Electroweak

SYMMETRIES

① Weak : $SU(2) \longrightarrow W^1, W^2, W^3$

same mathematical structure as SPIN

$\boxed{W^\pm, W^3}$

$$W^i{}_j = \frac{1}{2}\begin{pmatrix} W^3 & \sqrt{2}W^+ \\ \sqrt{2}W^- & -W^3 \end{pmatrix}$$

② Hypercharge : $U(1)$ $\boxed{Y}$

Matter :

DOUBLETS (under SU(2)) ← things that talk to $W^i{}_j$

$$Q^i_{1/6} = \begin{pmatrix} u \\ d \end{pmatrix}_{1/6} \leftarrow \text{hypercharge}$$

$$L^i_{-1/2} = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_{-1/2} \leftarrow \text{hypercharge}$$

(and associated antiparticles: $Q^\dagger{}_i$, $L^\dagger{}_j$)

↳ nb: these have opposite HYPERCHARGE

$$Q^\dagger{}_i \longrightarrow \left(e^{i \underset{\substack{\uparrow \\ 1/6}}{q_Q} \theta} Q^i\right)^\dagger = e^{-iq_Q\theta} Q^\dagger{}_i$$

$$H^i = \begin{pmatrix} h^u \\ h^d \end{pmatrix} \longleftarrow \text{HIGGS}$$

Singlets ← do not talk to $W^i{}_j$

$\bar{e}_{+1}$
$\bar{u}_{-2/3}$
$\bar{d}_{1/3}$

} look like antiparticles of our favorite matter particles.

LAST BIG TWIST

there are indices for SPIN / ANGULAR MOMENTUM

ROTATIONS ... BUT REALLY FULL SET OF LORENTZ (POINCARÉ)

it turns out that spin indices are tricky!! (technical: WIGNER DECOMPOSITION, LITTLE GROUP, ...)

RESULT: ADDITIONAL INDEX

↳ essentially: how does particle transform under ROTATIONS

BUT IT'S ALL WEIRD BECAUSE ACTUALLY WE CARE ABOUT FULL LORENTZ TRANSF AND TRANSLATIONS IN SPACETIME !!

SPIN-0 : does not transform under a rotation

two other cases for us:

spin 1/2
↳ $\alpha, \dot{\beta}$

spin-1
↳ μ ← vectors, like momenta

spin-1 : has a μ index

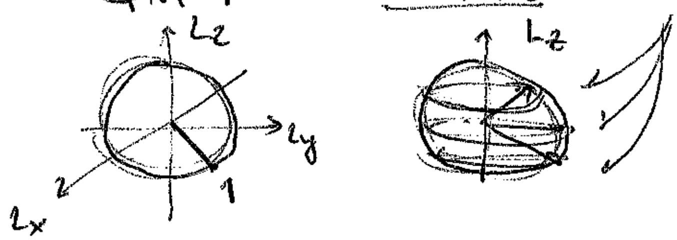
eg $A^\mu \longrightarrow \Lambda^\mu{}_\nu A^\nu$

$\uparrow$ photon under LORENTZ

QUANTUM : SPIN/ANGULAR MOMENTUM QUANTIZ.

QUANTUM #'s : L^2, L_z

spin-1 3 states: $m = \pm 1, 0$



these correspond to polarizations of the particle.

A HINT OF A PUZZLE :

$A^\mu \leftarrow \mu = 0, 1, 2, 3 \leftarrow$ LOOKS LIKE 4 DIFFERENT "PARTICLES" RELATED BY ROTATION

$L_z = \pm 1, 0 \longleftarrow$ seems like only 3 states from QM

E&M : 2 POLARIZ of EM field (LH & RH)

this will LEAD to
- WHAT IS A FORCE?
- MASSLESS VS. MASSIVE

FACT : ALL FORCE PARTICLES ARE VECTORS .

the weird one : spin 1/2 ← matter!

this is a deep rabbit hole —
WE CAN TALK AS MUCH AS YOU WANT
ABOUT WHAT IT "MEANS"

BUT THE KEY RESULT (RULE) IS:

THERE ARE 2 TYPES OF SPIN 1/2 PARTICLE
↳ often, they get smushed together
(they mix quantum mechanically)

LH CHIRALITY ⟶ α INDEX ($\alpha = 1, 2$)
RH CHIRALITY ⟶ $\dot{\beta}$ INDEX ($\dot{\beta} = \dot{1}, \dot{2}$)

they do not mix!
completely different

RELATED but different idea: HELICITY
↳ projection of spin (angular momentum) vector onto the momentum 3-vector.

WHAT DOES CHIRALITY TELL YOU?
for spin 1/2, rotation by 2π gives phase of $e^{i\pi} = -1$

chirality tells you the phase e π

under a rotation: $e^{\pm i\theta/2}$?

eg. LH, "spin up" : $e^{i\theta/2}$
spin down : $e^{-i\theta/2}$

RH spin up : $e^{-i\theta/2}$
spin down : $e^{+i\theta/2}$

Rules

conjugate of a spin 1/2 particle is opposite chirality

$$(\psi^\alpha)^\dagger = (\psi^\dagger)_{\dot\alpha}$$

$$(\bar\psi^{\dot\alpha})^\dagger = (\bar\psi^\dagger)_\alpha$$

DOTTED & UNDOTTED SPIN INDICES CANNOT CONTRACT (without additional help)

invariant tensor

$$(\sigma^\mu)^{\dot\alpha}{}_\alpha = (\mathbb{1}^{\dot\alpha}{}_\alpha, \underline{\sigma}^{\dot\alpha}{}_\alpha)$$

SPIN-1 INDEX! (vector)

PAULI MATRICES ... AGAIN!

$\mu=0: \begin{pmatrix} 1 & \\ & 1 \end{pmatrix}$ eg $\mu=1: \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}$

$\dot\alpha = 1$
$\alpha = 2$

this means I can combine 2 spin 1/2 into spin-1

$$(\psi^\dagger)_{\dot\alpha}(\sigma^\mu)^{\dot\alpha}{}_\alpha \psi^\alpha = \underbrace{(\psi^\dagger\sigma\psi)^\mu}_{\text{not invariant, but transforms like spin-1}}$$

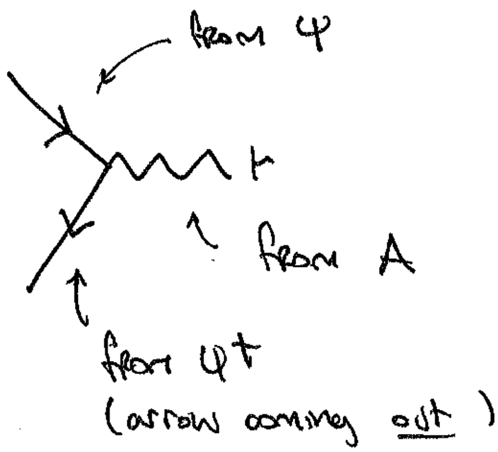
this makes sense from addition of angular momenta

suppose ψ^α has some electric charge.
then $(\psi^\dagger)_{\dot\alpha}$ has opposite electric charge.
these talk to a photon, A_μ, a <u>vector</u>.

how? $\underbrace{(\psi^\dagger \sigma \psi)^\mu A_\mu}$

all indices contracted, no net charge

$\Rightarrow$ invariant!



Π there's a <u>DIFFERENT</u> arrow notation which tracks CHIRALITY:

 is LH (α index)

 is RH ($\dot\alpha$ index)

can you raise & lower α, $\dot{\alpha}$ indices?

<u>YES</u>: RULE: $\varepsilon_{\alpha\beta}$, $\varepsilon_{\dot{\alpha}\dot{\beta}}$, $\varepsilon^{\alpha\beta}$, $\varepsilon^{\dot{\alpha}\dot{\beta}}$

are invariant tensors
that are METRICS.

$$\psi_\alpha = \varepsilon_{\alpha\beta}\psi^\beta$$

$$\varepsilon_{\alpha\beta} = \begin{pmatrix} & 1 \\ -1 & \end{pmatrix}$$ (up to overall sign)

So I can form invariants:

$$\psi^\alpha = \begin{pmatrix} \psi^1 \\ \psi^2 \end{pmatrix}$$ spin up/down

$$\psi^\alpha\psi^\beta\varepsilon_{\alpha\beta} = \psi^1\psi^2 - \psi^2\psi^1$$ nb this is a spin-0 combination

if ψ had some U(1) charge (electric charge), is $\psi^\alpha\psi^\beta\varepsilon_{\alpha\beta}$ invariant under this?

↳ no!

You can have LH ψ^α (w/ any other indices)

↳ Nature has ANTIMATTER: $(\psi^\alpha)^\dagger = (\psi^\dagger)_{\dot{\alpha}}$

RH ANTIPARTICLE
(& CONJUGATE ANY OTHER INDICES)

eg. LH electron (charge −1) & RH positron (charge +1) are related

... but LH positron / RH electron are totally different!

ELECTROWEAK: all the indices

SU(2) indices $i,j \in \{1,2\}$

FORCES: SU(2) $\longrightarrow (W_\mu)^i{}_j$

SPIN-1 INDEX

U(1) $\longrightarrow Y_\mu$ talks to hyper<u>charge</u>

MATTER

convention: WRITE EVERYTHING IN TERMS OF LH SPIN-1/2 INDEX

LH SPIN 1/2 SU(2)

$$Q^{\alpha i}_{Y=1/6} = \begin{pmatrix} u^\alpha{}_{1/6} \\ d^\alpha{}_{1/6} \end{pmatrix}$$

HYPERCHARGE

$$\bar{u}^\alpha{}_{Y=-2/3}$$

nb: $(\bar{u}^\dagger)_{\dot{\alpha},\, Y=2/3}$

LOOKS LIKE RH UP QUARK W/ CORRECT CHARGE

$$\bar{d}^\alpha{}_{Y=+1/3}$$

$$L^{\alpha i}_{Y=-1/2} = \begin{pmatrix} \nu^\alpha{}_{-1/2} \\ e^\alpha{}_{-1/2} \end{pmatrix}$$

$$\bar{e}^\alpha{}_{Y=+1}$$

"$Q\bar{u}\bar{d}L\bar{e}$" or "CUDDLY"

"<u>OTHER</u>" $H^i{}_{Y=1/2}$

Q: HOW CAN YOU CONNECT 3 of these?