LECTURE 1 Physics vs. Math ; DIM. ANALYSIS

REMINDER: DO COURSE INTRODUCTION

$$\boxed{\text{Physics} \neq \text{Mathematics}}$$

WHAT SEPARATES PHYSICS FROM MATH?
$\exists$ many answers, a few are very good.

- CONNECTION TO NATURE
  - empirical methodology
  - yet: PHYSICS IS UNIQUE IN THE SCIENCES IN THAT THERE'S A STRONG DIVISION BETWEEN Theory & Experiment

is this "math"?
no.
SEE OTHER BULLET POINTS

do I need this course?
yes.
1. YOU HAVE TO PASS QUALS
2. THIS IS ABOUT THINKING LIKE A PHYSICIST

eg. DOMAIN OF VALIDITY MAY FORMALLY BE 0!

- USE / MISUSE / RELIANCE on TAYLOR SERIES
  more generally: perturbation theory
  - make the right approximation.

• $\boxed{\text{UNITS}}$ *my favorite answer.*

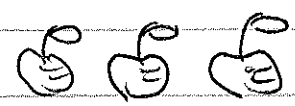
Physics relates/connects/predicts measurable <u>DIMENSIONFUL</u> quantities

cm, sec, GeV, watts, ...

ie. these are *UNITS*

TO BE VERY CLEAR: what is a unit?

DIMENSIONFUL QUANTITY:

eg. 3 <u>apples</u> 

DIMENSIONLESS NUMBER — math

PHYSICAL, AGREED UPON STANDARD } ~ PHYSICS

for what?

in fact: the "apple" is a <u>conversion</u> between all of these things.

- eg. COST
- eg. CALORIC INTAKE
- eg. MASS
- ⋮

eg: 1 $\boxed{\text{METER}}$

- DIST TRAVELLED BY LIGHT IN $(299\,792\,458)^{-1}$ sec ← time
- $1/10^{6}$ DIST FROM EQUATOR TO N. POLE
- SOME # OF WAVELEN OF Krypton-86..

DIMENSIONAL ANALYSIS

A PHYSICAL QUANTITY q HAS DIMENSION $[q]$ WHICH WE TYPICALLY WRITE AS

$$[q] = \underset{\text{length}}{L^a}\ \underset{\text{mass}}{M^b}\ \underset{\text{time}}{T^c}$$

COULD USE OTHER QUANTITIES (eg PRESSURE); BUT TYPICALLY CAN REDUCE TO THESE

eg. FORCE ... well, we know $F = ma$
$= m\ddot{x}$

then: $[F] = L^1 M^1 T^{-2}$

observe: $[F] = [m] \times [a]$

eg. what about ENERGY?

answ: $E = \frac{1}{2}mv^2$ or mc^2

$[E] = M L^2 T^{-2}$

Zeroth order Dim. Analysis

CHECK THE VALIDITY OF EXPRESSIONS

"sanity check"

eg $(1+x)$ LOOKS FINE

$(1+L)$ DOES NOT!

dim-less

LESSON: WRITE AS $\left(1 + L/L_0\right)$

identify dimensionless params.

eg. L is 2 cm. IS THERE A BIG CHANGE IF L is CHANGED TO 4 cm?

DOUBLED!!

not if $L_0 = 10$ m!

("basically zero to 1.0.")

even more egregious:

e^L or $\underbrace{\sin(3 \text{ cm})}$

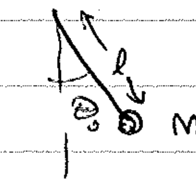
means nothing!

$1 + L + \frac{1}{2!}L^2 + \cdots$ ← each term had better have the same units.

First order DIM. Analysis

What is the period of a pendulum?

→ IDENTIFY RELEVANT QUANTITIES



L → ℓ LENGTH OF PENDULUM

M → M IS THE MASS OF THE WEIGHT

LT^{-2} → g IS THE GRAV. ACCELERATION

(1) → θ_0 IS THE INITIAL ANGULAR DISPLACEMENT

WANT AN EXPRESSION FOR A <u>TIME</u>

start w/ $g^{-1/2}$ since $[g^{-1/2}] = T\,L^{-1/2}$

then multiply by $\ell^{1/2}$ to cancel

$\tau \sim \cancel{\sqrt{g/\ell}}\ f(\theta_0)$ ← DIM'LESS FUN. OF θ_0

$\sqrt{\ell/g}\ \times$ DON'T KNOW FROM D.A.

<u>INDEP</u> OF M!

SO WE GET THE <u>PHYSICS</u> OUT:

ℓ GOES UP, τ GOES UP (LIKE SQRT)

g GOES UP, τ GOES DOWN (— " —)

M GOES UP, τ UNCHANGED

this is real physics

overall prefactors are measurements

Second order analysis

ESTIMATING QUANTITIES

eg the Hierarchy Problem, simplified

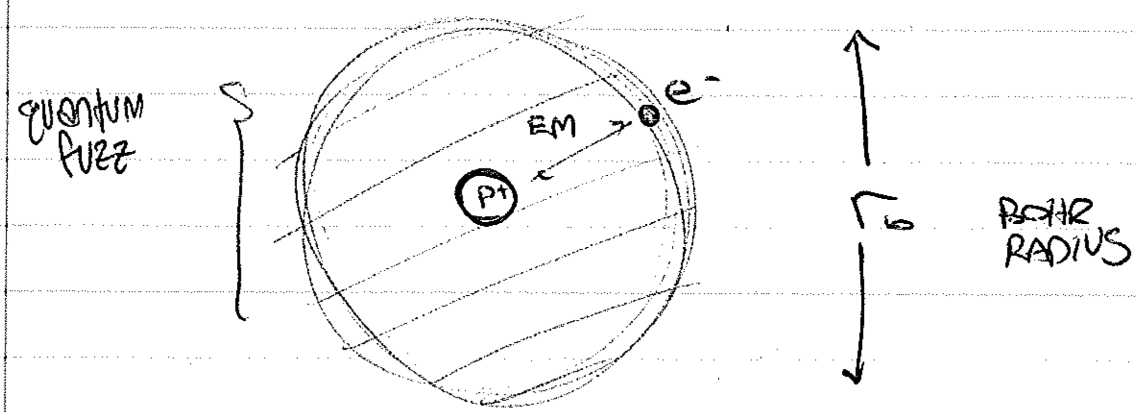
IN FIELD THEORY, EXTERNAL LINES IN A FEYNMAN DIAGRAM CORRESPOND TO FACTORS IN A TERM IN A LAGRANGIAN DENSITY.

↑ homework

↱ math.ucr.edu/home/baez/lengths.html

How large is hydrogen?

WHAT DOES THIS MEAN? SIZE IS SO SMALL THAT IT'S A QUANTUM QUESTION.



WHAT COULD THIS DEPEND ON?

- e^- ⟶ m_e ; e
- p^+ ⟶ m_p ; $q_p = -e$ ← not an indep. param

CONSTANTS ⟶ $\hbar$, c, G_N

OTHERS? ⟶ $B_\oplus$, $M_\oplus$, m_Z, ... ?

NOW STOP & THINK:
which of these quantities don't make sense?

M_p: PROTON MASS >> ELECTRON MASS
SO PROBABLY ONLY ONE MATTERS
(& other mass gives a $O(M_e/M_p)$ correction)

THE ELECTRON IS THE ONE IN A "CLOUD" & WE KNOW FROM QM THAT WHAT REALLY ENTERS IN A 2 BODY PROBLEM IS THE REDUCED MASS: $M_p M_e/(M_p+M_e) \approx M_e$

G_N: HAS TO DO W/ GRAVITY, BUT GRAVITY IS MUCH WEAKER THAN ELECTROMAGNETISM, SO LET'S IGNORE THIS.

SIMILARLY - other quantities don't matter.

LEFT W/ M_e, e, $\hbar$

M_e: MASS
e: CHARGE
$\hbar$: "QUANTUM-NESS"
MOMENTUM × ~~DIST~~ LENGTH
(recall this from stat mech!)

<u>CLAIM</u>: $[M_e] = M$
$[\hbar] = ML^2T^{-1}$ } easy

$[e] = \underbrace{M^{1/2}L^{3/2}T^{-1}}_{\text{homework}}$

HINT: USE FORCE LAW
$[q] = [F]^{1/2} L$

COMBINE THESE INTO SOMETHING W/ DIMENSION OF LENGTH:

$$\left[\frac{\hbar}{e}\right] = M^{1/2}L^{1/2} \quad \text{(get rid of } T\text{)}$$

$$\left[\frac{\hbar}{\sqrt{M_e}\, e}\right] = L^{1/2} \quad \text{(get rid of } M\text{)}$$

identify this w/ Bohr radius

$$\boxed{r_b = (\#)\,\frac{\hbar^2}{M_e e^2}}$$

coincidentally: $\# = 1$
could have had 2π's ...

THIRD ORDER DIM. ANALYSIS

Scaling & similarity

see: Arnold Math Methods. of Classical Mech. §11

VECTOR PDE: $m\ddot{\underline{r}} = -\partial U/\partial \underline{r}$

time dependence $\left[\frac{d}{dt}\right] = T^{-1}$

operator

(think of df/dt as $\Delta f/\Delta t = \frac{f_1 - f_0}{t_1 - t_0}$

$\Rightarrow [df/dt] = [f] T^{-1}$

$[U] = [F] L = ML^2T^{-2}$

$[\partial U/\partial \underline{r}] = MLT^{-2}$

SUPPOSE WE HAVE A GRAVITATIONAL SYSTEM

eg ELLIPTICAL ORBITS OF PLANETS ABOUT A STAR

SO WE HAVE A SET OF SOLUTIONS TO THE PDE ABOVE, $\underline{r}_0(t)$

WE CAN USE DIM-ANALYSIS TO UNDERSTAND OTHER SOLUTIONS.

IMAGINE DOING A SCALING TRANSFORM ON t

$$t \equiv \alpha t'$$

OLD VAR — NEW VAR

this is really just choosing units.

OBSERVE: ONLY LHS $(m\ddot{\underline{r}}_0(t))$ CHANGES!
WHY? RHS HAS T^{-2} DIM, BUT THIS COMES FROM $[G_N] \sim T^{-2}$, CONSTANT.

SO: $m\ddot{r}_0(t) \rightarrow \underbrace{m\,\alpha^{-2}}\,\ddot{r}_0(\alpha t)$

can "undo" this scaling if we also change $m \equiv \alpha^2 m'$

$\circ = \frac{d}{d(\alpha t)}$

encodes planet $\left\{ m\ddot{r}_0(t) \rightarrow m'\ddot{r}_0(\alpha t) \right.$

$\quad\quad\quad\quad\quad\; \shortparallel \quad\quad\quad\quad\quad \shortparallel$

encodes SYSTEM $\left\{ -\partial U/\partial r \quad\quad -\partial U/\partial r \right.$

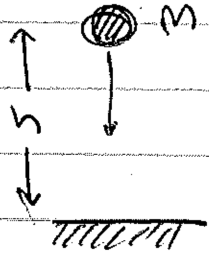
SO: GIVEN ~~SOLUTION~~ TRAJECTORY $r_0(t)$ FOR A PLANET OF MASS m; A PLANET THAT IS FOUR TIMES HEAVIER WILL TRAVERSE SAME TRAJECTORY TWICE AS QUICKLY.

FOURTH ORDER DIM. ANALYSIS

USE FOR ERROR ESTIMATE

cf. DIM. ANALYSIS, FALLING BODIES, & THE FINE ART OF NOT SOLVING DIFF EQ. Bohren
Am J. Phys 72 534

HIGH SCHOOL PROBLEM:



WHAT IS THE TIME t_0 FOR AN OBJECT TO HIT THE GROUND, DROPPED FROM HEIGHT h?

HIGH SCHOOL ANSWER:

~~$F = Mg$~~ g = GRAV. ACCEL

~~$M\ddot{x}$~~

$$\boxed{\ddot{x} = g}$$

0 init vel

INTEGRATE: $\overset{h}{x} = \frac{1}{2} g t^2 + \cancel{ct} + \cancel{D}$

picks coord. origin

$$\boxed{t_0 = \sqrt{\frac{2h}{g}}}$$

this is the easy answer.
often good enough.

IMPORTANT QUESTION:
HOW GOOD IS THIS APPROX?

BETTER: SAME Q, BUT: WITHOUT DOING HARD WORK!

SOLVING ANY MORE DIFF EQS.

OUR "ZEROTH ORDER" ESTIMATE IS $t_0 = \sqrt{2h/g}$
WANT TO KNOW THE ERROR:

$$\underbrace{\frac{t_{realistic} - t_0}{t_0}}$$

DIMENSIONLESS COMBINATION
THAT GIVES FRACTIONAL ERROR
FROM NEGLECTING "MICROPHYSICS"

→ by the way: this is a "deep" idea. THIS IS WHY A CHEF DOES NOT NEED TO KNOW SUBATOMIC PHYSICS.

UNDERLYING IDEA (FORMALIZED) IN MUCH OF THEORETICAL PHYSICS,

MAIN IDEA: ε IS SMALL
(otherwise t_0 was the wrong thing to calculate in the first place)

$$\frac{t_r - t_0}{t_0} \equiv f(\xi)$$

DIMENSIONLESS PARAM CHARACTERIZING UNACCOUNTED FOR MICROPHYSICS

which should also be small!

PICK ξ s.t. $\xi \to 0$ CORRESPONDS TO TURNING OFF THE MICROPHYSICS

THEN $f(0) = 0$

if this is not true, use $\xi' = 1/\xi$

THEN WE MAY TAYLOR EXPAND

SUPER SMALL

$$f(\xi) = f(0) + \left.\frac{df}{d\xi}\right|_0 \xi + \mathcal{O}(\xi^2)$$

0

SOME DIM'LESS NUMBER; PRESUMABLY $\mathcal{O}(1)$

TO TO LEADING ORDER IN THE MICROPHYSICS THE ERROR GOES LIKE

$$\boxed{\frac{t - t_0}{t_0} \sim \xi}$$

eg. g is not constant, varies w/ height (radial distance from center of earth!)

RELEVANT PARAMETERS TO INCLUDE? R, radius of earth!

(WHY NOT G_N? → ALREADY SECRETLY IN G_N)

$$\hookrightarrow m\ddot{x} = \frac{-mg}{(1 + z/R)^2}$$

TWO CHOICES FOR DIM'LESS ξ: $\frac{h}{R}$, $\frac{R}{h}$

$R \gg h$, so $\xi = h/R$

(when $R \to \infty$, expect naive result to be correct.)

then $\boxed{\frac{t - t_0}{t_0} \approx h/R}$

this symbol encodes physics.

DON'T FORGET MARKERS

NEXT WK: REVIEW OF LINEAR ALG/QM & INTRO TO DIFF EQ.