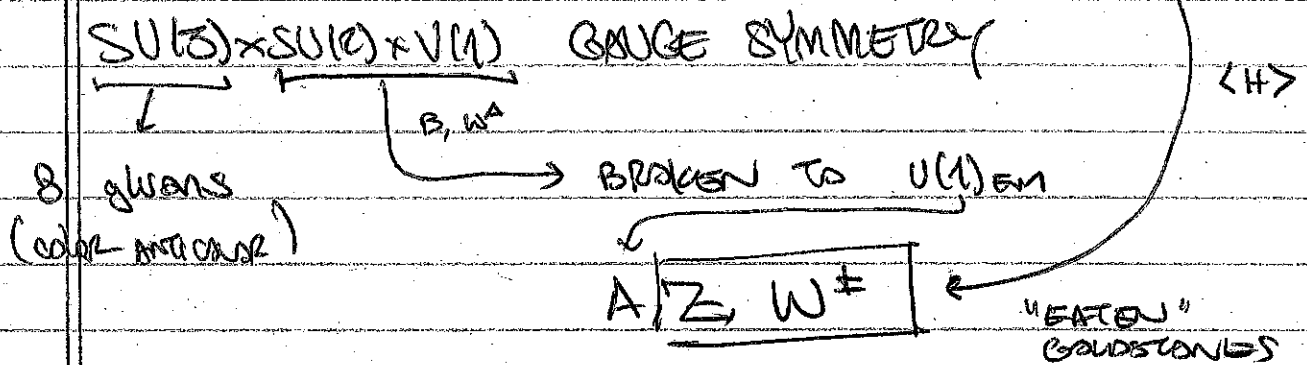P165: the 18+ PARAMETERS of the SM

Ref: "The 18 arbitrary parameters of the
Standard model & your everyday life"
ROBERT CAHN, RMP 68 JULY 1996

STANDARD MODEL

5 FERMIONS: Q u d L e
1 HIGGS : H ⊂ h + 3 Goldstone Bosons

$SU(3) \times SU(2) \times U(1)$ GAUGE SYMMETRY

B, W^A

8 gluons
(color-anticolor)

→ BROKEN TO $U(1)_{EM}$

⟨H⟩

$A, Z, W^{\pm}$

"EATEN"
GOLDSTONES

COVARIANT DERIVATIVE (GEOMETRY of GAUGE SYM)

$$D_\mu = \partial_\mu + i g_s T^M g_\mu^M + i g W_\mu^A T^A + i g' B_\mu$$

3 GAUGE COUPLINGS

③

RENORMALIZATION (briefly): DIMENSIONLESS COUPLINGS
ARE ATTACHED TO RENORMALIZABLE INTERACTIONS

g $\sim g^3 \log \Lambda$

DIMENSIONS²
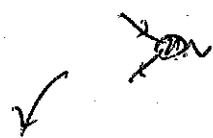
or "1PI" vertex
one particle irreducible

So: $g^{OBS} \sim g_0 + g_0^3 \log \Lambda$

$g_0 \sim g^{OBS}(1 - (g^{OBS})^2 \log \Lambda/\mu)$

$g_0 \approx g^{OBS}$

SCALE @ WHICH SUBTRACTION IS EXACT

$\Rightarrow g^{OBS} \sim g(1 + g^2 \log p^2/\mu^2)$

no Λ dependence,
last row depends on
ratio of kinematics
vs. scale μ

$4 \to 24$

<u>INTERPRETATION</u> : "COUPLING CONSTANTS ARE
NOT CONSTANT" (renormalization)
↳ eg WHEN WE ARGUED THAT VACUUM
POLARIZATION CHANGES $V(r)$ CORRECTION
TO ELECTRON MASS

WHAT IS HAPPENING: THE "BEST" THEORY @ A DIFFERENT SCALE
HAS SLIGHTLY DIFFERENT PARAMETERS.

BQU: SAME PHENS IN COND
MAT! (RENORMALIZE GROUP)
→ key of PHASE TRANSITIONS

FOR SU(3): $g_s(\mu)$ GETS LARGER AS μ GETS
SMALLER → PERTURBATION THRY BREAKS
DOWN

strong force is "stronger"
Feynman diagram expansion
of FREE PARTICLES w/ few
interactions is not accurate.

what we believe happens : confinement
↳ all color CHARGES CONFINE To color
Neutral states @ low energy

MASS GAP : $\langle q\bar{q} \rangle \sim 100$ MeV

CHIRAL CONDENSATE
(another VEV!)

confinement scale : $\Lambda \sim$ GeV

characteristic hadron scale
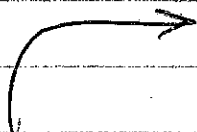comes from BINDING ENERGY

THE MASSIVE FERMIONS:

①
$$\left.\begin{array}{ccc} u & c & t \\ d & s & b \\ e & \mu & \tau \end{array}\right\} \quad \underline{9 \text{ mass terms}}$$

eigenvalues of
yukawa matrices

μb^i CAN ~~REPLACE~~ REPHASE u_L, u_R
TO MAKE eigenvalues $\pm R$.

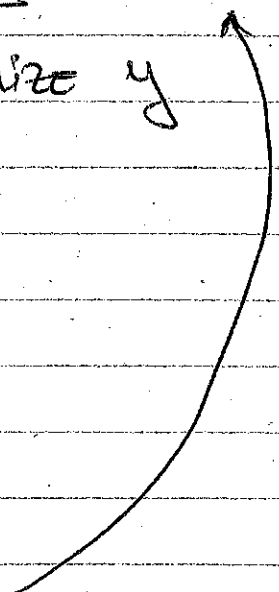$$y L^\dagger H e_R \rightarrow L^\dagger (U_L^\dagger y U_{eR}) e_R H$$

DIAGONALIZE y
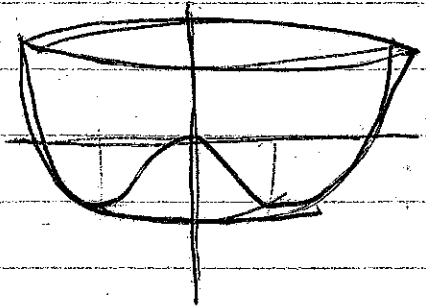
USING flavor (APPROX)
SYMMETRY: LET
y MATRICES DEFINE
"CORRECT" MASS EIGENBASIS

yukawa becomes MASS TERM
UPON $\langle H \rangle = v/2$ (HIGGS VEV)

②

$$V[H] \sim (\lambda)(|H|^2 - v^2/2)^2$$

two params

2 GIVES HIGGS MASS, VEV,
SELF INTERACTIONS.

nb: we could not formally diagonalize all Yukawas → $SU(2)$ locks u_L & d_L to be part of same Q doublet.

↳ but diagonalizing Y_u & Y_d require SEPARATE u_L & d_L ROTATIONS.

if we do these ROTATIONS, we pay the PRICE in the $SU(2)$ kinetic term

$$\bar{u}_L \sigma^r W^- V_{CKM} d_L + h.c.$$

CKM MATRIX: UNITARY MATRIX ENCODING MISMATCH of u_L & d_L ROTATIONS IN FLAVOR SPACE (to diagonalize both Y_u & Y_d) = $V_{CKM} = U_{u_L}^\dagger U_{d_L}$

$$V^\dagger V = 1$$

V_{CKM} is a 3×3 UNITARY MATRIX: 9 C PARAM → 9 R PARAM then REPHASE u_L & d_L (& CORRESPONDINGLY u_R, d_R to keep MASSES REAL)

Remaining 4 params

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

each V_{ij} can be written as a magnitude & phase: $V_{ij} = r_{ij} e^{\theta_{ij}}$

if we rephase each of the down-type quarks, we can remove the phases in, say V_{ud}, V_{us}, V_{ub} → but no more use be u rephasing

then we can use rephasing of c & t to remove 2 more phases

can remove 5 phases

(4) $\Rightarrow$ 9 params of V_{CKM} → |4 PHYSICAL PARAMS|

COMPARE to IR MATRIX (ORTHOGONAL):
3 EULER ANGLES ∴ the 4th PHYSICAL PARAMETER IS INTRINSICALLY COMPLEX

β decays

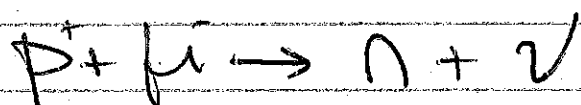↳ COMPLEX PHASE ⟺ matter—antimatter ASYM
(gives a difference in "+h.c." terms)

<u>PHYSICAL SIGNIFICANCE</u>: to macroscopic nature
examples from cahn article

- if $M_e \rightarrow m_\mu$, replace electrons
  w/ muons in atoms

  <u>then</u>: muon capture destabilizes
  ~~complex~~ atoms

  $$p^+ + \mu^- \rightarrow n + \nu$$

  mass difference $m_\mu - m_\nu$
  is larger than $m_n - m_p$

  so instead of hydrogen $\rightarrow$ neutrons   Bohr radius
  & neutrinos.   no chemistry
  $\rightarrow$ in gen: as we increase M_e, atomic matter shrinks

- if $M_p > m_n$ ∴ PROTON DECAY.
  $\rightarrow$ some light atoms stable (tritium)

- masses of other particles affects
  RG flow of couplings $\Rightarrow$ unification?

the missing PARAMETERS

in the SM (but not mentioned in
the Cahn article)

<u>θm angle</u> : CP phase / topological phase
↳ property of gauge theory
RELATED to MATTER—ANTIMATTER ASYMMETRY

EXPERIMENTAL SIGNATURE: MAGNETIC
MOMENT of NEUTRON ⟶ obs to
be consistent w/ zero to high
precision ... but no theoretical
REASON!

⌐ in contrast: fermion masses are
small compared to Λ_{UV} because
the m→0 limit enhances a symmetry.

ie for an angle, there is nothing
special about $\theta = 0$.

<u>POPULAR SOLUTION: AXION</u>

BEYOND the SM

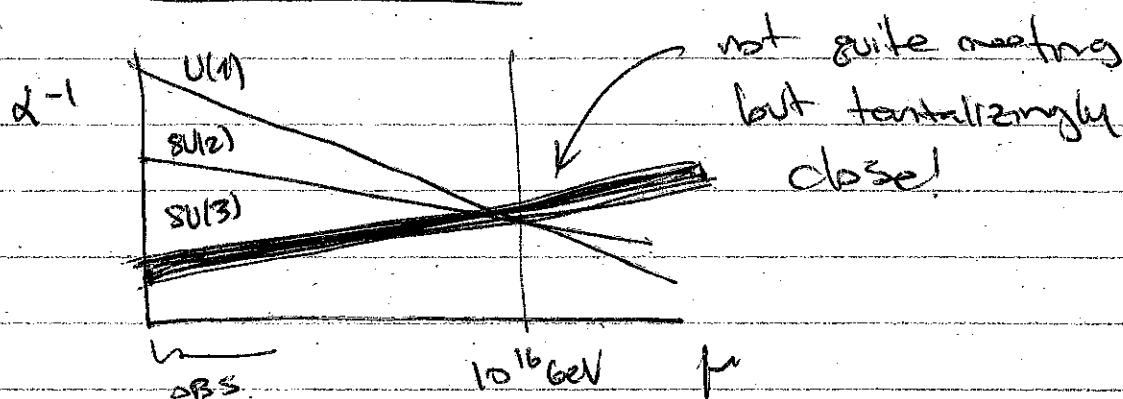- ν MASS $\longrightarrow$ ν YUKAWA

  $\searrow$ WEINBERG $\mathcal{L}$.

  ANTICIPATE 4 PHASES + 3 MASSES
  (in parallel w/ up/down quarks)

  analog of CKM matrix is PMNS matrix

  SIGNIFICANCE : CP phase in ν sector
  MAY EXPLAIN MATTER-ANTIMATTER ASYM
  (LEPTOGENESIS) $\Rightarrow$ IN GENERAL, BARYOGENESIS
  IS AN OPEN QUESTION

- GRAND UNIFICATION

α^{-1} ... U(1), SU(2), SU(3) vs μ ; 10^{16} GeV ; obs

not quite meeting but tantalizingly close!

COUPLING STRENGTHS SEEM TO CONVERGE @ HI SCALE
... POSSIBLE SIGNATURE of SU(5) $\rightarrow$ SU(3)×SU(2)×U(1)
SYMMETRY BREAKING ?!   (only one GAUGE SYM)

DARK MATTER

<u>ASTRO & COSMO</u> : MOST OF MATTER IN
UNIVERSE IS <u>NOT</u> "ORDINARY" SM
STUFF
⤷ WHAT IS IT?

QUANTUM GRAVITY / FORMAL QFT

@ M_{Pl}, WE EXPECT QFT OF GRAVITY
TO BE RELEVANT. <u>BUT</u> : GRAVITY IS NON-RENORM.
⤷ QUANTUM THEORY IS PLAGUED W/
apparent infinities — NO "UV" SCALE
TO POINT TO

GRAVITY IS A VERY DIFFERENT FORCE
⤷ REASON TO SUSPECT THAT QFT-AS-"WE-KNOW-IT"
MAY BREAK DOWN

BEST OPTION SO FAR: STRING THEORY(
BUT MAIN OUTPUT HAS NOT BEEN QG BUT
LOTS OF EXCITING FORMAL RESULTS IN QFT
eg. AdS/CFT