Review

FEYNMAN GAME ↜ DYNAMICS

- start w/ a theory: PARTICLES (lines) & INTERACTIONS (vertices)

you are allowed to make graphs using these lines & vertices ONLY.

↳ *the vertices encode conservation laws in the theory.*

- GIVEN AN INITIAL STATE & FINAL STATE,

LIST OF PARTICLES @ BEGINNING & END OF A REACTION

(w/ ASSOCIATED ~~state data: MOMENTUM,~~ SPIN, etc.)

the PROCESS $|\text{INITIAL}\rangle \longrightarrow |\text{FINAL}\rangle$ IS DYNAMICALLY ALLOWED IF YOU CAN DRAW A GRAPH THAT CONNECTS THE $|\text{IN}\rangle$ & $|\text{OUT}\rangle$ LINES.

- GRAPH MUST BE CONNECTED

↳ disconnected graph means some particles aren't participating.

- YOU MUST FIND THE SIMPLEST (fewest vertices) GRAPH

↳ AND ALL GRAPHS OF THE SAME ORDER (same # of VERTICES).

Kinematics: the initial & final states are on-shell ← $p^2 = m^2$ for EACH

- 4-Momentum is conserved @ each vertex ⇒ total 4-momentum is conserved in any graph — this is something that only depends on the initial & final momenta.

INDEXOLOGY GAME

INDICES TELL US HOW QUANTITIES TRANSFORM UNDER SYMMETRIES.

↑ generalizations of rotations

nb: actually, this is the statement that we always use a convenient basis w/ well defined transformation properties

- GIVEN A SYMMETRY, there are two kinds of ROTATIONS: $\boxed{U^i{}_j}$ $\boxed{(U^\dagger)^i{}_j}$

1st index up
2nd index down

conjugate

for almost all sym. we care about, this really is H.C. (BOOSTS ARE SUBTLE)

- OBJECTS THAT TRANSFORM HAVE INDICES < UPPER / LOWER

FOR EACH UPPER INDEX, CONTRACT WITH THE U ROTATION

FOR EACH LOWER INDEX CONTRACT W/ THE $U^\dagger$ ROTATION

$$(☺)^{ik}{}_\ell \mapsto U^i{}_a U^k{}_b (U^\dagger)^c{}_\ell ☺^{ik}{}_\ell$$

this generalizes matrix mult.

confirm that a "normal" matrix $M \to U^\dagger M U$ using this convention.

what is the index structure?

GOOD!

→ OBJECTS W/ NO INDICES are invariant (SYMMETRIC)

→ CONTRACTED INDICES ARE NOT COUNTED

eg $A^i B_i \equiv \sum_i A^i B_i = (AB)$ ← no matrices

INDEXOLOGY, part II

Sometimes we have special tensors that are invariant that we get to use to convert indices.

↳ the deep-dive of this is

a) ADDITION OF ANGULAR MOMENTA IN QM
b) REPRESENTATION THEORY OF LIE GROUPS

↳ see P262 notes

eg. METRIC: g_{ij} : takes upper index, spits out lower.

$$g_{ij} V^j \equiv V_i$$

can think of this as linear maps

(it's all linear algebra)

or contracts two upper indices:

$$g_{ij} V^j W^i = W \cdot V \leftarrow \text{dot product}$$

↳ which is same as $(gW)_j V^j$

g^{ij} : inverse metric: $g^{ij} g_{jk} = \mathbb{1}^i{}_k$

↳ raises indices, etc.

there will be other special tensors. ← come w/ def of SYMMETRY

eg ε_{abc} : totally antisym. tensor

$$\underline{V} \times \underline{W} \equiv \varepsilon_{abc} V^b W^c \leftarrow \text{evidently a lower index!}$$

SPECIAL CASE: LORENTZ GROUP

$$p^\mu = (E, \vec{p}) \qquad p_\mu = (E, -\vec{p})$$

for simplicity, 1+1 DIM. (Generalization should be clear!)

"ROTATIONS" ARE BOOSTS: up to SIGN (convention)

$$\Lambda^\mu{}_\nu = \begin{pmatrix} \gamma & \gamma\beta \\ \gamma\beta & \gamma \end{pmatrix}$$

$$\bar{\Lambda}^\mu{}_\nu = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \leftarrow \text{inverse (like } U^\dagger)$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

CHECK: $\Lambda^\mu{}_\nu \bar{\Lambda}^\nu{}_\rho = \begin{pmatrix} \gamma^2(1-\beta^2) & 0 \\ 0 & \gamma^2(1-\beta^2) \end{pmatrix} = \begin{pmatrix} 1 & \\ & 1 \end{pmatrix}$

GIVEN A MOMENTUM p^μ, IT BOOSTS TO

$$p^\mu \to (\Lambda p)^\mu = \Lambda^\mu{}_\nu p^\nu$$
$$= (\gamma p^0 + \gamma\beta p^1,\ \gamma p^1 + \gamma\beta p^0)$$

SO: GIVEN $p^\mu = (p^0, p^1)$, what BOOST GOES to the REST FRAME? $\leftarrow (\Lambda p)^1 = 0$

$$\Rightarrow \gamma p^1 + \gamma\beta p^0 = 0 \quad \Rightarrow \gamma p^0\left(\frac{p^1}{p^0} + \beta\right) = 0$$
$$\Rightarrow \boxed{\beta = -p^1/p^0}$$

sign. what is this? (velocity!)

nb: $\Rightarrow \gamma^2 = \frac{1}{1-\beta^2} = \frac{(p^0)^2}{(p^0)^2 - (p^1)^2}$ p^2

$$\boxed{\gamma = E/m}$$

IF YOU HAVE 2 PARTICLES, p^μ & k^μ

↳ what BOOST to the CM FRAME?

total spatial momentum = 0

$$(\Lambda(p+k))^1 = 0$$

$$\| $$

$$\gamma(p^1+k^1) + \gamma\beta(p^0+k^0)$$

again: solve for β, γ

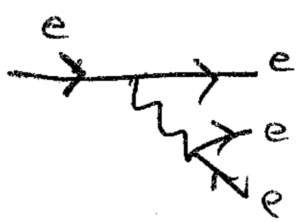
$$\boxed{\beta = -\frac{p^1+k^1}{p^0+k^0}} \qquad \gamma^2 = \frac{(p^0+k^0)^2}{(p+k)^2}$$

as LORENTZ VECTORS!

if something is kinematically not allowed in one frame, it is not allowed in any frame.

↳ some frames are easier!

ALTERNATIVELY: sometimes invariants help.



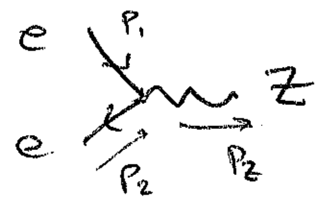
dynamics for $e^- \to e^- e^+ e^-$

not allowed kinematically.

in REST FRAME of initial state: $p^\mu = (m_e, \underline{0})$; final states: $k_1 = (E_1, \dots)$, $k_2 = (E_2, \dots)$, $k_3 = (E_3, \dots)$

but $E_i > m_e$ for EACH i!

PRODUCING A HEAVY PARTICLE:



in cm frame:

$$P_1 = (E, p') \qquad \text{s.t. } E^2 - (p')^2 = m_e^2$$
$$P_2 = (E, -p')$$

$$P_z = (2E, 0)$$

if $2E = M_Z$
then this is on shell
and allowed.

(must be <u>EXACTLY</u> M_Z)

what if we're not IN CM FRAME?

1) you can ~~boost~~ to cm FRAME
2) YOU CAN USE INVARIANTS.

cons of momentum: $P_1 + P_2 = P_Z$

on-shell-ness: $P_Z^2 = M_Z^2$

invariant, calc in any frame

$$\boxed{(P_1 + P_2)^2 = M_Z^2}$$

if this is satisfied, then
you can produce a Z
(it may be moving!)

HW.

for the <u>FUNDAMENTAL FORCES</u> (not counting gravity in this class)

WE CARE ABOUT "COMPACT LIE GROUPS"

↳ specifically: SU(N) & $\underset{\longleftrightarrow}{U(1)}$

rephasing

in a vertex

$$\xrightarrow{\text{symm}} e^{i\overbrace{(q_u - q_u + q_\gamma)}^{\equiv 0}\theta}$$

minus b/c anti-u going in

the net charge flowing <u>INTO</u> the vertex $= 0$

↳ nb: this is really invariance over $\mathbb{C}$ plane

<u>GENERATOR</u>:

$$e^{At} = \sum_{n=0}^{\infty} \frac{1}{n!} t^n A^n$$

matrix that GENERATES a transformation

eg: $\begin{pmatrix} & -1 \\ 1 & \end{pmatrix}$ generates 2D ROTATIONS

$$e^{\begin{pmatrix} & -1 \\ 1 & \end{pmatrix}\theta} = \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} + \begin{pmatrix} & -\theta \\ \theta & \end{pmatrix} + \frac{1}{2}\begin{pmatrix} -\theta^2 & \\ & -\theta^2 \end{pmatrix} + \dots$$

$$= \begin{pmatrix} 1 - \frac{1}{2}\theta^2 & -\theta \\ \theta & 1 - \frac{1}{2}\theta^2 \end{pmatrix} + \dots$$

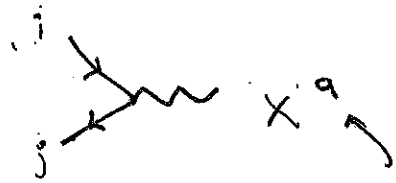
$$\longrightarrow \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$T^i{}_j$ HERMITIAN

$$U(\theta)^i{}_j = e^{iT\theta} \longleftarrow \exp\left[i \begin{pmatrix} & i \\ -i & \end{pmatrix} \theta\right]$$

WE WANT TO GENERALIZE <u>CHARGE CONS.</u> to <u>MATRICES</u>

invariance over $\mathbb{C}$ plane



a index

i, j INDICES

this is proportional to

$$(T^a)^i{}_j$$

<u>given</u> to us

there is some (unitary) transformation (A SYMMETRY) of this system where:

$$(T^a)^i{}_j \to (U_{Ad})^a{}_b \, U^i{}_k \, (U^\dagger)^l{}_j \, (T^b)^k{}_l$$

DIFF KIND OF ROTATION ASSOC. W/ SAME SYM!

FACT: RHS $= (T^a)^i{}_j$

it is invariant

CASE : SU(2) GAUGE symmetry ← force

doublet, $D^i = \begin{pmatrix} u \\ d \end{pmatrix}$

anti doublet : $(D^\dagger)_j = (u^\dagger, d^\dagger)$

h.c. : ANTIPARTICLE

"INVARIANTS (Generators)"

$$(T^1)^i{}_j = \frac{1}{2}\begin{pmatrix} & 1 \\ 1 & \end{pmatrix}$$

$$(T^2)^i{}_j = \frac{1}{2}\begin{pmatrix} & -i \\ i & \end{pmatrix}$$

$$(T^3)^i{}_j = \frac{1}{2}\begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$

$W^a \leftarrow W^1, W^2, W^3$ three GAUGE BOSONS
(3 PARTICLES) (force PARTICLES)

$$(W)^i{}_j = W^a (T^a)^i{}_j = \frac{1}{2}\begin{pmatrix} W^3 & W^1 - iW^2 \\ W^1 + iW^2 & -W^3 \end{pmatrix}$$

for a indices can sum uppers.

invariant: $(D^\dagger)_i W^a (T^a)^i{}_j D^j$ ← so this is allowed

WHAT THIS MEANS:

$$\frac{W^1 - iW^2}{\sqrt{2}}$$

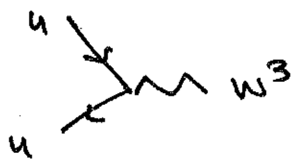
$$\frac{1}{2}\begin{pmatrix} u^+ & d^+ \end{pmatrix}\begin{pmatrix} W^3 & \sqrt{2}W^+ \\ \sqrt{2}W^- & -W^3 \end{pmatrix}\begin{pmatrix} u \\ d \end{pmatrix}$$

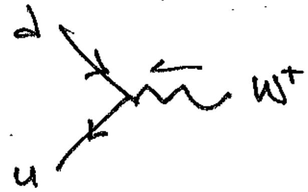
$$\frac{W^1 + iW^2}{\sqrt{2}}$$

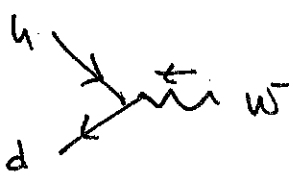
$$= \frac{1}{2} u^+ W^3 u + \frac{1}{\sqrt{2}} u^+ W^+ d$$
$$+ \frac{1}{\sqrt{2}} d^+ W^- u - \frac{1}{2} d^+ W^3 d$$

overall #'s don't matter for us

READ IT OFF: (all particles going in)







more to come!