

13 JUNE 2016

# UCR SUMMER PHYSICS TEACHERS' ACADEMY

↑ [theory.ucr.edu/SPTA](http://theory.ucr.edu/SPTA)

## QUANTUM ELECTRODYNAMICS with FEYNMAN DIAGRAMS

Prof. FLIP TANEDO (flip.tanedo@ucr)

↑ [physics.ucr.edu/~flip/](http://physics.ucr.edu/~flip/)

RESOURCES: link on SPTA website

[physics.ucr.edu/~flip/SPTA2017.html](http://physics.ucr.edu/~flip/SPTA2017.html)

## QUANTUM ELECTRODYNAMICS

↓  
"Maxwell's Equations" ← EM FIELD & WAVES

↑  
the quantum version  
↓

DISCRETE, like the energy levels  
of a hydrogen atom

- SCHRÖDINGER'S CAT  
superposition of possible realities
- DOUBLE SLIT EXPERIMENT  
simultaneous "sum" over paths

◦ PARTICLES VS. WAVES

"pop physics"

## QED HAS 2 PARTICLES

electrons & positrons

$e^\pm$

anti-electron

(NOT DISTINCT FROM  
ELECTRON.)

photons : quanta of light ,  $\boxed{\gamma}$   
↑  
electromagnetic field  
individual packets of light

## ANALOGIES (none are quite right, just poetic)

- ELECTROMAGNETIC FIELD IS THE OCEAN
- ELECTROMAGNETIC WAVES ~ OCEAN WAVES  
↑  
eg monochromatic light

- PHOTONS ~ WATER MOLECULES

↑ the collective behavior of quantum  
photons produces the classical  
electromagnetic field

↑ described by Maxwell's eq.

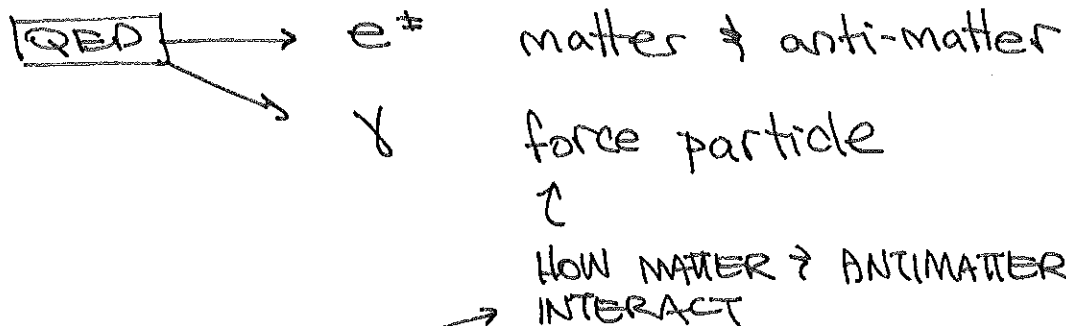
EM IS A FORCE

⇓

the photon is a  
force-carrier

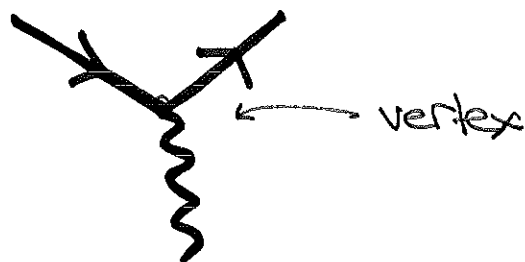
↖ nb: FORCES ARE DERIVATIVES  
OF POTENTIALS .

PHOTONS ARE ACTUALLY  
QUANTUM EXCITATIONS  
OF THE EM POTENTIAL



ah! interactions! this is the point.  
QED describes how matter & antimatter interact in a simple quantum theory of electromagnetism

### THE FUNDAMENTAL INTERACTION OF QED:



2 types of lines

SOLID & STRAIGHT  
↑

ELECTRON  
OR POSITRON

WIGGLY  
↑

PHOTON

# FEYNMAN DIAGRAMS

representation of particle interactions

↳ really scattering processes

"some particles from "far away" get "close-ish" together and then some particles exit & get far away from each other"

↑ HAS TO DO W/ BEING ABLE TO QUANTUM

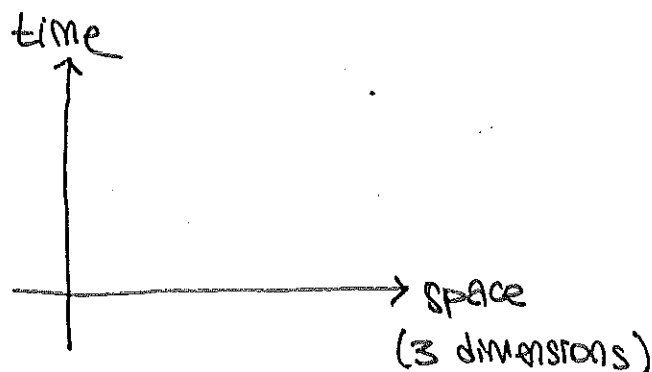
TECHNICALLY THESE ARE GRAPHS OF TRAJECTORIES IN SPACETIME

↑ hint that we're incorporating (special) relativity into this

NB: TECHNICAL JARGON:

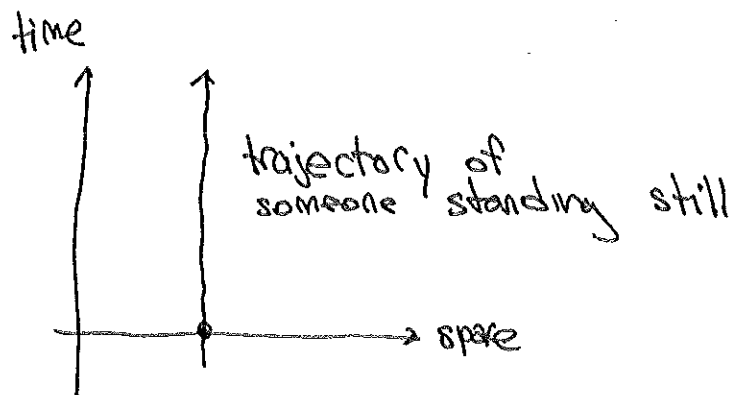
quantum field theory

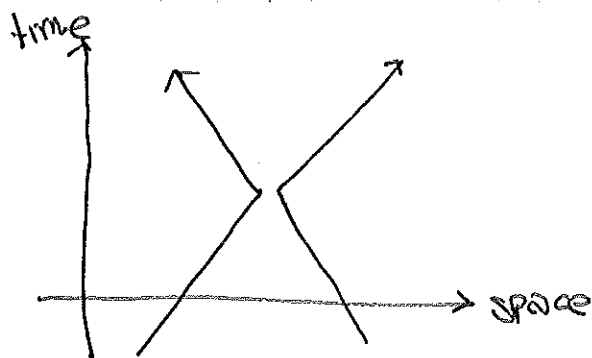
↳ QUANTUM MECH + RELATIVITY



} Aji: see the book VERY SPECIAL RELATIVITY, Sander Bas

↑ full course in relativity by pictures.

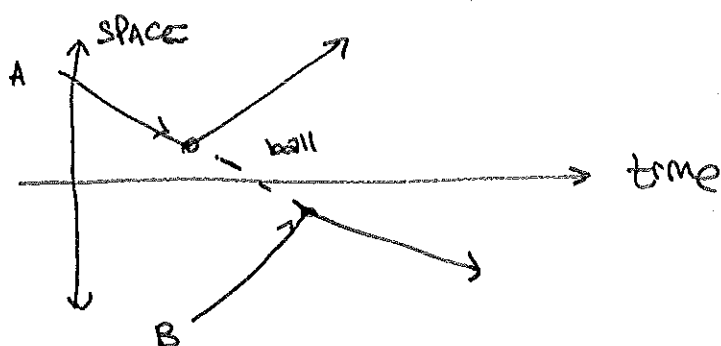




two people meet  
then go their separate  
ways

← SCATTERING

actually, we will draw things w/ time flowing  
from left-to-right:



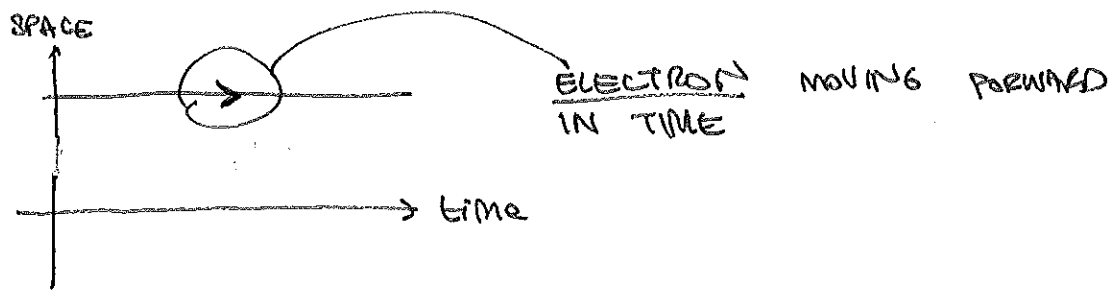
( ALICE & BOB SLIDING ON ICE (frictionless)  
TOWARD EACH OTHER.

THEN ALICE THREW A HEAVY BALL TO  
BOB. THIS EXCHANGES MOMENTUM "at a distance"  
SO THAT A & B ARE NOW MOVING AWAY  
FROM EACH OTHER

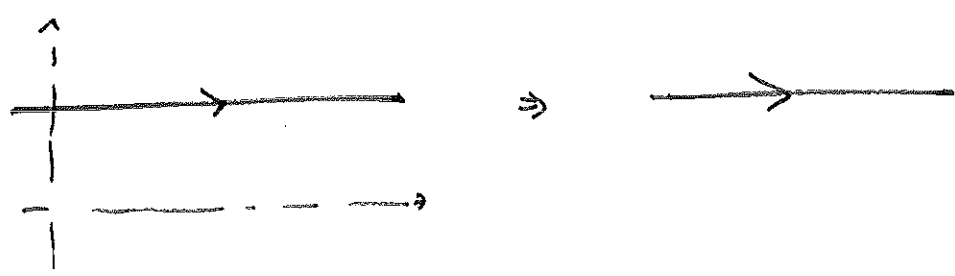


this cartoon will serve us well

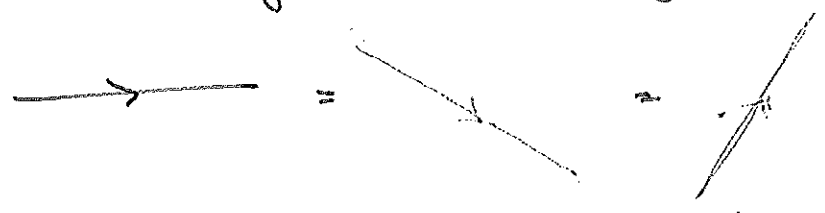
SOME BORING SPACETIME TRAJECTORIES  
USING QED NOTATION




in fact, let's just drop the axes!



in fact, angle doesn't really matter:



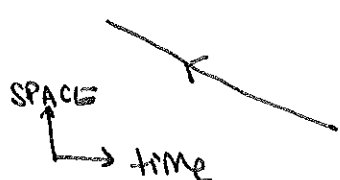
how much its moving in space doesn't matter:  
all we care about is: did it interact or not?  
↳ this is an unbothered electron.

by the way, think of  as a piece of rope w/ a sense of direction.  
MIGHT AS WELL DRAW LINES AS STRAIGHT AS POSSIBLE.



weird, unphysical trajectory ... but still "particle that moves forward in time w/ no interactions"

What if we rotate the line more than  $90^\circ$ ?



? looks like an electron moving backward in time.

- IDENTIFY THIS WITH A POSITRON (anti-electron) MOVING FORWARD IN TIME

does that make sense?

yes - think about the current of this particle

current of a positron differs from that of an electron by a minus sign

current is also a vector



so minus sign gives vector in opposite direction

as if electron were moving backward in time!



electron moving forward in time



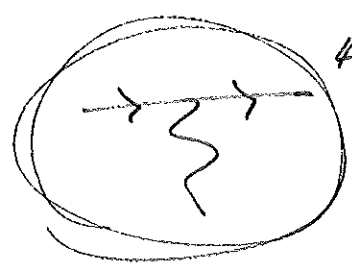
positron moving forward in time.

↑  
arrow gives direction of charge flow

nb: gives funny poetry: maybe  $\exists$  only one  $e$  in universe? it's zipping fwd + backward in time... (No.)

SO: HOW DO WE MAKE THINGS TALK?

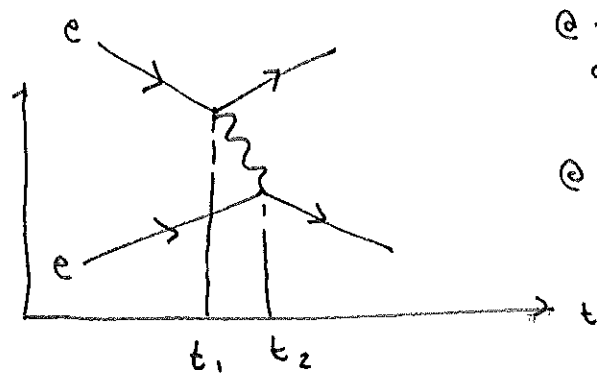
INTERACT?



the QED vertex

tells us that "charge goes in, charge goes out"  
 ↳ we attach a photon along an  $e^\pm$  line.

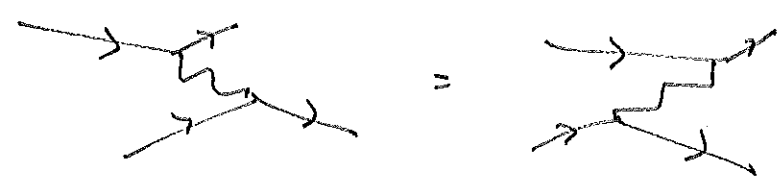
SO WE CAN DRAW TWO ELECTRONS INTERACTING W/  
 EACH OTHER



@ time  $t_1$ , top electron emits  
 a photon & changes  
 trajectory.

@ time  $t_2$ , bottom electron  
 "catches" the photon,  
 changes trajectory.

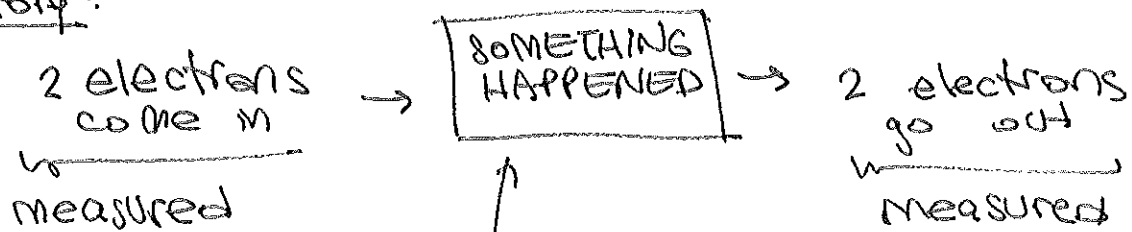
CONVENTION: WE DON'T KNOW (or care!!) ABOUT THE  
 ORDER OF  $t_1$  &  $t_2$ .



of course, these are both different  
 "PLOTS" to the same story:



Story:



NOT measured!

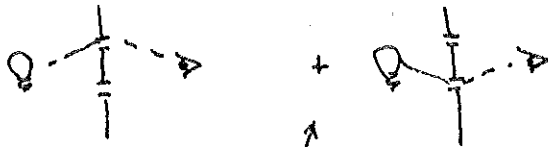
this is a black box!

the only reason why we know that something must have happened is

that

this is different from this!

IN FACT, THIS IS WHERE THE DOUBLE SLIT EXPERIMENT IS PERTINENT:



"SUM" BOTH POSSIBILITIES

(whatever that means...)

something related to probability

"amplitudes"! → WILL DISCUSS LATER)

IN THE SAME WAY, WE "SUM" OVER ALL POSSIBILITIES IN THE BLACK BOX:



↑ instantaneous !!

DRAW THIS SIMPLY AS:



one diagram (only, really) represents all possible "something happened" where one photon is exchanged between two electrons.

so we can really ignore time axis.

NOW WE CAN START PLAYING!

CAN YOU DRAW:

a) electron scattering w/ positron?

(nb: ice skater analogy breaks down!)

b) photon + electron  $\rightarrow$  photon + electron?

c)  $e^- + e^+ \rightarrow e^- + e^+ + e^- + e^+$

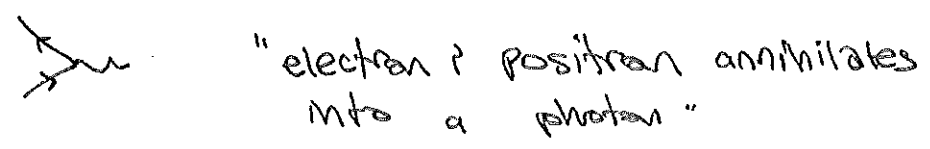
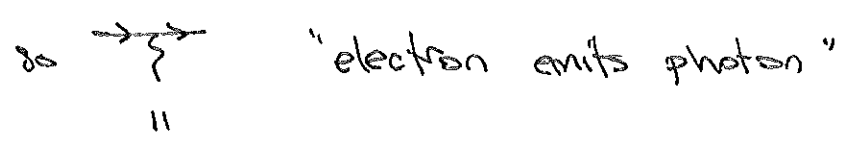
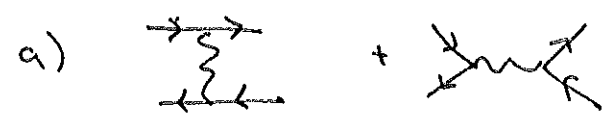
d)  $e^- + e^- \rightarrow e^- + e^- + \gamma$

f)  $e^- + e^- \rightarrow e^+ + e^+$

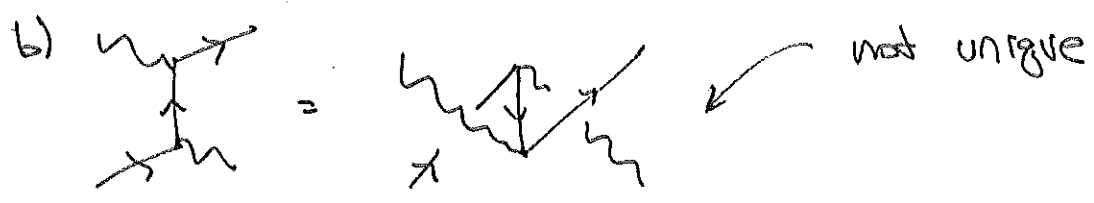
g)  $e^- \rightarrow e^- + e^+ + e^-$

# ANSWERS

↪ you can rotate the QED vertex!



↪ similar for &

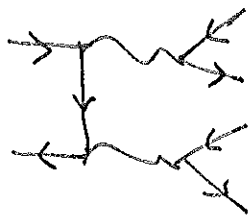


RULES: "PIA" INITIAL & FINAL PARTICLES

INTERNAL PARTICLES CAN BE MOVED AROUND, as long as you preserve topology

↪ no cutting!

c)



RULE: SIMPLER DIAGRAMS  
ARE BETTER  
(FEWER VERTICES)

WHY: THIS IS SECRETLY A  
TAYLOR EXPANSION

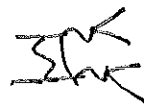
EACH VERTEX IS A SMALL #,  
SO MORE VERTICES  
MEANS HIGHER ORDER

↑  
(small #) higher power

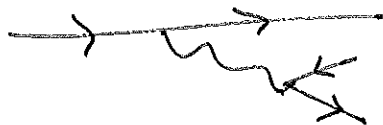


+ similar

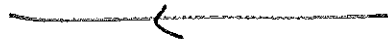
so not



RULE: ONLY CONNECTED DIAGRAMS

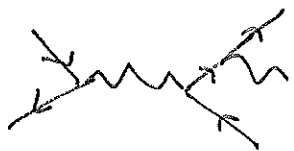


DOES NOT CONTRIBUTE

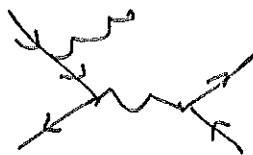


} nothing happened  
to this position.

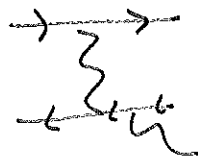
d)



+



+ similar



+

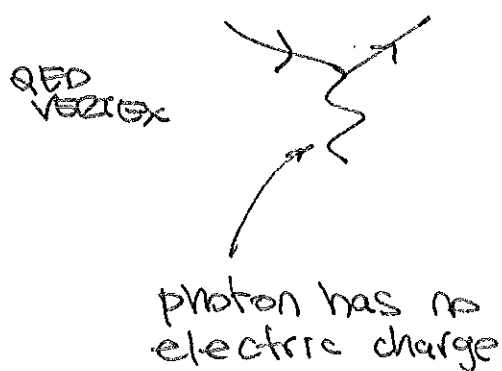
similar

f)  $e^- + e^- \rightarrow e^+ + e^+$

NOT POSSIBLE!

↳ why? CHARGE CONSERVATION

how does it show up in our theory?



} arrow is the direction of charge flow

CHARGE GOES IN  
CHARGE GOES OUT



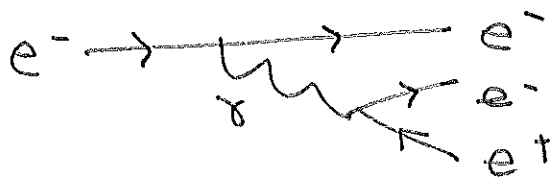
charge is conserved in each vertex



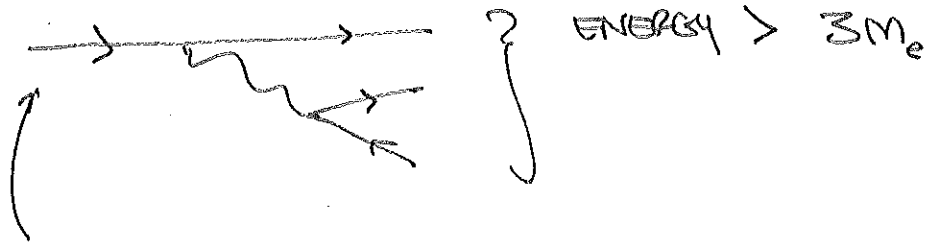
THEREFORE: CHARGE IS CONSERVED IN ANY DIAGRAM!

g) ALSO NOT POSSIBLE!

why? I can draw this:



BUT: this cannot conserve energy!



I can go to rest frame of  $e^-$

LIKE DRIVING UP NEXT TO A CAR ON THE FREEWAY. IN THAT FRAME, THE CAR HAS NO KINETIC ENERGY RELATIVE TO YOU.

of course, everything else (FREEWAY BARRIERS, SLOWER/FASTER CARS, ETC.) HAS KIN. ENERGY...  
... SO DRIVE SAFELY!!

IN THIS FRAME,  $E_e = m_e c^2$

We usually work  
in units where  $c^2 = 1$

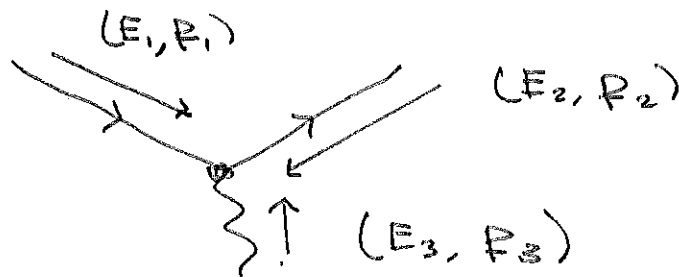
$$\left( \text{FULL EQ: } E^2 = (m_e^2 c^4) + \underset{\substack{\uparrow \\ \text{momentum}}}{p^2 c^2} \right)$$

then there is no way to conserve energy

→ one  $e$  does not have the energy in its rest frame to beget  $e^+ e^- e^-$

⇒ true in ANY frame  
(by relativity)

WE CAN BUILD THIS INTO OUR FUNDAMENTAL VERTEX



$$\begin{cases} E_1 + E_2 + E_3 = 0 \\ P_1 + P_2 + P_3 = 0 \end{cases}$$

→ SUM OF ENERGIES + MOMENTA = 0  
 ↳ conservation of energy / momentum

WAIT, WHAT?!  $E_1 + E_2 + E_3 = 0$  implies NEGATIVE E?!

WEIRD!

... but not so weird.  
 indeed,

$$e^- \rightarrow e^- + \gamma$$

is NOT KINEMATICALLY POSSIBLE

RULE: ANY INTERNAL LINE (inside the black box)

DOS NOT NEED TO BE ON-SHELL

$$E > 0$$

$$E^2 = m^2 c^4 + p^2 c^2$$

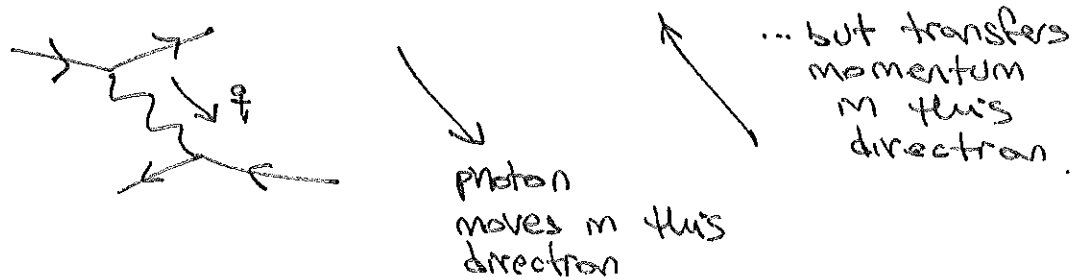
thus



this can have negative energy

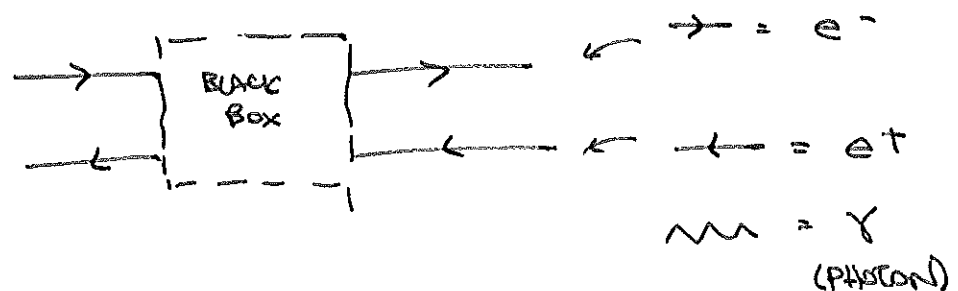
... or even transfer "negative momentum" relative to its motion

this is why  $e^+e^-$  can attract, even if they are "throwing photons" @ each other.



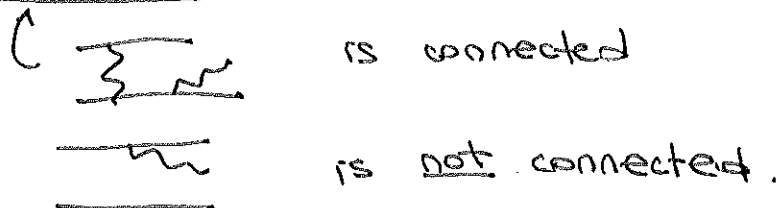
## THE "FEYNMAN RULES"

- TIME FLOWS FROM LEFT TO RIGHT
- FIX YOUR INITIAL & FINAL PARTICLES



- CONNECT LINES USING  ✓ guarantees charge conservation

- ONLY ACCEPT CONNECTED DIAGRAMS



- IMPOSE MOMENTUM & ENERGY CONSERVATION

↳ some otherwise valid diagrams are invalid processes

- "TIGHTEN" LINES (to make the diagrams look clean)  
→ only "topology" matters



EXTRA: WHAT ARE WE ACTUALLY DOING?

Feynman diagrams are a shorthand / code / hieroglyphics for mathematical expressions.

these evaluate to complex numbers

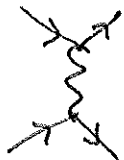
$$\uparrow$$

$$a+ib \quad \text{or} \quad re^{i\theta}$$

THESE ARE AMPLITUDES,  $\mathcal{M}$ :

↑ curly "M"... PROBABLY FOR "matrix" or some silly thing

So:



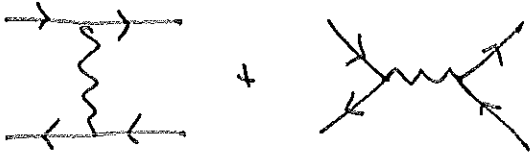
$$= \mathcal{M} = a+ib$$

or  
 $re^{i\theta}$

meaning: the probability of the scattering is

$$|\mathcal{M}|^2 = a^2 + b^2 = r^2$$

WHEN THERE ARE MULTIPLE DIAGRAMS, YOU CAN HAVE INTERFERENCE; eg



$$+ = (a+ib) + (c+id)$$

$re^{i\theta} + se^{i\phi}$

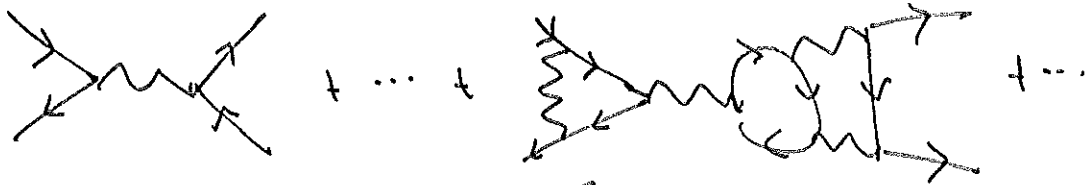
$$\text{PROBABILITY} = (a+c)^2 + (b+d)^2$$

$$= r^2 + s^2 + 2rs(\cos\theta \cos\phi + \sin\theta \sin\phi)$$

12

INTERFERENCE: TWO DIAGRAMS CAN ADD IN A WAY  
WHERE THE RESULTING PROBABILITY  
IS SMALLER THAN IF THE DIAGRAMS  
ONLY CONTRIBUTED INDIVIDUALLY.

WHAT ABOUT COMPLICATED DIAGRAMS?



many vertices!

this is small compared to  $\sum$

SO OFTEN, IT'S SUFFICIENT TO ONLY CONSIDER  
THE SIMPLEST DIAGRAM.

↳ PRECISE PREDICTIONS REQUIRE  
MANY MORE... BUT THE PHYSICAL  
INSIGHTS ARE OFTEN PRESENT  
IN THE SIMPLE DIAGRAMS BY THEMSELVES

HOW DO YOU CALCULATE THE  $\Phi$  NUMBER ASSOCIATED  
W/ A DIAGRAM?



THIS IS WHAT WE TEACH OUR  
GRAD STUDENTS IN  
QUANTUM FIELD THEORY

USE OPEN SOURCE CODE (MadGraph)  
TO CALCULATE FOR YOU!



possibility for collaborations  
between universities + high schools  
to analyze simulated data

WHAT ABOUT THE REST OF PARTICLE PHYSICS?

↳ VARIATIONS ON THIS THEME!

(matter) particles + antiparticles are solid lines... but now multiple types.

FORCE PARTICLES ARE VARIATIONS OF WIGGLY LINES

~~~~~  $\gamma$

PHOTON

~~~~~  $W^\pm$

~~~~~  $Z$

} WEAK NUCLEAR FORCES

~~~~~  $g$

STRONG NUCLEAR FORCE

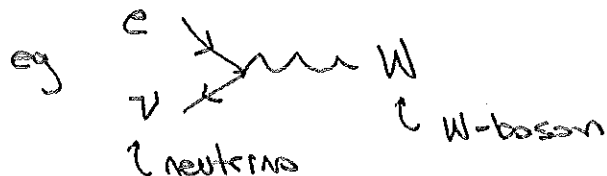
(~~~~~  $G$

) GRAVITATION

-----  $h$

HIGGS ← "kind of" a force

"THEORY" ↔ SET OF VERTICES

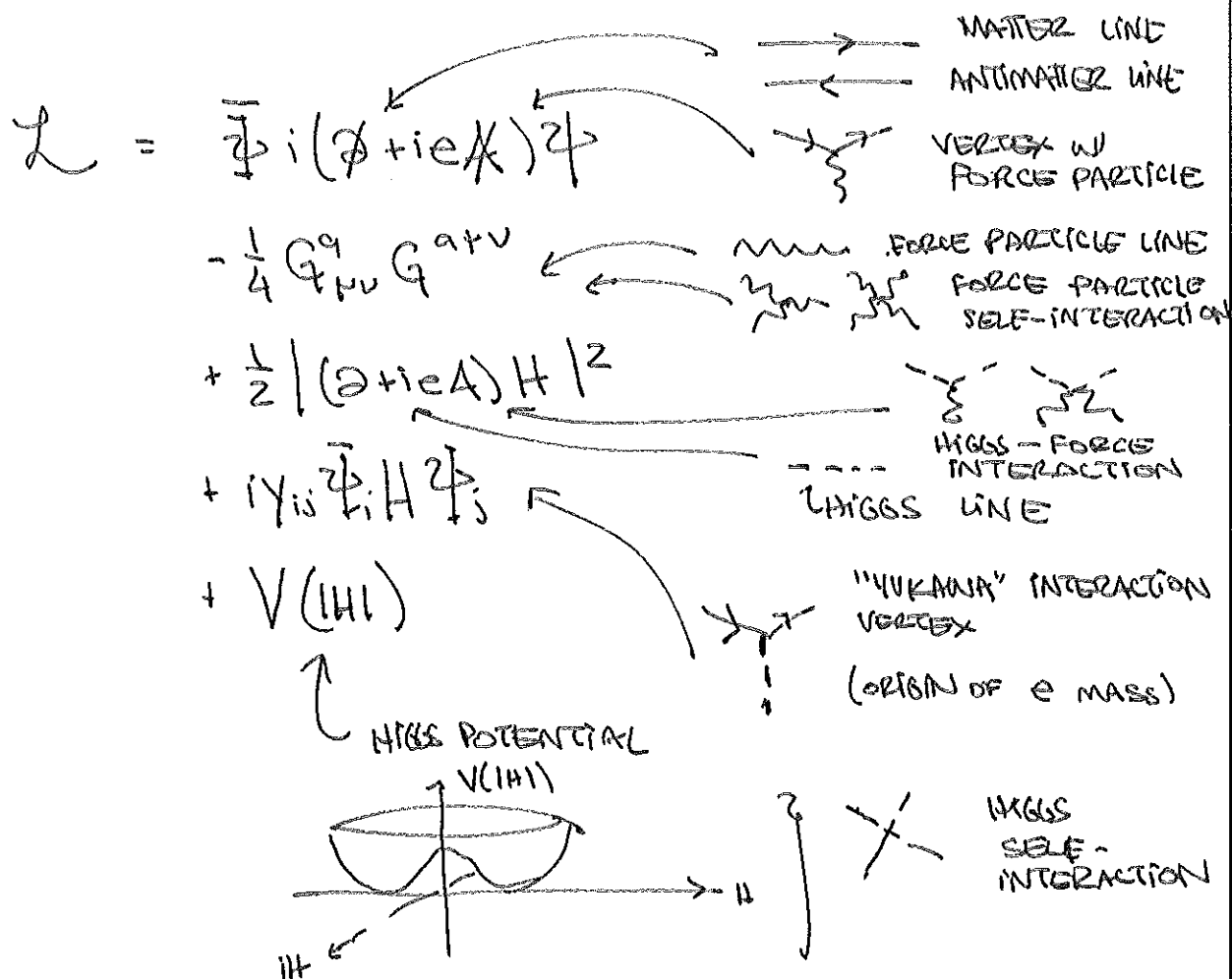


such that ENERGY, MOMENTUM, and ALL CHARGES ARE CONSERVED

generalizations of electric charge

IT IS POPULAR TO "WRITE THE STANDARD MODEL OF PARTICLE PHYSICS" ON T-SHIRTS, COFFEE MUGS, ETC.

HERE'S WHAT IT LOOKS LIKE SCHEMATICALLY, & THE FEYNMAN RULES ATTACHED.



BY USING FEYNMAN DIAGRAMS, YOU CAN EXPLORE WHAT IS POSSIBLE IN THE SUBATOMIC PERIODIC TABLE.

FUTURE APPLICATIONS IN HS CLASSROOM?

→ see website, contact me!

# ACTUAL CALCULATION OF A FEYNMAN DIAGRAM:

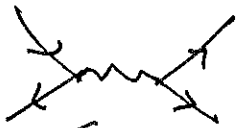


Diagram showing two incoming electron lines (solid with arrows) interacting via a photon (wavy line). The outgoing lines are also solid with arrows.


$$i\mathcal{M} = \bar{u}(p_2) ie\gamma^\mu u(p_1) \times \frac{-i\eta_{\mu\nu}}{(p_1 + p_2)^2 + i\epsilon} \times \bar{u}(k_1) ie\gamma^\nu u(k_2)$$


Diagram showing an incoming electron line (solid with arrow) and an outgoing electron line (solid with arrow) connected by a photon line (wavy line).




Diagram showing two incoming photon lines (wavy) and two outgoing photon lines (wavy).




Diagram showing an incoming electron line (solid with arrow) and an outgoing electron line (solid with arrow) connected by a photon line (wavy line), with a vertex correction loop (photon and electron lines).

$u$ 's are 4-component objects called spinors

↳ funny generalization of vectors (encodes spin)

$p$ 's are 4-momentum (RELATIVISTIC UNIFICATION OF ENERGY AND MOMENTUM)

$$p_\mu = (E, p_x, p_y, p_z)$$

$\gamma^\mu$  is a  $4 \times 4$  matrix

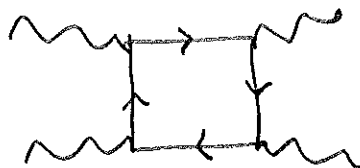
$\eta_{\mu\nu}$  is the metric of special relativity

↑  $4 \times 4$  diag. matrix:  $\begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$

$e$  is the electric charge

## OTHER FUN DIAGRAMS

LIGHT SCATTERING OFF LIGHT ?



+ reversed orientation

ADVANCED : IT TURNS OUT THAT  $e^\pm$  HAS SPIN



RIGHT HANDED ELECTRON

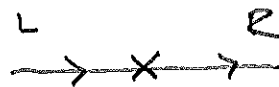


LEFT-HANDED ELECTRON

| similarly for  
RH & LH  
positron.

LABEL THIS SPIN BY "L" or "R"

THEN THE FEYNMAN RULES (ACCOUNTING FOR SPIN) ARE:



↑  
"mass insertions"

RULE : CHOP OFF ANY  $\times$  ATTACHED TO EXTERNAL LINES!



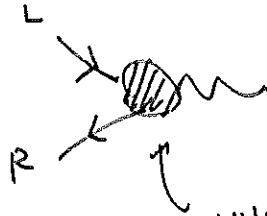
THE FUNDAMENTAL QED VERTEX LEAVES THE  $e^\pm$  SPIN UNCHANGED.

WE CAN ALSO IMAGINE A VERTEX THAT CHANGES SPIN. THIS HAS TO DO W/ TRANSFERRING ANGULAR MOMENTUM

→ From your intuition of classical electromagnetism,

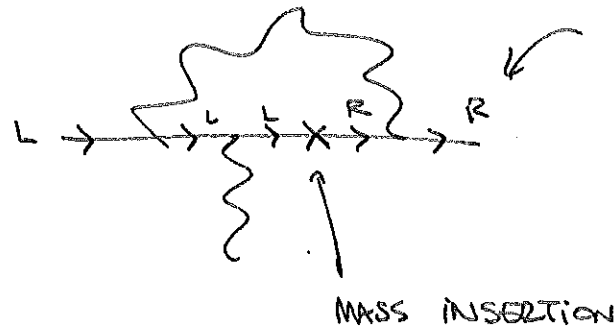
you may guess that the

$e_L e_R \gamma$  interaction is related to a DIPOLE MOMENT.



WHAT GOES IN HERE?

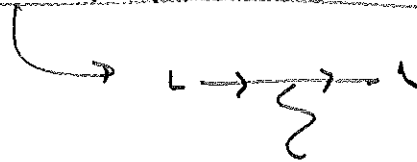
see next page for answer



nb:   
 DOESN'T COUNT   
 CHOP OFF   
 -X- ON EXTERNAL   
 LINES

WHAT WE OBSERVE :

DIPOLE INTERACTION IS MORE COMPLICATED   
 THAN ORDINARY INTERACTION



⇒ IT IS MUCH SMALLER.

further: DIPOLE → 0 AS MASS → 0   
 -X-

MY PHD THESIS HAD TO DO WITH UNDERSTANDING THIS   
 PROCESS IN FIVE DIMENSIONAL SPACETIME. ☺