	LEC 16: DIFFERENTIAL FORMS 78 OCT
	-> "Vector coloulus" is really "form coloulus"
The second secon	· SEE STONE ? GOLDBART
	* also: math. 97/0306194 ANV: A. MFINERNEY
management of the Middle of white	1st steps in Wiff GEO,
AND DESCRIPTION OF STREET	REVIEW_
de constant of the fi	BASIS OF DUAL VECTORS: CX
The state of the s	EY. 9x (9r) = 8hr
and the second second	L BASIS OF VECTORS
- Construction of the Cons	V= V+2+
11 - 12 - 13 - 13 - 13 - 13 - 13 - 13 -	can act on a function f:M>IR
dough control of	to give directional decinative
ellar-sallis alla anno sen	
and discourse the second	HOW DO WE GET THESE dxt's?
- disconsistent of the second	DIFFERENTIAL OPERATION / EXTERIOR DERIVATIVE
ALL AND	9: K-form -> (K+1) form
Continue of the same of the sa	
and the state of t	so: a o-form is just a function, f.
-	(has no form-ness.)
C	
	9f = 3xr 9xr
	Lbasis 1- Bim
-	takes a vector V to give directional deriv.
tens	

AF is a 1-form BERN TO BE INTEGRATED: So dF = ?
from ordinary calculus, you instructively want to write: Icaf = f(P,)-f(P=)
$E=\frac{1}{2}$ $E=\frac{1}{2}$ $E=\frac{1}{2}$ $X(0)=P_0$ $X(1)=P_1$
ONE WAY TO SEE THIS: PARAMETERIZE THE PATH: let x: IR > M s.t. 9 x(1) = P, ZWE A TIME PARAM (x(0) = Po
then: ladf = lo (af(xw)) at "signed integral"
3x; st 3t(xit) 9x;
Po P, = Po A P, SWITCH-BACK CANCELS
COMPARE THIS TO UNSIGNED INTEGER $1 \left \frac{df}{ds} \right ds$ FOR, eg, ARCHENGTH.

A MORE GEOMETRIC PICTURE: IMAGINE CONTOURS
OF CONSTANT & ON M.
DASHED LINES: POINTS & EM $s.t. \ f(x) = -1, \ =, 1, \dots$
F=10123
P, f=6 P(Pi) = 5.5
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$
P2 F=1 F(P0) = 1.5
then: I at = # times we move up a contain
- # times we make down
⇒ Jcdf = f(P,) - f(Po) f short out.
ORIBITIEP BAMDAM
= f /ac
TBOUNDARY

nb Malnomu	~ X' = X
up McIverna	$2-forms \qquad \qquad \chi' = \chi \\ \chi^2 = \chi$
	w= zi wpv dxt~dx
NOONS	= 2[Wxy dx ~ dy + Wyx dy ~ dx]
1975 3 ~17.5	= 2 Wxy - Wyx] dx ~ dy < dx ~ dy = -dg ~ dx
	200 kg ky impuæd Arraisym.
August overess finalmenterins spell (in the plant in the	= Wxy dx ~ dy
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n a market and leaves (framewommen on on tables of the leaves (1914 to 1914 to 1914 to 1914 to 1914 to 1914 to	MACHINE THAT TAKES 2 VECTORS
gy y ny ny na pang-fig na isan-inanainaina ita 22 tana mahin dhilladh 22 12 12 12 13 dhilladh i amara fha Ghardh dhibh dhibh	2 SPITS TO STIRS F
a ang atalamanan pananangan ana anama ah milili in milili an milili and milili ililih di 1900 (1900) di 1900 (1	PRODUCT OF COMPONIENTS
er van grogen en megenerale en med termeld med de elemente de men med men et met en de en de en de en de en de	
не дому допуснования дво до учено на 12 година 15 год до до до не-дъз на роменичение выполнения под дел во не на	dxrdy = dx @dy-dy @dx
gappan ang mangkan ang kalaman ang managan managan managan managan ang managan ang managan ang kalama ang mana	
	dxndy (V,W) = V'W2 - V2W1
nt artisen escada estados empresidades assistantes estados en entre con entr	WATT
adit awar swan swani ta di sani dada sini dada sini anda na panga sini anga na sini panda na sini panda na pan	gives val al
and a special control of the control	paravelph 2 1
pormo-do a plikika maki ma akuma naishi shaka shanna kombi dha a dhadhidh ili kalika 1854 dha shaka na kuma naishi shaka shanna kombi dha a dhadhidh ili kalika 1854 dha dha ma'	(OPIENZED)
mgruup saansangannessanskannii kaspussansessi ja kirintä kirintä kirinteela valla 1556 kiril 1994 kilantii Soksia kiril	
oky 200-lanny (2007) po je propovana od nejskom od oko privodnosta nejstých je pod 200 metričnich sed 200 sed 200 metri	
	if w= d(P(xy)) ~ d(g(xy))
discumptivity to proceed the process of the process	9=-1
gelegelegen eine Freihande seine Gelegen eine Gelegen erwalt aus die eine eine de dat weren ammy war weren wyg a em	3^{-1}
	f tan
PTII James (Specific Collins and Specific Collins and Proposition Association Collins (Specific Collins Collin	(8.8)
a. la kilong jelejen od jaki 1904. dilikah prinsekt and serekt anak ilah jelejen kilondori e 1800. jil 1809 (1909) di	(x, y)

Itm. Spe \$ 11.2.2.

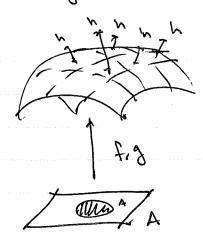
2-form is infinitesimal area et a monifold. (ORIENTED) 3-form is so inal volume, also oriented

THIS STORY GENERAUZES

Jah(x,y) dfrdg th,g: R2 > M

W: R2 -> R

(Weights EACH AREA DIEMBN7

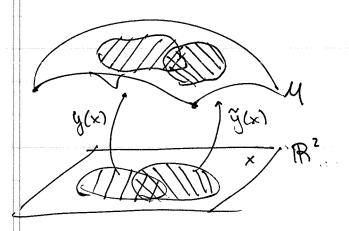


then: Inhalanda a 2D RIEMANN INTEGRAL OF h

this is a nice way to PARAMERSLIZE

induced neter p. 421

> ALSO GIVES A PICTURE FOR US TO EXPLORE A BETTER WORKING DEFINITION OF A MANIPOND



MJG-NSPACE WHICH IS EUBOUWHERE vocacy (in patches) DIFFEDMORPHIC WI 1RM S.t. THESE MAPS AGREE WHERE they overlap

eg. 52, 2-SPHERE. ONE POTCH WON'T DO.

P. 420 STO GO	SPECIAL CASE: N-DIM MANIFOLD IS EMBEDDED
Stownes	IN A HIGHER-DIM EVOLIDEAN SPACE
$p_{i_{1},i_{2},i_{3},i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i_{4}+i$	C'increal" ul usual sense
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nd deliging descriptions of the contract of th	ND: "MANIFOLDS BON'T HAVE TO BE EMBEDOINGS
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2007-ni Spressensensen villade királ sírál sírál vilk elle helyélén kenner ni kerneg POJ si 2007-ni 2007-ni 2007-ni	m this case:
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Tanggi projek kalandaka isabehi dangga perunanan melika isabi SS ISS pendahangga pengganangga penganan kanan k	SURFACE M IS A RESCRICTION
$g_{ab}(x,x,y) = \frac{1}{2} \left(\frac{1}$	80 x' = x'(y',, y')
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indicated the control of the control	
570 G P.421	INDUCED WERRIC: $4e^2 = \sum_{m>0} \left(\frac{3\lambda_m}{9x_i} q_{n} \right) \otimes \left(\frac{3\lambda_n}{9x_i} q_{n} \right)$
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HW:

IN THE CASE, THE DIFFERENTIAL VOLUME

1'S

d(Vol) = Ndet gru dy'n -- ~ dy'

jacobson.

Invariant under change

of osordinates

Stokes' theorem

INTEGRALS TAKE DW OVER A "VOLUME"

TO W OVER Q "SURFACE OFFACE"

So W W

K-MM J ORIENTED

KHIDE-DIM WANIF.

RELL AREA J

P(P1)-P(P0)

20 CASE: CONSIDER W= A: dx = Axdx+Azdy+Azdr dul = d (Axdx) +---= (DAx dx dx + DA dy dy dx + DA dex dex dx + A ddx by ontison $= \left(-\frac{\partial A_{x}}{\partial y} dx \wedge dy + \frac{\partial A_{x}}{\partial z} dz \wedge dx\right) + \cdots$ = (3Ay - 3Ay) dx rdy + (34 - 3Ay) dyrdz Chk: + (DAX - DAZ) dZAdX RECOGNIZE: the components are (TXA); the 2-form basis in the (2st line are the (+ oriented) areas of parallelograms normal vectors (dx rdy ~ dx x dy = dz) = ((TXA) dA $\int_{\Lambda} d \left(A : d \times_{i} \right)$ = | & A - Al 12 W Green's 4hm

SUGGESCIVE

 $dW = \left(\frac{3f_x}{3x} + \frac{3f_y}{3z} + \frac{3f_z}{3x}\right) dx - dy - dz$

 $\int_{\mathcal{V}} d\omega = \int_{\mathcal{V}} \nabla \cdot \vec{f} \, d(vol)$ $\int_{\partial V} \omega = \int_{\partial V} \vec{f} \cdot d\vec{A}$

identifying, of dy-dz = nx dA

FURTHER REMINDING US OF "VECTOR CALCULUS":

FOR W A 0-FORM (FUNCTION)

2 W 13 A 1-FORM (Dxfdx+--)

2 W 15 A 2-FORM (TX(DF)) & dy x d 2 + ---]

(ey blc Broy of Broy = 0)

[0= barg-/rw] 0= 7×7 6

POR W A 1- FORM $W = f_{\times} \frac{d^{2}x}{dx^{2}} + \cdots$ dw is A 2- FORM $(\nabla \times \vec{f})_{z} dx^{2} dy^{2} + \cdots$ $d^{2}w$ is A 3- FORM $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{f}) dx^{2} dz^{2}$ $v = f_{\times} \frac{d^{2}x}{dx^{2}} + \cdots$ $d^{2}w$ is A 3- FORM $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{f}) dx^{2} dz^{2}$ $v = f_{\times} \frac{d^{2}x}{dx^{2}} + \cdots$ $d^{2}w$ is A 3- FORM $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{f}) dx^{2} dz^{2}$

CONTEXT: We "generalized" vector

calculus.

To non-plat Uninep DIFFERENTIAL

STRUCTURE

Killer app for vector calculus? E7M!

first, a quick result:

& M oomes from potential

EXACT FORM: W = dA CLOSGO FORM: dw = 0

obvirous: EXACT → cuosED

Pomocolé lemma

Pomocolé lemma

"nice" spaces j most of
our coses.

(contractible)

POTENTIALS Pancaré: SUPPOSE "VEC" E; dx' s.t. 7 x E = 0 → dE=0 () E=dY C E = VY i.e. E HAS NO CURL >> E IS GRAP OF SCALAR. SIMILARY: SUPPOSE "VEC." BANDO B. Lyndz +... Nb: very diff. object! marnell's 08. dB =0 B=dA 7 7.B=0 7v2 on this note: an interesting operator HODGE ETAR: 4 on n 2m monifold * 9x, v... 9x, = (V-K) [5!"... " drien v-vyx

turns 2-born -> 1-born M 3-SPACE.

80: IN MINKOUSKI SPACE, EM PIEUDS LIVE IN Fru

? one key point is F = dA

4- potential
A+= (4, A)

next time: pushing vectors