P3327 SECTION 9

26 OCT 2012

<u>REMARK</u>: these are <u>ROUGH</u> notes which may or may not correspond to what we actually did in section! - FLIP

ANNOUNCEMENTS

- ENCOURAGED: READ §4.9 & §4.10
  - §4.9: for culture
  - §4.10: for later

- HW EXTENSIONS

  We've been fairly lax — BUT JUST BECAUSE YOU GET AN EXTENSION, IT DOESN'T MEAN THAT THE CLASS IS SLOWING DOWN!

  → EACH EXTRA DAY SPENT ON OLD HW IS ONE LESS DAY FOR CURRENT HW

  → PRELIM SOON!

  WE'RE GIVING YOU WIGGLE ROOM BECAUSE YOU'RE GROWN UPS, BUT MAKE SURE YOU DON'T END UP SCREWING YOURSELF.

- REPEAT: WORK WITH OTHER PEOPLE!!
  it's a matter of efficiency

- HW9 HINT - to be posted
  USE KELVIN Bes Kelvin Bei

- HW: HAND WRITING, BE HONEST W/ ?

OH MISTAKE

(Seong)

coulomb: $\nabla \cdot A = 0$

$\nabla \cdot \int \frac{J}{|r-s|} = \nabla \cdot j + \frac{\delta}{\cdot\cdot}$

So: ρ is not t-indep.

WARM UP

Getting used to ε. (PERMITTIVITY)

Q: is $\varepsilon \geq 1$

or ≤ 1 ?

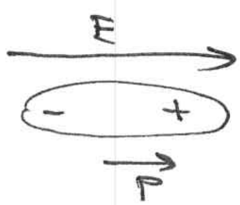
$$\boxed{D = \varepsilon E}$$

$\varepsilon = (1 + 4\pi \chi_e)$ ← elec. susceptibility

what is $\underline{D}$? this is the "ELECTRIC FIELD" THAT is sensitive to (SOURCED BY) only FREE CHARGE ie NOT BOUND CHARGE. (d.o.f. of the medium)

So: WHICH is BIGGER, D or E?

D is! BOUND DIPOLES ALIGN ACCORDING TO E, CREATE A CONTRIBUTION THAT WANTS TO CANCEL E.



CREATES A SMALL MICROSCOPIC field AGAINST $\underline{E}$

(do not confuse w/ $r \gg p/q$ field @ large distances)

so: $\varepsilon \geq 1$, $D \geq E$

↳ $\varepsilon \to 0$ doesn't make sense.

Remarks on μ

$$D = \varepsilon E \quad \text{but} \quad H = \frac{1}{\mu} B$$

$$D = E + 4\pi P \qquad\qquad H = B - 4\pi M$$

b/c $\nabla \cdot E = 4\pi(\rho_f + \rho_b)$

$\rho_b = -\nabla \cdot P$

opp dir

why: $\nabla \times B = \frac{4\pi}{c}(J_f + J_b)$

$J_b = c \nabla \times M$

IN fact:

$\mu \gtrsim 1$ diamagnetic

$\mu \lesssim 1$ paramagnetic

($\mu \gg 1$ FERROMAGNETIC)

BUT for TYPICAL MATERIALS, $\mu \approx 1$ so for NOW WE STICK TO THIS REGIME.

DIELECTRIC (ε) MEDIA

$\mu \approx 1$

$\sigma = 0$ why? $\sigma \neq 0 \to \exists\ J_{free}$

HW will be all about this.

$\left(\to \nabla \times B - \frac{\varepsilon\mu}{c}\dot{E} = \frac{4\pi\sigma\mu}{c}E \quad ; \text{later} \right)$

PUNCHLINE: LIGHT TRAVELS SLOWER IN MEDIA

index of refraction n

$v = c/n$

EASY TO SEE

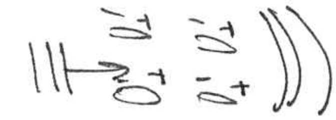
the whole point of PLANE WAVES $\nabla \to ik$, $\partial_t \to i\omega$ (WHY FOURIER IS SO USEFUL)

$$\nabla \times B = \frac{1}{c}\dot{E} \longmapsto \nabla \times B = \frac{\varepsilon\mu}{c}\dot{E}$$

$$\left(\text{then } \nabla \times \nabla \times B = \nabla(\nabla \cdot B) - \nabla^2 B = \frac{\varepsilon\mu}{c}\nabla \times E = -\frac{\varepsilon\mu}{c}\left(\frac{\partial}{\partial t}\right)^2 \right)$$

but why? (microscopically)

~~PROBLEM: ~~Show~~ Show, BY SUPERPOSITION, THAT $v = \frac{\varepsilon\mu}{c}$ ASSUMING THE INCIDENT WAVE TRAVELS @ $v_{vac} = c$. SUPERIMPOSE THE INDUCED WAVES.~~

~~PROBE~~ $\boxed{\text{IDEA}}$:  ← SUPERPOSITION

WAVE @ $v=c$ CAUSES OSCILLATIONS IN DIPOLES ~~..~~ OF MEDIUM

SUPERPOSITION OF INITIAL WAVE + ~~WAVE~~ INDUCED WAVES GIVES NEW MONOCHROMATIC WAVE @ ~~...~~

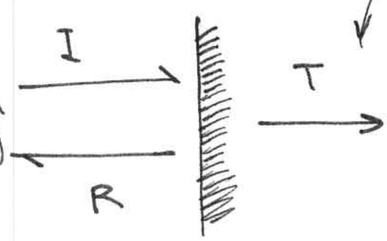
$v = c/\sqrt{\mu\epsilon} = c/n$

TWO PART PROBLEM

(1) ("nonperturbative")

ASSUME $v = \sqrt{\mu\epsilon}/c$ HERE (we'll ~~pr~~ prove this later)

$$\left[\begin{array}{l} E_I = E_0 e^{ikx} e^{-i\omega t} \hat{z} \\ B_I = \frac{E_0}{c} e^{ikx} e^{-i\omega t} \hat{y} \end{array}\right.$$



we will study this in ch. 6 (BASIS of OPTICS)

$\boxed{\text{FIND } E_T \text{ \& EXPAND IN } h = 4\pi\chi_e .}$

$$\underline{E}_T = E_T e^{ik'x - i\omega t} \hat{z} \qquad B_T = \frac{1}{c}(\text{—})\hat{y}$$

$$\underline{E}_R = E_R e^{-ikx - i\omega t} \hat{z} \qquad B_R = \frac{1}{c}(\text{—})(-\hat{y})$$

$$k' = \omega\sqrt{\mu\epsilon}/c = n/c$$

$$k = \omega/c$$

(2) show that superposition above gives same expansion in h.

$\boxed{BC}$ (from Maxwell @ interface)

$$\nabla\cdot D = 0 \Rightarrow \varepsilon_1 E_1^{\perp} = \varepsilon_2 E_2^{\perp}$$
$$\nabla\times E = 0 \Rightarrow E_1^{\parallel} = E_2^{\parallel}$$
$$\nabla\cdot B = 0 \Rightarrow B_1^{\perp} = B_2^{\perp}$$
$$\nabla\times H = 0 \Rightarrow \frac{1}{\mu_1}B_1^{\parallel} = \frac{1}{\mu_2}B_2^{\parallel}$$

trivial, no $\perp$ component for normal incidence

So: E-field : $E_0 + E_R = E_T$

B-field : $E_0 - E_R = n E_T$

$$2E_0 = (n+1)E_T \Rightarrow \boxed{E_T = \frac{2}{n+1}E_0}$$

similarly : $E_R = -\left[\frac{n-1}{n+1}\right]E_0$
but we don't care here.

Now: ASSUME $\mu = 1 \Rightarrow n = \sqrt{\varepsilon} = \sqrt{1+4\pi\chi_e} = \sqrt{1+h}$

$$E_T = \frac{2}{\sqrt{1+h}+1} E_0 \underline{e^{i\sqrt{1+h}\,kx}} \, {}^{-i\omega t}\, \hat{y}$$

$$e^{inkx} = e^{i(n-1)kx}e^{ikx}$$

$$\sqrt{1+h} = 1 + \tfrac{1}{2}h - \cancel{\ldots}\ \tfrac{1}{2}\tfrac{1}{4}h^2$$

$$= \underline{\left(\frac{2}{2+\frac{1}{2}h - \frac{1}{8}h^2 + \ldots}\right)} \cancel{E_0}\, e^{i\left(\frac{1}{2}h - \frac{1}{8}h^2+\ldots\right)kx} E_0 e^{ikx - i\omega t}$$

$$\cancel{= 1 - \tfrac{1}{2}h}$$

$$\cancel{2\left(\frac{1}{1+\frac{1}{4}-\frac{1}{16}h^2+\ldots}\right)\left(1+\frac{1}{2}h-\frac{1}{8}h^2+\ldots\right) E_0 e^{\ldots}}$$

FOR SIMPLICITY, TAKE ONLY THE LO IN h

↳ YOU CAN CHECK THE REST USING Mathematica
(USE SERIES FUNCTION)

$$E_T \approx \left(1 - \frac{1}{4}h(1-2ikx) + \cdots\right) E_0 e^{ikx} e^{i\omega t} \hat{y}$$

Part II: ITERATIVE SOLUTION FROM SUPERPOSITION

INCIDENT PLANE WAVE $\boxed{E_I}$ as before
INDUCES A POLARIZATION

$$\underline{P} = \chi_e \underline{E}_I = \chi_e E_0 e^{ikx - i\omega t} \hat{z}$$

time varying ⇒ INDUCES A DISPLACEMENT CURRENT

$$J_P^{(1)} = \dot{P} = -i\omega \chi_e E_0 e^{ikx} e^{-i\omega t} \hat{z}$$

LEMMA: GIVEN A NEUTRAL PLANE SURFACE CURRENT $K(t)$ IS

$$\underline{E} = -\frac{2\pi}{c}\,\underline{K}(\underbrace{t - s/c}_{\text{finite time effects}})$$

More on this later!

I'M not 100% sure about factors of 2π & c

pf/ $$A^{(x)} = \frac{1}{c}\int \frac{K(t_R)}{\sqrt{s^2+r^2}}\,da \leftarrow 2\pi s\,ds$$

$$t_R = t - \sqrt{s^2+r^2}/c$$

RETARDED TIME

$$= \frac{2\pi}{c}\int_0^\infty K\left(t - \frac{\sqrt{s^2+r^2}}{c}\right)\frac{s}{\sqrt{s^2+r^2}}\,ds$$

$$u = \frac{1}{c}\left(\sqrt{s^2+r^2} - r\right)$$
$$du = \frac{1}{c}\frac{s}{\sqrt{s^2+r^2}}\,ds$$

$$t - \frac{\sqrt{s^2+r^2}}{c} = t - \frac{r}{c} - u$$

$$= 2\pi\int_0^\infty K\left(t - \frac{r}{c} - u\right)du$$

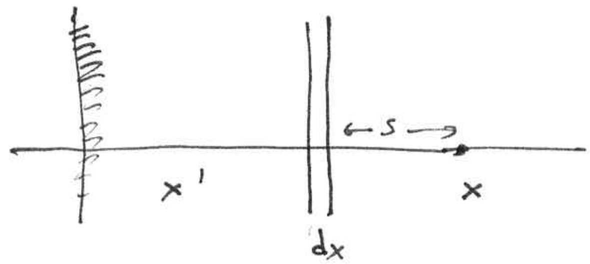
$$\underline{E} = -\frac{1}{c}\frac{\partial A}{\partial t} = -\frac{2\pi}{c}\int_0^\infty \frac{\partial}{\partial t}K\left(t - \frac{x}{c} - u\right)du$$

$$= +\frac{2\pi}{c}\int_0^\infty \frac{\partial}{\partial u}K\left(t - \frac{x}{c} - u\right)du$$

$$= -\frac{2\pi}{c}K(t - x/c) - 0 \leftarrow \text{ASSUMES!}$$

somewhat inconsistent (force by giving $\omega \to \omega + i\epsilon$)

So: $E = -\frac{2\pi}{c} K(t - s/c)$



So: INCIDENT → POLZ → ~~E~~ CURRENT → E

$$E^{(1)} = \left(-\frac{2\pi}{c}\right)\left(-i\omega \chi_e E_0 \hat{z}\right)\left[\int_0^x e^{ikx' - i\omega\left(t - \frac{x-x'}{c}\right)} dx' + \int_x^\infty e^{ikx' - i\omega\left(t - \frac{x'-x}{c}\right)} dx'\right]$$

K ↘

$$= i\frac{\omega}{c}\frac{h}{2}E_0\hat{z}\left(e^{i(kx-\omega t)}\int_0^x dx' + e^{-ikx-i\omega t}\int_x^\infty e^{2ikx'}\, dx'\right)$$

~~lost factor of 2~~

$$= \underbrace{i\frac{kh}{2u}E_0\hat{z}\,e^{ikx-i\omega t}}_{E_s}\left(x + e^{-2ikx}\cdot\left(\frac{\cancel{e^{2ik\infty}}}{2ik} - \frac{e^{2ikx}}{2ik}\right)\right)$$

WAVE REFL BACK FROM FAR SIDE OF DIELECTRIC.

ARTIFACT of PURE PLANE WAVE

→ 0 if attenuated

$$E_{\Phi}^{(1)} = E_I \frac{h}{4} ik \left(2x - \frac{1}{ik}\right)$$

$$= E_I \frac{h}{4} \left(2ikx - 1\right)$$

$$\boxed{= -E_I \frac{h}{4} \left(1 - 2ikx\right)}$$

cf E_T !!

WOW! EXPLAINS TRANSPARENCY