Preliminaries

ANNOUNCE:

Kuntal on THU.
CROSS SEC.

- "PERIODIC TABLE"
  ↳ 1st gen matter
  4 fund. forces
  HIGGS

  + ELECTRIC CHARGES,

- RULES for QED + Z + μ
  ↑ e-ness → e#
  μ-ness → μ#

- HW 1b: #1 ERIC
  #2 NICK

- NEW RULES:

p^+: uud ≈ 1 GeV

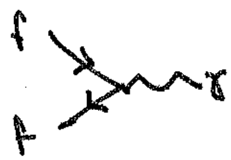
×3		
u	+2/3	≈ 5 MeV
d	-1/3	≈ 5 MeV
ν_e	0	≈ 1 eV ?
e	-1	≈ ½ MeV

γ	0	
Z	~100 GeV	
$W^\pm$	~100 GeV	
g	0	

h ~100 GeV

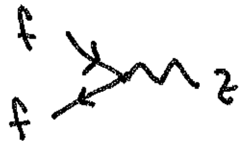
spin ½ spin-1 spin-0

PHOTON:



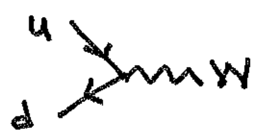
for: f = u, d, e (anything w/ electric charge)

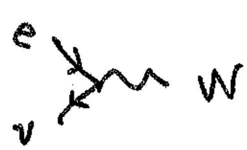
Z-BOSON:



for: f = u, d, ν, e
↳ is there "Z charge"?

$W^\pm$ boson:

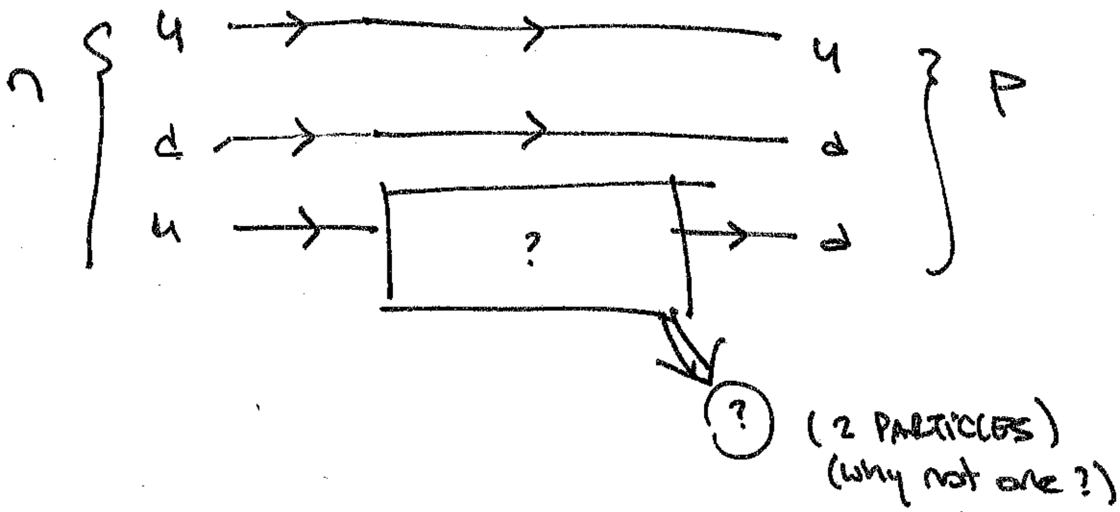




CHECK: CHARGES. (W^- GOING IN = W^+ OUT)

Notecard:

draw the diagram for neutron decay



q: why not $P \to n + (?)^*$

LAST TIME *vectors & tensors*

1. DEFINE A SYMMETRY eg 2D ROTATIONS $(SO(2))$

   nb: continuous symmetries only ("Lie Groups")

2. there are UPPER & LOWER INDICES

   UPPER: COL VECTOR
   LOWER: ROW VECTOR

3. there are ROTATIONS & "anti-ROTATIONS"

   $R^i{}_j$ — 1st upper, 2nd lower

   $\bar{R} = R^\dagger$

   for 2D ROT: $\bar{R}^i{}_j = R^j{}_i$

4. given an object w/ indices, each upper index transforms w/ R

   lower index ——— " ——— $\bar{R}$

reminder: REPEATED UPPER & LOWER INDICES MEAN A SUM.

eg. ROTATING A VECTOR

$$R^i{}_j = \begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix}$$ eg this is $R^1{}_2$

$$V^k = \begin{pmatrix} v^1 \\ v^2 \end{pmatrix}$$ ← upper index

$$V^k \to (RV)^k = R^k{}_j V^j = R^k{}_1 V^1 + R^k{}_2 V^2$$

DUMMY INDEX

this just reproduces matrix mult.

eg. ROW VECTORS TRANSFORM "OPPOSITELY"

$$W_k \to (W\bar{R})_k = \bar{R}^j{}_k W_j = \bar{R}^1{}_k W_1 + \bar{R}^2{}_k W_2$$

in "matrix" notation, ORDER MATTERS.

no ORDER DOESN'T MATTER, these are 'just #'s'

$$\bar{R} = \begin{pmatrix} c_\theta & -s_\theta \\ s_\theta & c_\theta \end{pmatrix}$$

$$(W\bar{R})_1 = \bar{R}^1{}_1 W_1 + \bar{R}^2{}_1 W_2$$

$$= c_\theta W_1 + s_\theta W_2$$

$$(W\bar{R})_2 = -s_\theta W_1 + c_\theta W_2$$

5. AN OBJECT W/ NO INDICES AFTER CONTRACTIONS IS INVARIANT W/RT THE SYMMETRY.

REPEATED UPPER/LOWER

$$\text{eg.} \quad W_k V^k = \underbrace{W_1 V^1 + W_2 V^2}$$

$$(W_1 \; W_2)\begin{pmatrix} V^1 \\ V^2 \end{pmatrix} = \underline{W}^T \underline{V}$$

if WE TRANSFORM:

$$(W\bar{R})_k (RV)^k = (cW_1 + sW_2, \; -sW_1 + cW_2)\begin{pmatrix} cV^1 + sV^2 \\ -sV^1 + cV^2 \end{pmatrix}$$

$$= c^2 W_1 V^1 + csW_1V^2 + csW_2V^1 + s^2 W_2 V^2$$
$$+ s^2 W_1 V^1 - csW_1 V^2 - csW_2 V^1 + c^2 W_2 V^2$$

$$= W_1 V^1 + W_2 V^2 \quad \checkmark$$

alternatively: $W_j \bar{R}^j{}_k R^k{}_\ell V^\ell = W_j V^j \quad \checkmark$

UNITARY (ORTHOGONAL)

BUT $\bar{R} = R^\dagger = R^{-1}$

$$\bar{R}^j{}_k R^k{}_\ell = \delta^j{}_\ell$$

$$\begin{pmatrix} c & -s \\ s & c \end{pmatrix}\begin{pmatrix} c & s \\ -s & c \end{pmatrix} = \begin{pmatrix} 1 & \\ & 1 \end{pmatrix}$$

object w/ more indices:

$$(\ddot\smile)^{ik} \longrightarrow R^i{}_m R^k{}_n \, (\ddot\smile)^{mn}$$

$$= R^1{}_1 R^1{}_1 (\ddot\smile)^{11} + R^1{}_2 R^1{}_1 (\ddot\smile)^{21} + \cdots$$

NEW OBJECT: <u>METRIC</u> (& inverse metric)

g_{ij} : takes upper index, converts to lower index

$$\boxed{g_{ij} V^j \equiv V_i}$$

similarly: inverse metric

$$\boxed{g^{ij} V_j \equiv V^i} \quad \& \quad g^{ij} g_{jk} = \delta^i{}_k$$

for $\mathbb{R}^2$: $g_{ij} = \delta_{ij}$ $\quad g^{ij} = \delta^{ij}$

these are different!
different still from $\delta^i{}_j$!

from $\mathbb{R}^2 \to$ $\underbrace{\text{Minkowski SPACE}}_{\text{SPACETIME}}$

$\mathbb{R}^2 \to \mathbb{R}^3$: $R^i{}_j$ is more complicated.

$\bar{R}^i{}_j$ is still $\bar{R} = R^T$

g_{ij} is still δ_{ij} (& so forth)

... but $i, j = 1, 2, 3$

one space, one time

from $\mathbb{R}^1 \to \mathbb{R}^{1,1}$:

$R^i{}_j \longrightarrow \Lambda^\mu{}_\nu$ ← BOOST/ROT.

$$\Lambda^\mu{}_\nu = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$\bar{\Lambda}$ is now Λ^{-1} — A LITTLE DIFFERENT (most of the time its DAGGER) · SPACETIME SYM.

$$\bar{\Lambda}^\mu{}_\nu = \begin{pmatrix} \gamma & \gamma\beta \\ \gamma\beta & \gamma \end{pmatrix}$$

$$g_{ij} \longrightarrow \eta_{\mu\nu} = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$

eg $p^\mu = \begin{pmatrix} E \\ p \end{pmatrix}$

$$p_\mu = \eta_{\mu\nu} p^\nu = (E, -p)$$

$$p^\mu \longrightarrow \begin{pmatrix} \gamma & \gamma\beta \\ \gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} E \\ p \end{pmatrix}$$

$$= \Lambda^\mu{}_\nu p^\nu$$

$$p^2 = p_\mu p^\mu = p_0 p^0 + p_1 p^1$$

$$= E^2 + (-p)p$$

$$= E^2 - p^2 \quad \longleftarrow \text{invariant!}$$

transf.

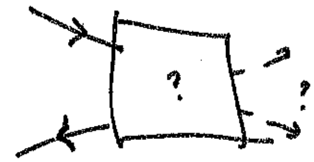
$$\bar{\Lambda}^\nu{}_\mu p_\nu \Lambda^\mu{}_\sigma p^\sigma = p_\nu \underbrace{\bar{\Lambda}^\nu{}_\mu \Lambda^\mu{}_\sigma}_{\equiv \delta^\nu{}_\sigma} p^\sigma$$

check:

$$\begin{pmatrix} \gamma & \gamma\beta \\ \gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} = \begin{pmatrix} \gamma^2 - \gamma^2\beta^2 & 0 \\ 0 & '' \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1-\beta^2}{1-\beta^2} & 0 \\ 0 & '' \end{pmatrix}$$

eg. $\xrightarrow{e^-} \quad \xleftarrow{e^+}$ $\longleftarrow$ in FEYNMAN DIAG:



What is a quantity that is invariant & conserved?

$$S \equiv (P_{e^-} + P_{e^+})^2 \quad \longleftarrow \text{TOTAL 4-momentum, squared}$$

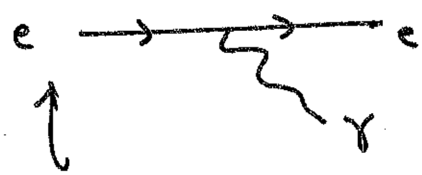
↑ stupid name, but that's what it is

IN CM FRAME: $\quad P_{e^-} = (E^{cm}, \underline{p})$

$$P_{e^-} = (E^{cm}, -\underline{p})$$

$$S = (2E^{cm}, \underline{0})^2$$
$$= 4(E^{cm})^2$$

can an electron spontaneously emit a γ?



$$\left.\right\} \quad K_e = (E', \underline{k}) \quad \longleftarrow \quad K_e^2 = M_e^2$$
(on shell)

$$k_\gamma = (E_\gamma, E_\gamma \hat{n})$$

↑

$P = (E, \underline{p})$

$S = p^2 = M_e^2$

in CM FRAME,
$P = (M_e, \underline{0})$

$$E_\gamma \hat{n} = -\underline{k}$$

$$S = (k_e + k_\gamma)^2 = k^2 + k_\gamma^2 + 2k \cdot k_\gamma$$
$$= M_e^2 + 0 + \underbrace{2(E' E_\gamma + E_\gamma^2)}_{>0}$$

<u>INTERNAL SYMMETRIES</u> : generaliz. of ROTATIONS.

for us: SU(N) symmetries

$N \times N$ <u>UNITARY</u> MATRICES w/ det $= 1$

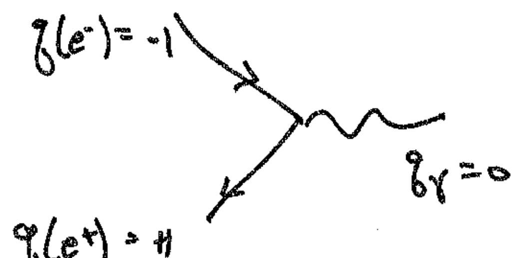
$$U^{\dagger} U = \mathbb{1} \quad \text{s.t.} \quad \tilde{U} = U^{-1} = U^{\dagger}$$

UPPER INDEX: U TRANSF
LOWER INDEX: $U^{\dagger}$ TRANSF

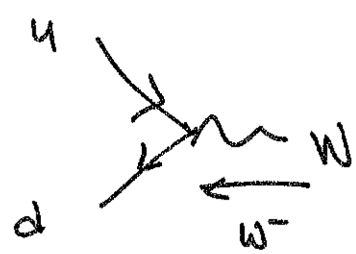
simplest case: $U = e^{i\theta} \leftarrow N = 1$

<u>RULE</u>: a particle w/ charge q going <u>into</u> a vertex picks up a phase $e^{i\theta q}$.

<u>RULE</u>: if a symmetry is good, then the phases going into a vertex must be zero.



$$e^{iq_{e^-}\theta} e^{iq_{e^+}\theta} e^{iq_{\gamma}\theta} = 1$$



$$q_u + q_d + q_{W^-} = 0 \quad \checkmark$$