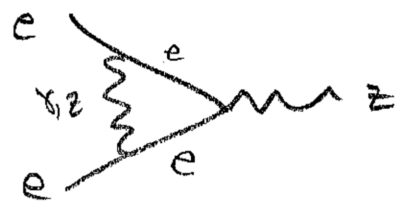
1.1

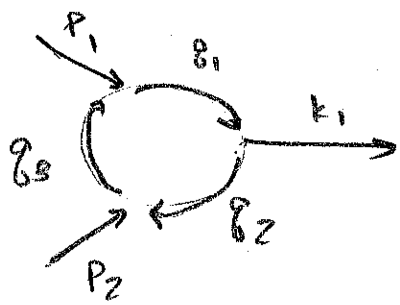


1.2 $(p_1+p_2)^\mu = k_1^\mu \rightarrow (E_1, \mathbf{p}) + (E_2, -\mathbf{p}) = (M_Z, \mathbf{0})$

$$E_1^2 = E_2^2 = |\mathbf{p}|^2 + M_e^2 = M_Z^2 \times \frac{1}{4}$$

$$\Rightarrow \boxed{|\mathbf{p}|^2 = \frac{1}{4} M_Z^2 - M_e^2}$$

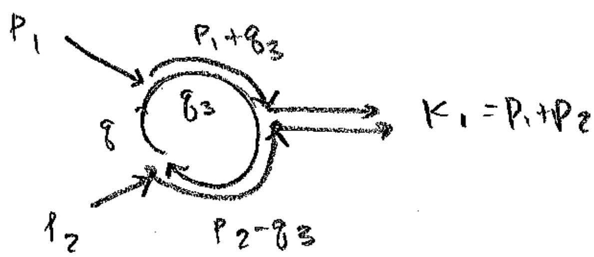
1.3



$$\begin{aligned} p_1 + q_3 &= q_1 \\ q_1 &= k_1 + q_2 \\ q_2 &= p_2 + q_3 \end{aligned}$$

$$\boxed{k_1 = p_1 + p_2}$$

constraint



$$\Rightarrow \boxed{\begin{aligned} q_1 &= p_1 + q_3 \\ q_2 &= p_2 - q_3 \\ q_3 &= \text{undetermined} \end{aligned}}$$

1.4. q_3 UNDETERMINED → HAVE TO SUM (INTEGRATE) OVER ALL ALLOWED VALUES (of "LOOP MOMENTUM")

effectively infinite number of terms to add

2.1 $(V')^1 = c V^1 + s V^2$
$(V')^2 = -s V^1 + c V^2$
$(W')_1 = c W_1 + s W_2$
$(W')_2 = -s W_1 + c W_2$

matches matrix multiplic.

2.2 THIS IS JUST AN EXPLICIT CALCULATION
WE'LL DO THE $i=1,\ j=2$ ELEMENT
AS AN EXAMPLE. THE OTHERS ARE IDENTICAL
IN METHODOLOGY.

TENSOR NOTATION:

$$(M')^1{}_2 = (R^{-1})^k{}_2 \, R^1{}_l \, M^l{}_k$$

$$\begin{aligned}
&= (R^{-1})^{k=1}{}_2 \, R^1{}_{l=1} \, M^{l=1}{}_{k=1} \\
&+ (R^{-1})^{k=1}{}_2 \, R^1{}_{l=2} \, M^{l=2}{}_{k=1} \\
&+ (R^{-1})^{k=2}{}_2 \, R^1{}_{l=1} \, M^{l=1}{}_{k=2} \\
&+ (R^{-1})^{k=2}{}_2 \, R^1{}_{l=2} \, M^{l=2}{}_{k=2}
\end{aligned}$$

$$\begin{aligned}
&= (+s)(c)\, m^1{}_1 \\
&\quad (-s)(s)\, m^2{}_1 \\
&\quad (c)(c)\, m^1{}_2 \\
&\quad (c)(-s)\, m^2{}_2
\end{aligned}$$

signs of $(R^{-1})^1{}_2$ $(R^{-1})^2{}_1$ wrong in eq (B)

$$= \boxed{-cs\, m^1{}_1 - s^2 m^2{}_1 + c^2 m^1{}_2 + cs\, m^2{}_2}$$

COMPARE TO MATRIX MULTIPLICATION

$$\begin{pmatrix} c & -s \\ s & c \end{pmatrix} \begin{pmatrix} m^1{}_1 & m^1{}_2 \\ m^2{}_1 & m^2{}_2 \end{pmatrix} \begin{pmatrix} c & s \\ -s & c \end{pmatrix}$$

$$= \left(\begin{array}{c|c} cm^1{}_1 - sm^2{}_1 & cm^1{}_2 - sm^2{}_2 \\ \hline n/a & n/a \end{array} \right) \begin{pmatrix} c & s \\ -s & c \end{pmatrix}$$

$$= \left(\begin{array}{c|c} n/a & \star \\ \hline n/a & n/a \end{array} \right)$$

$$\star = \begin{array}{l} (cm^1{}_1 - sm^2{}_1)s \\ +(cm^1{}_2 - sm^2{}_2)c \end{array}$$

$$= \boxed{cs\, m^1{}_1 - s^2 m^2{}_1 + c^2 m^1{}_2 - csm^2{}_2}$$

2.3 $(v')^i (w')_i = w_l \underbrace{(R^{-1})^l{}_i \, R^i{}_k}\, v^k$

$(R^{-1})^l{}_i R^i{}_k = \delta^l{}_k$ ✓

2.4 $\delta_{ij} \delta^{jk} = \delta_{i1} \delta^{1k} + \delta_{i2} \delta^{2k}$

δ^{1k}: 1 if $k=1$, 0 if $k=2$ — δ^{2k}: 1 if $k=2$, 0 if $k=1$

δ_{i1}: 1 if $i=1$, 0 if $i=2$ — δ_{i2}: 1 if $i=2$, 0 if $i=1$

$$= \boxed{\begin{array}{l} 1 \text{ if } i=k \\ 0 \text{ otherwise} \end{array}}$$

2.5 $$\eta_{\mu\nu}\eta^{\nu\sigma} = \begin{pmatrix}-1 & \\ & 1\end{pmatrix}\begin{pmatrix}-1 & \\ & 1\end{pmatrix} = \underbrace{\begin{pmatrix}1 & \\ & 1\end{pmatrix}}_{\delta^{\sigma}_{\mu}}$$

makes from this.

2.6 $$p^2 = p^\mu p_\mu = p^0 p_0 + p^1 p_1$$

$$= +p_0 p_0 - p_1 p_1$$

$$= \boxed{E^2 - p^2 = m^2}$$

INVARIANT.

2.7 ① $\cosh w = \gamma$
$\sinh w = \gamma\beta$

$$c^2 - s^2 = \gamma^2(1-\beta^2) = 1 \quad ✓$$

$\gamma^2 = \frac{1}{1-\beta^2}$ from DEFINITION.

② $$\tanh w = \frac{\sinh w}{\cosh w} = \beta$$

$$\boxed{w = \tanh^{-1}\beta}$$ RAPIDITY

① $(R_1)^i{}_k (R_2)^k{}_j \leftarrow \begin{pmatrix} c_1 & s_1 \\ -s_1 & c_1 \end{pmatrix} \begin{pmatrix} c_2 & s_2 \\ -s_2 & c_2 \end{pmatrix}$

$$= \begin{pmatrix} c_1c_2 - s_1s_2 & c_1s_2 + s_1c_2 \\ -s_1c_2 - c_1s_2 & -s_1s_2 + c_1c_2 \end{pmatrix} \qquad (*)$$

RECALL: $e^{i\theta_1}e^{i\theta_2} = e^{i\theta_3} \leftarrow \theta_3 = \theta_1 + \theta_2$

$(c_1 + is_1)(c_2 + is_2)$

$$\underbrace{(c_1c_2 - s_1s_2)}_{c_3} + i\underbrace{(s_1c_2 + c_1s_2)}_{s_3} = c_3 + is_3$$

$$= \begin{pmatrix} c_3 & s_3 \\ -s_3 & -s_3 \end{pmatrix} = \boxed{R_{(1+2)}{}^i{}_k} \checkmark$$

② LET: $c \mapsto \cosh$, $s \mapsto \sinh$

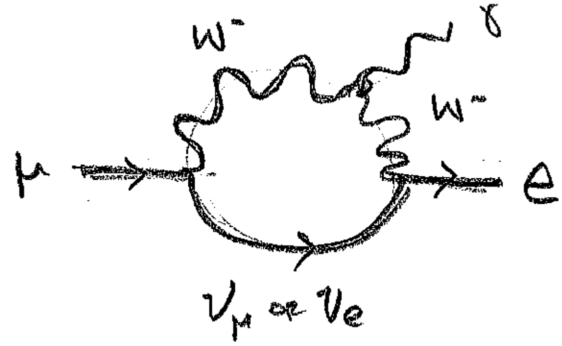
$$(\Lambda_1)^\mu{}_\rho (\Lambda_2)^\rho{}_\nu = \begin{pmatrix} c_1 & s_1 \\ s_1 & c_1 \end{pmatrix} \begin{pmatrix} c_2 & s_2 \\ s_2 & c_2 \end{pmatrix}$$

hyperbolic trig: $\sinh(x+y) = \sinh(x)\cosh(y) + \cosh(x)\sinh(y)$
$\cosh(x+y) = \cosh(x)\cosh(y) + \sinh(x)\sinh(y)$

$$= \begin{pmatrix} c_1c_2 + s_1s_2 & c_1s_2 + s_1c_2 \\ s_1c_2 + c_1s_2 & s_1s_2 + c_1c_2 \end{pmatrix} = \boxed{\begin{pmatrix} c_3 & s_3 \\ s_3 & c_3 \end{pmatrix}} \checkmark$$

$w_3 = w_1 + w_2$

3.



COULD NOT DRAW IN QED BECAUSE YOU NEED SOMETHING TO TURN MUON NUMBER INTO ELECTRON NUMBER.

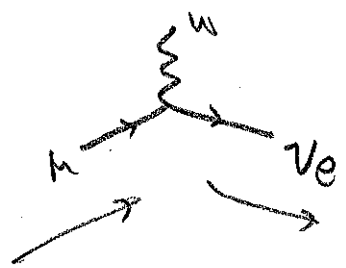
eg. μ & ν_μ HAVE MUON NUMBER 1

e & ν_e —— ELECTRON NUMBER 1

QED:

conserves e #

IN QED + μ + W:



BREAKS e # & μ #

μ# = 1
e# = 0

e# = 1
μ# = 0