The SM:

GENERATION (DIFF MASSES)

QUARKS $\begin{cases} u & c & t \\ d & s & b \end{cases}$

LEPTONS $\begin{cases} \nu_e & \nu_\mu & \nu_\tau \\ e & \mu & \tau \end{cases}$

EA ROW HAS SAME PROPERTIES ... EXCEPT MASS ... & interaction w/ HIGGS

$\underbrace{\qquad\qquad}$ FERMIONS (MATTER) SPIN 1/2

ongoing mysteries (soon)

FORCE: "GAUGE BOSONS" SPIN-1

$A \quad Z \quad W^+ \quad W^-$ $\qquad$ g

$Y \qquad \underbrace{W^3 \quad W^1, W^2}$ $\qquad$ g

HYPERCHARGE	WEAK	STRONG
1D $\mathbb{C}$ ROT.	2D $\mathbb{C}$ ROT	3D $\mathbb{C}$ ROT

$\underbrace{\qquad\qquad\qquad}$ ELECTROWEAK : HYPER + WEAK is <u>BROKEN</u>.

COLOR "NATURE DOESN'T SEE COLOR" → true symm.

Key phrase: *electroweak symmetry breaking*

RULES: MATTER-FORCE INTERACTIONS: conserve <u>charge</u> ↩ may be like "RADIUS"

we'll write rules in <u>broken</u> phase ... (& see <u>why</u> later)

$$f^i, \ f^\dagger_j \ \text{—} \ V^i{}_j = V_a (T^a)^i{}_j$$

different kind of index

for photon: $T^i{}_j = e\, q_f\, \delta^i_j$

HW: r URS

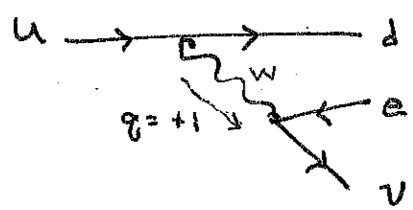
Phy. stackx 31514 ↑ factors of π

EXAMPLE: neutron lifetime estimate ← for DEMONSTRATION

useful: $\frac{1}{s} \approx 10^{-24}$ GeV

$\hbar = $ eV $\cdot$ m $= 10^{-15}$ GeV m $\times \underbrace{(3\times10^{8}\ m/s)}_{c}{}^{-1}$

$= \frac{1}{3}\ 10^{-23}$ GeV s

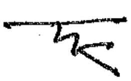


$\approx \dfrac{g^2}{M_W^2}$

g ← $f^i f^j$ W vertex $\sim g/2\sqrt{2}$ ← WEAK COUPLING ↑ silly

M_W^2 ← "PROPAGATOR" (cost of internal line @ low momentum)

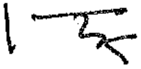
AS COUPLING ↑, AMPLITUDE ↑
AS M_W^2 ↑, AMPLITUDE ↓ (MORE OFF-SHELL)

PROB ~ AMP² ~ ||² $\sim 10^{-2}\dfrac{g^4}{M_W^4}$ ← $g^2 \sim 2/3$, $M_W \sim 100$ GeV

$\sim 10^{-11}\ \dfrac{1}{\text{GeV}^4}$

↑ DIMENSIONS WEIRD

to get a decay RATE, need (MASS)^5

$\Gamma \sim$ ||$^2 \times (\Delta m)^5$

$m_u - m_d - \underbrace{m_e - m_\nu}_{\approx 0} \approx 10^{-3}$ GeV

why this? when $m_u - m_d \to 0$, there is no phase space

PROCESS IS KINEMATICALLY NOT ALLOWED

$10^{-11} \times 10^{-15}$ GeV $= 10^{-26}$ GeV $= 10^{-2}/s$

$\Gamma \sim 1/\tau_n \Rightarrow \boxed{\tau_n \sim 100\ s}$

compare to τ lifetime.

INDEXOLOGY : unbroken electroweak theory

RULES : WEAK MATTER HAS $\rightarrow$ UPPER INDEX D^i

anti-MATTER HAS $\rightarrow$ LOWER INDEX $D^{\dagger}{}_j$

these will be modified as we go.

$\hookrightarrow$ analog of "OPPOSITE CHARGE"

$(\text{if } Uz = e^{i\theta} z \;;\; (Uz)^* = e^{-i\theta} z^*)$

$D^i = \begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} \nu \\ e \end{pmatrix}$

$D^{\dagger}_i = (\bar{u}^{\dagger}, \bar{d}^{\dagger}), (\nu^{\dagger}, e^{\dagger}) \quad \rightarrow (UD)^i = U^i{}_j D^j \;;\; (UD)^{\dagger}_i = (D^{\dagger})_j (U^{\dagger})^j{}_i$

WEAK GAUGE BOSON (force particle)

$\hookrightarrow$ one for each "axis of rotation"

SU(2) has 3 "axes" $\rightarrow W^1, W^2, W^3$

(SU(3) has 8)

for our purposes: each gauge boson has 1 up, 1 low index

$(W^1)^i{}_j, \; (W^2)^i{}_j, \; (W^3)^i{}_j$

there are strict rules for what these matrices are.

they are $\propto$ $\boxed{\text{PAULI MATRICES}}$

$$\text{fact: } W^i{}_j = \frac{1}{2}\begin{pmatrix} W^3 & \overbrace{W^1 - iW^2}^{\sqrt{2}W^+} \\ \underbrace{W^1 + iW^2}_{\equiv\sqrt{2}W^-} & -W^3 \end{pmatrix}$$

PACKAGE OF ALL 3 GAUGE BOSONS

3 fields

WHY? GAUGE BOSONS ~ GENERATORS $= \frac{1}{2}\begin{pmatrix} W^3 & \sqrt{2}W^+ \\ \sqrt{2}W^- & -W^3 \end{pmatrix}$

nb: $W^\pm$ is just a different way of packaging W^1 & W^2

→ it will <u>turn out</u> that this is the "correct" basis in the "broken phase" [when this thy → SM]

HOW DOES $W^i{}_j$ transform under $\underbrace{SU(2)}_{\text{"weak symmetry"}}$?

$$W^i{}_j \mapsto U^i{}_k W^k{}_\ell (U^\dagger)^\ell{}_j$$

where $U^i{}_k$ is a <u>unitary, 2x2 matrix</u> (preserves length in $\mathbb{C}$ 2x2 space)

$$[\; U = e^{i\theta^a (T^a)^i{}_j}$$

generators of SU(2) (same as Pauli)

HOW DOES MATTER TRANSFORM?

$$D^i \to U^i{}_j D^j \qquad (D^\dagger)_i \to D^\dagger{}_i (U^\dagger)^j{}_i$$

So here is an invariant:

$$\boxed{(D^\dagger)_i \; W^i{}_j \; D^j} \to D^\dagger \underbrace{U^\dagger (U}_{=\mathbb{1}} W \underbrace{U^\dagger) U}_{=\mathbb{1}} D$$

this combo respects SU(2) sym!

⇒ VALID FEYNMAN RULES

NOW DO THE MULTIPLICATION

$$(u^\dagger \ d^\dagger)\ \frac{1}{2}\begin{pmatrix} W^3 & \sqrt{2}W^+ \\ \sqrt{2}W^- & -W^3 \end{pmatrix}\begin{pmatrix} u \\ d \end{pmatrix}$$

$$= \frac{1}{2}\left[\left(u^\dagger W^3 + \sqrt{2} d^\dagger W^- ,\ \sqrt{2} u^\dagger W^- - d^\dagger W^3 \right) \begin{pmatrix} u \\ d \end{pmatrix} \right]$$

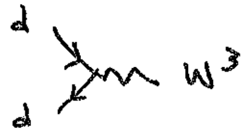
$$= \frac{1}{2}\left[u^\dagger W^3 u + \sqrt{2}\, d^\dagger W^- u + \sqrt{2}\, u^\dagger W^- d - d^\dagger W^3 d \right]$$

interesting: these have a $\sqrt{2}$

(RELATED TO FEYNMAN RULE HAVING A $1/\sqrt{2}$)

not interesting: overall phase ($-1 = e^{i\pi}$)

<u>curious</u>: what the hell is W^3 particle ?



comes w +1 comes w/ −1

RELATED TO CHARGE

$$\sim D^\dagger T^3 D \sim (u^\dagger, d^\dagger) \begin{pmatrix} 1/2 & \\ & -1/2 \end{pmatrix} \begin{pmatrix} u \\ d \end{pmatrix}$$

generator

(analog of $e^{iq\theta}$)

evidently: diff charges

... diff by one unit

... sounds like EM !

... but #'s are wrong

<u>the Higgs</u>: introduce a new <u>DOUBLET</u>

$$H = \begin{pmatrix} h_u \\ h_d \end{pmatrix}$$

$$H^\dagger = (h_u^\dagger, h_d^\dagger)$$

H is different from other doublets.

IN FACT: let's give them names:

$$Q = \begin{pmatrix} u \\ d \end{pmatrix} \qquad \text{quark doublet}$$

$$L = \begin{pmatrix} \nu \\ e \end{pmatrix} \qquad \text{lepton doublet}$$

<u>RULE</u>:

gives same rules as Q w/ $u \to h_u$, $d \to h_d$

there is something special about the Higgs: it <u>breaks</u> SU(2) symmetry.

↳ how? it has a <u>BACKGROUND value</u>.

$$\langle H \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

↳ a preferred direction in this $\mathbb{C}$ 2D space!

WEAK thy $\longrightarrow$ $\underbrace{\text{ELECTROWEAK}}$ thy

$$SU(2) \times U(1)$$

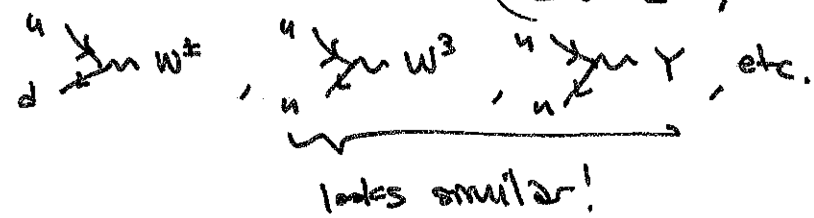
$SU(2)$	$U(1)$
WEAK (3 GAUGE BOSONS)	HYPERCHARGE ($\sim$ EM) (1 GAUGE BOSON)
$W^{1,2,3}$ $= W^{\pm}, W^3$	Y

<u>Matter</u>: format: $D^i_{(Y)} = \begin{pmatrix} u \\ d \end{pmatrix}_Y$

hypercharge

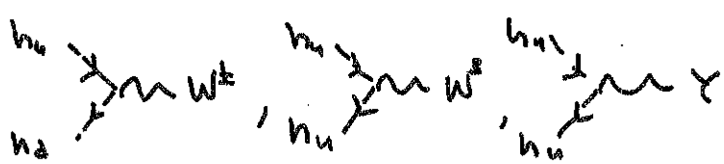
s.t. under hypercharge, $D^i_q \to e^{iq\theta} D^i_q = \begin{pmatrix} e^{iq\theta} u \\ e^{iq\theta} d \end{pmatrix}$

$Q_{1/6} = \begin{pmatrix} u \\ d \end{pmatrix}_{1/6}$



$L_{-1} = \begin{pmatrix} \nu \\ e \end{pmatrix}_{-1/2}$

$H = \begin{pmatrix} h_u \\ h_d \end{pmatrix}_{1/2}$



so far just adding additional type of charge to <u>WEAK</u> thy.

Now ADD <u>SINGLETS</u> ← invariant under SU(2) but not hypercharge

$$S \xrightarrow[\text{(scalar)}]{SU(2)} S \qquad S \xrightarrow{U(1)} e^{i y \theta} S$$

~~one separate particle for EACH SPIN 1/2 MATTER~~

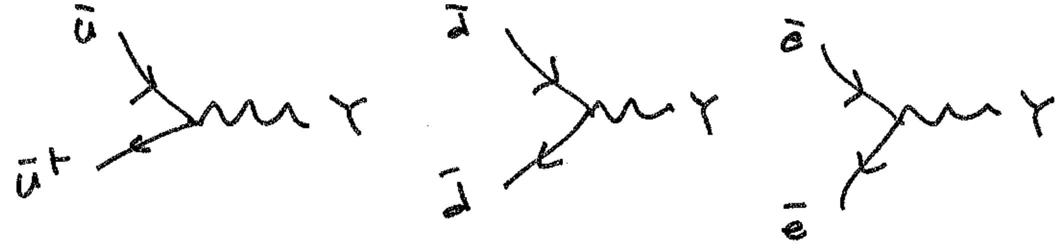
$\bar{u}_{-2/3}$ ⟵ nothing to do w/ Q (<u>yet</u>)

$\bar{d}_{1/3}$

$\bar{e}_{+1}$ ⟵ nothing to do w/ L (<u>yet</u>!)

<u>CURIOUS</u>: no $\bar{\nu}$?!

?



this is a weird mess that does not look like the SM!!

<u>MATTER</u>	<u>FORCES</u>	<u>HIGGS</u>
$Q^i_{1/6}$	$W^{\pm,3}$	$H^i_{1/2}$
$L^i_{-1/2}$	γ	
$\bar{u}_{-2/3}$		
$\bar{d}_{1/3}$		
$\bar{e}_{+1}$		

$\underbrace{\qquad}_{\text{anti}}$

Key so far: weird redundancy in matter
... nobody has "the RIGHT" charge
W^3 & Y instead of γ, Z
four HIGGSES
↑ 2 $\mathbb{C}$ numbers

WHAT ABOUT LORENTZ INVARIANCE?
↓
have to deal w/ SPIN in earnest

PREVIEW: Mo' indices!
LORENTZ × SU(2) × U(1)

scalar (SPIN-0) → no index

vector (SPIN-1) → μ ← hey, we know this
eg $W = (W^\mu)^i{}_j$

SPINOR → α LH SPINOR $\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$
SPINOR → $\dot{\beta}$ RH SPINOR $\begin{pmatrix} \bar{\chi}_1 \\ \bar{\chi}_2 \end{pmatrix}$

there are invariant tensors that convert indices.