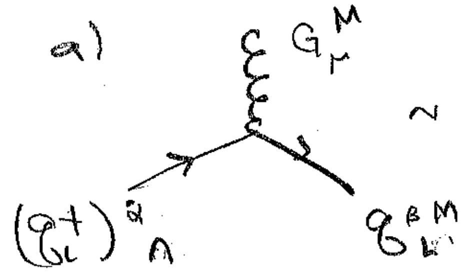
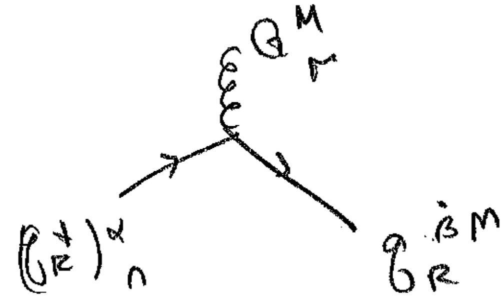
$\boxed{1.1}$ a)



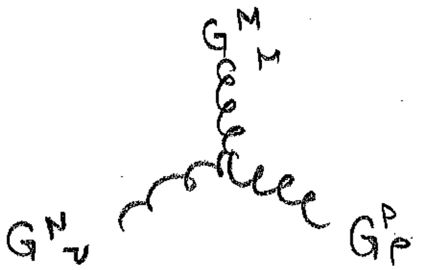
$$\sim \underbrace{\bar\sigma^{\mu}{}_{\dot\alpha\beta}}_{\text{SPIN}}\ \underbrace{(T^M)^n{}_m}_{\text{COLOR}}$$

b)



$$\sim \sigma^{\mu}{}_{\alpha\dot\beta}\,(T^M)^n{}_m$$

c)



$$\sim \underbrace{\varepsilon^{MNP}}_{\text{color}}\ \underbrace{\eta_{\mu\nu}\,p_\rho}_{\text{spin}}$$

$\boxed{1.2}$ a) $q_L\ (q_R^\dagger)\ G$ ✗

$\alpha \qquad \beta$

two undotted indices

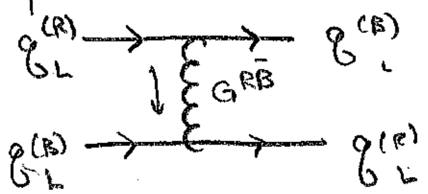
(can contract w/ $\varepsilon_{\alpha\beta}$, but then uncontracted μ index on G)

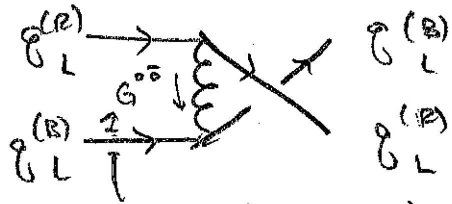
b) $(q_L)^\ell (q_L)^m (q_L)^n\, \varepsilon_{\ell m n}$ ✗

three undotted indices. no tensor to contract all three.

$\boxed{1.3}$

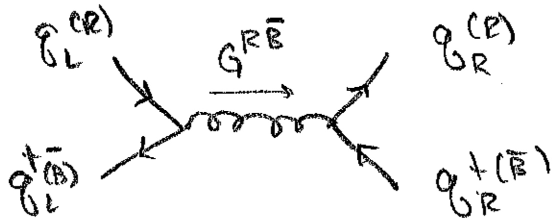
a)





any color/anti (same) color combination

b)



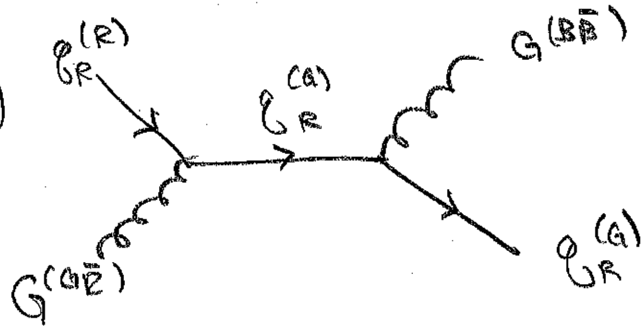
UPDATE 2/27

the HW had a typo! :(

↳ SHOULD BE

$q_L^{(R)}\, q_L^{\dagger(\bar{B})} \to q_R^{(R)}\, q_R^{\dagger(\bar{B})}$

c)



$\boxed{1.4}$ a) $q_L^{(R)}\, q_L^{(B)} \longrightarrow q_L^{(G)}\, q_L^{(B)}$

Does not conserve color
LHS has 1 unit of red ness, no green
RHS has 1 unit of green-ness, no red.

b) $q_L^{(R)}\, q_L^{(B)} \longrightarrow q_R^{(R)}\, q_L^{(B)}$

GLUONS CONSERVE CHIRALITY, BUT THIS PROCESS DOES NOT

(⌒ only connects $q_L^\dagger q_L$, never $q_L^\dagger q_R$, for eg)

c) $q_R^{(B)}\, G^{(G\bar{R})} \longrightarrow q_R^{(G)}\, G^{(B\bar{B})}$

DOES NOT CONSERVE REDNESS / BLUE-NESS

2. $\Delta^{++}_{3/2} = |\uparrow\rangle \otimes |\uparrow\rangle \otimes |\uparrow\rangle$

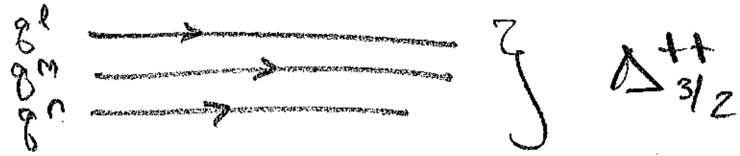
these are fermions, so wavefunction must be antisymmetric.

HOWEVER, w/o an additional index, this configuration is <u>symmetric</u>.

SOLUTION: EACH QUARK IS A DISTINCT COLOR. THEN CONTRACT COLOR INDICES W/ ε_{lmn}.

$$|\uparrow\rangle^l \otimes |\uparrow\rangle^m \otimes |\uparrow\rangle^n \underbrace{\varepsilon_{lmn}}_{\text{totally antisym.}}$$

nb: this is <u>not</u> a vertex. this is a particle



3. FIELD THEORY

3.1 $L = \frac{1}{2}\sum \Delta x \left[\frac{m}{\Delta x} \dot{q}_i^{\,2} - K\Delta x \frac{(q_{i+1} - q_i)^2}{\Delta x^2} \right]$

See addendum at end of this document! 3/12/2018

have the same mass dim IN NATURAL UNITS

(because $[\dot{q}] = [\frac{\Delta q}{\Delta x}]$)

$[\frac{m}{\Delta x}] = [K\Delta x]$ ← note $[\Delta x] = -1$

$$[K] = [\Delta x^2 \, m] = 2[\Delta x] + [m] = -2 + 1$$

$\cancel{\boxed{[K] = -1}}$

[k] = +3

OR:

$\cancel{[m(\frac{\Delta q}{\Delta t})^2] = [K \Delta q^2]}$

$\cancel{\Rightarrow [K] = [m] - 2[\Delta t]}$

$\cancel{= \boxed{-1}}$

3.2 $S = \int dt \, L \rightarrow \underbrace{[L] + [dt] = 0}$

$e^{iS} \rightarrow [S] = 0$ $\Rightarrow \boxed{[L] = 1}$

IN THE DERIVATION, $\rho = m/\Delta x \rightarrow \boxed{[\rho] = 2}$

then: (eq (6)) → $\cancel{[\frac{\Delta q^2}{\Delta t^2}]} = [\frac{K}{\rho}] \cancel{[\frac{\Delta q^2}{\Delta x^2}]}$

$$[\frac{K}{\rho}] = 0$$

$[K] - [\rho] \Rightarrow \boxed{[K] = [\rho] = 2}$

3.3 the trick here is:

$$L = \frac{\rho}{2} \sum_i \Delta x \left[\dot{q}_i^{\,2} - \left(\frac{\kappa}{\rho}\right) \left(\frac{\Delta q}{\Delta x}\right)^2 \right]$$

these 2 terms must have the same units.

$$[\dot{q}^2] = 2[q] - 2[t]$$

$$\left[\left(\frac{\kappa}{\rho}\right) \frac{\Delta q^2}{\Delta x^2} \right] = 2[q] - 2[\Delta x] + \left[\frac{\kappa}{\rho}\right]$$

EQUATING THESE

$$-2[t] = -2[x] + \left[\frac{\kappa}{\rho}\right]$$

$$\Rightarrow \left[\frac{\kappa}{\rho}\right] = 2\left([x] - [t]\right)$$

$$\boxed{\left(\frac{\kappa}{\rho}\right) \sim (\text{length})^2 (\text{time})^{-2}}$$

$$\sim \left(\frac{\text{length}}{\text{time}}\right)^2$$

this is a <u>SPEED</u>

$\Rightarrow \frac{\kappa}{\rho} = \frac{1}{c^2}$ } the speed of "massless" waves in the medium.

HW5 - LONG ADDENDUM (3.1)

$$L = \sum_i \tfrac{1}{2} m \dot{q}_i^{\,2} - K(q_{i+1} - q_i)^2$$

$$\underset{\overset{\shortparallel}{0}}{[S]} = [L\,dt] = \underset{+1}{[m]} + 2[q] - \underset{-1}{[t]}$$

$$\longrightarrow \quad 0 = 2 + 2[q] \quad \hookrightarrow \quad [q] = -1$$

$$\underset{\overset{\shortparallel}{0}}{[dt\,K\,(q_{i+1} - q_i)^2]} = [K] - 1 + 2[q]$$

$$= [K] - 3$$

$$\Rightarrow \boxed{[K] = 3}$$

3.4 $$S = \frac{1}{2} \int \underbrace{dx\, dt}_{\text{MASS DIM } -2} (\partial \theta)^2$$

$[\partial] = +1$

$\Rightarrow \boxed{[\theta] = 0}$ ← only in 2D spacetime!

from dimensional analysis,
ripples travel w/ speed $= \boxed{c = \kappa/\rho = 1}$