<u>today</u>: HW PRESENTATIONS

<u>IDE</u>: Kuntal

<u>TODAY</u>: the RULES for MAKING THEORIES

$Q^{\alpha i a}_{1/6}$

$\bar{u}^{\alpha}_{-2/3\, a}$

$\bar{d}^{\alpha}_{+1/3\, a}$

$L^{\alpha i}_{-1/2}$

$\bar{e}^{\alpha}_{1}$

$H^{i}_{1/2}$

one important ADDENDUM: for SU(2), one more invariant tensor:

$$\varepsilon_{ij}\ ,\quad \varepsilon^{ij}$$

raises + lowers indices ... BUT <u>ANTISYMMETRIC</u>

$$H^i = \begin{pmatrix} h^u \\ h^d \end{pmatrix} \to H_i = \varepsilon_{ij} H^j = (-h^d, h^u)$$

<u>notice</u>: (particle)† (particle) is invariant ... except for spin

need $(\sigma^\mu)^{\dot\alpha}{}_{\alpha}$ to convert spin 1/2 indices into spin-1 index

... contracts naturally w/ <u>GAUGE BOSON</u>

so 

always works.

WHAT'S MISSING

1. Gauge boson self interactions

as if W has "W^3 charge" ... HM!

(etc)

index structure? [this is "extra"]

- FACT: YOU CAN USE DERIVATIVES TO CONTRACT μ INDEX. CAN ALSO USE METRIC, $\eta^{\mu\nu}$.
- fact: each symmetry has a set of invariant tensors called structure constants.

f^{ABC} ← totally antisymmetric $\qquad A = (1, \ldots, \#\text{ generators})$

for SU(2): $f^{ABC} = \epsilon^{ABC}$

- fact: these A's (index GENERATORS) ARE CALLED ADJOINT INDICES

the GENERATORS CONVERT ADJOINT INDICES INTO THE VECTOR INDICES:

$$(T^A)^i{}_j$$

$$\text{s.t. } (W)^i{}_j = (W^A \underbrace{\tfrac{1}{2}\sigma^A}_{(T^A)^i{}_j \text{ of } SU(2)})^i{}_j$$

... with these tensors you can form the GAUGE BOSON 3 & 4 POINT VERTICES.

extra material — confusing on 1st read

2. MASS

so far, mass is just a property of a particle.

turns out: you can only have a mass if you can have a "2 point vertex" at zero momentum

is $|\psi|^2$ invariant?

PARTICLE
eg $\psi^\dagger\psi$

analog: H.O. freq / energy set by potential.

ENERGY $\longleftrightarrow$ MASS

but... it looks like we cannot do that for fermions!

$\hookrightarrow$ spin index gets in the way!

could try $\psi^\alpha \psi^\beta \varepsilon_{\alpha\beta}$ ... but then other charges get in the way!!

GAUGE BOSONS ... seems like $V^\mu V^\nu \eta_{\mu\nu}$ is ok ... but not allowed for more subtle reasons.

$\hookrightarrow$ gauge invariance

[related to an observation:

V^μ has 4 components...

but EM waves only have 2 polarizations...]

Higgs can have a mass.
↳ ... turns out it's tachyonic!! :O

3. Higgs interactions

$|H|^2$, $|H|^4$ ⟶

more interesting: interactions w/ matter

$H^i_{1/2}$ ← doublet → $\underbrace{\text{needs another doublet}}$

hypercharged

Q or L

have spin... need another fermion

(BUT NOT DOUBLET)

start w/: $Q^{i\alpha}$

i index

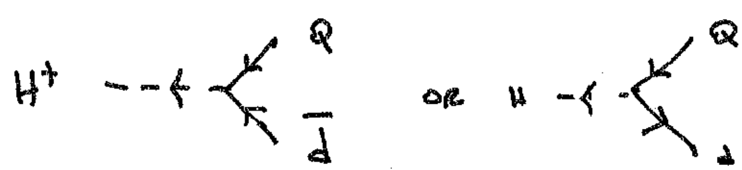
need singlet w/ α index: $\bar{u}^\alpha$ or $\bar{d}^\alpha$ or $\bar{e}^\alpha$

$(H^+)_i \quad Q^{i\alpha} \quad \bar{d}^\beta \varepsilon_{\alpha\beta}$ ... let HYPERCHARGE decide

$Y: -1/2 \qquad \underbrace{1/6 \qquad 1/3}_{1/2}$

sum to zero.

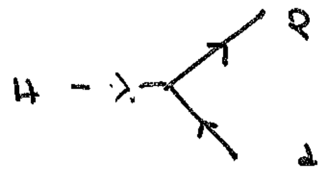
wouldn't have worked w/ $\bar{u}, \bar{e}$



<u>note:</u> if $H^\dagger Q \bar{d}$ is invariant, so is $(H^\dagger Q \bar{d})^\dagger$

$$\Rightarrow \quad H^i \underbrace{(Q^\dagger)_{i\dot\alpha} (\bar{d}^\dagger)_{\dot\beta}}_{} \, \varepsilon^{\dot\alpha\dot\beta}$$

Y: $\quad 1/2 \quad\quad -1/2$

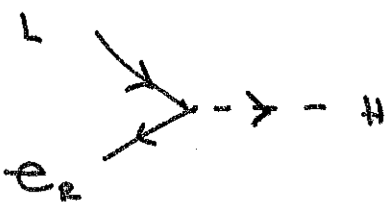


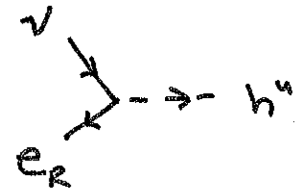
gives "anti-" version of Feynman rule.

<u>another contraction</u> :

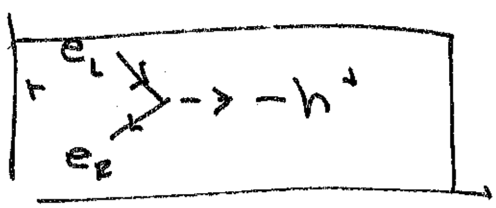
$$(H^\dagger)_i \, L^{i\alpha} \, \bar{e}^{\beta} \, \varepsilon_{\alpha\beta}$$

Y: $\quad -1/2 \quad -1/2 \quad +1$





to distinguish it from e in L



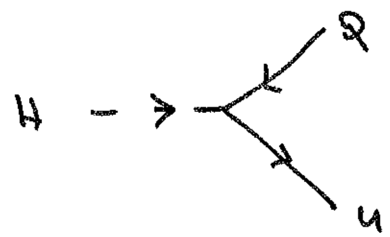
& also conjugate version w/ arrows flipped

<u>challenge</u>: there's one more!

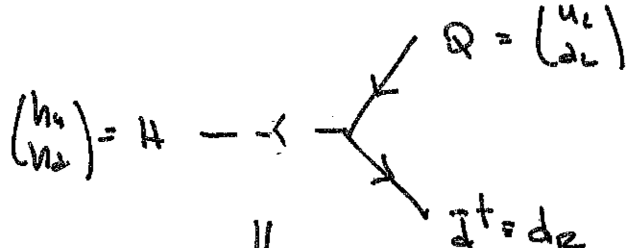
hint: need to use ε_{ij}

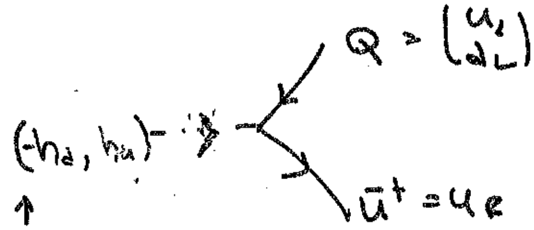
$$\varepsilon_{ij} H^{j} Q^{i\alpha} \bar{u}^{\beta} \varepsilon_{\alpha\beta} \quad + \text{h.c.}$$

$$Y: \quad \tfrac{1}{2} \quad \tfrac{1}{6} \quad -\tfrac{2}{3}$$

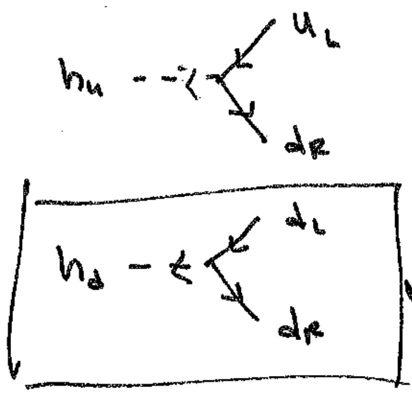


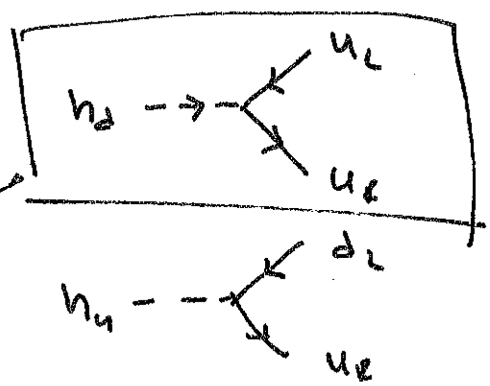
compare to "down-type" Yukawa:





SIGN NOT IMPORTANT FOR US YET





hint: h_d connects pairs...

BROKEN SYMMETRY

ferromagnet analogy.

A FIELD of spins

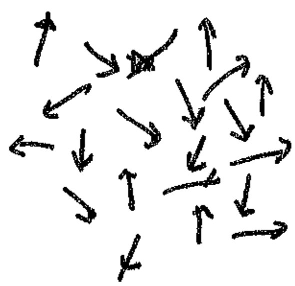
@ every position there is a spin

in this case, an ACTUAL spin.

theory doesn't prescribe any preferred orientation

↳ ROTATIONAL SYMMETRY (SO(3))

indeed, @ hi temp:



} no net spin

@ low temp: magnetic moment of nearby spins influence each other.

low ENERGY!

↳ "REALIZE" that they can minimize energy by aligning

overall alignment is random, but all spins align to that orientation.

GROUND STATE of thy has a preferred direction

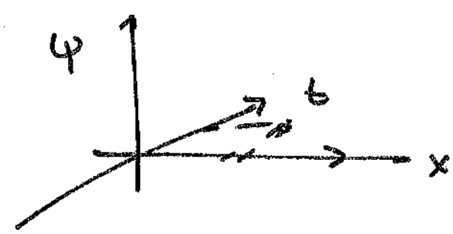
→ BREAKS SO(3) symmetry!!

ther: there is some direction $\underline{V}$ that physically means something.
↳ OBSERVATIONS DEPEND on $\underline{V}$.

PARTICLES ARE ALSO "FIELDS"
↳ quantum, rather than "ACTUAL"

@ each position in spacetime, there is something like a wavefunction $\psi(x)$ that expresses the "existence" of a ψ at that point.

↳ WIGGLES of this field are quantum particles.



ψ @ x,t ← $\psi(x^\mu)$
is A HARMONIC OSCILLATOR (QM)
excitations = PARTICLE @ x^μ.

the field is RELATED TO but not the particle.

usually, field is zero in the absence of excitations.

Higgs: it has a potential that is minimized for nonzero vacuum expectation value (vev)

ie: the minimum of the potential is $\underbrace{H(x)}_{\text{Higgs field}} \neq 0$ everywhere.

↳ not the same as having particles everywhere!

ANALOG: in swimming pool (on surface): there is water everywhere, but RIPPLES only when we "excite" the surface.

so if $H(x) =$ const ... but H is a doublet ... and hyperchanged ... then its vev breaks these symmetries:

$$\boxed{\langle H(x) \rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}}$$

... hmm ...