LEC 13 : FIELD THEORY ONCE MORE — 20 FEB 2018

TO DO: MIDTERM DOODLE POLL

So far:

Model

1. Symmetries → Global, Gauge
2. Fields (PARTICLES)
3. $\mathcal{L} = \underbrace{(\text{QUADRATIC})}_{\text{mass \& kinetic term.}} + (\text{INTERACTIONS})$ ← Vertices

FACTS

① $\mathcal{L}$ is REAL.

eg $\; y H^{\dagger} \cdot L E + (h.c.)$ ← $y^* H L^{\dagger} E$

SWAP ALL PARTICLES W/ ANTIPARTICLES

(charge-parity symmetry ... if $y^* = y$)

② $\mathcal{L}$ is made of couplings, invariant tensors, fields

couplings, invariant tensors: what the "rule" is for the vertex

fields: particles that go into vertex

③ $\mathcal{L}$ IS INVARIANT UNDER THE SYMMETRIES

4-component $\bar{\psi} = \psi^{\dagger}\gamma^{0}$

④ THE QUADRATIC TERMS ARE STANDARD:

$(\partial\phi)^2 - m^2\phi^2$ for scalar ($\frac{1}{2}$ if $\mathbb{R}$)

$\psi^{\dagger}\gamma^{0}\gamma^{\mu}\partial_{\mu}\psi - m\psi^{\dagger}\gamma^{0}\psi$ for fermion

$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - m^2 A^2$ for gauge boson

$\partial_{[\mu}A_{\nu]} - g[A_{\mu}, A_{\nu}]$

not resp for this

BUT KNOW THE MASS TERMS

FACT: GAUGE INTERACTIONS come "for free"

HOW? PROMOTE $\partial_\mu \to D_\mu \equiv \underset{\delta^\Delta_\nabla}{\partial_\mu} - ig A^a_\mu T^{a\Delta}{}_\nabla$

where Δ, ∇ are indices of object it's acting on w/r/t GAUGED sym.

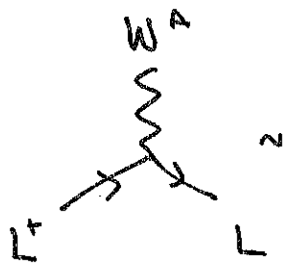
e.g. $\underset{\text{for } L^a_{Y=-1/2}}{D_\mu} = \Big(\delta^a_b \partial_\mu - \underbrace{\underbrace{ig W^A T^{Aa}{}_b}_{SU(2)} - \underbrace{\delta^a_b i g' Y q_Y}_{U(1)_Y}}\Big)$

then: g's are COUPLINGS, typically small!

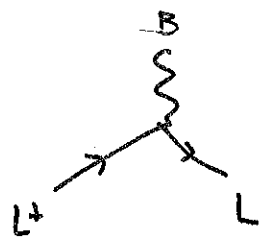
won't be careful w/ γ^0 →

USUAL KINETIC TERM

$$i L^\dagger_a \gamma^\mu \underbrace{(D_\mu L)^a}_{D_\mu{}^a{}_b L^b} = L^\dagger \gamma^\mu \partial_\mu L \longrightarrow L$$
$$- i g W^A_\mu L^\dagger \gamma^\mu T^A L$$
$$- i g' q_L B_\mu L^\dagger \gamma^\mu \underset{\delta^a_b}{L}$$



$\sim ig\gamma^\mu T^A$



$\sim ig'\gamma^\mu \delta q_L$ CHARGE UNDER Y!

connects different components of SU(2) doublet

$L = \begin{pmatrix} \nu_e \\ e_L \end{pmatrix}$

connects ν_e to ν_e, e_L to e_L

(doesn't "see" SU(2))

GAUGE BOSONS

not caring about indices

$$\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \sim (\partial A - g A^2)$$

$$\sim (\partial A)^2$$

$$- g(\partial A) A^2$$

some momentum

$\sim gP$

$$- g^2 A^4$$

$\sim g^2$

HIGHER ORDER!

this is why we didn't care about the 4-point int. too much.

eg: GLUONS :

color neutral

INDEX CARD :

QUARK DOUBLET :

$$Q^m = \begin{pmatrix} u_L^m \\ d_L^m \end{pmatrix} \qquad Y = 1/6 \qquad SU(3) \text{ TRIPLET } (m = 1,2,3)$$

KINETIC TERM : $Q^\dagger i D_\mu \gamma^\mu Q$

$$D_\mu = \partial_\mu - i g W_\mu^A T^A - i g' B - i g_c G^M T^M$$

HIGGS DOUBLET :

$$H = \begin{pmatrix} H^1 \\ H^2 \end{pmatrix} \qquad Y = 1/2 \qquad \text{KIN}: (\partial H)^2 \to |DH|^2$$

what interactions?

<u>Higgs</u> : $DH \sim \partial H - igW^A T^A H \quad - ig' g_H B H$

$$
\begin{aligned}
\Rightarrow |DH|^2 \sim\ & |\partial H|^2 && \text{I} \\
& igW^A (\partial H)^\dagger T^A H + \text{h.c.} && \text{II} \\
& ig' g_H B (\partial H)^\dagger H + \text{h.c.} && \text{III} \\
& g_H g g' (W^A T^A H)^\dagger B H + \text{h.c.} && \text{IV} \\
& g^2 |W^A T^A H|^2 && \text{V} \\
& g_H^2 (g')^2 |BH|^2 && \text{VI}
\end{aligned}
$$

I  H

II 

$\sim gPT^A$

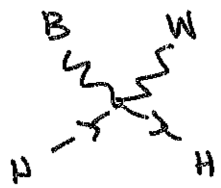
the h.c. swaps which momentum you pick up ...

RESULT IS $\sim g(P_1 - P_2) T^A$

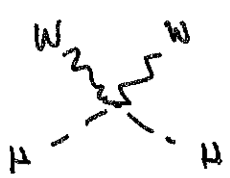
III 

$\sim g' g_H P$

nb: why no g_H in W vertex? it's hidden in T^A ... factors of $1/2$

IV 

$\sim g g' g_H T^A$

V 

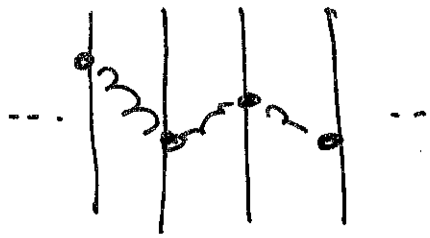
$\sim g^2 (T^A)^2$ ← ends up as $\sim \mathbb{1} \cdot \frac{1}{4}$ ·

↑ from $(\frac{1}{2})\sigma^i$

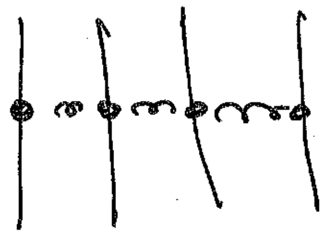
VI 

$\sim g_H^2 (g')^2$

SO FAR:

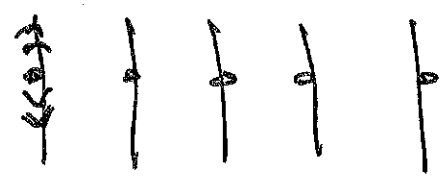


costs energy

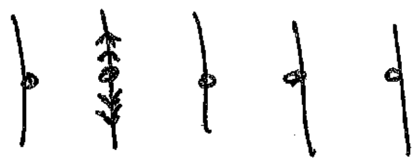


"ground state"
(all oscillators the same in space-time)

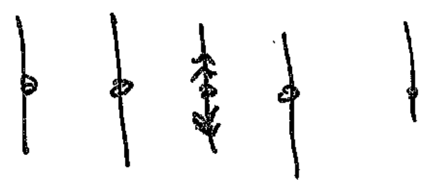
A PARTICLE IS A QUANTUM WIGGLE THAT DISTURBS THIS



$a^\dagger_i$



$a^\dagger_{i+1}$



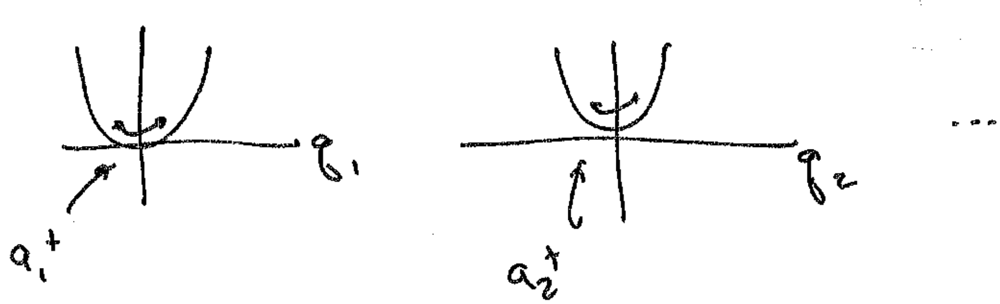
$a^\dagger_{i+2}$

BUT WHAT SETS THE ZERO VALUE OF THE FIELD?

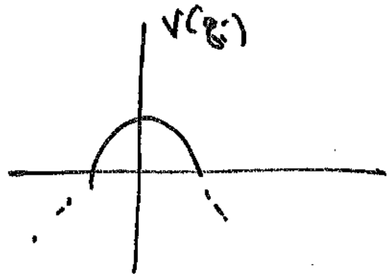
$L = T - V$

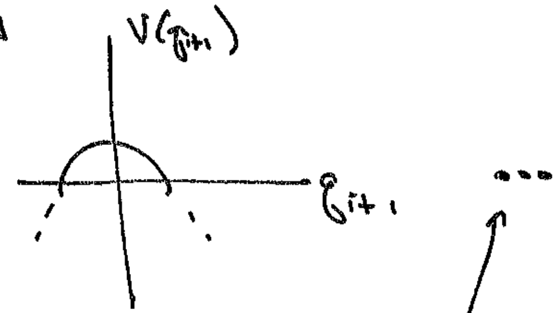
kinetic, POTENTIAL ← eg $\int d^4x \; \underbrace{-m^2 q(x)^2}_{V \sim m^2 q^2}$

$$\int d^4x \, |q(x)|^2 = q_i^2 + q_{i+1}^2 + q_{i+2}^2$$



BUT WHAT IF ... $\mathcal{L} = \pm m^2\phi^2$
?
"WRONG" SIGN





$q_i = 0$ is not vacuum.

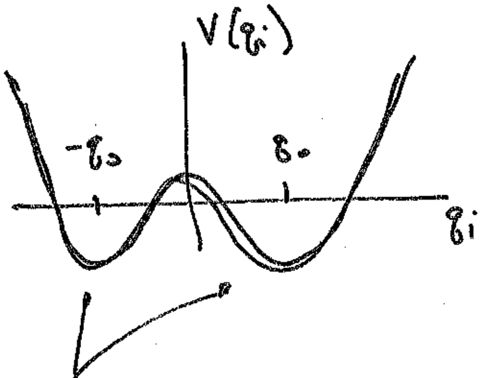
ARE THEY ALL THE SAME? YES → BY ~~SPACETIME~~ SYMMETRY!
?
doesn't always have to be the case

... no vacuum ... doesn't make sense

FC?

[nb: looks tachyonic]

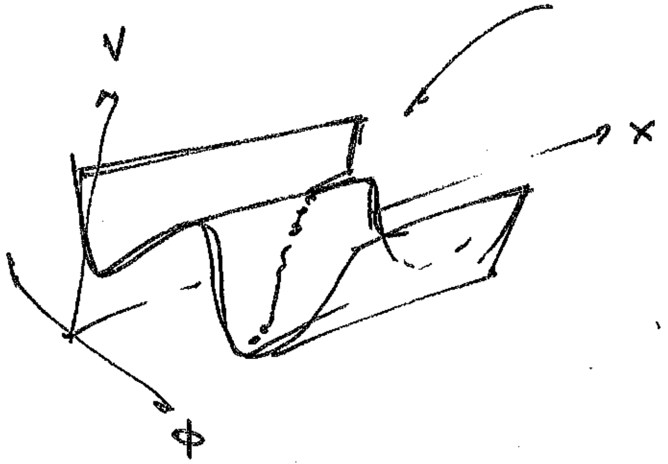
to make sense, need $\mathcal{L} = + m^2\phi^2 - \lambda\phi^4$



note: $\langle q \rangle = \pm q_0$
CAN BREAK symmetries if q has quantum #!

equivalent, but distinct vacua.

$\mathcal{L} = (\partial_0\phi)^2 - (\partial_i\phi)^2 + m^2\phi^2 - \lambda\phi^4$



IF YOU TRY TO PULL INTO OTHER VACUUM, YOU COST ENERGY HERE.

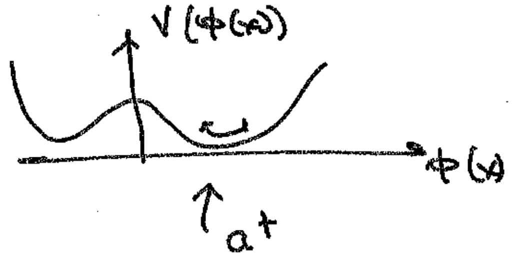
so: $\mathcal{L}(\phi)$ can have ground state where minimum energy config has $\langle \phi \rangle \neq 0$

scalar field

ie springs are "compressed"

→ the field is classically non-zero.

→ particles are excitations about the ground state



IF ϕ HAS QUANTUM #'s, these symmetries are SPONTANEOUSLY BROKEN.

$\mathcal{L}$ IS COMPOSED OF INVARIANTS. BUT VACUUM IS NOT INVARIANT.

eg.

$$\mathcal{L}[H] = +m^2 |H|^2 - \lambda |H|^4$$

s.t. $\langle H \rangle = v/\sqrt{2}$.

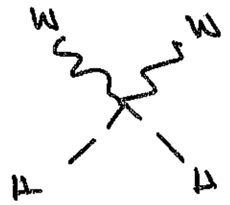
so: $H(x) = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} + \begin{pmatrix} H^1(x) \\ H^2(x) \end{pmatrix}$

$\langle H \rangle$

excitations about vacuum

nb $\partial \langle H \rangle = 0$

GO BACK TO LAGRANGIAN:



← from $\underbrace{g^2 \, |W^A T^A H|^2}$

$$g^2 \left| W^A T^A \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \right|^2$$

$$\to g^2 W^A W^A \cdot \frac{v^2}{2} \cdot \frac{1}{2}$$

$$\to \underbrace{\boxed{\frac{g^2 v^2}{4}}}_{M_W^2} (W^2)$$

BREAKING OF GAUGE SYMMETRY GAVE MASS TO W.

↳ & ALSO B, & MASS MIXING BETWEEN THEM.