LECTURE 5 : CANMUSS

13 OUTOBER

REFS.

" Stone & Goldbort - BATHER ADVANCED DIECUSSION

OHES! ANAUTIC PROPERTIES OF GREEN'S FINS

CM. A: B ANMYSI->

Ch. 18: APPLICATIONS , MO. PLEMELS (P. 678), KK

JORDAN'S LEMMA P. 672

· WHAR 6

CO. 5:1150 OF EXAMPLES, KIND OF SLANOW?

INC. PRINCIPAL VALUE \$5-18

ONF MAP I WAY APPLAUSS FLY

- BURON PRUTE MATHEMATICS of CLOSSICO/ 7

QUANTUM Physics

- see 865: HUB! CANOTH PHINC VAZ.

- see \$65: HUB! CANGTH PMUC VAR.
\$6.6 INTRO. TO MERBRISION & really great!

TUGOS notes on Adv. Solid State Phys. Causality i Kranors- Kroning relations http://lampx.tugraz.at/~hadley/ssz/linearresponse/ ausality.ph.p

NOTE: DIS SEC IF YOU WANT MORE POE DISC!

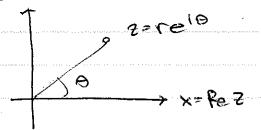
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	COMPLEX ANALYSIS				
	IMMEDIATE GOAL", CONTOUR INTEGRALS				
	Calle solution (see) Grassia Place				
	if you get nothing else from this unit, then learn to do these!				
	unit, then learn to do these!				
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(and the state of a community of an analysis of the state of a sta	STHER IMBRITANT THINGS				
"nice" 6	-> analyticity is important in physics				
TO BE DEFINED	0, 0				
Andre and an antique million and and	eg CHUSAUTY IN DISPERSION RELATIONS				
***************************************	29 behavior of [scatterry] amplitudes				
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reaction and the second control of the secon	also: SHORTCUT FOR HARMONIC FUNCTIONS				
	IN 20 " CONF MAPPING				
	applications to Electrostatics)				
	FWIP PYNAMICS				
	1				
a ya mara aya ka galifa (1865) aya magayinan Handan Bag Mili (1844) aya maraka ka 16 minin	Culty do airplanes fly?				
	WHY DO C # SHOW UP IN THE PR WARED?				
	1. PR PHENOMENA (USE WAVES) WHOSE PANAUTIC PROPERTIES				
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	7. PHYSICS THAT IS INHERENTLY O, eg QUANTLYM				
ir i ar enement en l'immense e enclàtric i l'aux Chada (N) (200 min 11 famille un de martin et l'article et en	The second second the second s				

COMPLEX VARIABLES! Z = X + 14 Z = X - 14 Los 2*

this is really a 2D vector space with ADDITIONAL STRUCTURES (MUTIPUCATION): V·V → V)

(x) (x') (xx'-yy')
(y) (xy + yx')



COMPLEX PUNCTIONS

4=1m2

m general: f(z,z) - gives generic function
who there is well have
written fixy)

SPECIAL CLASS OF PUNCTIONS

ANAUSTIC: f(2) W/ DERV. DEFINED.

Dice 2 type of thing you see or physics: sufficiently differentiable, smooth, etc. SUPPRICIENT TO ONISIDER 2 BASIS

DIRECTIONS Y=IM 2 AZ=IDY

2 AZ=DX

OR IN LANGUAGE OF TANGENT SPACE (later!)

12 2 PASIS OF DIFFERENTIAL

OPERATIONS ON THIS SPACE.

I tell that these should be portiols until we've established availabled...

ď

CAUCHY-RIEMANN

4 SAME FOR IM

x = Re Z

MD: IN pass or motor coming from J. H. Juscylens.

C DIFFERENTIABLE (S) CAUCHY REMANN (S) F(Z,Z) = f(Z)

COULD IMAGINE A DIFFERENT BASIS FOR THE TANGENT SPACE @ 20

$$\frac{\partial}{\partial z} = \Delta x + i \Delta y$$

$$\frac{\partial}{\partial z} = \Delta x - i \Delta y$$

$$\frac{df}{dz} = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} = \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + i\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$\frac{df}{dz} = \frac{2f}{2x} + \frac{2f}{-i2y} = \left(\frac{2u}{2x} - \frac{2v}{2y}\right) + i\left(\frac{2v}{2x} + \frac{2u}{2y}\right)$$

PREDIETY WHAT JANISHES
UNDER CAUCHY "PREMANN. 1]

dead one "manlocal"

TO: AN ANALAC EMECION (6 5-)

13 C DIFFERENTIABLE [IN SOME NEW of 20]

Nos to be finite @ 20, etc.

aran kaniliyan di Gerin Makada di Makada ya kaniliya ka a da	CAUCHY-RIEMANN 4-> 4 ? V are 20 HARMONZ					
	16 PM = PN = 0					
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	A/ BU = 2,24 + 23V					
, grant est est est est est est est est est es	= 8x(2yv) + 2y(-3xv)					
	2 these derivatives commute on that sp					
	30: C ANALYSIS () ANALYTIC FUNCTIONS) ARE A					
	SHORTCUT FOR 2D EVECTROSTATICS,					
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Byron 1	@ functions as maps of P->C					
Fuller						
det is now up and account and account a select solid framework as per property of the significant features if an	LETTE CONSIDER A SIMPLE DOMAIN / PRE-INVEE					
er der 1800 der der Stellen der Verleit der	- 2 = e'0					
1900) y niwaja wikawi 1904 (Shakali 1914 nj. 1814 niwat 1 niw						
att til sa staten er sammer sessa hande å kan disense men men sen er i förstatte sente framstyrtet e	S IO					
	7-12=Gel0 W 1671					
ant describe and such success of the suit of the suck and an extension of the extension of the extension of the	(consider) = 52 f = 52 + 20					
polyton nadlikiliji dele operiji ganjeti jeji kina uma politik në njëpjetitë e artiku u atti e ë e e e e e e e	CONSIDER: f(2) = Z2 f= 22+20					
	PROUND TWICE					
	E RADINS HOURING					

WHOT ABOUT & (2)= 12?
J
CISSISO PATH
~ (()) × - + > - + +) / ,
NOT A CLASSOO PAFUN!
HOW BO WE BET WHERE WALF PLANE IN MIGHT?
WELL O GOES FROM ZA -> LA?
-> MUSTIVALUED!
1
BETTER: DEPINE DOMAIN WIPT PLEMANN SHEET
$f_i = NF e^{i\theta/2}$
57 1 F
1(0124)/2
$f_2 = \sqrt{\frac{e^{i(\theta + 2\pi)}}{2}}$
alus out is collect
could that cas so and 8° for my
I and a second of the commence of the commenc
note: f(e) B ANAUTAZ! (everywness!)
CUT only sidns up as a guesse
PROPERTY IF YOU WIND MOVIND THE

BUGN.

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	leoparthm: 菌z=lnr+io
	$ 722 = \ln(r_1r_2) + i(\theta_1 + \theta_2) $
	$\log = \ln(r_1) + \ln r_2 + \cdots$
***************************************	= 109 21 + 109 22
	obs: every time you go around the
	oxigin, you must go to new
	Riemann sheet blc 0 just
	keeps moreasing:
EBUS AVOSTANCES	19 log 2 analyte?
	YES, EVERYWITERE BUT AT Z=0,
A CONTRACTOR OF THE PROPERTY O	WITT
	ON DEPME IT "PIECENISE" SINGLE VANDO PINS:
WAS ASSESSED TO THE PARTY OF TH	fn(2) = Inr +10 +2001
	1 D @ + 27T \
	The second secon
	FAN 2003 MILLIAM Mr
AND THE PROPERTY OF THE PROPER	
	NOTATION: Log 2 : fo st. elog = = =
	NOTATION: Log & : fo st. e = = =
uat a' ainm liath faoille na 1911 ban a	
2	So comply atte: DON'T MODES THEM
	DO: PRANCH OUTS: DON'T CROSS "CHESM" WHEN YOU INTERNATE.

ANAUTIC -> DICE

NPE ISN'T ALWAYS NICE. I BUT THAT'S WHOP PHYSICS ISN AWAYS ME. I THINGS COST NOW

TOWNE IS PHINGE? I AWAYS ANALYTIC

BUT OUR DESCRIPTIONS ARE WET.

SO NONANAUTICITY IS OUR THEORY

TOWNE US THAT THERE'S MORE

PHYSICS TO DESCRIBE?

SOMETIMES YOU HAVE SINGULARITIES

eg (2-1)2

1 here the singularity

or 1+172 is only over 9

Single Riemann sheef!

WE'LL MOSTLY ONLY DEAL WI ISSATED SINGULARITIES WHERE IT IS ANADOUGH EXCEPT AROUND A SWALL NEIGHBORHOOD OF THE SINGULAR POINT.

SINGULORITY

SINGULORITY

D & F IS ANALUTUR IN REST OF DOMAN

OF HAS A TAYLOR SERIES FORM THAT VONUTROES

OWNER FROM THE SINGULARITY.

	The state of the s
१८०५	MORE GENERALY, CAN WRITE LAWRENT SERVES
entral and the latest the latest through the second or second and the latest through the	f(z) = max = an (z-z-)" + = bm (z-z-)"
angerina ang mangrapa na ang Transpiral (1944 milihanti manahatan manahatan manahatan manahatan manahatan manah	
	SINGULAL CERINS
a kalanda kan palangan sagat ang kalanda dalam da antaha pilah pilah dalam da palam da palam da palam da palam	if all bm=0, fis ANAUUTUICE Z= CPA NBD AROUNDIT)
	if bm=0 + m> m for sufficiently large m,
erente en	for mo> m, then we say
د از در داد با داد داد این این این این داد داد داد داد با داد با داد با داد داد	f has a pole of order m @ 30
	l'if in =1, simple pole.
والمستوالة والمستوادة	if an as number of los are nonzero,
	then essential singularity @ 20
Accepted to the state of the st	the osethicient that by is the RESIDUE
	of f(z) at Zz
a soften som spoken tillgam (plensyster) for another som at som an accordance of bester motion	A company of the comp
	hey to contour integrals
	Key to contour integrals
Annuar de la caractería d La caractería de la caractería	
rang pangaman na manana na manana na manana di Amerika na manana di Amerika na manana di Amerika na manana na m	
Prillipe i i mangra canara manana canara manana ang pa canara pa	

INTEGRALS

"THESE ARE THE JOVAL 2D UNE PATH INTEGRALS.

but note P(Z)= U(Xy) + i V(X,y)
WERE TREATING THE INTEGRAL AS IR2 -> C
OR CON THING OF THIS AS TWO IR2 -> IR INTEGRALS

CAVOLLY'S THEOREM IF \$ 13 analytic in R 1 C= DR

Se f(Z) dz = 0

C = DR C 13 BOUNDARY OF

A PEGRON R

1 13 BNAYTIC IN R

on ware

Sketch proof: ANAMURE A 3 TOMUNESTER & OD DIFF.

Then break up R INTO LITTLE REGIONS

EACH PATH IS

FIRE PROBLEMS

FROM PATH

FROM

(f(z)) + p'(z-)e-) (e-e +ie-ie) = 0

CONFIRMS ONE INTUITION: IF THINGS AGE TOO NICE,

CAUDAY'S INTEGRAL FORMULA: If his ANAMOR IN RIZER
$f(Z_0) = \frac{1}{2\pi i} \theta \frac{f(Z)}{Z'-Z_0} dZ^2 e^{-\frac{1}{2}} e^{-\frac{1}{2}}$
A. define: $+(z^*) = \frac{f(z)}{z-ga}$
That andmars 6 \$ 9
CNOT ANIAM 92 (2 9
20 1 2 + Pelo
$g_c + (z) dz + g_c, + (z) dz = 0$ by carcian run
Copp. outentarion
Po Φ(2) de = 90 Φ(2) d2
C SAME SUBURACION
2 = Se ¹⁰ +2=
$\frac{1}{2} = \frac{1}{2} = \frac{1}$
1/2)
= 1/c, Peio 18e 20 10 take poo
Company of the contract of the
= 2TTi[F(2-)]
The RIWY of
, 1 15 4 5 5 1 1 4 7 1 27 1 5 1
F(3) = 271 8 +(w) dw

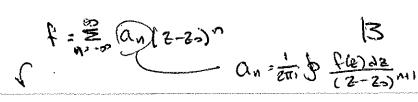
BESIDIE THEOREM GO BAGK TO PROSP OF CAVOHY'S TUM IF & (WHICH WAS & IN ONUM THAN PR) IS SOME FUNCTION WI A SIMPLE POLE @ 20, THEN: Jef(2)dz = Je = an(2-2-) + = bm (2-2-) m contains 2. anguar Jobs dz + Ef (2-20) m dz C other-than 2. (2-20) -> Peio Pt: [not be lee] if t= = col2-a), then Cu = 11 & (3-0) NH 95

Pcf(z) de = 271 Res (f0zo)

Butlon R 74

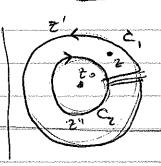
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Resers is by coeff.



Adidity;	ьţ	of	LAVE	ENT	WAST

CAHHU:



no poles in annulus

COUCHU TOWN: f(2) = 2TT Jaiz 21-2 22

= 1 f(z1) = 12 (2-2-) -(2-2-) de (2-2-) -(2'-2-) de

 $\frac{f(2')}{(2'-20)\left[1-\frac{2-20}{2'-20}\right]} \frac{f(2n)}{(2-20)\left[1-\frac{2''-20}{2-20}\right]}$

note: 2"-2- (1)

USE: $\left(1 - \frac{2 - 20}{2! - 20}\right)^{-1} = \frac{20}{2! - 20}\left(\frac{2 - 20}{2! - 20}\right)^{1/2}$

TAMILAR SIGNES IN (2-2-/21-2-)

= E (2-2-)" 2III)c, (2'-2-)" de'

+ = (2"-2-) (2"-2-) (2"-2-) (2") (2")

Eder hare.

BUDGETT IN ANNUES

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	14
	COSCONETURS: take Ci > C
	$ \frac{1}{2} \left\{ \frac{1}{2} \right\} = \frac{1}{1200} \left(\frac{1}{2} - \frac{1}{200} \right)^{1/2} \left(\frac{1}{2} - \frac{1}{200} \right)^{1/2} d^{2} $
	CAPPLLIENT OF LANDENT EXPANSION
Bobs	FINAING RESIDUES
14.6	8D LAURENT SERIES, CHEFF OF ZITTERM 3 SIMPLE POLE
9.13 -	1 take 2→2.

of $f(z) = \frac{7}{(22+1)(5-7)}$ and (22+1) Res(-1/2) $(2+\frac{1}{2})f(2)|_{2=2-2} = \frac{-1/2}{(5+1/8)}$

@ muchiple poles: if Pole of dote in:

- MUCUPU BM (2-2-) M (M=n)
 - DIFFERENTIATE (M-1) TIMES
- NWINE PAY (M-1)!
- = EVANTE & Z=Zs.