COURSE INFO: P3327 ADVANCED E&M

LECTURER: MAXIM PERELSTEIN ← away, will return FRIDAY
TA: FLIP TANEDO ← this is me
GRADER: SHIVAM GOSH

SECTION: F 2:30-3:20pm R321

SEE SYLLABUS FOR MAIN INFO; MAXIM WILL GO OVER IT ON FRIDAY.

FOR NOW, SOME NUTS & BOLTS:

① REGISTER FOR THIS CLASS (OTHERWISE NO ACCESS TO BLACKBOARD)

② 327 (ADV E&M) VS. 323 (INT E&M)

for physics concentrators — eg. main academic focus is physics

eg. 2x majors not focusing on physics

→ REFER TO ADVICE FROM DUS, etc.

IF YOU'RE NOT SURE WHICH COURSE TO TAKE, FEEL FREE TO SAMPLE BOTH. SEE MAXIM IF YOU HAVE PROBLEMS REGISTERING FOR BOTH.

③ TEXTBOOK → OUT OF PRINT! RELEVANT CHAPTERS WILL BE PUT ON THE BLACKBOARD SITE.

feel free to look for used copies, other ...

④ HOMEWORK & OFFICE HRS → MONDAY 5:15-7:15, PSB 470 "HW PARTY STYLE"

HW1 POSTED, DUE WED. IN CLASS. (50% CREDIT IF TURNED IN FRIDAY IN SECTION)

GENERAL ADVICE: PHYSICS IS A PARTICIPATION SPORT. WORK TOGETHER ON HWs (& TAKE THEM SERIOUSLY), LEARN TO DISCUSS PHYSICS w/ COLLEAGUES.

⑤ MATHEMATICA: VERY USEFUL TOOL; WE WILL HAVE TUTORIAL SESSIONS STARTING ~WK 2. I SUGGEST BUYING THE STUDENT SOFTWARE LICENSE SO YOU CAN USE IT PAST THIS SEMESTER.

THIS COURSE: "INTRO TO FIELD THEORY"
ELECTROMAGNETIC

- THIS IS ONE OF THE FIRST "REAL" PHYSICS COURSES THAT WILL TIE INTO MOST (ALL?) OF YOUR FUTURE STUDIES.

Q'S: WHAT IS THE EM FIELD & WHAT ARE ITS PROPERTIES?
HOW DOES IT BEHAVE IN DIFFERENT MATERIALS, SETTINGS, ETC?
HOW IS RELATIVITY BUILT INTO EM?

YOU ALL HAVE ALREADY MET MAXWELL'S EQNS IN SOME FORM

↳ you already "know" the main part of this course

BUT THE GOAL IS TO DEVELOP SOME FAMILIARITY & SOPHISTICATION W/ THE LANGUAGE & TOOLS THAT BOLSTER THE IDEAS.

the MATH

YOU'RE EXPECTED TO HAVE SEEN LOTS OF THE MATH ALREADY
↳ VECTOR CALCULUS, DIFFERENTIAL EQS, & ANALYSIS..

BUT THIS IS WHERE WE PUT IT ALL TOGETHER & USE THEM TO UNDERSTAND PHYSICS.

note: THE ONLY WAY TO REALLY GET A FEEL FOR ALL OF THIS IS TO WORK THROUGH IT. TAKE YOUR HW SERIOUSLY!

→ THE INTUITION THAT YOU DEVELOP HERE WILL CARRY OVER TO THE REST OF YOUR PHYSICS (+ OTHER?) LIFE.

TODAY: VECTOR CALCULUS REVIEW (tbc in section Fri)

- ASK QUESTIONS!
- MY GOAL IS TO ASSESS YOUR FAMILIARITY W/ THESE IDEAS; WE'LL REVIEW AS NECESSARY IN SECTION

VECTORS : WE WILL ASSUME 3-DIMENSIONS

CARTESIAN COORDINATES:

$$\underline{V} = V_x \hat{x} + V_y \hat{y} + V_z \hat{z} \leftrightarrow \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}$$

$\mathbf{V}$ OR $\vec{V}$

$\hat{e}_x$ OR $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, UNIT BASIS VECTOR

eg can also use radial vector

$$\underline{r} = x\hat{x} + y\hat{y} + z\hat{z} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\hat{r} = \frac{\underline{r}}{r} = \frac{\underline{r}}{\sqrt{x^2+y^2+z^2}}$$

no underline:
SCALAR QUANTITY
$= |\underline{r}|$, length of vector

remark: $\hat{r}$ WILL BE IMPORTANT AS A BASIS VECTOR IN SPHERICAL COORDINATES.

WHAT IS A VECTOR? "$\underbrace{\text{MAGNITUDE \& DIRECTION}}_{\text{usual response}}$"

BETTER: AN OBJECT WHICH TRANSFORMS IN A SPECIFIC WAY UNDER SPATIAL ROTATIONS.

WE WILL GENERALIZE LATER IN THIS COURSE TO SPACE-TIME "ROTATIONS"; IN OTHER COURSES YOU'LL SEE EVEN MORE GENERAL 'ROTATIONS'.

eg: rotation about the $\hat{z}$ AXIS by $\theta$
IN MATRIX NOTATION:

$$\begin{pmatrix} V_x' \\ V_y' \\ V_z' \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{rotation matrix, } R} \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}$$

A MORE USEFUL WAY OF WRITING THIS:

$$V'_i = \sum_{j=1}^{3} R_{ij} V_j \quad \leftarrow \quad i=1 \leftrightarrow x,\ i=2 \leftrightarrow y, \text{ etc}$$

eg $R_{11} = \cos\theta$, $R_{12} = \sin\theta$, etc.

sometimes we omit the explicit $\Sigma$ symbol (EINSTEIN SUMMATION NOTATION – repeated indices are assumed to be summed over)

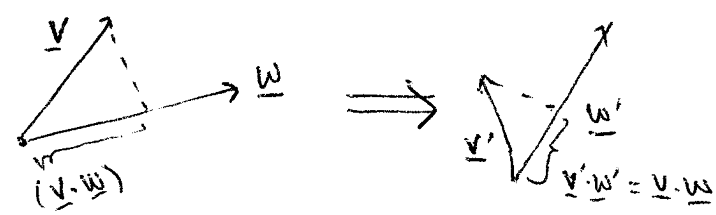
HOW TO THINK ABOUT THIS: A <u>VECTOR</u> IS AN OBJECT WITH ONE INDEX. ROTATIONS ACT ON THIS INDEX W/ A ROTATION MATRIX ($R_{ij}$). THE INDEX ENCODES HOW THE OBJECT TRANSFORMS.

YOU CAN ALSO CONSIDER OBJECTS W/ NO INDICES: <u>SCALARS</u>

eg: # of STUDENTS IN THE ROOM IS INVARIANT UNDER ROTATIONS

BUT THERE ARE OTHER WAYS TO FORM OBJECTS W/ NO INDICES!

↳ DOT PRODUCT / INNER PRODUCT of VECTORS




HOW WE WROTE THIS IN KINDERGARDEN:

$$\underline{V} \cdot \underline{W} = \underbrace{\text{"}\underline{V}^T\text{"}\ \underline{W}}_{\text{"matrix mult"}} = \underbrace{(V_x\ V_y\ V_z)}_{\text{ROW VECTOR}} \begin{pmatrix} W_x \\ W_y \\ W_z \end{pmatrix}$$

Q: HOW DOES A ROW VECTOR TRANSFORM UNDER ROTATIONS?

$$\underline{V} \longrightarrow \underline{V}' = \underline{RV} \quad \leftarrow \quad \begin{pmatrix} c\theta & s\theta & 0 \\ -s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}$$

$$\Rightarrow \underline{V}^T \longrightarrow \underline{V}^{T\prime} = (\underline{RV})^T$$

$$= \underline{V}^T R^T \quad \longleftarrow \quad (V_x\ V_y\ V_z) \begin{pmatrix} c\theta & s\theta & 0 \\ -s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

SO COLUMN VECTORS: TRANSF BY MULT $R$ FROM LEFT
ROW VECTORS: TRANSF BY MULT $R^T$ FROM RIGHT

DOES THIS MAKE SENSE?

$$\underline{V} \cdot \underline{W} \rightarrow \underline{V}^T \underbrace{R^T R}\, \underline{W}$$

← using this silly "matrix" multiplication notation

PROPERTY of ROTATION MATRICES: $R^T = R^{-1}$
SO THIS PRODUCT IS $\mathbb{1}$.

IN "GROWN UP" NOTATION:

check that this is $\mathbb{1}_{ij}$

$$V^i W_i = \text{"}V_i W_i\text{"} \rightarrow V_i (R^T)_{ik} R_{kj} W_j = V_i \boxed{R_{ki} R_{kj}}\, W_j$$

implied sum

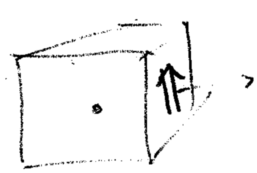
UPPER INDEX TO DENOTE $V^T$
DON'T WORRY, WE'LL GO INTO THIS WHEN WE DO RELATIVITY

for those w/ some linear algebra: col & row vectors should be thought of as vectors & <u>DUAL VECTORS</u>

IN ADDITION TO SCALARS & VECTORS, THERE ARE MORE GENERAL OBJECTS WITH MULTIPLE INDICES: TENSORS. (2 INDICES → MATRIX)

eg. later in this course you'll meet maxwell's stress tensor

$T_{ij}$ : the FORCE/AREA ACTING IN THE $i^{th}$ DIRECTION acting on a surface in the $j^{th}$ DIRECTION

$T_{zx}$ ~ 

HOW DOES THIS TRANSFORM UNDER A ROTATION?

$$T_{ij} \rightarrow T'_{ij} = R_{ii'} R_{jj'} T_{i'j'}$$

ROTATES DIR of THE FORCE

ROTATES DIR of THE PLANE

SANITY CHECK: HOW DO MATRICES TRANSFORM UNDER A CHANGE of BASIS? [need to be careful about row vs col indices]

REMARKS ('for CULTURE')

THE FUNDAMENTAL PRINCIPLE HERE (and in all of physics) is SYMMETRY.

in this case, rotational symmetry

@ THIS LEVEL: THE LAWS of NATURE ARE INVARIANT UNDER ROTATIONS.

⇒ OBJECTS (actual or mathematical) MUST BE COVARIANT, ie transform in a particular way.

[this is so intuitively obvious that you probably never had to actually say it.]

this is not true for other symmetries!

eg: CP sym: SWAP PARTICLES w/ ANTIPARTICLES [not obvious.]

SCALE INVARIANCE: physics @ galactic scales is very different from physics @ subatomic scales.

LATER IN THIS COURSE: EXTEND TO SPACE-TIME SYMMETRY.

↳ Relativity.

then we work with spacetime vectors w/ 4 COMPONENTS.

& the idea of a 3-vector becomes worthless (it's not even covariant!)

## CALCULUS ON VECTOR SPACES

GRADIENT: $\underline{\nabla} = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$

... that's all there is to it. NOW THERE'S A BUNCH of THINGS YOU CAN DO w/ $\underline{\nabla}$:

- DIRECTIONAL DERIVATIVE: $\underline{v} \cdot \underline{\nabla}$

  "how much is a function changing as i head in the $\underline{v}$ direction?"

  → note $(\underline{v} \cdot \underline{\nabla})$ is A SCALAR OPERATOR

- GRADIENT of A FUNCTION: $\underline{\nabla} f(x)$

  takes a function, returns vector of derivatives

- DIVERGENCE: $\underline{\nabla} \cdot \underline{V}(x)$

  takes vector function, returns scalar
  YOU SHOULD ALREADY KNOW THAT THIS IS INTERPRETED AS "SOURCE-Y-NESS"

- CURL: $\underline{\nabla} \times \underline{V}(x)$

  takes vector function, returns vector
  "circulation"

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SANITY CHECKS: MAKE SURE YOUR OPERATORS MAKE SENSE

eg: $\underline{\nabla} \times \underline{\nabla} \cdot \underline{V}(x)$ no!

$\underline{\nabla} \times f(x)$ no!

$\underline{\nabla} \cdot \underline{\nabla} f$ → yes! LAPLACIAN

can also form things like the Hessian...

What about: $\underline{v} \times \underline{\nabla}$ ? TECHNICALLY ok, BUT NOT CLEAR WHY IT'S USEFUL. NOTE:

$$\underline{v} \times \underline{\nabla} \neq -\underline{\nabla} \times \underline{v} \quad !$$

REMARK: CURVILINEAR COORDINATES

WE CAN WRITE OUR VECTORS IN OTHER ORTHONORMAL COORDINATE SYSTEMS

- CYLINDRICAL → eg. COAXIAL CABLES, SHEETS
- SPHERICAL → eg. POINT CHARGES, SHELLS

MAKES LIFE MUCH EASIER WHEN A SYSTEM HAS PARTICULAR SYMMETRIES.

OUR CONVENTIONS:

CYLINDER: $d\underline{l} = d\rho\,\hat{\rho} + \rho\, d\phi\,\hat{\phi} + dz\,\hat{z}$

SPHERE: $d\underline{l} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\, d\phi\,\hat{\phi}$

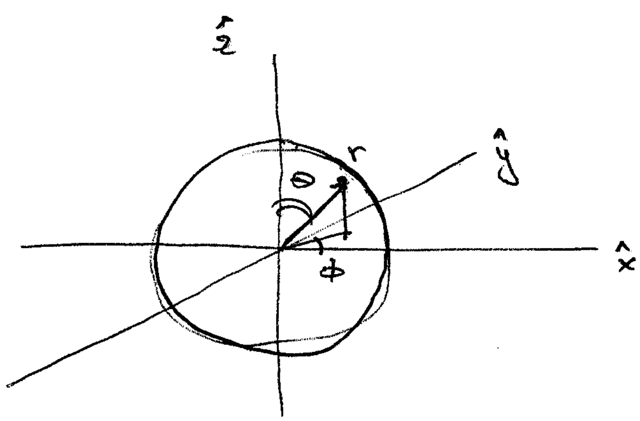
YOU SHOULD FAMILIARIZE YOURSELF WITH THE EXPRESSIONS FOR $\underline{\nabla}, \underline{\nabla}\cdot, \underline{\nabla}\times, \nabla^2$ IN THESE COORDINATES

→ don't have to memorize, but know how to derive them!

ALSO: UNDERSTAND WHY THEY LOOK SO DIFFERENT FROM CARTESIAN COORDINATES!

→ "position-dependent metric"

IF YOU DON'T KNOW HOW TO DO THIS, I RECOMMEND SPENDING TIME REVIEWING IT.



## INTEGRATION

NOW "MANY" OPTIONS : 1D INTEGRAL: $\int_a^b \underline{V}(r) \cdot d\underline{l}$ ← differential LINE ELEMENT $= dx\,\hat{x} + \cdots$

↳ typically PATH DEPENDENT

PATH CONSTRAINS, SAY, $z$ & $y$ TO BE FUNCTIONS OF $x$, SO INTEGRAL IS REALLY ONE DIMENSIONAL.

2D INTEGRAL: $\int_S \underline{V}(r) \cdot d\underline{a}$ ← diff. surface elem char by. normal vector. eg $d\underline{a} = dx\,dy\,\hat{z}$

3D INTEGRAL: $\int_V f(r)\, dV$ ← diff volume elem $= dx\,dy\,dz$

REMARK: IN DIFF COORDINATES, THE "differential — element" TAKES DIFFERENT FORMS!

$$dx\,dy\,dz \neq dr\,d\theta\,d\phi$$

[eg: dimensional analysis]

↘ $r^2 dr\, d(\cos\theta)\, d\phi$

THERE ARE A BUNCH OF INTEGRATION THEOREMS eg GREEN'S THM, STOKES' THM...

→ THEY'RE ALL THE SAME.

fancy formulation: [GENERAL STOKES' THM]: $\int_{\partial \Omega} \omega = \int_\Omega d\omega$

(differential form → $\omega$; BOUNDARY → $\partial\Omega$)

~~[illegible]~~

~~[illegible]~~

~~IS EQUAL TO~~

~~INTEGRAL OVER THE~~

[should be familiar to those who know differential geometry — we'll explain heuristically for those who don't]

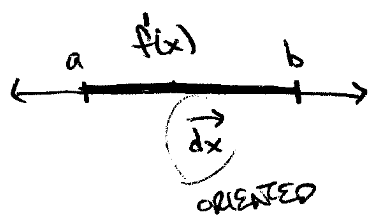
**MORE COLLOQUIALLY:**

THE INTEGRAL OF AN OBJECT $\omega$ OVER THE ORIENTED BOUNDARY OF A REGION "$\partial\Omega$"

IS EQUAL TO

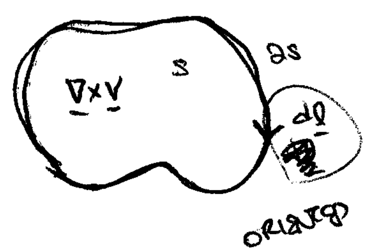
THE INTEGRAL OF THE DERIVATIVE OF THAT OBJECT $d\omega$ OVER THE ENTIRE REGION $\Omega$.

eg. FUNDAMENTAL THM OF CALCULUS: $\Omega$ = INTERVAL IN 1D




$$\int_a^b \underbrace{dx\, f'(x)}_{df(x)} = f(x)\Big|_a^b$$

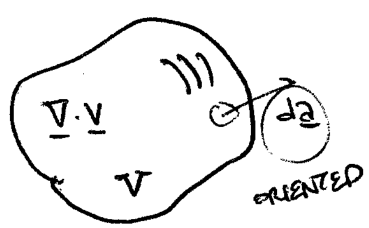
eg. GREEN'S THM




$$\int_S (\underline{\nabla} \times \underline{V}) \cdot d\underline{a} = \oint_{\partial S} \underline{V} \cdot d\underline{l}$$

WHY $\nabla \times$? GIVES VECTOR. ALSO, THE ANTISYMMETRY IS ACTUALLY PART OF THE DEF OF INTEGRAL! (diff. form)

eg. GAUSS' THM




$$\int_V (\underline{\nabla} \cdot \underline{V})\, d\tau = \oint_{\partial V} \underline{V} \cdot d\underline{a}$$

MAXWELL'S EQS, FOR EG

**PHYSICS:** PHYSICAL LAWS ARE DIFFERENTIAL EQS WHICH GOVERN SPACETIME DYNAMICS OF A SYSTEM.

THIS CONSTRAINS THE "DATA" OF A SYSTEM: GIVEN INFO ABOUT THE BOUNDARY, WE CAN LEARN WHAT'S HAPPENING IN THE BULK.

THE DIRAC DELTA FUNCTION

not really a function

$$\int_{-\infty}^{\infty} f(x)\,\delta(x)\,dx = f(0)$$

kills integrals.

FORMALLY: $\delta(x) = \begin{cases} 0 & \text{if } x \neq 0 \\ \infty & \text{if } x = 0 \end{cases}$ in such a way that...

WORKING W/ $\delta$: WHAT IS $\int_{-\infty}^{\infty} f(x)\,\delta\left(\underbrace{e^{2x-3} - 4}\right) dx$ ?

I ALWAYS FORGET THE RULES... JUST CHANGE VARS: $y = \cdots$

but: BE CAREFUL W/ ABSOLUTE VALUES.
SIGNS COME FROM INTERCHANGING $x = +\infty \rightarrow y = -\infty$.

eg: $\int_{-\infty}^{\infty} f(x)\,\delta(kx)\,dx = \frac{1}{|k|} f(0)$

MORE GENERALLY: $\delta^{(3)}(\underline{r}) = \delta(x)\,\delta(y)\,\delta(z)$

s.t. $\int_{\text{SPACE}} f(\underline{r})\,\delta^{(3)}(\underline{r})\,dx\,dy\,dz = f(0)$

YOU SHOULD ALREADY BE VERY FAMILIAR W/ $\delta$ FUNCTIONS AS THE ORIGIN OF THE "POINT ELECTRIC SOURCE"