Weekly HW 1: **Kinematics and QED**

Course: Physics 165, *Introduction to Particle Physics* (2018)
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Due by: Tuesday, January 16

This is the main weekly homework set. Unless otherwise stated, give all responses in natural units where $c = \hbar = 1$ and energy is measured in electron volts (usually MeV or GeV).

1 Everything in natural units

Write the following quantities in natural units with energy measured in GeV. You may write everything to one significant figure.

- The mass of the sun, $M_\odot$.
- The present day Hubble expansion rate, H_0.
- The classical electron radius, r_e.
- The Schwarszchild radius of the sun, $2G_N M_\odot/c^2$.

You may look up the information anywhere you want, but I suggest the first few pages of the PDG.

2 Special Relativity and Kinematics

2.1 A relativistic electron

In some frame, the electron has *momentum* equivalent to its rest mass, m_e. Use the value of the rest mass to one significant figure. I shouldn't have to tell you where to look it up. Write out the components of the **momentum four-vector** p_μ.

2.2 A symmetric particle collider

Imagine a symmetric electron–positron collider. At the collision point, it collides a beam of electrons and positrons with one another so that these have four-momenta:

$$p_\mu^{e^-} = (E, 0, 0, p) \qquad \text{and} \qquad p_\mu^{e^+} = (E, 0, 0, -p) \ . \tag{.1}$$

What is the expression for p as a function of E and m_e? What is the **center of mass energy** of the collision in the lab frame?

Suppose that this collider was invented to produce a 91 GeV particle, Z, through the process $e^+e^- \to Z$. What energy E is required for each beam? What is the momentum of the Z particle in the lab frame?

Extra credit: Suppose the Z is unstable and decays. This means that you don't get to measure it directly. Without knowing anything else about how the Z interacts, what is one **decay mode** that is guaranteed to exist? In other words, what types of particles should you make sure you can detect?

2.3 A fixed target experiment

Imagine a very asymmetric kind of collider called a **fixed target experiment**: a high-energy beam of particles hits a stationary target. Assume that both the beam and the target are composed of protons and that the collision occurs head-on[1]. Write the four-momenta of a beam particle and the target particle in the lab frame.

Suppose you wanted to produce some completely made up particle—let's call it a *Flippon*[2]—that has a mass of 14 GeV. To one significant figure, what proton beam energy E is required to produce the Flippon through $pp \to$ Flippon? (Assume that such a process is possible.) What is the momentum of the Flippon in the lab frame?

HINT: This problem is constrained by kinematics. There's an easy way and a hard way of doing this. One involves doing a Lorentz transformation to a more convenient frame. The other involves realizing that the quantity in the convenient frame can be written as a Lorentz invariant. I don't care which way you do this, though you should probably to understand how to do it both ways.

DISCUSSION: Fixed target experiments are nice because you don't have to worry about engineering two beams to collide with one another. They also have a very useful feature that the new particle is produced *boosted* relative to the lab frame. This can be very useful for untangling the decay products of the new particle from other particle debris from the beam hitting the target.

3 Stability of the electron

Draw the Feynman diagram for the process $e^- \to e^-\gamma$. Based on this, I foolishly propose that a 1 GeV electron can 'decay' into a lower-energy electron and a photon. Prove that this is impossible kinematically.

HINT: There's an easy way and a hard way of doing this. They both involve conservation of energy and momentum. I don't care which way you do this.

4 Feedback

Approximately how long did it take you to complete the non-extra credit parts of this assignment?

Extra Credit

If you do any of these problems, please write a short note giving your thoughts on the reading: did you like them? Were they too simple / difficult? I do not expect you to be able to complete all (or necessarily any) of the extra credit.

[1] This is a classical idea, but for now we can live with this kind of deceit. Relevant: https://www.youtube.com/watch?v=AnaQXJmpwM4

[2] Unrelated to this: https://arxiv.org/abs/1602.01377

1 Minkowski Diagrams

The mathematical basis of relativity is geometry. This is most simply seen in what are called **Minkowski diagrams**. I'm pretty sure A good introduction to these are in https://arxiv.org/abs/1508.01968 by Boxiang Liu and Thushara Perera[3]. Consider two reference frames with some non-zero relative velocity. Sketch the axes of the Minkowski diagram this system: that is, draw the (x, t) axes and the (x', t') axis where (x', t') are related to (x, t) by a Lorentz transformation. Draw two spacetime events and their respective light cones. Comment on the idea of causality using these diagrams. Those who are mathematically inclined may enjoy https://doi.org/10.1119/1.4997027.

2 Impact of Special Relativity on Physics: Compton Scattering

Look over David Jackson's article "The Impact of Special Relativity on Theoretical Physics" from the May 1987 issue of *Physics Today*, https://doi.org/10.1063/1.881108. Focus on the section "Waves and particles," where the author discusses **Compton scattering**. You drew the Feynman diagrams for this on your short homework assignment #1. Use special relativity to derive the author's expression for $\delta\lambda$, the shift in the photon wavelength.

3 Relativistic mass

There is an antiquated notion of *relativistic mass* that people used to talk about. Lev Okun gives a nice overview in "The Concept of Mass" in the June 1989 issue of *Physics Today*, https://doi.org/10.1063/1.881171. Read the article and explain why there is only *one* useful notion of mass and that it is the *rest mass*.

[3] A somewhat more polished reference is the book *Very Special Relativity* by Sander Bais. There's also a Khan Academy video, https://youtu.be/nEqexIckVCM.