$\boxed{1.1}$ (a) $[S] = 0$

$\uparrow \; S = \int \underbrace{d^4x}_{} \, \mathcal{L}$ $\;\longleftarrow\; [\mathcal{L}] + [d^4x] = [S]$

$[d^4x] = 4 \cdot [dx] = -4$

$\Rightarrow \boxed{[\mathcal{L}] = 4}$

(b) $S = \int d^4x \; \frac{1}{2} (\partial \phi)^2 + \cdots$

$[d^4x] = -4$

$[\partial] = 1$

$\Rightarrow \underbrace{[d^4x]}_{-4} + \underbrace{2[\partial]}_{2} + 2[\phi] = 0$

$\Rightarrow \boxed{[\phi] = 1}$

(c) $\Delta\mathcal{L} = -\frac{1}{2} A \phi^2$

$\underset{4}{[\mathcal{L}]} = [A] + 2\underbrace{[\phi]}_{2} \;\Rightarrow\; \boxed{[A] = 2}$

$\boxed{1.2}$ $\boxed{A = m^2}$

$$\varphi = c\,e^{ip^\mu x_\mu}$$

$$0 = -(\partial^2 + A)\varphi = -(\partial^2+m^2)\varphi$$

$$= (p^2 - m^2)\,\varphi$$

$$\Rightarrow \boxed{p^2 - m^2 = 0} \longleftrightarrow \boxed{p^2 = m^2}$$

Einstein relation

$$E^2 - |\vec{p}|^2 = m^2$$

so $A = m^2$ is the <u>MASS</u> of φ-PARTICLES.

$\boxed{2}$ $$\mathcal{L} = \frac{1}{2}(\partial\varphi_1)(\partial\varphi_1) + \frac{1}{2}(\partial\varphi_2)(\partial\varphi_2) - \frac{1}{2}m^2\varphi_1\varphi_2$$

$$\frac{1}{2}(\varphi_A + \varphi_B)^2$$
$$= \frac{1}{2}\left(\varphi_A^2 \boxed{+2\varphi_A\varphi_B} + \varphi_B^2\right)$$

$$\frac{1}{2}\left(\varphi_A^2 \boxed{-2\varphi_A\varphi_B} + \varphi_B^2\right)$$

cancel

$$\frac{1}{2}(\varphi_A + \varphi_B)(\varphi_A - \varphi_B)$$
$$= \frac{1}{2}(\varphi_A^2 - \varphi_B^2)$$

combining these:

$$\mathcal{L} = \frac{1}{2}(\partial\varphi_A)^2 + \frac{1}{2}(\partial\varphi_B)^2$$

$$- \frac{1}{2}\left(\frac{m}{\sqrt{2}}\right)^2 \varphi_A^2 \quad + \underline{\frac{1}{2}\left(\frac{m}{\sqrt{2}}\right)^2 \varphi_B}$$

$$\boxed{M_A^2 = \frac{m^2}{2}}$$

$\boxed{M_B^2 \text{ is } \underline{\text{negative}} \text{ !}}$

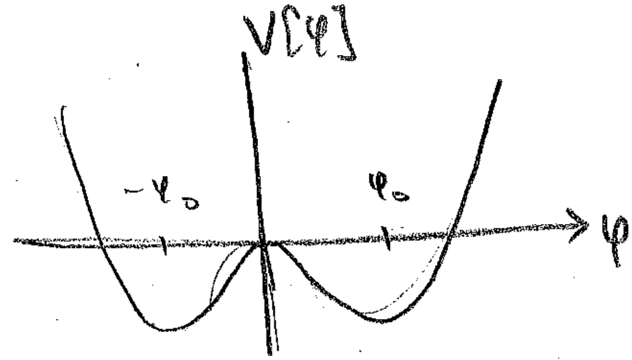
masses of scalar fields always take the form

$$\mathcal{L} = \cdots - (\text{mass})^2 (\text{field})^2$$

UP TO FACTORS OF 1/2

$\boxed{5}$ $V = -\frac{1}{2}m^2\varphi^2 + \frac{\lambda}{4}\varphi^4$

ⓐ



(b) $V' = -m^2\varphi + \lambda\varphi^3$

$= -\lambda\varphi\left(\varphi^2 - \frac{m^2}{\lambda}\right)$

$\varphi = 0$ is local max

local min @ $\boxed{\varphi_0 = \pm\sqrt{\frac{m^2}{\lambda}}}$

can check 2nd derivative

(c) $V[\varphi_0 + \phi] = -\frac{1}{2}m^2(\varphi_0+\phi)^2 + \frac{\lambda}{4}(\varphi_0+\phi)^4$

$= -\frac{1}{2}m^2\left(\varphi_0^2 + 2\varphi_0\phi + \phi^2\right)$

$+ \frac{\lambda}{4}\left(\varphi_0^4 + 4\varphi_0^3\phi + 6\varphi_0^2\phi^2 + 4\varphi_0\phi^3 + \phi^4\right)$

$= \left(-\frac{1}{2}m^2\varphi_0^2 + \frac{\lambda}{4}\varphi_0^4\right)$

$=0 \longrightarrow + \left(-\varphi_0 m^2 + \lambda\varphi_0^3\right)\phi$

$+ \boxed{\left(-\frac{1}{2}m^2 + \frac{3}{2}\lambda\varphi_0^2\right)}\phi^2$

$+ 4\varphi_0\phi^3$

$+ \frac{\lambda}{4}\phi^4$

$\frac{1}{2}M_\phi^2 = \frac{1}{2}\left(3\lambda\varphi_0^2 - m^2\right)$

$= \frac{1}{2}\left(2m^2\right)$ $\boxed{>0}$ ✓

$\boxed{M_\phi = \sqrt{2}\,m}$ ✓

(4) FROM EQN (60) on page 66 of the Los Alamos Primer:

$$e = \frac{gg'}{\sqrt{g^2+(g')^2}}$$

FROM BELOW EQN (78) on page 68 (or p. 48)

$$M_W = \frac{g\varphi_0}{2}$$ ← φ_0 is what we call v

SO: IF WE DOUBLE g:

$$\boxed{e \rightarrow \frac{(2g)\,g'}{\sqrt{(2g)^2+(g')^2}}}$$ ← ALMOST DOUBLE e

$$M_W \rightarrow \boxed{2\,\frac{g\varphi_0}{2}}$$ ← DOUBLE M_W.