leftover from <u>ELECTROWEAK SYM BREAKING</u>

Where we are: EW sym

<u>MASS BASIS</u> : $W^\pm$, Z, A

↑ HAMILTONIAN IS DIAGONAL

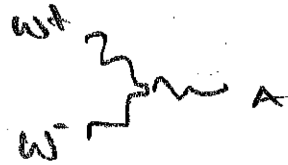
$W^\pm$, Z: massive

A: unbroken, massless, long range

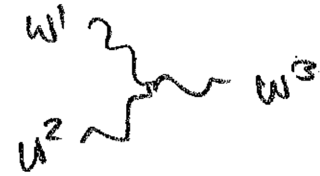
$\boxed{\text{FACT}}$ $W^\pm$ HAS ELECTRIC CHARGE ± 1

comes from transformation of "ADJOINT REP"

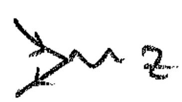
W^A ← A is an index ... it transforms ... has "charge" under SU(2)



from

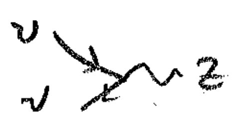


Z & A have similar interactions



↳ but the coefficient is different (charge)

eg



is not zero.

$W^\pm$ only talks to <u>DOUBLETS</u> (L, Q)

only to LEFT-CHIRAL FERMIONS!