LAST TIME: X-SEC W/ ADAM

THE GAME:

- THY = FEYNMAN RULES
  - LINES (PROPAGATORS)
  - VERTICES

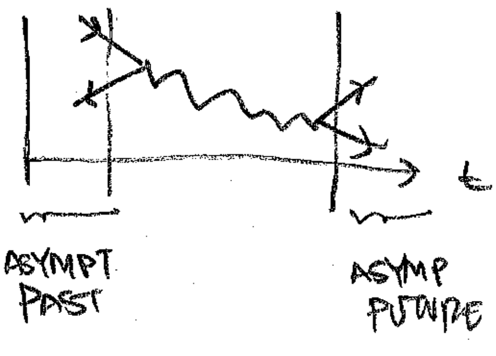
eg 

some lines oriented
RELATES TO CHARGE FLOW
IE PARTICLE VS ANTIPARTICLE

some lines UNORIENTED
(eg if PARTICLE IS ITS OWN ANTIPARTICLE)

[sometimes we drop the orientation if it is "obvious" or we label in some other way]

- SCATTERING PROCESS: CONNECTED GRAPH
  implied: spacetime graph



only topology matters
IMPLIED SUM OVER VERTEX POSITIONS
(btw: necessary for relativity)

EXT STATES ARE ON SHELL & HAVE FIXED OBSERVABLE QUANTITIES

eg ALL Q'M #s (SPIN, MASS, CHARGES ...)
& KINEMATICS (eg 4-MOMENTUM)

- for a given $|in\rangle$ & $|out\rangle$,

PATH INTEGRAL $\leftrightarrow$ SUM over (∞) # of diagrams that connect $|in\rangle$ to $|out\rangle$

$\underbrace{\text{PATH INTEGRAL}}_{\Sigma \text{ "HISTORIES"}}$

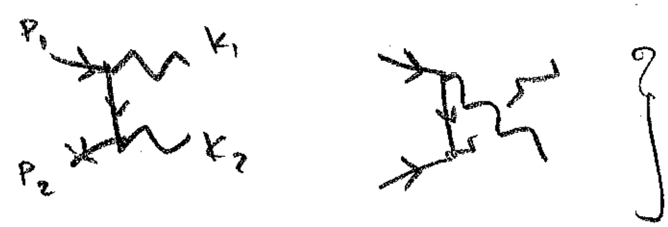
notes

- if no diagrams, then process is not [PERTURBATIVELY] possible
- USUALLY ∞ # DIAGRAMS!
- SOME NONPERTURBATIVE EFFECTS ARE NOT CAPTURED BY DIAGRAMS

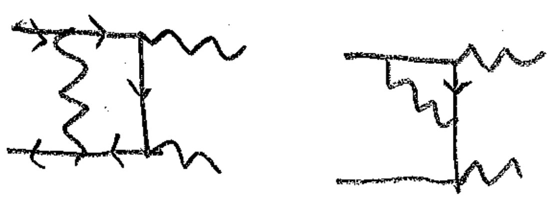
- USUALLY, ONLY CARE ABOUT LEADING ORDER DIAGRAMS

ALL diagrams w/ fewest # of vertices

eg.



$\mathcal{O}(\text{vertex}^2)$



etc. $\mathcal{O}(\text{vertex}^4)$

the ∞ # of diagrams are terms in a TAYLOR EXPANSION. THE SMALL PARAM. IS THE VERTEX ?

sometimes vertex is not small … then DYNAMICS ARE NONPERTURBATIVE

- KINEMATICS
  - DIAGRAM → DYNAMICALLY POSSIBLE
    QUANTUM
  - still need to check if kinematically POSSIBLE
    eg. RELATIVITY

RULE: ALL EXT PARTICLES HAVE FIXED 4-MOMENTA

RULE: 4-MOMENTUM FLOWS THROUGH DIAGRAM LIKE CURRENT IN A CIRCUIT

eg. PROVE THAT TOTAL 4-MOMENTUM IS CONSERVED

RULE: ALL EXT PARTICLES MUST BE ON-SHELL

↳ internal particles ARE (IN GENERAL) NOT ON-SHELL, but they do have well def. 4-momentum

... funny things happen when they go on shell.

eg. DIAGRAMS W/ LOOPS HAVE AN UNCONSTRAINED INTERNAL MOMENTUM. WE SUM (integrate) OVER ALL POSSIBLE MOMENTA

- WHAT DOES IT MEAN?
  - EA DIAGRAM IS A $\mathbb{C}$ #, A CONTRIBUTION TO A SCATTERING AMPLITUDE

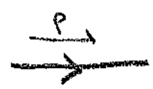
    " PROB $\sim |\Sigma$ DIAGRAMS$|^2$ "

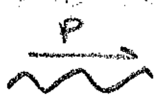
    MORE CAREFULLY: CROSS-SECTION GIVES A USEFUL QUANTITY FOR SCATTERING LIKELIHOOD

  - FACTORS in the NUMERICAL VALUE of A DIAGRAM ARE ENCODED IN THE FEYNMAN RULES.

eg


$\sim ie$


$\sim \frac{i}{p^2 - m_e^2}$


$\sim \frac{i}{p^2}$

this is ok for BASIC ESTIMATES

... but we're missing info

note: internal lines appear to diverge when they go on-shell

(AMPLITUDE GETS LARGE)

WHAT'S MISSING: <u>TENSOR STRUCTURE</u>

TENSORS.

PARTICLE PHYSICS : ultimately about SYMMETRY

eg cons. laws
limit kinematics

ALSO RESTRICTS → RULES

DYNAMICS

YOU ALREADY KNOW THIS :

p^μ ← UPPER INDEX

LORENTZ : $p^\mu \longrightarrow \boxed{\Lambda^\mu{}_\nu} p^\nu$

$\begin{pmatrix} \gamma & \gamma\beta \\ \gamma\beta & \beta \end{pmatrix}$

index tells you how it transforms

eg $T^{\mu\nu} \rightarrow \underbrace{\Lambda^\mu{}_\alpha}_{\text{one for ea upper index}} \Lambda^\nu{}_\beta \, T^{\alpha\beta}$

OTHER MACHINES :

METRIC: $g_{\mu\nu}$ ← LOWERS INDICES

SPECIAL: DEFINES GEOMETRY
- symmetric
- INVARIANT UNDER LORENTZ

$p_\mu = g_{\mu\nu} p^\nu \longrightarrow (\Lambda^{-1})^\nu{}_\mu \, p_\nu$ | in Φ vec space $T \rightarrow T$

s.t. $\underbrace{p \cdot k}_{\text{no free index}} \rightarrow \underbrace{p_\nu (\Lambda^{-1})^\nu{}_\mu}_{p'_\mu} \underbrace{\Lambda^\mu{}_\alpha k^\alpha}_{k'^\mu} = \underbrace{p \cdot k}_{\text{DOES NOT TRANSFORM}}$

<u>MESSAGE</u>: the GAME is <u>ALWAYS</u> to find INVARIANTS.

eg. I <u>SEE</u> A PARTICLE w/ 4-MOMENTUM $p^\mu = (E, \vec{p})$

YOU ARE IN A DIFF FRAME; I MEASURE YOUR 4-VELOCITY TO BE $v^\mu = \left(\frac{dt}{d\tau}, \frac{dx}{d\tau}\right)$

in <u>your</u> frame, $v'^\mu = (1, \underline{0})$

& YOU MEAS PARTICLE w/ $p'^\mu = (E', \vec{p})$

HOW DO I CALCULATE WHAT ENERGY, E', YOU MEASURE?

COULD DO LORENTZ TRANSFORMATION w/ $\beta = (dx/d\tau)/c$

<u>OR</u>: $\underbrace{E' = v' \cdot p'}_{\text{trivially}} = \underbrace{v \cdot p}_{\text{BY INVARIANCE}}$

LAWS of PHYSICS: WRITTEN WRT <u>INVARIANTS</u>

↳ SYMMETRIES BECOME MORE <u>ABSTRACT</u>

<u>INTERNAL</u> SYMMETRY

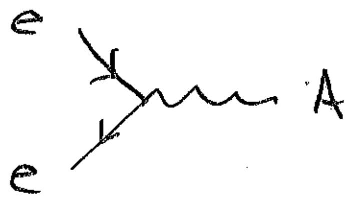
↳ not a spacetime symm.

a symmetry of the $\mathcal{L}$

2 kinds: { GLOBAL - normal

GAUGED/LOCAL - comes w/ FORCE PARTICLE

DUMB EXAMPLE: U(1) GAUGE SYM



$\sim \underbrace{e^\dagger e}\, A$ ← gauge boson.

these are $\mathbb{C}$ objects

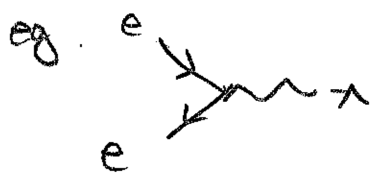
$e(x) \to e(x)\, e^{iq\theta}$ ← CHARGE

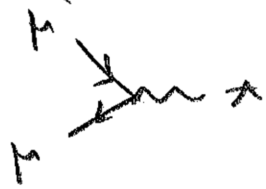
global version.
$\left\{ \begin{array}{l} e(x) \to e(x)\, e^{iq\theta} \\ \text{then: } e^\dagger \to e^\dagger e^{-iq\theta} \\ \text{and } e^\dagger e \text{ is INVARIANT} \end{array} \right.$

we'll develop the GAUGE version soon.

MUON, copy of electron.

eg.





w/ SAME INTERACTION strength & charge

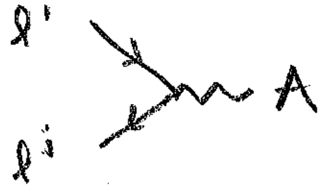
SYMMETRY: $e \longleftrightarrow \mu$

in fact:
$$e \to \cos\theta\, e + \sin\theta\, \mu$$
$$\mu \to -\sin\theta\, e + \cos\theta\, \mu$$

$$\begin{pmatrix} e \\ \mu \end{pmatrix} \to \begin{pmatrix} c & s \\ -s & c \end{pmatrix} \begin{pmatrix} e \\ \mu \end{pmatrix} \leftarrow \ell^i \text{ w/ } \left(\begin{array}{l} \ell^1 = e \\ \ell^2 = \mu \end{array} \right)$$

nb: of course: we can tell e & μ APART
so this symmetry is "broken"
... more on this w/ HIGGS.

→ for now, suppose e & μ have some mass one way to write this <u>FAMILY</u> symmetry is:



$\sim ieq\,\delta^i_j$

$\ell^i \underbrace{\ell^\dagger_j}_{} A$

H.C. HAS LOWER INDEX

not invariant: BUT $\underbrace{\ell^i (\ell^\dagger)_i}_{\text{"}\ell^\dagger \ell\text{"}}$ <u>is</u> invariant (no free indices)

So: $\ell^i (\ell^\dagger)_j\, \delta^i_j\, A \times \underbrace{eq}_{}$

some NUMERICAL PREFACTOR

$= (e^\dagger e A + \mu^\dagger \mu A)\, eq$

turns out, this is a term in the LAGRANGIAN!