I must warn you, now, of a small fraud that tarnishes many applications of Faraday's law: Electromagnetic induction, of course, occurs only when the magnetic fields are *changing*, and yet we would like to use the apparatus of magneto*statics* (Ampère's law, the Biot-Savart law, and the rest) to *calculate* those magnetic fields. Technically, any result derived in this way is only approximately correct. But in practice the error is usually negligible unless the field fluctuates extremely rapidly, or you are interested in points very far from the source. Even the case of a wire snipped by a pair of scissors (Prob. 7.18) is *static enough* for Ampère's law to apply. This régime, in which magnetostatic rules can be used to calculate the magnetic field on the right hand side of Faraday's law, is called **quasistatic**. Generally speaking, it is only when we come to electromagnetic waves and radiation that we must worry seriously about the breakdown of magnetostatics itself.

Example 7.9

An infinitely long straight wire carries a slowly varying current $I(t)$. Determine the induced electric field, as a function of the distance s from the wire.[10]

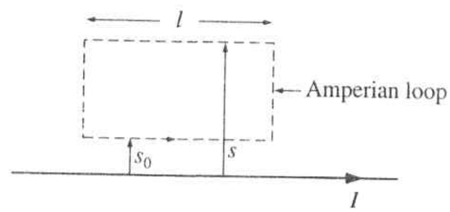

Figure 7.26

Solution: In the quasistatic approximation, the magnetic field is $(\mu_0 I/2\pi s)$, and it circles around the wire. Like the **B**-field of a solenoid, **E** here runs parallel to the axis. For the rectangular "Amperian loop" in Fig. 7.26, Faraday's law gives:

$$\begin{aligned}\oint \mathbf{E}\cdot d\mathbf{l} &= E(s_0)l - E(s)l = -\frac{d}{dt}\int \mathbf{B}\cdot d\mathbf{a} \\ &= -\frac{\mu_0 l}{2\pi}\frac{dI}{dt}\int_{s_0}^{s}\frac{1}{s'}\,ds' = -\frac{\mu_0 l}{2\pi}\frac{dI}{dt}(\ln s - \ln s_0).\end{aligned}$$

Thus

$$\mathbf{E}(s) = \left[\frac{\mu_0}{2\pi}\frac{dI}{dt}\ln s + K\right]\hat{\mathbf{z}}, \tag{7.19}$$

where K is a constant (that is to say, it is independent of s—it might still be a function of t). The actual *value* of K depends on the whole history of the function $I(t)$—we'll see some examples in Chapter 10.

[10]This example is artificial, and not just in the usual sense of involving infinite wires, but in a more subtle respect. It assumes that the current is the same (at any given instant) all the way down the line. This is a safe assumption for the *short* wires in typical electric circuits, but not (in practice) for *long* wires (transmission lines), unless you supply a distributed and synchronized driving mechanism. But never mind—the problem doesn't inquire how you would *produce* such a current; it only asks what *fields* would result if you *did*. (Variations on this problem are discussed in M. A. Heald, *Am. J. Phys.* **54**, 1142 (1986), and references cited therein.)

Equation 7.19 has the peculiar implication that E blows up as s goes to infinity. *That* can't be true ... What's gone wrong? *Answer:* We have overstepped the limits of the quasistatic approximation. As we shall see in Chapter 9, electromagnetic "news" travels at the speed of light, and at large distances $\mathbf{B}$ depends not on the current *now*, but on the current *as it was* at some earlier time (indeed, a whole *range* of earlier times, since different points on the wire are different distances away). If τ is the time it takes I to change substantially, then the quasistatic approximation should hold only for

$$s \ll c\tau, \tag{7.20}$$

and hence Eq. 7.19 simply does not apply, at extremely large s.

Problem 7.15 A long solenoid with radius a and n turns per unit length carries a time-dependent current $I(t)$ in the $\hat{\boldsymbol{\phi}}$ direction. Find the electric field (magnitude and direction) at a distance s from the axis (both inside and outside the solenoid), in the quasistatic approximation.

Problem 7.16 An alternating current $I = I_0 \cos(\omega t)$ flows down a long straight wire, and returns along a coaxial conducting tube of radius a.

(a) In what *direction* does the induced electric field point (radial, circumferential, or longitudinal)?

(b) Assuming that the field goes to zero as $s \to \infty$, find $\mathbf{E}(s, t)$. [Incidentally, this is not at all the way electric fields *actually* behave in coaxial cables, for reasons suggested in footnote 10. See Sect. 9.5.3, or J. G. Cherveniak, *Am. J. Phys.*, **54**, 946 (1986), for a more realistic treatment.]

Problem 7.17 A long solenoid of radius a, carrying n turns per unit length, is looped by a wire with resistance R, as shown in Fig. 7.27.

(a) If the current in the solenoid is increasing at a constant rate ($dI/dt = k$), what current flows in the loop, and which way (left or right) does it pass through the resistor?

(b) If the current I in the solenoid is constant but the solenoid is pulled out of the loop and reinserted in the opposite direction, what total charge passes through the resistor?

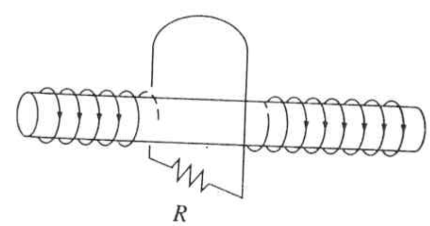

Figure 7.27