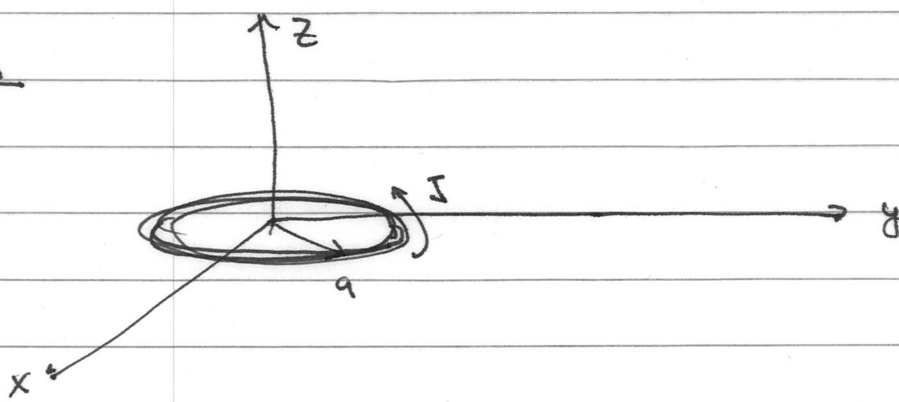
SECTION 7 12 OCT 2012

ANNOUNCEMENTS

- BETHE LECTURES NEXT WEEK
- ADVICE REQ for POSTDOCS: great American frat party

TODAY: BY REQUEST, focus MAINLY ON EXAMPLES.

eg 1



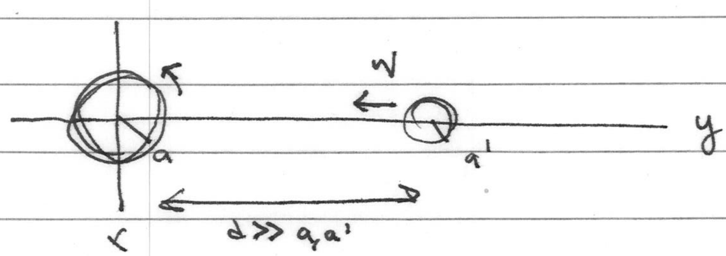
a) WHAT is THE MAGNETIC FIELD FAR AWAY?,
⇒ JUST THE DIPOLE TERM

$$\underline{m} = \frac{I\pi a^2}{c}\hat{z}$$

$$\underline{B} = \frac{1}{r^5}\left[3(\underline{m}\cdot\underline{r})\underline{r} - \underline{m}r^2\right]$$

$$\text{cf} \quad \underline{E}^{(2)} = \frac{1}{r^5}\left(3(\underline{p}\cdot\underline{r})\underline{r} - \underline{p}r^2\right)$$

b) NOW ADD SECOND CONDUCTING LOOP w/ radius a'
MOVING TOWARDS FIRST LOOP w/ VELOCITY $\underline{v} = -v\hat{y}$
; $v \ll c$.



WHAT IS EMF IN MOVING LOOP?

ALONG xy PLANE:

$$\underline{B} = \frac{1}{r^5}\left[3\cancel{(\underline{m}\cdot\underline{r})}\,\underline{r} - \underline{m}r^2\right]$$

0

$$= \frac{-m}{r^3} \longleftarrow \quad m = I\pi a^2/c$$

SO IF 2^{ND} LOOP CENTERED @ $s\hat{y}$ $\quad (s \gg a^{(\prime)})$

THEN THE FLUX IS

$$\Phi_B(s) = \int_S \underline{B}(s')\cdot dA = \cancel{\ldots}$$

$$= -m\int_S \frac{dA}{(s')^3} \longleftarrow \cancel{\ldots}$$

INTEGRAL OVER SURFACE
CENTERED @ s

$\longrightarrow$ to leading order:

$$\Phi_B(s) = -m\cdot\frac{\pi(a')^2}{s^3} = \frac{-I\pi^2 a^2 (a')^2}{c\,s^3}$$

NOW USE $s = \text{const} - vt$

$$\mathcal{E} = \frac{1}{c}\frac{\partial \Phi_B(s)}{\partial t} = \boxed{3I\pi^2\frac{v}{c^2}\frac{a^2(a')^2}{s^4}}$$

c) SUPPOSE THE SMALLER LOOP HAS RESISTANCE R.
WHAT IS THE POWER DISSIPATED IN THE LOOP?

$$P = \frac{\mathcal{E}^2}{R} \qquad \text{(OHM'S LAW)}$$

$$= \frac{9I^2\pi^4}{R}\left(\frac{v}{c^2}\right)^2\left(\frac{qa'}{s^2}\right)^4$$

d) WHAT IS THE FORCE ON THE LOOP?

MAGNETIC FIELDS DO NO WORK.
SO POWER MUST BE DISSIPATED AS
A CHANGE IN KINETIC ENERGY. (SEE GRIFFITHS)

$$d(KE) = \underline{F}\cdot d\underline{x} = F\,dy$$

$$\underline{F} = q\underline{E} + \frac{q}{c}\underline{u}\times\underline{B}$$

then: $P = \frac{d(KE)}{dt} = F\frac{dy}{dt} = -Fv \leftarrow \underline{v} = -v\hat{y}$

[WE ASSUME LOOP'S VELOCITY IS BEING KEPT CONST
BY AN EXTERNAL FORCE]

$$\Rightarrow F = \frac{9I^2\pi^4}{R}\frac{v}{c^4}\left(\frac{qa'}{s^2}\right)^4 \qquad \text{in the } \hat{y} \text{ DIR.}$$

HTM 4.14 COULOMB GAUGE

$$\nabla \cdot \underline{A} = 0$$ USEFUL WHEN NO NET CHARGE.

(a) CLAIM: SCALAR SATISFIES POISSON EQ.

Gauss' law:
$$4\pi\rho = \nabla \cdot \underline{E}$$
$$= \nabla \cdot \left(-\nabla\phi - \tfrac{1}{c}\dot{\underline{A}}\right) \quad \nabla \cdot A = 0$$
$$= -\nabla^2\phi \quad \checkmark$$

(b) WHAT is EQN for $\underline{A}$?

Ampere $\Rightarrow$
$$\nabla \times B = \frac{4\pi}{c} J - \frac{1}{c}\partial_t\left(\nabla\phi - \frac{1}{c}\partial_t A\right) = \nabla \times \nabla \times A$$
$$= \nabla(\underbrace{\nabla \cdot A}_{0}) - \nabla^2 A$$

$$\Rightarrow \underbrace{\nabla^2 A - \frac{1}{c^2}\ddot{A}}_{\text{wave eqn.}} = \frac{1}{c}\nabla\dot{\phi} - \frac{4\pi}{c} J$$

(c) w/o loss, WRITE: $\underline{J} = \underline{J}_1 + \underline{J}_2$ w/

$\nabla \times J_1 = 0$ LONGITUDINAL

$\nabla \cdot J_2 = 0$ TRANSVERSE

CLAIM: $\nabla\dot{\phi} = 4\pi J_1$ INDEP of ϕ