LEC 14: MASSES once MORE

see LEC 13 notes.

MODEL →
- symmetries
- fields
- $\mathcal{L}$ $\xrightarrow{\text{terms}}$
  - QUADRATIC → PROPAGATORS; GAUGE int.
  - HIGHER ORDER → other vertices

$\boxed{\text{GAUGE int.} \quad \text{other vertices}}$ vertices

IR func. $[\mathcal{L}] = 4$

RECALL: $L = T - V$ (T: kinetic, V: potential)

HIGHER ORDER TERMS

nb. $(\partial\phi)^2 = (\partial_t\phi)^2 - \underbrace{(\nabla\phi)^2}_{\text{P.E. of neighboring springs.}}$ ← $\mathcal{L}$ (DENSITY)

V is really a classical potential
[that is being used for quantum purposes]

$(\nabla\phi)^2$ term: V increases if all "springs" not @ same config

costs "gradient" energy — costs no gradient energy

so grad term (propagation): field wants to take a classical/background const. value.

WHAT SETS THIS CONST VALUE?

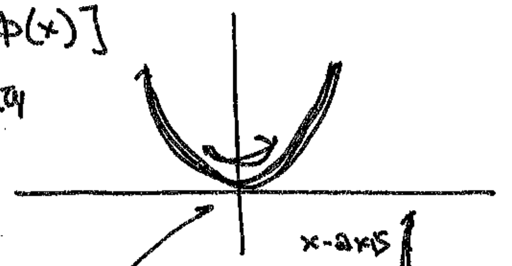
eg. MASS TERM : $\mathcal{L} = \cdots - m^2 \phi(x)^2$

for EACH x

FIELD (SPRING) @ x

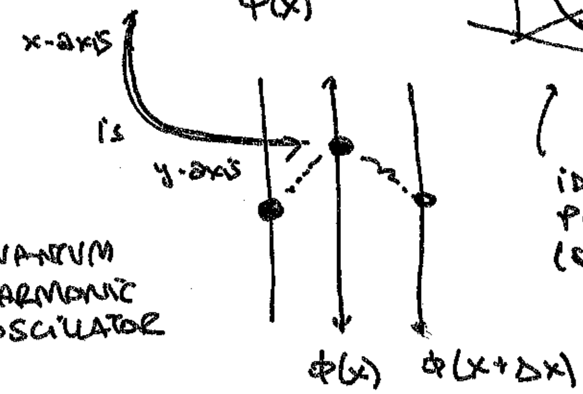
$V[\phi(x)]$

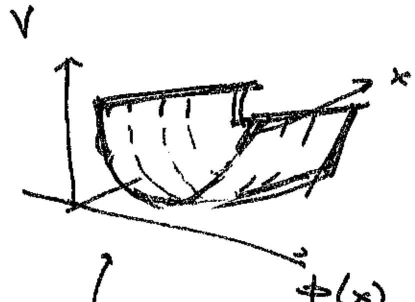
POT DENSITY



$\phi(x)$ wants to be @ zero.

"TAYLOR EXP" ~ QUANTUM HARMONIC OSCILLATOR

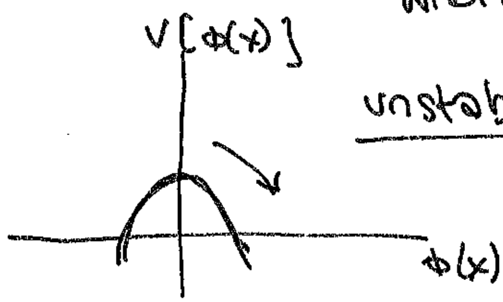




IDENTICAL POT. @ EA x (SPACETIME SYM)

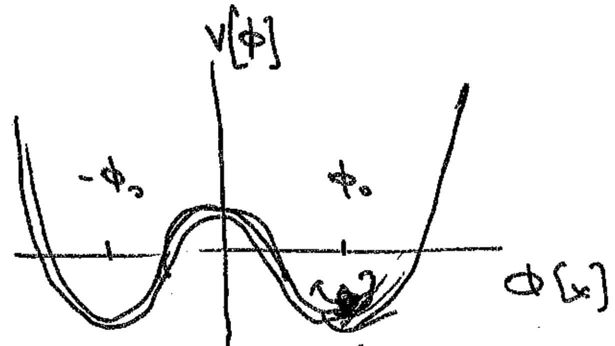
HIGHER ORDER TERMS MODIFY POTENTIAL ... BUT NEAR $\phi(x) = 0$, this term dominates.

on hw last wk: what if m^2 term had wrong sign? $p^2 = E^2 - \vec{p}^2 = -m^2$?! $\vec{p}^2 > E^2$!! → tachyon!



unstable! $\phi(x) = 0$ is not a minimum E configuration!

$\mathcal{L} = \cdots - m^2\phi^2 + \lambda\phi^4$ ← to give stable min.



PARTICLES ("a†")

minima : $\pm\phi_0$

PROB : what is ϕ_0

PARTICLES : EXCITATIONS ABOUT ϕ_0.

We write: $\langle \phi \rangle = \phi_0$ ← vacuum expec. value

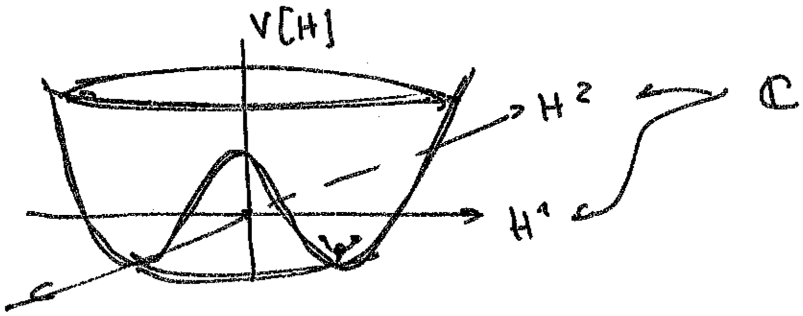
"expectation value of $\phi(x)$ is ϕ_0"

ϕ @ x

FIELD IS CLASSICALLY NON-ZERO. / spontaneous symmetry breaking

CASE of HIGGS:

$$\mathcal{L} = \cdots \underbrace{+\mu^2 |H|^2 - \lambda |H|^4}_{-V(H)}$$



$$\langle H \rangle = H_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$$

OBSERVE: $\mathcal{L}$ is TOTALLY INVARIANT

$\langle H \rangle$ is NOT.

minimum E (classical) CONFIG.

ANALOG: FERRO MAGNET: spins

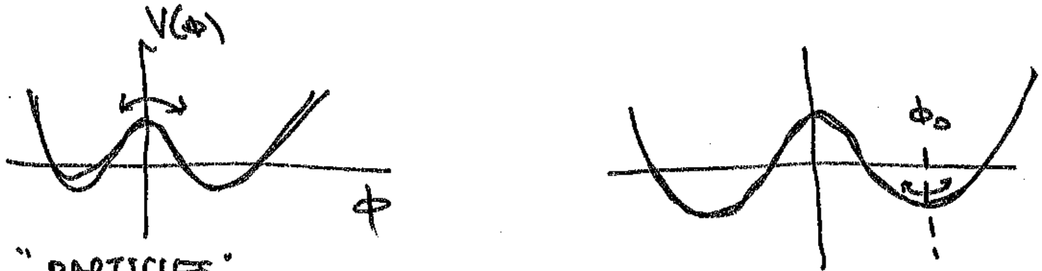
$$\begin{matrix} \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \end{matrix}$$

GROUND STATE BREAKS ROT. SYM EVEN THOUGH THY IS ROT. INVARIANT

HW: how this gives MASS.

So What

everywhere there is $H(x)$,
replace it w/ a field <u>expanded about H_0</u>



"PARTICLES"
DON'T MAKE
SENSE ABOUT $\phi = 0$

$\phi(x) \to \phi_0 + \phi(x)$

vacuum

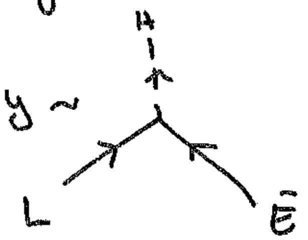
excit. about vacuum.

which <u>means</u>: if I have a term in $\mathcal{L}$

$$y \cdot H^\dagger L \bar{E} \longrightarrow \boxed{\underbrace{y H_0}_{\text{const.}} L \bar{E}} + y H L \bar{E}$$

$\boxed{\text{MASS TERM (off diagonal)}}$

$y \sim$



(Weyl arrow)

$$L \longrightarrow \underset{y H_0}{\bullet} \longleftarrow \bar{E}$$

$$\downarrow$$

$$e_L \longrightarrow \underset{y \frac{v}{\sqrt{2}}}{} \longleftarrow e_R$$

$$H_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

SM MUG

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$
$$+ i\bar{\psi}\not{D}\psi$$
$$+ |D\phi|^2$$

QUADRATIC TERMS

(D is GAUGE COV. DER)

HIGGS.
(GIVES MASS TO GAUGE BOSONS IF VEV.)

$$+ y_{ij}\psi^i\psi^j\phi \; + h.c.$$
$$- V(\phi)$$

YUKAWA
2 FERMIONS + HIGGS
GIVES MASS TO FERMIONS WHEN VEV TO HIGGS.

HIGGS POTENTIAL.
(self interactions)
... more important:
sym. breaking
from min of potential

Thoughts

- this would be good exam Q.
- all couplings here: dimensionless or Mass^2 ...
  - ↳ could write more complicated junk.

    eg ψ^4 ... but coupling would have negative mass.

    if $[g] = D$ $\quad \frac{g'}{g} \sim \left(\frac{L'}{L}\right)^{-D} \sim \left(\frac{L}{L'}\right)^{D}$

    when $L' \to 0$, $g' \to \infty$
    pert thy breaks.

- no tunneling in QFT.