- final plan ; solutions

TODAY : REN – SIDM / LUIS – BRAZIL | TOMMY – MET

THU : ADAM C / STEVEN / ELI

<u>A MODEL</u> :

- symmetry → global, gauge
- particles
- $\mathcal{L} = (\text{QUAD}) + (\text{HIGHER } \mathcal{O})$

(QUAD) → KINETIC TERMS (spacetime propag.); gauge interactions

(HIGHER $\mathcal{O}$) → POTENTIAL (vevs); other interactions

<u>SM mug</u>:

$$\mathcal{L} = -\tfrac{1}{4}F_{\cdot\cdot}F^{\cdot\cdot} + i\bar{\psi}\not{D}\psi + |DH|^2 \quad \text{(QUAD)}$$

$D = \partial + \underline{igA}$ GIVES GAUGE INTERACT.

yukawa → $+ \, y_{ij}\psi^i\psi^j H + \text{h.c.}$

$- V(H)$ ← potential for higgs

<u>notes</u>: $V(H)$ has a <u>minimum</u> @ $\langle H \rangle = H_0 \neq 0$

$$H_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\Rightarrow |DH|^2 = \cdots \underbrace{\tfrac{1}{2}g^2 v^2}_{\text{mass!}} A^2$$ ← gives mass (QUAD TERM w/ NO DERIV.)

<u>nb</u>: this is the <u>only</u> way we know to give FUNDAMENTAL SPIN-1 A MASS.

$\Rightarrow$ similarly, YUKAWA $\rightarrow y\frac{v}{\sqrt{2}}\psi\psi$ ← QUAD TERM, NO DERIV. → MASS

The Fermi Interaction (approximately)

LONG BEFORE SM, fundam. physics
$\downarrow$
$\boxed{\text{FERMI THY}}$: thy of weak int. @ energies $\ll$ 100 GeV

<u>sym</u> : SPACETIME
ELECTRIC CHARGE $\longrightarrow$ PHOTON

<u>particles</u> :

<u>proton</u>	$q = +1$	p	all spin $\frac{1}{2}$
<u>neutron</u>	$q = 0$	n	
<u>electron</u>	$q = -1$	e	
<u>neutrino</u>	$q = 0$	ν ← MASSLESS	

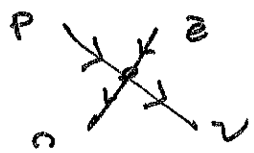
<u>$\mathcal{L}$</u> : $-\frac{1}{4}F_{..}F^{..}$

$+ i\bar{\Psi}\not{D}\Psi$

actually: $\bar{e}_L \nu_L$!

$-G_F(\bar{p}n)(\bar{e}\nu) + \text{h.c.}$

COUPLING CONST.

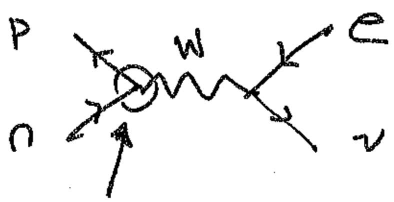


CARD?

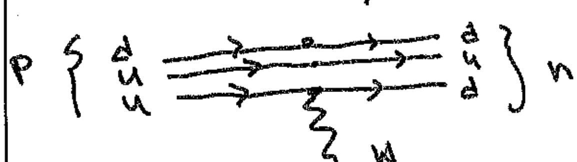
$\boxed{\text{Q}}$ what is mass dim of G_F?

① $[p] = [n] = [e] = [\nu] = 3/2$
FROM KIN TERM OF FERMION
$\rightarrow [\mathcal{L}] = 4$, so $\boxed{[G_F] = -2}$ $\sim 1/M^2$

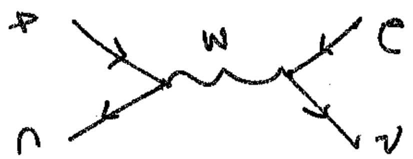
② I know where this comes from in SM!



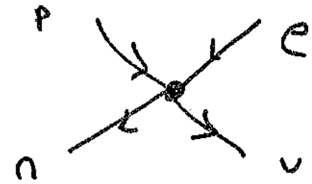
$\sim \boxed{\dfrac{g^2}{p^2 - M_W^2}}$ $\sim$ MASS DIM -2 ✓



so now I can match these theories:



$\sim$



$\sim \frac{g^2}{M_W^2}$ in $p^2 \ll M_W^2$ limit

all energies $\ll$ 100 GeV

$\sim G_F$

$$\boxed{G_F \sim \frac{g^2}{M_W^2}}$$

if W MASS HEAVIER, this int. is WEAKER

if g LARGER, this int is STRONGER

* if $E \gtrsim 100$ GeV, then DYNAMICS OF VIRTUAL W BECOME IMPORTANT, the FERMI THEORY BECOMES A POOR APPROXIMATION!!

<u>Deep Idea</u>: FERMI thy is AN <u>EFFECTIVE</u> THEORY.

MAKES SENSE for $E \ll M_W$, BUT @ HIGH E, YOU PROBE NEW <u>DYNAMICS</u>

W BOSON.

but this is disconcerting – how do we know if a theory (eg SM) is "it" or just effective??

Related to <u>MASS DIM</u> of couplings.

<u>In SM</u>: why no additional interactions?

$$y \psi \psi H + \text{h.c.}$$ } Yukawa is the <u>only</u> "additional" interaction we put in.

$[y] = 0$ ← hmm...

well, we have $-\mu^2 |H|^2 + \lambda |H|^4$

<u>other terms</u> (constrained by gauge invariance)

$$|H|^6 \, , \; (\underbrace{L^a \varepsilon_{ab} H^b}_{})^2 \, , \; \cancel{(L E)^2} \, , \; \ldots$$

$(H^\dagger L \bar{E})^2$

SU(2) singlet
U(1)$_Y$ neutral
BUT HAS SPIN

square to contract spin indices

these are all higher dim.

$[|H|^6] = 6$

$[(L\varepsilon H)^2] = 5$

$[(HLE)^2] = 8$

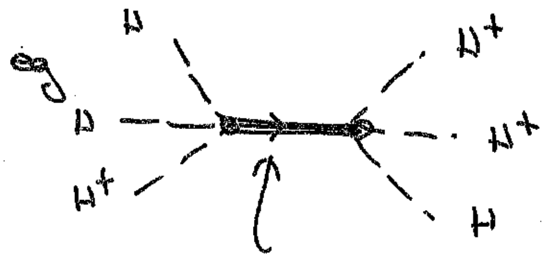
BUT $[\mathcal{L}] = 4$, so: $A |H|^6 + B (L\varepsilon H)^2 + C (HLE)^2$

$[A] = -2 \qquad [B] = -1 \qquad [C] = -4$

NEGATIVE MASS DIMENSION COUPLING

↳ looks like a heavy particle propagator

eg



$\sim \dfrac{1}{M_h^2} \leftarrow$ if $E \ll M_h$

h (what qm #'s?)

spin-0
& same qm # as H

> <u>turns out</u>: any time [coupling] < 0,
> theory is necessarily <u>effective</u>
> (<u>not</u> fundamental)

<u>WHY</u>: (this is kind of ~~deep~~)

↳ see article on allometry.

COUPLING c w/ MASS dimension $-D$

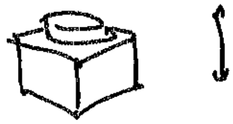
$c \sim M^{D} \sim (\text{Length})^{-D}$

SUPPOSE (as is the case) A THEORY IS DEFINED W/RT SOME "FUNDAMENTAL" SCALE, Λ or ℓ

Λ ↑ MASS, ℓ ↑ DISTANCE

any physics "smaller" THAN ℓ is "ignored"

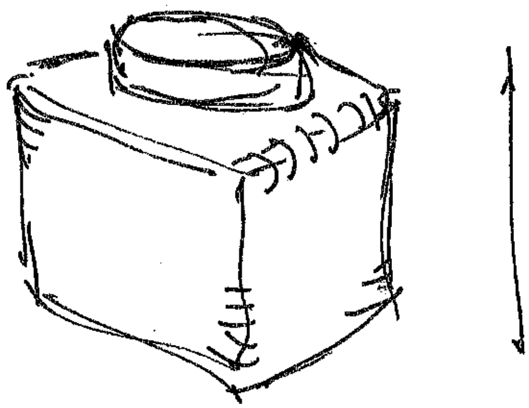
eg. UNIVERSE IS MADE OF LEGO BRICKS.
Some discrete unit of lego.



$l \sim \Lambda^{-1}$

what if we then replaced lego $\leftrightarrow$ duplo

(KIDS' VERSION W/ BIGGER BLOCKS)



$l' \sim (\Lambda')^{-1}$

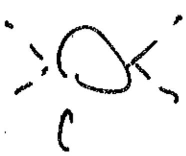
HOW DOES VALUE of COUPLING <u>CHANGE</u> IN ORDER TO DESCRIBE THE <u>SAME</u> PHYSICS?

@ large length scales $L \gg l, l'$
I should get the same physics.

BUT IN one case I include quantum effects of all energies up to Λ

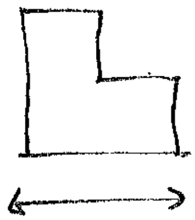
in the other I include quantum effects up to (Λ')

SO THE THEORY W/ DUPLO BLOCKS HAS A <u>DIFFERENT COUPLING</u> TO ACCOUNT FOR THE <u>SAME</u> PHYSICS



LOOP MOMENTUM UNCONSTRAINED ... goes up to Λ.

IMAGINE SOME BASELINE LENGTH SCALE IN A THEORY



← use this to build something

(theory, or "lego thing")

$L \sim M^{-1}$

$\{ L \gg \ell$ ← cutoff (fundamental lego size)

$M \ll \Lambda \sim \ell^{-1}$ ← maximum quantum momentum scale

so COUPLING GOES LIKE $\boxed{c \sim M^{D} \sim L^{-D}}$

BUT THIS GIVES FUNDAMENTAL SCALE $\ell \sim \Lambda^{-1}$

(cf. fundamental speed, c
ang. momentum, $\hbar$)

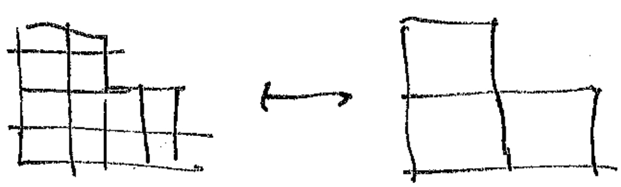
↳ conversion factor!

can make c DIM-LESS BY COUNTING IN UNITS OF ℓ!

$c \sim 1/L^{D} \to (\ell/L)^{D}$

$\boxed{c \sim (L/\ell)^{-D}}$

if we change ℓ, we should still get some "large scale" physics (low energy)



ℓ $\qquad\qquad$ ℓ'

changing ℓ from lego size to duplo size.

$\Rightarrow \boxed{\dfrac{c'}{c} \sim \left(\dfrac{\ell'}{\ell}\right)^{D}}$ → so if ℓ' gets smaller (more fundamental?), and D is positive → coupling gets smaller / less important.

<u>makes sense</u> : in SM, "couplings" w/ $\Delta > 0$ are masses

eg $m^2 A^2$, $m\psi\psi$

As $E \gg m$, the masses don't do much.

$$\frac{1}{p^2 - m^2} \to \frac{1}{p^2}$$

$\boxed{\text{BUT}}$ if $\boxed{\Delta < 0}$

then defining the theory @ <u>smaller</u> fundamental length scales l

$\to \frac{c'}{c}$ gets <u>bigger</u>

BUT c is a COUPLING...
IT CONTROLS OUR PERTURBATION THEORY!

if c'/c gets big, <u>our pert. thy breaks!</u>

theory does not make sense.
↳ new dynamics must show up.

<u>think about this</u> :

$$[G_N] = -2$$