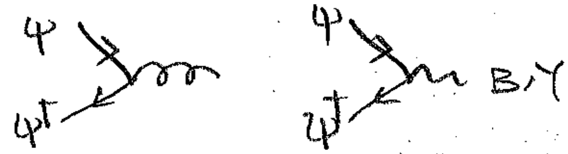
THE SM: all our PARTICLES

INTERACTIONS:



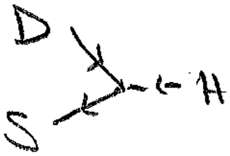
$\psi = (Q, u, d)$ $\quad$ $\psi = (Q, u, d, L, e)$

SU(2) DOUB. D

SU(2) SINGLET S

↑

no index



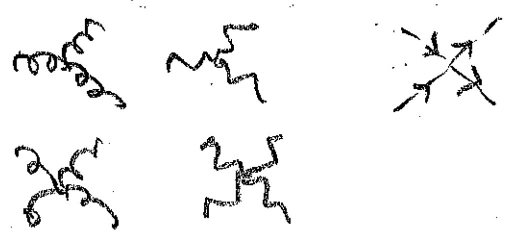
YUKAWA S

$L^\dagger H e_R$

$Q^\dagger H d_R$

$Q^\dagger H^\dagger u_R$

SELFPING:



outstanding puzzles:

1. FLAVOR (×3 # PARTICLES)
2. WHERE is PHOTON? (EWSB) $SU(2) \times U(1)_Y \to U(1)_{EM}$
3. MASS of PARTICLES:

thus far: everything is massless!?

see HW 7a: 2-LINE VERTICES 
w/o DERIVATIVES

→ MASS TERM

in SM: these are PROHIBITED?!

<u>why?</u>

<u>EXERCISE</u>: state space $q = \underbrace{(q^1, q^2, \ldots, q^N)}_{\text{"INDEP DOF"}}$

$$L = \frac{1}{2} q^T A q = \frac{1}{2} q_i A^i{}_j q^j$$

$$= \sum_{i,j} \frac{1}{2} q_i \underbrace{A_{ij}}_{} q_j$$

ASSUME SYMMETRIC (HERMITIAN)

eg: if $q_i \sim q(t_i)$, then

LOCALITY IN TIME

$$\frac{1}{2} \dot{q}_i^2 = -\frac{1}{2} q_i \left(\frac{d}{dt}\right)^2 q_i \qquad \frac{1}{\Delta t^2}\begin{pmatrix} \ddots & & & \\ -2 & 1 & & \\ 1 & -2 & 1 & \\ & 1 & -2 & 1 \\ & & & \ddots \end{pmatrix}$$

$m=1$

integ by parts $\quad A = \left(\frac{d}{dt}\right)^2$

(ALLOWED? YES: this lives in $\int dt\, L = S$)

$$(Aq)_i = \frac{q^{i+1} - 2q^i + q^{i-1}}{\Delta t^2}$$

~ SERIES of COUPLED HARMONIC OSCILLATORS IN TIME. STRAIGHTFORWARD to EXTEND TO COUPLED H.O. IN SPACETIME.

↳ excitations ⟷ PARTICLES

<u>EQUATION of motion</u>: $\frac{\delta S}{\delta q_i} = 0 \rightarrow \boxed{\frac{\delta L}{\delta q_i} = 0}$

↳ COMPARE to EULER LAGRANGE
WE TREAT d/dt AS PART OF OPERATOR

↳ so no $\frac{d}{dt}\frac{\delta L}{\delta \dot{q}_i}$ term (this is equivalent)

$$\Rightarrow \boxed{(Aq)_i = 0}$$

HW: write in components
use symm. of A_{ij}

FOR US: $A = \partial^2 + m^2$ (UP to a sign...)

GO FROM LATTICE $\to$ CONTINUUM SPACETIME

$$S = \int dt\, d^3\underline{x}\ \tfrac{1}{2}\phi(x)\,\mathcal{O}\,\phi(x) \qquad \text{(UP to a sign...)}$$

$$\mathcal{O} = \partial^2 + m^2 = \partial_t^2 - \underline{\nabla}^2 + m^2$$

<u>EOM</u>: $\mathcal{O}\phi(x) = (\partial^2 + m^2)\phi = 0$

<u>FOURIER</u>: $\phi(x) = \int d^4p\ e^{-ip\cdot x}\,\tilde{\phi}(p)$

$$\mathcal{O}\phi(x) = \int d^4p\,\underbrace{(-p^2 + m^2)}\,\tilde{\phi}(p) = 0$$

$$\text{EOM} \Rightarrow \boxed{p^2 = m^2}$$

on-shell condition!

<u>SO WHAT?</u> we just started w/ A QUADRATIC LAGRANGIAN

∂^2 term we identified w/ KINETIC TERM

$m^2 + p^2$

m^2 is JUST SOME non-derivative QUADRATIC TERM; 2-point vertex

SUGGESTIVELY NAMED, BUT <u>NOT</u> IDENTIFIED w/ MASS

& WE FOUND EOM $\leftrightarrow$ on-shell condition

if m^2 = MASS^2 of PARTICLE!

<u>one step further</u>: we never write 2-point vertices ANYWAY.

QUADRATIC L → $\boxed{\text{LINEAR EOM} \quad \mathcal{O}\phi = 0}$

we can solve these! eg GREEN'S FUNC.

AS OPPOSED to if $\exists$ NONLIN TERMS, eg $L \sim \phi^3, \phi^4, \ldots$

in fact, usually L includes <u>SOURCE</u>: $J(x)\phi(x)$

excitation of

if $J = \delta(x) \sim \int d^4p \; e^{-ip\cdot x}$

then EOM: $(p^2 - m^2)\tilde{\phi} = 1$

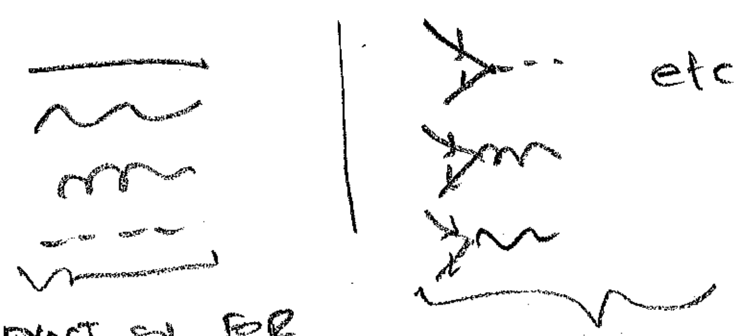
$$\tilde{\phi}(p) = \boxed{\frac{1}{p^2 - m^2}}$$

<u>PROPAGATOR</u>

rule for internal line ——— $\sim \dfrac{1}{p^2 - m^2}$

propagation through space time

do FEYNMAN RULE GAME:



<u>EXACT</u> SOL. FOR UNPERTURBED PROPAG

PERTURBATIONS AWAY FROM EXACT PROPAG

(interactions)

our n-PARTICLE states are not exact eigenstates of H

MASSLESSNESS of SM

HIGGS: $H^\dagger H$ is allowed. may have mass.

GAUGE BOSONS

$B_\mu B^\mu$ is NOT ALLOWED

why: under LOCAL SYMMETRY

$$B_\mu \to B_\mu - \frac{1}{g}\partial_\mu \underbrace{\theta(x)}_{\text{symm transf PARAM}} \quad (*)$$

$(\text{eg } \psi \to e^{ig\theta}\psi)$

& similar for W, g

so $B_\mu B^\mu$ is not INVARIANT UNDER $(*)$.

$\longrightarrow$ no mass term for GAUGE BOSONS!

FERMIONS

SPIN INDICES: ψ^α $(\psi^\dagger)_{\dot\alpha}$ LH CHIRAL

$\chi_{\dot\alpha}$ $(\chi^\dagger)^\alpha$ RH CHIRAL

so: $\psi^\dagger\psi$ cannot be contracted

$(\psi^\dagger)_{\dot\alpha}(\bar\sigma^\mu)^{\dot\alpha}{}_\alpha\psi^\alpha$ allowed ... but μ index needs to be contracted

Either: new field (eg B_μ) $\longrightarrow$ not QUADRATIC, INTERACTION!

or: $\partial_\mu \to$ GIVES KINETIC TERM, NOT MASS what IS allowed: $\psi^\alpha \psi^\beta \varepsilon_{\alpha\beta} = \psi^2$.

↳ BUT PROHIBITED BY $SU(3)\times SU(2)\times U(1)$!

$$\boxed{\psi^\alpha \psi^\beta \varepsilon_{\alpha\beta}}$$

if ∃ GAUGE indices, then all upper!

or, maybe SU(2) is ok:

$$L^{\alpha a} L^{\beta b} \varepsilon_{\alpha\beta} \varepsilon_{ab}$$

BUT HYPERCHARGE not CONSERVED!

$$Y[LL] = 2Y[L] = -1 \underbrace{\neq 0}_{\text{not } U(1)_Y \text{ INVARIANT}}$$

conclusion: none of the QudLe's can have mass!!

in fact, no CHIRAL spin ½ field w/ CHARGE CAN HAVE MASS!

↳ what IS allowed:

$$\psi^\alpha \chi^\dagger_\alpha \quad \text{two DIFFERENT particles}$$

LH ↗ ↖ RH

SUPPOSE QED: $\psi = e_L \quad \chi = e_R$
$q_\psi = -1 \quad q_\chi = -1$

then: $\psi\chi^\dagger$ is BOTH SPIN & $U(1)_{EM}$ INVARIANT

→ BUT THERE ARE TWO spinors: e_L^α & e_R;

(we call this a DIRAC SPINOR VS. WEYL)