[1] the key to this problem is observing that:

$$\bar{\Psi}\gamma^\mu\Psi = \underbrace{\Psi^\dagger\gamma^0}_{(\psi_R^\dagger \;\; \psi_L^\dagger)}\gamma^\mu\Psi$$

$\Psi = \begin{pmatrix}\psi_L \\ \psi_R\end{pmatrix}$, $\gamma^\mu = \begin{pmatrix} & \sigma^\mu \\ \bar{\sigma}^\mu & \end{pmatrix}$

$\bar{\sigma}^\mu = (\mathbb{1}_{2\times2}, -\vec{\sigma})$

$$\Rightarrow \boxed{\bar{\Psi}\gamma^\mu\Psi = \psi_R^\dagger\sigma^\mu\psi_R + \psi_L^\dagger\bar{\sigma}^\mu\psi_L}$$

thus: this type of interaction connects fermions of the <u>same</u> chirality.

e_L will <u>never</u> talk to e_R.

WHAT ABOUT SPINS?

$\sigma^0 = \begin{pmatrix}1 & \\ & 1\end{pmatrix}$, $\sigma^1 = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}$, $\sigma^2 = \begin{pmatrix} & -i \\ i & \end{pmatrix}$, $\sigma^3 = \begin{pmatrix}1 & \\ & -1\end{pmatrix}$

σ^0 & σ^3 are <u>diagonal</u>: couple spin up to spin up
" down " down

σ^1 & σ^3 are <u>off diagonal</u>: couple spin up w/ spin down.

		SAME CHIRAL?	σ COUPLING → CONSISTENT
a	$(e_R^{\dagger})^{\uparrow} \sigma^{\mu} e_L^{\uparrow}$	✗	
b	$(e_L^{\dagger})^{\uparrow} \sigma^{3} e_L^{\uparrow}$	✓	DIAGONAL → ✓ OK
c	$(e_L^{\dagger})^{\uparrow} \sigma^{1} e_L^{\uparrow}$	✓	OFF DIAGONAL → ✗
d	$(\mu_L^{\dagger})^{\uparrow} \sigma^{2} \mu_L^{\uparrow}$	✓	OFF DIAGONAL → ✗

(e) $(e_L^{\dagger})^{\uparrow} \sigma^{\mu} e_L^{\uparrow}$ ✓

$(\mu_L^{\dagger})^{\downarrow} \sigma^{\nu} \mu_L^{\uparrow}$ ✓

REQ. DIAGONAL $\Rightarrow \mu = 0$ or 3

REQ. OFF DIAGONAL $\Rightarrow \nu = 1$ or 2

contract w/ $\boxed{\eta_{\mu\nu}}$ → REQUIRES $\mu = \nu$

these cannot all be satisfied ✗

$\boxed{2}$ $D_\mu H_0 = \cancel{\partial H_0} - ig W^A T^A H_0 - ig' q_H B H_0$

0 b/c H_0 is constant.

$H_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$

$$= -ig\frac{1}{2}\begin{pmatrix} W^3 & \sqrt{2}W^+ \\ \sqrt{2}W^- & -W^3 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$-ig'\cdot\left(\frac{1}{2}\right)\cdot\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ Bv \end{pmatrix}$$

nb: $W^\pm = \frac{1}{\sqrt{2}}(W^1 \mp iW^2)$

$$W^A T^A = W^1 T^1 + W^2 T^2 + W^3 T^3$$

$$= \frac{1}{2}\begin{pmatrix} W^3 & W^1 - iW^2 \\ W^1 + iW^2 & -W^3 \end{pmatrix}$$

$$D_\mu H_0 = \frac{-ig}{2\sqrt{2}}\begin{pmatrix} \sqrt{2}W^+ v \\ -W^3 v \end{pmatrix} + \frac{-ig'}{2\sqrt{2}}\begin{pmatrix} 0 \\ Bv \end{pmatrix}$$

$$= \boxed{\frac{-iv}{2\sqrt{2}}\begin{pmatrix} g\sqrt{2}W^+ \\ g'B - gW^3 \end{pmatrix}}$$

$$|DH|^2 = \frac{v^2}{8}\left[2g^2 W^+ W^- + (g'B - gW^3)^2\right]$$

$$(g')^2 B^2 - 2gg' BW^3 + g^2 (W^3)^2$$

$$= \boxed{\left(\frac{gv}{2}\right)^2 W^+ W^- + (B \;\; W^3)\begin{pmatrix} (g')^2 & gg' \\ gg' & g^2 \end{pmatrix}\begin{pmatrix} B \\ W^3 \end{pmatrix}}$$

M matrix

[3] $\mathcal{L} = y H^{\dagger} L \bar{E} + \text{h.c.}$

v is $\mathbb{R}$ by choice

$$y H_0^{\dagger} L \bar{E} = \frac{y}{\sqrt{2}} (0 \;\; v) \begin{pmatrix} \nu_e \\ e_L \end{pmatrix} e_R^{\dagger}$$

$$= \left(\frac{yv}{\sqrt{2}}\right) e_L e_R^{\dagger}$$

$$\boxed{m_e = \frac{yv}{\sqrt{2}}}$$