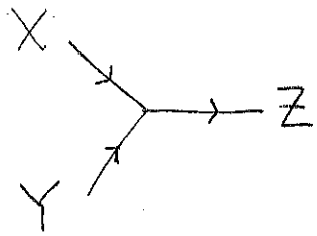
Long HW 3: Symmetries

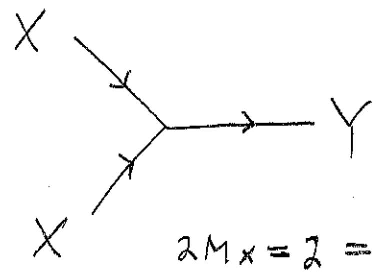
Solutions

1) $M_i \equiv$ Megacharge of particle i, $M_X = 1$, $M_Y = 2$ $M_Z = 3$

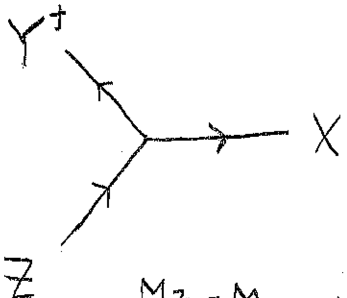
<u>1.1</u>



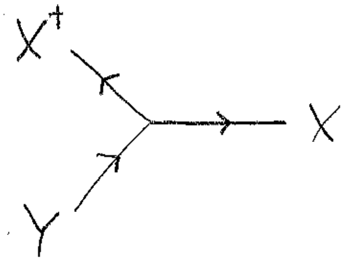
$M_X + M_Y = 1 + 2 = 3 = M_Z$



$2M_X = 2 = M_Y$

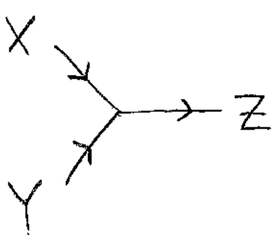


$M_Z - M_Y = 1 = M_X$

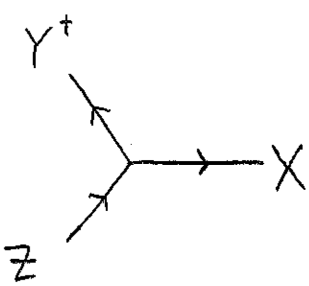


$M_Y - M_X = 1 = M_X$

<u>1.2</u> $U_i \equiv$ Ultracharge of particle i, $U_X = 2$ $U_Y = 1$ $U_Z = 3$

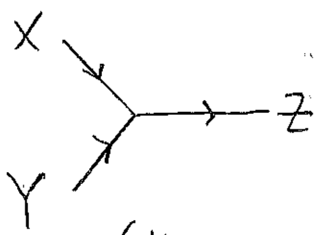


$U_X + U_Y = 3 = U_Z$



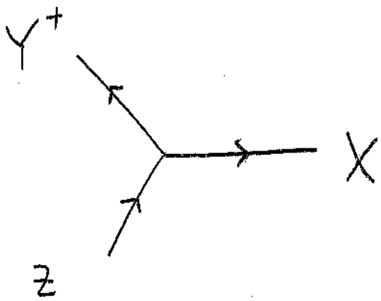
$U_Z - U_Y = 2 = U_X$

1.3 The same ones! Nothing changes



$$(M_X + U_X) + (M_Y + U_Y) = 6 = M_Z + U_Z$$

$$(M_X - U_X) + (M_Y - U_Y) = 0 = M_Z - U_Z$$

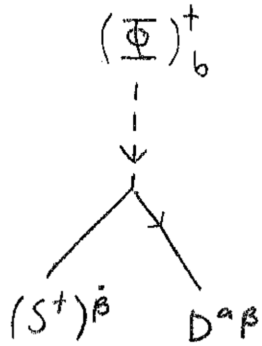


$$(M_Z + U_Z) - (M_Y + U_Y) = 3 = M_X + U_X$$

$$(M_Z - U_Z) - (M_Y - U_Y) = -1 = M_X - U_X$$

2)

2.1



$$(S^{\dagger})^{\dot\beta}(\Phi)^{\dagger}_{b} D^{a\beta} \delta^{b}{}_{a} \bar\sigma^{\mu}{}_{\dot\beta\beta} p_{\mu}$$

2.2

If $q = -1$ then the first two rules are allowed $(q+1 = -1+1 = 0)$ and the previous one isn't $(-q+1 = 1+1 \neq 0)$.

3)

$$D^{a} \to \exp\left[i\theta_3 (T^3)^{a}{}_{b}\right] D^{b}$$

$$= \exp\left[\frac{i\theta_3}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1\end{pmatrix}\right]\begin{pmatrix} D^1 \\ D^2 \end{pmatrix}$$

$$= \begin{pmatrix} e^{\frac{i\theta_3}{2}} D^1 \\ e^{-\frac{i\theta_3}{2}} D^2 \end{pmatrix}$$

$$= \begin{pmatrix} e^{iq_1\theta_3} D^1 \\ e^{iq_2\theta_3} D^2 \end{pmatrix} \Rightarrow \quad q_1 = \frac{1}{2} \quad , q_2 = -\frac{1}{2} = -q_1$$

T^3 generates a $U(1)$ symmetry where $D^a \to e^{i\theta_3 q_a} D^a$

Extra Credit

$$(\sigma^{03})_\alpha{}^\beta = \frac{1}{2}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$(\bar{\sigma}^{03})^{\dot\alpha}{}_{\dot\beta} = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\psi_\alpha \to \exp\left(-\frac{i}{2}\omega_{\mu\nu}\sigma^{\mu\nu}\right)_\alpha{}^\beta \psi_\beta$$

$$= \exp\left[-\frac{i\omega_{03}}{2}\sigma^{03} + \frac{i\omega_{03}}{2}\sigma^{30}\right]_\alpha{}^\beta \psi_\beta$$

$$= \exp\left[i\frac{\theta}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\right]\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

$$= \begin{pmatrix} e^{i\theta/2}\psi_1 \\ e^{-i\theta/2}\psi_2 \end{pmatrix} \Rightarrow \operatorname{Im} e^{i\theta/2} = \sin\theta/2$$

Similarly,

$$\bar{\chi}^{\dot\alpha} \to \exp\left(-\frac{i}{2}\omega_{\mu\nu}\bar{\sigma}^{\mu\nu}\right)^{\dot\alpha}{}_{\dot\beta}\bar{\chi}^{\dot\beta}$$

$$= \exp\left[i\theta/2\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\right]\begin{pmatrix} \bar{\chi}^1 \\ \bar{\chi}^2 \end{pmatrix}$$

$$= \begin{pmatrix} e^{-i\theta/2}\bar{\chi}^1 \\ e^{i\theta/2}\bar{\chi}^2 \end{pmatrix} \Rightarrow \operatorname{Im} e^{-i\theta/2} = \sin\theta/2$$