<u>SU(2)</u>: $\underbrace{(T^A)^a{}_b}_{\text{GENERATOR}}$ $\quad f^{ABC}$ $\quad \underbrace{\varepsilon^{ab}, \varepsilon_{ab}}$

Still writing hw.

A: <u>ADJOINT</u> INDEX
1, 2, 3

$\boxed{T^A \sim \sigma^A}$

fundamental
+ anti-fundamental
$a, b \in 1,2$

STRUCTURE CONSTANT

CARRIES the non commutation information.

$$[T^A, T^B] = i f^{ABC} T^C$$

LEVICIVITA w/ 2 INDICES
B/C SU(2)

> <u>note</u>: <u>DIAGONAL</u> GENERATOR IS SPECIAL. IT CAN BE USED AS A LABEL for components
>
> $$T^3 = \frac{1}{2}\begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$
>
> $$\left.\begin{aligned} T^3\,|\nu\rangle &= \tfrac{1}{2} \\ T^3\,|e\rangle &= -\tfrac{1}{2} \end{aligned}\right\} \begin{aligned} \nu &\sim |\tfrac{1}{2}\rangle \\ e &\sim |{-\tfrac{1}{2}}\rangle \end{aligned}$$
>
> $|\nu\rangle, |e\rangle$: doublet states

fundamental, eg LEPTON DOUBLET $\underbrace{L^a}$

complex 2-component vector of SU(2)

$(L^\dagger)_a \leftarrow$ LOWER INDEX: ANTI-FUNDAMENTAL

$\dagger$: CHARGE CONJUGATION
(<u>ALMOST</u> ANTI-PARTICLE ... sometimes we say anti ... it's almost)

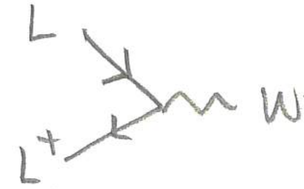
<u>INVARIANTS</u>: $(L^\dagger L)^n \leftarrow$ $L \to\bullet\to L^\dagger$

evidently, ∞ number of possible terms

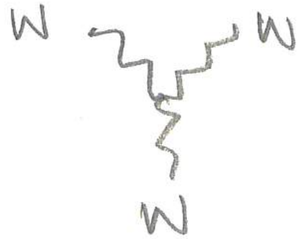
... focus on small #'s
(we will justify later)

<u>other INVARIANTS</u>: W^A is <u>GAUGE BOSON</u> (<u>FORCE</u>)

one for EA GENERATOR



from $(L^\dagger)_a (T^A)^a{}_b L^b W^A$



from $f^{ABC} W^A W^B W^C$

$\boxed{\text{BTW}}$: can treat T^A as conversion btwn adjoint to pair of fund/antifund. → nb: 3 gauge bosons, even if expect 4!

<u>weird ones</u>:

from $\varepsilon_{ab} L^a L^b$

from $\varepsilon^{ab} L^\dagger_a L^\dagger_b$

<u>MULTIPLE SYMMETRIES</u>: $SU(2) \times U(1)$

L^a & $q_L = \frac{1}{2}$

SU(2): $L^a \to U^a{}_b L^b$ ← $U = e^{i\theta^A T^A}$

U(1) $L^a \to e^{i q_L \varphi} L^a$

rot by θ about 3-axis

eg: $T^3 = \frac{1}{2}\begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$ s.t. $e^{i\theta T^3} = \begin{pmatrix} e^{i\theta/2} & \\ & e^{-i\theta/2} \end{pmatrix}$

then: SU(2) rot by θ along 3-axis &
U(1) rot by φ gives

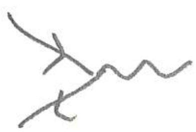
$$\begin{pmatrix} \nu \\ e \end{pmatrix} \to e^{i(\frac{1}{2})\varphi} \begin{pmatrix} e^{i\theta/2} & \\ & e^{-i\theta/2} \end{pmatrix} \begin{pmatrix} \nu \\ e \end{pmatrix} = \begin{pmatrix} e^{i(\varphi+\theta)/2}\, \nu \\ e^{i(\varphi-\theta)/2}\, e \end{pmatrix}$$

MORE IMPORTANTLY: additional symmetries <u>RESTRICT</u> allowed invariants

eg: UNDER U(1), <u>W^A IS INVARIANT</u>

intrinsically an SU(2) object

invariant



invariant

→ can see: just treat arrow as U(1) CHARGE!

invariant: obviously.

<u>not</u> invariant under U(1)!

$\varepsilon LL \to e^{2i\varphi q_L} \varepsilon LL$

under ROT BY φ w/rt U(1)

so imposing U(1) removes these terms from consideration!

SU(3)×SU(2) "strong-weak thy"

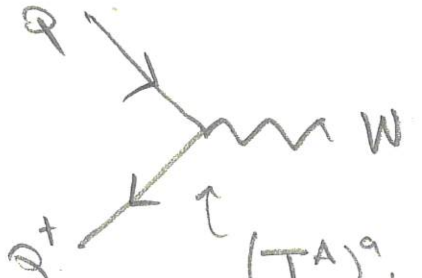
$Q = \left[\begin{pmatrix} u \\ d \end{pmatrix}_r, \begin{pmatrix} u \\ d \end{pmatrix}_g, \begin{pmatrix} u \\ d \end{pmatrix}_b \right]$ + also $(Q^\dagger)_{aM}$

$= Q^{a\,m}$ (a: SU(2), m: SU(3))

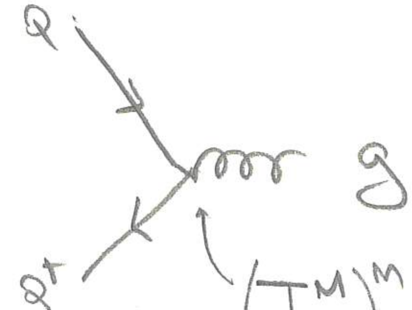
SU(3) → $(g^M)^m{}_n$ GLUON

SU(2) → $(W^A)^a{}_b$ W boson

INVARIANTS:



$\underbrace{(T^A)^a{}_b}_{SU(2)}$



$\underbrace{(T^M)^m{}_n}_{SU(3)}$

OBVIOUSLY SU(2) INVT.
IS IT SU(3) INVT? YES.

$(Q^\dagger)_{am} (T^A)^a{}_b W^A Q^{bm}$

DIRECT CONTRACTION

MIXES UP SU(3) INDICES

$(Q^\dagger)_{am} (T^M)^m{}_n g^M Q^{an}$

DIRECT SU(2) CONTRACTION

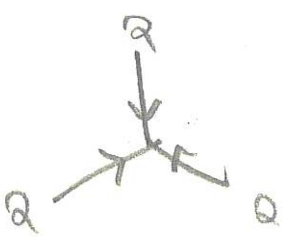
$\sim (Q^\dagger)_{am} Q^{am}$ ✓

OBVIOUSLY INVT
→ constructed to be INVT under one sym, does not transform under the other

$\sim Q^a Q^b \varepsilon_{ab}$ ... what about color?

$Q^{am} Q^{bn} \varepsilon_{ab} \boxed{?}$

no ε_{mn}! only ε_{mnl}



$\sim Q^m Q^n Q^l \varepsilon_{mnl}$

... but SU(2) DOESN'T WORK!

only ε_{ab} + $\underbrace{\varepsilon^{ABC}}_{\text{ADJOINT INDICES}}$

PREVIEW: the standard Model (REAL VERSION)
→ one generation

Q^{am} $\quad q = 1/6 \quad$ LH quark doublet $\quad Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$

u^{m} $\quad q = +2/3 \quad$ RH up quark

d^{m} $\quad q = -1/3 \quad$ RH down quark

L^{a} $\quad q = -\frac{1}{2} \quad$ LH lepton doublet $\quad L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$

e $\quad q = -1 \quad$ RH electron

H^{a} $\quad q = \frac{1}{2} \quad$ HIGGS DOUBLET $\quad H = \begin{pmatrix} \cdots \\ \cdots + h \end{pmatrix}$

+ all conjugates (ϕ^{+})
+ GAUGE BOSONS:

SU(3)	g^{M}	
SU(2)	W^{A}	
U(1)	Y	← "photon-like"

↑ forces

what's missing: ① GENERATION SYM.
② SPIN SYM (fermions are spin-$\frac{1}{2}$)
③ SYMMETRY BREAKING

examples of invariants: all the 
(obvious)

$(L^{\dagger})_a H^a e$  ← "YUKAWA COUPLING"

$q = +\frac{1}{2} \quad q = \frac{1}{2} \quad q = -1$

also $L^a H^{\dagger}_a e^{\dagger}$  ← given a rule w/ arrows, can always flip all arrows

(just † whole thing)

$\boxed{\text{SPIN}}$ LORENTZ GROUP:

$$V^\mu \longrightarrow \Lambda^\mu{}_\nu V^\nu$$

in fact, all <u>gauge bosons</u> (SM FORCE PARTICLES) have a vector index.

$$(D_\mu)^a{}_b = \delta^a_b \partial_\mu + ig (T^A)^a{}_b W^A_\mu$$

ea term must have same index structure

YOU ALREADY SAW this:
the PHOTON, A_μ, is THE <u>QUANTUM</u> excitation of the classical 4-POTENTIAL.

... then wtf is a <u>fermion</u>?
spin-1/2 (IN GEN'L half-int spin)

<u>FIRST</u>: how it works (not WHY it works)

two kinds of spin-1/2: LH & RH

↳ two kinds of indices

$(LH)^\alpha$ $\quad\quad$ $(RH)_{\dot\alpha}$ DOT! CANNOT CONTRACT DOTTED & UNDOTTED

$(LH^\dagger)_{\dot\alpha}$ $\quad\quad$ $(RH^\dagger)^\alpha$ $\Big\}$ $\dagger$ turns LH into RH

$\alpha, \dot\alpha$ RUN OVER 1,2

↳ very SIMILAR TO SU(2)!

TENSORS of SPIN SYM (LORENTZ GROUP/POINCARÉ)

$\varepsilon^{\alpha\beta}$, $\varepsilon_{\alpha\beta}$, $\varepsilon^{\dot\alpha\dot\beta}$, $\varepsilon_{\dot\alpha\dot\beta}$ } like in SU(2), we got these for free

instead of the usual GENERATORS (traceless sym...) WE HAVE

$$(\sigma^\mu)^\alpha{}_{\dot\beta} \quad \sigma^0 = \begin{pmatrix} 1 & \\ & 1 \end{pmatrix}, \quad \sigma^i = \sigma^i$$

$$(\bar\sigma^\mu)^{\dot\alpha}{}_\beta \quad \bar\sigma^0 = \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \quad \bar\sigma^i = -\sigma^i$$

↑

USUAL VECTOR INDEX

WE CAN CONVERT $\underbrace{\alpha, \dot\alpha}_{\text{PAIR of LH + RH}}$ to μ

PAIR of LH + RH ⟶ VECTOR

note: we are not GAUGING spin sym
... but it IS VERY ATYPICAL + IS NOT USUALLY COVERED IN A GROUP THY COURSE.

WE MAY ALSO USE $\partial_\mu \longleftrightarrow \underbrace{ip_\mu}$

b/c we work w/ fourier transformed fields

$$\phi(x) = e^{ip\cdot x}\tilde\phi(p)$$

so $\partial_\mu \phi \sim ip_\mu \phi$

LH PARTICLE $\psi^{\alpha} = \begin{pmatrix} \psi^1 \\ \psi^2 \end{pmatrix} \begin{matrix} \leftarrow \text{spin up} \\ \leftarrow \text{spin down} \end{matrix}$ } LH CHIRALITY

RH PARTICLE $\bar{\chi}_{\dot{\alpha}} = \begin{pmatrix} \bar{\chi}_1 \\ \bar{\chi}_2 \end{pmatrix} \begin{matrix} \leftarrow \text{spin up} \\ \leftarrow \text{spin down} \end{matrix}$ } RH CHIRALITY

$\underbrace{\qquad\qquad}_{\sim \text{ HELICITY}}$

<u>HELICITY</u> : quantize along direction of motion

$\vec{p}$

RH HELICITY ~ spin up

LH HELICITY ~ spin down

<u>CHIRALITY</u> : what <u>PHASE</u> does it pick up
<u>(QUANTUM)</u> under a rotation?

$\underset{\sim\sim}{\text{analogy}}$: in $SU(2)$: $T^3 = \frac{1}{2}\begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$

↑
b/c full math is tedious

$U_3 = \begin{pmatrix} e^{\frac{i}{2}\theta} & \\ & e^{-\frac{i}{2}\theta} \end{pmatrix}$

so $\begin{pmatrix} L^1 \\ L^2 \end{pmatrix} \to \begin{pmatrix} e^{+\frac{i}{2}\theta} L^1 \\ e^{-\frac{i}{2}\theta} L^2 \end{pmatrix}$

for massless particles : helicity = CHIRALITY

massive particles :

- HELICITY doesn't make sense (FRAME DEPENDENT)
- CHIRALITY MIXES

We use <u>CHIRALITY</u> to DEFINE PARTICLES

INVARIANTS

$\psi^\alpha \psi^\beta \epsilon_{\alpha\beta}$; similar

← obviously forbidden if ψ has any ℓ CHARGE
(related to MAJORANA mass)

$(\psi^\dagger)_{\dot\alpha} (\bar\sigma^\mu)^{\dot\alpha}{}_{\beta} \psi^\beta \, i p_\mu$

↑ proportional to 4-momentum

eg. PURE SU(3) theory

$\underline{Q^m Q^n Q^\ell \epsilon_{mn\ell}}$

BUT: no way to contract spin indices!

$Q^{m\alpha} Q^{n\beta} Q^{\ell\gamma} \epsilon_{mn\ell}$

↑ can only use $\epsilon_{\alpha\beta}$
σ^μ just introduces more indices!

eg USUAL GAUGE interaction?

$L^{a\alpha}$

W^A_μ

$\leftrightsquigarrow (L^\dagger)_{b\dot\beta} (T^A)^b{}_a W^A_\mu \underbrace{(\bar\sigma^\mu)^{\dot\beta}{}_{\alpha}}_{\text{aha!}} L^{a\alpha}$

$(L^\dagger)_{b\dot\beta}$