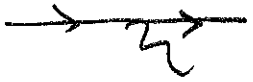
LAST TIME: QED RULES | TWO ADAM

today: on shell off shell E path

puzzle:  $(e \to e\gamma)$

2 → allowed diagram

but doesn't seem to make sense...

check: cons of 4-momentum ↔ kinematics vs DYNAMICS

initial electron: $(E, \vec{p})$

↳ could be anything subject to $m_e^2 = E^2 - p^2$ (p^2)

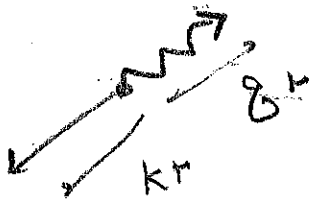
DIFF VALUES of p^μ ↔ DIFF FRAMES

ONE frame is most convenient

$p^\mu = (m_e, \vec{0})$ ← e^- REST FRAME

↳ stick to this frame the whole calc.

then e turns into $|\gamma + e|$



[PASS THRU: check for ANY FRAME PERFORM BOOST

[to do cons of p^μ @ EA VERTEX

[what is m?

$$p^\mu = k^\mu + q^\mu$$

net 3-momentum cancels ⇒ BACK to BACK

CHOOSE FRAME (ROTATE) S.T. MOMENTUM IS IN z-DIR

$$k^\mu = \begin{pmatrix} E' \\ 0 \\ 0 \\ k \end{pmatrix} \qquad q^\mu = \begin{pmatrix} k \\ 0 \\ 0 \\ -k \end{pmatrix}$$ ← B/C $q^2 = 0$ from on-shell

ENERGY CONS:

$$m_e = \boxed{E'} + k$$

$\uparrow$

$(E')^2 - k^2 = m_e^2$ from on-shell

So LET'S SEE THIS THROUGH

$$E'^2 = (m_e - k)^2$$

$$\underbrace{m_e^2 + k^2}_{E'^2} = m_e^2 - 2m_e k + k^2$$

$$\Rightarrow \boxed{0 = -2m_e k} \rightsquigarrow \boxed{k = 0}$$

then $q^\mu = 0$
PHOTON w/ NO ENERGY
(not a photon)
↳ undetectable

so process is really $e \to e$ ☺

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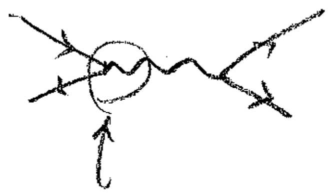
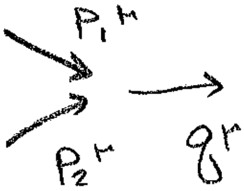
but now we know $\to\!\!\!\!\!\xi\!\!\to$ has problems w/ kinematics!



how does this make sense?



go to cm FRAME

$$p_1^\mu = (E, 0, 0, p)$$
$$p_2^\mu = (E, 0, 0, -p)$$

$$q^\mu = p_1^\mu + p_2^\mu = (2E, 0)$$

$\uparrow$ not on shell

do now what? AT LEAST 2 OPTIONS

① E and/or momentum not conserved

② Relax "on-shell" RULES

1. but spacetime symmetries still valid
even in QM → E + p still conserved

2. ... only physical particles need to be on shell

↑ C vs. VIRTUAL

OBSERVABLE (observed) — UNOBSERVABLE

↑ in/out states — ↑ internal lines

VIRTUAL: does not satisfy $E^2 - p^2 = M^2$

↳ but that's okay. This is accounted for.

In fact: FEYN. DIAGRAMS ARE ℂ #'s

the factor coming from internal lines goes like $\frac{i}{p^2 - M^2 + \cdots}$

BIG WHEN $p^2 = E^2 - p^2 \approx M^2$

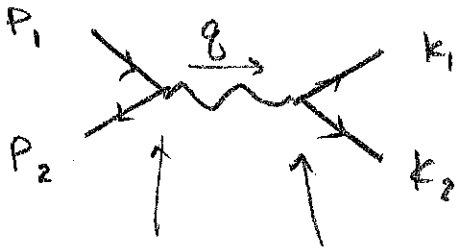
SMALL OTHERWISE.

So: RULE: ext lines on-shell

RULE: ea line carries 4-momentum

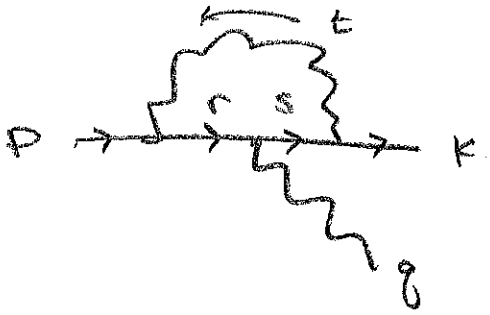
RULE: 4-MOMENTUM CONSERVED @ EA VERTEX.

HW show that total 4momentum is conserved in any graph.



$q = p_1 + p_2 \quad q = k_1 + k_2 \Rightarrow \boxed{p_1 + p_2 = k_1 + k_2}$

HW show that in Loop diagrams, there is an UNCONSTRAINED INTERNAL 4-momentum ... but total 4-momentum is still conserved

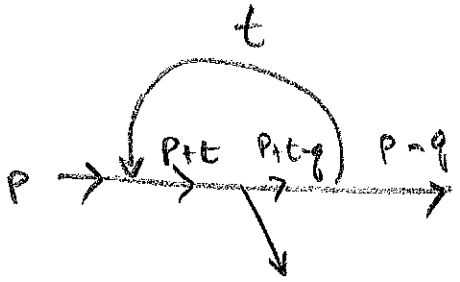


1 $r = p + t$
2 $s = r - q$
3 $k = s - t$

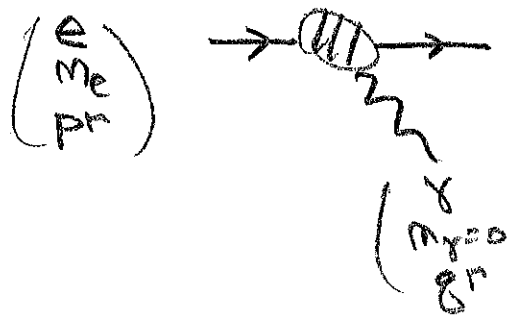
} are per Vertex

23: $r - q = k + t$

11: $p + \not{k} - q = k + \not{k}$ ✓



NB: EXTERNAL LINES NOW HAVE INFORMATION



} PROPERTIES of PARTICLE
- PROPERTY of KINEMATICS

... we will gradually add more info (spin, charge, ...)

this information is used to DISTINGUISH EXT. STATE PARTICLES

↳ USUALLY WE ARE IN MOMENTUM EIGENSTATES

"PHOTON w/ momentum k_1"

"PHOTON w/ momentum k_2"

tells different story

"γ w/ momentum k_2 connects to incoming electron"

SO WHAT ARE THESE DIAGRAMS? ℂ #

↳ REVIEW ∞ SUM "EXPT"

↳ REVIEW SUM over PATHS

eg dog fetching ball on beach
+ Index of REFRACTION
(remnant of least action)
↑ $S = \int dt\, L$

1 principle of all paths

→ [PHASORS]

$$\langle f | e^{iHt} | i \rangle$$

equivalent to sum over all possible ways that $|i\rangle \to |f\rangle$

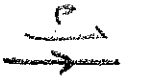
each "way" is a ℂ #
encoded in the FEYNMAN diagram expansion

some estimates:

 $= ie$ ← every VERTEX PICKS UP AN e

$(\alpha = \frac{1}{137} = \frac{e^2}{4\pi})$

 $= \frac{i}{q^2}$

 $= \frac{i}{\sqrt{p^2 - m_e^2}}$

} not quite, but approx