PS327 FALL 2012 SECTION 6 28 SEPT 2012

→ WRITTEN DURING PRELIM I... THIS IS THE MOST SERIOUS THAT I'VE EVER SEEN YOU GUYS.

HOUSE KEEPING

I. PRELIM — COMMENTARY

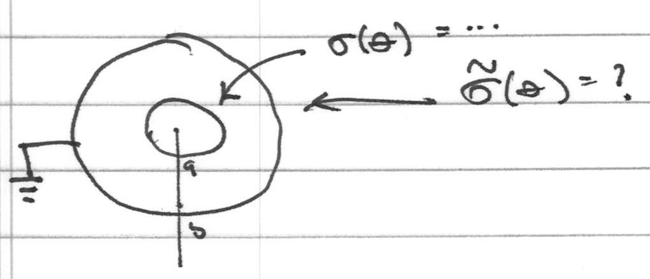
REGRADE POLICY: REVIEW EXAM NOW

II. "I got a C in analytical mechanics & was very happy about it" — THOMAS B.

PROBLEM #2 INTUITION

You could have solved part (f) based on physical intuition (IF YOU'RE VERY CLEVER)

way more clever than me!



$P_0(\cos\theta)$, etc.

FULL CREDIT FOR PART (a): $\sigma(\theta) = \sigma_0 P_0 + \sigma_1 P_1 + \sigma_2 P_2$

THEN WE EXPECT $\tilde{\sigma} = \tilde{\sigma}_0 P_0 + \tilde{\sigma}_1 P_1 + \tilde{\sigma}_2 P_2$

BUT YOU CAN GET MORE INTUITION FOR THE VALUES of $\tilde{\sigma}_i$ IN TERMS of σ_i, a, b.

inside

<u>PHYSICS</u> : TOTAL INDUCED CHARGE IS $\boxed{Q_{ind} = -Q}$

$$Q_{tot} = \int d^3s\, \rho(\underline{s}) \leftarrow \rho(s) = \sigma(\theta)\,\delta(s-a)$$

$$\int d\Omega\, s^2 ds$$

$$= \boxed{a^2} \int d\Omega\, \sigma(\theta)$$

DEPENDS ON RADIUS OF THE SPHERE AS A <u>SQUARE</u>

GIVEN DENSITY, TOTAL CHARGE

why? DEPENDS ON <u>SURFACE AREA</u>.

$$Q_{ind} = b^2 \int d\Omega\, \tilde{\sigma}(\theta)$$

TOTAL CHARGE COMES ONLY FROM <u>MONOPOLE TERM</u>

THUS WE MUST HAVE

$$\tilde{\sigma}_0 = -\left(\frac{a^2}{b^2}\right)\sigma_0$$

s.t. $Q_{ind} = -Q$

<u>PHYSICAL ORIGIN</u>: SURFACE AREA OF EA SPHERE.

@ this point, may guess: $\tilde{\sigma}_i = -\left(\frac{a^2}{b^2}\right)\sigma_i \quad \forall i$

(THAT'S WHAT I GUESSED)

$\hookrightarrow$ more subtle!

<u>DIPOLE</u>: $$\underline{p} = \int d^3\underline{s}\ \underline{s}\ \rho(\underline{s})$$
$$= \int d\Omega\, s^2\ \underline{s}\ \sigma(\theta)$$

SURFACE AREA

$\underline{s} = s\hat{e}_s$

ADDITIONAL RESCALING!

IN ADDITION TO SURFACE AREA FACTOR, THE DIPOLE KNOWS ABOUT SPATIAL CHARGE SEPARATION — THIS IS AN ADDITIONAL FACTOR OF LENGTH RESCALING

LOGIC: CAN PROJECT OUT $P_1(\cos\theta)$ TERM.
SAME AS MONOPOLE: NO DIPOLE OUTSIDE THE GROUNDED SPHERE, SO $\tilde{\sigma}_1$ HAD BETTER CANCEL σ_1.

SINCE $\underline{p} \sim a^3$, $\underline{p}_{ind} \sim b^3$.
NEED $$\boxed{\tilde{\sigma}_1 = -\left(\frac{a}{b}\right)^3 \sigma_1}$$

<u>QUADRUPOLE</u>: SAME SCHTICK!

$$Q_{ij} \sim \int d^3\underline{s}\ \underbrace{(3s_is_j - s^2\delta_{ij})}\ \rho(\underline{s})$$

$\sim d\Omega\, s^2$ AS USUAL

I DON'T CARE ABOUT EXACT FORM
ALL THAT MATTERS IS THAT IT DEPENDS ON s AS $\boxed{\sim s^2}$

$$\boxed{Q_{ij} \sim a^4} \Rightarrow Q_{ij}^{ind} \sim b^4 \Rightarrow \boxed{\tilde{\sigma}_2 = -\left(\frac{a}{b}\right)^4 \sigma_2}$$

IN FACT, YOU CAN SEE THAT THIS WOULD WORK FOR EVERY TERM IN THE MULTIPOLE MOMENT

IN GENERAL: GIVEN $\sigma(\theta) = \Sigma \sigma_\ell P_\ell(\cos\theta)$ @ $r=a$,
THE INDUCED CHARGE DENSITY @ $r=b$ IS:

$$\boxed{\tilde{\sigma}(\theta) = -\left(\frac{a}{b}\right)^2 \Sigma \left(\frac{a}{b}\right)^\ell \sigma_\ell P_\ell(\cos\theta)}$$

EXERCISE: WHAT IF WE LIVED IN 5 DIMENSIONS?
(OR BETTER: d DIMENSIONS)

then $\int d^3s \longrightarrow \int d^ds \sim \underbrace{\int d\Omega_d \, s^{d-1} \, ds}_{\text{BY DIM. ANALYSIS!}}$

BUT MONOPOLE IS STILL INDEP OF SCALING.
DIPOLE STILL SCALES W/ ADDITIONAL FACTOR OF s
QUAD ——— " ——— s^2, etc.

$$\boxed{\underset{\text{IN } d \text{ DIM}}{\tilde{\sigma}(\theta)} = -\left(\frac{a}{b}\right)^{d-1} \Sigma \left(\frac{a}{b}\right)^\ell \sigma_\ell P_\ell(\cos\theta)}$$

General Exam comments

- ORDER MATTERS!
  ↳ ESP IN BC PROBLEMS

PROBLEM 2: $\Phi_{mid}(b) = 0$ RELATES 2 COEF.

$\phi_{in}(a) = \phi_{mid}(b)$ RELATES 3 COEF.

EASIER!

$\underbrace{\partial_r \phi_{in}(a) - \partial_r \phi_{mid}(a)}$ $= -4\pi\sigma$

UGLY ... EASIER WHEN
MUCH FEWER UNDET. COEF!

- TIMING – MY APOLOGIES.

→ WRITE EQNS! QUALITATIVE = NICE

QUANTITATIVE = BETTER.

REMARKS: MAGNETISM

ELECTROSTATICS

$$\nabla \cdot E \sim \rho$$
$$\nabla \times E = 0$$

$\Longrightarrow$

MAGNETOSTATICS

$$\nabla \cdot B = 0$$
$$\nabla \times B \sim j$$

RELATION TO ELECTRO?

$$\boxed{\dot{\rho} + \nabla \cdot j = 0}$$
$$\Downarrow$$
$\nabla \cdot j = 0$ for STATIC config

GAUGE TRANSFORMATION

$$\nabla \cdot B = 0 \Rightarrow B = \nabla \times A$$
$$\nabla \times B = 0 \Rightarrow (\nabla \times)^2 A \sim j$$

CHECK: $= \nabla \underbrace{(\nabla \cdot A)}_{\text{SCALAR}} - \nabla^2 A$ $\leftarrow$ $\boxed{\text{UGLY!!}}$

BUT: IF PHYSICAL QUANTITY is $\underline{B}$, THEN WE CAN SHIFT:

$$\underline{A} \rightarrow \underline{A}' = \underline{A} + \underline{\nabla} \chi$$

w/o CHANGING $\underline{B}$

one piece of info

IN PARTICULAR, CAN CHOOSE χ s.t. $\nabla \cdot \underline{A}' = 0$

$$\nabla \cdot \underline{A}' = \nabla \cdot A + \nabla^2 \chi \Rightarrow \underbrace{\nabla^2 \chi = -\nabla \cdot \underline{A}}_{\text{POISSON}}$$
$$\Downarrow$$
$$\chi \sim \int d^3 s \, \frac{\nabla \cdot A(\underline{s})}{|\underline{r} - \underline{s}|}$$

END UP w/ MUCH SIMPLER :

$$\nabla^2 \underline{A}' \approx j$$

$$\nabla^2 A_i \approx j_i \qquad \text{POISSON FOR EACH COMP.}$$

$$\Rightarrow \quad A(\underline{r}) \approx \int d^3\underline{s}\, \frac{j(\underline{s})}{|\underline{r}-\underline{s}|}$$

FUN STUFF TO DISCUSS :

MEANING OF GAUGE REDUNDANCY $\chi(x)$!

HW 6 , USEFUL DATA

$$R_{EARTH} = 6.37 \times 10^8 \text{ cm}$$

CAREFUL: