TODAY : Midterm

1. Antonio - WEAK NEUTRAL
2. Joy - CMS cosmic
3. Jesus - Z boson

- HW5 1.3 / red

REBOOT : HOW TO WRITE A THEORY

1. IDENTIFY SYMMETRIES (charges & indices)
2. IDENTIFY FIELDS & how they transform under #1
3. WRITE LAGRANGIAN DENSITY , $\mathcal{L}(x)$

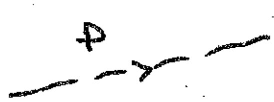
POLYNOMIAL IN THE FIELDS , $\varphi(x)$ @ a point
→ INVARIANT
→ REAL
→ COEFFICIENTS ⟷ #

1 & 2 are as usual.

Step 3 : FANCY WAY OF "WRITE ALL INVARIANT VERTICES"

$$\mathcal{L} = [\text{QUADRATIC TERMS}] + [\text{HIGHER ORDER}]$$

CAN SOLVE EXACTLY
USUALLY OF THE FORM
"$(\partial\phi)^2 - m\phi^2$"
from "network of springs"
GIVES: PROPAGATOR

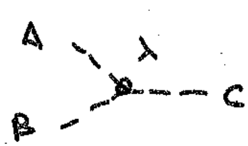


$$\sim \left(\frac{1}{p^2 - m^2}\right)$$

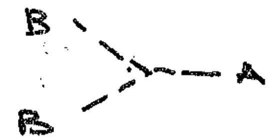
have to do Taylor expansion

PRACTICALLY :

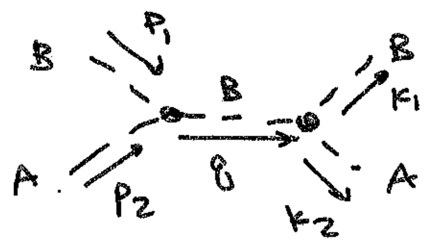
$\lambda A(x) B(x) C(x)$



<u>eg</u>: $\mathcal{L} = \frac{1}{2}(\partial A)^2 - \frac{1}{2} M_A^2 A^2 + \frac{1}{2}(\partial B)^2 - \frac{1}{2} M_B^2 B^2 + \lambda \underbrace{A(x) B(x)^2}_{\text{3 fields meeting @ a point}}$

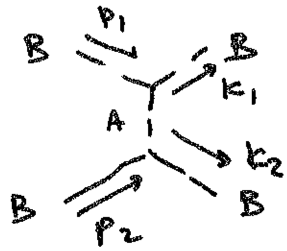

$\sim \lambda$

SO I CAN DRAW: $BA \to BA$

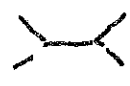


$$\sim \frac{\lambda^2}{\underbrace{(p_1+p_2)^2}_{q^2} - M_B^2}$$

CAN ALSO DRAW: $BB \to BB$



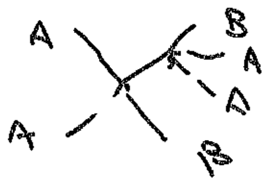
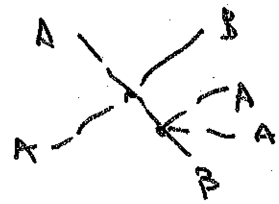
$$\frac{\lambda^2}{(p_1-k_1)^2 - M_B^2} + \frac{\lambda^2}{(p_1+p_2)^2 - M_B^2} + \frac{\lambda^2}{(p_1-k_2)^2 - M_B^2}$$

eg if $(p_1+p_2)^2 \approx M_B^2$,  DOMINATES!

eg: $\mathcal{L} = (\text{QUADRATIC TERMS}) + g \underbrace{A(x)^2 B(x)^2}$


$\sim g$

$\overset{p_1 p_2}{AA} \to \overset{k_1 k_2 k_3 k_4}{BBAA}$

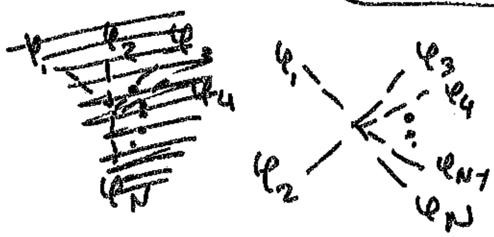
$$= \frac{g^2}{(p_1+p_2-k_4)^2 - M_B^2} + \frac{g^2}{(p_1+p_2-k_3)^2 - M_B^2}$$

<u>QUADRATIC TERMS</u>: PROPAGATORS, HOW INTERNAL LINES GO FROM ONE SPACETIME POINT TO ANOTHER

<u>HIGHER ORDER TERMS</u>: $(\#)\ \underbrace{\varphi_1(x)\varphi_2(x)\cdots\varphi_N(x)}$

coupling const. PRODUCT OF FIELDS @ SAME POINT.

<u>RULE</u>:



$\sim (\#)$

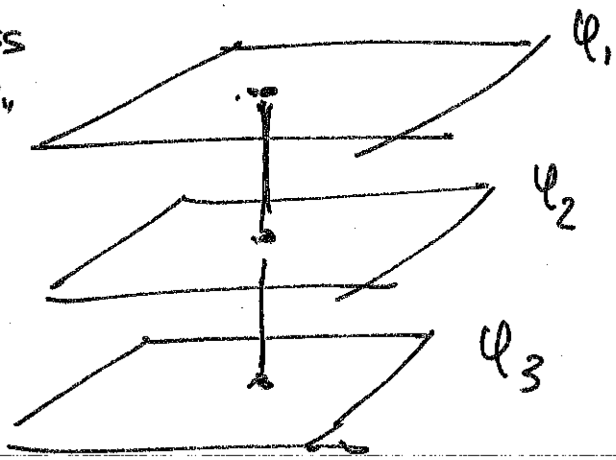
It's as simple as that.

EACH <u>FIELD</u> $\longrightarrow$ LINE GOING INTO THE VERTEX

if it's conjugate, then

- line going out or (EQUIVALENTLY)
- anti-line going in.

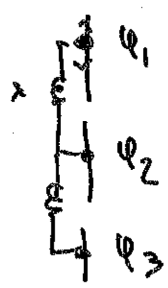
MATTRESS "SPRING THEORY"



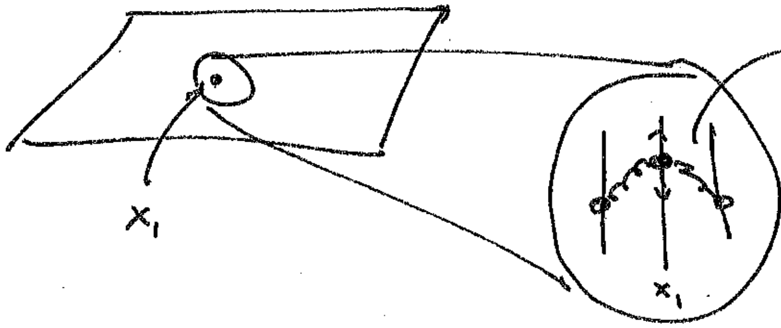
"$\lambda\varphi_1\varphi_2\varphi_3$" means that the quantum fields for these 3 particles are coupled.

QUANTUM ~~TEEF~~ EXCITATIONS OF ONE CAN PRODUCE EXCITATIONS IN THE OTHER TWO.

λ: HOW COUPLED THE SPRINGS ARE.



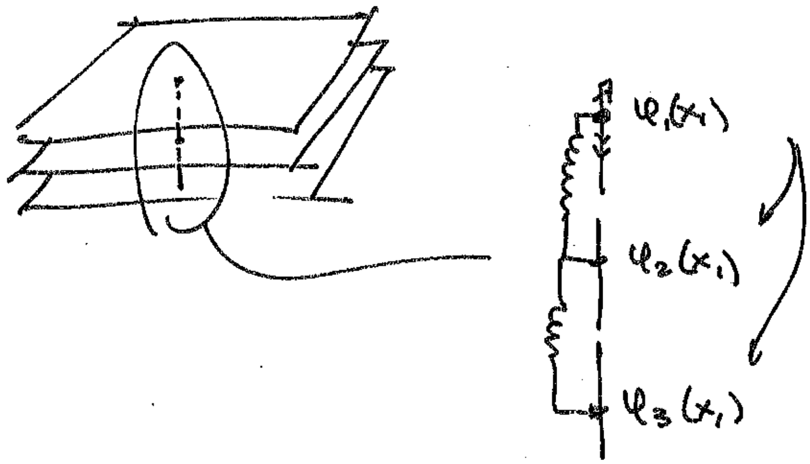
EXCITATION IN $\varphi_1(x_1)$ (QUANTUM)



can transfer excitation to neighboring spring of <u>same</u> field

(PROPAGATION IN SPACETIME)

↳ QUADRATIC PART



can transfer excitation into <u>other</u> fields @ same spacetime point

(interaction)

<u>propagator</u> (internal line)

↳ excitation in one field propagating in spacetime

<u>Vertex</u>: excitations exchanged between fields @ same position

SO FAR, NO INDICES.

↳ indices & charges just serve to restrict what terms we can write down.

We are free to use whatever tensors are available

eg.

LH FERMION $\psi^{\alpha}(x)$

↳ can write $\#\, \underbrace{\psi^{\alpha}(x)\,\psi^{\beta}(x)}_{\text{QUADRATIC}}\, \varepsilon_{\alpha\beta}$

$\#$ ← MASS

cannot write: $\#\, \psi^{\alpha}(x)\,\psi^{\beta}(x)\,\psi^{\gamma}(x)$

no tensor available to contract them!

YUKAWA:

$L^{\alpha a}(x)_{Y=-1/2}$ $\qquad L^{\dagger\dot\alpha}{}_{a}(x)_{Y=1/2}$

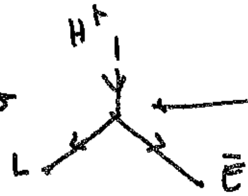
$\bar{E}^{\alpha}(x)_{Y=1}$ $\qquad \bar{E}^{\dagger\dot\alpha}(x)_{Y=-1}$

$H^{a}(x)_{Y=1/2}$ $\qquad H^{\dagger}{}_{a}(x)_{Y=-1/2}$

can form:

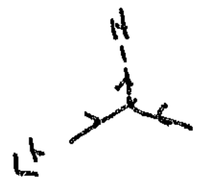
$$y\, H^{a}(x)\, L^{\dagger\dot\alpha}{}_{b}(x)\, \bar{E}^{\dagger\dot\beta}(x)\, \underbrace{\boxed{\varepsilon_{\dot\alpha\dot\beta}\, \delta^{b}_{a}}}_{\text{tensors}}$$

$y\,\varepsilon_{\dot\alpha\dot\beta}\,\delta^{b}{}_{a} \rightarrow$



$y^{\dagger} H^{\dagger}{}_{a} L^{\alpha b}\, \bar{E}^{\beta}\quad \varepsilon_{\alpha\beta}\,\delta^{a}_{b}$

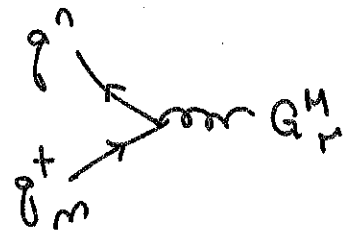
$\boxed{\text{BUT } \mathcal{L} \text{ is } \mathbb{R}, \text{ so include } + \text{c.c.}}$



$y^{\dagger}\varepsilon_{\alpha\beta}\,\delta^{a}{}_{b}$

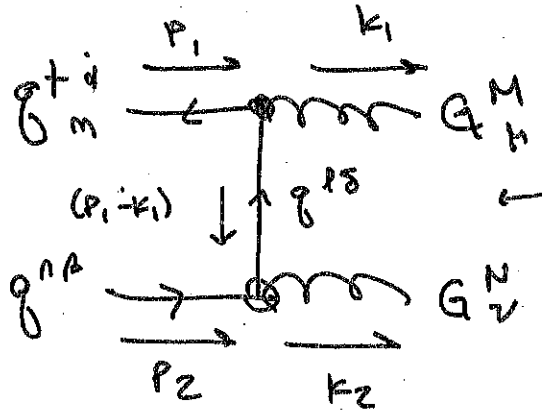
$\underline{\text{QCD}}$ interactions : $g_c\, q^{\dagger\dot\alpha}_{L\,m}\, G^M_\mu\, \underbrace{\sigma^\mu_{\dot\alpha\beta}}_{\text{spin tensor}}\, \underbrace{(T^M)^m{}_n}_{\text{color tensor}}\, q^{\beta n}_L$

LH QUARK
W/ SPIN β
& COLOR n



$\sim g_c\, \underbrace{\sigma^\mu_{\dot\alpha\beta}(T^M)^m{}_n}$

PART OF THE
FEYNMAN RULE



QUADRATIC TERM
HAS INDICES, TOO!

$\Delta(p_1-k_1)^{\ell\delta}{}_{\ell'}{}^{\dot\delta}$

$\sim \dfrac{\cdots^{\ell\delta}{}_{\ell'}{}^{\dot\delta}}{(p_1-k_1)^2-m_q^2}$

$$\mathcal{M} \sim g_c(\sigma^\mu)_{\dot\alpha\delta}\,(T^M)^m{}_\ell\, \frac{\cdots^{\ell\delta}{}_{\ell'}{}^{\dot\delta}}{(p_1-k_1)^2-m_q^2}$$

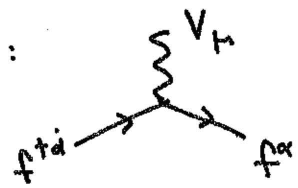
$$(\sigma^\nu)_{\dot\delta\beta}\,(T^N)^{\ell'}{}_n$$

(just a giant matrix multiplication)

$\underline{\text{QFT}}$: practical output - set of algorithms to resolve the contracted indices.

SOME GUIDELINES

SPIN :

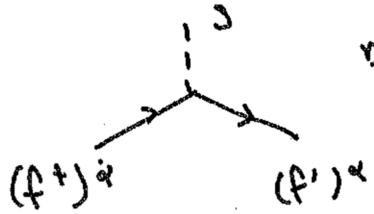


$\sigma^\mu{}_{\dot\alpha\beta}$

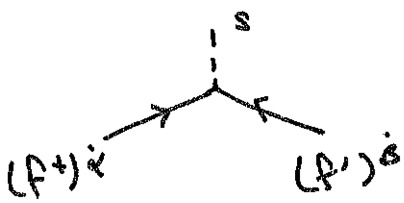
$\boxed{f^\dagger V_\mu \sigma^\mu f}$

similarly for RH.

same f

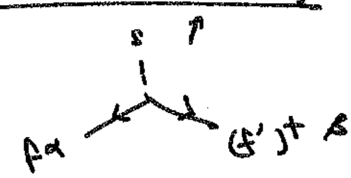


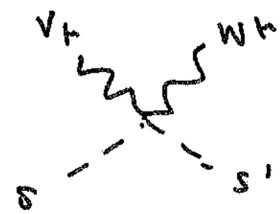
not allowed

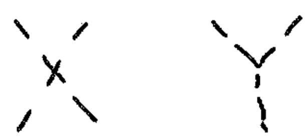


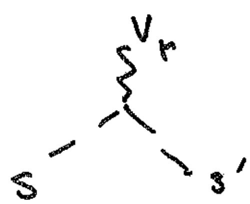
$\varepsilon_{\dot\alpha\dot\beta}$

$\boxed{s f^\dagger f' \ + h.c.}$



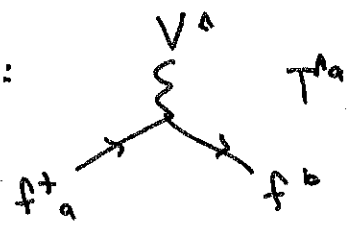






$\partial^\mu \longleftrightarrow p^\mu$

GAUGE :

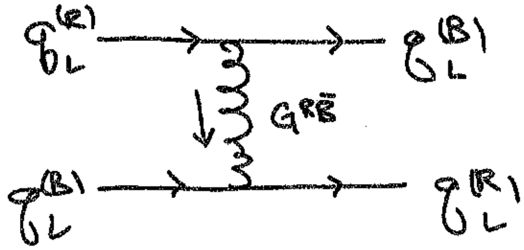


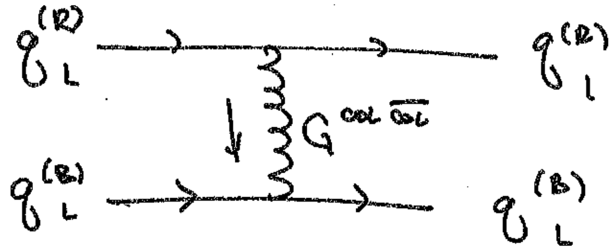
$T^a{}_b$

HW 5b 1.3

$\boxed{\text{need laptop}}$

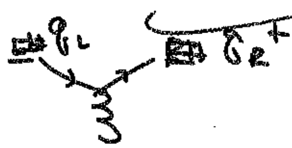
(a) $q_L^{(R)} q_L^{(B)} \longrightarrow q_L^{(R)} q_L^{(B)}$

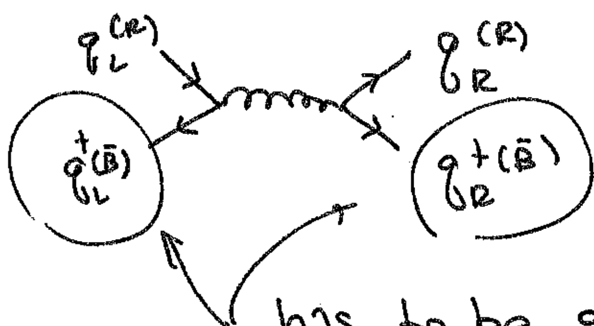




(b) $q_L^{(R)} q_L^{(B)} \longrightarrow q_R^{(R)} q_R^{(B)}$

cannot have





has to be <u>anti</u> quark

(c) $q_R^{(R)} G^{(G\bar{R})} \rightarrow q_R^{(G)} G^{(B\bar{B})}$