TODAY: Quantum Review
FEYNMAN RULES : APPLYING KINEMATICS, RED VARIABLES
OFF SHELL

- COLLECT HW
- QUESTIONS 2 "is it obvious".

Break: quantum #'s / $e^+e^- \to \gamma\gamma$ DIAGRAMS / MK

GOAL: highlight what's important & what is not

Quantum Mechanics

$\boxed{Q}$ What are the main ideas?

Some pop physics ideas:

- quantum $\longleftrightarrow$ discrete

IN QM: lots of things are quantized
eg. angular momentum (units of $\hbar$)

... some things are not! eg. POSITION OF A PARTICLE

... energy — quantized or no?

$\hookrightarrow$ DEPENDS on the system!
QUANTIZED for "PARTICLE IN A BOX"
OR HARMONIC OSCILLATOR
... but not for a free particle

[for some reason we don't talk about this enough!]

PARTICLE PHYSICS: something more than ordinary (NR) QM $\longrightarrow$ QUANTIZE $\underline{\text{no. \# of PARTICLES}}$

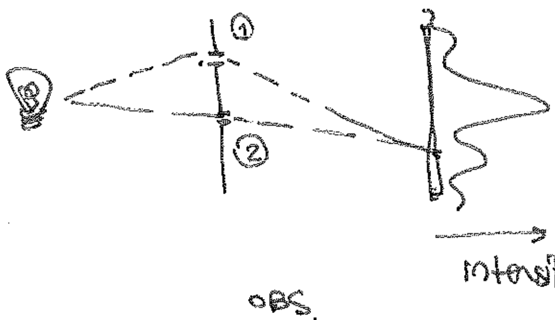
$\hookrightarrow$ really a harmonic oscillator w/ creation & annihilation operators

- Schrödinger's cat

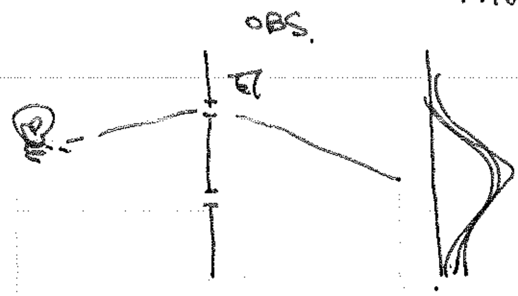
① YOU OBSERVE $|\,\text{alive}\,\rangle$ OR $|\,\text{dead}\,\rangle$

② IN BETWEEN — not really classically defined in general —
SUPERPOSITION of POSSIBLE states

- DOUBLE SLIT EXPERIMENT



INTERFERENCE



NO INTERFERENCE

"observation collapses the wavefunction"

INTERFERENCE $\longleftrightarrow$ SUPERPOSITION

like sum of sine waves

BEST WAY TO DESCRIBE THIS IS WITH COMPLEX NUMBERS

"wavefunction" is not a useful concept for us... but AMPLITUDE is useful.

$a\psi_1 + b\psi_2$

PROBABILITY DISTRIBUTION:

$|a\psi_1 + b\psi_2|^2$

has interference terms

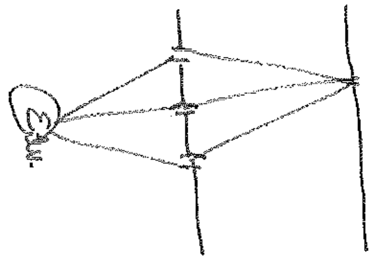
ψ_1 is the amplitude for upper path.

$$\langle \text{DET} | \text{💡} \rangle = \langle \text{DET} | \Big[\underbrace{|1\rangle\langle 1| + |2\rangle\langle 2|}_{= \mathbb{1}} \Big] |\text{💡}\rangle$$

$$= \underbrace{\langle \text{DET} | 1 \rangle \langle 1 | \text{💡} \rangle}_{\psi_1} + \underbrace{\langle \text{DET} | 2 \rangle \langle 2 | \text{💡} \rangle}_{\psi_2}$$

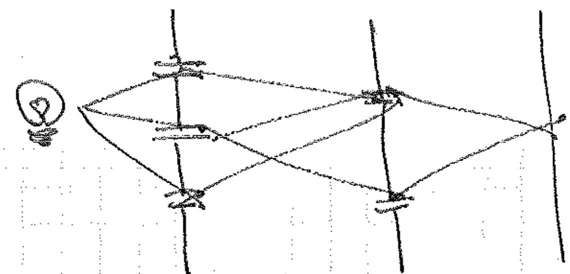
these are "scattering amplitudes"

if 3 slits?



sum over all 3 possible intermediate steps

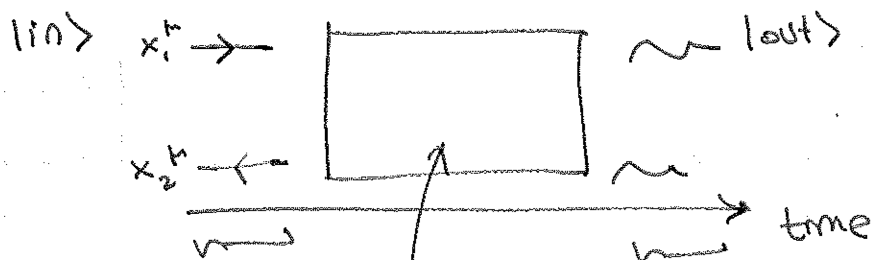
if many slits?



if ∞ many slits, ∞'ly densely packed?
↳ "SUM OVER ALL PATHS."
(between $|in\rangle$ & $|out\rangle$

↑
sum of amplitudes

THIS SHOULD SEEM FAMILIAR: $e^+e^- \to \gamma\gamma$

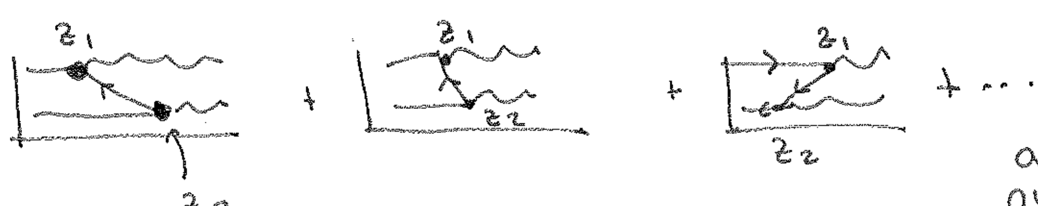


e^- @ x_1^μ
e^+ @ x_2^μ

sum over all possib

γ @ y_1^μ
γ @ y_2^μ

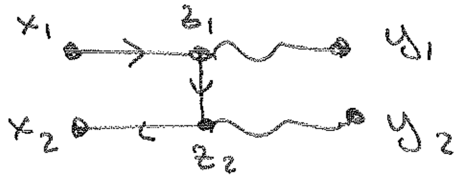
} spacetime coords



an ∞ number!

→ really $d^4z_1\, d^4z_2$

<u>WRITE</u>:



$= \mathcal{M}(x_1, x_2, z_1, z_2, y_1, y_2)$ ... etc.

where you obs the e^-

where you obs e^+

$\mathcal{M}$ is the analog of ψ: it's an <u>amplitude</u>

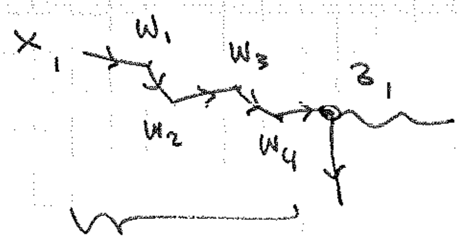
$$\mathcal{M} = \left\langle \gamma(y_1)\,\gamma(y_2) \left| \underline{\int d^4z_1\, d^4z_2}\; \Theta(z_1, z_2) \right| \quad \right\rangle$$

$e^-(x_1)\; e^+(x_2)$

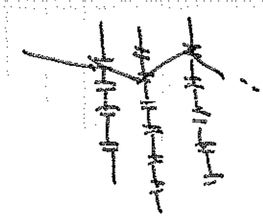
analog of $\psi = \sum a_i \psi_i$

$\mathcal{M}$ is a $\mathbb{C}$ number.

it's the sum of many complex numbers



analog of

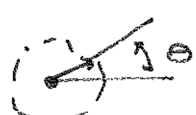


∞ slit
∞ walls

BREAK IT UP INTO INFINITESIMAL STEPS

EACH POSSIBLE PATH: one $\mathbb{C}$ number in the sum.

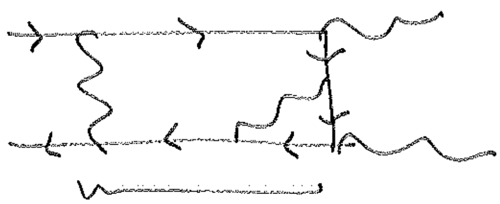
complex #: $re^{i\theta}$ (length; direction)



"see Feynman lec on PHASORS"

adding complex #

other kinds of paths :

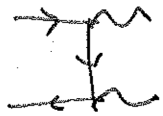
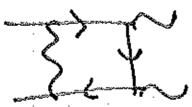
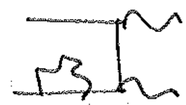


A DIFFERENT SET OF $\mathbb{C}$ #'s SUMMED TOGETHER

implicitly includes all configurations of vertices

so :

$e^+e^- \to \gamma\gamma$ is something like

$\mathcal{M}_{FULL} =$  +  +  + ...

increasingly complicated

more vertices, more internal lines

for now: we'll stick to the <u>simplest</u> option.

why this is okay: it turns out that the sum of diagrams is a Taylor series

↳ the more complicated diagrams are higher order

IN WHAT? ... VERTICES!

Right now you may want to learn how to calculate the $\mathbb{C}$ numbers associated with these diagrams ... it's not illuminating yet — need to get there.

one more remark about quantum mechanics:
Don't get hung up about the <u>WAVEFUNCTION</u>.

$\psi(x)$ ← PROB AMPLITUDE TO OBSERVE THE PARTICLE @ POSITION x

$$\mathbb{P}(\text{PARTICLE within } dx \text{ of } x) = |\psi(x)|^2 dx$$

What's the probability of observing <u>two</u> particles?

cannot answer in QM: particle # is conserved.

SR: When ENERGY is high enough, can produce new particles ← MASS–ENERGY RELATION

so to build a mathematical thy for particle physics, need SR + QM ← QUANTUM <u>FIELD</u> THEORY

some object that takes values on spacetime,

eg $\phi(x)$

LOOKS LIKE A "WAVEFUNCTION" ... IT IS <u>NOT</u>.

the distinction is (in old parlance) referred to as "SECOND QUANTIZATION"

one more thing about quantum mechanics:
the operators that get sandwiched by bras & kets are <u>matrices</u>.

$$\langle out | \mathcal{O} | in \rangle$$

OPERATOR DOES SOMETHING
- Time evolution

$$\mathcal{O} = e^{-iHt}$$

- Rotation

$$\mathcal{O} = e^{i \underline{\sigma} \cdot \underline{r}}$$

$\underline{\sigma}$ = PAULI MATRICES

this is a vector in state space

it has indices

eg spinor

$$|in\rangle = a\begin{pmatrix}1\\0\end{pmatrix} + b\begin{pmatrix}0\\1\end{pmatrix}$$

$|\uparrow\rangle$ $|\downarrow\rangle$

$|a|^2 + |b|^2 = 1$

SPINOR IN STERN GERLACH

$$H = -\underline{\mu} \cdot \underline{B} \sim S_z$$

pick in $\hat{z}$

$$\sim \begin{pmatrix}1 & \\ & -1\end{pmatrix}$$

so the AMPLITUDE $\langle out | \boxed{} | in \rangle$ is a matrix element

$$\underbrace{\begin{pmatrix} a & b \\ c & d \end{pmatrix}}_{M} \qquad M_{12} = (1 \; 0) \, M \begin{pmatrix}0\\1\end{pmatrix}$$

... so there are indices in our states
some are more abstract than others

Indices $\longleftrightarrow$ quantum numbers

a state (ket) is described by a list of quantum numbers

not nec. discrete in our usage

eg Stern Gerlach

$$|\text{state}\rangle = |\text{spin}\rangle$$

$\uparrow$ or $\downarrow$

(REALLY SPIN IN z-DIRECTION)

MORE PRECISE: $|S, S_z\rangle$

total spin   spin in z

eg: $|\tfrac{1}{2}, \pm\tfrac{1}{2}\rangle$

rotations leave this alone

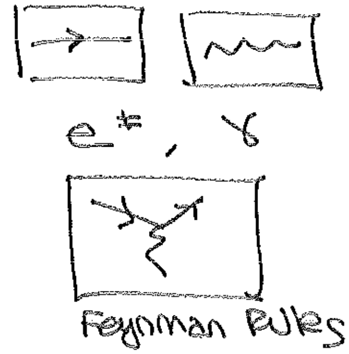
but act on this index

INDICES $\longleftrightarrow$ TRANSFORMATION INSTRUCTIONS
we will use this to understand the role of symmetry in this whole business

other types of labels:
position? momentum? charge?

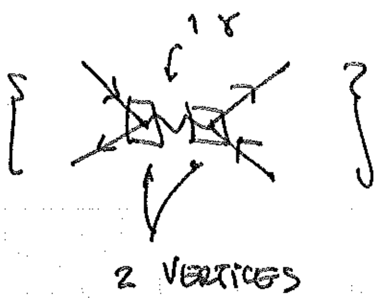
QED: once more.

A. MODEL: → particles
→ interactions



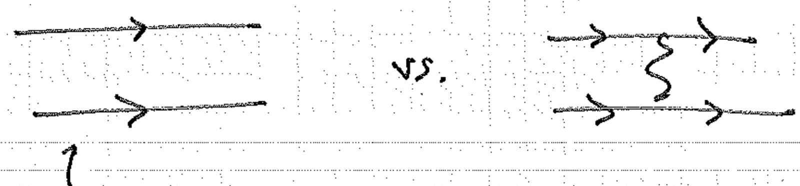
RULES OF THE DYNAMICS:

- YOU CAN USE AS MANY OF THE "CARDS" ABOVE TO DRAW DIAGRAMS.

e^-e^+  e^-e^+

DO NOT THINK OF THIS AS "ADDING LINES"

- IGNORE DISCONNECTED DIAGRAMS

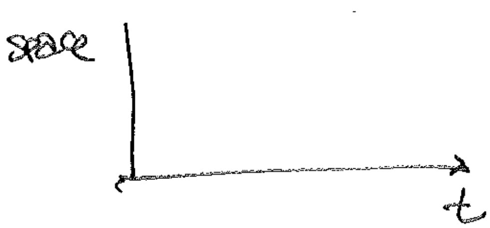


why?: ~~#~~ discon. diag ~~is~~ may not be allowed kinematically, ~~#~~

even if it is — why consider two separate scatterings?

→ this is not a deep thing
more on it later.

- INTERPRET AS A SPACETIME DIAGRAM

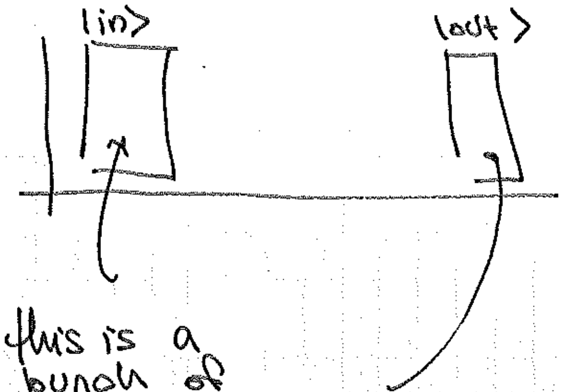


- consider all possible diagrams in principle
  - ↳ BUT IN PRACTICE, only consider the <u>simplest ones</u>

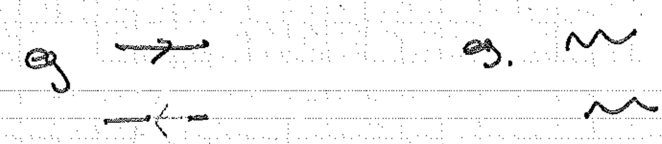
EFFECTIVELY: "same # of vertices" is same order in perturbation theory

THE GAME

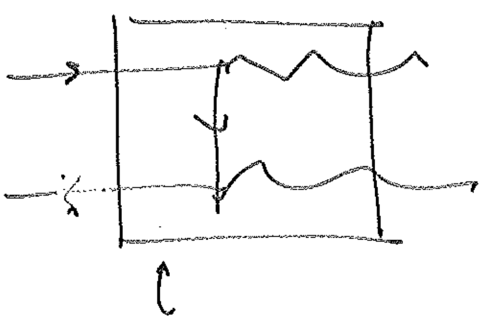
- GIVEN $|in\rangle$ & $|out\rangle$,



this is a bunch of particle lines



DRAW VALID DIAGRAMS THAT CONNECT THEM



$\langle out | i\mathcal{M} | in \rangle$

nb: this is a matrix

it connects two different states.

TECHNICAL NOTE:

QM: Hilbert space
QFT: Fock SPACE ← generalizes to incl. particle # non cons.

$e^+e^- \to \gamma\gamma$ REALLY MEANS

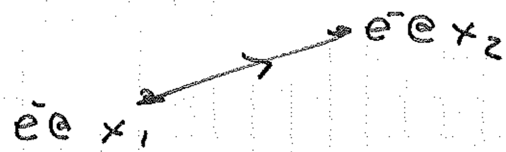
$|e^- @ x_1, e^+ @ x_2\rangle$ goes to

$|\gamma @ y_1, \gamma @ y_2\rangle$

where x_i, y_i ARE 4 VECTOR COORD.

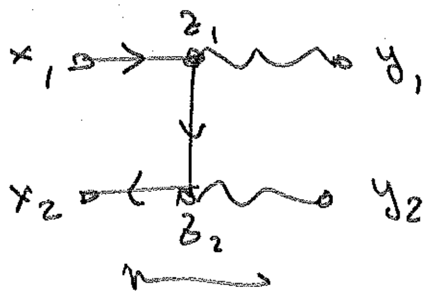
↳ they are events : (t, x, y, z)
(EVENT : observation of these things)

so eg: here's a "dumb" Feynman diagram :

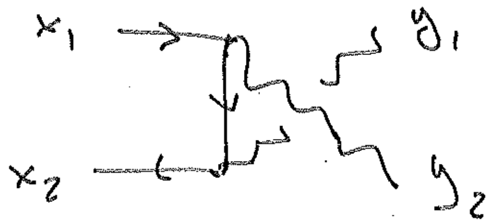


Represents a matrix element that gives amplitude that an electron that we obs @ x_1 ends up being observed @ x_2.

back to $e^+e^- \to \gamma\gamma$

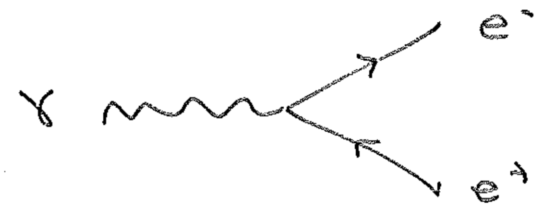


implicitly sum



convince yourself that this is different.

something you may find using the rules of <u>Dynamics</u>



I'll omit spacetime points ... they're tedious

(+ also, momentum space.)

Q What's wrong with this?

$$P_\gamma = (E_\gamma, 0, 0, p)$$

align along $\hat{z}$ axis

frequency

$E_\gamma = p$
BY EINSTEIN REL
+ $m_\gamma = 0$

so: $P_\gamma = P_{e^+} + P_{e^-}$ } as <u>4-vectors</u>

can you solve this?. GIVEN p, $\exists$ p_+ & p_- s.t.

$$E_\gamma = E_+ + E_- \qquad E_\pm = \sqrt{p_\pm^2 + m_e^2}$$

$$p = p_+ + p_- \qquad E_\gamma = p$$

in this case, <u>NO</u>.

<u>EASY WAY</u>: IN e^+e^- REST FRAME...
well... $\exists$ REST FRAME.

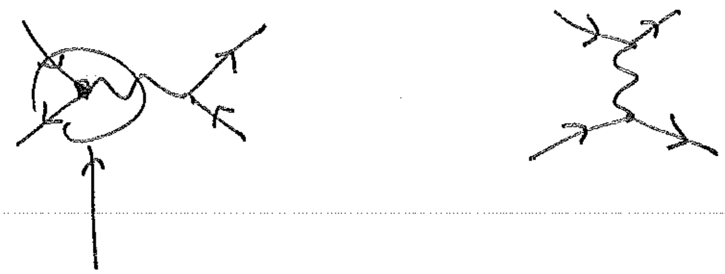
so: there's more information to include in our rules

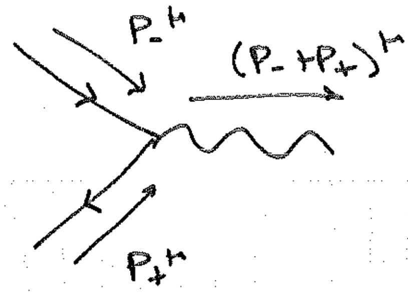
<u>Rules of KINEMATICS</u>

1. @ each vertex, conserve 4-momentum
2. $|in\rangle$ & $\langle out|$ particles are on-shell

nb: trivial that each line cons. 4-momentum eg: $e^-(p_-)\quad e^+(p_+) \longrightarrow e^-(k_-)\quad e^+(k_+)$

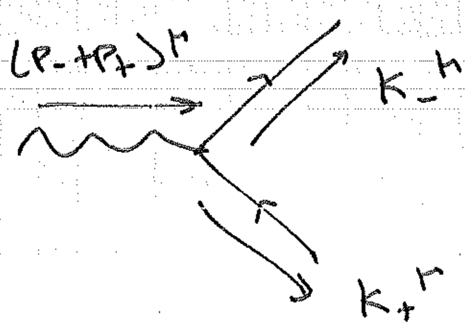
& DIAGRAMS :





4-MOMENTUM CONS. MEANS WE HAVE SPECIFIED THE MOMENTUM OF THE INTERNAL LINE.

next vertex :



WE REQUIRE

$$(P_- + P_+)^\mu = K_-^\mu + K_+^\mu$$

THIS IS 4 EQ.

$$\boxed{\begin{aligned} (P_-)^0 + (P_+)^0 &= (K_-)^0 + (K_+)^0 \\ \underline{p}_- + \underline{p}_+ &= \underline{k}_- + \underline{k}_+ \end{aligned}}$$

E & p cons. of the process .

THINGS TO OBSERVE

① ~~[illegible]~~ assuming that you know the momentum of the ext. states

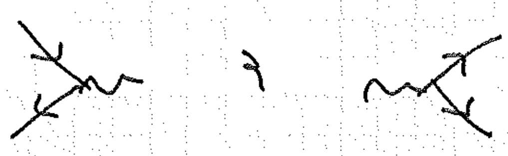
↓

you do not know the position.

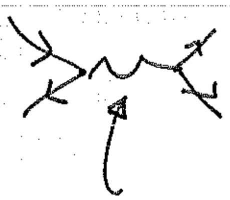
(PLANE WAVE LIMIT)

so from now on: external states are 4-momentum eigenstates.

[INTERNAL STATES: doesn't matter integrating over internal points $\longleftrightarrow$ integrating over internal momenta via Fourier transform]

② 

not allowed by kinematics



is allowed ... what gives?

E, p cons.

ASSUME e^+e^- REST FRAME

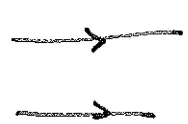
then $(p^+ + p^-) = (2E, \underline{0})$

not a valid p_γ!

VIRTUAL PARTICLE.

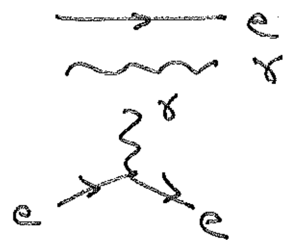
quantum:

$\Delta E \Delta t \sim \hbar$

③ 

REQ. $p^\pm = k^\pm$ } if not, then these diagrams fail the kinematics

VARIANTS of $\boxed{\text{QED}}$ →



ADD MORE STUFF

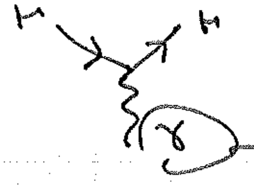
1. PARTICLES
2. INTERACTIONS

QED + μ

muon. LIKE THE ELECTRON, BUT NOT.

Q HOW IS IT DIFFERENT ? PDG?

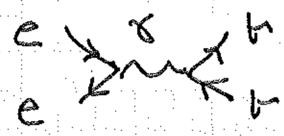
$\longrightarrow \mu$



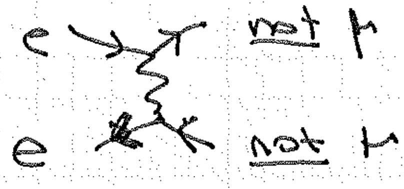
So every QED process w/ e can be replaced w/ QED process w/ $e \rightarrow \mu$

same photon

So: $e^+e^- \rightarrow \mu^+\mu^-$



compare to



so you do not have this diagram for $e^+e^- \rightarrow \mu^+\mu^-$!!

EFFECT : NO QUANTUM INTERF.

AMPLITUDES DIFFERENT!

Why ?

no  rule !!

conserves electric charge

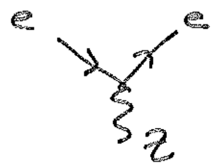
but not "electron-ness" or "muon-ness" } FLAVOR

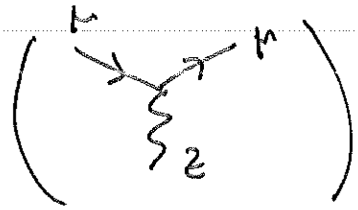
... can think of these as quantum #

QED + Z

← like the photon, but not.

(Q) How is it different? PDG

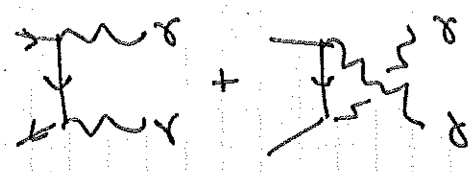




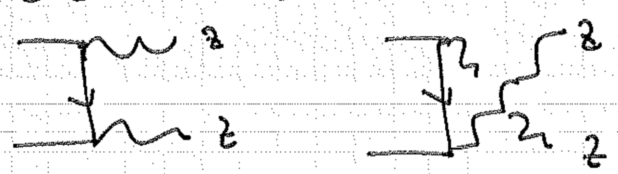
← QED + Z + μ

SO WHAT?

$e^+e^- \to \gamma\gamma$



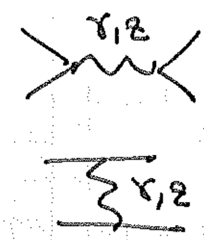
$e^+e^- \to ZZ$



BUT: KINEMATICS DIFFERENT

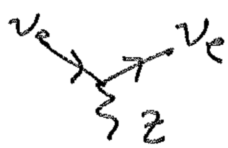
if $E^{\pm}$ too small, kinem. prohibited!
(same for $e^+e^- \to \mu^+\mu^-$)

$e^+e^- \to e^+e^-$



more diagrams!

QED + Z + ν_e



$Q_\nu = 0$, no electric charge

$M_{\nu_e} \ll M_e \ll M_\mu \ll M_Z$

but no coupling to γ