LOGISTICS → COURSE FORMAT & WEBSITE, BOOKS
ASSIGNMENTS
Q&A EXPECTATIONS
TEACHING TEAM

SM

BIG PICTURE : ELEMENTARY PARTICLE PHYSICS

Reductionist science - BIG THINGS ARE MADE of SMALL THINGS
... PROPERTIES of SMALL THINGS MUST BE MORE FUNDAMENTAL

this class: Standard Model of PARTICLE PHYSICS

→ PARTICLES → matter (fermions)
→ forces (bosons)

"quantum fields"

& interactions (DYNAMICS)
+ kinematics

many diff approaches, traditionally:

- HISTORICAL / EXPERIMENTAL – long! (BUT IMPORTANT)
- QUANTUM FIELD THEORY – many prereqs
- TAXONOMY – no depth

→ $\boxed{\text{theory first}}$

→ go STRAIGHT to FEYNMAN DIAGRAM LANGUAGE & BUILD SM AND INTERPRETATION AS WE GO

idiosyncratic! YOU GET WHAT YOU PUT IN

this is a deep subject - our approach will be playful (AS ALL GOOD SCIENCE), but there IS A LOT JUST under the SURFACE

UNITS

- REVIEW of UNITS (eg 1 🍎 = $1 = 100 cal)

why don't we use apples for currency?
or dollars for energy?

on the other HAND, AS OF TODAY:

$$\$1 = 0.75 \text{ £}$$

btw: in this class, LO is usually good enough

units of LHS ≠ units of RHS

there is a conversion:

$$1 = \boxed{0.75 \text{ £}/\$}$$

currently £/\$ is DIMENSIONLESS ... both are "money"

nuances of GLOBAL ECONOMICS NOTWITHSTANDING

conversion constant SAME EVERYWHERE

so if 1 🍎 = $1, how much in £?

↳ $\boxed{\text{MULTIPLY BY ONE}}$ ... in the RIGHT WAY

THIS IS EVERYTHING YOU NEED TO KNOW re: UNITS

$$1 🍎 = \$1 \times (0.75 \text{ £}/\$) = 0.75 \text{ £}$$

value of 🍎 hasn't changed. just diff units

... of COURSE, EXCHANGE RATES ARE <u>NOT</u> CONSTANT
they change w/ time

NATURE GIVES US A FEW CONSTANTS

or hints that we made up too many units eg k_B

NATURAL UNITS

$c = 3\times10^8$ m/s $= 1$ SPECIAL REL.

$\hbar = 6.6\times10^{-22}$ MeV·s $= 1$ QUANTUM MECH

Mega electron volt : 10^6 eV

"quantumness" of universe
(what happens if $\hbar$ changes?)

PARTICLE PHYSICS = SR + QM

NATURE HAS GIFTED US conversion factors!

$1 = 3\times10^8$ m/s $\Rightarrow$ 1 sec $= \underbrace{3\times10^8 \text{ m}}_{\text{"LIGHT SECOND"}}$ ← LEN ⟺ TIME

sure, can say that we're using light beams as clocks & rulers ... pretty mundane

↳ BUT REFLECTS A MUCH DEEPER TRUTH:

Space & time are "the same"

eg. Han Solo Kessel RUN: $\boxed{12 \text{ pc}}$ ← pc is a distance $1\text{pc} = 3\times10^{16}$m

$12 \text{ pc} = 12\times(3\times10^{16}\text{m}) \approx 4\times10^{17}\times\left(\frac{1}{c}\right)$

$\approx 10^9$ sec ← 1 yr $\approx \pi\times10^7$ sec

≈ 30 yrs

$\hbar = 1$? $[\hbar] = \text{ENERGY}\times\text{TIME} = \underbrace{\text{MASS}\times\text{LEN}^2/\text{TIME}}_{\text{ANGULAR MOMENTUM}}$

ENERGY: $\text{MASS}\left(\frac{\text{LEN}}{\text{TIM}}\right)^2$

(better: units of ACTION, $S = \int dt\, L$)

$\hbar = 1$ converts $E \longleftrightarrow t$

mnemonic: $\Delta E \Delta t \sim \hbar$

$\Delta x \Delta p \sim \hbar$

IN PARTICLE PHYSICS, SIMPLIFY EVERYTHING BY USING UNITS OF ENERGY

how long since class started? approx?

$$\sim 10^3\, s \times \left(\frac{1}{\hbar}\right) \sim 10^3 \cdot \frac{s}{10^{21}\,\text{MeV}\,s}$$

$$\sim 10^{-18}\ {}^1/_{\text{MeV}}$$

eg. LARGE HADRON COLLIDER (LHC) → A microscope!

$$\underbrace{E \sim 10\ \text{TeV}}_{\text{ENERGY}} \times \underbrace{\left(\frac{1}{\hbar}\right)}_{\frac{1}{\text{EN.TIM}}} \times \underbrace{\left(\frac{1}{c}\right)}_{\frac{\text{TIM}}{\text{LEN}}}$$

what RESOLUTION?

$$\sim 10^7\,\text{MeV} \times 10^{21}\,\frac{1}{\text{MeV}\cdot\text{sec}} \times 10^{-8}\,\frac{\text{sec}}{\text{m}}$$

$$\sim \frac{1}{10^{-20}\ \text{m}}$$

characteristic LENGTH is $\ell \sim {}^1/_E \sim \boxed{10^{-20}\ \text{m}}$

is that BIG OR SMALL?

$Å \sim 0.1\ \text{nm} \sim 10^{-10}\ \text{m} \sim$ atomic

[note: LHC actually probes slightly larger scales than $1/E$ ... why?]

HINT: collides protons

observe: $\boxed{\uparrow E \quad \downarrow \ell}$ Hi E ⟷ short DIST (small)

PARTICLE PHYS

- KINEMATICS (SR) how things move ← LAWS of SPACE + TIME
  eg ENERGY + MOMENTUM RELATIONS

- DYNAMICS (QM) how things interact
  eg theory of particles, LAGRANGIAN

nb: GR is the DYNAMICS of spacetime itself

for us: kinematics → conservation of E, $\underline{p}$
→ invariance of "MASS"

DYNAMICS ⟶ FEYNMAN DIAGRAMS
(feynman RULES)
that encode a theory
⊕ QM: sum over amplitudes

"scattering" — weird part of mechanics. but this is 99% of what we do to study small things.

eg Rutherford foil expt.

KINEMATICS:

1. E conserved
2. $\underline{p}$ 〃 } from SPACETIME SYM.

3. $E^2 = m^2 + \underline{p}^2$ ← $E^2 = m^2c^4 + \underline{p}^2c^2$

when $\underline{p}^2$ is small w.r.t. m^2c^2

$E \approx mc^2\left(1 + \frac{1}{2}\frac{\underline{p}^2}{m^2c^2}\right)$

4-VECTOR NOTATION ← big theme in this class is indexology

four momentum

$$\underbrace{p^{\mu}}_{\text{index, } \mu} = (\underset{\mu=0}{E}, \underbrace{p^x, p^y, p^z}_{\underset{\mu=1,2,3}{\mathbf{p}}}) = (E, \mathbf{p})$$

$\frac{E}{c}$

(differences in) position: $x^{\mu} = (\overset{\mu=0}{t}, \underbrace{x, y, z}_{\underline{x} \quad \mu=1,2,3})$

↑ ct

DOT PRODUCT (metric, inner product)

$$p \cdot k = \underbrace{p^{\mu} k_{\mu}}_{\text{Einstein summation convention}} = E_p E_k \underbrace{- p^x k^x - p^y k^y - p^z k^z}_{-(\mathbf{p} \cdot \mathbf{k})_{\text{EUCLIDEAN}}}$$

REP. UPPER/LOWER INDEX SUMMED

LOWER INDEX OBJECT IS A "ROW VECTOR"

if $k^{\mu} = (E_k, \underline{k}) \longrightarrow k_{\mu} \equiv (E_k, -\underline{k})$

OR: $k_{\mu} = \underbrace{g_{\mu\nu}}_{\begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}} k^{\nu}$

metric of Minkowski SPACE

NORM: $p^2 = p \cdot p = p^{\mu} p_{\mu} = E^2 - \mathbf{p}^2 = m^2$

↑ invariant

why indices? INDICES TELL US HOW VECTORS (& TENSORS) TRANSFORM.

start w/ simpler case: $\mathbb{R}^3$

$$\underline{V} = \begin{pmatrix} V^x \\ V^y \\ V^z \end{pmatrix}$$

components

general matrix: $A = A^i{}_j$

$$A\underline{V} = A^i{}_j V^j = A^i{}_1 V^1 + A^i{}_2 V^2 + A^i{}_3 V^3 = (AV)^i$$

cf. $\begin{pmatrix} a^1{}_1 & a^1{}_2 \\ a^2{}_1 & a^2{}_2 \end{pmatrix}\begin{pmatrix} V^1 \\ V^2 \end{pmatrix} = \begin{pmatrix} a^1{}_1 V^1 + a^1{}_2 V^2 \\ a^2{}_1 V^1 + a^2{}_2 V^2 \end{pmatrix}$

$= \begin{pmatrix} a^1{}_j V^j \\ a^2{}_j V^j \end{pmatrix} = \begin{pmatrix} (aV)^1 \\ (aV)^2 \end{pmatrix}$

ROW VECTORS: multiply by metric: $g_{ij} = \delta_{ij}$

$\begin{pmatrix} 1 & \\ & 1 \end{pmatrix}$ $\qquad \delta_{ij} = \begin{cases} 1 & \text{if } i=j \\ 0 & \text{otherwise} \end{cases}$

eg $V_i \equiv g_{ij} V^j$ ← in this case, trivial

MATRICES → LINEAR TRANSFORM.
1st O of any transform.

some transformations "preserve" the inner product (dot)

← doesn't matter if you ROTATE

$\underline{V} \cdot \underline{W} = \cos\theta \times |\underline{V}| \times |\underline{W}|$

in 2D : $R^i{}_j = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

s.t $\underline{v} \to R\underline{v}$ ← $v^i \to R^i{}_j v^j \equiv v'^i$

$\underline{v}^T \to \underline{v}^T R^T$ ← $R^T = R^{-1}$ for ROTATIONS

$v_k \to v_\ell (R^T)^\ell{}_k \equiv v'_k$

then: $v^i w_i \to v'^i w'_i = R^i{}_j v^j (R^T)^\ell{}_i v_\ell$

$\underline{v}\cdot\underline{w}$

$= \underbrace{(R^T)^\ell{}_i R^i{}_j}\, v^j v_\ell$

$R^T R = \mathbb{1}$

$(R^T R)^\ell{}_j = \delta^\ell{}_j$

inner prod. is PRESERVED

$= \underline{v}\cdot\underline{w}$

now we know how tensors TRANSFORM ← obv w/ some # of upper/lower indices

$$T^{i_1 i_2}{}_{j_1 j_2}{}^{i_3} \to R^{i_1}{}_{a_1} R^{i_2}{}_{a_2} (R^T)^{b_1}{}_{j_1} (R^T)^{b_2}{}_{j_2} \times R^{i_3}{}_{a_3}\, T^{a_1 a_2}{}_{b_1 b_2}{}^{a_3}$$

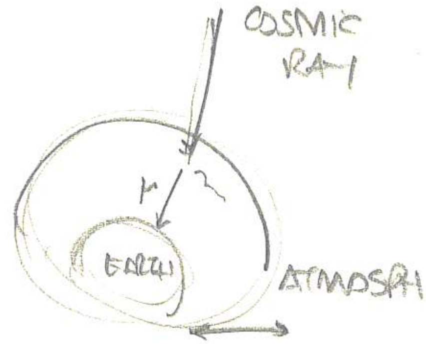
⇝ can generalize to SR.

muon lifetime

↳ see pdg.lbl.gov

$$\boxed{\tau_{(\mu)} \approx 2\times10^{-6}\,s}$$

$m_{(\mu)} \approx 100$ MeV



~8000 M

MUONS PRODUCED IN ATMOSPHERE BY COSMIC RAYS

do they REACH EARTH?

moving @ ≈ .998 c
OR E ~ 1-5 GeV

WRONG: $v \approx c$ & $d \approx \tau_{(\mu)} c = 2\times10^{-6}s \cdot 3\times10^{8}\,m/s$

$= 600\,m \ll 8000\,m$

TIME DILATION:

$$\begin{pmatrix}\Delta t\\ \Delta x\end{pmatrix}_{\oplus} = \begin{pmatrix}\gamma & -\beta\gamma\\ -\beta\gamma & \gamma\end{pmatrix}\begin{pmatrix}\tau\\ 0\end{pmatrix}_{(\mu)} = \begin{pmatrix}\gamma\tau\\ -\beta\gamma\tau\end{pmatrix}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \qquad \beta = \frac{v}{c} = 0.998$$

for us: $\gamma \approx 15 \Rightarrow \Delta t_{\oplus} = \gamma\tau = 15\,(600\,m) = \boxed{9000\,M}$ ✓

why? $\begin{pmatrix}E\\ p\end{pmatrix}_{\oplus} = \begin{pmatrix}\gamma & -\beta\gamma\\ -\beta\gamma & \gamma\end{pmatrix}\begin{pmatrix}M\\ 0\end{pmatrix} = \begin{pmatrix}M\gamma\\ -\beta\gamma M\end{pmatrix}$

$\rightsquigarrow \boxed{\gamma = E/M}$