Long HW 4: **Flavor Indices II**

Course: Physics 165, *Introduction to Particle Physics* (2020)
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Due by: **Tuesday**, March 10

This is our last real homework. Homework 5 will be a semi-final exam.

1 Fixing the Flavor Indices

I screwed up last week on the short homework. Together we will suffer through fixing the flavor indices. Sorry about that.

Recall that the three generations of matter can be written as an additional index on our matter particles. let $I, J = 1, 2, 3$ be generation (or *flavor*) indices. These indices behave similarly to color indices ($a, b = 1, 2, 3$) except that there is no force associated with flavor[1].

The situation is actually more complicated. In the absence of the Yukawa interactions, *each* of the 'cuddly' fields can rotate in its own flavor space *independently* of the others. So in fact, there are *five* types of flavor rotations:

$$Q^I \to (U_Q)^I{}_J Q^J \quad \bar{u}_I \to \bar{u}_J \left(U^\dagger_{\bar{u}}\right)^J{}_I \quad \bar{d}_I \to \bar{d}_J \left(U^\dagger_{\bar{d}}\right)^J{}_I \quad L^I \to (U_L)^I{}_J L^J \quad \bar{e}_I \to \bar{e}_J \left(U^\dagger_{\bar{e}}\right)^J{}_I \ . \tag{1.1}$$

Each of the U_X are independent *unitary* matrices. Unitary means that

$$(U^\dagger_X)^I{}_J (U_X)^J{}_K = \delta^I_K \ .$$

We have suppressed all of the other indices of the 'cuddlies.' The fact that each of the U_X is independent means that there is actually a $\mathrm{U}(3)^5$ flavor symmetry. Because we've run out of alphabets, we're writing them all with the same types of indices, but this means that you *cannot* contract the flavor indices of one type of 'cuddly' with different type of 'cuddly.' For example:

$$Q^I \bar{d}_I \to Q^J \bar{d}_K (U^\dagger_{\bar{d}})^K{}_I (U_Q)^I{}_J \ . \tag{1.2}$$

The right-hand side is *not* $Q^I \bar{d}_I$ because U_Q and $U_{\bar{d}}$ are the *same* rotation. But one is free to rotate the Q flavors and the $\bar{d}$ flavors totally independently, that's why they have different unitary matrices. (*This is important.*)

Writing the flavor indices on the Yukawa interactions gives (suppressing non-flavor indices because you already know how they contract):

$$(y_u)^I{}_J H Q^J \bar{u}_I + (y_d)^I{}_J H^\dagger Q^J \bar{d}_I + (y_e)^I{}_J H^\dagger L^J \bar{e}_I + \text{h.c.} \ . \tag{1.3}$$

Note that the Higgs has no flavor indices and doesn't want to have anything to do with any of this.

The Yukawa matrices $(y_X)^I{}_J$ are a set of nine constants that tell how how strongly one flavor of 'cuddly' talks to a different flavor of 'cuddly'. For example, the $(y_u)^1{}_3$ element tells me how much the *third generation* quark doublet (composed of a t_L and a b_L) talks to the *first generation* right-handed up (composed of a $u^\dagger_R$). When you insert the Higgs vev (which projects out one component of the doublet), this is an apparent mass term that connects the t_L to the u_R. Weird.

[1] We call flavor a **global symmetry**, whereas the symmetries associated with forces are called **gauge symmetries**. The significance of gauge symmetries is discussed in sites.google.com/ucr.edu/p230b/.

1.1 The name game

What pairs of particles are connected by the following Yukawa matrix terms:

1. $(y_d)^2{}_1$
2. $(y_e)^3{}_2$

Which two particles are glued together in a mass term once we insert the Higgs vev to project out one of the doublets?

1.2 General flavor transformation

The Yukawa matrices are constants: they have indices, but they do not transform. Only the 'cuddlies' transform under a flavor transformation. Under the general flavor transformation (1.1), write down how each term in (1.3) transforms. As a hint, see (1.2).

1.3 Too many masses

The general set of Yukawa matrices induce all sorts of masses. For example, each flavor of left-handed up-type quark $(u_L)^I$ is glued to each flavor of right-handed up-type quark, $\bar{u}_J$ through $(y_u)^J{}_I$.

However each flavor of 'cuddly' particle has identical charges and indices. The only thing that distinguishes them are the Yukawa couplings. We can thus use the general flavor transformation from the previous section to *diagonalize* the Yukawa terms.

FACT: A complex square matrix can be diagonalized by a **biunitary** transformation. For any matrix $M^I{}_J$, there exist unitary matrices U and V such that UMV^{-1} is diagonal.

Argue that you can diagonalize the charged lepton masses, but that you can at most diagonalize *either* the up-type masses or the down-type masses.

COMMENT: This is the origin of what particle folks call *flavor physics*. The issue is that the Q has only one flavor rotation to donate, but the up-type and down-type Yukawas each need an independent rotation to contribute to the biunitary transformation that diagonalizes the interactions. There is no such discrepancy for the leptons as long as we do not include a right-handed neutrino.

2 The origin of funny $W^\pm$ interactions

What do we do when we it seems like we cannot diagonalize a mass matrix? We diagonalize it anyway. In order to diagonalize both the up-type and down-type mass matrices, we need the u_L^I and the d_L^I particles to rotate independently:

$$u_L^I \to (U_u)^I{}_J u^J \qquad\qquad d_L^I \to (U_d)^I{}_J d^J \ . \tag{2.1}$$

These really *should* only be allowed to rotate by the *same* matrix $U_u = U_d = U_Q$, but we *really* want to diagonalize our masses so that we can have a diagonal Hamiltonian and all that. Maybe we just diagonalize now and ask for forgiveness later, eh?

Here's where the other shoe drops. The interactions of the Q with the W boson are of the form:

$$\frac{g}{2}(Q^\dagger)_i(W)^i{}_jQ^j = \frac{g}{2}(Q^\dagger)_i(W)^i{}_jQ^j \begin{pmatrix} u_L^\dagger & d_L^\dagger \end{pmatrix} \begin{pmatrix} W^3 & \sqrt{2}W^- \\ \sqrt{2}W^+ & -W^3 \end{pmatrix} \begin{pmatrix} u_L \\ d_L \end{pmatrix} . \tag{2.2}$$

We have suppressed the flavor indices, which you should now restore. Show that the W^3 interactions are invariant under (2.1). Show, further, that the $W^\pm$ interactions are *not* invariant under (2.1).

The $W^\pm$ intraction is of the form

$$\frac{g}{\sqrt{2}}(u_L^\dagger)_I(V_{\text{CKM}})^I{}_J d_L^J , \tag{2.3}$$

explicitly write out V_{CKM} in terms of the matrices U_u and U_d that were needed to diagonalize the mass matrix.

Comment: this implies that a generation of up-type quarks can talk to a different generation of down-type quark through a $W^\pm$ boson in the basis where all of the masses are diagonal.

Hint: for all of these problems, you may find inspiration in the high-level discussion in Section 3.1 of `1711.03624`[2].

3 No neutrino oscillations

Based on this homework, comment on why the Standard Model with only the 'cuddly' matter particles does not predict neutrino oscillations.

4 CLASS survey (not for credit)

As a favor, I request that you fill out the CLASS survey on iLearn. In order not to bias participation, this survey will not affect your grade.

5 Complex Phases in the Yukawa Couplings (Extra Credit)

Complete Problem 2.1 in `1711.03624`. Specifically, for $N = 3$ generations, how many physical complex phases are in the Standard Model Yukawa matrix?

Comment: A complex phase in a parameter of the Standard Model implies a violation of matter–antimatter asymmetry. You may have a sense of this from noticing that particles and anti-particles are related by Hermitian conjugation. It turns out that the complex phase in the Standard Model is too small to explain the imbalance between matter and antimatter in our universe. This is an open question in the Standard Model. If you have a really good solution to this that is predictive, testable, and elegant, then you are well on your way towards a Nobel prize.

[2] https://arxiv.org/abs/1711.03624