<u>MIDTERM TALKS</u> : Eli
Steven
AdamC

<u>Theory</u> :
1. symm.
2. particles
3. $\mathcal{L}$

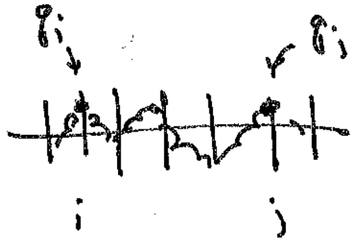
shows up in $\boxed{e^{iS}}$

like a <u>partition function</u>
this is a weight

$\sim \mathcal{M}$, amplitude (almost)

eg: $\langle q_i \, q_j \rangle = \frac{1}{Z} \int dq_1 \cdots dq_N \; q_i \, q_j \; e$

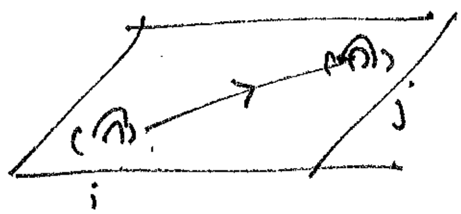
normaliz.

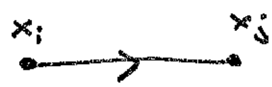


correlation function
(wiggles of field @ different places)

HOW DO WIGGLES @ i
<u>PROPAGATE</u> TO WIGGLES @ j

comes from <u>explicitly</u> (<u>exactly</u>) solving
quadratic part of S
(kinetic terms)





What about vertices?

$$\langle \cdots \rangle = \frac{1}{Z}\int \underbrace{dq_1 \cdots dq_N}_{\equiv \mathcal{D}q} (\cdots)\, e^{iS_{(2)}}\, e^{iS_{int}}$$

integ over field config.

☺ solvable

screws it up

eg $S_{int} = \int d^4x \, c \, q(x)^3 + \cdots$

coupling, want "small"

$$e^{iS_{int}} = 1 + \int d^4x \; c\, q(x)^3 + \frac{1}{2}\left(\int d^4x \, c q(x)^3\right)\left(\int d^4y \, c q(y)^3\right) + \cdots$$

different spacetime points

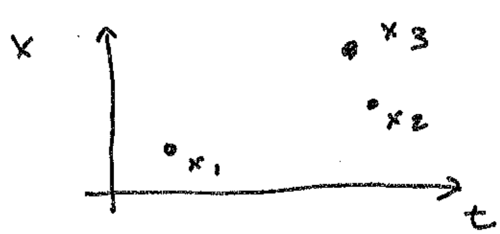
$$\langle \cdots \rangle = \frac{1}{Z}\int \mathcal{D}q \; (\cdots)\left[1 + \int d^4x \; cq(x)^3 + \cdots\right] e^{iS_{(2)}}$$

EACH TERM is SOLVABLE !!

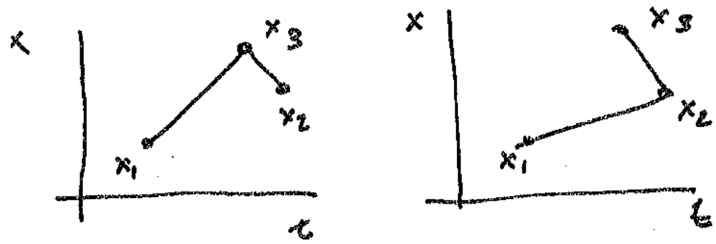
eg: $\langle q(x_1)\, q(x_2)\, q(x_3) \rangle$ ← $\langle q_1 q_2 q_3 \rangle$ for short

correlation of seeing "wiggle in q" @ spacetime points x_1, x_2, x_3

if completely unrelated $\longrightarrow 0$
[nb: connected correlation function]



<u>1st term</u> $\langle q_1 q_2 q_3 \rangle = \frac{1}{Z} \int \mathcal{D}q \; q_1 q_2 q_3 \, e^{iS_{(2)}}$



<u>2nd term</u>: $\langle q_1 q_2 q_3 \rangle = \frac{1}{Z} \int \mathcal{D}(q) \; q(x_1) q(x_2) q(x_3)$

$$\times \; c \int d^4x \, q(x)^3$$

$$\times \; e^{iS_{(2)}}$$

$$= \frac{1}{Z} \int \mathcal{D}q \int d^4x \; c \, [q(x_1) q(x) \times q(x_2) q(x) \times q(x_3) q(x)] \, e^{iS_{(2)}}$$

" cluster decomposition "

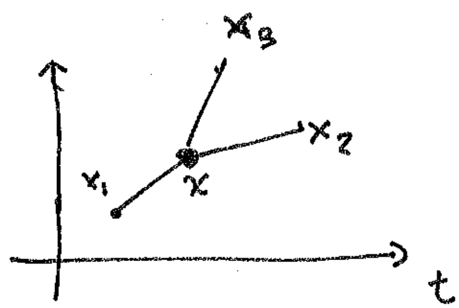
coupling

$$= c \int d^4x \; \langle q(x_1) q(x) \rangle \langle q(x_2) q(x) \rangle \times \langle q(x_3) q(x) \rangle$$

integ over vertex point.

each $\langle \; \rangle$: propagate from x_i to x

$\left(\int d^4x \right)$



coupling

$(\times \; c)$

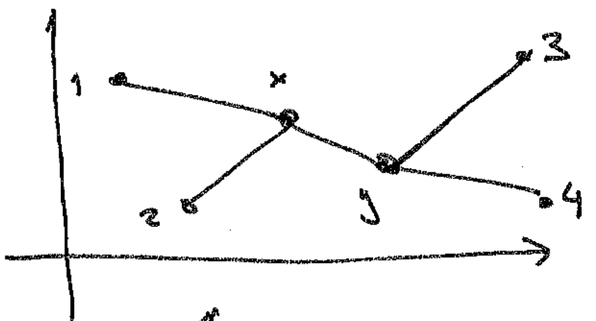
eg. $\langle q_1 q_2 q_3 q_4 \rangle = \frac{1}{2} \int \mathcal{D}q \; q_1 \cdots q_4 \; \underset{\substack{\uparrow \\ \text{taylor exp}}}{e^{iS_{int}}} \; e^{iS_{(2)}}$

<u>2nd order term</u> :

$$\frac{1}{2} \int \mathcal{D}q \; q_1 q_2 q_3 q_4 \times \frac{1}{2} \underbrace{\int d^4x \; c\, q(x)^3}\; \underbrace{\int d^4y \; c\, q(y)^3} \; e^{iS_{(2)}}$$

$\downarrow$

$$= \frac{c^2}{2} \underbrace{\langle q_1 q(x) \rangle \langle q_2 q(x) \rangle}_{\text{ext. line}} \underbrace{\langle q(x)\, q(y) \rangle}_{\substack{\uparrow \\ \text{PROPAGATE} \\ \text{FROM } x \to y}} \underbrace{\langle q_3 q(y) \rangle \langle q_4 q(y) \rangle}_{\text{ext. line}}$$

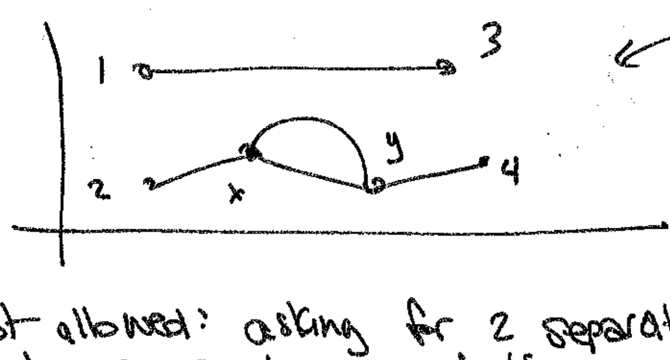
$\int d^4x \, d^4y$ 

$\times \left(\frac{c^2}{2} \right)$ coupling per vertex

this is a feynman diagram!!

(rule: chop off propagator for external legs)

<u>not allowed</u>



DISCONNECTED DIAGRAM ✗

loop is ok

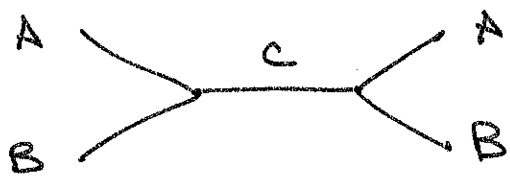
✗ : not allowed: asking for 2 separate correlations, not one single correlation eg. ABC theory

$$S_{int} = \int d^4x \; g \, A(x) B(x) C(x)$$

$$e^{iS_{int}} = 1 + \left(\int d^4x \; g ABC\right) + \frac{1}{2}\left(\int d^4x \; g ABC\right)\left(\int d^4y \; g ABC\right) + \cdots$$

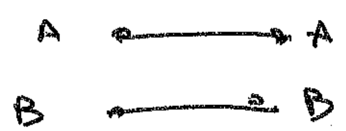
ASK : $AB \to AB$

FEYNMAN DIAGRAM :

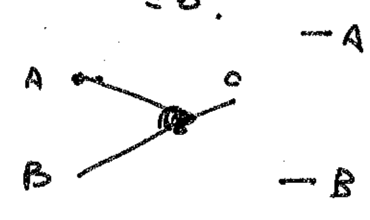


$$\mathcal{M} \sim \langle A(x_1) B(x_2) \; A(x_3) B(x_4) \rangle$$

$$= \frac{1}{Z} \int \underbrace{\mathcal{D}A \mathcal{D}B \mathcal{D}C}_{\text{over all field configurations}} \; \overbrace{A_1 B_2 A_3 B_4} \left(1 + \int d^4x \; gABC + \cdots\right) e^{iS_2}$$



$= 0$!



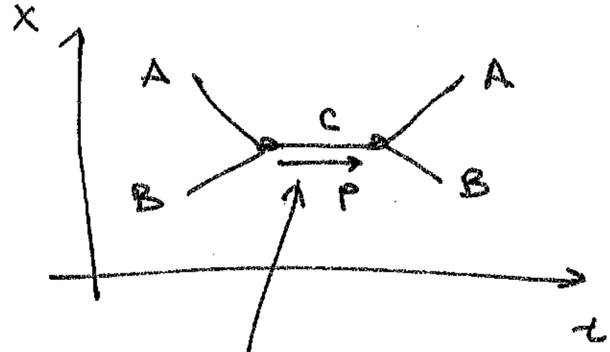
2ND O term :

$$\frac{1}{Z} \int \mathcal{D}A \mathcal{D}B \mathcal{D}C \; A_1 B_2 A_3 B_4 \frac{1}{2}\left(\int d^4x \; gABC\right)\left(\int d^4y \; gABC\right) e^{iS_{(2)}}$$

$$\frac{g^2}{2} \int d^4x \, d^4y \, \langle A_1 A_x \rangle \langle B_2 B_x \rangle \langle C_x C_y \rangle \langle A_3 A_y \rangle \langle B_3 B_y \rangle$$

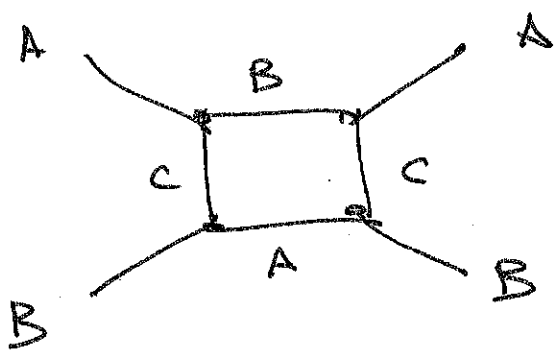
$$\rightarrow \int d^4x\, d^4y \times \frac{g^2}{2}$$

integ over internal vertex positions



$$\frac{1}{p^2 - m_c^2}$$

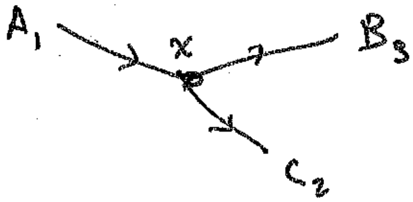
<u>CORRECTIONS</u> : HIGHER ORDER TERMS



$\sim g^4 \times$ (PROPAGATORS)

PERT. IF $g^4 \ll g^2$

<u>MOMENTUM CONSERVATION</u>



comes w/ $\int d^4x\; A_1 A_x B_3 B_x C_2 C_x$

$$A_1 = \int e^{iP_1 x_1} \tilde{A}(P_1)\, dP_1$$

$$\left.\begin{aligned} A_x &= \int e^{-iP_1 x} \tilde{A}(P_1)\, dP_1 \\ B_x &= \int e^{iP_2 x} \tilde{B}(P_2)\, dP_2 \\ C_x &= \int e^{iP_3 x} \tilde{C}(P_3)\, dP_3 \end{aligned}\right\} d^4x$$

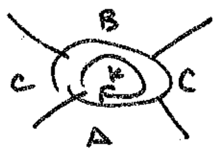
<u>RECALL</u> : $\int dx\; e^{ipx}\, dx = \delta(p)$ UP TO FACTORS OF 2π

$$\int d^4x\;\; e^{-i(P_1 - P_2 - P_3)x}\, \tilde{A}(P_1)\, \tilde{B}(P_2)\, \tilde{C}(P_3) = \delta(P_1 - P_2 - P_3)\, \tilde{A}\tilde{B}\tilde{C}$$

TREE DIAGRAM: (no loops)

the δ functions kill all integrals over momentum.

LOOP DIAGRAM:



one leftover momentum integral per loop.

$$\sim \int d^4k \, \frac{1}{k^2 - m_B^2} \left(\frac{1}{k^2 - m_C^2} \right)^2 \left(\frac{1}{k^2 - m_A^2} \right) g^4$$

Q: is this finite?