THE SM

	SPIN (& CHIRALITY)	$SU(3)$	$SU(2)_L$ (ELECTROWEAK)	$U(1)_Y$ (ELECTROWEAK)	Q
$Q^{\alpha m a}$	$\frac{1}{2}$ L	✓	$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\frac{1}{6}$	$\begin{pmatrix} 2/3 \\ -1/3 \end{pmatrix}$
$u_{R\dot{\alpha}}^{m}$	$\frac{1}{2}$ R	✓		$\frac{2}{3}$	$\frac{2}{3}$
$d_{R\dot{\alpha}}^{m}$	$\frac{1}{2}$ R	✓		$-\frac{1}{3}$	$-\frac{1}{3}$
$L^{\alpha a}$	$\frac{1}{2}$ L		$\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	$-\frac{1}{2}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
$e_{R\dot{\alpha}}$	$\frac{1}{2}$ R			-1	-1
H^{a}			$\begin{pmatrix} h^1 \\ h^2 \end{pmatrix}$	$\frac{1}{2}$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
$\bar{\sigma}^{\mu\dot{\alpha}}{}_{\alpha}$, $\sigma^{\mu}{}_{\alpha\dot{\alpha}}$ (convert spin 1/2 → SPIN 1)	$\varepsilon_{\alpha\beta}$ $\varepsilon^{\alpha\beta}$ $\varepsilon^{\dot{\alpha}\dot{\beta}}$ $\varepsilon_{\dot{\alpha}\dot{\beta}}$	$T^{M m}{}_{n}$, g^{M}_{μ}, ε^{mnp}, ε_{mnp}	$T^{A a}{}_{b}$, W^{A}_{μ}, ε^{ab}, ε_{ab}	B_μ	DERIVED FROM $SU(2)_L \times U(1)_Y$ A_μ : lin COMB of B + W^3

CAN ALSO USE ∂_μ

<u>SPIN</u> : PUZZLE: spin sym is LORENTZ SYM

↳ we <u>know</u> the indices of LORENTZ!
4-VECTOR INDICES!!

MORE SUBTLE. (WIGNER REPS)

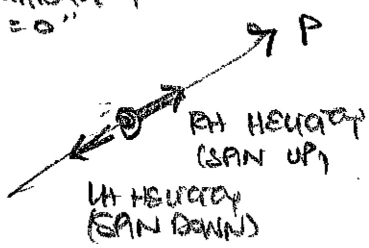
we have $\boxed{\text{SPINORS}}$ ↔ fermions

$\boxed{\text{SPIN } 1/2}$ → LEFT CHIRAL
→ RIGHT CHIRAL

not to be confused w/ <u>HELICITY</u>
↓
PROJECTION of SPIN ON DIRECTION OF MOTION

nb: "HELICITY = CHIRALITY WHEN MASS = 0"

eg all ν's ARE LEFT HANDED ($\bar{\nu}$ ARE RH)



⇒ DEPENDS ON DIRECTION OF MOTION BEING QUANTIZATION AXIS.

<u>CHIRALITY</u>: SIGN of the phase when you rotate the particle.

PUNCHLINE: DIFFERENT KINDS OF INDICES

LEFT CHIRAL: $\alpha, \beta = 1, 2$
RIGHT CHIRAL: $\dot{\alpha}, \dot{\beta} = 1, 2$
} spin up/down w/RT QUANTIZ AXIS

<u>TENSORS</u>: $\varepsilon_{\alpha\beta}$, $\varepsilon^{\alpha\beta}$, & same w/ dots

$$(\bar{\sigma}^\mu)^{\dot{\alpha}}{}_{\beta} \qquad (\sigma^\mu)^{\alpha}{}_{\dot{\beta}}$$

& (RULE) $(\psi^\alpha)^\dagger = (\psi^\dagger)_{\dot{\alpha}}$

SPIN/LORENTZ: see notes from LEC 10

note: $\psi^\dagger\psi$ is **not** contractible

$(\psi^\dagger)_{\dot\alpha}\psi^\alpha$ or $(\chi^\dagger)^\alpha\chi_{\dot\alpha}$

L CHIRAL R CHIRAL

BUT: $\psi\psi = \psi^\alpha\psi^\beta\varepsilon_{\alpha\beta}$ ← we can write $\psi_\beta = \varepsilon_{\beta\alpha}\psi^\alpha$

is allowed ... BUT NOT INVARIANT UNDER ANY OTHER SYMMETRY! eg REPHASING

HERMITICITY

ALLOWED: $(\psi^\dagger)_{\dot\alpha}(\bar\sigma^\mu)^{\dot\alpha}{}_\beta\, i\partial_\mu\psi^\beta$

$\not{p}$ ← $p_\mu\bar\sigma^\mu$ notation

$\partial_\mu\psi \sim ip_\mu\psi$

because $\psi(x) = \int\frac{d^3p}{2E}e^{ip\cdot x}\psi(p)$

PLANE WAVE OF DEF MOM.

VERY SIMILAR STRUCTURE TO $(T^A)^a{}_b$

THIS IS THE KINETIC TERM. TELLS YOU THAT THE PARTICLE CAN PROPAGATE IN SPACETIME.

↳ analog of $\frac{1}{2}mv^2$ in NON REL MECHANICS.
IT IS ALWAYS THERE

IT IS THE PIECE THAT HAS (anti-particle), particle and one or TWO DERIVATIVES

spin 1/2 spin 0, 1

eg for a spin-0 field (no LORENTZ INDEX)

KINETIC TERM: $(\partial_\mu\phi)^\dagger(\partial^\mu\phi)$

AN ASIDE on PROPAGATORS

———————— $\sim \frac{i}{p^2 - m^2}$

why? GREEN'S FUNCTION of EQN of MOTION

SIMPLEST LAGRANGIAN: (up to factors of 1/2)

$$\mathcal{L} = \tfrac{1}{2}\left[(\partial_\mu \phi)^2 - m^2\phi^2\right]$$ ← ℝ function to "FIELD"

$S = \int d^4x\, \mathcal{L}$, so we may integrate by parts

$$\mathcal{L} = -\phi(\partial^2 + m^2)\phi$$

EQUATION of motion: $(\partial^2 + M^2)\phi = 0$ ← no source

KLEIN GORDON EQ.

(LIKE SO MUCH of PHYSICS, IT'S A LAPLACIAN.)

in general: $\mathcal{O}\phi = j$ ← SOURCE

$\mathcal{O}$: DIFFERENTIAL OPERATOR

ϕ: FIELD ← want to solve for this

ANALOG TO LIN ALG: $A\underline{v} = \underline{w}$ ← SOLVE FOR THIS ($\underline{v}$)

$$\underline{v} = A^{-1}\underline{w}$$

← need to find A^{-1}

what is A^{-1}? def by $AA^{-1} = \mathbb{1}$

for functions: $\mathcal{O}_x G(x,x') = \delta(x - x')$

$\mathcal{O}_x$: $\left(\frac{\partial}{\partial x}\right)^2 + m^2$

$G(x,x')$: "A^{-1}" GREEN'S FUNCTION

x': POSITION OF UNIT SOURCE

eg. $\nabla \Phi = \delta^{(3)}(\underline{x}-\underline{x}') \Rightarrow \Phi = \underbrace{\frac{1}{4\pi}\frac{1}{|\underline{x}-\underline{x}'|}}$

this is the PROPAGATOR of ELECTRIC POTENTIAL FROM $\underline{x}'$ to $\underline{x}$

why: once you have $G(x,x')$ or $(A^{-1})^i{}_j$;

then you may construct solution to inhomog eq:

$$V^i = (A^{-1})^i{}_j\, W^j \qquad \Phi(x) = \int dy\; G(x,y)\; j(y)$$

SUM OVER SOURCE PS.

INTEGRATE OVER SOURCE POSITION.

SO: in FOURIER SPACE (MOMENTUM SPACE),

$\partial^2 + m^2 = -p^2 + m^2$ (DIAGONAL MATRIX)

then if "A" $= -p^2 + m^2$

"A^{-1}" $= \boxed{-(p^2 - m^2)^{-1}}$

this is why we have the ODD FEYNMAN RULE for PROPAGATION, (UP to $-i$)

for SPIN-1/2, END UP WITH

$$\frac{1}{\not{p}} = \frac{\not{p}}{p^2} \quad \leftarrow (p\cdot\bar{\sigma})(p\cdot\sigma) = p^2$$

or: $$\frac{1}{\not{p}-m} = \frac{\not{p}+m}{p^2-m^2} \quad \leftarrow \text{mult by } \frac{\not{p}+m}{\not{p}+m}$$

IN FACT: this is where fundamental force interactions come in.

$$\partial_\mu \longmapsto D_\mu = \partial_\mu + ig\underbrace{T^{\circ}\left(\begin{smallmatrix}\text{FORCE}\\ \text{PARTICLE}\end{smallmatrix}\right)^{\circ}}$$

term like this for EACH FORCE

(IF) OBJECT D_μ ACTS ON has charge.

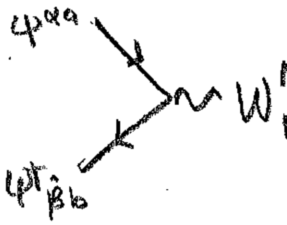
START W/ KINETIC TERM

$$\psi^\dagger i\bar{\sigma}^\mu \partial_\mu \psi \mapsto \psi^\dagger i \bar{\sigma}^\mu D_\mu \psi$$

$$= \psi^\dagger i\bar{\sigma}^\mu \partial_\mu \psi$$

$$- \underbrace{\psi^\dagger \bar{\sigma}^\mu g T^A W^A \psi}$$

the fundamental vertex



$$\sim g(\bar{\sigma}^\mu)^{\dot\beta}{}_\alpha (T^A)^b{}_a$$

nb: I'M BEING SLOPPY W/ factors of i — they don't matter for our PURPOSES eg QUARK DOUBLET: $Q^{\alpha m a} = \begin{pmatrix} u_L^{\alpha m} \\ d_L^{\alpha m} \end{pmatrix} \begin{matrix} \leftarrow a=1 \\ \leftarrow a=2 \end{matrix}$

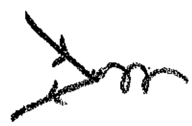
SPIN 1/2, LH, $\alpha = 1,2$; color $m = 1,2,3$; SU(2): 1,2 } & $Y = 1/6$

$(\bar{\sigma}^\mu)^{\dot{\alpha}}{}_\beta$

$$(Q^\dagger)_{\dot{\alpha} m a}\, i\Big(\underset{\delta^m_n \delta^a_b}{\partial_\mu} + i g_s (T^M)^m{}_n\, g^M_\mu\, \delta^a_b + i g (T^A)^a{}_b\, W^A_\mu\, \delta^m_n + i g' \underset{1/6}{q_a}\, B_\mu\, \delta^a_b \delta^m_n\Big) Q^{\alpha n b}$$

U(1) CASE
generator is 1
no indices.

always Q & $Q^\dagger$

 $g^M \sim (\bar{\sigma}^\mu)^{\dot{\alpha}}{}_\beta (T^M)^m{}_n \delta^a_b$ $\quad \times g_s$

 $W^A \sim (\bar{\sigma}^\mu)^{\dot{\alpha}}{}_\beta (T^A)^a{}_b \delta^m_n$ $\quad \times g$

 $B \sim (\bar{\sigma}^\mu)^{\dot{\alpha}}{}_\beta \delta^a_b \delta^m_n$ $\quad \times g'$

EX: SPIN-1 KINETIC TERM is MORE COMPLEX

$$\underbrace{(\partial_\mu A_\nu - \partial_\nu A_\mu)}_{F_{\mu\nu}}(\partial^\mu A^\nu - \partial^\nu A^\mu)$$

$$F_{\mu\nu} = \left(\begin{array}{c|ccc} 0 & & (-E) & \\ \hline & 0 & -B_z & B_y \\ & & 0 & -B_x \\ & & & 0 \end{array}\right)$$

& GAUGE VERSION IS A BIT MORE TRICKY

- invariant under $A \to A + \partial_\mu f \quad \forall f$
- will give (and !)