P165 LECTURE 6 | 25 January 2018 | 1

TODAY: ALL THE SYMMETRY (for this class)

- questions
- LIST OF SYMMETRIES
- LEPTONIC, UNBROKEN EW

SYMMETRIES

- TRANSLATION in SPACETIME

quantum #/ index : p^μ ← $\eta_{\mu\nu}, \eta^{\mu\nu}, \varepsilon_{\mu\nu\rho\sigma}$ CAN USE THIS TO MAKE INVT.

conserve total 4-momentum @ vertex

eg. of how symmetry acts:

$$\text{time translation in QM: } |\psi\rangle \to e^{\overset{\text{OPERATOR (GENERATOR)}}{iH}\overset{\text{PARAMETER}}{t}}|\psi\rangle$$

- LORENTZ (ROTATIONS + BOOSTS)

quantum #/ index : (REPRESENTATIONS) — contract all indices @ vertex

force → VECTOR : μ $\mathbb{R}$

matter { LH SPINOR: α $\mathbb{C}$ } CP = T (†)

matter { RH SPINOR: $\dot\alpha$ $\mathbb{C}$ }

up to interpret. → scalar : no index $\mathbb{R}$ or $\mathbb{C}$

tensors: metrics to raise & lower

$$\sigma^\mu{}_{\alpha\dot\alpha}$$

$$\varepsilon_{\mu\nu\rho\sigma}$$

↑ SPACETIME

↓ ~~INTERNAL SYMMETRY~~

- ~~ABELIAN / U(1) (HW prob 1)~~

~~QUANTUM # : q, CHARGE ← cons. charge @ vertex~~

eg of how symmetry acts

$$V^\mu = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \xrightarrow[\text{ROT. ABOUT } \hat{z}]{} \underbrace{\begin{pmatrix} 0 & & & \\ & c & s & \\ & -s & c & \\ & & & 0 \end{pmatrix}}_{e^{i\theta(-1')}} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ c \\ -s \\ 0 \end{pmatrix}$$

$e^{i\theta(-1')}$ ← hmm... exp showing up a lot...

now the INTERNAL SYMMETRIES : two kinds ↱ ABELIAN ↳ NONABELIAN

ABELIAN / U(1)

← unitary 1×1 matrix ↳ $e^{i\theta}$ ⟷ QUANTUM PHASE

↳ $M^\dagger M = \mathbb{1}$

eg. electric charge
hypercharge

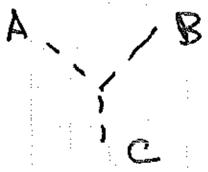
baryon #? lepton #?
electron # ? ...

quantum # : charge, q ← conserve @ vertex

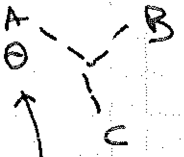
A PARTICLE W/ CHARGE q transforms ("rotates") as

$$\psi \to e^{iq\theta}\psi$$

↳ eg



→ $e^{i\underbrace{(q_A+q_B+q_C)}_{\text{SHOULD SUM TO ZERO}}\theta}$



↑ parameter (anything)

NON-ABELIAN ← these have indices

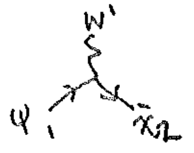
what do indices of internal symmetry mean?
multiplicity of particles.

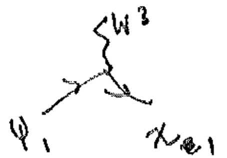
ψ^i ← $i = 1,2,3$ ⇒ THERE ARE 3 PARTICLES THAT ALL BEHAVE SYMMETRICALLY ("the same")

$$W^A \sigma^{A\,i}{}_j\, \bar{\chi}_i \psi^j = W^1 (\bar{\chi}_1\ \bar{\chi}_2)\begin{pmatrix} & 1 \\ 1 & \end{pmatrix}\begin{pmatrix}\psi^1\\ \psi^2\end{pmatrix} + \cdots + W^3 (\bar{\chi}_1\ \bar{\chi}_2)\begin{pmatrix} 1 & \\ & -1 \end{pmatrix}\begin{pmatrix}\psi^1\\ \psi^2\end{pmatrix}$$

(SUPPRESS OTHER INDICES)

so $W^1 \bar{\chi}_1 \psi_1$ allowed, $W^3 \bar{\chi}_1 \psi_2$ allowed





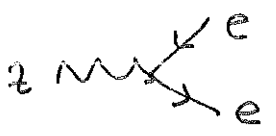
@ this point; "philosophical question":

if nature is really <u>symmetric</u>, then we can't tell them apart!

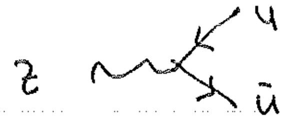
two answers

① can tell that there's a multiplicity

eg



<u>convenient lie</u>: ↳



almost same mass, almost same interaction strength ... expect this to happen @ same rate.

$Z \to u\bar{u}$ happens ~ 3× more often.

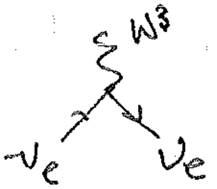
↑

this is a convenient fairy tale that is <u>not</u> representative of SM.

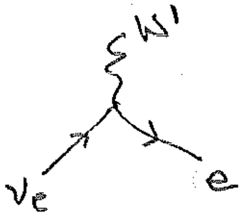
② often, the symmetry isn't ~~exact~~.

eg. $\begin{pmatrix}\nu_e\\ e\end{pmatrix}$ are in an SU(2) doublet

so:



(looks like Z)



looks like $W^\pm$

these are SU(2) symmetric ... but not <u>electrically symmetric</u>.

$\left.\begin{pmatrix}\nu_e\\ e\end{pmatrix}\right\}$ even if they (interaction) cons. electric charge...

this is something to dig into!!

the non Abelian symmetries in this class:

SU(2) & SU(3) ← general: SU(N)

special unitary matrices, N×N

↑ det = 1 ↑ $U^\dagger U = 1$

$$e^{i\,(\text{param})\,(\text{generator})} = U$$

there are others: SO(N), Sp(2N), ... (&10))

← phase space

↑ SO(4) is ~ Lorentz

let's hammer down the critical info for SU(2), SU(3)

SU(2)

fundamental rep: doublet $\psi^a = \begin{pmatrix}\psi^1 \\ \psi^2\end{pmatrix}$

↑ this is complex, so antiparticles are in the anti-fundamental:

$$(\psi^\dagger)_a = \cancel{\begin{pmatrix}\psi^\dagger_1 \\ \psi^\dagger_2\end{pmatrix}}\,(\psi^\dagger_1,\ \psi^\dagger_2)$$

so that $\psi^\dagger\psi$ is invariant if you contract.

adjoint rep: triplet: $W^A = (W^1, W^2, W^3)$

↑ REAL REP, no need to distinguish particle & antiparticle

↳ either allow δ_{AB} metric, or just write everything upper index & allow them to contract (like 3-vectors)

(mnemonic: a 'rotation' on the fundamental is given by the ADJOINT.)

Remark: ADJOINT REP is WHAT [fundamental] FORCE PARTICLES ARE IN

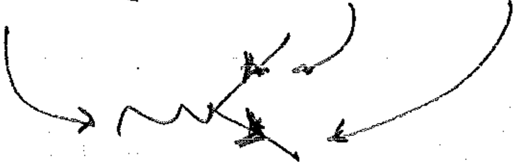
tensors: $(\mathbb{T}^{A})^{a}{}_{b}$ ← fundamental/anti fund.

Adjoint

this lets me connect

(ADJOINT) (FUNDAMENTAL) (ANTI-FUNDAM)

reminds you of:



this is not a coincidence.

note: we've assumed vector - spinor - spinor^{+}

FORCE PARTICLE

can also have vector - scalar - scalar^{+}

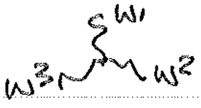
or scalar - spinor - spinor

not a "force interaction"

other tensors:

ε_{ab} ε^{ab} ← metrics. — special to SU(2)

$f^{ABC} \sim \varepsilon^{ABC}$ ← "structure constant" gives



SU(3) very similar

fundamental : $\psi^i = \begin{pmatrix}\psi^1\\ \psi^2\\ \psi^3\end{pmatrix}$

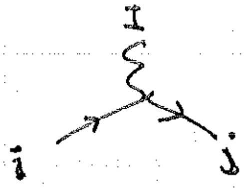
ℂ, so † is ANTIFUNDAMENTAL

$(\psi^\dagger)_i = (\psi^\dagger{}_1, \psi^\dagger{}_2, \psi^\dagger{}_3)$

adjoint : $W^I = (W^1, \dots, W^8)$ ← why 8?!

ℝ rep, allow repeated upper index contraction

$(T^I)^i{}_j \longrightarrow$

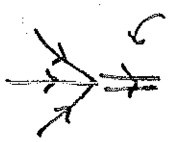


ε^{ijk}

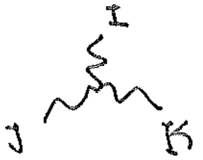
$\varepsilon_{\cancel{abc}\,ijk}$ ← no ε_{ab}! $\varepsilon_{a_1 \dots a_n}$ allowed for SU(N)

$f_{\cancel{ABC}\,IJK}$

so I can connect 3 fundamentals, or
3 antifundam., or
3 adjoints



something w/ spin, but SU(3) neutral

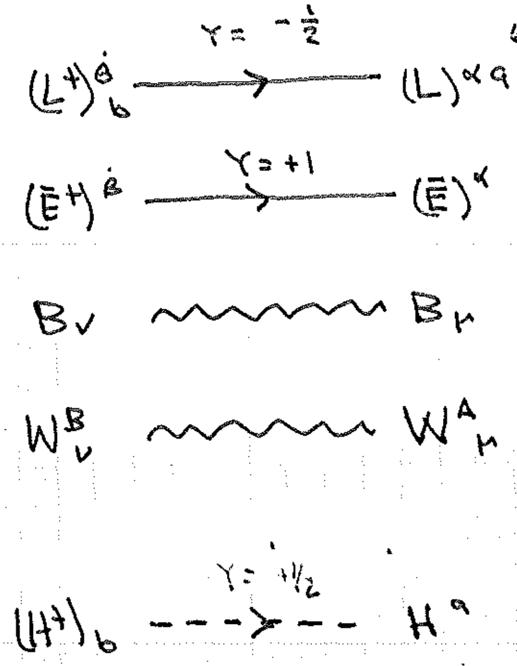


ELECTROWEAK LEPTONIC SECTOR - 1 flavor

SYMMETRY : TRANSLATION } spacetime → always have
LORENTZ

internal : HYPERCHARGE (U(1)/abelian) Y
SU(2) 'WEAK'
↳ $(T^A)^a{}_b$

PARTICLES :



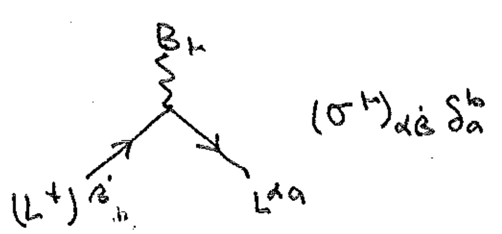
$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$ ← LH electron

looks like LH positron

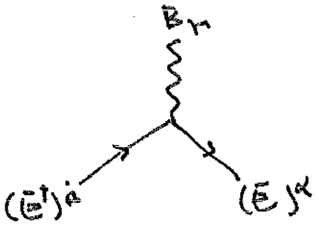
talks to anything w/ hypercharge

talks to things w/ SU(2) indices

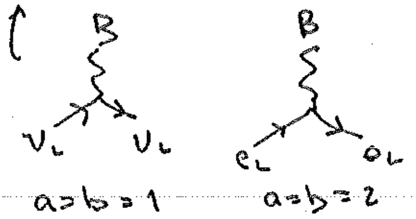
HYPERCHARGE ~ similar to electric

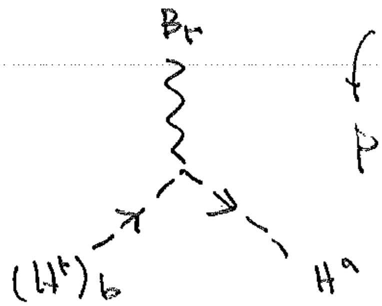


$(\sigma^\mu)_{\alpha\dot\beta}\,\delta^b_a$



$(\sigma^\mu)_{\alpha\dot\beta}$





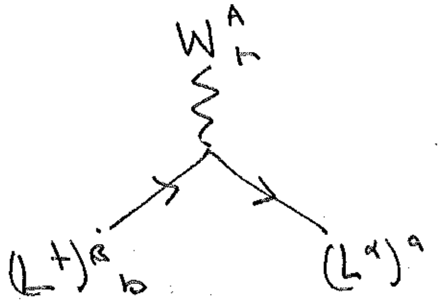
some momentum
... see how momentum flow matters

$p^\mu\,\delta^b_a$

WEAK: $(T^A)^a{}_b = \frac{1}{2}(\sigma^A)^a{}_b$

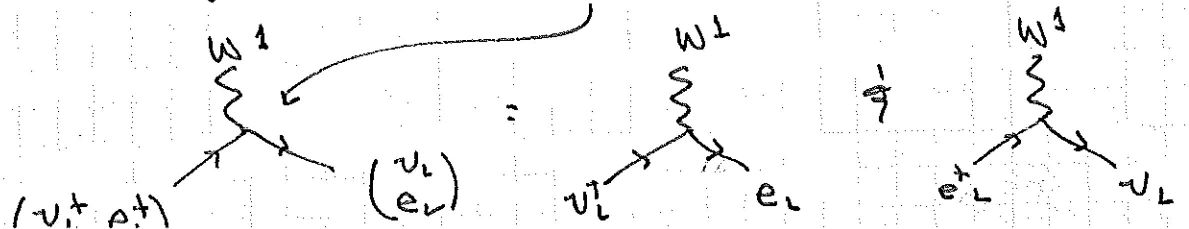
$\uparrow$ pauli

$\begin{pmatrix}0&1\\1&0\end{pmatrix}, \begin{pmatrix}&-i\\i&\end{pmatrix}, \begin{pmatrix}1&\\&-1\end{pmatrix}$

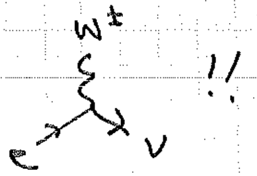


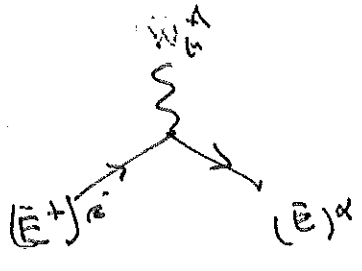
$\sigma^\mu{}_{\alpha\dot\beta} \cdot \frac{1}{2}(\sigma^A)^b{}_a$

(eg: $A = 1 \rightarrow \sigma^A = \begin{pmatrix}&1\\1&\end{pmatrix}$



looks like



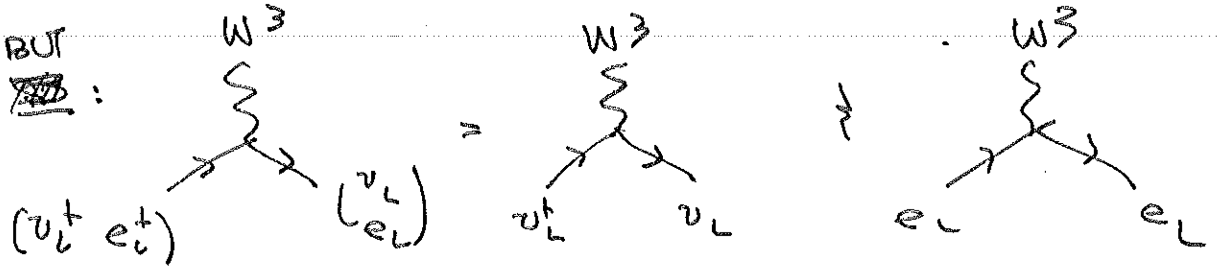


note: not allowed

← not SU(2) charged

no SU(2) index
cannot contract
W's A index!

BUT:



QUESTIONS

- ? TENSOR → HOW TO VISUALIZE ? → vs. matrix?
- ? ANTIMATTER ⟷ something to do w/ Hermitian conjugate

  rule: † : CHARGE, PARITY

  fact: $\boxed{CPT = 1}$ so CP = T

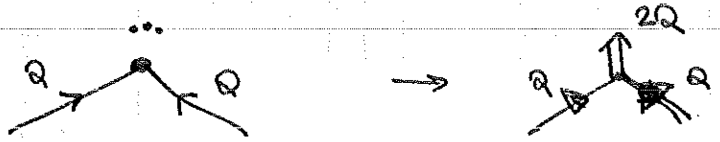
- why the lies? ← see Hogfather.
- what about <u>pauli exclusion</u> ?.

  ↳ relation to parity?

  eg. $e_{RH}^{\uparrow}$ $e_{LH}^{\uparrow}$ can they coexist?

  ↳ I think the answer is yes... but it gets more complicated

- why can't we H.C. $(e^{\dagger})^{\dot\alpha}$ s.t. ~~[illegible]~~ it can contract w/ e^{α} ?



← what spin?

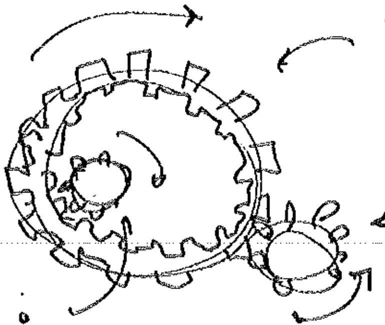
0 works

1 doesn't

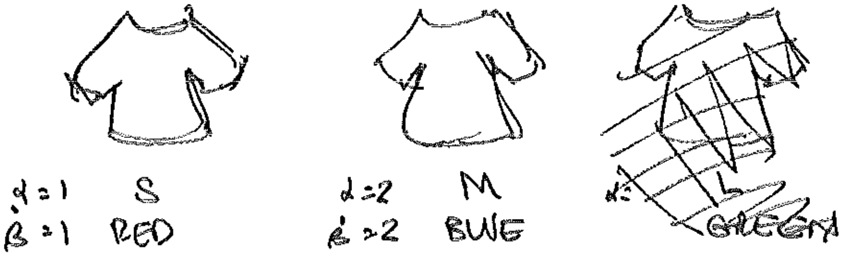
1/2 doesn't

↑ related to <u>MASS</u>.

- $\sigma^{\mu}{}_{\alpha\dot\beta}$



- can $\alpha = \dot\beta$? ← $\alpha = 1$ & $\dot\beta = 1$ BUT THESE HAVE "NOTHING" TO DO W/ EACH OTHER think of it this way



SIZE & COLOR ARE RELATED,
BUT INDEPENDENT QUANTUM NUMBERS

- Do different $\mu, \alpha, \dot{\beta}$ indices ⟷ multiple particles?

  eg SU(2) doublet : $D = \begin{pmatrix} N \\ C \end{pmatrix}$ ⟵ two particles

  symmetry: rotate between them.

  eg. ~~3D~~ 2D ROTATIONS

$$= \begin{pmatrix} V_x \\ V_y \end{pmatrix}$$

but I can rotate between V_x & V_y

is this particle "different" from this:

?

- Was the goal to find invariants?

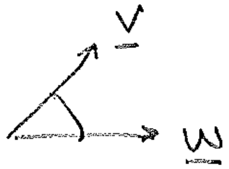
  ↳ yes.

- ? when you contract indices?

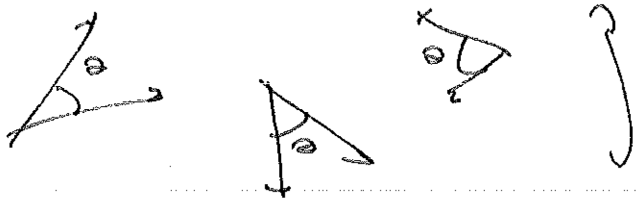
  physically ?

  eg. 2D rotations

  $v^i w_i$



  $= |\underline{v}||\underline{w}| \cos\theta$



all related by rotations, $v^i w_i$ unchanged

for 4-vectors :

$$P^2 = E^2 - \underline{p}^2 = m^2$$

~~$(P_1 \cdot P_2)$~~ $(P \cdot K)$ also invariant

different momenta.

meaning? RELATED TO "what is the Energy of P-PARTICLE in K-REST FRAME"

$$(E_p, \underline{p}) \cdot \begin{pmatrix} m \\ \underline{0} \end{pmatrix} = P \cdot K \text{ in any frame.}$$

-

- LH & RH completely different?

  ↳ yes QUANTUM PHENOMENON.

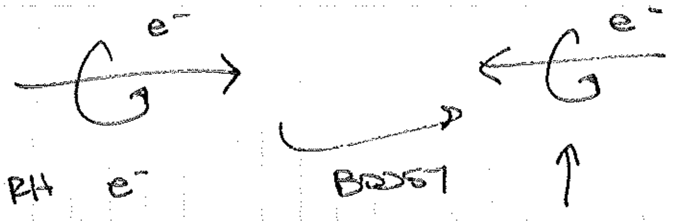
  & coincidence of 4D spacetime.

  analog of arm is quantum phase

  CANNOT LORENTZ TRANSFORM ONE CHIRALITY TO THE OTHER.

- relation to mass?

  ↳ first we have to figure out what mass is.



iff $M_e > 0$

compare to $(e^+)^\beta \longrightarrow e^\alpha$ ← LH particle

RH anti particle

- μ index on γ?

  ↳ $A_\mu = (V, \underline{A})$

  electric pot.

  vector pot.

↳ think of LIGHT WAVE:

2 polz ... but μ has 4 components!!

- ~~Where do $\sigma^{\mu}_{\alpha\beta}$ come from?~~
- fundamental reason why 720° rotation for electron?

  $\psi \to e^{i\theta}\psi$ "invariance" of wavefunction for one particle

  Projective representation

  ↳ see my notes.

  & try HW extra credit.

- How to use magic tensors?

  → constrain allowed interactions.

  otherwise, any interaction is allowed.

- $\underset{\mu=0,1,2,3}{\underset{\uparrow}{(\sigma^{\mu})_{\alpha\beta}}}$? No $\underset{i=1,2,3}{\underset{\uparrow}{(\sigma^{i})_{ab}}}$

- grap theory stuff : structure constants

  ↳ they're there.