Weekly HW 1: **Kinematics and QED**

Course: Physics 165, *Introduction to Particle Physics* (2018)
Instructor: Prof. Flip Tanedo (flip.tanedo@ucr.edu)
Due by: Tuesday, January 16

This is the main weekly homework set. Unless otherwise stated, give all responses in natural units where $c = \hbar = 1$ and energy is measured in electron volts (usually MeV or GeV).

1 Everything in natural units

Write the following quantities in natural units with energy measured in GeV. You may write everything to one significant figure.

- The mass of the sun, $M_\odot$.
- The present day Hubble expansion rate, H_0.
- The classical electron radius, r_e.
- The Schwarszchild radius of the sun, $2G_N M_\odot / c^2$.

You may look up the information anywhere you want, but I suggest the first few pages of the PDG.

2 Special Relativity and Kinematics

2.1 The non-relativistic limit

A particle is **non-relativistic** if its 3-momentum is much smaller than its energy, $|\mathbf{p}| \ll E$. In this limit, show that $E^2 = m^2 + \mathbf{p}^2$ reduces to the familiar $E = mc^2$ upon restoring factors of c. In ths non-relativistic limit, what is the *leading-order correction* to $E = mc^2$? You'll want to Taylor expand—make sure you do this with respect to a small, dimensionless parameter.

2.2 A relativistic electron

In some frame, the electron has *momentum* equivalent to its rest mass, m_e. Use the value of the rest mass to one significant figure. I shouldn't have to tell you where to look it up. Write out the components of the **momentum four-vector** p_μ.

2.3 A symmetric particle collider

Imagine a symmetric electron–positron collider. At the collision point, it collides a beam of electrons and positrons with one another so that these have four-momenta:

$$p_\mu^{e^-} = (E, 0, 0, p) \qquad \text{and} \qquad p_\mu^{e^+} = (E, 0, 0, -p) \ . \tag{.1}$$

What is the expression for p as a function of E and m_e? What is the **center of mass energy** of the collision in the lab frame?

Suppose that this collider was invented to produce a 91 GeV particle, Z, through the process $e^+e^- \to Z$. What energy E is required for each beam? What is the momentum of the Z particle in the lab frame?

Extra credit: Suppose the Z is unstable and decays. This means that you don't get to measure it directly. Without knowing anything else about how the Z interacts, what is one **decay mode** that is guaranteed to exist? In other words, what types of particles should you make sure you can detect?

2.4 A fixed target experiment

Imagine a very asymmetric kind of collider called a **fixed target experiment**: a high-energy beam of particles hits a stationary target. Assume that both the beam and the target are composed of protons and that the collision occurs head-on[1]. Write the four-momenta of a beam particle and the target particle in the lab frame.

Suppose you wanted to produce some completely made up particle—let's call it a *Flippon*[2]—that has a mass of 14 GeV. To one significant figure, what proton beam energy E is required to produce the Flippon through $pp \to$ Flippon? (Assume that such a process is possible.) What is the momentum of the Flippon in the lab frame?

HINT: This problem is constrained by kinematics. There's an easy way and a hard way of doing this. One involves doing a Lorentz transformation to a more convenient frame. The other involves realizing that the quantity in the convenient frame can be written as a Lorentz invariant. I don't care which way you do this, though you should probably to understand how to do it both ways.

DISCUSSION: Fixed target experiments are nice because you don't have to worry about engineering two beams to collide with one another. They also have a very useful feature that the new particle is produced *boosted* relative to the lab frame. This can be very useful for untangling the decay products of the new particle from other particle debris from the beam hitting the target.

3 Why $\mu \to e\gamma$ doesn't happen.

[**Flip**: I have made a terrible mistake. $\mu \to e\gamma$ is indeed kinematically possible. Thanks to Adam G. for pointing this out on the Tuesday that this is due; please find the momentum four-vectors of the outgoing e and γ in the rest frame of the μ. Homework due Thursday. (This is repeated in the Week 2 short homework.)]

The muon is a heavy version of the electron. It has the same electric charge, but is 200 times heavier. In the Standard Model it can decay—what does it decay into? (Give the decay mode that happens $\approx 100\%$ of the time.) You don't have to draw a Feynman diagram for this, we haven't gotten there yet.

You should be able to see from the dynamics (Feynman rules) of the "QED+μ theory" that $\mu \to e\gamma$ cannot occur: you cannot draw a Feynman diagram for it. However, imagine some crazy theory

[1] This is a classical idea, but for now we can live with this kind of deceit. Relevant: https://www.youtube.com/watch?v=AnaQXJmpwM4

[2] Unrelated to this: https://arxiv.org/abs/1602.01377

that does have a vertex that connects a photon, electron, and muon so that you could draw a Feynman diagram for $\mu \to e\gamma$. Prove that this process is still impossible due to kinematics.

HINT: There's an easy way and a hard way of doing this. They both involve conservation of energy and momentum. I don't care which way you do this.

DISCUSSION: This is a useful reminder of the difference between *dynamics* (a model of particles) and *kinematics* (relations between "motion variables" like four-momenta).

4 Feedback

Approximately how long did it take you to complete the non-extra credit parts of this assignment?

COMMENT: Homework will become more challenging as the quarter progresses.

Extra Credit

If you do any of these problems, please write a short note giving your thoughts on the reading: did you like them? Were they too simple / difficult? I do not expect you to be able to complete all (or necessarily any) of the extra credit.

1 Minkowski Diagrams

The mathematical basis of relativity is geometry. This is most simply seen in what are called **Minkowski diagrams**. I'm pretty sure A good introduction to these are in `https://arxiv.org/abs/1508.01968` by Boxiang Liu and Thushara Perera[3]. Consider two reference frames with some non-zero relative velocity. Sketch the axes of the Minkowski diagram this system: that is, draw the (x, t) axes and the (x', t') axis where (x', t') are related to (x, t) by a Lorentz transformation. Draw two spacetime events and their respective light cones. Comment on the idea of causality using these diagrams. Those who are mathematically inclined may enjoy `https://doi.org/10.1119/1.4997027`.

2 Impact of Special Relativity on Physics: Compton Scattering

Look over David Jackson's article "The Impact of Special Relativity on Theoretical Physics" from the May 1987 issue of *Physics Today*, `https://doi.org/10.1063/1.881108`. Focus on the section "Waves and particles," where the author discusses **Compton scattering**. You drew the Feynman diagrams for this on your short homework assignment #1. Use special relativity to derive the author's expression for $\delta\lambda$, the shift in the photon wavelength.

[3]A somewhat more polished reference is the book *Very Special Relativity* by Sander Bais. There's also a Khan Academy video, `https://youtu.be/nEqexIckVCM`.

3 Relativistic mass

There is an antiquated notion of *relativistic mass* that people used to talk about. Lev Okun gives a nice overview in "The Concept of Mass" in the June 1989 issue of *Physics Today*, `https://doi.org/10.1063/1.881171`. Read the article and explain why there is only *one* useful notion of mass and that it is the *rest mass*.