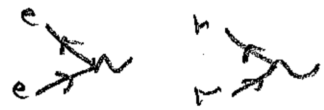
PREVIEW: SHORT HW:  MOMENTUM CONS.

$\boxed{\text{QED}+\mu}$ 

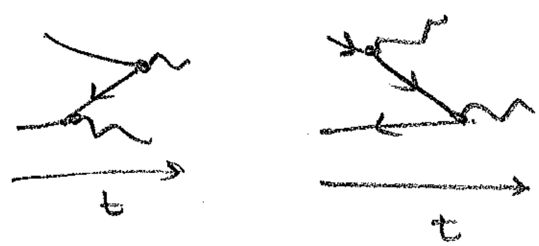
↳ Q: $\mu \to e\gamma$?

QUESTIONS FROM LAST TIME

interpreting spacetime diagram

question was re: VERTICAL LINES

 ← what is this?



← sum over all positions of internal vertices

SEE: QED+μ FROM LEC 6 (we didn't get to it LAST TIME)

SU(2) INTERNAL GAUGE SYMMETRY

↳ unlike QED+μ, which has a ROTATIONAL SYMM when $M_\mu = M_e$

(GLOBAL/NORMAL)

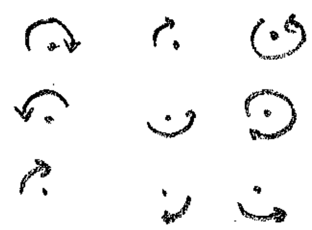
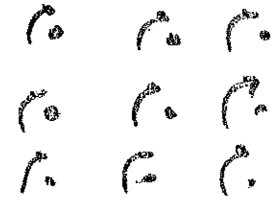
GAUGE symmetry: FUNDAMENTAL FORCE
mathematical REDUNDANCY

↳ eg GRAVITY, E&M

$$A_\mu \to A_\mu + \partial_\mu f$$

DOES NOT CHANGE MAXWELL

ASIDE: how it works mathematically:

ALLOW LOCAL SYMMETRY:  VS. GLOBAL: 

(nb both are internal

the LAGRANGIAN for E&M:

$$\mathcal{L} = i\bar{\psi}(\partial_\mu + \underbrace{ieA_\mu}_{\text{GAUGE FIELD}})\psi + m\bar{\psi}\psi$$

$\sim \psi^\dagger$

s.t. VARIATION GIVES

$i\partial_\mu\psi + m\psi = 0$

(up to signs, etc)

if only this:

$$\left.\begin{aligned} \psi &\to e^{i\theta}\psi \\ \bar{\psi} &\to e^{-i\theta}\psi \end{aligned}\right\}$$ GLOBAL SYM. IS INVT.

BUT: if $\theta = \theta(x)$

$$\bar{\psi}\partial_\mu\psi \to \bar{\psi}e^{-i\theta(x)}\partial_\mu(e^{i\theta(x)}\psi)$$

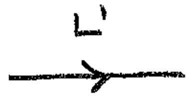
$$= \bar{\psi}e^{-i\theta(x)}\Big(\underbrace{i\theta'(x)\,e^{i\theta}\psi}_{\substack{\text{new term:} \\ \bar{\psi}(i\partial_\mu\theta)\psi}} + e^{i\theta}\partial_\mu\psi\Big)$$

BUT: IF $A_\mu \to A_\mu + \frac{1}{e}\partial_\mu\theta$

then the new term CANCELS.

that's what GAUGE FIELD DOES.

WEAK THEORY (does not resemble SM!)

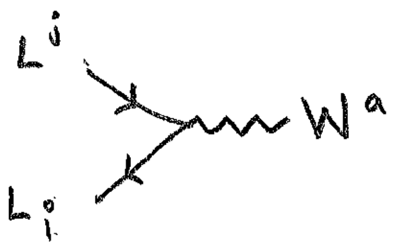


LEPTON DOUBLET

$$= \begin{pmatrix} \nu \\ e \end{pmatrix} \begin{matrix} \leftarrow L^1 \\ \leftarrow L^2 \end{matrix}$$

W TRIPLET

$$= \begin{pmatrix} W^1 \\ W^2 \\ W^3 \end{pmatrix}$$



$$= i g (T^a)^i{}_j$$

coupling strength (analog of e)

LOWER INDEX FOR ARROW MOVING AWAY FROM VERTEX

"GENERATOR of SU(2)"

T^a IS A MATHEMATICAL OBJECT "GIVEN TO US" FROM THE SYMMETRY IN ORDER TO HELP US MAKE INVARIANTS.

idea: IN A LAGRANGIAN:

$$i\, \bar{L}_i \, (\underbrace{\partial_\mu + i g (T^a)^i{}_j W^a}_{\text{COVARIANT DERIVATIVE } D = \mathbb{1}^i{}_j \partial_\mu + \dots}) L^j$$

RULES: REPEATED a indices ARE <u>CONTRACTED</u> (SUMMED over, "cancel" the index)

REPEATED UPPER/LOWER $i j$ INDICES CONTRACTED eg UNDER A $\underset{SO(2)}{\underline{\text{ROTATION}}}$:

$$V \to RV$$
$$A \to RAR^T$$

$$\underline{W}^T A \underline{V} \to \underline{W}^T \underbrace{R^T R}_{\mathbb{1}} A \underbrace{R^T R}_{\mathbb{1}} \underline{V}$$

$= \underline{W}^T A \underline{V}$, INVARIANT.

SU(2) DIFFERS B/C "ROTATIONS" ARE NOW UNITARY TRANSFORMS

↳ unitary = $\mathbb{C}$ version of ROTATION

⇒ PRESERVES $\mathbb{C}$ INNER PRODUCT

→ ie PRESERVES PROBABILITY.

The ∂_μ term is not invariant under LOCAL transformations, BUT THIS is compensated by the transformation of the W^a.

↳ for now: analogy to U(1) is sufficient

what are the $(T^a)^i{}_j$? the PAULI MATRICES

$$T^1 = \tfrac{1}{2}\sigma^1 = \tfrac{1}{2}\begin{pmatrix} & 1 \\ 1 & \end{pmatrix}$$
$$T^2 = \tfrac{1}{2}\sigma^2 = \tfrac{1}{2}\begin{pmatrix} & -i \\ i & \end{pmatrix}$$
$$T^3 = \tfrac{1}{2}\sigma^3 = \tfrac{1}{2}\begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$

BASIS of HERMITIAN 2×2 MATRICES

← the factors of 1/2 ARE IMPORTANT MATHEMATICALLY (NORMALIZING a BASIS)

.. but not a big deal for us.

UNITARY TRANSFORMATION on this $\mathbb{C}$ 2×2 SPACE: $\underbrace{U(\underline{\theta})}_{\text{"}R^i{}_j\text{"}} = e^{i\theta^a (T^a)^i{}_j}$

so that under SU(2) symmetry

$$L \to \mho L = e^{i\Theta^a (T^a)_{ij}} \begin{pmatrix} \nu \\ e \end{pmatrix}$$

eg. easy one: $\Theta^a = (0,0,\Theta)$

$$\mho = e^{i\frac{\Theta}{2}\begin{pmatrix}1 & \\ & -1\end{pmatrix}} = \begin{pmatrix} e^{i\Theta/2} & \\ & e^{-i\Theta/2}\end{pmatrix}$$

$$\begin{pmatrix} \nu \\ e \end{pmatrix} \to \begin{pmatrix} e^{i\Theta/2}\,\nu \\ e^{-i\Theta/2}\, e \end{pmatrix}$$

REPHASING BY OPPOSITE AMT!

eg. let's do a small rotation in a=1 direction

$$\Theta^a = (\varepsilon, 0, 0)$$

$$\mho = e^{i\frac{\varepsilon}{2}\begin{pmatrix} & 1 \\ 1 & \end{pmatrix}} \approx \mathbb{1} + \frac{i\varepsilon}{2}\begin{pmatrix} & 1 \\ 1 & \end{pmatrix}$$

$$\begin{pmatrix} \nu \\ e \end{pmatrix} \to \begin{pmatrix} 1 & \frac{i}{2}\varepsilon \\ \frac{i}{2}\varepsilon & 1 \end{pmatrix}\begin{pmatrix} \nu \\ e \end{pmatrix} = \begin{pmatrix} \nu + \frac{i\varepsilon}{2} e \\ \underbrace{\frac{i\varepsilon}{2}\nu + e} \end{pmatrix}$$

ν & e mix into each other!

evidently, SU(2) JUMBLES UP COMPONENTS of L!
↳ IN WEAK THY ν & e MUST be "the same"

what about W^a? W^a LIVES IN A MATRIX, $W = W^a (T^a)_{ij}$

$$W^a T^a \to \mho(\Theta)\, W^a T^a\, \mho(\Theta)^\dagger$$

Jumbles up components.

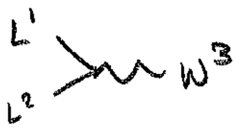
$$W T^a = \frac{1}{2}\begin{pmatrix} W^3 & W^1 - iW^2 \\ W^1 + iW^2 & -W^3 \end{pmatrix}$$

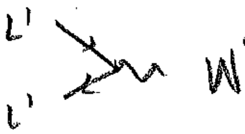
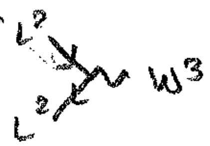
HOW to READ FEYNMAN RULE:

 $= ig(T^a)^i{}_j$

↑ this is a # once you fix i, j, a
... may be zero!

eg $T^3 = \frac{1}{2}\begin{pmatrix}1 & \\ & -1\end{pmatrix}$ ← DIAGONAL

SO  $= 0$ ⇐ no $\nu e W^3$ interaction
⇒ not symmetric under $SU(2)$

eg  $= -\Big($  $\Big)$

↑ B/c $(T^3)^1{}_1 = -(T^3)^2{}_2$