SECTION 8 OR SOMETHING

19 OCT 2012

ANNOUNCEMENTS:

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~~[illegible]~~ SOME REMARKS ABOUT HW 7

HYM 4.2 $B(t) = B_0 t \hat{z}$ "ASSUME AXIAL SYM"

CALC $\underline{A}$ $\underbrace{\text{"ASSUMING } \Phi = 0\text{"}}$

this is implicitly a gauge choice!

AXIAL SYM → WANT YOU TO PICK CIRCULAR STOKESIAN PATH s.t. $A \sim \hat{\varphi}$. CHOICE of A, COULD HAVE HAD UN COMP IN $\hat{\varphi}, \hat{\rho}$ DIR. BUT, $\hat{\rho}$ COMP NOT COMPATIBLE w/ $\Phi = 0$.

IN GENERAL (for $A \sim A_\varphi \hat{\varphi} + A_\rho \hat{\rho}$), WOULD NEED TO USE FARADAY'S LAW & $E = -\nabla\phi - \frac{1}{c}\dot{A}$ TO DETERMINE Φ.

BUT THEN THE QUESTION IS STUPID B/C IT ASKS TO FIND ~~[illegible]~~ E & THEN CONFIRM FARADAY. BUT YOU NEED FARADAY TO (in general gauge) FIND E.

NEW ("macroscopic") MAXWELL EQNS

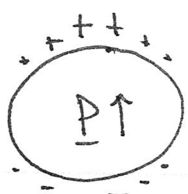
$$\nabla\cdot\underline{D} = 4\pi\rho_f \qquad\qquad \underline{D} = \underline{E} + 4\pi\underline{P} = \varepsilon\underline{E}$$
$$\nabla\cdot\underline{B} = 0 \qquad\qquad \underline{H} = \underline{B} - 4\pi\underline{M} = \tfrac{1}{\mu}\underline{B}$$
$$\nabla\times\underline{E} = -\tfrac{1}{c}\dot{\underline{B}}$$
$$\nabla\times H = \tfrac{4\pi}{c}\underline{J}_f + \tfrac{1}{c}\dot{\underline{D}}$$

note: $\nabla\cdot\underline{H} = -4\pi\nabla\cdot\underline{M}$
not "analogous to $\underline{B}$"

DIVERGENCE LAWS $\longrightarrow$ (dis) CONTINUITY of NORMAL comp.
CURL LAWS $\longrightarrow$ (dis) CONTINUITY of TANGENTIAL.

eg. in absence of <u>FREE</u> CHARGE,
$D_\perp$ is CONTINUOUS $\Longrightarrow$ $E = \frac{1}{\varepsilon}D$ is <u>NOT</u>.

<u>HW 1.13</u>



WHAT is $\underline{E}(0)$?

INTEGRATE COULOMB FIELD of BOUND SURFACE CHARGE σ_b. (or ρ_b)

$$\rho_b = \underline{n}\cdot\underline{P} = P\cos\theta$$

$$E(\underline{r}) = \int \frac{\rho(s)}{|\underline{r}-\underline{s}|^2}\,\widehat{(\underline{r}-\underline{s})}\,d^3s \xrightarrow{r=0} \int \frac{\rho_b}{a^2}(-\hat{r})\,(2\pi)\,a^2\,d(\cos\theta)$$

GO AHEAD + PROJECT OUT E_z, ONLY NONZERO COMP

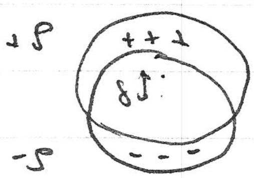
$$E_z(0) = \int \frac{1}{a^2}P\cos\theta\cdot\underbrace{(-\hat{r}\cdot\hat{z})}_{-\cos\theta}\,(2\pi)\,a^2\,d(\cos\theta)$$

$$E_z(0) = -2\pi P \underbrace{\int_{-1}^{1} \cos^2\theta \, d(\cos\theta)}_{\frac{1}{3}u^3\Big|_{-1}^{1} = \frac{2}{3}} = \boxed{-\frac{4\pi}{3} P}$$

$$\rightarrow \boxed{\underline{E}(0) = -\frac{4\pi}{3} \underline{P}}$$

b) DOES THIS LOOK REASONABLE? WHAT ABOUT OTHER $r < a$?

CLAIM: COMPARE TO OVERLAP of OPP CHARGED UNIFORM SPHERES



CHOOSE ρ & δ s.t. OVERLAP CONFIG MATCHES PART (a)

note: PART a HAS TOTAL DIPOLE MOMENT $\boxed{P \frac{4\pi a^3}{3}} = P_{tot}$

s.t. SUPERIMPOSED CONFIG HAS DIPOLE MOMENT

$$p = q\delta \qquad \text{w/ } q = \frac{4\pi a^3}{3}\rho$$

the DIPOLE MOMENTS MATCH WHEN $P = \rho\delta$.

check!

CLAIM: IN A UNIFORM SPHERE $\underline{E}(\underline{r}) = \frac{4\pi}{3}\rho\, r$

then: $\underline{E} = \frac{4\pi}{3}\rho\left[\left(\underline{r} - \frac{\delta}{2}\right) - \left(\underline{r} + \frac{\delta}{2}\right)\right]$

$= -\frac{4\pi}{3}\rho\,\underline{\delta}$

$$\boxed{\underline{E} = -\frac{4\pi}{3}\underline{P}\ .}$$

what we found @ $r=0$
in fact, $\underline{E}$ is constant everywhere inside sphere!

ALTERNATE DERIVATION: SPHERICAL HARMONICS

$$\rho_s = P_0 P_1(\cos\theta)$$

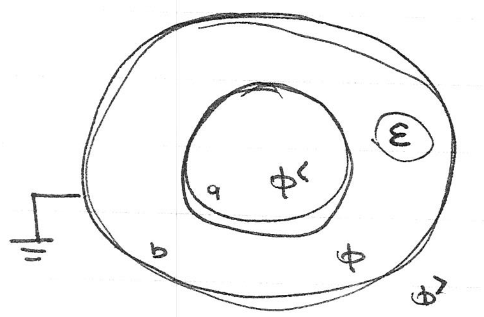
(c) NOW CONSIDER A POLARIZED MEDIUM w/ A SPHERICAL CAVITY CARVED OUT. WHAT IS E_c IN THE CAVITY W/RT $\underline{E}$ IN THE MEDIUM? ($\underline{E}$ is EXTERNAL)

SAME TRICK: SUPERIMPOSE A POLARIZED SPHERE w/ OPP POLZ TO DIELECTRIC. ie POLARIZE $\underline{P}_{sup} = -\underline{P}$

$$\underline{E}^{ext} + \underline{E}^{sup} = \underline{E} - \frac{4\pi}{3}\underline{P}^{sup} = \boxed{\underline{E} + \frac{4\pi}{3}\underline{P}}$$

<u>note</u> : we are ignoring dipole contribution!

<u>EXAMPLE</u> : MIDTERM w/ DIELECTRIC



$\cancel{\sum (\ldots) P_\ell}$

$\sum \left(C_\ell r^\ell + D_\ell \frac{1}{r^{\ell+1}} \right) P_\ell$

$\sum A_\ell^< r^\ell P_\ell$

<u>BC</u> : CONTINUITY of $E_{\parallel}$: Φ IS CONTINUOUS

DISCONTINUITY of $D_\perp$: $\varepsilon^< \frac{\partial \Phi^<}{\partial r} - \varepsilon^> \frac{\partial \Phi^>}{\partial r} = 4\pi\sigma$

<u>START @ BOUNDARY r = b</u> , SIMPLEST.

$$\Phi(b) = 0 \Rightarrow \boxed{D_\ell = -C_\ell\, b^{2\ell+1}}$$

$$\Phi = \sum C_\ell \left(r^\ell - b^{2\ell+1}/r^{\ell+1} \right) P_\ell$$

<u>NEXT SIMPLEST: CONTINUITY @ r = a</u>

$$\Phi^<(a) = \Phi(a) \Rightarrow A_\ell a^\ell = C_\ell \left(a^\ell - \frac{b^{2\ell+1}}{a^{\ell+1}} \right)$$

$$\boxed{A_\ell = C_\ell \left(1 - \left(\frac{b}{a}\right)^{2\ell+1} \right)}$$

so far: <u>SAME!</u>

HARD BC

$@\ r=a\ ,\quad \underline{\underline{\varepsilon\partial_r\phi(a)}} - \partial_r\phi^<(a) = -4\pi\sigma(\theta)$

$$\sum_\ell \varepsilon C_\ell\left[\ell a^{\ell-1}+(\ell+1)\frac{b^{2\ell+1}}{a^{\ell+2}}\right]P_\ell - \sum_\ell C_\ell\left[1-\left(\frac{b}{a}\right)^{2\ell+1}\right]\ell a^{\ell-1}P_\ell$$

$$= -4\pi\sigma_0\left[\frac{4}{3}P_0 + P_1 + \frac{2}{3}P_2\right]$$

ℓ^{th}:

$$\underline{\underline{\varepsilon C_\ell \ell a^{\ell-1}}} + \varepsilon C_\ell(\ell+1)\frac{b^{2\ell+1}}{a^{\ell+2}} - \underline{C_\ell \ell a^{\ell-1}} + C_\ell \ell\frac{b^{2\ell+1}}{a^{\ell+2}} = \cdots$$

$$C_\ell\left[(\varepsilon+1)\ell+1\right]\frac{b^{2\ell+1}}{a^{\ell+2}} + (\varepsilon-1)C_\ell \ell a^{\ell-1} = \cdots$$

SO MATCHING COMPONENTS :

$$C_0\frac{b}{a^2} = -4\pi\sigma_0\frac{4}{3} \longrightarrow \boxed{C_0 = (-4\pi\sigma_0)\frac{4}{3}\frac{a^2}{b}}$$

$$C_1\left[(\varepsilon+1)+1\right]\frac{b^3}{a^3} + (\varepsilon-1)C_1 = (-4\pi\sigma_0)$$

$$\hookrightarrow C_1\left[(\varepsilon+2)b^3/a^3 + (\varepsilon-1)\right] = -4\pi\sigma_0$$

$$\boxed{C_1 = -4\pi\sigma_0\left[(\varepsilon+2)b^3/a^3 + (\varepsilon-1)\right]^{-1}}$$

$$C_2\left[(\varepsilon+1)2+1\right]\frac{b^5}{a^4} + (\varepsilon-1)C_2 2a = -4\pi\sigma_0\frac{2}{3}$$

$$\boxed{C_2 = -4\pi\sigma_0\frac{2}{3}\left[(2\varepsilon+3)b^5/a^4 + 2(\varepsilon-1)a\right]^{-1}}$$

INDUCED CHARGE @ $r=b$

$$-\partial_r \phi(b) = -4\pi\tilde{\sigma} \leftarrow \text{INDUCED} \qquad \rightarrow \quad \tilde{\sigma} = \frac{\partial_r \phi(b)}{4\pi}$$

$$\partial_r \phi(r) = \Sigma\, C_\ell \left[\ell r^{\ell-1} + (\ell+1)\frac{b^{2\ell+1}}{r^{\ell+2}} \right] P_\ell$$

$$\frac{\partial_r \phi(b)}{4\pi} = -\sigma_0 \Big\{ \frac{4}{3}\frac{a^2}{b}\left(0 + \frac{1}{b}\right) P_0$$

$$+ \left[(\varepsilon+2)\frac{b^3}{a^3} + (\varepsilon-1) \right]^{-1} (1+2)\ P_1$$

$$+ \frac{2}{3}\left[(2\varepsilon+3)\frac{b^5}{a^4} + 2(\varepsilon-1)\,a \right]^{-1} P_2 \Big]$$

UUUGLY! but can see: ε affects different multipoles differently.