Short HW 8: **The theoretical origin of electromagnetism**

Course: Physics 165, *Introduction to Particle Physics* (2022)
Instructor: Prof. Flip Tanedo (flip.tanedo@ucr.edu)
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This "short homework" is a little more involved. You may find the Lecture 15 notes helpful. The goal for this short homework is to go through the manipulations we presented in class.

Useful Reference Ideas

The Higgs vacuum expectation value ("vev") is

$$\langle H \rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} . \tag{0.1}$$

The electroweak gauge bosons of $\text{SU}(2)_L \times \text{U}(1)_Y$ pick up a mass from the kinetic term,

$$\left(D_\mu \langle H \rangle \right)^\dagger \left(D^\mu \langle H \rangle \right) , \tag{0.2}$$

where the covariant derivative[1] is

$$D_\mu = \partial_\mu + igT^A W_\mu^A + ig' q_Y B_\mu . \tag{0.3}$$

Note that we have not written any implicit unit matrices. For example, when D_μ acts on a doublet, the T^A are 2×2 Hermitian matrices while the ∂_μ and B_μ terms implicitly have a 2×2 unit matrix acting on the SU(2) indices. Recall that

$$T^1 = \frac{1}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad T^2 = \frac{1}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad T^3 = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ & -1 \end{pmatrix} , \tag{0.4}$$

when acting on doublets like the Higgs.[2] It is obvious that $\partial_\mu \langle H \rangle = 0$ because the vev is constant.

1 The mass terms

Normalizing the gauge bosons

[**Flip**: This subsection added 5/17.] You can think of $W^{1,2,3}$ and B as an orthonormal basis of fields. This means that

$$\langle W^i, W^j \rangle = \delta^{ij} \qquad \langle W^i, B \rangle = 0 \qquad \langle B, B \rangle = 1 . \tag{1.1}$$

[1] Where did this come from? We motivated the covariant derivative as the natural 'promotion' of the ordinary derivative that was required to make $\text{SU}(2)_L \times \text{U}(1)_Y$ a *local* symmetry. This meant that we had to introduce new position-dependent objects (fields... which are particles) that we identified with the force particles.

[2] When acting on a singlet like the right-handed particles, T^A gives zero since the singlets do not transform and T^A is the matrix that generates an infinitesimal transformation.

The definition of the inner product is not important here, but it is $\langle \psi, \phi \rangle = \int d^4x\, \psi^*(x)\phi(x)$. The relevant bit here is that terms that look like $|W^i|^2$ are normalized to one and cross terms like $B^* W^1$ go to zero. Consider the W^+ boson. We know that it is

$$W^+ = \mathcal{N}\left(W^1 + iW^2\right) \ , \tag{1.2}$$

we can determine the normalization $\mathcal{N}$ by fixing $|W^+|^2 = \mathcal{N}^2\left(|W^1|^2 + |W^2|^2\right) = 1$. This means

$$\mathcal{N} = \frac{1}{\sqrt{2}} \ . \tag{1.3}$$

The linear combination for the Z boson is proportional to the combination that gets a mass.

1.1 Inserting the vev

Show that [**Flip**: Corrected 5/17, thanks Robert V.]

$$D_\mu \langle H \rangle = \frac{iv}{2\sqrt{2}} \begin{pmatrix} g\left(W^1 - iW^2\right) \\ g'B - gW^3 \end{pmatrix} \equiv \frac{iv}{2\sqrt{2}} \begin{pmatrix} g\sqrt{2}W^+ \\ -g_z Z \end{pmatrix} \ , \tag{1.4}$$

where $g_z^2 = g'^2 + g^2$ is the characteristic Z-boson interaction strength. In the last step we just defined the properly normalized W^+ and Z bosons. Based on the above note, convince yourself that $g'B - gW^3 = -g_z Z$ with respect to the properly normalized Z. The minus sign is conventional.

1.2 Masses

When you take $|D_\mu \langle H \rangle|^2$, you end up with masses

$$M_W^2 W^+ W^- + \frac{1}{2} M_Z^2 Z^2 \ . \tag{1.5}$$

The factor of $1/2$ is convention for terms with two identical particles.[3] Show that the masses are

$$M_W^2 = \frac{g^2 v^2}{4} \qquad\qquad M_Z^2 = \frac{g_Z^2 v^2}{4} \ . \tag{1.6}$$

Which particle is heavier, the Z or the W? [**Flip**: Corrected 5/17, thanks Robert V.]

2 Mixing Angles

The Z boson is a linear combination of W^3 and B,

$$Z = \frac{-g'}{g_Z} B + \frac{g}{g_Z} W^3 \equiv -\sin\theta_W B + \cos\theta_W W^3 \ , \tag{2.1}$$

where θ_W is called the Weinberg angle. You can think of this as a two-dimensional real vector space with basis vectors $|B\rangle$ and $|W^3\rangle$. The Z-boson, $|Z\rangle$ is a different basis vector. It is the state that explicitly picked up a mass from the Higgs vev. The photon is the other basis vector in this

[3] For those interested in doing further reading, this is called a symmetry factor and it has to do with the different permutations of particle creation/annihilation operators.

new (mass eigenstate) basis. It did not pick up a mass, but we can infer the linear combination by requiring that it is orthonormal to the Z. Show that (up some choice of signs), the photon is

$$A = \frac{g}{g_Z}B + \frac{g'}{g_Z}W^3 \ . \tag{2.2}$$

COMMENT: this only a line or two.

Extra Credit: photon couplings

The interactions of matter with the photon are governed by their kinetic terms.

Right-handed matter

Right-handed matter particles are SU(2) singlets (i.e. no a, b indices). Consider the kinetic term for the right-handed up quark:[4]

$$u_R^\dagger \sigma^\mu D_\mu u_R \ = u_R^\dagger \sigma^\mu \left(\partial_\mu + ig' q_Y B_\mu\right) u_R \ , \tag{2.3}$$

where q_Y is the hypercharge of the u_R. Show that the interaction with the photon is

$$ie q_{\text{EM}} u_R^\dagger \sigma^\mu A u_R \qquad\qquad e = \sin\theta_W \cos\theta_W g_Z = \cos\theta_W g' \qquad\qquad q_{\text{EM}} = 2/3 \ . \tag{2.4}$$

We have thus found an expression for the electric coupling e and the electric charge of the (right-handed) up quark, q_{EM}.

Left-handed matter

Left-handed matter are SU(2) doublets. The covariant derivative is thus a 2×2 matrix in SU(2) space. We will not bother with the off diagonal terms (the $W^\pm$ bosons). The diagonal parts are

$$D_\mu = \partial_\mu + \frac{ig}{2}\begin{pmatrix} W^3 & \\ & -W^3 \end{pmatrix} + ig' q_Y \begin{pmatrix} B & \\ & B \end{pmatrix} \ , \tag{2.5}$$

where we have now written the explicit 2×2 unit matrix on the B term. Show that the photon coupling to the quark doublet Q is

$$ieQ^\dagger \bar{\sigma}^\mu A_\mu (T^3 + q_Y) Q \ , \tag{2.6}$$

where e is the same electric coupling defined above and the electric charge $q_{\text{EM}} = T^3 + q_Y$ is the sum of the T^3 eigenvalue ($\pm 1/2$) of the doublet with the hypercharge. Show that the left-handed up quark has the same electric charge as the right-handed up quark. This had to be true since the Yukawa couplings pair the u_L and u_R together into a massive charged fermion (a so-called Dirac fermion).

And all the rest

Show that the electric charges for the down quark, electron, and neutrino are as you expect. Show that unlike the photon, the Z boson will interact with neutrinos.

[4] I'm being sloppy with overall factors of i. Those aren't the point here. I am also suppressing several indices. Once you know they're there, you only need to write them out when you need them.