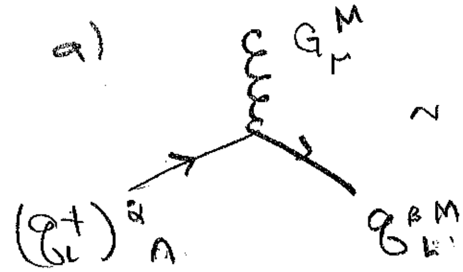
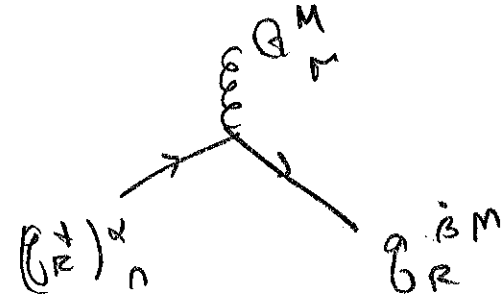
$\boxed{1.1}$ a)



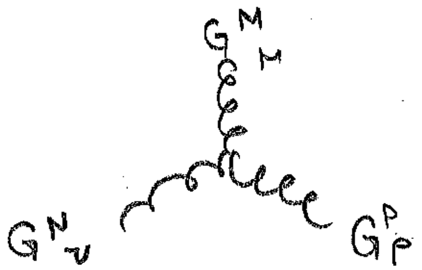
$\sim \underbrace{\sigma^{\mu}{}_{\dot\alpha\beta}}_{\text{SPIN}} \underbrace{(T^M)^n{}_m}_{\text{COLOR}}$

b)



$\sim \sigma^{\mu}{}_{\alpha\dot\beta} (T^M)^n{}_m$

c)



$\sim \underbrace{\varepsilon^{MNP}}_{\text{color}} \underbrace{\eta_{\mu\nu} p_\rho}_{\text{spin}}$

$\boxed{1.2}$ a) $q_L \; (q_R^\dagger) \; G$ ✗

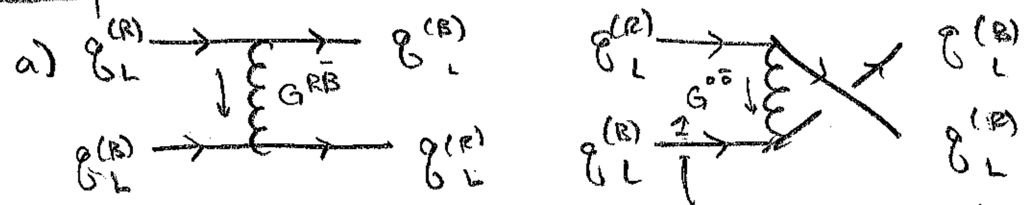
α β two undotted indices

(can contract w/ $\varepsilon_{\alpha\beta}$, but then uncontracted μ index on G)

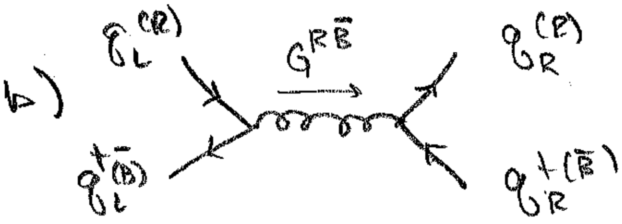
b) $(q_L)^\ell (q_L)^m (q_L)^n \varepsilon_{\ell m n}$ ✗

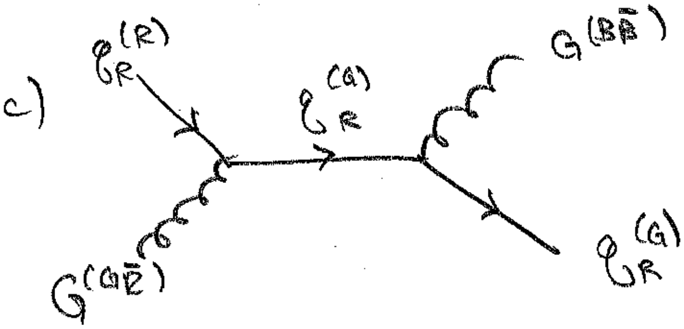
three undotted indices. no tensor to contract all three.

1.3

a) 

any color/anti (same) color combination

b) 

c) 

1.4 a) $q_L^{(R)}\, q_L^{(B)} \longrightarrow q_L^{(G)}\, q_L^{(B)}$

Does not conserve color
LHS has 1 unit of redness, no green
RHS has 1 unit of green-ness, no red.

b) $q_L^{(R)}\, q_L^{(B)} \longrightarrow q_R^{(R)}\, q_L^{(B)}$

GLUONS CONSERVE CHIRALITY, BUT THIS PROCESS DOES NOT

(γ^μ only connects $q_L^\dagger q_L$, never $q_L^\dagger q_R$, for eg)

c) $q_R^{(B)}\, G^{(G\bar{R})} \longrightarrow q_R^{(G)}\, G^{(B\bar{B})}$

DOES NOT CONSERVE REDNESS / BLUE-NESS

2. $\Delta^{++}_{3/2} = \underbrace{|\uparrow\rangle \otimes |\uparrow\rangle \otimes |\uparrow\rangle}$

these are fermions,
so wavefunction must be
antisymmetric.

HOWEVER, w/o an additional
index, this configuration
is <u>symmetric</u>.

SOLUTION: EACH QUARK IS A DISTINCT
COLOR. THEN CONTRACT
COLOR INDICES w/ ε_{lmn}.

$$|\uparrow\rangle^{l} \otimes |\uparrow\rangle^{m} \otimes |\uparrow\rangle^{n} \underbrace{\varepsilon_{lmn}}_{\text{totally antisym.}}$$

nb: this is <u>not</u> a vertex.
this is a particle

$$\left.\begin{array}{l} q^{l} \longrightarrow \\ q^{m} \longrightarrow \\ q^{n} \longrightarrow \end{array}\right\} \Delta^{++}_{3/2}$$

3. FIELD THEORY

3.1 $$L = \frac{1}{2}\sum_i \Delta x \left[\frac{m}{\Delta x} \dot{q}_i^{\,2} - K\Delta x \frac{(q_{i+1}-q_i)^2}{\Delta x^2} \right]$$

have the same mass dim
IN NATURAL UNITS
(because $[\dot{q}] = [\frac{\Delta q_i}{\Delta x}]$)

$[\frac{m}{\Delta x}] = [K\Delta x]$ $\longleftarrow$ note $[\Delta x] = -1$

$$\begin{aligned}[K] &= [\Delta x^2\, m] \\ &= 2[\Delta x] + [m] \\ &= -2 + 1 \\ \boxed{[K]} &\boxed{= -1}\end{aligned}$$

OR :

$$[m(\tfrac{\Delta q}{\Delta t})^2] = [K\Delta q^2]$$
$$\Rightarrow [K] = [m] - 2[\Delta t] = \boxed{-1}$$

3.2 $S = \int dt\, L \longrightarrow \underbrace{[L] + [dt] = 0}$

$e^{iS} \rightarrow [S] = 0$ $\Rightarrow \boxed{[L] = 1}$

IN THE DERIVATION, $\rho = m/\Delta x \rightarrow \boxed{[\rho] = 2}$

then : (eq (6)) $\longrightarrow$ $\cancel{[\frac{\Delta q_i^2}{\Delta t^2}]} = [\frac{K}{\rho}]\cancel{[\frac{\Delta q_i^2}{\Delta x^2}]}$

$$[\frac{K}{\rho}] = 0$$
$$[K] - [\rho] \Rightarrow \boxed{[K] = [\rho] = 2}$$

3.3 the trick here is:

$$L = \frac{\rho}{2} \sum_i \Delta x \left[\dot{q}_i^{\,2} - \left(\frac{K}{\rho}\right) \left(\frac{\Delta q}{\Delta x}\right)^2 \right]$$

these 2 terms must have the same units.

$$[\dot{q}^2] = 2[q] - 2[t]$$

$$\left[\left(\frac{K}{\rho}\right) \frac{\Delta q^2}{\Delta x^2} \right] = 2[q] - 2[\Delta x] + \left[\frac{K}{\rho}\right]$$

EQUATING THESE

$$-2[t] = -2[x] + \left[\frac{K}{\rho}\right]$$

$$\Rightarrow \left[\frac{K}{\rho}\right] = 2\left([x] - [t]\right)$$

$$\boxed{\left(\frac{K}{\rho}\right) \sim (\text{length})^2 (\text{time})^{-2}}$$

$$\sim \left(\frac{\text{length}}{\text{time}}\right)^2$$

this is a <u>SPEED</u>

$\frac{K}{\rho} = \frac{1}{c^2}$ } the speed of "massless" waves in the medium.

3.4 $S = \frac{1}{2}\int \underbrace{dx\,dt}_{\text{MASS DIM } -2} (\partial \theta)^2$

$[\partial] = +1$

$\Rightarrow \boxed{[Q] = 0}$ ← only in 2D spacetime!

from dimensional analysis,
ripples travel w/ speed $= \boxed{c = K/\rho = 1}$