LECTURE 2: VECTOR SPACED 26 SEPT GOOD REFS: Matthews ? Walker Matth ? Methods of Phy. La chapter C., mol. Mass exercise: rel to so day.

also GEETTER ROSERTREL & TRALLI
"Some Moth. Medhods of Physics"

CH.4, B

You should already be familian w/ the idea that QM = CF) Linear Algebra.

- WE WILL USE TH'S AS A BASIS TO UNDERSTAND

(LINEAR) DIFFERENTIAL EQUATIONS

V -> VECTOR SPACE in am state space

MANY OHOICES FOR A BASIS

1x>= ZX/1e;> = ZX/1f;>

1 BUT THE STATE IS BASS- INDEP

31

not a great almost a solution

| an and a second             |  |
|-----------------------------|--|
|                             | THEAL SOME PROPERTIES 2) See, Eg. APPENDIX                   |
|                             | of TextBook  |
|                             | VECTOR ADDITION: 2 VECTORS -> VECTOR                         |
| important<br>for subscribit | +: V,V > V COMMUTATIVE, ASSOCIATIVE                          |
|                             | RESCAUNG: VECTOR ? SCHAR -> VECTOR  O:V, # -> V DISTRIBUTIVE |
|                             | WI IDENTITY & INVERSE ELEMENTS FOR BOTH OP'S                 |
|                             | DUAL SPACE ~ ROW VECTORS (UP TO COMBUG)  KETS                |
|                             | DEFINED AS MYPS THAT TAKE ELEMENTS  OF A VECTOR SPACE TO #.  |
|                             | (x14) is a number<br>& (x1:14) > #,                          |
|                             | similarly: 14>; (x1 -> # (OVAL)                              |
|                             | (** X,                   |
| Ĭ                           |  |

|   | DUAL SPACE REACHL IS A DIFFERENCE SPACE   |
|---|---|
|   | ((x) + 14>) BOESN'T MEAN ANYTHING,  |
|   | 1   |
|   | DUAL BASIS: et or (eil + lei)   |
|   | USUALLY WE HAVE AN INNER PRODUCT BUT USUALLY TO RELATE $V + V^* - 1$ (e; le;)   |
|   | BOT PRODUCT" Si   |
|   | (·1·): V×V* ->#   |
|   | such that < xly> = <ylx>** and:</ylx>   |
|   | UNEAR IN $V: \langle x ay_1+by_2\rangle = \alpha\langle x y_1\rangle + b\langle x y_2\rangle$<br>ANTILINEAR IN $V^{\mu}: \langle \alpha x_1+bx_2 y_2\rangle = \alpha^{\mu}\langle x_1 y_2\rangle + b^{\mu}\langle x_2 y_2\rangle$   |
|   | SO WE WRITE (IN OM) DAGGER:   |
|   | $\langle x   \sim lx \rangle^{+}$ in the sense<br>that:<br>$t \in lx \rangle \rightarrow \langle x, \circ \rangle$  |
| • |   |
|   | In. funtion that takes to #.  |
|   | Total |

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|  | SO NAMED BECAUSE IT GIVES US A WAY TO MEASURE ANGLES ! DISTANCES.   |
|  |   |
|  | eg llx>ll2 = (xlx>  |
| houristic  | (x14) = "000 0" / (x1x) (y14)   |
|  | OPERATORS/MATRICES/TRANSFORMETIONS  |
| n marka ilasahka iz zennikan kalanda filik landah kalanda kalanda kelanda kelanda kelanda kelanda kelanda kela   | OPERAUS / MAIRIUS / URMOSTORINE Y UNE   |
|  | $A \times = \sum_{i} A_{i,i} \times i \qquad (A_{i1} A_{i2}) \times (X_{i1} A_{i2} \times X_{i2}) \times (X_{i2} A_{i2} \times X_{i3} A_{i2} \times X_{i3}) \times (X_{i1} A_{i2} \times X_{i3} A_{i2} \times X_{i3} A_{i3} \times X_{i3}) \times (X_{i2} A_{i3} \times X_{i3} A_{i2} \times X_{i3} A_{i2} \times X_{i3}) \times (X_{i1} A_{i2} \times X_{i3} A_{i2} \times X_{i3} A_{i3} \times X_{i3} A_{i2} \times X_{i3}) \times (X_{i1} A_{i2} \times X_{i3} A_{i2} \times X_{i3} A_{i3} \times X_{i3} A_{i2} \times X_{i3}) \times (X_{i1} A_{i2} \times X_{i3} A_{i3} \times X_{i3} A_{i3} \times X_{i3} A_{i2} \times X_{i3} A_{i3} \times X_{i4} A_{i4} \times X_{i4} A_{i4}$ |
|  | = ZA; le; >(e; \ Z×klek)  |
|  | = i Anx leixeles les  |
|  | PECKL<br>1 = \( \)  |
|  | = 5 Aij Xi lei>   |
|  |   |

|  | EIGEN VE CLOP ?                              |
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| a gold (1) the second of the s   |  |
| $- \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{j=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{j=1$ | a person # 2 . endername, operation          |
| parameter and a second control of the second   | C AND THE TOTAL I HAD O CONNIC               |
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|  | 26/N of 11> 12. (1/89) 1>=+1                 |
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| and the state of t   | WEFOL REVIEW-                                |
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|  | s.t. (y/Ax) = (A+y)                          |
|  |  |
|  | HORMICIAN MATCHIX: AT = A                    |
|  | COBSERVARIES -> DAS R EIGENVANCES            |
| and the state of t   | CENERATES CONTRACY TRANSF                    |
|  | Luta =1                                      |
|  | t t  |

## HERMITIAN MATRICES (CORERATIONS) O REAL EIGENVALUES (AXIX) = (XIAX) = X\*(x/x) = X(x/x)

" tigen/lectors ARE ORTHOGONAT (A x; 1 xs) = (x; 1 Ax; ) = >; (x; 1xs) = >; (x; 1xs)

but if \i \ i \ \ i, Heer (xil x5) =0

NOW WE WANT TO CONVECT ALL OF THIS TO PUNCTION SPACES (50 DIMENSIONAL!)

of course, this is what relates were mechanics.

eg consider (0,11 m) functions f at f(o) = f(1) = 0

DENTARY: 100) = 1/2 SIN (NTX)

discrete basis of so dim for space.

|  | f(x) = 5 C, 12 SIN (NITX) BEW: X= 1/2           |
|--|---|
|  | 1   |
|  |   |
| $= 1.015 \times 10^{-6} \times 10^{-2} \times 10^{$ | $C_n = \int_0^\infty SN(n\pi x) f(x) dx$        |
| and the second and th   |   |
| - gay gay 2000 kg ha 200 kg g ha sagaidh a air gaill dha bha ann 11 air an ann an air ain air air air air air a  | 1/f> = 2 cn/en>                                 |
|  | Cn= (enlf)                                      |
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| , and the second   |   |
|  | INNER PRODUCT                                   |
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|  |   |
|  | Some THINGS YOU NOTICE:                         |
| - SQUARGERICA - A ACCORDANCE CONTROL CONTROL CONTROL OF A   | FUNCTION SPACE HAS A DOMAIN - 20 CO. 1)         |
|  | 3. BRUNDARY COMD> f(0) = f(1) = 0               |
|  |   |
|  | need sufficiently ruce" & what come or wrong it |
| a transport and the second of the second property and a second second second second second second second second  | functions; maybe co WE USED DIFFERENT           |
|  | BOUNDARY COMO?                                  |
| e rom en   | → 15 入f, (x) + >2 ( ) (x)                       |
| and to had to his to a supply in the principle of the high high period to the supply to the supply of the supply o   | STILL IN THIS SPACE?                            |
| enterprise planting described and the second of the Second Company of the Second Company of the Second Company   | THE WHER CONDICT IS AN WITHOUM IN THIS OBE      |
| na di salah da salah   | THE INNER PRODUCT IS AN INTERRAL IN THIS OSE    |
| to, Payl Marie also and the Cartes and the second s   |   |
| granger germakker grang gjagog grang meta yang managang kamalak sekketapa  | (31+) = 10 3(x) +(x) 9x 1 1 now                 |
| and the company of the contract of the contrac   | eg it we used ein                               |

SUGARLY MARE GENERALLY,

(917) = 10 M(x) 9\*(x) f(x) dx

I WEIGHT FUNCTION

things I am not too womed about 2000

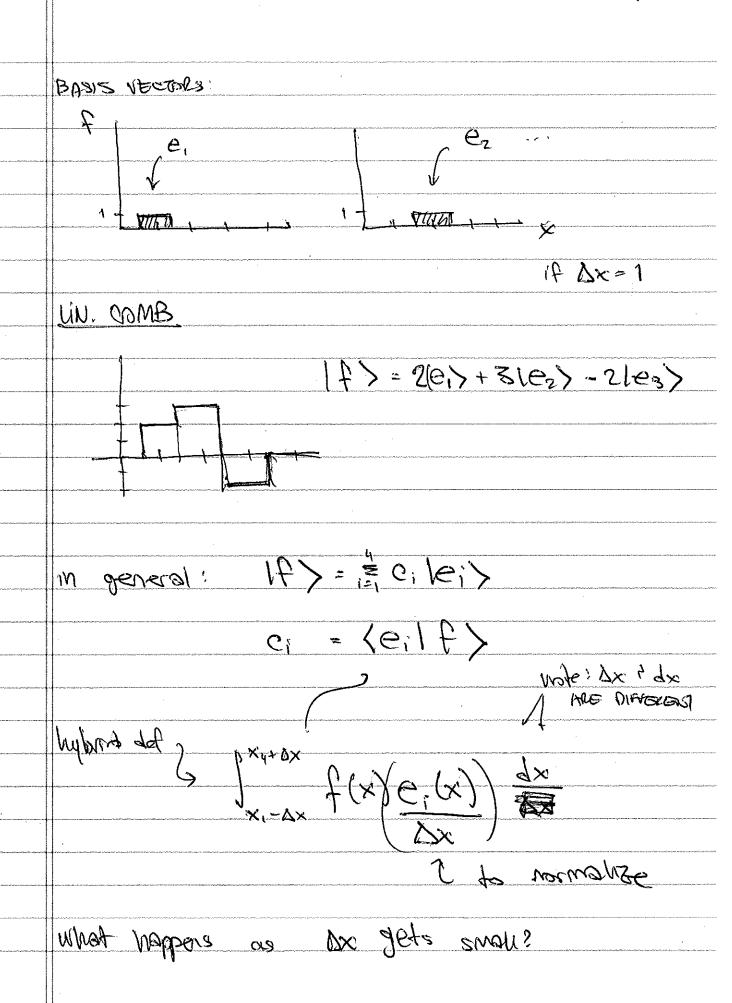
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- " OTHER DEF. OF MORMS
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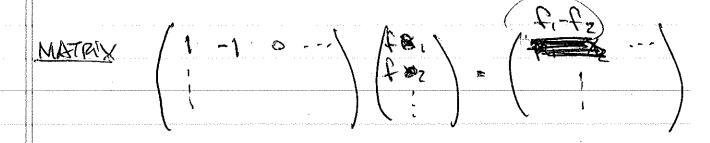
DISCRETIZED VERSION OF CONTINUOUS INCORNAR

THIS IS A 4D  $\Rightarrow$  8PACE W

PARTS  $e_1 = (1, 0, 0, 0) \leftarrow value of f(x,)$   $e_2 = (0,1,0,-)$ 

UNBAL SPACE, CLENTY





I CAN IMAGINE SUCH AN OPERATOR ACTING ON OUR DESCRETIZED PUNCTURY SPACE

Two POINTS

So: 
$$\frac{1}{4} \left( \frac{1}{2} - \frac{1}{2} \right) \left( \frac{1}{4} - \frac{1}{2} \right) = \frac{1}{4} \left( \frac{1}{4} - \frac{1}{4} \right)$$

paps out.

OBSERVATION: DIFFERENTIAL OPERATIONS

ARE "MATRICES" ACTING ON

FUNCTION SPACE.

usup color reighbor... how to get

T BUT THE MOS EXPLAINS WHY PHYSICS IS WELTER WINT DIFF & OF WIN DIM

| INTUINA OPERATOR  |
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| $1 = 7 / e_i / e_i $                                      |
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|   |
| = (=) file; \ \(e_i \right) \\                            |
| I I xut Dx ei(x)ei(x)dx Sis for FINITE DIM.               |
|   |
| GENERALIZATION 1  |
| f(p) = Jo) 3(x-y) + (y) dy                                |
|   |
| flx) is the coeff of lx>                                  |
| re this is the exp for a small or in Ecilei>              |
| NEXT: this afternoon: Disc- SEC.                          |
| Wed: DIFF EQ AS OR ACTIVE                                 |
| Ciro voctorios aux  |
| In particular gives $Ax = y \longrightarrow (x = A^{2}y)$ |
|   |

| ASIDE: HOW I'VE USED THIS   |
|---|
| WINDFUNCTIONS (really: FIELDS)  WIND SHOETIME ORDUND WHOLE SPACE  BC. QUEZ COMPACT KD |
| WE CARE ABOUT STORES OF WELL DEF. ENERGY  -> KK TOWER C-> OCCUVES                     |
| eg sines. — Just Fourier series.  |
| MAKE IT WORE COMPICATION.  WHEPINCIONS HE POSHIPTED                                   |
| Co get more complicated basis of Pris  BUT SALL ORTHOGENIAL -> (1).                   |
| overlap integral ~ Sis.   |