Final Exam: This is the End

Course: Physics 165, *Introduction to Particle Physics* (2018)
Instructor: Flip Tanedo (`flip.tanedo@ucr.edu`)
Due: Wednesday, March 21, 3:00pm; submit to Ian Chaffey's office (3004S)

Rules for this exam

Please read these carefully.

1. **This exam is an *take home exam*.** You are encouraged to use the course notes, the homework solutions, and any of the course references.
2. **You may only use *non-interactive* resources**. This means any book, academic paper, magazine, or article online. You are required to cite any sources that you use.
3. **You may *not* ask for help online or in person from anyone**. You may not use online forums, question & answer sites, social media sites, etc.
4. **Write all your responses on *one side* of each page.** You should be able to fit your responses on this exam. If you may add additional pages as needed, but only write on *one side* of the page. (The exams are scanned one-sided.)
5. **Write clearly and *box* or *circle* your final answers.**
6. **You may ask questions in class** on Tuesday, March 13 and Thursday, March 15. Outside of these times, I will only answer logistical questions via e-mail.

By signing this you agree to abide by the rules of the take home exam. Failure to uphold these rules—especially the UCR standards for academic integrity[1]—will result in a strict fail on this examination.

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Name (print) Signature

[1] `http://conduct.ucr.edu/policies/academicintegrity.html`

1 Helicty versus Chirality [10 points]

For the following statements, **circle** whether the correct word is *helicity* or *chirality*.

1.a Coupling of the W boson

In the Standard Model, the W boson couples to particles of left-handed *helicity / chirality*.

1.b Transformations under spacetime symmetries

If I rotate a fermion, then I may change its *helicity / chirality*.

1.c Preserved quantum number

A massive fermion has a well defined *helicity / chirality*.

1.d Angular Momentum

Th angular momentum of a particle is associated with its *helicity / chirality*.

1.e Higgs

The Higgs boson acquires a vacuum expectation value. The Yukawa couplings of the Higgs with the Standard Model fermions necessarily connects fermions of different *helicity / chirality*, which would otherwise be totally unrelated particles.

1.f Indices

We have indexed chiral fermions with dotted and undotted indices, e.g. ψ^α or $\bar{\chi}^{\dot{\beta}}$. The choice of dotted or undotted indices refers to the *helicity / chirality* of the particle.

2 Three Scalars Outside Ebbing, Missouri [30 points]

Consider a theory with three real scalar fields, $A(x)$, $B(x)$, $C(x)$. The Lagrangian is

$$\mathcal{L} = \frac{1}{2}(\partial A)^2 + \frac{1}{2}(\partial B)^2 + \frac{1}{2}(\partial C)^2 - \frac{m^2}{2}(A^2 + B^2) + \frac{\mu^2}{2}C^2 - \lambda\left[\left(A^2 + B^2\right)^2 + C^4\right] - yABC^2\ .$$

The parameters of the theory are all positive: $m^2\,,\ \mu^2\,,\ \lambda\,,\ y\ > 0$.

2.a The vacuum

What are the **vacuum expectation value**s of the each of the fields in terms of the parameters of the theory? Call these A_0, B_0, and C_0. For example $\langle A(x)\rangle = A_0$. (Pick the positive vev.)

2.b The spectrum

Expand the fields with respect to the vacuum expectation values:

$$A(x) = A_0 + a(x) \qquad B(x) = B_0 + b(x) \qquad C(x) = C_0 + c(x)\ . \tag{2.1}$$

Identify the mass eigenstates of the theory as $c(x)$ and two linear combinations of $a(x)$ and $b(x)$; call these combinations $\alpha_\pm(x)$. What is the mass of each state in terms of μ^2, m^2, y, and C_0?

2.c Feynman Rules, Batman Drools, Robin Laid an Egg

Write down all of the three- and four-point Feynman vertices with respect to the $\alpha_{\pm}(x)$ and $\xi(x)$ fields. For each vertex, write down the proportionality of the Feynman rule to the parameters λ, C_0, and y.

2.d $\alpha_+\alpha_+ \to cc$ annihilation

Draw the three leading order (simplest) diagrams for the annihilation of two α_+ particles into two c particles.

3 Beyond the Standard Model [30 points]

3.a Brave New Fields, I

In addition to the usual Standard Model fields, add the following fields:

Field	Spin	SU(3)	SU(2)	U(1)
ψ	ψ^{α}	none	ψ^{a}	$\psi_{Y=1/2}$
χ	χ^{α}	none	χ^{a}	$\chi_{Y=-1/2}$
a	scalar	a_m	none	$a_{Y=-2/3}$
b	scalar	b_m	none	$b_{Y=1/3}$

Draw all allowed three-point vertices between combinations of these fields and the quark doublet, $Q^{\alpha m a}_{Y=1/6}$. (There are two.)

3.b Brave New Fields, II

What is the only allowed three-point interaction that includes the a field and two Standard Model fields? Write out the Lagrangian term that gives this interaction; explicitly write the indices and show that they contract. You may use any of the invariant tensors: $(T^A)^a{}_b$, $(T^M)^m{}_n$, δ^a_b, δ^m_n, ϵ_{ab}, ϵ^{ab}, ϵ^{ABC}, ϵ^{MNL}, $\epsilon_{mn\ell}$, $\epsilon^{mn\ell}$, $\sigma^{\mu}{}_{\alpha\dot{\beta}}$, $\epsilon_{\alpha\beta}$, $\epsilon_{\dot{\alpha}\dot{\beta}}$, etc.

3.c SU(4) gauge theory

In QCD, the SU(3) symmetry *confines* and the resulting **baryons** and **mesons** are color neutral bound states of quarks. Now imagine a theory based on SU(4) symmetry with 'hyper-quarks' that have four 'hyper-colors.' For simplicity, instead of 'hyper-color' we'll use the phrase *suites*[2]. As in QCD, the hyper-mesons are made up of particle–anti-particle pairs that are suite-neutral.

The **hyper-baryon** is composed of a suite-neutral combination of hyper-quarks. What is the hyper-quark composition of a hyper-baryon? HINT: In SU(N), you are allowed to use the $\varepsilon_{m_1 \cdots m_N}$ tensor. Based on this, are hyper-baryons fermions (half-integer spin) or bosons (full integer spin)?

3.d Dark Matter, Part I

The local density of dark matter is 0.3 GeV/cm^3. What is this in natural units?

3.e Dark Matter, Part II

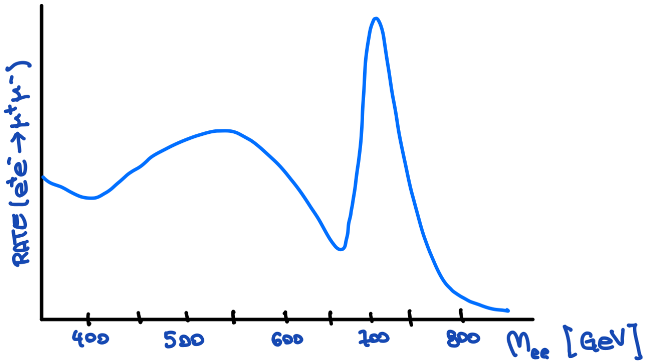
Suppose dark matter, χ, is a part of an SU(2) doublet, X. This means that there are actually two new particles, $X^a = (\xi, \chi)^T$. Given that *dark matter is dark* (has no electric charge), what is the hypercharge of the dark matter doublet, X? (The ξ particle may have electric charge.)

[2]... like ♡, ♢, ♠, ♣

4 "Experiments" [15 points]

Imagine that there are two new gauge bosons, Z' and Z'', that talk to e and μ. These have coupling strengths g' and g'' respectively. Assume that the Z' is the lighter of the two.

Here is a sketch of the rates for $e^+e^- \to \mu^+\mu^-$ in this imaginary universe.



The x-axis is the invariant mass of the e^+e^- pair, $\sqrt{(p_{e^+} + p_{e^-})^2}$, the y-axis is the rate for $e^+e^- \to \mu^+\mu^-$. The sketch reflects the presence of the Z' and Z'' bosons.

4.a Spectrum

What are the approximate masses of the Z' and Z''?

4.b Lifetimes

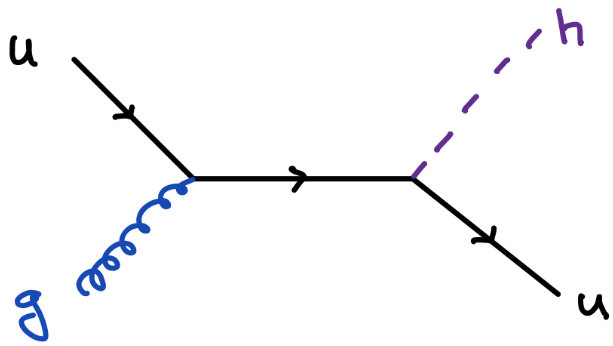
Which particle has an *longer* lifetime? How can you tell from the plot?

4.c Couplings

Suppose Z' and Z'' only differ by their coupling strengths (assume the mass difference is negligible). Based on the lifetimes of the Z' and Z'', which coupling is larger, g' or g''?

5 Reading a Diagram [15 points]

Below is a diagram for $ug \to uh$. For this diagram, answer the following questions.



Recall that the gluon is a spin-1 particle, the Higgs is a spin-0 particle.

5.a Chirality

If the initial up quark is left-chiral, what is the chirality of the final state up quark?

5.b Colors

If the initial up quark is red, what are the possible colors of the final up quark?

5.c Kinematics

The gluon is massless, the up-quark has mass m_u, and the Higgs has mass m_h. In the center of mass frame for the initial particles, suppose that the initial up quark and initial gluon each have energy $E > m_h$. What is the energy of the final state Higgs as a function of m_u, m_h, and E?