1. NATURAL UNITS – LOOK UP IN PDG

- $M_\odot \approx 2\times10^{30}\ \text{kg} \approx \boxed{1\times10^{57}\ \text{GeV}}$

  $\uparrow$

  $g = 6\times10^{23}\ \text{GeV}$

- $H_0 = 6\times10^{-2}\ \frac{1}{\text{Gyr}} \approx \boxed{1\times10^{-42}\ \text{GeV}}$

  Giga $= 10^9$

  $\uparrow$

  $\text{yr} = 3\times10^{7}\ \text{sec}$

  $= 3\times10^{7}\times(7\times10^{-25}\ \text{GeV})^{-1}$

  $= 5\times10^{31}\ \frac{1}{\text{GeV}}$

- $r_e = 3\times10^{-15}\ \text{m} \approx \boxed{20\ \frac{1}{\text{GeV}}}$

  $\uparrow$

  $\text{cm} = (2\times10^{-14})^{-1}\ \text{GeV}^{-1}$

  $= 5\times10^{13}\ \frac{1}{\text{GeV}}$

  $\downarrow$

- $\frac{2G_N M_\odot}{c^2} \approx 3\ \text{km} \approx \boxed{2\times10^{19}\ \frac{1}{\text{GeV}}}$

  $\uparrow$

  $\text{km} = 10^{5}\ \text{cm}$

  $= 5\times10^{18}\ \text{GeV}^{-1}$

2. KINEMATICS

2.1

$$E^2 = m^2 + p^2$$
$$= m^2(1 + p^2/m^2)$$
$$E = m\sqrt{1 + p^2/m^2}$$

small param, Taylor expand

$$= m\left(1 + \frac{1}{2}\frac{p^2}{m^2} + \mathcal{O}\left(\frac{p^4}{m^4}\right)\right)$$
$$= m + \boxed{\frac{1}{2}\frac{p^2}{m}} + \ldots$$

non relativistic: $\underline{p} = m\underline{v}$

so this term is $\boxed{\text{KINETIC ENERGY}}$

$$\frac{1}{2}\frac{p^2}{m} = \frac{1}{2}m\underline{v}^2$$

2.2 $|\underline{p}| = m_e = 5 \times 10^{-1}$ MeV

$$p_\mu = \left(\sqrt{m_e^2 + |\underline{p}|^2},\ m_e\hat{\underline{p}}\right)$$

unit vector in dir of motion

$$= \boxed{\left(0.7\text{ MeV},\ (0.5\text{ MeV})\hat{\underline{p}}\right)}$$

2.3 from Einstein relation (on-shell):

$$\boxed{E^2 = p^2 + m_e^2}$$

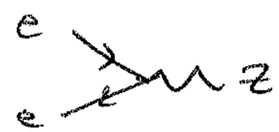
cm energy: $\boxed{2E}$

to produce a 91 GeV particle, need

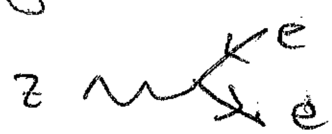
$$2E = 91 \text{ GeV}$$

$$\boxed{E = 45.5 \text{ GeV}}$$

in this frame, Z boson produced @ rest; $p_z = 0$.

TC: if  allows production,

then Z must be allowed to decay via



2.4

$$p \longrightarrow p$$

$$(E_b, 0, 0, p) \qquad (m_p, \underline{0})$$

$$E_b = \sqrt{m_p^2 + p^2}$$

proton mass (1 GeV

EASY WAY: OBSERVE:

in the center of mass frame, this reduces to 2.3.

you need $E_{cm} = M_{\text{new particle}}$
but in cm frame,

$$E_{cm}^2 = (P_1 + P_2)^2 \leftarrow \text{INVARIANT!}$$

beam 4 momenta

since (P_1+P_2) is invariant, it can be calculated in any frame.

$$\Rightarrow m_{new}^2 = \left[(E_b,0,0,p) + (M_p,0)\right]^2$$

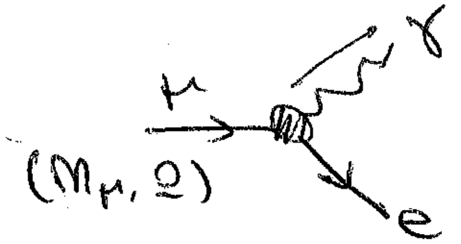
$$= (E_b+M_p)^2 - p^2$$

$$= M_p^2 + \cancel{p^2} + 2\sqrt{M_p^2+p^2}\,M_p + M_p^2 - \cancel{p^2}$$

$$= 2M_p^2 + 2\sqrt{M_p^2+p^2}\,M_p$$

$$\Rightarrow \boxed{P = 97 \text{ GeV}}$$

3. $\mu \to e\gamma$ KINEMATICS



$$\boxed{P_\gamma = (p, 0, 0, p)}$$

$$\boxed{P_e = (E_e, 0, 0, -p)}$$

$$E_e = \sqrt{m_e^2 + p^2}$$

cons. of 3-mom.

Solve: $m_\mu = p + E_e$

$$= p + \sqrt{m_e^2 + p^2}$$

$$\boxed{p = \frac{m_\mu^2 - m_e^2}{2 m_\mu}} \approx 100 \text{ MeV}$$