P165 MTG 3 14 Jan '20

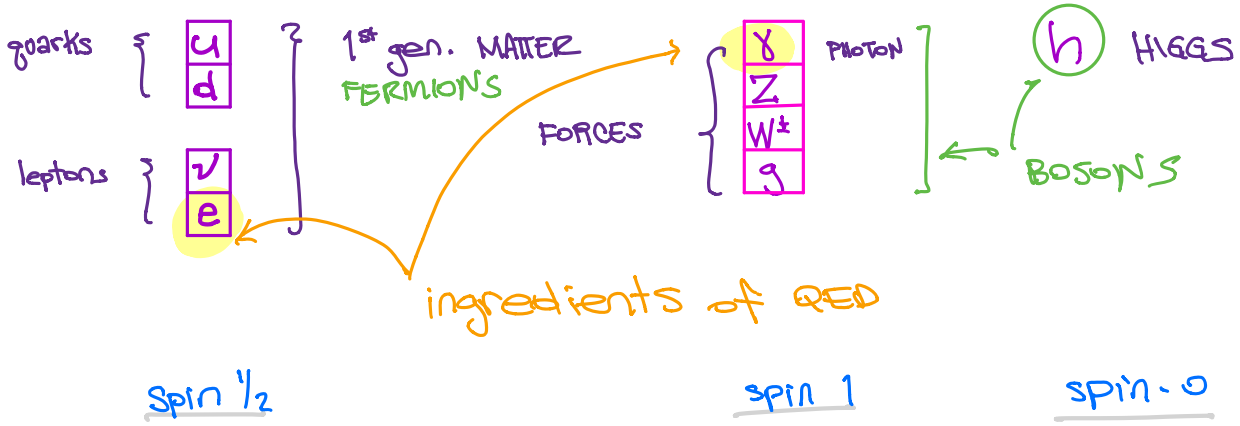
Learning: sometimes you have to try many "wrong" paths to get to a "right" one. OFTEN YOU NEED THE INSIGHTS OF THE WRONG PATHS – I WELCOME THEM IN THIS CLASS!

PRELIMS

① KINEMATICS + DYNAMICS
↑ 3 pillars? ↑ list of what 2 ingredients

② RULES for QED

③ PREVIEW of Standard Model
"CHEMISTRY" LESSON – LEARN THESE PARTICLES



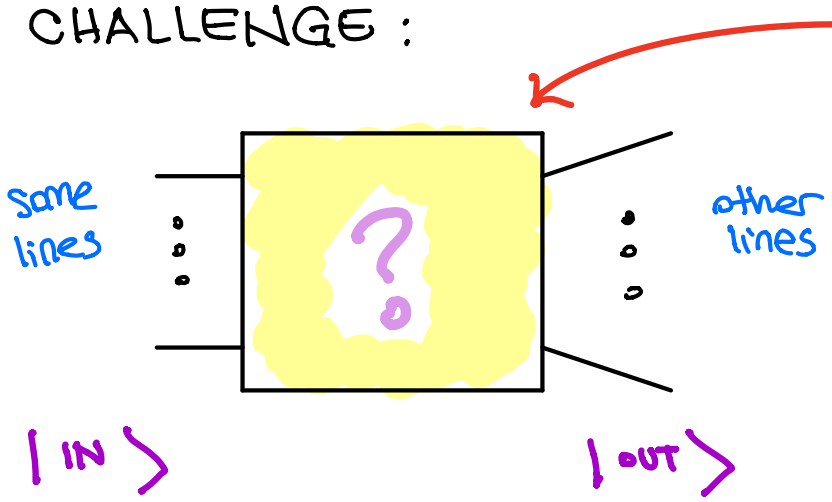
WHAT DID WE LEARN?

REMINDER – the GAME:

GIVEN: a theory (DYNAMICS) → PARTICLES (LINES); VERTICES (VERTICES (ways to connect))

you may use as many lines & vertices, but only these lines & vertices.

CHALLENGE:



FILL IN THE BOX TO CONNECT THE LINES!

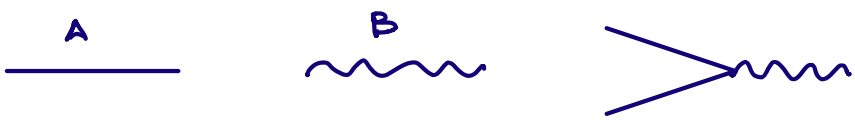
→ ONLY CONNECTED DIAGRAMS

→ SIMPLER IS BETTER
fewest vertices

IF IT IS POSSIBLE, then |IN⟩ → |OUT⟩ is AN ALLOWED DIAGRAM

otherwise: is there a reason why it is not allowed?

PROMPTS FROM LAST TIME:

1. THEORY B: 

CLAIM: A → 3A, 5A, 7A, ... ALLOWED

CLAIM: A ↛ 2A, 4A, 6A, ...

WHY? DOES # B IN IN/OUT STATE AFFECT THIS?

ANSWER:  PRESERVES "# of A mod 2"
(no matter what orientation)

2. QUANTUM ELECTRODYNAMICS

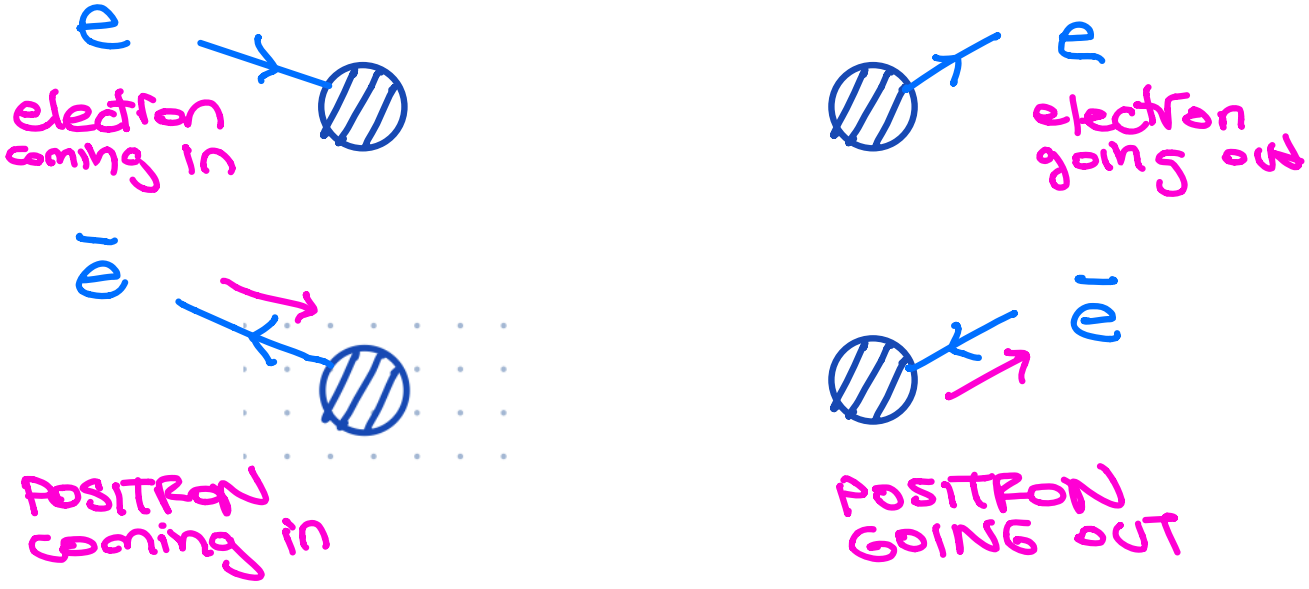
COURSE WEBSITE: FEYNMAN'S AUCKLAND LECS.

RULES:

think: conservation laws?

↑ REMARK: We're doing science "backwards"!
USUALLY OBSERVE PROCESS, THEN DEDUCE LAWS.

conventions for in/out states



WARM UP:

1. $ee \to ee$
2. $e\bar{e} \to e\bar{e}$
3. $e\gamma \to e\gamma$
4. $e\gamma \not\to \bar{e}\gamma$ ← why not?

} allowed if can

CHALLENGES ← note cards

1. $e\bar{e} \to \gamma\gamma\gamma$

   DRAW TWO DIAGRAMS

   AND: HOW MANY TOTAL DIAGRAMS "@ LOWEST ORDER" (SIMPLEST; SAME # of VERTICES)

2. $e \to e\bar{e}e$

   comment on what's "weird" about this.

   ↳ DIAGRAM IS FINE, BUT WHAT'S WRONG WITH THE PHYSICS?

EXTRA

3. $e\bar{e} \to$ 100 γ ?

4. $e\bar{e} \to$ 100 e ?

5. $e\bar{e} \to$ 100 e + 100 $\bar{e}$?

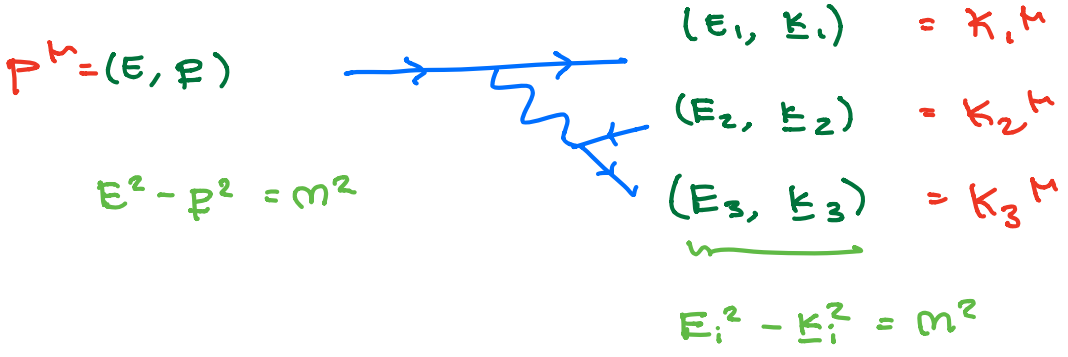
↳ WHAT'S THE CONSERVATION LAW?

DISCUSSION : KINEMATICS IDENTICAL PARTICLES

→ one last twist:

(new) A physical process obeys the rules of kinematics

eg. $e \to e\bar{e}e$



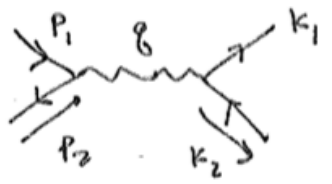
GO TO A NICE FRAME: $\underline{p} = 0 \to E = m$

eg

$0 = \underline{k}_1 + \underline{k}_2 + \underline{k}_3$

$m = E_1 + E_2 + E_3$ ← BUT $E_i > m$

CANNOT HAPPEN

eg 

$(p_1+p_2)^\mu = q^\mu$

$q^\mu = (k_1+k_2)^\mu$

$\Rightarrow (p_1+p_2)^2 = (k_1+k_2)^2$

↑ in any frame

eg in CM FRAME:

$p_1 = (E, 0, 0, p)$ $\quad E^2 = m^2 + p^2$

$p_2 = (E, 0, 0, -p)$

$k_1 = (E, p\hat{k})$ ← $\hat{k}$ is some unit 3-vector

$k_2 = (E, -p\hat{k})$ ELASTIC collision IN CM FRAME

long way: $k_1 = (E', \underline{k}_1)$

$k_2 = (E'', \underline{k}_2) = (E'', -\underline{k}_1)$

↑ CM FRAME: $\underline{k}_1 = -\underline{k}_2$

on shell: $(E')^2 = m^2 + |\underline{k}_1|^2$

$(E'')^2 = m^2 + (-\underline{k}_1)^2$

↑ $E' = E''$

total momentum conservation

$(p_1+p_2)^\mu = (2E, 0, 0, 0)$

$(k_1+k_2)^\mu = (2E', 0, 0, 0)$

$\Downarrow$

$E = E'$

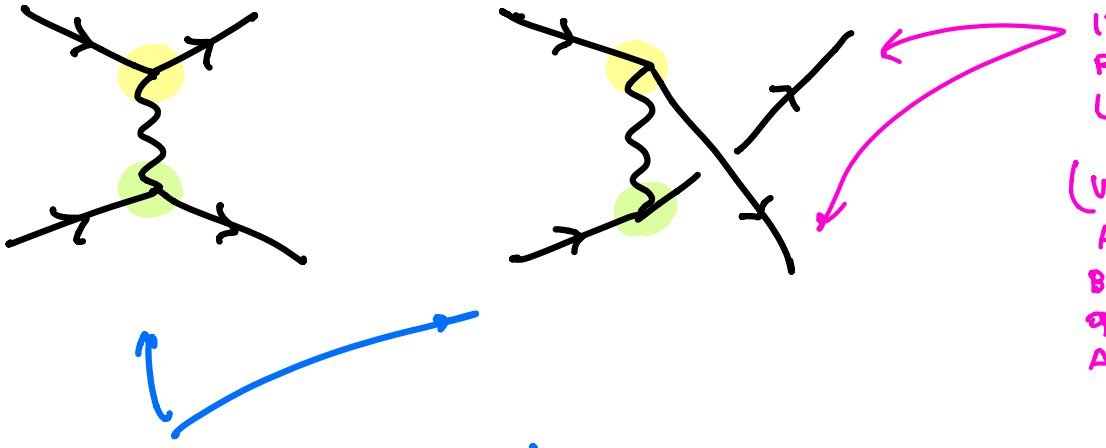
$|\underline{k}_1| = p$

$m^2 + p^2$ = $m^2 + |\underline{k}_1|^2$

TOPOLOGIES & IDENTICAL PARTICLES

(& quantum mechanics)

$ee \to ee$



IDENTICAL, BUT DIFFERENT LABELS

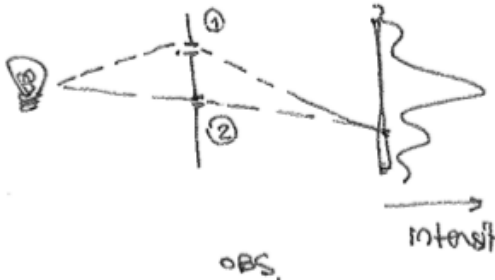
(WAVEFUNC IS ANTISYMMETRISED, BUT STILL A SENSE OF "THIS ONE" AND "THAT ONE")

two distinct histories

REMINDER : QM

- Schrödinger's cat
  - ① YOU OBSERVE $|\uparrow\rangle$ OR $|\overset{\times}{\downarrow}\overset{\times}{\downarrow}\rangle$
  - ② IN BETWEEN — not really classically defined in general — SUPERPOSITION of (possible states)

- DOUBLE SLIT EXPERIMENT



INTERFERENCE → SUPERPOSITION
↑ like sum of sine waves

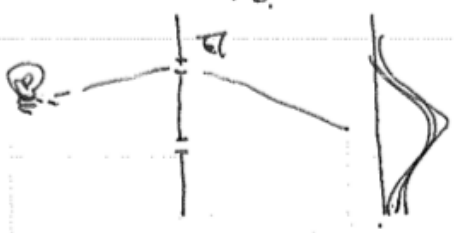
BEST WAY TO DESCRIBE THIS IS WITH COMPLEX NUMBERS

intensity

$a\psi_1 + b\psi_2$

PROBABILITY DISTRIBUTION:
$|a\psi_1 + b\psi_2|^2$
↑ has interference terms

OBS.



NO INTERFERENCE

↑ "observation collapses the wavefunction"

BRA-KET NOTATION

$$\langle \text{DET} | \Psi \rangle = \langle \text{DET} | \left[|①\rangle\langle①| + |②\rangle\langle②| \right] |\Psi\rangle$$

$\langle$OUT STATE | IN STATE$\rangle$

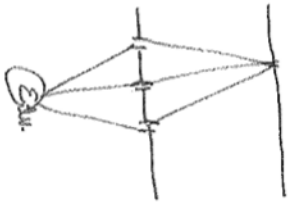
$= \mathbb{1}$

$$= \langle \text{DET} | ① \rangle \langle ① | \Psi \rangle + \langle \text{DET} | ② \rangle \langle ② | \Psi \rangle$$

ψ_1 ψ_2

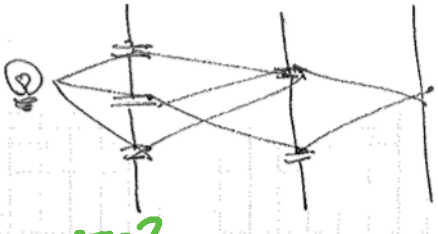
these are "scattering amplitudes"

if 3 slits?



sum over all 3 possible intermediate steps

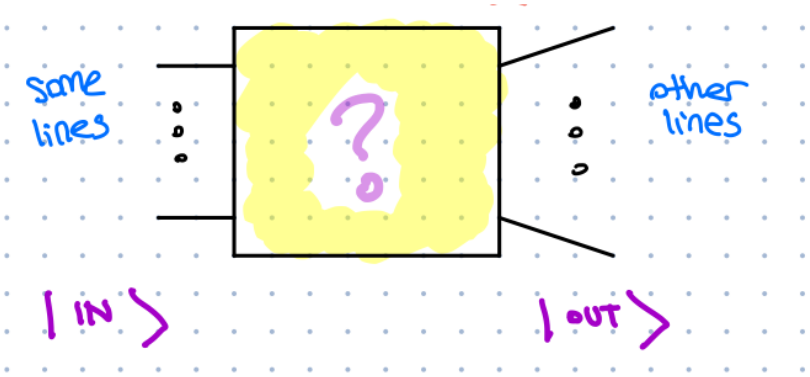
if many slits?



IF ∞ SLITS?

SUM OVER ALL POSSIBLE HISTORIES

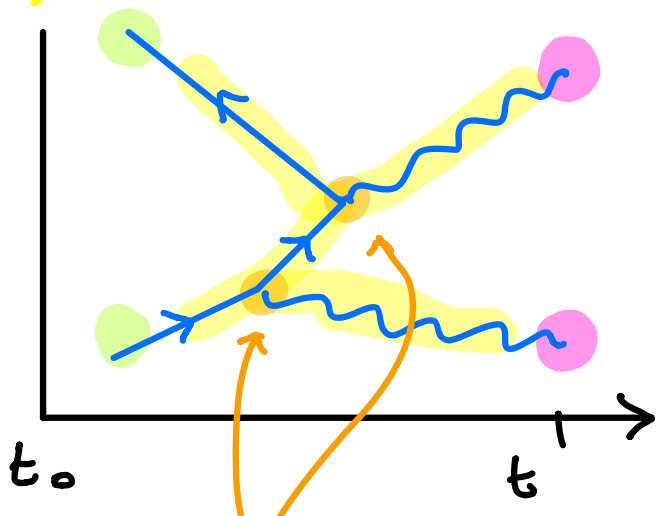
The Feynman Game



$\Leftrightarrow \langle \text{OUT} | \; ? \; | \text{IN} \rangle$

the diagrams encode OPERATORS that enact time evolution

$\langle \gamma\gamma \; | \; \; | \; e\bar{e} \rangle = \mathbb{C}$ number

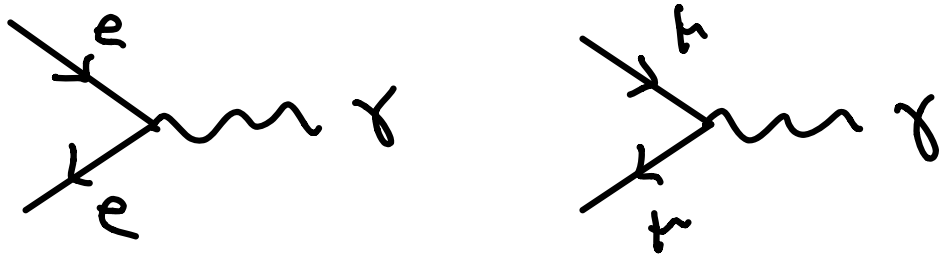


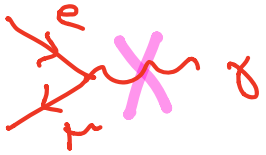
SUM OVER VERTEX POSITIONS IMPLIED
↳ like ∞ SLIT

A new theory: QED + μ

A NEW PARTICLE!

two types of "line w/ arrow"
HAVE TO LABEL THEM TO
KEEP DISTINCT



NOT A VERTEX:  no!

conservation laws?

$e\bar{\mu} \rightarrow \mu\bar{e}$?

$e\bar{\mu} \rightarrow \gamma\gamma$?

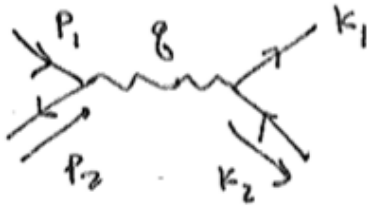
$e\bar{e} \rightarrow \mu\bar{\mu}$?

Real vs. Virtual

OBSERVED — "quantum" ← INTERNAL STATES DO NOT NEED TO SATISFY THE EINSTEIN RELATION!

in & out states obey LAWS of KINEMATICS

@ each vertex, 4-momentum is conserved

eg 

$(p_1+p_2)^\mu = q^\mu$

$q^\mu = (k_1+k_2)^\mu$

$\Rightarrow (p_1+p_2)^2 = (k_1+k_2)^2$ in any frame

eg: for a photon, $m_\gamma = 0$

BUT: $q^2 = (p_1+p_2)^2$

$= (E_1+E_2)^2 - (\vec{p}_1+\vec{p}_2)^2$ → 0 in cm frame

$\neq 0$ in general!