ELECTROWEAK / CHIRAL SYMMETRY BREAKING

ELECTROWEAK ↑ $SU(2)\times U(1)$

CHIRAL ↑ α, $\dot{\beta}$ indices

BREAKING ↑ universe isn't pretty.

RECALL: GAUGE (by fiat) → automatically have

B (talks to anything w/ $U(1)$ Y)

W^A_μ ← Adjoint, talks to anything w/ $SU(2)$ indices

Symmetry breaking

symmetric universe:

empty except a perfectly rotationally symmetric planet @ origin

→ can calculate $V(r)$

slightly asymmetric.

LITTLE MOUNTAIN, BREAKS ROT. SYM.

$V(r) \to V(x,y,z)$

BUT: $V(\underline{r}) = V(r) + \underbrace{f\left(\frac{h}{R}\right)}_{\text{DESCRIBES MAGNITUDE OF PERTURBATION}} g(x,y,z)$

FYI: this is multipole exp.

LESSON: the thing that breaks the symmetry is an order parameter or spurion

if the mountain is bigger, the departure from $V(r)$ is more pronounced.

more importantly: any asymmetric effect is proportional to the order parameter

FACT: Higgs BREAKS $SU(2) \times U(1)$ and chiral sym

DIRECTLY — somewhat indirectly

HOW: the thing we've been calling $- \overset{H}{\rightarrow} -$ is not a "good" particle

It can end $H^+ - - > - - \otimes$

$\otimes^+ - - > - - H$

$$\otimes = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

silly normalization

BUT: $- \rightarrow -$ { carries $SU(2)$ CHARGE (an "a" index along the arrow)

carries $U(1)$ CHARGE ($Y = +1/2$)

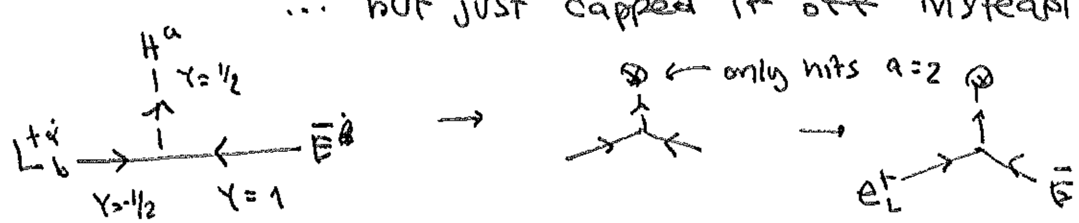
$SU(2)$: $T^1 = \frac{1}{2}\begin{pmatrix} & 1 \\ 1 & \end{pmatrix}$, $T^2 = \frac{1}{2}\begin{pmatrix} & -i \\ i & \end{pmatrix}$, $T^3 = \frac{1}{2}\begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$ } $e^{i\theta T^{1,2}}$ ROTATE $\otimes$

$$\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \rightarrow \begin{pmatrix} e^{i\theta/2} & v/\sqrt{2} \end{pmatrix}$$

So: $- > - \otimes$ BREAKS $SU(2)$. IT IS NOT INVARIANT.

ATTACHING IT TO A DIAGRAM MAKES THAT DIAGRAM NOT INVARIANT

↳ should have contracted on 'a' index ... but just capped it off instead.



IN $W^{\pm,3}$ BASIS:
$$T^+ = \begin{pmatrix} 0 & 1 \\ & \end{pmatrix} \leftarrow T^+ \otimes$$
$$T^- = \begin{pmatrix} & \\ 1 & 0 \end{pmatrix} \leftarrow \otimes^\dagger T^-$$

~~RAISES~~ ~~MATCH~~

KEY OBSERVATION:

T^3 rotation: $\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \to \begin{pmatrix} 0 \\ e^{-i\theta_3/2}(v/\sqrt{2}) \end{pmatrix}$

acts as a phase like a U(1) on ~~only~~ lower component.

U(1): $H \to e^{i\theta_Y \overset{\substack{\text{charge of H} \\ \downarrow}}{Y}} H$

$\langle H \rangle \to \begin{pmatrix} 0 \\ e^{+i\theta_Y/2}(v/\sqrt{2}) \end{pmatrix}$

phase for θ_Y transf. of object w/ charge $+\frac{1}{2}$

even more "key" OBSERVATION: indeed: $T^\pm$ & $T^{1,2}$ TOTALLY MESS UP $\langle H \rangle$, SO IF $\langle H \rangle$ IS A RULE, THEN SU(2) IS BROKEN

(not a good sym)

BUT the T^3 & Y rotations both act as "only" a phase...

combined rotation:

$$\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \longrightarrow \begin{pmatrix} 0 \\ e^{i(\theta_Y - \theta_3)/2}(v/\sqrt{2}) \end{pmatrix}$$

IF $\theta_Y = \theta_3$: invariant!!

SO: A COMBINATION ($\theta_x = \theta_3$) of T^3 & Y LEAVES $\langle H \rangle$ UNCHANGED

↑

recall: $\langle H \rangle$ breaks symmetries
how do we know? It is a preferred direction

c.f. ↗ breaks rotational symmetry

Q: so what! It transforms covariantly
└── how is $\langle H \rangle$ different from H ?

$\langle H \rangle$ exists everywhere

ANALOGY [FERROMAGNET vs. INDIVIDUAL SPIN

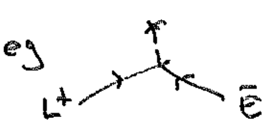
~~↑↑↑↑↑↑~~ ↗

if I try to rotate one spin,
it breaks the order

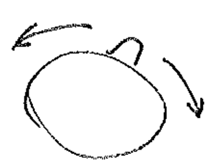
(~~Q: why not rotate all?~~
~~→ GLOBAL vs. GAUGE~~)

$\langle H \rangle$ does not rotate. It's fixed.
IT IS PACKAGED IN A COVARIANT OBJECT
└── s.t. I CAN FORM "SPURIOUS" COVARIANTS

eg



BUT: that's just a trick to see HOW sym is broken.



can rotate, sure

BUT ROT SYM of this universe is broken.

SU(2) : BROKEN

└ THREE ROTATION AXES

T^1 or T^+ } BROKEN
T^2 or T^-

T^3 BROKEN ... BUT JUST PHASE

U(1) Y BROKEN ... as a phase.

$\boxed{(T^3 + Y) \longrightarrow \text{UNBROKEN}} \longrightarrow$ EM Q

$(T^3 - Y) \longrightarrow$ BROKEN

<u>SO</u> : $\exists\ U(1)_Q$ (comb of U(1) in SU(2)
U(1) hypercharge)

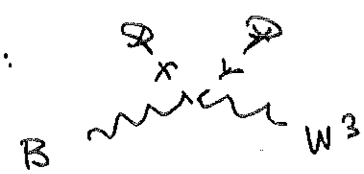
that is a <u>good</u> symmetry
& it's a <u>gauge</u> sym.

$$A_\mu = \boxed{\cos\theta_W}\ B_\mu + \boxed{\sin\theta_W}\ W^3_\mu$$

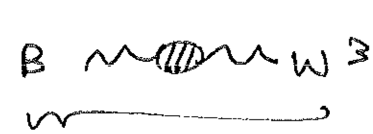
Weinberg Angle

$$|A\rangle = c_W\,|B\rangle + s_W\,|W^3\rangle$$

<u>DIAGRAMS</u> :

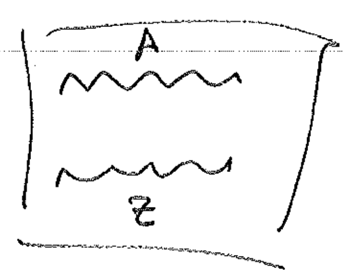


$\rightarrow$



you didn't know about this vertex ... we won't use it
(we will explain where it came from)

these mix



WHAT SETS θ_W?

if Higgs talks to W^3 & Y w/ equal strength ⟵ some # in the theory, then $\theta_W = \pi/4$

BUT: Y & $SU(2)$ are totally different! their characteristic strengths are unrelated... just like

G_N vs. α

(actually not "just like" → DIMENSIONS!)

CARD: H has "equal" coupling to $W^{1,2,3}$, $W^{\pm,3}$

H has separate coupling to ~~#~~ B

$W^{\pm}$, Z, A

massless b/c GOOD ~~sym~~ GAUGE SYM HAS MASSLESS PARTICLE

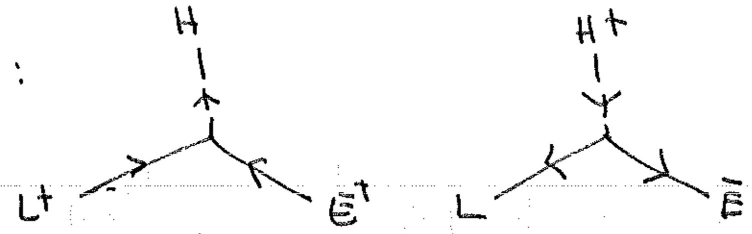
MASSIVE: what are masses?

~~DOES #~~

is the interaction strength FOR SU(2) OR U(1)$_Y$ STRONGER?

MASS	massless	massive
spin 0 →	1 dof (2 if $\mathbb{C}$)	1 dof (2 if $\mathbb{C}$)
spin 1	2 dof	3 dof
spin 1/2	2 dof ← LH OR RH	4 dof = $\underbrace{2 \times (2 \text{ dof})}_{\text{LH AND RH}}$

SPIN 1/2 :



$\Downarrow$

$$e_L^\dagger \longrightarrow \overset{\Phi}{—} \longleftarrow \bar{E}^\dagger$$

$\bar{E}^\dagger \leftarrow e_R^*$

$$e_L^\dagger \longrightarrow \overset{e_L \quad e_R}{\circledast} \longleftarrow e_R^*$$

Higgs breaks a spacetime sym!

↓

CHIRAL SYM BREAKING

$$(e_L^\dagger)^{\dot\beta} \longrightarrow \otimes \longleftarrow (e_R^*)^{\dot\alpha}$$

↑

compare to

$$(e_L^\dagger)^{\dot\beta} \longrightarrow e_L^\alpha$$

(ANTI PART.) ———— (PARTICLE)

↑ wants to have opp. dot/undot index if chiral sym is respected

$$E^A = \begin{pmatrix} e_L^\alpha \\ e_R^{\dot\alpha} \end{pmatrix} \quad (A: \alpha \text{ or } \dot\alpha)$$

← this is how most people learn electrons.

A note about lines & arrows

OUR CONVENTION (used in theory)

$\dot{\beta} \longrightarrow \alpha \quad e_L$

$\beta \longleftarrow \dot{\alpha} \quad e_R$

there are so many charges in the theory (Y, Q, ..) that we'll let the arrow follow LH charge

POINT TO THE UNDOTTED INDEX OR AWAY FROM DOTTED INDEX

Weyl notation

MOST P. PHYSICISTS (used in phenomenology)

$\longrightarrow \quad e$

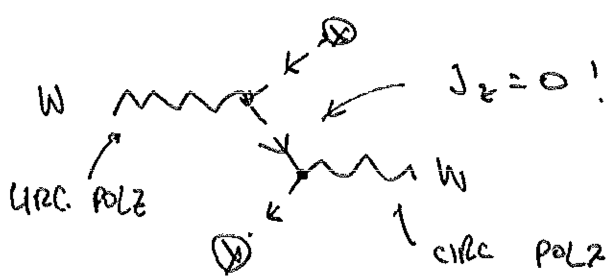
Dirac notation

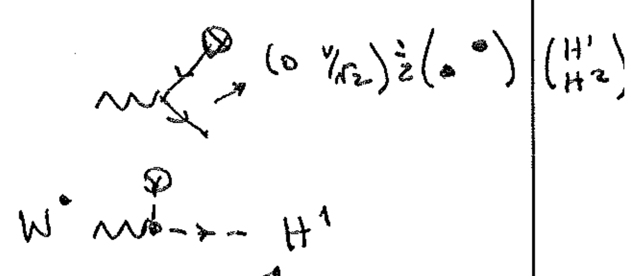
Pick a charge. Usually EM. the arrow follows the charge.

this is what we did in weeks 1-2

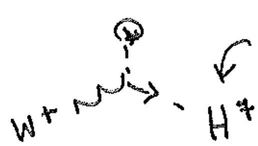
SPIN-1 : puzzle: 3 dof. $|s=1, j_z = \pm 1, 0\rangle$

WE HAVE $W^\pm$ (combo of $W^{1,2}$) PICKS UP MASS $\longleftrightarrow$ LONGITUDINAL MODE





$W^{\circ} = W^1 + iW^2 \longrightarrow$



$H^+ = \mathrm{Re}\, H^1 + i\, \mathrm{Re}\, H^2$

SIMILARLY : Z boson (Real) eats part of H_2

$$H = \begin{pmatrix} H^1 \\ H^2 \end{pmatrix} = \begin{pmatrix} a+ib \\ c+id \end{pmatrix}$$

W^+ EATS THIS
W^- EATS $a - ib$
2 $\mathbb{R}$ DOF

Z EATS THIS

Leftover : $c = h$

the Higgs, 1 $\mathbb{R}$ DOF.

Q: ? ELECTRIC CHARGE ?

HOW DOES HIGGS INTERACT?.
↳ RESPECTING $SU(2) \times U(1)$
BUT USING $\langle H \rangle$ AS ORDER PARAMETER FOR ITS BREAKING

Does h talk to ν ?
Why not?