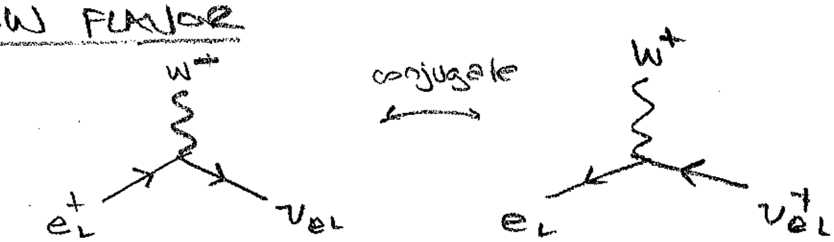
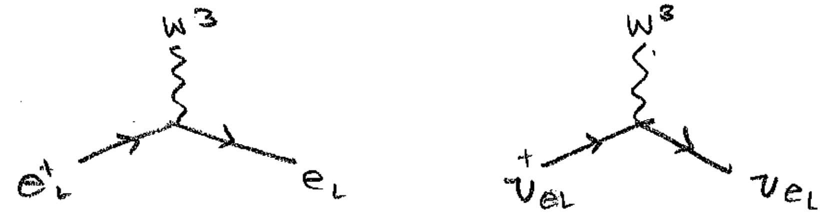
1. EW FLAVOR

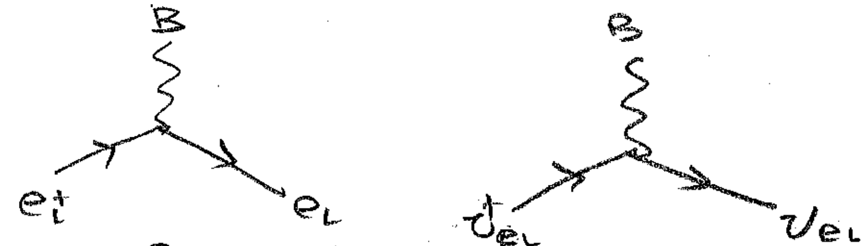
1.1



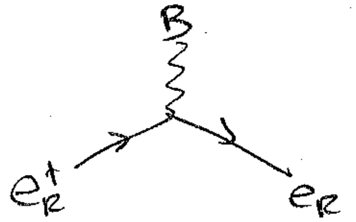
$W^\pm$ GOES BETWEEN DOUBLET COMPONENTS



W^3 connects the same component to itself

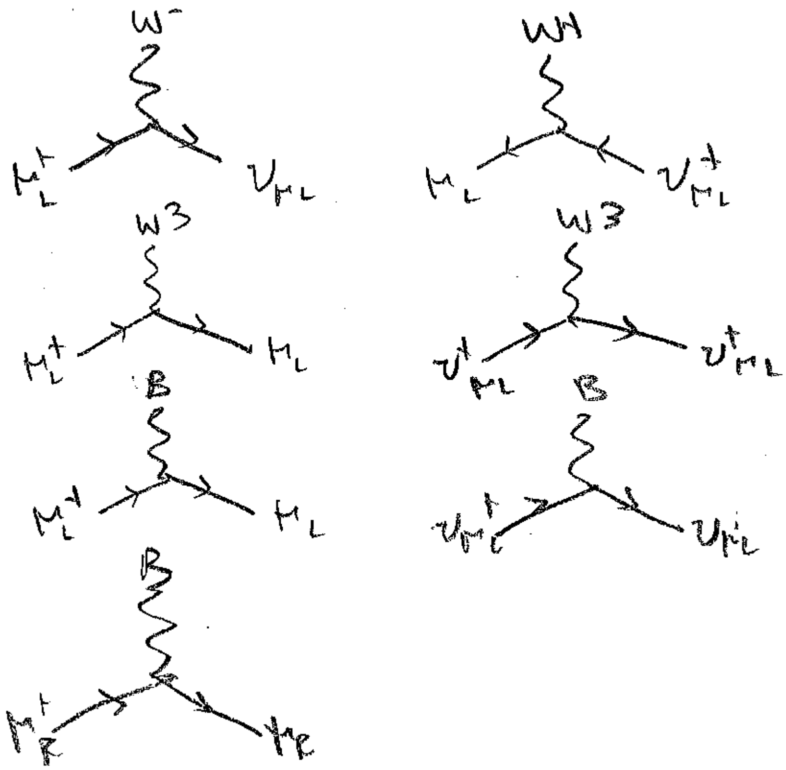


L HAS $Y = -1/2$, So B talks to each component

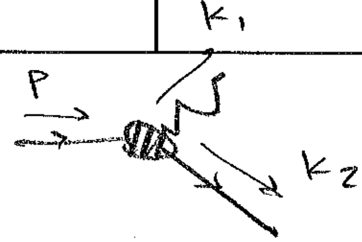


e_R HAS $Y = -1$ SO B TALKS TO IT.

THE MUON VERTICES ARE IDENTICAL ($e \to \mu$)



1.2



IF ALL PARTICLES MASSLESS, THEN THE 4-MOMENTA ALL HAVE:

$E_p = |\vec{p}|$

$E_1 = |\vec{k}_1|$ $\qquad$ $E_2 = |\vec{k}_2|$

FOR k_1 not collinear w/ k_2, YOU CAN ALWAYS GO INTO THE 2-BODY REST FRAME, WHERE

$k_1 + k_2 = (E_{12}, \vec{0})$

then $(k_1 + k_2)^2 = (E_{12})^2$

BUT $P = k_1 + k_2$

$p^2 = 0$ if massless.

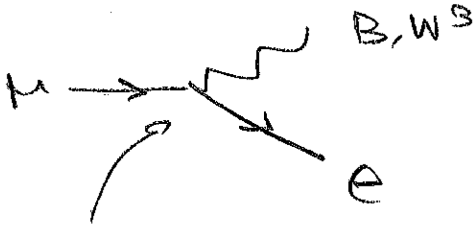
IF ok: you could have $\vec{k}_1$ parallel to $\vec{k}_2$,

eg: $P = (E_p, 0, 0, |\vec{p}|)$

$K_1 = K_2 = (\frac{1}{2}E_p, 0, 0, \frac{1}{2}|\vec{p}|)$

then $p = k_1 + k_2$

1.3 NOT POSSIBLE



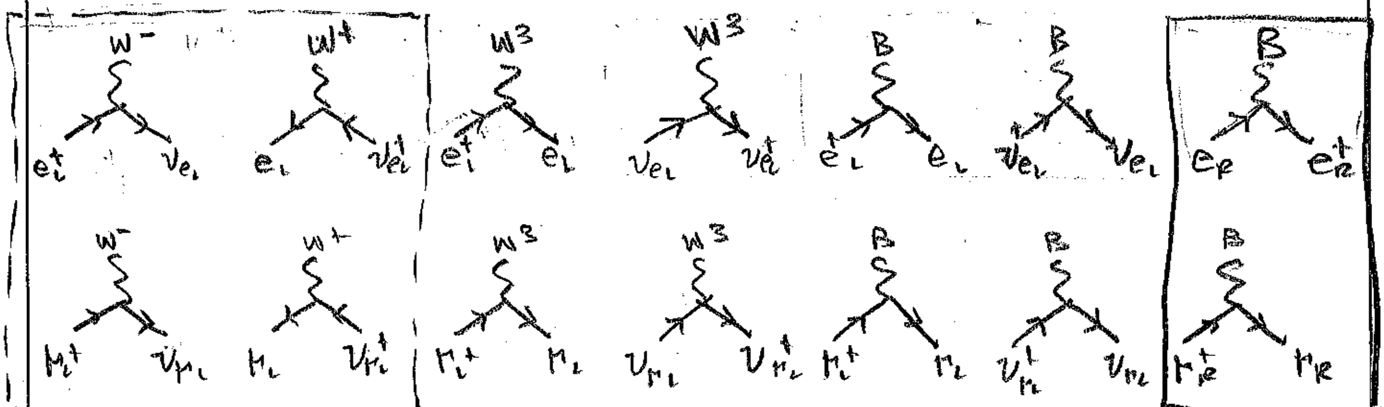
no vertex like this

The gauge interactions connect particle & antiparticle

(W± may connect e_L & ν_{eL} but that's because e_L & ν_L are both part of same $L_{(e)}$ doublet)

thus: no way to convert a μ line to a e line using only spin-1 vertices.

1.4 THE GAUGE FEYNMAN RULES ARE



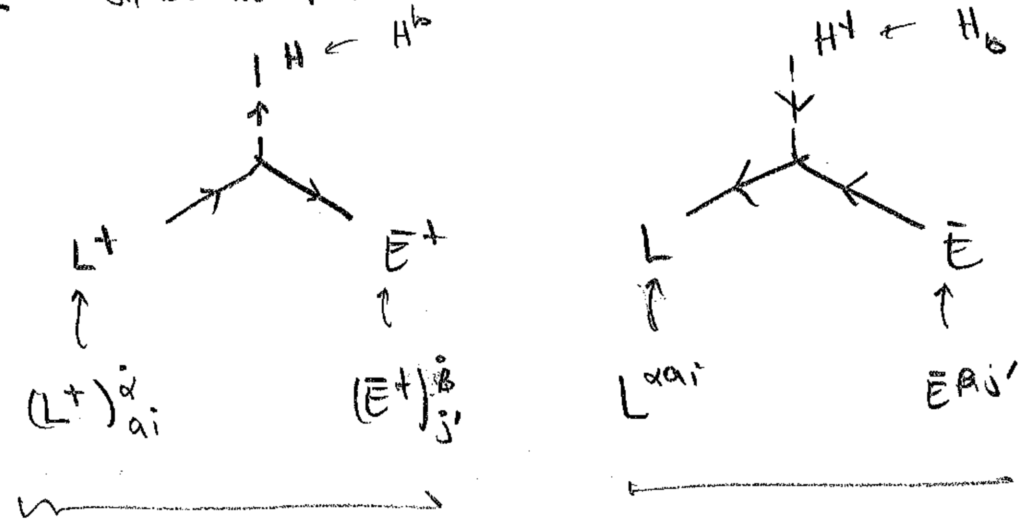
if you just rotate $e_L \leftrightarrow \mu_L$, these rules get messed up. [similarly if you just rotate $\nu_{eL} \leftrightarrow \nu_{\mu L}$]

IF YOU "ROTATE" $e \leftrightarrow \mu$ AMONG LEFT-HANDED, THESE RULES STAY THE SAME

IF YOU ROTATE $e_R \leftrightarrow \mu_R$, THESE RULES STAY THE SAME

two separate SU(2) sym.

1.5 SHORT HW 4



you can contract the spin ($\epsilon_{\dot\alpha\dot\beta}$), SU(2) ($\delta^a_b$) indices

ditto

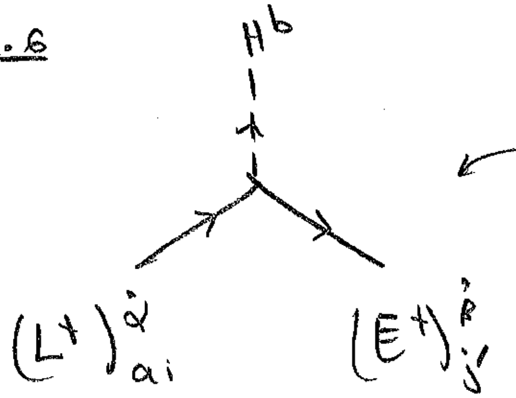
BUT LEFT OVER w/ TWO LEFTOVER INDICES i, j'

$SU(2)_L$ $\quad$ $SU(2)_E$

DIFFERENT SYMMETRIES; CANNOT CONTRACT.

So $SU(2)_L \times SU(2)_E$ symmetry is incompatible w/ the Yukawa term.

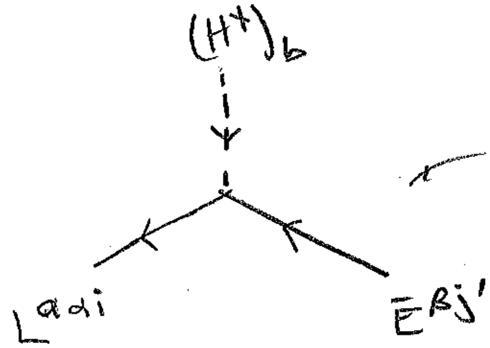
1.6



$$\varepsilon_{\dot\alpha\dot\beta}\;\delta^a_b\;(y^\dagger)^{ij'}$$

SPIN — SU(2) — $SU(2)_L \times SU(2)_E$ FLAVOR SYM.

$$\varepsilon_{\alpha\beta}\;\delta^b_a\;y_{ij'}$$



1.7 $\quad L^i y_{ij'} \bar{E}^{j'} \rightarrow L^i (U_L)^k{}_i\, y_{k\ell'}\, (U_E)^{\ell'}{}_{j'}\, \bar{E}^{j'}$

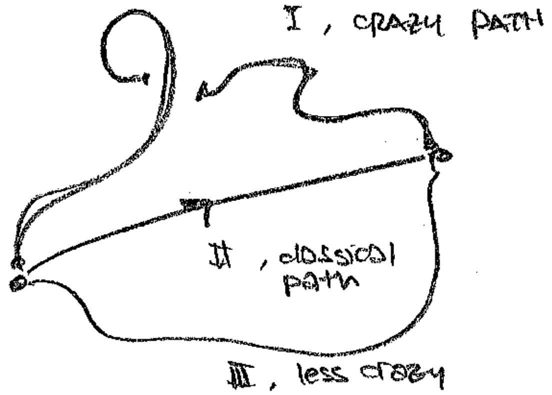
- $y_{ij'}$ can be diagonalized by judicious choice of U_L & U_E matrices.

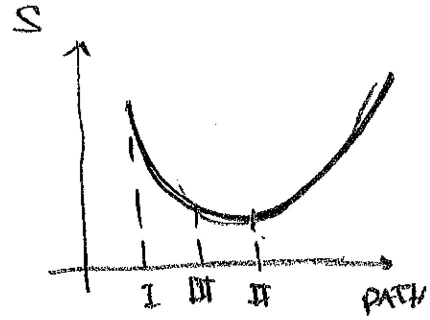
  (CAN DIAGONALIZE A ? MATRIX W/ A BIUNITARY TRANSFORMATION)

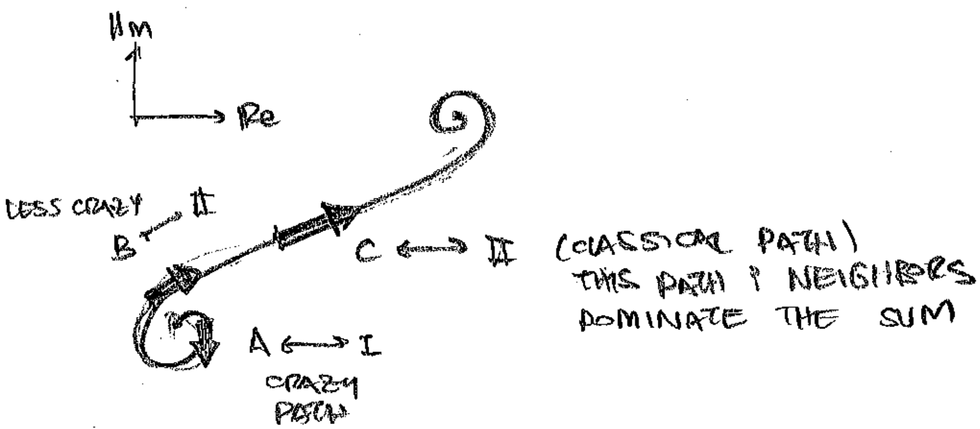
- IN GENERAL, YOU CANNOT MAKE $U_L y U_E \propto \mathbb{1}$.

2 see attached notes.

3







P165: These are lecture notes from P231 (Fall 2017), provided as a hint for P165 (Win 2018) homework #4b.

GAUSSIAN INTEGRALS

something different. We will relate to DIFF EQ. & PROBABILITIES @ END.

REF: Zee, QFT in a Nutshell APPENDIX 1

$$G = \int_{-\infty}^{\infty} dx\, e^{-\frac{1}{2}x^2} \quad \leftarrow \text{ how to solve?}$$

TRICK:

$$G^2 = \int \underbrace{dx\,dy}_{r\,dr\,d\theta}\, e^{-\frac{1}{2}\underbrace{(x^2+y^2)}_{r^2}}$$

let $u = \frac{1}{2}r^2$, $du = r\,dr$

$$= \int_0^{\infty} du\, e^{-u}\,(2\pi)$$

$$= (-e^{-\infty}+1)(2\pi) = 2\pi \Rightarrow \boxed{G = \sqrt{2\pi}}$$

SIMILARLY: SUPPOSE $x \equiv \sqrt{a}\,y$

$$G = \int_{-\infty}^{\infty} \sqrt{a}\,dy\, e^{-\frac{1}{2}ay^2} = \sqrt{2\pi}$$

$$\rightarrow \int_{-\infty}^{\infty} dy\, e^{-\frac{1}{2}ay^2} = \boxed{\sqrt{\frac{2\pi}{a}}}$$

could guess $a^{-1/2}$ from dimensional analysis.

MORE VARIANTS

$$\int_{-\infty}^{\infty} dx \; e^{\underbrace{-\frac{1}{2}ax^2 + Jx}}$$

COMPLETE THE SQUARE

$$= -\left(\frac{a}{2}\right)\left(x^2 - 2Jx/a\right) = -\left(\frac{a}{2}\right)\underbrace{\left(x - \frac{J}{a}\right)}_{y = x - J/a}^2 + \frac{J^2}{2a}$$

$$= \underbrace{\int_{-\infty}^{\infty} dy \; e^{-\frac{1}{2}ay^2}}\; \underbrace{e^{J^2/2a}}_{\text{const}}$$

$$= \boxed{\sqrt{\frac{2\pi}{a}}\; e^{J^2/2a}}$$

IN N DIMENSIONS : N×N SYMMETRIC MATRIX A_{ij}

invertible

$$\int_{-\infty}^{\infty} dx_1\, dx_2 \cdots dx_N \; e^{-\frac{1}{2}\underbrace{\underline{x}^T \cdot \underline{A} \cdot \underline{x}}_{x_i A^i{}_j x^j} + \underbrace{\underline{J} \cdot \underline{x}}_{J_i x^i}}$$

LET $\underline{A}$ BE DIAGONALIZED BY AN ORTHOGONAL TRANSFORMATION, $\underline{R}$

$$\underline{A} = \underline{R}^{-1} \cdot \underbrace{\hat{\underline{A}}}_{\text{DIAGONAL}} \cdot \underline{R} \qquad \hat{A} = \begin{pmatrix} \hat{a}_1 & & & \\ & \hat{a}_2 & & \\ & & \hat{a}_3 & \\ & & & \ddots \end{pmatrix}$$

THEN CHANGE VARIABLES : $\underline{y} \equiv \underline{R} \cdot \underline{x}$

MEASURE UNCHANGED

$$\prod_i dx_i = \prod_i dy_i$$

$$= \int_{-\infty}^{\infty} dy_1 \cdots dy_N \; e^{-\frac{1}{2}\underbrace{\underline{y} \cdot \hat{\underline{A}} \cdot \underline{y}}_{\hat{a}_{11} y_1^2 + \hat{a}_{22} y_2^2 + \cdots} + \underbrace{\underline{J} \cdot \underline{R}^{-1}}_{\underline{J}'} \cdot \underline{y}}$$

$$J'_1 y_1 + J'_2 y_2 + \cdots$$

$$= \left(\int dy_1\, e^{-\frac{1}{2}\hat{a}_{11} y_1^2 + J'_1 y_1}\right)\left(\int dy_2 \cdots\right)\cdots$$

$$= \prod_i^N \sqrt{\frac{2\pi}{\hat{a}_{ii}}}\, e^{(J'_i)^2/2\hat{a}_i}$$

$$\sum_i (J'_i)^2 = \underline{J}'\cdot\underline{J}' = \underline{J}^T \underline{R}^T \cdot \underline{R}' \underline{J} = \underline{J}^T \cdot \underline{J}$$

I'LL BE EXPLICIT W/ T FOR THIS LINE

note! $J^T \to J$

$$\prod_i^N \hat{a}_i = \det \hat{A} = \det A$$

FURTHER :

$$\sum_i (J'_i)\frac{1}{\hat{a}_i} = \underline{J}^T \cdot \underline{R}^T (\underline{\hat{A}})^{-1} \underline{R}\, J$$

$$\underline{A}^{-1} = \underline{R}^{-1} \cdot \underline{\hat{A}}^{-1} \cdot \underline{R}$$

$$\sum_i (J'_i)\frac{1}{\hat{a}_i} = \boxed{\underline{J}\cdot\underline{A}^{-1}\cdot\underline{J}}$$

$$= \boxed{\sqrt{\frac{(2\pi)^N}{\det \underline{A}}}\, e^{\frac{1}{2}\underline{J}\cdot\underline{A}^{-1}\underline{J}}}$$

cute result. so what?

Gaussians show up all the time as distributions

WE OFTEN TAKE MOMENTS OR CORRELATION FUNCTIONS OF DISTRIBUTIONS.

eg. $\int_{-\infty}^{\infty} dx \; x \; e^{-\frac{1}{2}ax^2} = 0$ by symmetry

NEGATIVE PART CANCELS POSITIVE

eg $\int_{-\infty}^{\infty} dx \, x^2 \, e^{-\frac{1}{2}ax^2} = ?$

LIKE AN EXPECTATION OF x^2

OBSERVE: $? = -2 \frac{d}{da} \boxed{\int_{-\infty}^{\infty} dx \; e^{-\frac{1}{2}ax^2}}$

$$= -2 \frac{d}{da} \sqrt{\frac{2\pi}{a}}$$

$$= \sqrt{2\pi} \; \boxed{a^{-3/2}}$$

COULD GUESS FROM DIM. ANALYSIS

DEFINE EXPECTATION VALUE OF x^2 WRT DIST

$$\langle x^2 \rangle = \frac{1}{Z} \left(-2 \frac{d}{da}\right) Z$$

$$Z = \int dx \; e^{-\frac{1}{2}ax^2}$$

$$= \boxed{\frac{1}{a}}$$

SIMILARLY

$$\langle f(x^2) \rangle = \frac{1}{Z} \underbrace{f\left(-2 \frac{d}{da}\right)}_{\text{as taylor expansion}} Z$$

when we take the INTEGRAL W/ A SOURCE:

$$Z = \int dx \; e^{-\frac{1}{2}ax^2 + Jx}$$

$$\sqrt{\frac{2\pi}{a}} \, e^{J^2/2a}$$

$$\langle x \rangle = \frac{1}{Z} \int dx \; x \; e^{-\frac{1}{2}ax^2 + Jx} = \frac{1}{Z} \frac{d}{dJ} Z = \frac{J}{a}$$

J IS A SOURCE FOR x

LET ME NOW CHANGE VARIABLES FROM x TO q
CONSIDER OUR MOST COMPLICATED GAUSSIAN

$$\underbrace{Z}_{\text{some kind of distribution}} = \int \underbrace{dq_1 \cdots dq_N}_{} \, e^{-\frac{1}{2} q \cdot A \cdot q + J \cdot q} = \sqrt{\frac{(2\pi)^N}{\det A}} \, e^{\frac{1}{2} J \cdot A^{-1} \cdot J}$$

some kind of distribution

bunch of objects like sequence of coupled springs that have heights distributed according to Z

Z encodes physics

THEN CAN ASK: $\langle q_i q_j \rangle = \frac{1}{Z} \int dq_1 \cdots dq_N \, (q_i q_j) \, e^{\cdots}$

What is the correlation btwn spring i & j?
IF SPRING i IS SQUISHED, IS SPRING j LIKELY TO BE

- SQUISHED ALSO $(\langle q_i q_j \rangle > 0)$
- PULLED INSTEAD $(\langle q_i q_j \rangle < 0)$
- COMPLETELY INDEP $(\langle q_i q_j \rangle = 0)$

$$\langle q_i q_j \rangle = \frac{1}{Z} \frac{d}{dJ_i} \frac{d}{dJ_j} Z$$

$$= \frac{1}{Z} \sqrt{\frac{(2\pi)^N}{\det A}} \frac{d}{dJ_i} \frac{d}{dJ_j} \exp\left[\frac{1}{2} J_a A^{-1}_{ab} J_b\right]$$

$$\cdots + \frac{1}{2} J_i A^{-1}_{ij} J_j + \frac{1}{2} J_j A^{-1}_{ji} J_i + \cdots$$

(no sum over repeated)

nb: $A^{-1}_{ij} = A^{-1}_{ji}$ BY SYM. OF A

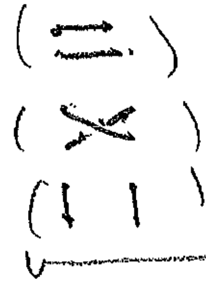
$$= \underbrace{\left(\frac{1}{Z} \left(\frac{(2\pi)^N}{\det A}\right) e^{\frac{1}{2} J A^{-1} J}\right)}_{=1} \cdot \boxed{A^{-1}_{ij}}$$

SO (A^{-1}_{ij}) TELLS YOU HOW INFORMATION AT q_i PROPAGATES TO q_j.

YOU CAN ALSO CALCULATE

$$\begin{pmatrix} i & \bullet & \bullet & j \\ k & \bullet & \bullet & l \end{pmatrix}$$

$$\begin{aligned} \langle x_i x_j x_k x_l \rangle &= A^{-1}_{ij} A^{-1}_{kl} \\ &+ A^{-1}_{il} A^{-1}_{jk} \\ &+ A^{-1}_{ik} A^{-1}_{jl} \end{aligned}$$



? PROTO FEYNMAN DIAGRAMS

4-point correlation

breaks into pairs of 2-point correl.

YOU MAY RECOGNIZE Z AS A PARTITION FUNCTION IN STATISTICAL MECHANICS.

$$Z = \sum_i e^{-\beta E_i}$$

promoted to an integral

EXACT SAME STRUCTURE CARRIES OVER TO QUANTUM MECHANICS

↳ quantum randomness ~ thermal randomness

<u>CLAIM</u>: $Z = \int dq_1 dq_2 \cdots e^{iS(q)}$

$q_i = q(t_i)$

$$S = \int_0^T dt \sum m\dot{q} - V(q)$$

HOW TO DO THESE INTEGRALS:

$$I = \int dq \, e^{-\frac{1}{\hbar} f(q)}$$

$\hbar$ SMALL PARAM

$$f(q) = \underset{\substack{\uparrow \\ \text{MINIMUM}}}{f(a)} + \frac{1}{2} f''(a)(q-a)^2 + \dots$$

$$= \int dq \, e^{-\frac{1}{\hbar} f(a)} \, e^{-\frac{1}{\hbar}\frac{1}{2} f''(a)(q-a)^2} \, e^{\cdots}$$

$$= e^{-\frac{1}{\hbar} f(a)} \sqrt{\frac{2\pi\hbar}{f''(a)}} \, e^{O(\hbar^{1/2})}$$

for $\vec{q}$

$$= \underbrace{e^{-\frac{1}{\hbar} f(\vec{a})} \sqrt{\frac{(2\pi\hbar)^N}{\det f''(\vec{a})}}}_{\text{can do quadratic part.}} \, e^{O(\hbar^{1/2})}$$

WAIT: WHAT ABOUT i?! $Z = \int \mathcal{D}q \, e^{iS}$

PHASOR: STILL DOMINATED BY STEEPEST DESCENT.

RELATION OF $\mathbb{R}$ & $\mathbb{I}\mathrm{m}$ IS USED OFTEN

↓

Analytic continuation

EXTREMUM OF S → EULER-LAGRANGE EQ.

→ WHERE ALL OF OUR GREEN'S FUNCTIONS COME FROM.

SERIES OF SPRINGS (like a bed mattress)

$$L = \frac{1}{2}\sum_i m\dot{q}_i^2 - \sum_{ij} K_{ij}\, q_i q_j$$

$[q(t) - q_i(t-\varepsilon)]^2$

usually only nearest neighbors to good approx (COUPLED H.O.)

$$\frac{1}{2} K_{ij} (q_i - q_j)^2$$

$$\downarrow$$

$$\frac{1}{2} K (q(t,x) - q(t, x-\varepsilon))^2$$

$$L = \int dx \;\; \frac{1}{2}\dot{q}(t,x)^2 - \frac{1}{2} q'(t,x)^2$$

UP TO NORMALIZ

$$S = \int dx\, dt \; \frac{1}{2}\Big[\underbrace{(\partial_t q)^2 - (\partial_x q)^2}_{(\partial q)^2} \Big]$$

WHERE THE WAVE EQ COMES FROM IT'S ALL SHO.

VARIATIONAL PRINCIPLE GIVES US KG EQ: $\boxed{\partial^2 q = 0}$

WHEN THERE IS A SOURCE

$$Z = e^{iS + iJq} \quad \leftarrow \text{up to factors of } i$$

$$\downarrow$$

$$\boxed{\partial^2 q = J}$$

We've sketched a functional approach to this problem.