TODAY: QUESTIONS? / RECAP
HW1a DEBRIEF
QED VARIANTS, $W^\pm$, FLAVOR
MORE SPECIAL RELATIVITY / KINEMATICS

QUESTIONS?

comment: lots of great questions on short HW1 ... why didn't anyone ask @ start of class on thursday?

Remarks & responses

- $m_p = 938.\ldots$ MeV
  $= 9.38 \times 10^2$ MeV

$\xrightarrow{\text{ORDER OF MAGNITUDE}}$ 10^3 MeV $=$ GeV ← "0 sig fig"

$\xrightarrow{\text{one sig fig}}$ 9×10^2 MeV

If you want to be fancy: keep track of propagation of errors, eg.

$$0.5 \times 0.5 = 0.25 \sim 10^{-1}$$

$$\mathcal{O}(1) \times \mathcal{O}(1) = \mathcal{O}(1)$$

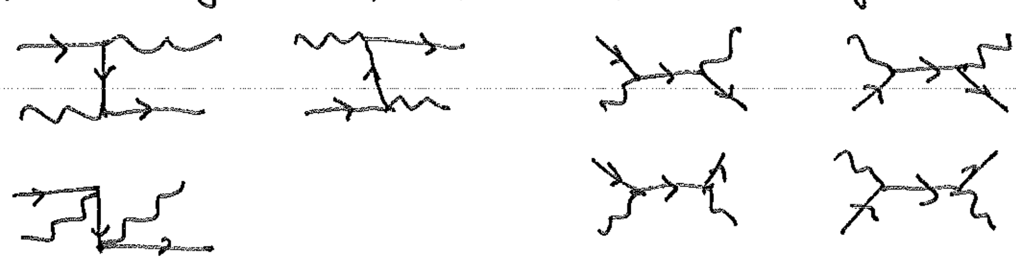
... we won't make a big deal about this. BUT BE AWARE OF IT.

↳ until recently, cosmologists would do silly things like

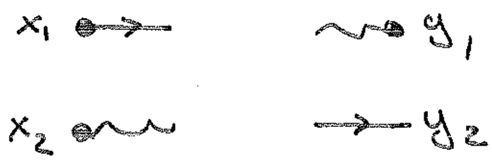
$$\pi \sim \mathcal{O}(1) \ldots \Rightarrow \pi^{100} \sim \mathcal{O}(1)$$

- Feynman Rules & Diagrams

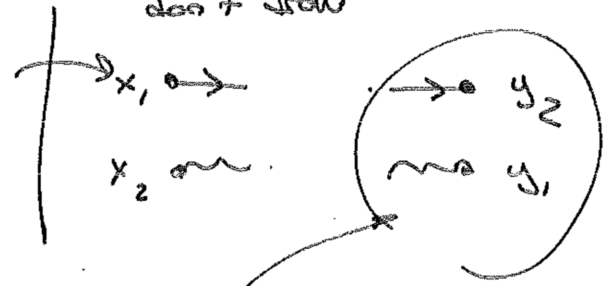
which diagrams are the same? WHY?



issue: be consistent w/ how you draw in/out states

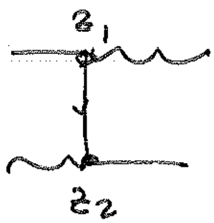


don't draw



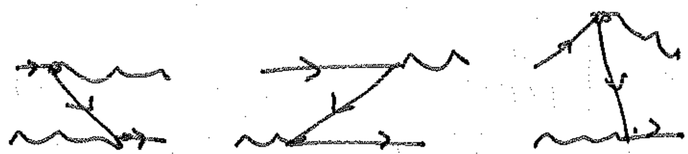
no! once you pick a convention ("y_1 in top right"), stick to it — avoids confusion.

<u>then</u>:



reads like a story

↑ <u>includes</u>

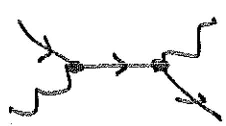


etc.

but always: the e^- @ x_1 goes to z, then emits a γ that goes to y_1 ...

never: the e^- @ x_1 goes to z where it meets the γ from x_2 ...

<u>so</u>:



is a second, unique diagram

↑ includes

etc.

- Are the rules correct?

this is a good & deep question

RELATED: is Newtonian physics correct?
Gen. Relativity?

⟹ physics (science!) is scale-dependent

QED is a model ← distinguish from "theory"

↳ Bohr model of hydrogen is a model
... that's why it gets so many corrections...

QFT is a framework for models

there is something remarkable about many models ⟶ they tell you when they break down.

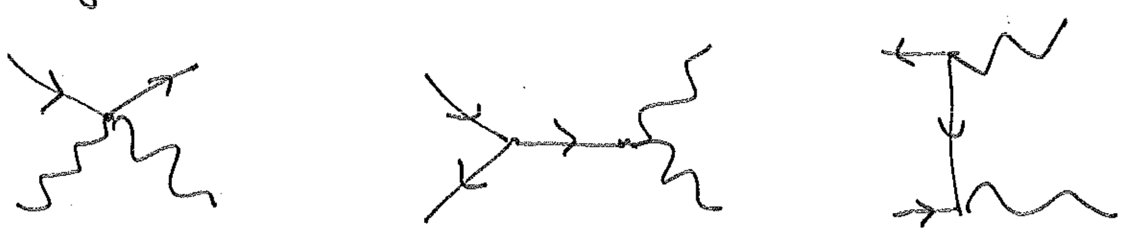
↓

WHY WE THINK QUANTUM GRAVITY IS DIFFERENT

WHY WE THINK THERE'S MORE TO THE HIGGS

- SPEAKING OF THE RULES: some anonymous misconceptions:

Why are these incorrect?



Feynman Recap

Diagram drawing game

List of lines → PARTICLES

List of rules to connect → INTERACTIONS

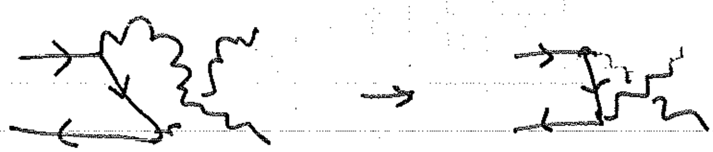
↳ imagine these as cards in some board game

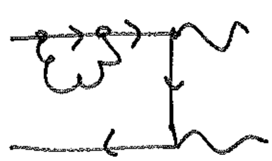
you can use as many cards as you need to connect an $|\text{in}\rangle$ to an $|\text{out}\rangle$

... but only the cards you have.

to draw diagrams, lines don't have to be straight. May even "pass under."

↳ BUT TIDY THEM UP ONCE YOU'VE DRAWN THE TOPOLOGY





← can't tidy up further

INTERPRETATION : spacetime diagram

We draw distinct topologies. (why we can "tidy")

↳ so moving internal vertices around is implicit

↳ we're summing over all internal spacetime events eg: QED, succinctly: $\longrightarrow$ e $\quad$ ~~ γ 

I DON'T EVEN BOTHER DISTINGUISHING PARTICLE FROM ANTIPARTICLE .. SAME! JUST ROTATE ON DIAGRAM

CONSERVATION LAWS BUILT IN!

e^- moving forward in time

e^+ moving forward in time

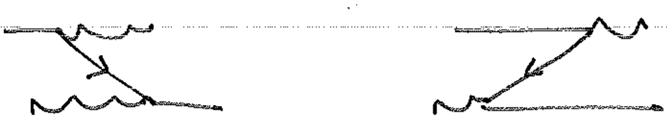
WHAT IS ANTIMATTER? WHY DO WE NEED IT?

for now: OPPOSITE CHARGE
full answ: OPPOSITE CHARGE + PARITY

CP symmetry

WHY DO WE NEED IT?

HOKEY INTERP: SR + QM



intermediate (virtual) e^- exchanged, moving fwd in time

either e^- exchanged superluminally (!) or e^+ exchanged.

Better: (via QUORA: Why does SR req matter to mirror antim.)

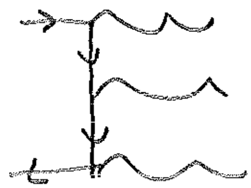
quantum field theory

FIELDS $\leftrightarrow$ PROGENITOR OF PARTICLES

we'll get to it. $\mathcal{L}$ is $\mathbb{R}$ func. of $\mathbb{C}$ fields

ϕ & ϕ^* are different & not analytically related.

eg. $e^+e^- \to \gamma\gamma\gamma$ ← possible? Sure... no conservation of γ number.



(total of 3! diagrams)

RULES: KINEMATICS

from now on, all observed states are <u>momentum</u> eigenstates

↳ definite p_μ ⟷ integrate over all x^μ

so when we say x_1^μ •⟶

really we do a fourier transform

there is <u>something</u> that we multiply by $e^{-ix\cdot p}$ & integrate d^4x_1.

namely: AMPLITUDE: $\psi(x_1, \dots)$

→ momentum space amplitude

$$\tilde{\psi}(p_1, \dots) \sim \int d^4x_1\, e^{-ix_1\cdot p}\, \psi(x_1, \dots)$$

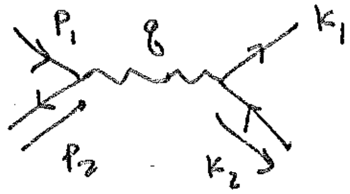
- ALL LINES PRESERVE 4-MOMENTUM
- ALL VERTICES CONSERVE TOTAL 4-MOMENTUM
- ALL EXTERNAL PARTICLES ARE ON-SHELL
  - ↳ PREVENTS $e^+e^- \to \mu^+\mu^-$ @ low energies

no matter how complicated the diagram, this enforces <u>total</u> 4-momentum cons.

↳ SAME WAY local Q cons. @ vertex enforces total Q cons in any diagram.

WE CAN DETERMINE THE 4-MOMENTUM IN EACH LEG

eg



$$\left.\begin{aligned}(p_1+p_2)^\mu &= q^\mu \\ q^\mu &= (k_1+k_2)^\mu\end{aligned}\right\} \quad (p_1+p_2)^\mu = (k_1+k_2)^\mu$$

in any frame

eg in CM FRAME:

$$p_1 = (E, 0, 0, p) \qquad E^2 = m^2 + p^2$$

$$p_2 = (E, 0, 0, -p)$$

$$K_1 = (E, p\hat{k})$$

$$K_2 = (E, -p\hat{k})$$

$\hat{k}$ is some unit 3-vector
ELASTIC COLLISION IN CM FRAME

long way: $K_1 = (E', \underline{k}_1)$

$$K_2 = (E'', \underline{k}_2) = (E'', -\underline{k}_1)$$

CM FRAME: $\underline{k}_1 = -\underline{k}_2$

on shell: $(E')^2 = m^2 + |\underline{k}_1|^2$

$$(E'')^2 = m^2 + (-\underline{k}_1)^2$$

$$E' = E''$$

total momentum conservation

$$(p_1+p_2)^\mu = (2E, 0, 0, 0)$$

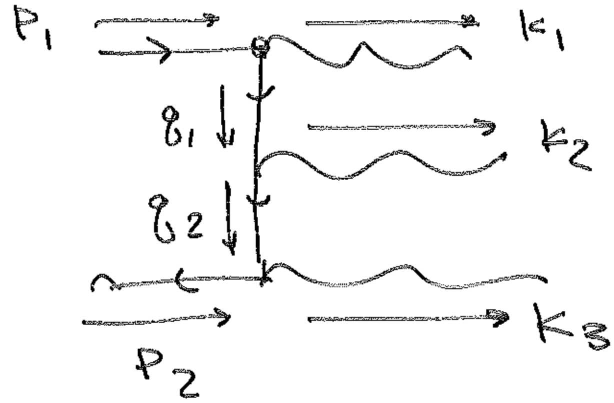
$$(k_1+k_2)^\mu = (2E', 0, 0, 0)$$

$$\Downarrow$$

$$E = E'$$

$$\begin{array}{ccc} \shortparallel & & \shortparallel \\ m^2 + p^2 & & m^2 + |\underline{k}_1|^2 \end{array}$$

$|\underline{k}_1| = p$



$(E_1, \underline{k}_1)$ $\quad E_1^2 = |\underline{k}_1|^2$

$(E_2, \underline{k}_2)$ $\quad E_2^2 = |\underline{k}_2|^2$

$(E_3, \underline{k}_3)$ $\quad E_3^2 = |k_3|^2$

GIVEN SOME P_1 & P_2 , GO TO CM FRAME

$$P_1 = (E, 0, 0, p)$$
$$P_2 = (E, 0, 0, -p)$$

$\boxed{\text{HW}}$

$$\left.\begin{aligned} p_1{}^\mu &= k_1{}^\mu + q_1{}^\mu \\ q_1{}^\mu &= k_2{}^\mu + q_2{}^\mu \\ p_2{}^\mu + q_2{}^\mu &= k_3{}^\mu \end{aligned}\right\}$$

do you see how we write these?

(4 eqns each.)

<u>Prove</u>: $\underbrace{(p_1 + p_2)^\mu = (k_1 + k_2 + k_3)^\mu}_{\text{total 4-momentum cons.}}$

<u>observe</u>: what are constraints on 3-momenta?

$$\underbrace{E_1 + E_2 + E_3}_{\sum_i |\underline{k}_i|} = 2E$$

$\sum_i |\underline{k}_i|$ $\leftarrow$ sum of lengths

$\sum_i \underline{k}_i = 0$ $\leftarrow$ vector sum

INTERPRETATION

Dynamics: DIAGRAMS ARE A TAYLOR EXPANSION OF THE AMPLITUDE

↳ simplest diagrams are the most dominant contributions

 $\sim \mathcal{O}(\varepsilon)$

Q: WHAT IF $\varepsilon \sim 1$?
THEN DIAGRAMS FAIL TO BE USEFUL TO CALCULATE AMPLITUDE

Kinematics: BUT EVEN A NON-ZERO AMPLITUDE (@ least @ diagram-drawing level)

may be zero b/c not kinematically allowed ↳ ext. states cannot be on-shell

eg. $e \to e\gamma$

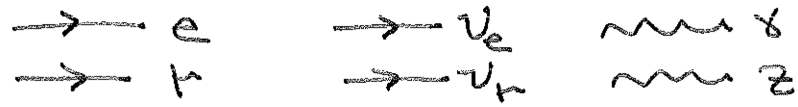
$e^+e^- \to \mu^+\mu^-$ if energy too low.

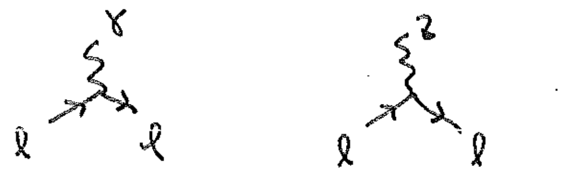
but if I boost?

INVARIANTS

the menagerie of Rules:

ADDING MORE TILES DOESN'T CHANGE THE RULES OF THE GAME. THIS IS LIKE DLC.





$\ell = e, \mu$ $\qquad$ $\ell = e, \mu, \nu_e, \nu_\mu$

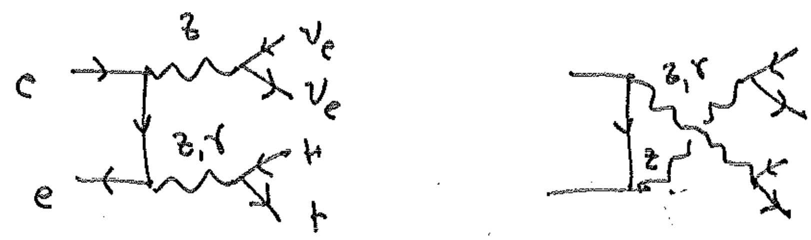
<u>conservation laws</u>: electric charge

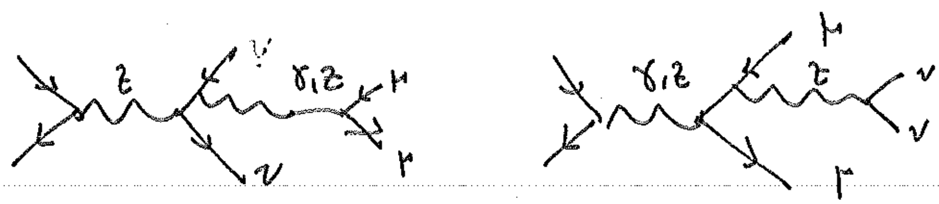
"Z" charge (eh?!)

$$\left.\begin{array}{l} e\text{-ness} \\ \mu\text{-ness} \\ \nu_e\text{-ness} \\ \nu_\mu\text{-ness} \end{array}\right\} \begin{array}{l} e\ \# \\ \mu\ \# \\ \nu_e\ \# \\ \nu_\mu\ \# \end{array}$$

<u>BREAK</u>: $e^+e^- \to \nu_e \bar{\nu}_e \mu^+ \mu^-$, leading order assuming known.

anti





etc. $\qquad$ etc.

obs: different momentum routing!

INVARIANCE & COVARIANCE

why do we have vectors & matrices?

2D EUCLIDEAN SPACE:

$$\underline{v} = v^i \hat{\underline{e}}_{(i)} \longrightarrow \begin{pmatrix} v^1 \\ v^2 \end{pmatrix}$$

BASIS VECTOR $\quad v^i$

eg $\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

transform under rotations in a well defined way.

$$\underbrace{\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}}\begin{pmatrix} v^1 \\ v^2 \end{pmatrix} = \begin{pmatrix} w^1 \\ w^2 \end{pmatrix}$$

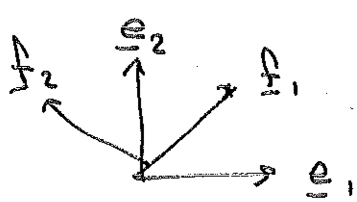
$$\underbrace{R^i{}_j \quad v^j} = w^i$$

einstein convention: repeated upper/lower indices are summed over

→ this is just to make equations simpler to write

OBS: BOTH SIDES HAVE ONE ~~FREE~~ INDEX

alternatively: $\underline{V} = v^j \underline{e}_{(j)} = w^i \underline{f}_{(i)}$



$$\underline{e}_{(j)} = R^i{}_j \underline{f}_{(i)}$$

$$\underline{V} = \underbrace{V^j R^i{}_j}_{R^i{}_j V^j} \underline{f}_{(i)} = \underbrace{W^i}_{= W^i} \underline{f}_{(i)}$$

VECTOR HAS WELL DEFINED TRANSFORMATION RULES

$$R^i{}_j \, V^j$$

lower index contracts w/ upper index

spits out object w/ upper index

"ROW VECTORS" (bra vs. ket)

$\underline{V}^2 = V_j V^j$ no free indices

HOW DO THEY TRANSFORM?

$$V_j \to (R^{-1})^j{}_i \, V_j$$

Why? b/c we know $\underline{V}^2$ is a scalar → DOES NOT TRANSFORM.

$$\underline{V}^2 = V_j V^j \longrightarrow V_j \underbrace{(R^{-1})^j{}_i \, R^i{}_j}_{} V^j$$

PRECISELY $R^{-1}R = \mathbb{1}$

SO: INDEX STRUCTURE TELLS US HOW AN OBJECT TRANSFORMS W/ RT ROTATIONS

$$T^{ij}{}_k \longrightarrow R^i{}_{i'} \, R^j{}_{j'} \, (R^{-1})^{k'}{}_k \, T^{i'j'}{}_{k'}$$

etc.

WE PRESERVE THE EUCLIDEAN INNER PRODUCT

↳ lengths of vectors

Minkowski space $\longleftrightarrow$ SR

preserve $\boxed{E^2 - p^2}$ $\leftarrow$ $\begin{pmatrix} E \\ p \end{pmatrix}$ $\leftarrow$ 1+1 dim

compare to $E^2 + p^2$ for EUCLIDEAN.

then the Rotations $\longrightarrow$ Lorentz transforms

IN PRACTICE:

AN OBJECT W/ LORENTZ INDICES TRANSFORMS

$$p^\mu \longrightarrow \Lambda^\mu{}_\nu \, p^\nu$$

$$\begin{pmatrix} \gamma & \gamma\beta \\ \gamma\beta & \gamma \end{pmatrix}$$

AN OBJECT W/O LORENTZ INDICES IS INVARIANT:

$$p^2 = p^\mu p_\mu = \boxed{E^2 - p^2 \equiv m^2}$$

so I can talk about the MASS of an electron w/o specifying what frame it's in.

Q: $e^+e^- \to \mu^+\mu^-$: how can I tell if I have enough energy?

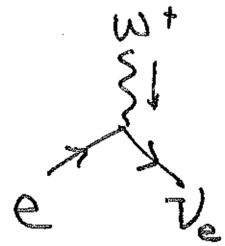
$$(P_1 + P_2)^2 = E_{cm}^2 \quad \text{in CM FRAME} \to \text{in ANY FRAME}$$

$$\equiv s$$

THINGS TO THINK ABOUT

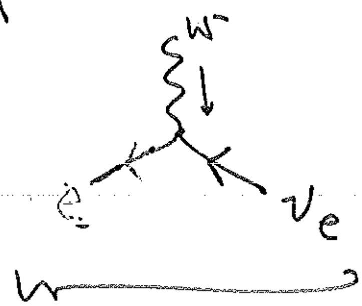
could draw

W BOSON → CHARGED $W^{\pm}$ ... but most people don't

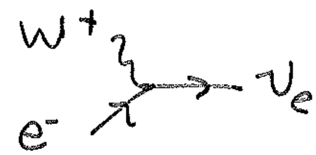


such that charge is conserved

two different rules



re



e # & ν_e # are violated!!