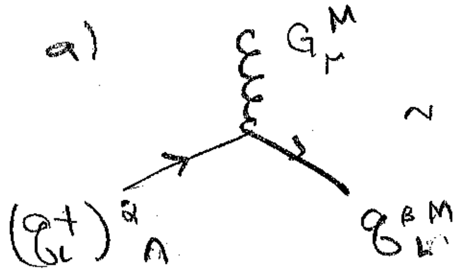
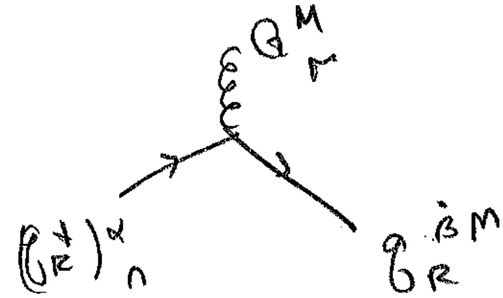
$\boxed{1.1}$ a)



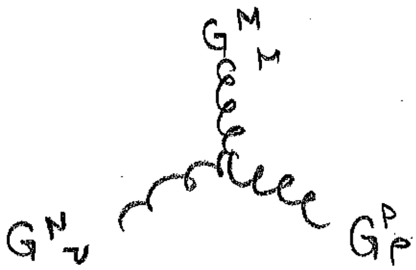
$$\sim \underbrace{\sigma^\mu{}_{\dot\alpha\beta}}_{\text{SPIN}}\ \underbrace{(T^M)^n{}_m}_{\text{COLOR}}$$

b)



$$\sim \sigma^\mu{}_{\alpha\dot\beta}\ (T^M)^n{}_m$$

c)



$$\sim \underbrace{\varepsilon^{MNP}}_{\text{color}}\ \underbrace{\eta_{\mu\nu}\, p_\rho}_{\text{spin}}$$

$\boxed{1.2}$ a) $q_L\ (q_R^\dagger)\ G$ ✗

α β two undotted indices

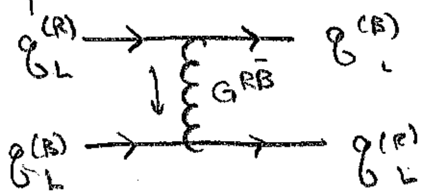
(can contract w/ $\varepsilon_{\alpha\beta}$, but then uncontracted μ index on G)

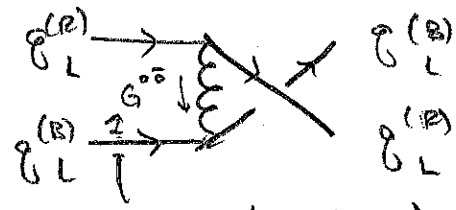
b) $(q_L)^l (q_L)^m (q_L)^n\, \varepsilon_{lmn}$ ✗

three undotted indices. no tensor to contract all three.

1.3

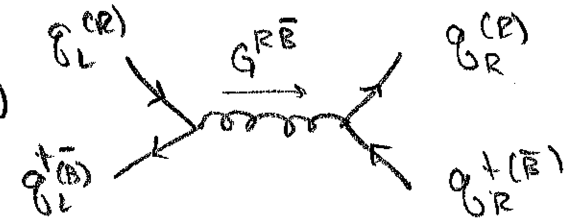
a)





any color/anti (same) color combination

b)



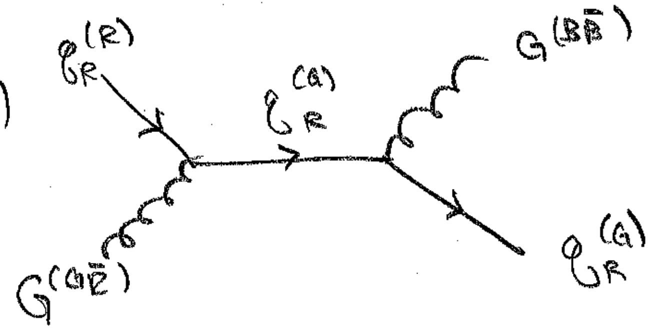
UPDATE 2/27

the HW had a typo! :(

↳ SHOULD BE

$q_L^{(R)} q_L^{\dagger(\bar B)} \to q_R^{(R)} q_R^{\dagger(\bar B)}$

c)



1.4 a) $q_L^{(R)} q_L^{(B)} \longrightarrow q_L^{(G)} q_L^{(B)}$

Does not conserve color
LHS has 1 unit of redness, no green
RHS has 1 unit of green-ness, no red.

b) $q_L^{(R)} q_L^{(B)} \longrightarrow q_R^{(R)} q_L^{(B)}$

GLUONS CONSERVE CHIRALITY, BUT THIS PROCESS DOES NOT

(~ only connects $q_L^{\dagger} q_L$, never $q_L^{\dagger} q_R$, for eg)

c) $q_R^{(B)} G^{(G\bar R)} \longrightarrow q_R^{(G)} G^{(B\bar B)}$

DOES NOT CONSERVE REDNESS / BLUE-NESS

2. $\Delta^{++}_{3/2} = \underbrace{|\uparrow\rangle \otimes |\uparrow\rangle \otimes |\uparrow\rangle}$

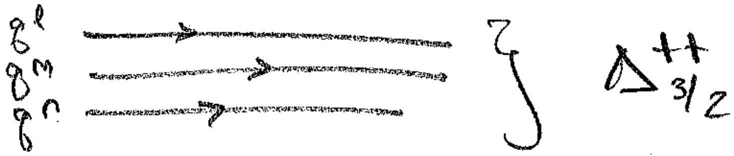
these are fermions, so wavefunction must be antisymmetric.

HOWEVER, w/o an additional index, this configuration is <u>symmetric</u>.

SOLUTION: EACH QUARK IS A DISTINCT COLOR. THEN CONTRACT COLOR INDICES w/ ε_{lmn}.

$$|\uparrow\rangle^{l} \otimes |\uparrow\rangle^{m} \otimes |\uparrow\rangle^{n} \; \underbrace{\varepsilon_{lmn}}_{\text{totally antisym.}}$$

nb: this is <u>not</u> a vertex. this is a particle



3. FIELD THEORY

3.1 $L = \frac{1}{2}\sum_i \Delta x \left[\frac{m}{\Delta x} \dot{q}_i^{\,2} - K\Delta x \frac{(q_{i+1} - q_i)^2}{\Delta x^2} \right]$

have the same mass dim IN NATURAL UNITS

(because $[\dot{q}] = [\frac{\Delta q_i}{\Delta x}]$)

$[\frac{m}{\Delta x}] = [K\Delta x]$ ⟵ note $[\Delta x] = -1$

$$\begin{aligned}[K] &= [\Delta x^2 \, m] \\ &= 2[\Delta x] + [m] \\ &= -2 + 1 \\ \boxed{[K]} &\boxed{= -1}\end{aligned}$$

OR:

$$\begin{aligned}[m(\tfrac{\Delta q}{\Delta t})^2] &= [K\Delta q^2] \\ \Rightarrow [K] &= [m] - 2[\Delta t] \\ &= \boxed{-1}\end{aligned}$$

3.2 $S = \int dt \, L \longrightarrow \underbrace{[L] + [dt]}_{} = 0$

$e^{iS} \rightarrow [S] = 0 \qquad \Rightarrow \boxed{[L] = 1}$

IN THE DERIVATION, $\rho = m/\Delta x \rightarrow \boxed{[\rho] = 2}$

then: (eq (6)) ⟶ $\cancel{[\frac{\Delta q^2}{\Delta t^2}]} = [\frac{K}{\rho}] \cancel{[\frac{\Delta q^2}{\Delta x^2}]}$

$$[\frac{K}{\rho}] = 0$$

$$[K] - [\rho] \Rightarrow \boxed{[K] = [\rho] = 2}$$

3.3 the trick here is:

$$L = \frac{\rho}{2} \sum_i \Delta x \left[\dot{q}_i^{\,2} - \left(\frac{K}{\rho}\right) \left(\frac{\Delta q}{\Delta x}\right)^2 \right]$$

these 2 terms must have the same units.

$$[\dot{q}^2] = 2[q] - 2[t]$$

$$\left[\left(\frac{K}{\rho}\right) \frac{\Delta q^2}{\Delta x^2} \right] = 2[q] - 2[\Delta x] + \left[\frac{K}{\rho}\right]$$

EQUATING THESE

$$-2[t] = -2[x] + \left[\frac{K}{\rho}\right]$$

$$\Rightarrow \left[\frac{K}{\rho}\right] = 2\left([x] - [t]\right)$$

$$\boxed{\left(\frac{K}{\rho}\right) \sim (\text{length})^2 (\text{time})^{-2}}$$

$$\sim \left(\frac{\text{length}}{\text{time}}\right)^2$$

this is a <u>SPEED</u>

$\hookrightarrow \frac{K}{\rho} = \frac{1}{c^2}$ } the speed of "massless" waves in the medium.

3.4 $$S = \frac{1}{2}\int \underbrace{dx\, dt}_{\text{MASS DIM } -2} (\partial\theta)^2$$

$[\partial] = +1$

$\Rightarrow \boxed{[\theta] = 0}$ ← only in 2D spacetime!

from dimensional analysis,
ripples travel w/ speed $= \boxed{c = \kappa/\rho = 1}$