WHERE WE ARE?

PRESENTATIONS

QUIZ | HIGGS | GAUGE

HW Q&A

$\langle H \rangle$ PAIRS UP LH & RH CHIRALITY PARTICLES TO FORM MASS EIGENSTATES

NO "GOOD" SU(2) or U(1) QUANTUM NUMBERS

$\langle H \rangle$ ALSO GIVES GAUGE BOSONS MASS

GIVES TOGETHER W^+W^- & W^3 Y

$$\frac{1}{2}\begin{pmatrix} W^3 & \sqrt{2}(W^1 - iW^2) \\ \sqrt{2}(W^1 + iW^2) & -W^3 \end{pmatrix} \quad Y$$

↑ MASSIVE CHARGED $W^\pm$

↓ MASSIVE Z & ORTHOGONAL A (MASSLESS)

$$\begin{aligned} Z &= \cos\theta_W W^3 - \sin\theta_W Y \\ A &= \sin\theta_W W^3 + \cos\theta_W Y \end{aligned}$$

$$\tan\theta_W = g'/g$$

W^3_μ, Y_μ ← each have 2 DOF ("MASSLESS" GAUGE PARTICLES)

↓ $\langle H \rangle$

A, Z ← 2 + 3 DOF

↑ LONGITUDINAL POL. OF Z IS ACCOUNTED FOR

$W^\pm$ ← 2 DOF EACH

↓ $\langle H \rangle$

$W^\pm$ MASSIVE ← 3 DOF EACH

↑ 2 LONG. POL. MISSING!

where did these missing DOF come from?

↳ the Higgs!

$$H = \begin{pmatrix} g^+ \\ \left(\frac{v}{\sqrt{2}} + h(x)\right) + i g^0 \end{pmatrix}$$

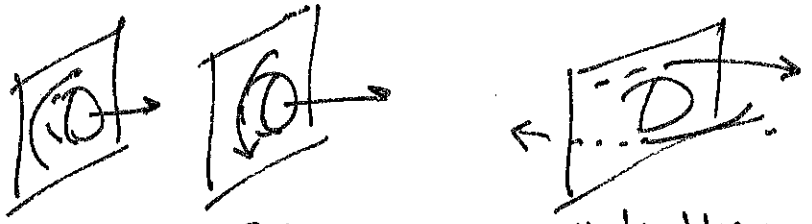
g^+ ← 1 C dof = 2 R dof

$\left(\frac{v}{\sqrt{2}} + h(x)\right) + i g^0$ ← 1 R dof

GOES INTO $W^\pm, Z$ DOF.

Indeed: massless spin-1 has 2 DOF

(general: massless of any non-zero spin has 2 DOF: helicity.



→ not allowed:
massless → $v = c$
then "top" is moving faster than c.

PROHIBITED for MASSLESS PARTICLES
ALLOWED for MASSIVE

nb: the spin-1 state of Hydrogen has three values of L_z: $\pm 1, 0$

± 1: LH, RH HELICITY

0: LONGITUDINAL
(when you BOOST INTO REST FRAME.)

So: photon only has 2 DOF.
Z boson has 3 DOF.

WHERE DID THEY COME FROM?

the 'deep' statement is that A_μ contains more dof than the natural phenomena it is supposed to describe.

↳ can shift A by the gradient of any function & describe the SAME photon ... this is REALLY JUST GAUGE/EM POTENTIAL

$$A_\mu = (\Phi, \underline{A})$$

$$\Phi \to \Phi + \partial_t f$$

$$\underline{A} \to A + \nabla f$$

$$\underline{E} \sim \nabla \Phi$$

$$\underline{B} \sim \nabla \times \underline{A}$$

RESULT: SPIN-1 GAUGE PARTICLES HAVE A μ index (4 DOF), BUT ONE IS FAKE. ↳ count as $\boxed{3 \text{ DOF}}$

is this correct?

HOW MANY "DOF" IN EM WAVE (CLASSICAL)?

↳ two: LH, RH polarized light

ie light is transversely polarized

SPIN-1 particles

FACT: the GAUGE BOSONS are REAL (self-conjugate)

can see this in, eg, W:

$$\begin{pmatrix} W^3 & \sqrt{2}W^- \\ \sqrt{2}W^+ & -W^3 \end{pmatrix}^\dagger = \begin{pmatrix} W^3 & \sqrt{2}W^- \\ \sqrt{2}W^+ & -W^3 \end{pmatrix}$$

$W^\pm$ is complex comb of W^1, W^2 (REAL).

how many dof?

$\mu = 0, 1, 2, 3$

so, eg: W^3_μ is 4 DOF

Y_μ is 4 DOF

Z_μ ——————

A_μ ——————

HOW MANY SHOULD BE THERE?

"DEEP" FACT: GAUGE SYMMETRIES ARE ACTUALLY REDUNDANCIES

↳ they "TAKE AWAY" A DOF IN the GAUGE BOSONS.

You've seen this:

$A_\mu \longrightarrow A_\mu + \partial\alpha$ any function

Degrees of freedom

↑ # REAL SCALAR functions

eg Higgs field: $\begin{pmatrix} h_u(x) \\ h_d(x) \end{pmatrix} = \begin{pmatrix} a(x)+ib(x) \\ c(x)+id(x) \end{pmatrix}$

each are $\mathbb{C}$
HAD TO BE: has HYPERCHARGE

$\langle c \rangle = v/\sqrt{2}$

actually this is where the $\sqrt{2}$ comes from: norm of 2 states

HIGGS HAS 4 DOF
EA DOF IS IN PRINCIPLE A PARTICLE

↳ analogs: $Q = \begin{pmatrix} u(x) \\ d(x) \end{pmatrix}$

BUT ALSO 3 COLORS (a index)
AND ALSO 2 SPINS (α index)

↳ $2 \times 3 \times 2 = 12$ DOF ("PARTICLES")
i a α

nb most people (REASONABLY) do not call a spin-up particle as different from a spin-down particle ... aren't they related by a rotation? YES.
That's the point.

Part of Higgs Mechanism

(GAUGE) ↙ w/ SPIN-1 GAUGE BOSONS

"SPONTANEOUS BREAKING OF SYM

⇒ GAUGE BOSONS PICK UP MASS

⇒ AND (✗ AS REQ by consistency)

EAT PIECES of the FIELD

that BREAK THE SYMMETRY;

"pieces" are called GOLDSTONE BOSONS

? show up all over the place

(eg. "spinons" in cond-mat)