TODAY: QUESTIONS
QED w/ INDICES
SU(2) w/ INDICES
EW THEORY: LEPTONS, unbroken phase
probability density

QED: from scratch, once again

1. PARTICLES: $\overrightarrow{}$ e $\sim\sim$ A   I'm going to use A instead of γ this lecture

2. SYMMETRY

- TRANSLATIONS IN SPACETIME
  ↳ dynamics conserve p^μ ← "DERIVATION" of p-cons @ vertex

  ie Σp^μ conserved @ ea. vertex

- ELECTROMAGNETIC: total electric charge conserved @ vertex
  ⇒ we won't have much to say about this
  ... turns out that we almost get this 'for free'

- LORENTZ: ROTATIONS + BOOSTS

  this is the interesting one
  HOW DO WE ENFORCE LORENTZ INVARIANCE?

  Lorentz symmetry gives us tensors:

Vector index (spin-1) → $\eta_{\mu\nu}$ $\eta^{\mu\nu}$ METRIC FOR 4-VECTOR INDEX

but we also have spinors ← "spin-$\frac{1}{2}$"

2-component objects (spin up / spin down) that also transform in a well-defined way w/rt Lorentz.

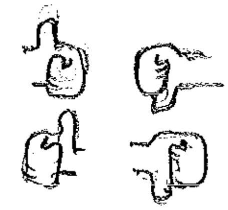
these are a totally different kind of index!

→ FERMIONS

IN FACT in 4 dimensions, 2 kinds of spinors

~~LEFT-HANDED~~

SPIN UP SPIN DOWN



LEFT - CHIRAL ψ^α $\alpha = 1, 2$

RIGHT - CHIRAL $\bar{\chi}^{\dot\beta}$ $\dot\beta = 1, 2$

these are two different representations of the Lorentz symmetry group.

↳ the vector representation, eg p_μ, is a third

$\mu = 0,1,2,3$

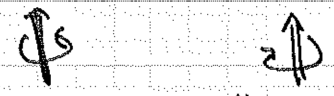
CP

FACT: an anti-LEFT-CHIRAL particle is RIGHT-CHIRAL

$$(\psi^\alpha)^\dagger = (\psi^\dagger)^{\dot\alpha}$$

intuition: just like e^+ looks like e^- moving "backward in time"

a LH particle w/ time moving backward is really spinning in the opposite direction



SPIN AXIS "rewind"!

IMPORTANT REMARK: there is a very closely related idea called helicity

in fact: for massless particles

CHIRALITY = HELICITY

but technically: helicity is angular momentum

chirality is representation

⤳ we'll dig into this more later.

BTW: you also have scalars (spin-0) that tensors for spinors:

$\boxed{\sigma^{\mu}{}_{\alpha\dot\beta}}$ ← rosetta stone: converts μ index to α and $\dot\beta$

$\varepsilon^{\alpha\beta}$ $\varepsilon_{\alpha\beta}$ METRIC & INVERSE METRIC FOR LEFT-CHIRAL INDICES

$\varepsilon^{\dot\alpha\dot\beta}$ $\varepsilon_{\dot\alpha\dot\beta}$ METRIC & INVERSE FOR RIGHT-CHIRAL INDICES

related: $\bar\sigma^{\mu\dot\alpha\beta} = \varepsilon^{\dot\alpha\dot\beta}\varepsilon^{\beta\alpha}\sigma^{\mu}{}_{\alpha\dot\beta}$

these are the tools that we have

↳ a Feynman vertex is allowed by this symmetry (Lorentz) if one can form an <u>invariant</u> out of the particles it connects using only the tensors that the symmetries give us.

PARTICLES, once again: ← assume $m_e = 0$

A LEFT-CHIRAL ELECTRON / RIGHT-CHIRAL POSITRON

$\psi^{\dot\beta}$ ——▶—— ψ^{α}

RH LH

A <u>VECTOR</u>:

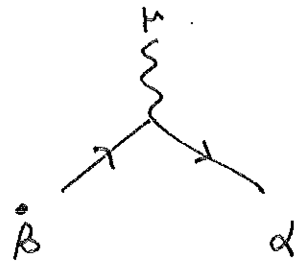
$A_\nu \sim\sim\sim\sim A_\mu$

↳ $A_\mu = (\varphi, \underline{A})$ ↲ $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

contains $\vec E$, $\vec B$

↑ electric pot. ↑ vector pot

A SCALAR - - - - - - sometimes they have arrows if charged try the QED vertex:

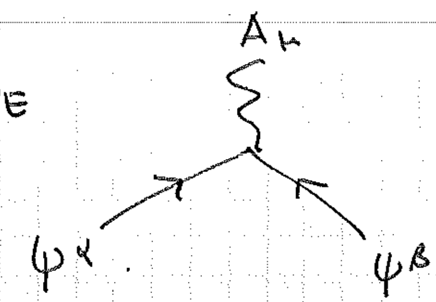


do we have a combination of objects w/ these indices?

$$A_\mu (\psi^\dagger)^{\dot\beta} \psi^\alpha \boxed{\sigma^\mu{}_{\alpha\dot\beta}}\, ie$$

some # that tells us the strength

CAN WE HAVE



?

$$A_\mu \psi^\alpha \psi^\beta \boxed{?}^\mu{}_{\alpha\beta}$$

convert $\dot\beta \to \beta$

$$\underbrace{\sigma^\mu{}_{\alpha\dot\beta}}_{\text{not the right index!}} \bar\sigma^{\nu\,\delta\dot\beta}\, \varepsilon_{\beta\delta}$$

BUT now a third VECTOR INDEX & nothing to contract w/!

could try $\eta_{\nu\rho}$ or $\sigma^\nu{}_{\alpha\dot\beta}$ ...
but cannot get rid of the indices!!

so: No : you cannot have this

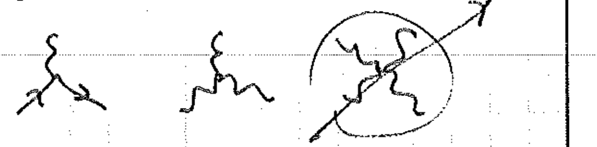
other vertices

$\leftarrow$ eg $\psi\psi = \psi^\alpha \psi^\beta \varepsilon_{\alpha\beta}$

anything w/ just 2 particles: ignore for now.
$\hookrightarrow$ has ~~to~~ do w/ how particle propagates in spacetime

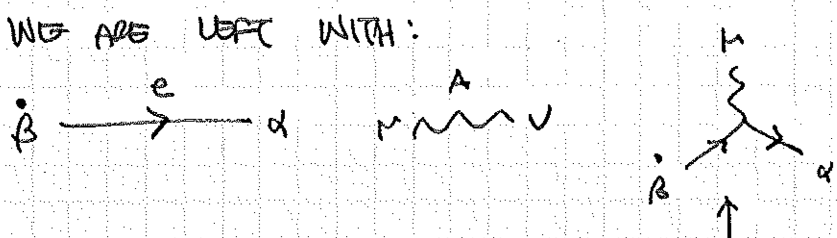
there are vertices you can form for ≥ 4 particles
$\hookrightarrow$ these are "non-renormalizable"

for now: we only focus on



WE WILL JUSTIFY FROM DIM. ANALYSIS LATER.

SO: WE ARE LEFT WITH:



somewhere in the theory $\exists$
$A_\mu (\psi^\dagger)^{\dot\beta} \psi^\alpha \sigma^\mu{}_{\alpha\dot\beta} \cdot ie$

REMARKS:

this theory is "anomalous" ... QUANTUM MECHANICS SAYS IT DOESN'T MAKE SENSE.

we'll want to understand why.

lorentz transformation:

$$(\psi')^\alpha = e^{i M_{\mu\nu} (\bar\sigma^{\mu\dot\beta\alpha} \sigma^\nu{}_{\beta\dot\beta} - \bar\sigma^{\nu\dot\beta\alpha} \sigma^\mu{}_{\beta\dot\beta})} \psi^\beta$$

$\hookrightarrow$ antisym 4x4 matrix.
$\rightarrow$ 6 parameters: 3 rot, 3 boosts

A TOY SU(2) THEORY

symmetry : TRANSLATIONS ✓
~~EM~~ (ignore! no electric charge)
$\boxed{SU(2)}$
LORENTZ ✓
fermions have $\alpha, \dot{\beta}$
vectors have μ

A NEW KIND OF SYMMETRY
ALSO collects particles into PAIRS
we call them SU(2) doublets

$$D^a = \begin{pmatrix} C \\ N \end{pmatrix} \begin{matrix} \leftarrow D^1 \\ \leftarrow D^2 \end{matrix} \qquad a = 1, 2$$

if D is a fermion, it also has a spinor index

$$D^{a\alpha} = \begin{pmatrix} C^\alpha \\ N^\alpha \end{pmatrix}$$

each of these is a spinor... LH, apparently

$$\text{or } D^{a\dot{\beta}} = \begin{pmatrix} C^{\dot{\beta}} \\ N^{\dot{\beta}} \end{pmatrix}$$

THERE IS ALSO A TRIPLET REPRESENTATION:

$$W^A = (W^1, W^2, W^3)$$

← these are always upper. eg $W^A W^A$ is contracted

ANTI-DOUBLETS: LOWER INDEX

$$(D^a)^\dagger = (D^\dagger)_a \quad \text{or} \quad \bar{D}_a$$

can have antidoublet LH fermion: $\bar{D}_a^\alpha$

or a triplet RH fermion $W^{A\dot{\beta}}$
or a triplet vector $W^{A\mu}$

INVARIANTS: $T^{Aa}{}_{b}$ (A: 1,2,3; a: 1,2; b: 1,2) ε^{ab} ε_{ab} f^{ABC}

in fact: $T^{A\,a}{}_{b} = \sigma^{i}$ matrices!
but this is not a rotation in space, it's a rotation in "inner space"

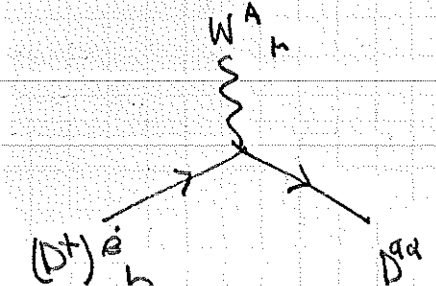
PARTICLES.

$(D^\dagger)^{\dot\beta}_{b} \longrightarrow D^{a\alpha}$

note: this theory is incomplete... but you don't know why yet.

$W^B_\nu \sim\sim\sim\sim W^A_\mu$

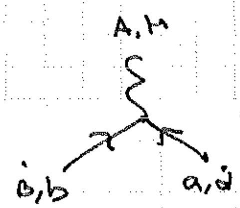
ALLOWED RULES



$$T^{Ab}{}_{a}\,\sigma^{\mu}_{\alpha\dot\beta}\,\boxed{ig}\,W^A_\mu (D^\dagger)^{\dot\beta}{}_{b} D^{a\alpha}$$

#

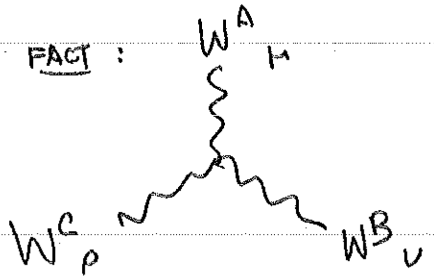
other rules don't work:



$\overset{?}{\sim}\ T^{Ab}{}_{c}\,\varepsilon^{ca}\,\sigma^{\mu}_{\alpha\dot\beta}$

can't convert into an $\dot\alpha$!

FACT:



$$f^{ABC}\,\eta^{\mu\nu}\,(?)^{\rho}\,W^A_\mu W^B_\nu W^C_\rho$$

turns out that you can use derivatives.
$(\partial/\partial x)^{\rho}$

some <u>representations</u> are $\mathbb{R}$, others $\mathbb{C}$

"what indices I carry / how I transform"

$\mathbb{R}$ rep $\leftrightarrow$ its own antiparticle

$\mathbb{C}$ rep $\leftrightarrow$ complex (Hermitian) conjugate is antiparticle

$(\text{field})^\dagger \xrightarrow{P} (\text{field})$ ← field : "particle"

<u>LORENTZ</u> :	scalar (no index)	$\mathbb{R}$
	spinor (RH $\overset{\dagger}{\leftrightarrow}$ LH)	$\mathbb{C}$
	vector	$\mathbb{R}$
<u>SU(2)</u>	singlet (scalar)	$\mathbb{R}$
	doublet / fundamental	$\mathbb{C}$
	triplet	$\mathbb{R}$

~~fields~~

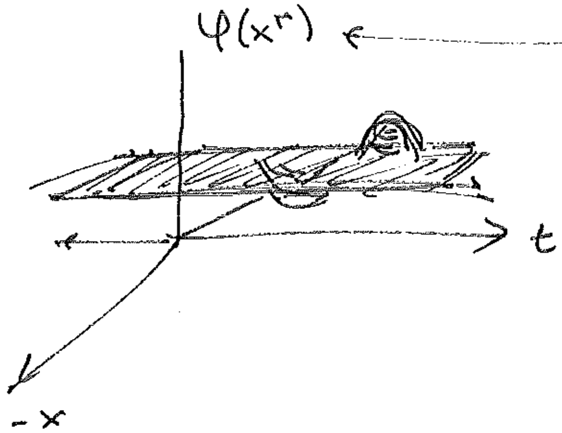
A particle has a representation under <u>each</u> symmetry of the theory

if <u>any</u> is $\mathbb{C}$, then the particle $\neq$ antiparticle

eg a triplet spinor

$$(\psi^\dagger)^{\dot{\beta} B} \longrightarrow \psi^{\alpha A}$$

A CARTOON of the QUANTUM FIELD



φ has values @ each point in spacetime

encodes prob. amplitude to find a particle there... but "more" than a wavefunction.

Why QUANTUM FIELD? Why not just QM?

QM + SPECIAL RELATIVITY

observables are operators

$\boxed{\text{causality}}$

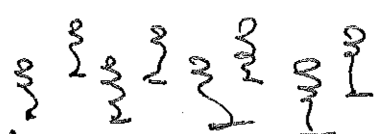
commute if they don't affect each other.

$[\mathcal{O}_1, \mathcal{O}_2] \neq 0$ if they're related & in the same lightcone

$[\mathcal{O}_1, \mathcal{O}_2] = 0$ even if they are related if outside the LIGHT CONE

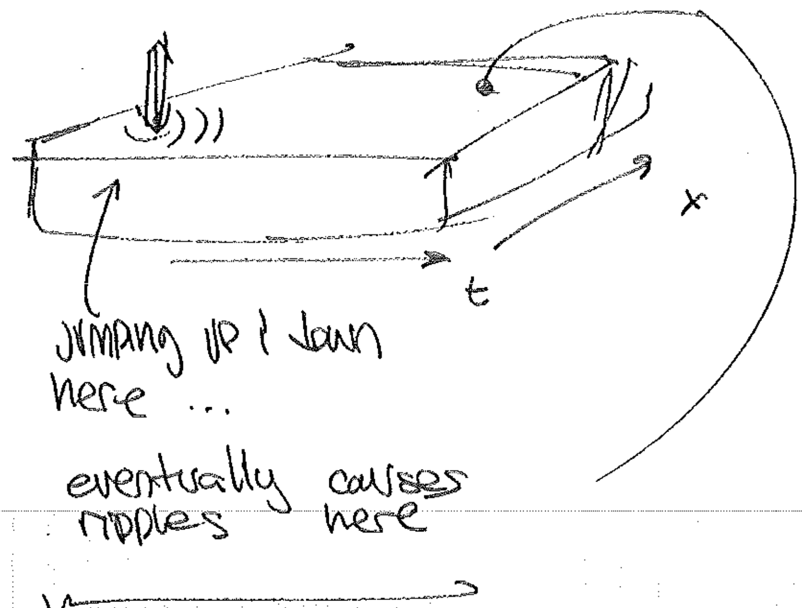
What the FIELD DOES:

Like a box spring



wiggle this → causes wiggles in others.

⇝ waves that propagate



how the information of
a particle being here
propagates in space-time.

What we care about:
What is the correlation of observing
a particle @ x_1^μ & another @ x_2^μ?

QUANTUM WAY OF ASKING: DID THE PARTICLE
MOVE FROM x_1^μ TO x_2^μ?

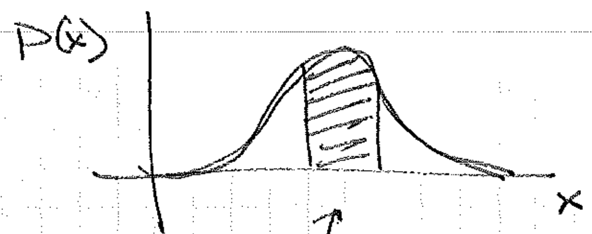
CORRELATION FUNCTIONS ARE THE BREAD & BUTTER
OF STATISTICAL FIELD THEORY.
↳ no surprise, essentially some mathematics

USEFUL: probability distribution function (pdf)
(not particle physics ... but we'll need it)

$p(x)\,dx$ s.t. $\int_D p(x)\,dx = 1$

over domain

continuous version of histogram



eg you can draw a continuum number

$\int_{x_1}^{x_2} p(x)\,dx$ = PROB DRAWING $x \in (x_1, x_2)$

$\int_D x\,p(x)\,dx$ = MEAN OF x OVER THE DISTRIBUTION

eg if $p(x)$ is distribution of GPA, x
then $\int_D x\,p(x)\,dx$ is AVG GPA
of population.

$\langle x \rangle$, "moment of $p(x)$"