TODAY: 1. how to do $\boxed{HW}$
2. HIGGS & MASS

$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$ $\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$

$$Q^{\alpha i a}_{Y=1/6} \quad \bar{u}^{\alpha a}_{Y=-2/3\,a} \quad \bar{d}^{\alpha a}_{Y=1/3\,a} \quad L^{\alpha i}_{Y=-1/2} \quad \bar{e}^{\alpha}_{Y=+1}$$ $\boxed{\text{MATTER}}$

$\boxed{CP}$

$$(Q^\dagger)^{Y=-1/6}_{\dot\alpha i a} \quad (u_R)^{Y=2/3\,a}_{\dot\alpha a} \quad (d_R)^{Y=-1/3\,a}_{\dot\alpha a} \quad (L^\dagger)^{Y=1/2}_{\dot\alpha i} \quad (e_R)^{Y=-1}_{\dot\alpha}$$

PARTICLE/ ANTIPARTICLE SYMM.

$u_R = \bar{u}^\dagger \quad d_R = \bar{d}^\dagger \quad e_R = \bar{e}^\dagger$ } just names for PARTICLE v. ANTI

you can think of it as "h.c."
analog of a & $a^\dagger$
one creates particle, other creates 'hole'

$\boxed{\text{Higgs}}$: $H^i_{Y=1/2} \qquad \langle H \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$

$(H^\dagger)^{Y=-1/2}_i \qquad \langle H^\dagger \rangle = \begin{pmatrix} 0 & \frac{v}{\sqrt{2}} \end{pmatrix}$

$\boxed{\text{GAUGE}}$: Y_μ

$$(W_\mu)^i{}_j \leftarrow \frac{1}{2}\begin{pmatrix} W^3 & \sqrt{2}W^- \\ \sqrt{2}W^+ & -W^3 \end{pmatrix}$$

$$(g_\mu)^a{}_b \leftarrow \sim \begin{pmatrix} \bar{r}r & \bar{r}g & \bar{r}b \\ \bar{g}r & \bar{g}g & \bar{g}b \\ \bar{b}r & \bar{b}g & \bar{b}b \end{pmatrix}$$

no additional antiparticles
(FACT FOR FORCE PARTICLES)
... they're Hermitian

INVARIANTS

SU(2): ε_{ij} ε^{ij}

SU(3): ε_{ijk} ε^{ijk}

} in gen: $\varepsilon_{i_1 \cdots i_N}$ for SU(N)

spin ½: $(\sigma^\mu)^{\dot\alpha}{}_\alpha$ $(\bar\sigma^\mu)^\alpha{}_{\dot\alpha}$ $\varepsilon_{\alpha\beta}$ $\varepsilon^{\alpha\beta}$ $\varepsilon_{\dot\alpha\dot\beta}$ $\varepsilon^{\dot\alpha\dot\beta}$

spin 1: $\eta^{\mu\nu}$ $\eta_{\mu\nu}$

fact: GAUGE BOSONS HAVE A SPECIAL TRANSFORMATION UNDER THE GAUGE SYMMETRY

eg $\Upsilon_\mu \to \Upsilon_\mu + \partial_\mu f(x)$

analog of $A_\mu(x) \to A_\mu(x) + \partial_\mu \underbrace{\alpha(x)}_{\text{GAUGE}}$

has to do w/ REDUNDANCY IN MATHEMATICAL DESCRIPTION OF THEORY

↑

analog of CONNECTION / COVARIANT DERIVATIVE in classical GR.

FACT: CAN have mass if you can contract particle & antiparticle

↳ not possible for MATTER ← CHARGE & SPIN INDEX GETS IN WAY

possible for HIGGS

... not possible for GAUGE b/c of GAUGE symm.

⇒ everything but HIGGS is massless!

~~[illegible]~~

but: HIGGS VEV (vacuum expectation value) gives an ORDER PARAMETER for SYMMETRY BREAKING

how it works: ① write down SYMMETRIC theory. AS USUAL.

② REPLACE H w/ $\langle H \rangle + H'$

$\langle H \rangle$: constant — does not transform ... even though it has indices.

YUKAWAS: $y_d \, Q^{\alpha i a} H^{\dagger}{}_i \, \boxed{\bar{d}^{\beta}{}_a} \, \varepsilon_{\alpha\beta} + \text{h.c.}$

Q: $1/6$, $H^\dagger$: $-1/2$ → $-1/3$; $\bar{d}$: $1/3$

y_d: just a number. strength of this interaction

$\bar{d}$: or $(d_R^{\dagger})^{\beta}{}_a$

$$y_d\, Q^{\alpha i a} \langle H^+ \rangle_i \, (d_R^+)^\beta_a \, \varepsilon_{\alpha\beta} + \text{h.c.}$$

$$\begin{matrix} Q^{i=1} \to \\ Q^{i=2} \to \end{matrix} \begin{pmatrix} u_L^{\alpha a} \\ d_L^{\alpha a} \end{pmatrix} \qquad \left(\underset{\langle H^+ \rangle_{i=1}}{0}, \ \underset{\langle H^+ \rangle_{i=2}}{\frac{v}{\sqrt{2}}} \right)$$

$$= \underbrace{y_d \frac{v}{\sqrt{2}}}_{\text{just a number}} \underbrace{d_L^{\alpha a} (d_R^+)_{a\alpha}}_{(\text{particle})(\text{particle})^+} + \text{h.c.} \leftarrow y_d^+ \frac{v}{\sqrt{2}} \underline{d_L^+ d_R}$$

LOWERED INDEX W/ EXP

completely different!

$\boxed{d_L \text{ + } d_R^+ \text{ ARE ANTIPARTICLES}}$

this is a mass!

WHAT IS THE VALUE OF THE MASS?

$$m_d = y_d \frac{v}{\sqrt{2}} \longleftarrow v = 246 \text{ GeV}$$

note: no mass for UP QUARK from this term!
↳ comes from other terms

$$\overset{\text{some \#}}{y_u} \ \varepsilon_{ij} H^j Q^{i\alpha} \bar{u}^\beta e_{\alpha\beta} + \text{h.c.}$$

gives: $\frac{y_u v}{\sqrt{2}} \, u_L u_R^+ + \text{h.c.}$

finally: $y_e H^{+}{}_i L^{\alpha i} \bar{e}^{\beta} \varepsilon_{\alpha\beta} + h.c.$

$$\hookrightarrow \frac{y_e v}{\sqrt{2}} e_L e_R^{+} + h.c.$$

what about $y_N H^j \varepsilon_{ij} L^i \bar{N}^{\beta} \varepsilon_{\alpha\beta}$

INTRODUCE NEW PARTICLE
RH NEUTRINO
$\nu_R = \bar{N}^{+}$

this seems important for writing NEUTRINO MASS
... which we now know must exist.
why don't we write it?. $\exists$ few REASONS

IN HW: 1. $\bar{N}$ can get mass "by itself"
... SUSPECT THAT IT MAY BE VERY HEAVY

... SEE SAW MECHANISM

2. Weinberg operator:

$|HL|^2$ can give ν_L MASS w/o ν_R!

PREFACTOR

SYMMETRY POST MORTEM

$$\langle H \rangle_{Y=1/2} = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}_{Y=1/2}$$

$SU(2)_L$: no longer a good sym.
$U(1)_Y$: —————— " ——————

SU(3) COLOR UNAFFECTED.

if I do a rotation, the VEV <u>moves</u>, so the ground state is <u>not</u> invariant.

<u>FACT</u>:
IF THESE SYMMETRIES ARE BROKEN, THEN THE GAUGE BOSONS ARE NO LONGER MASSLESS.

GAUGE SYM IS PRESERVED BY VACUUM $\Longleftrightarrow$ GAUGE BOSON IS MASSLESS

<u>DIG DEEPER</u>

imaginary version ← analog of 2D IR ROT

SU(2) ROTATIONS : 3 axes → $\frac{1}{2}\begin{pmatrix} & 1 \\ 1 & \end{pmatrix}$, $\frac{1}{2}\begin{pmatrix} & -i \\ i & \end{pmatrix}$

$e^{i\theta^a T^a}$

$\boxed{\frac{1}{2}\begin{pmatrix} 1 & \\ & -1 \end{pmatrix}}$

rephasing

clearly BROKEN

$T^3 = \frac{1}{2}\sigma^3$

U(1) hypercharge rotation: one axis
↳ simply rephasing

$$\boxed{e^{i\theta_Y q_Y}}$$

↑ charge of thing you're rotating

LET US take the ORDER PARAMETER of $SU(2)\times U(1)$ BREAKING ... and see if there is any surviving symmetry:

$$\langle H \rangle \xrightarrow{e^{i\theta_Y q_Y} e^{i\theta^3 T^3}} e^{i\frac{\theta_Y}{2}} \begin{pmatrix} e^{i\theta^3/2} & \\ & e^{-i\theta^3/2} \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

$\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$

$$= \begin{pmatrix} 0 \\ \underbrace{e^{i(\theta_Y - \theta^3)/2}}_{=1 \text{ when } \theta_Y = \theta^3} \frac{v}{\sqrt{2}} \end{pmatrix}$$

... then $\langle H \rangle$ is invariant!

⇒ there is a "SUB-SYMMETRY" of the theory that is UNBROKEN!

it is an overall REPHASING. → U(1)

it is a combination of $U(1)_Y$ and T^3 ROT of $SU(2)_L$

...

<u>SO</u>: what is the charge of MATTER under this leftover "GOOD" symmetry?

eg. $\boxed{Q} = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \xrightarrow{T^3} \begin{pmatrix} e^{i\theta/2} u_L \\ e^{-i\theta/2} d_L \end{pmatrix}$

SO UNDER T^3 ROT of $SU(2)$

u_L has charge $+1/2$
d_L has charge $-1/2$

UNDER $U(1)_Y$, BOTH HAVE CHARGE $1/6$.

the SURVIVING SYMMETRY is:
"EQUAL ROT IN Y & T^3 DIRECTIONS"

so: $q^u_{\text{SURVIVE}} = \frac{1}{2} + \frac{1}{6} = \frac{2}{3}$

$q^d_{\text{SURVIVE}} = -\frac{1}{2} + \frac{1}{6} = -\frac{1}{3}$

ELECTRIC CHARGES!

this 'leftover' sym is E&M!!