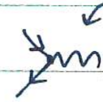
4/12/22 Particle lecture - Cross Section

Review short HW

QED rules $\rightarrow$ electron

$\sim\sim\sim$ photon



one arrow in
one arrow out

a). $e\bar{e} \to e\bar{e}$ d). $e\gamma \to e\gamma$ g). $e \to e\gamma$

b). $ee \to ee$ e). $e \to \bar{e}\gamma\gamma$ h). $\gamma \to ee\bar{e}\bar{e}$

c). $ee \to eee$ f). $\gamma\gamma \to \gamma\gamma$

Cross Sections

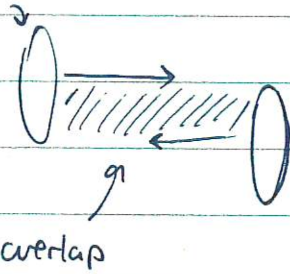
- Derive the cross section in ~~classical mechanics~~ macroscopic view, develop intuition
- ~~Attempt~~ Construct / Build the particle physics cross section (microscopic view)

The Main Punchline $\rightarrow$ cross sections connect theory to experiment

[Macroscopic] LHC works by colliding bundles of protons into eachother and measuring the stuff that comes out. ~~We want to quantify the likely~~ want to define a unit that is a measure of ~~the~~ how likely an interaction is.

If we took two individual protons and ~~took~~ watched them collide, it would look similar to this:

Lorentz contracted



$\mathbb{P}$

The interaction is likely if the overlap is large and small if the overlap is small.

Notably, [overlap] = Area.

[Idea] σ has something to do with overlap of states.

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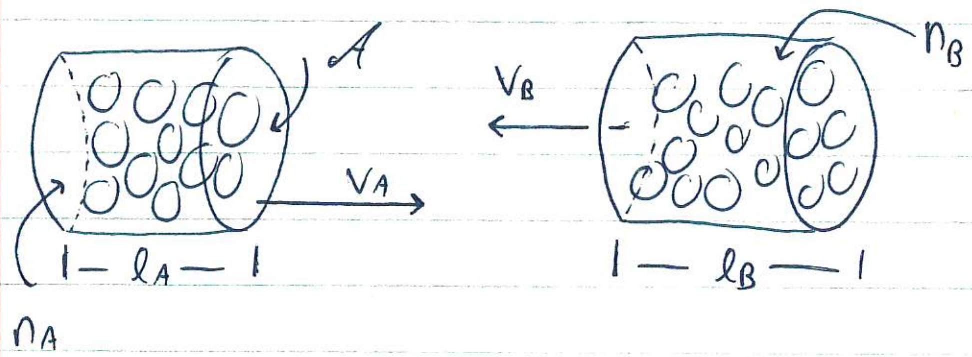
Now more realistic, take two bunches of protons and shoot them at eachother.

[Goal] We want the # of collisions / time. (or interactions / time)



[Ask]

What we know		
l_A : length	length	
n_A : # density	$1/\text{length}^3$	
V_A : speed	length / time	
$\mathcal{A}$: beam Area	length^2	

Game plan

① Pick a single proton from B. how many protons from A does it pass

② Multiply by # of B protons

① For a single B proton, we want $[N_A/t] = 1/\text{time}$.

[Ask]

$$\left[\frac{N_A}{t}\right] = \frac{1}{\text{time}}$$, how can we assemble what we know into something with dimensions $1/t$?

$$\frac{N_A}{t} \propto \underset{\frac{\text{length}}{\text{time}}}{(\text{relative velocity})}\underset{\frac{1}{\text{length}^3}}{(\#\text{ density})}\underset{\text{length}^2}{(\text{Beam Area})} = |V_A - V_B|\, n_A \mathcal{A}$$

This is the # of A's a single B sees per time

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Now we need the # of B's available to interact

$$N_B^{eff} \propto (\#\text{ density})(\text{Volume of "ONLY" protons})$$

$$n_B \times l_B \times \sigma$$

$\hookrightarrow [\sigma] : \text{length}^2$

$\sigma \sim$ "cross sectional area of each proton"

$\sim$ "How much one proton sees another proton"

Now we can construct what we want : events/time

$$\frac{\text{Events}}{\text{time}} = \frac{N_A}{t} * N_B = \underbrace{n_A n_B\, l\, \mathcal{A}\, |v_A - v_B|}_{\text{"Flux Factor" or "Luminosity"}} \times \underbrace{\sigma}_{\text{Cross - Section}}$$

Measure these

"Flux Factor" or "Luminosity"
- depends ONLY on experimental parameters
- Control this

Cross - Section
- depends ONLY on the interacting particles
- intrinsic to particle interactions
- Calculate this

The rest of today will be focused on diving into what σ is.

[Microscopic] **Cross Sections**

In QM, we know the initial and final states in question.

$|in\rangle$ and $\langle out|$

[Ask] ~~We can cut~~ what is $\mathbb{P}$ that we measure ~~$|out$~~ $|in\rangle$ to be in state $\langle out|$?

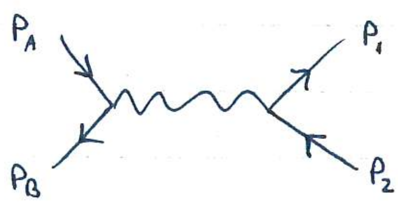
$$\mathbb{P} = |\langle out|in\rangle|^2$$

In particle physics, we know the 4-momenta of all the particles in the in state $|P_A P_B\rangle$ and the 4-momenta of all the particles in the out state $\langle P_1 P_2 \cdots P_n|$

[Ask] If you had to guess, what is the $\mathbb{P}$ to find the in-state in the out-state?

$$\mathbb{P} = |\langle P_1 P_2 \cdots P_n | P_A P_B\rangle|^2$$

eg: $e\bar{e} \rightarrow e\bar{e}$



In $|P_A P_B\rangle$ Out $\langle P_1 P_2|$

So the $\mathbb{P}$ for an electron with momentum P_A^μ and Positron P_B^μ to turn into an electron with momentum P_1^μ and Positron P_2^μ IS

$$\mathbb{P} = |\langle P_1 P_2 | P_A P_B\rangle|^2$$

Know that σ is specific to kinds of particle interactions, intrinsic to ~~parts~~ interactions so we expect

$$\sigma \sim |\langle P_1 P_2 \cdots P_n | P_A P_B\rangle|^2$$

[Clarify] Change in notation $\langle P_1 P_2 \cdots P_n |$ is a lot to write so use $\langle out |$ instead.
Similarly for $| in \rangle$, the set of all 4-momenta for our particles

[Ask] $$\underbrace{\sigma}_{\text{Length}^2} \propto \underbrace{|\langle out | in \rangle|^2}_{\text{Dimensionless}}$$ Are we done?
Are we missing anything? (Units)

We have to scale the IP by something dimensionful.
What should it be? (Energy, size, mass, ...)

Recall from earlier today that the cross section has to do with overlap.
Interpret this in QM, and you would say that the cross section depends on how much their wavelengths overlap. Are they in the same region of space.

We are dealing with IP and states overlaping
For two particles to interact, they need to be near eachother in space
[Ask] How do we quantify how localized something is in space?

- de Broglie wavelength $\lambda_{DB} = \hbar / |\vec{p}|$ only in high $|\vec{p}|$ limit
- Compton wavelength $\lambda_c = \hbar / mc$ only in low $|\vec{p}|$ limit

This suggests that: $\sigma \propto \lambda_A \lambda_B |\langle \cdots | \cdots \rangle|^2$

$$\sigma \propto \frac{1}{(\text{Momentum or Mass})^2} |\langle out | in \rangle|^2$$

How do we write one thing down that is valid in both regimes?

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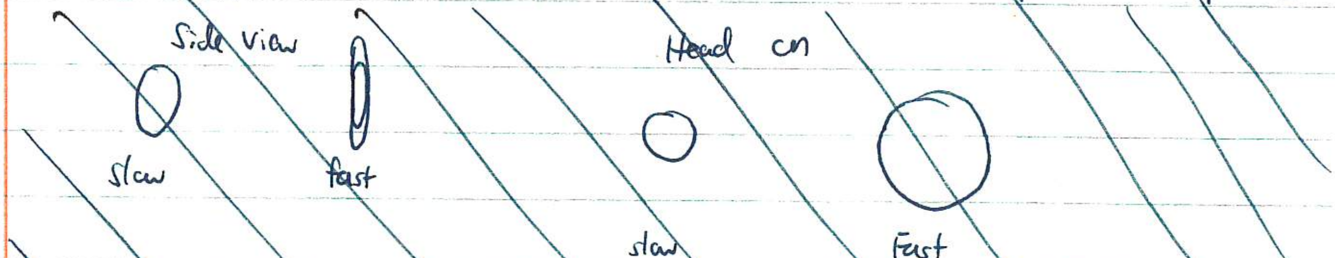
Use energy, $E^2 = m^2 + p^2$

$$\sigma \propto \frac{1}{2E_A} \frac{1}{2E_B} |\langle out | in \rangle|^2 \quad , \quad \begin{matrix} \text{if } p >> m, \; E \sim p \\ \text{if } p << m, \; E \sim m \end{matrix}$$

We want the cross section to be Lorentz invariant. (LI)
More specifically, Boost invariant

[Ask] Is our σ LI? (Boost Invariant)

- Mathematically $E = p^0$ and p^μ transforms under Lorentz boosts so no. (Look at Lorentz indices)
- Physically, the amount of length contraction depends on the speed of the protons



* this picture may be wrong but the idea is that our σ right now depends on how fast the protons are moving which is not what we want *

Look at how each chunk of σ transforms under boosts.
Define the beam direction as the z-direction so we only ever boost in $\hat{z}$

① $\mathbb{P} = |\langle out | in \rangle|^2$ is Lorentz invariant, no Lorentz indices.

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② E : $E \xrightarrow{\text{Z Boost}} E' = \gamma(E + \beta P_z)$

P_z : $P_z \xrightarrow{\text{Z Boost}} P_z' = \gamma(P_z + \beta E)$

So the energy factors transform as

$$\frac{1}{E_A E_B} \xrightarrow{\text{Z - Boost}} \frac{1}{E_A E_B}\left[\frac{1}{\gamma^2(1+\beta V_A)(1+\beta V_B)}\right], \quad V = P_z/E$$

Recap : ~~σ as written is not L.I. we~~

We want σ to be L.I.

Currently, it is not, and it transforms as above.

To fix this, we need something that

a). is dimensionless in natural units

b). Transforms inversely of $1/E_A E_B$ under Boosts

We only know the 4-momenta and we've already used energy (and momentum)

The only thing we have left is velocity, the initial velocities

$$V = P_z/E \xrightarrow{\text{Z Boost}} V\left(\frac{1 + \beta/V}{1+\beta V}\right)$$

Playing with various combinations of V_A and V_B,

$(V_A + V_B,\ V_A - V_B,\ V_A V_B,\ V_A/V_B)$

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We eventually find that $|v_A - v_B|$ transforms the way we want.

$$|v_A - v_B| \xrightarrow{z\ \text{Boost}} \frac{|v_A - v_B|}{\gamma^2(1+\beta v_A)(1+\beta v_B)}$$

This is exactly what we need to cancel out how $1/E_A E_B$ transforms.

The full Boost (Lorentz) invariant cross section is now

$$\sigma = \frac{1}{2E_A}\frac{1}{2E_B}\frac{1}{|v_A - v_B|}|\langle \text{out}|\text{in}\rangle|^2$$

4.2.2 We are now left with, wtf is $|\langle \text{out}|\text{in}\rangle|^2$?

$|\text{in}\rangle$ is the set of initial-state 4-momenta, p_A^μ, p_B^μ
$\langle \text{out}|$ " final-state 4-momenta, p_1^μ p_2^μ ... p_n^μ

why momenta? Colliders measure momentas and energies and ~~completely~~ basically ignore positions.

Colliders measure a $\underbrace{\text{particular configuration}}$ of final state momenta.

a single choice in the phase space of final momentums

eg: 2 → 3