LECTURE 27: a taste of GAUGE THEORY 28 NOV

how I use gauge sym.

REMINDERS:

(1) please fill out course evaluations on iEval
→ feel free to email DETAILED feedback / constr. crit. directly to me

(2) Grading: exact rubric TBD;
but if you've completed most of the homework, I'll grade from (B+, A)
↳ NEED TO HAVE DONE ENOUGH TO SHOW: Green's func & Contour Int!
→ if not: B.

LAST TIME: SPACETIME SYMMETRY
↳ remarkably subtle how we understand particles of a given SPIN
rep of WHAT?

WE DIDN'T GO IN DETAIL, BUT POINTED OUT WHERE THINGS ARE TRICKY & YOU GET NEAT BEHAVIOR.

<u>TODAY</u>: FORCES (more gen: interactions)

<u>JARGON</u>: Some related ideas

<u>LOCAL</u> sym.

[FUNDAMENTAL] FORCE ⟷ GAUGE THEORY ⟷ FIBER BUNDLE / PRINCIPAL BUNDLE

GAUGE FIELD

proper geometric description

"CONNECTION" COVARIANT DERIVATIVE

$\underbrace{\text{FORCE, GAUGE THEORY, GAUGE FIELD}}_{\text{OUR FOCUS TODAY}}$

$\underbrace{\text{FIBER BUNDLE / PRINCIPAL BUNDLE, CONNECTION}}_{\text{MORE LIKE GR \& IT'S MATHEM. EXTENSIONS.}}$

→ goal: Standard Model

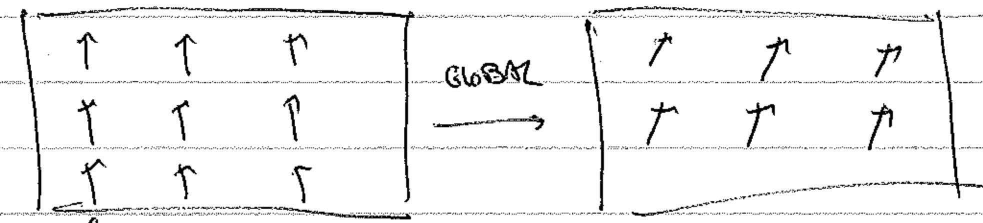
ALL SYMMETRIES THAT WE HAVE CONSIDERED THUS FAR ARE GLOBAL.

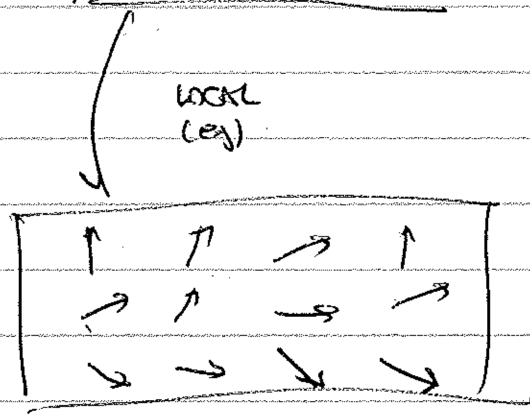
$$e^{i t^a T^a} \quad \text{v.s.} \quad e^{i t^a(x) T^a}$$

constant parameter in spacetime

twist a bit here, twist a lot there

eg





different rotation @ each point in space(time)

eg. for a lattice of spins: B-field all messed up!

GAUGE SYMMETRIES ARE SPECIAL SYM. THAT HOLD LOCALLY.

SIMPLEST EXAMPLE: $U(1) \leftarrow$ rep: $e^{i\alpha(x)}$
(1 phase)

this is an ∞ # of transformations
$\rightarrow$ it's a field full of transformations

GAUGE SYM if $\exists$ some physics that is "charged" under this U(1) but for which ~~nature is~~ the laws of physics are unchanged.

WE HAVE A FAVORITE EXAMPLE OF THIS

$$A_\mu \rightarrow A_\mu + \partial_\mu \alpha(x)$$

DOES NOT CHANGE $\underline{E}$ OR $\underline{B}$

<u>physical</u> observables

WE RECALL WHY: PHYSICAL EFFECTS LIVE IN $F = dA$

"cohomology" { BUT $ddf = 0$
SO $A \rightarrow A + d\alpha$
LEAVES $F = dA$ UNCHANGED

$A_\mu(x)$ IS A 4-vector field

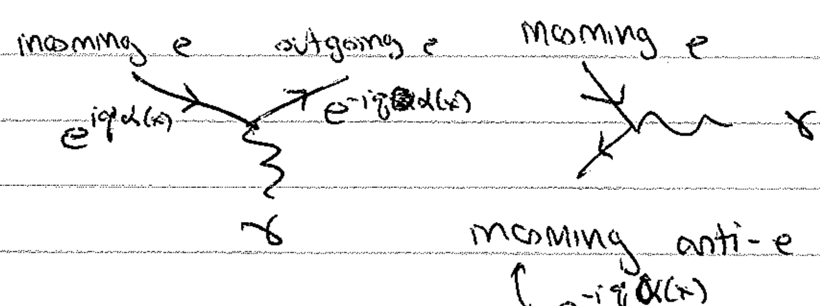
QUANTUM EXCITATIONS OF THIS FIELD ARE CALLED <u>PHOTONS</u>

[identified w/ green's functions in matrix elements.]

electrons: have charge q under this $U(1)$
meaning $\psi \rightarrow e^{iq\alpha}\psi$
under $U(1)$

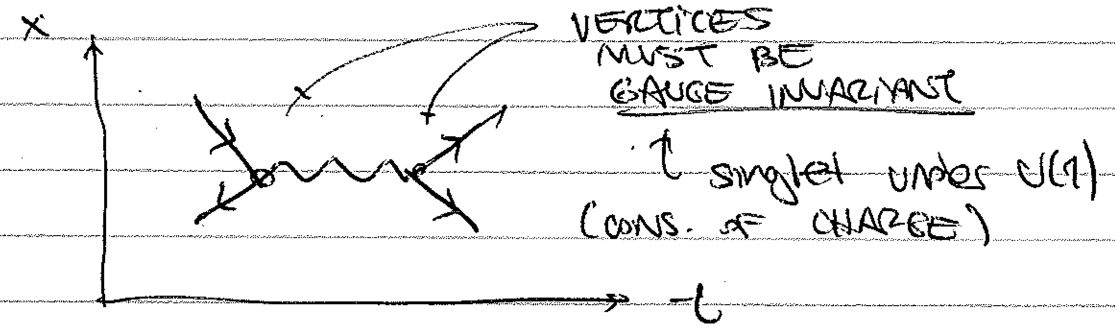
photons: uncharged (just funny shift)

INTERACTIONS



these "Feynman diagrams" encode quantum interactions.

the vertex is some point in spacetime @ which the interaction occurs.



"electron & positron annihilate into a virtual photon, before popping back out w/ re-arranged momenta"

REMARK: the existence of the A_μ (photon)

$\Updownarrow$

local/gauge symmetry

↳ in fact: A_μ is the connection (part of covariant derivative)

WHEN YOU MAKE TRANSF. PARAM LOCAL (a field), YOU ALSO BRING A_μ TO LIFE.

ALTERNATIVE VIEW: A MASSLESS PARTICLE THAT IS A SPIN-1 CAN BE DESCRIBED BY A 4-COMPONENT FIELD ... BUT IT'S NOT LORENTZ COVARIANT.

⇒ to describe theory in a MANIFESTLY lorentz invariant way, NEED TO USE A 4-VECTOR.

↑

introduces extra deg. of freedom that you have to remove by GAUGE FIXING

CHOICE IN HOW TO DO THIS IS SAME GAUGE SYM.

→ REDUNDANCY in theory.

commutators vanish

$SU(\cancel{2}\ 3)$: NON - ABELIAN

commutators don't vanish

MORE GENERAL CASE.

FACT: GAUGE FIELDS IDENTIFIED W/ ADJOINT

"force mediators"

representation of the group

USUALLY: MATTER FIELDS $\Longleftrightarrow$ FUNDAMENTAL

$(T)(\vdots)$

matrix

ANTIMATTER $\Longleftrightarrow$ ANTIFUNDAMENTAL

$(T^{\dagger})(\vdots)$

equiv. to $(\cdots)(T)$

~~FOR SU(2): GENERATORS $\sim \sigma^1, \sigma^2, \sigma^3$~~

$\cancel{\sigma^{1}}, \cancel{\sigma^{3}}$

~~W~~ ~~Z~~ ~~bosons~~

~~HAPPENS TO HAVE ELEC. CHARGE...!~~

MATTER: $\underline{3}$ of $SU(3)$

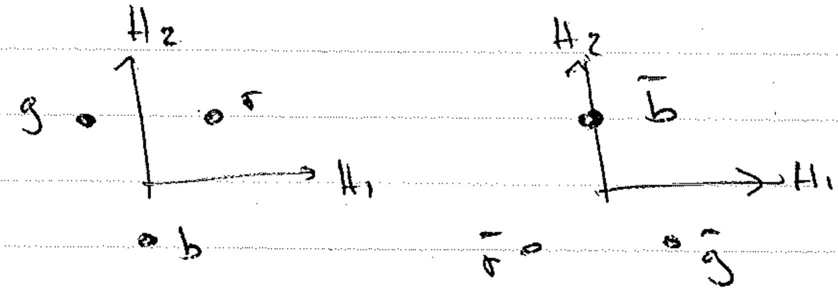
3 DIM VECTOR SPACE:

$$\begin{pmatrix} r \\ g \\ b \end{pmatrix}$$

← 'red' quark
← 'green' quark
← 'blue' quark

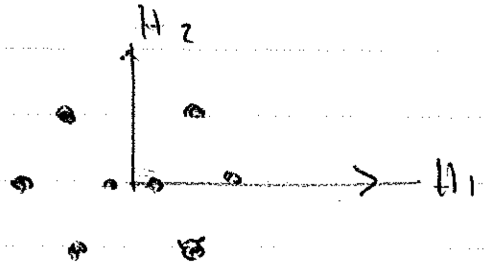
ANTIMATTER: $\bar{\underline{3}}$ of $SU(3)$ $\begin{pmatrix} \bar{r} \\ \bar{g} \\ \bar{b} \end{pmatrix}$

RECALL:

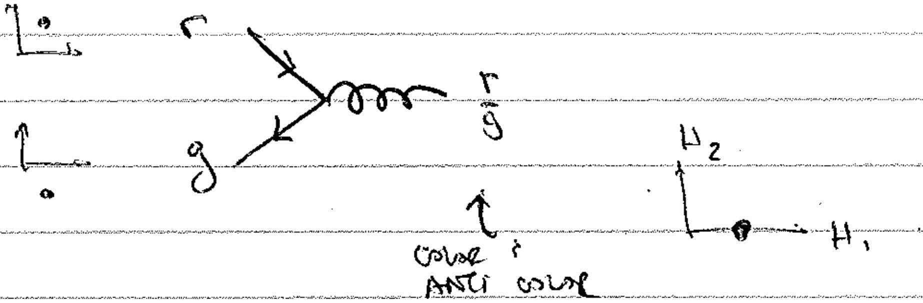


GLUONS: 8 of them, one for each generator.

live in the **ADJOINT**



I WANT TO UNDERSTAND



"A red quark & anti-green anti-quark interacts w/ a $r\bar{g}$ gluon"

REPS: $\underline{3} \otimes \underline{\bar{3}} = \underline{8} \oplus \underline{1}$

there is some overlap w/ $\underline{3} \otimes \underline{\bar{3}}$ And the ADJOINT.

$\Rightarrow \exists$ some ~~invariant~~ matrix that connects these reps

the generators themselves

$$(T^a)^i{}_j$$

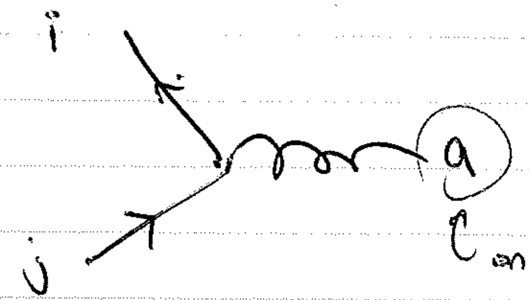
index for $\underline{8}$

index for $\bar{3}$

index for 3

$\bar{q}_i \, G^a \, (T^a)^i{}_j \, q^j$

all indices contract
s.t. this combination
is gauge invariant



$\sim (T^a)^i{}_j$

only 8 of these

if I transform, then the vertex
does not do anything.

A starting point for the Standard Model of particle physics.

(THE PERIODIC TABLE)

	LH spinor $(\frac{1}{2}, 0)$	$\underline{SU(3)}$ (8 gluons)	$\times$ $\underline{SU(2)}$ ($W^{\pm}, Z$ — REALLY W^a)	$\times$ $\underline{U(1)_Y}$ (γ — REALLY Y)
quarks	$Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\underline{3}$	$\underline{2}$	$+\frac{1}{6}$
	$\overline{u}_R$	$\underline{\overline{3}}$		$-\frac{2}{3}$
	$\overline{d}_R$	$\underline{\overline{3}}$		$\frac{1}{3}$
leptons	$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	$\cancel{1}$	$\underline{2}$	$\frac{1}{2}$
	$\overline{e}_R$	$\cancel{1}$		$+1$
Higgs	H		$\underline{2}$	$-\frac{1}{2}$

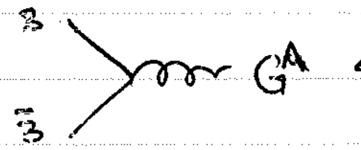
ELECTRIC CHARGE : $T^3 + Y$

$$\text{eg. } \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \text{ has } T^3 \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} = \begin{pmatrix} -\frac{1}{2}\, \nu_L \\ +\frac{1}{2}\, e_L \end{pmatrix}$$

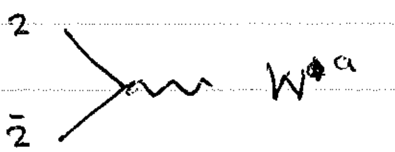
So: SU(2) is KIND OF SPECIAL,
IN FACT: SU(2) × U(1) is SPECIAL;
"electroweak" symmetry is "broken" to ELECTROMAG.

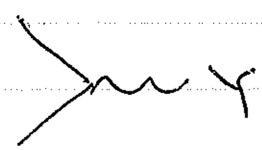
but this is a question of the phase of our vacuum.

WHAT THE TABLE TELLS US:
WHAT INTERACTIONS ARE ALLOWED



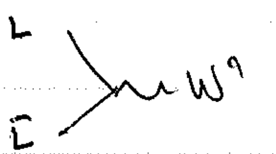
← 8 generators

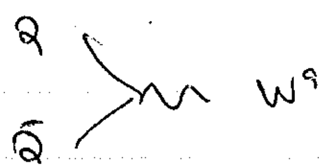




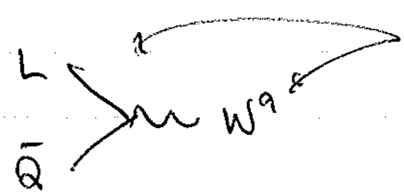
as long as other quantum numbers allow!

e.g.





BUT NOT



SU(3) SINGLETS

3 of SU(3) ← UNCONTRACTED INDEX.

NB: ALL SU(2) INDICES CONTRACTED
"$2 \times 2 = 3 + 1$"

Yukawa couplings

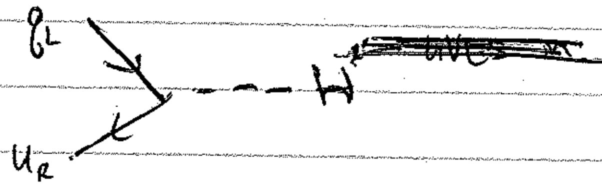
interaction w/ Higgs

↑ SPIN-0

" $\underline{2} \times \underline{2} = \underline{3} + \underline{1}$ "

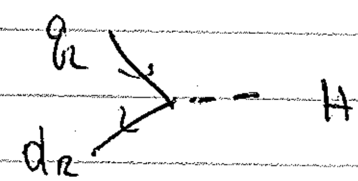
color su(2)

$\Sigma Y = 0 \rightarrow \underbrace{(\bar{Q}_L)_{Ii}}_{-1/6} \underbrace{H^j}_{-1/2} \underbrace{(u_R)^J}_{2/3} \underbrace{\delta^i_j \, \delta^I_J}_{\text{invariant tensor}}$



have to be same color

SIMILARLY



$\underbrace{(\bar{Q}_L)_{Ii}}_{-1/6} \underbrace{H^j}_{-1/2} \underbrace{(d_R)^I}_{-1/3} \delta^i_j \longrightarrow \neq 0$ ✗

$\underbrace{(\bar{Q}_L)_{Ii}}_{-1/6} \underbrace{H^\dagger_j}_{+1/2} \underbrace{(d_R)^I}_{-1/3} \varepsilon^{ij} \longrightarrow = 0$ invariant! ✓

↑ observe: $\underline{2} = \bar{\underline{2}}$

H^i vs $H^\dagger_j \varepsilon^{ij}$

IN GORY DETAIL: ADDING SPIN

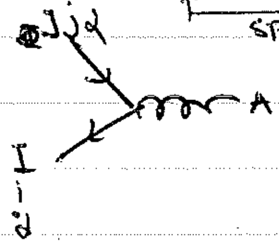
$$\left(\tfrac{1}{2},0\right)\otimes\left(0,\tfrac{1}{2}\right) = \text{Vector} \oplus \text{Scalar}$$

LH PARTICLE REP — ANTIPARTICLE REP

α $\qquad\dot\alpha\qquad$ $(\sigma^\mu)_\alpha{}^{\dot\alpha}$ $\qquad\delta^{\dot\alpha}{}_\alpha$

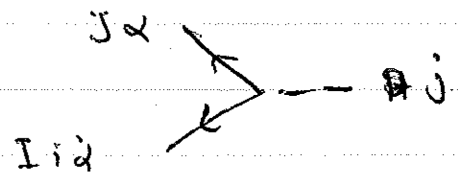
$(\bar\sigma^\mu)^\alpha{}_{\dot\alpha}$

$$(\bar Q_L)_{Ii\dot\alpha}\,\underbrace{(G^A_\mu)(T^A)^I{}_J(\sigma^\mu)^{\dot\alpha}{}_\alpha}_{\text{spin}}\,(Q_L)^{Ji\alpha}\quad\underbrace{\delta^i{}_j}_{SU(2)}$$



$\sim (T^A)^I{}_J(\sigma^\mu)^{\dot\alpha}{}_\alpha$

$$(\bar Q_L)_{Ii\dot\alpha}\,H^j\,(u_R)^{J\alpha}\quad\underbrace{\delta^I_J\,\delta^i{}_j\,\delta^{\dot\alpha}{}_\alpha}$$



$\sim \delta^I_J\,\delta^i{}_j\,\delta^{\dot\alpha}{}_\alpha$

BONUS

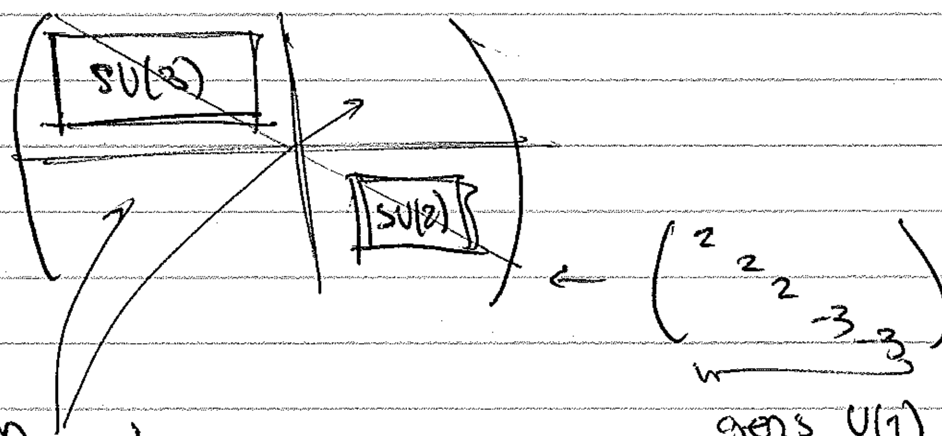
UNIFICATION

Why $SU(3) \times SU(2) \times U(1)$?

maybe these all live in
a bigger, simple group?

one candidate: $SU(5)$

$g \sim$



stuff in here
is weird → MAYBE HEAVY

<u>NOT MENTIONED</u>: GRAVITY AS GAUGE THY?
→ see COLLOQ. THIS THU!
(I think)