

# SHORT HW 3: Indices

COURSE: Physics 165, *Introduction to Particle Physics* (2018)  
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 DUE BY: **Thursday**, January 25

Note that this short assignment is due in class on Thursday. You have only *two days* to do it. This should be quick, I recommend doing it right after class on Tuesday.

## 1 The Yukawa interaction

Consider the following toy theory of spin-1/2 particles interacting with a spin-0 particle. The theory has an SU(2) symmetry with doublet indices  $a, b, c, \dots \in \{1, 2\}$  and triplet indices  $A, B, \dots \in \{1, 2, 3\}$ . You have the following tensors:

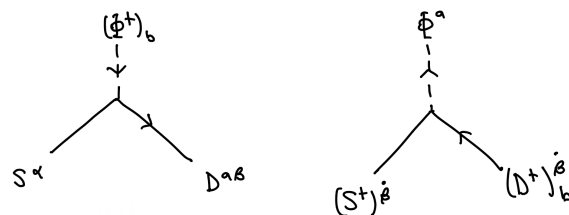
- Lorentz metrics: metric for vectors  $(\eta^{\mu\nu}, \eta_{\mu\nu})$ , metric for spinors  $(\varepsilon^{\alpha\beta}, \varepsilon_{\alpha\beta}, \varepsilon^{\dot{\alpha}\dot{\beta}}, \varepsilon_{\dot{\alpha}\dot{\beta}})$
- Lorentz tensor:  $\sigma$ -matrices  $(\sigma^\mu_{\alpha\dot{\beta}})$
- SU(2) metrics:  $\epsilon_{ab}, \epsilon^{ab}, \delta_{AB}$  (i.e. you can contract two upper  $A$  indices)
- SU(2) tensors:  $f^{abc}, T^{Aa}_b$
- The identity:  $\delta_b^a, \delta_\beta^\alpha$ , etc.
- Momenta:  $p^\mu$  of any particle going into the vertex

You have the following particles:

$$(\Phi^\dagger)_b \dashrightarrow \Phi^a \quad (S^\dagger)^{\dot{\beta}} \text{---} S^\alpha \quad (D^\dagger)^{\dot{a}}_b \longrightarrow D^{a\alpha}$$

Here  $\Phi^a$  is an SU(2) doublet, meaning it has two components. It has an anti-particle,  $(\Phi^\dagger)_b$ . Observe that  $\Phi$  is a Lorentz scalar: it doesn't have any spinor or vector index. The particle  $S$  is a spin-1/2 particle that is an SU(2) singlet ("SU(2) scalar"), it has a spinor index but does not have any SU(2) indices. It has an antiparticle  $(S^\dagger)^{\dot{\beta}}$ . The particle  $D$  is an SU(2) doublet and a spin-1/2 particle with spinor indices. It has an anti-particle  $(D^\dagger)^{\dot{\beta}}_b$ .

Show that you can write down the following Feynman rules:



That is, for each proposed Feynman rule, write down an invariant using the tensors above and the particles. For example, your answer for the first diagram should look like:

$$S^\alpha (\Phi^\dagger)_b D^{a\beta} [\text{some combination of tensors}]^b_{\alpha\beta a}$$

REMARKS: Don't confuse  $a$  and  $\alpha$ . The  $S$  has no arrow because it carries no charge.