

LONG HW 1: Kinematics and QED

COURSE: Physics 165, *Introduction to Particle Physics* (2022)

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DUE BY: Thursday, April 7

This is the main weekly homework set. Unless otherwise stated, give all responses in natural units where $c = \hbar = 1$ and energy is measured in electron volts (usually MeV or GeV).

1 Everything in natural units

Write the following quantities in natural units with energy measured in GeV. You may write everything to one significant figure. The relevant constants (as well as everything we know about particle physics) is available free at the website of the Particle Data Group (**the PDG**), <http://pdg.lbl.gov>. Look at the pages for ‘Physical Constants’ and ‘Astrophysical Constants.’

- The mass of the sun, M_\odot .
- The present day Hubble expansion rate, H_0 .
- The classical electron radius, r_e .
- The Schwarzschild radius of the sun, $2G_N M_\odot / c^2$.

COMMENTS:

- If you’re not familiar with what a Schwarzschild radius is, look it up. Even without knowing general relativity, you could have guessed the order of magnitude of the Schwarzschild radius simply from dimensional analysis.
- What do you think the meaning of the classical electron radius is?

2 Dynamics: QED

2.1 Compton Scattering

Compton scattering is the process $e^- \gamma \rightarrow e^- \gamma$, an electron and a photon turning into an electron and a photon. You can assume that the incoming particles have different momenta than the outgoing particles so that some interaction must have happened. Draw *all* of the **leading order** Feynman diagram(s) for Compton scattering in quantum electrodynamics.

COMMENT: recall that a valid diagram is (1) connected and (2) uses only the lines and vertices of QED. *Leading order* means simplest: we want the diagrams with the fewest number of vertices.

2.2 Inverse Compton Scattering

A process called *inverse* Compton scattering is important in astroparticle physics. What are the leading order Feynman diagrams for inverse Compton scattering? How is this process different from Compton scattering?

COMMENT: Go ahead, use Google. If you find a source that explains something well, be sure to cite it.

3 Special Relativity and Kinematics

3.1 The non-relativistic limit

A particle is **non-relativistic** if its 3-momentum is much smaller than its energy, $|\mathbf{p}| \ll E$. In this limit, show that $E^2 = m^2 + \mathbf{p}^2$ reduces to the familiar $E = mc^2$ upon restoring factors of c . In the non-relativistic limit, what is the *leading-order correction* to $E = mc^2$? You'll want to Taylor expand—make sure you do this with respect to a small, *dimensionless* parameter. (There's no such thing as a 'small parameter' that has dimensions.)

3.2 A relativistic electron

In some frame, the electron has *momentum* equivalent to its rest mass, m_e . Use the value of the rest mass to one significant figure. I shouldn't have to tell you where to look it up. Write out the components of the **momentum four-vector** p^μ .

3.3 A symmetric particle collider

Imagine a symmetric electron-positron collider. At the collision point, it collides a beam of electrons and positrons with one another so that these have four-momenta:

$$p_{e-}^\mu = (E, 0, 0, p) \quad \text{and} \quad p_{e+}^\mu = (E, 0, 0, -p) . \quad (.1)$$

What is the expression for p as a function of E and m_e ? What is the **center of mass energy** of the collision in the lab frame?

Suppose that this collider was invented to produce a 91 GeV particle, Z , through the process $e^+e^- \rightarrow Z$. What energy E is required for each beam? What is the momentum of the Z particle in the lab frame?

Extra credit: Suppose the Z is unstable and decays. This means that you don't get to measure it directly. Without knowing anything else about how the Z interacts, what is one **decay mode** that is guaranteed to exist? In other words, what types of particles should you make sure you can detect?

3.4 A fixed target experiment

Imagine a very asymmetric kind of collider called a **fixed target experiment**: a high-energy beam of particles hits a stationary target. Assume that both the beam and the target are composed of protons and that the collision occurs head-on¹. Write the four-momenta of a beam particle and the target particle in the lab frame.

Suppose you wanted to produce some completely made up particle—let's call it a *Flippon*²—that has a mass of 14 GeV. To one significant figure, what proton beam energy E is required to produce the Flippon through $pp \rightarrow \text{Flippon}$? (Assume that such a process is possible.) What is the momentum of the outgoing Flippon in the lab frame?

¹This is a classical idea, but for now we can live with this kind of deceit. Relevant: <https://www.youtube.com/watch?v=AnaQXJmpwM4>

²Unrelated to this: <https://arxiv.org/abs/1602.01377>

HINT: This problem is constrained by kinematics. There's an easy way and a hard way of doing this. The hard way is to perform a boost to the center of mass frame where it's easy. The easy way is to identify that the Einstein relation tells us that squares of momenta are *invariant*, and so you don't need to do the boost.

DISCUSSION: Fixed target experiments are nice because you don't have to worry about engineering two beams to collide with one another. They also have a very useful feature that the new particle is produced *boosted* relative to the lab frame. This can be very useful for untangling the decay products of the new particle from other particle debris from the beam hitting the target.

4 Feedback

Approximately how long did it take you to complete the non-extra credit parts of this assignment?

COMMENT: Homework will become more challenging as the quarter progresses.

Extra Credit

If you do any of these problems, please write a short note giving your thoughts on the reading: did you like them? Were they too simple / difficult? I do not expect you to be able to complete all (or necessarily any) of the extra credit. You can access all journal articles through the UCR library.

1 Minkowski Diagrams

The mathematical basis of relativity is geometry. This is most simply seen in what are called **Minkowski diagrams**. I'm pretty sure a good introduction to these are in <https://arxiv.org/abs/1508.01968> by Boxiang Liu and Thushara Perera³. Consider two reference frames with some non-zero relative velocity. Sketch the axes of the Minkowski diagram for this system: that is, draw the (x, t) axes and the (x', t') axes where (x', t') are related to (x, t) by a Lorentz transformation. Draw two spacetime events and their respective light cones. Comment on the idea of causality using these diagrams. Those who are mathematically inclined may enjoy <https://doi.org/10.1119/1.4997027>.

2 Impact of Special Relativity on Physics: Compton Scattering

Look over David Jackson's article "The Impact of Special Relativity on Theoretical Physics" from the May 1987 issue of *Physics Today*, <https://doi.org/10.1063/1.881108>. Focus on the section "Waves and particles," where the author discusses **Compton scattering**. Use special relativity to derive the author's expression for $\delta\lambda$, the shift in the photon wavelength.

3 Relativistic mass

There is an antiquated notion of *relativistic mass* that people used to talk about. Lev Okun gives a nice overview in "The Concept of Mass" in the June 1989 issue of *Physics Today*, <https://doi.org/10.1063/1.881108>.

³A somewhat more polished reference is the book *Very Special Relativity* by Sander Bais. There's also a Khan Academy video, <https://youtu.be/nEqexIckVCM>.

[//doi.org/10.1063/1.881171](https://doi.org/10.1063/1.881171). Read the article and explain why there is only *one* useful notion of mass and that it is the *rest mass*.