ELECTROWEAK LEPTONIC SECTOR

- spacetime symmetry $\left\{ \begin{array}{l} \text{translation} \\ \text{Lorentz} \end{array} \right.$

- U(1) Hypercharge
- SU(2) Weak

$\mapsto$ GAUGE SYMMETRIES — these are special ← new

remark: GAUGE SYM $\Leftrightarrow$ FORCE (fundamental)

↳ $\exists$ spin-1, massless particle that talks to things charged under the gauge symmetry

further, the spin-1 is in the ADJOINT REP.

so:

U(1): $B_\nu \sim\sim\sim B_\mu$ $Y=0$ ← B is not itself charged

SU(2): $W_\nu^B \sim\sim\sim W_\mu^A$ ← Adjoint of SU(2) → triplet

remark: SO BEFORE WE'VE MADE ANY CHOICES ABOUT PARTICLE CONTENT, WE AUTOMATICALLY GOT 2 PARTICLES (really 4) FOR FREE!

particles

LH LEPTON DOUBLET $\quad (L^\dagger)^{\dot\beta}_b \xrightarrow{Y=-\frac{1}{2}} L^{\alpha a}$ $L = \begin{pmatrix} L^{a=1} = \nu_L \\ L^{a=2} = e_L \end{pmatrix}$

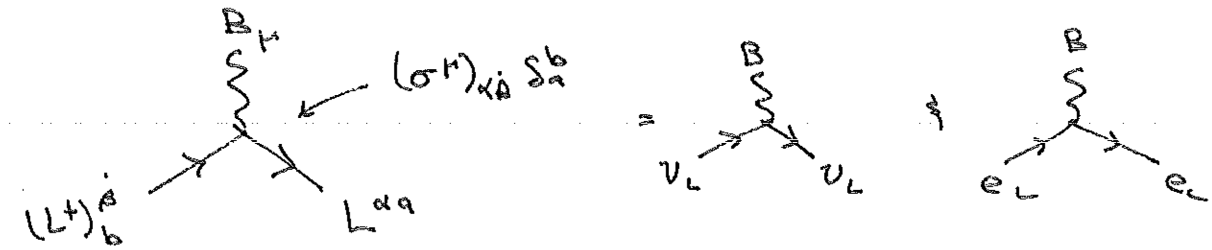
RH (ANTI)-ELECTRON $\quad (\bar{E}^\dagger)^{\dot\beta} \xrightarrow{Y=+1} (\bar{E})^\gamma$

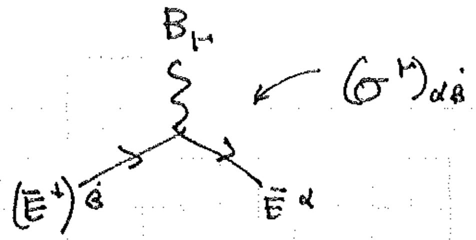
Higgs Doublet $\quad (H^\dagger)_b \xrightarrow{Y=+\frac{1}{2}} H^a$ ← $H = \begin{pmatrix} H^1 \\ H^2 \end{pmatrix}$

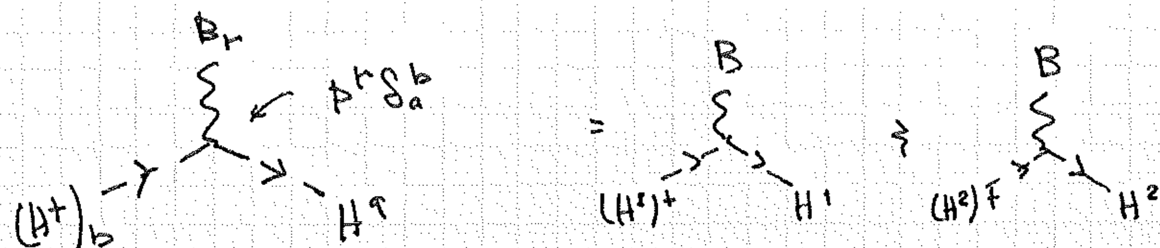
Card: What are the Feynman rules?

Drawing the rules

- Hyperchage boson talks to hypercharged particles (↙ all of them)

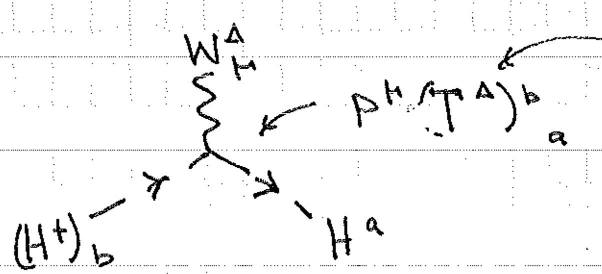






- ok. THIS ALL LOOKS LIKE QED.
- W^A ARE MORE INTERESTING. TALKS TO SU(2) CHARGED OBJECTS: L & H

HW: ACTION OF SU(2) ON SU(2)? (EC.)



$$T^A = \frac{1}{2}\begin{pmatrix} & 1 \\ 1 & \end{pmatrix},\ \frac{1}{2}\begin{pmatrix} & -i \\ i & \end{pmatrix},\ \Big\}\ \text{off diag.}$$
$$\frac{1}{2}\begin{pmatrix} 1 & \\ & -1 \end{pmatrix}\ \}\ \text{diag!}$$

observe & recall (HARMONIC OSCILLATOR)
CAN DEFINE:

$$T^+ \equiv \tfrac{1}{2}\sigma^+ \equiv \tfrac{1}{2}\cdot(\sigma^1 + i\sigma^2) = \begin{pmatrix} & 1 \\ 0 & \end{pmatrix}$$

$$T^- \equiv \tfrac{1}{2}\sigma^- \equiv \tfrac{1}{2}(\sigma^1 - i\sigma^2) = \begin{pmatrix} & 0 \\ 1 & \end{pmatrix}$$

Why this should look familiar:

$$T^+ \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

"$|\downarrow\rangle$" "$|\uparrow\rangle$"

RAISING OPERATOR

$$T^- \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

LOWERING OP.

$$T^3 \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} a \\ -b \end{pmatrix}$$

⟵ funny "H operator" ~~ASKS: WHAT IS YOUR CHARGE UNDER SU(2)?~~

unsurprisingly, these are related by commutation relations ...

... in turn, related to how W's talk to each other.

REMARK: the raising/lowering structure in the T^A matrices is an intimate part of what it means to be non-Abelian.

SU(3) is more complicated. other groups are more complicated still ...

but the structure of the symmetry is encoded in how the T's commute w/ each other.

and they decompose into raising/lowering in more [internal] directions.

In the same way, one may write

$$W^\pm = \underbrace{(W^1 \pm i W^2)}_{\mathbb{R}} / \sqrt{2}$$

normalize

$\mathbb{C}$! ⟶ can have charge

Why would you do this? We'll see... but @ this level we're just picking a weird basis.

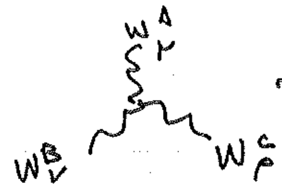
WHAT DOES THE ± CHARGE REFER TO?

W^1 ± i W^2 ← like $|W^1\rangle \pm i|W^2\rangle$

mixed state

neither of these have Y charge
so ± isn't Y charge.

<u>BUT</u> we know

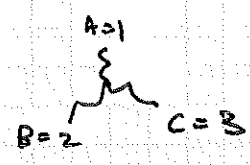


$\sim \varepsilon^{ABC} \underbrace{(p^\mu \eta^{\nu\rho} + \dots)}$

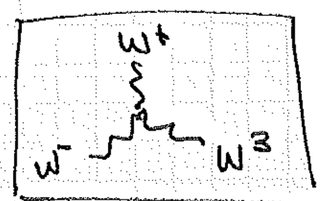
details here don't matter, only that we can form an invariant

totally antisym.

so:



→



AH! IT LOOKS LIKE $W^\pm$ IS CHARGED UNDER W^3.

remember this week's homework:

$$(T^3)^a{}_b \sim \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$

acts on doublets like a charge on each component

$$e^{i\theta T^3} \begin{pmatrix} H^1 \\ H^2 \end{pmatrix} = \begin{pmatrix} e^{i\theta} H^1 \\ e^{-i\theta} H^2 \end{pmatrix}$$

in fact: ABELIAN!

<u>observe</u>: there is a U(1) symmetry inside SU(2)

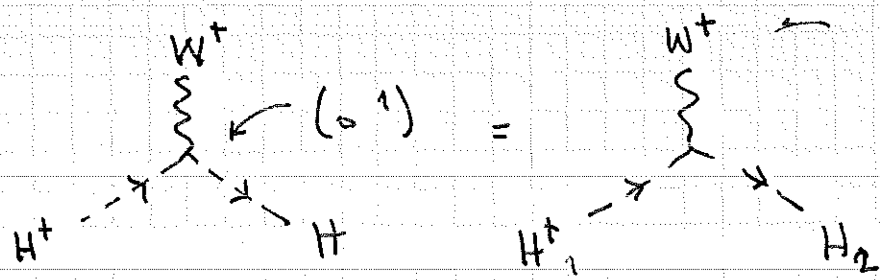
THUS FAR :

- nothing has mass
  $\rightarrow$ GAUGE sym $\Rightarrow$ spin-1's ARE MASSLESS
- fermions (spin-½) dont have mass because their chiral quantum #'s are respected
  $\hookrightarrow$ α & $\dot{\beta}$ indices are separate.
- Higgs (spin-0) could have mass. we're not touching that yet.
- We have chosen a weird basis for W's
  $\hookrightarrow$ $W^{1,2,3} \rightarrow W^{\pm}, W^3$

  breaks the obvious sym. but @ this point, sym is still there.

HOW THE $W^{\pm}$ INTERACTS W/ DOUBLETS :

$$(T^{\pm})^a{}_b = \begin{pmatrix} & 1 \\ 0 & \end{pmatrix} \text{ or } \begin{pmatrix} & 0 \\ 1 & \end{pmatrix}$$



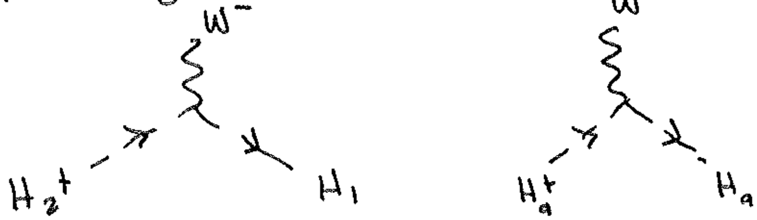
+1 CHARGE UNDER W^3

+½ CHARGE UNDER W^3

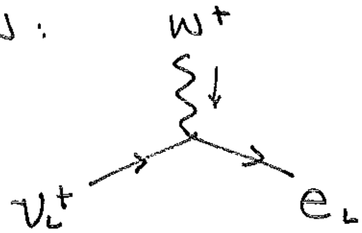
$$\text{So } H = \begin{pmatrix} H^1 \\ H_2 \end{pmatrix}$$

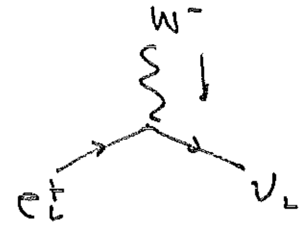
-½ CHARGE UNDER W^3

similarly :



for LEPTON:





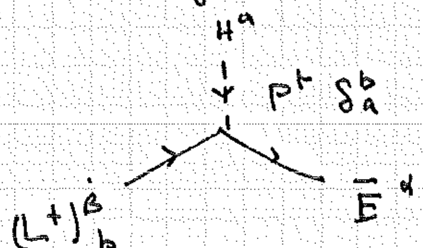
now we're getting somewhere! this looks like a theory that we've seen before.

<u>QUESTION</u>: what are allowed Higgs - fermion 3-particle interactions?

<u>LORENTZ</u>: either $\sigma^\mu{}_{\alpha\dot\beta}$ OR δ^α_β $\delta^{\dot\alpha}_{\dot\beta}$

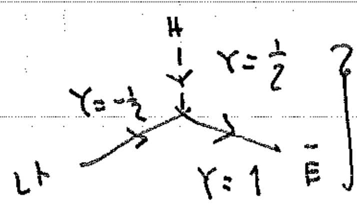
$\varepsilon_{\alpha\beta}$ $\varepsilon_{\dot\alpha\dot\beta}$

μ REQUIRES SOME 4-MOMENTUM (as you know from HW)



LOOKS GOOD for LORENTZ & SU(2) (as you know from HW!)

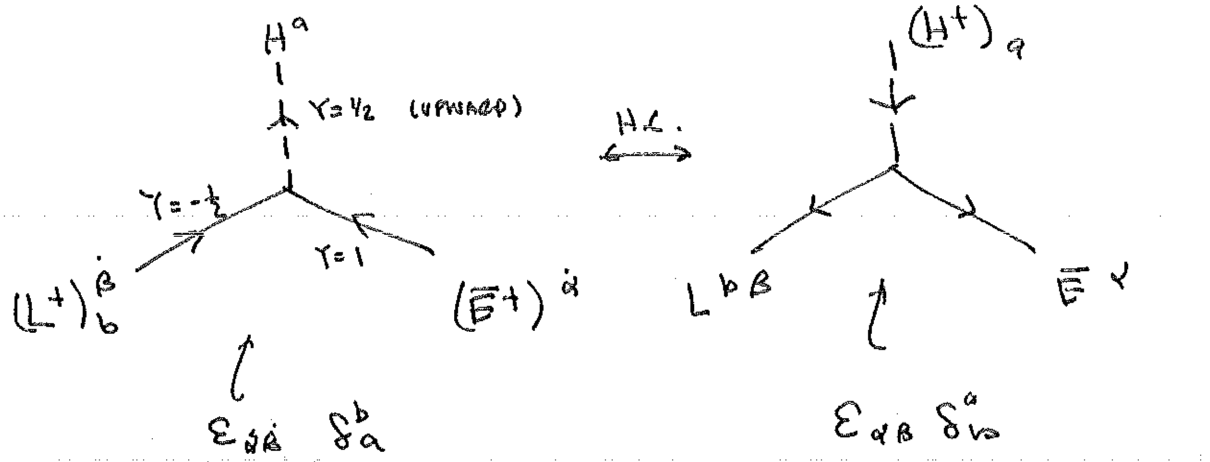
<u>BUT</u>: HYPERCHARGE?



? does not work! Y <u>not</u> conserved!!

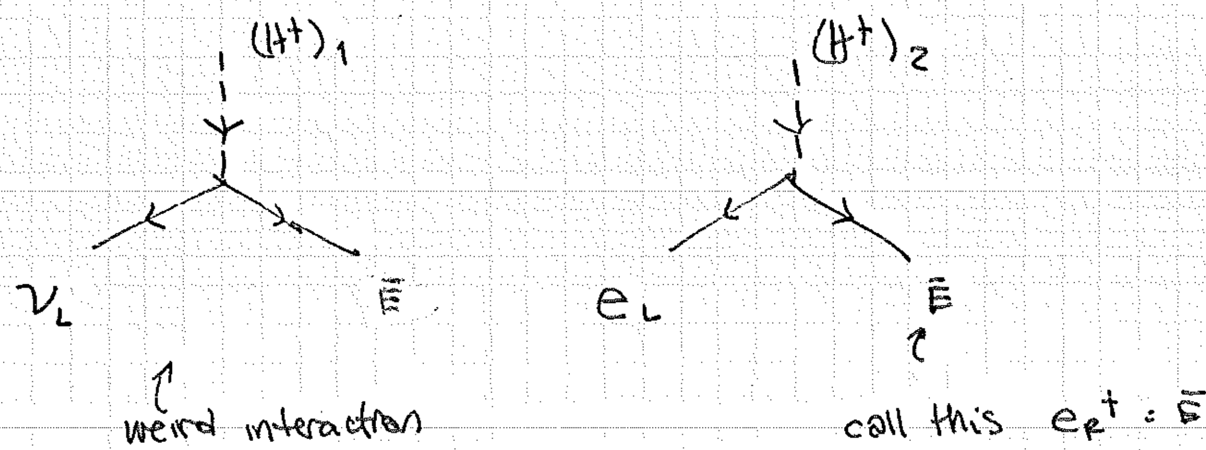
so this vertex is <u>not</u> allowed.

<u>Other Lorentz structure</u> contract chiral spinors



obs: arrows on spinors hit each other!

LET'S UNPACK THE DOUBLETS (not shown: H.c.)



very interesting

connects LEFT HANDED "electron" to RH. "electron"

We haven't yet shown that these are our favorite particles IRL

HERE'S WHERE IT GET'S CRAZY – <u>the</u> BIG TWIST OF THE CLASS

<u>new rule</u>: the Higgs line can end.

$$H^{+}_{b} \; - \rightarrow - \times \quad \langle H \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

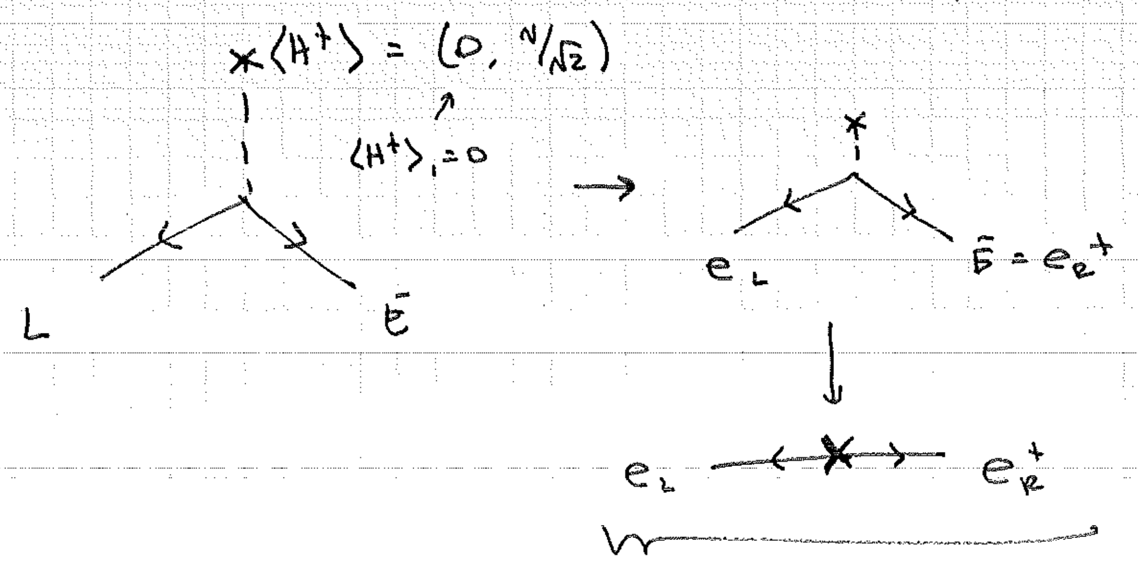
vacuum expectation value

$$(0 \quad v/\sqrt{2}) \; - \rightarrow \; - H^{?}$$

<u>SPECIFICALLY</u>: $H = \begin{pmatrix} a + ib \\ c + id \end{pmatrix}$

this thing "ends"

IMPLICATIONS



in some frames its LH, in others it is RH ... so chirality not a good quantum #!

("chiral sym breaking")

mixes $e_L \longleftrightarrow e_R^{+}$
(MIXING CHIRALITIES)

SPIN 1/2 LH $\longleftrightarrow$ RH

... this is MASS!