ANNOUNCEMENTS

- PRELIM #1 NEXT WK → SUGGESTION: DO PRACTICE PROBLEMS eg. GRIFFITHS EXAMPLES
  - ↳ SHORT HW #5
    MON OH: ASK QUESTIONS ABOUT THE COURSE
- HOMEWORKS

Review

1. LAPLACE EQ IN $\left\{\begin{matrix}\text{RECT}\\ \text{SPHR}\\ \text{CYL}\end{matrix}\right\}$ COORDINATES

2. SEPARATION ANSATZ $\Phi(\underbrace{x_1, x_2, x_3}) = X_1(x_1)\,X_2(x_2)\,X_3(x_3)$

   x, y, z
   r, θ, φ
   ρ, φ, z

3. SEPARATE (AS MUCH AS POSSIBLE) LAPLACE EQ., ~~CST~~ WRITE OUT CONSTANTS (frequencies!)

eg. $\frac{1}{X}X'' + \frac{1}{Y}Y'' + \frac{1}{Z}Z'' = 0$

EACH IS CONSTANT.
IN FACT, $\alpha^2 + \beta^2 + \gamma^2 = 0$.

eg. $\frac{1}{r^2 R}\frac{d}{dr}\left(r^2 R'\right) + \frac{1}{P r^2 \sin\theta}\frac{d}{d\theta}\left(\sin\theta\, P'\right) + \frac{1}{Q r^2 \sin^2\theta} Q'' = 0$

↓ RESHUFFLE

$\left[r ; \theta \right] = \boxed{-\frac{1}{Q}Q''}$

↑ SAME TYPE OF EQ. AS CARTESIAN CASE $\rightarrow e^{im\varphi}$

SOLUTION: $\left(A_{\ell m} r^\ell + B_{\ell m}\frac{1}{r^{\ell+1}}\right) Y_\ell^m$

↳ $\propto P_\ell(\cos\theta)$ for $m = 0$

4. USE BC TO SOLVE FOR:

① COEFFICIENTS – EASY ONES

↳ eg. NO DIVERGENCE @ $r = 0 / \infty$
eg. $\Phi = 0$ @ SURFACE → KILL cosh OR cos term

② FREQUENCIES

↳ eg ONCE ONE BC FIXES REL COEF, PARALLEL BC CAN ONLY FIX FREQ.

OR MATCHING TO BNDY w/ FIXED P_ℓ OR Y_ℓ^M EXPANSION.

③ COEFFICIENTS – HARD ONES USING FOURIER'S TRICK

5. IF NEEDED, SOLVE FOR ρ USING DISCONTINUITY IN Φ'

CYLINDRICAL COORDS

LAPLACE + SEPARATION:

$$\underbrace{\frac{r}{R}\frac{d}{dr}\left(r\frac{dR}{dr}\right)}_{\text{cf. SPHERICAL CASE!}} + \frac{r^2}{Z}\frac{d^2Z}{dz^2} = \underbrace{\frac{-1}{Q}\frac{d^2Q}{d\theta^2}}_{}$$

$Q \sim e^{\pm in\theta}$

[making an assump on sign of n^2]

SOLUTION TO LHS: $Z \sim e^{\pm kz}$. IF $k = 0$, then:

$$R_n(r) = \underset{\text{CONST.}}{A_0} + \underbrace{B_0 \ln r}_{\substack{\text{physical sig?} \\ \infty \text{ line charge}}} + \left(A_n r^n + B_n \frac{1}{r^n}\right)$$

not $(n+1)$!

MORE GENERAL SOLUTION IN CYLINDRICAL COORDS

↳ $k \neq 0$, MAKES r EQN MORE DIFFICULT

GEN SOLUTION

$$\sum_{m,n} \left[A_{mn} J_n(k_m r) + B_{mn} N_n(k_m r) \right] e^{\pm i n \theta} \, e^{\pm k_m z}$$

$A \cos(n\theta) + B \sin(n\theta)$

$A \sinh(k_m z) + B \cosh(k_m z)$

KEY PROPERTIES.

ORTHOGONALITY FOR FOURIER'S ~~THE~~ TRICK:

$$\int_0^a J_n(k_m r)\, J_n(k_{m'} r)\, \underbrace{r\, dr}_{\text{RECALL: CYL. COORD.}} = \underbrace{\frac{a^2}{2} J^2_{n+1}(k_{m'} a)}_{\text{const.}} \delta_{mm'}$$

IN THIS CLASS: MOSTLY FOCUS ON $n=0$

N DIVERGES @ ORIGIN: $B_{mn}=0$ INSIDE CYLINDERS.

UPSHOT: THERE ARE ONLY A HANDFUL OF SUFFICIENTLY TRACTABLE BESSEL FUNCTION PROBLEMS @ THIS LEVEL.

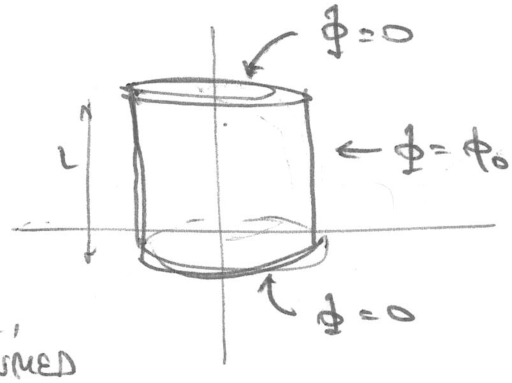
① EXAMPLE 3.5 IN THE BOOK
② PROBLEMS 3.36 – 3.38 IN BOOK

1st 2 ON HW

LAST ONE IS PROBABLY A BIT TOO DIFFERENT.

BOOK : 3-38

FIND Φ INSIDE.



DIFFERENT FROM CLASS / ~~eqs~~ 3-5!

↓

CAN YOU SEE WHY?
Z(Z) MUST NOW BE PERIODIC, WHEREAS WE PREVIOUSLY ASSUMED EXPONENTIAL DECAY.

↳ SEPARATION CONSTANT IS <u>NEGATIVE</u> → ? MEANS for BESSELS?

BY NOW MAYBE YOU CAN JUST READ off THE Z DEPENDENCE

$$Z(z) = A \sin\left(\frac{m\pi}{L} z\right) \leftarrow m \in \mathbb{Z}_{\geq 0}$$

BC @ L

no B cos term by BC @ z = 0

<u>RECALL</u> : IN RECTANGULAR SYSTEM (SAY, 2D), ONE SINUSOID & ONE EXPONENTIAL.

↳ relation btwn separation constants

SO: BEFORE : $Z(z) \sim e^{kz}$

NOW : $Z(z) \sim e^{ikz}$

$k \to ik$

END UP w/ SOMETHING LIKE ~~J_n~~ $J_n(kr) \to J_n(ikr)$

TURNS OUT THERE'S A NAME for THESE:

$$\boxed{I_n(kr) = i^{-n} J_n(ikr)}$$

"Modified Bessel"

~ exponential

(I_n, K_n)

↳ DIVERGE @ ORIGIN!

~ SINUSOID

(J_n, N_n)

↕

Y_n

WHAT ABOUT ~~[illegible]~~ ANGULAR DEPENDENCE?,

↳ no θ DEPENDENCE $\Leftrightarrow$ $n=0$ in $e^{in\theta}$ term
SAME AS PROJECTING OUT COSINE.

$$\Phi = \sum_{m \text{ odd}} A_m \sin(k_m z)\, I_0(k_m r)$$

$k_m = \frac{m\pi}{L}$, not zero of BESSEL.

B/C @ $r=a$: $\Phi(r=a,z) = \Phi_0$

$$\hookrightarrow = \sum_{m \text{ odd}} \underbrace{A_m I_0(k_m a)}_{\equiv B_m} \sin(k_m z)$$

FOURIER'S TRICK:

$$\underbrace{\int_0^L \Phi_0 \sin(k_n z)\, dz}_{\frac{2L}{\pi n}\Phi_0} = \int_0^L \sum_m B_m \sin(k_m z) \sin(k_n z)\, dz = B_n \frac{L}{2}$$

$\boxed{n \in \text{ODD}}$ ✓

$$\Rightarrow \boxed{B_m = \frac{4\Phi_0}{\pi m}} \qquad m \in \text{ODD}, \text{ OTHERWISE}$$

$$A_m = \frac{4\Phi_0}{\pi m}\frac{1}{I_0(k_m a)}$$

$$\boxed{\Phi = \sum_{m \text{ odd}} \frac{4\Phi_0}{\pi m} \frac{I_0(k_m r)}{I_0(k_m a)} \sin(k_m z)}$$