CROSS - SECTION

→ Connects theory with experiments.
→ In very layman's term → slice of an object.
In quantum/particle physics → the probability that two particles will collide and react in a certain way. (under certain conditions.

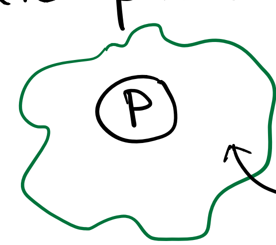
$\boxed{\text{For Ex - For } pp \to t\bar{t}}$ cross section means we are trying to count how many $t\bar{t}$ pairs were created when a given no of protons were fired at each other.

↳ But there is a problem - no. of final states is uncountably infinite and in most cases and the set of final states is not even fully known. → look for relative probability.

$\boxed{\text{classically}}$ - particles thought to be indestructible balls, their probability is α to their size

For subatomic particles, size is something not well defined.
↓
De-Broglie wavelength ← wave particle duality

$$\boxed{\lambda = \frac{h}{P}}$$



more like a cloud than a hard sphere

So depending on the energy / momentum outcome of a collision vary.

$\boxed{\text{So what do we do ?}}$

The best shot is we shoot a lot of particles in a narrow confined area. Then the probability is just area covered by the particles divided by the total area of the cloud.

→ independent of accelerators
→ depends on energy and spin

LUMINOSITY → How many particles we are able to squeeze in a given space in a given amount of time.

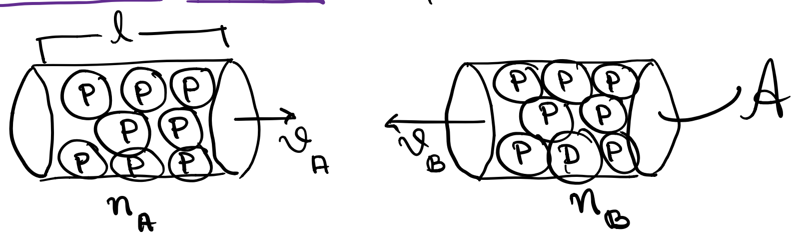
$L \propto$ cross section of one particular process – rate for that process

$L \propto$ cross section of all possible outcomes – total no. of collisions. $\left[\frac{N}{t} = \sigma L\right]$

Units → Barn – $10^{-24}\,cm^2$ → Area !!

Cross sectional area of a proton is about $\frac{1}{100}$ barn.

COLLISION RATE



Total no. of protons N_A in bunch A ~~that~~ a slice protons in bunch B sees in unit time

$$\boxed{\frac{N_A}{t} = n_A \mathcal{A} \cdot |v_A - v_B|}$$

length per unit time of bunch A ~~that~~ passes through bunch B

Total no. of protons in bunch B ~~that~~ could possibly interact with the protons of bunch A is

$$\boxed{N_B^{eff} = n_B l \sigma}$$

Total no. of scattering events is then

$$\boxed{\frac{N_A N_B^{eff}}{t} = \underbrace{n_A n_B \mathcal{A} l |v_A - v_B|}_{\text{flux factor}} \sigma}$$

Luminosity.

SCATTERING AMPLITUDE

Initial state $\quad |P_A P_B\rangle_{in} \quad — \quad t = -\infty$

Final state $\quad {}_{out}\langle P_1 P_2 \cdots P_n| \longrightarrow t = +\infty$

Probability of scattering / collision

$$\sigma \propto |{}_{out}\langle P_1 P_2 \ldots P_n | P_A P_B\rangle_{in}|^2$$

dimensions wrong

Corrected by the de-broglie wavelength

$$\lambda_{dB} = \frac{\hbar}{|\vec{p}|}$$

$$\boxed{\sigma \propto \lambda_A \lambda_B \left| {}_{out}\langle P_1 P_2 \cdots | P_A P_B \rangle_{in} \right|^2}$$

A proton at rest the size is determined by Crompton wavelength

$$\boxed{\lambda_c = \frac{h}{mc}}$$

$\therefore$ For higher energies

$$\sigma \propto \frac{1}{|P_A||P_B|} \left| {}_{out}\langle P_1 P_2 \cdots | P_A P_B \rangle_{in} \right|^2$$

whereas in the lower energy scheme

$$\sigma \propto \frac{1}{m_A m_B} \left| \langle P_1 P_2 \cdots | P_A P_B \rangle_{in} \right|^2$$

Combining both

$$\boxed{\sigma \propto \frac{1}{2E_A E_B} \left| {}_{out}\langle P_1 P_2 \cdots | P_A P_B \rangle_{in} \right|^2}$$

1) Dimensions are correct
2) Probability amplitude included
3) Still not Lorentz invariant $\rightarrow \frac{1}{E_A E_B}$

Insert factor $|v_A - v_B|$ to make it Lorentz invariant.

$$\boxed{\sigma = \frac{1}{2E_A E_B |v_A - v_B|} \left| {}_{out}\langle P_1 P_2 P_3 \cdots | P_A P_A \rangle_{in} \right|^2}$$