Preliminaries

- Tyler → HW1a short. (nb: correction, Bryant)
- PERIODIC TABLE : 1^{st} gen matter
  4 fundamental particles

  EX: CHARGES

- RULES FOR QED ← *the game*
- QED + ?? → our laws?

· |IN>, |OUT>
· draw the box in between
· connected diagrams
· fewest vertices better

Last time :

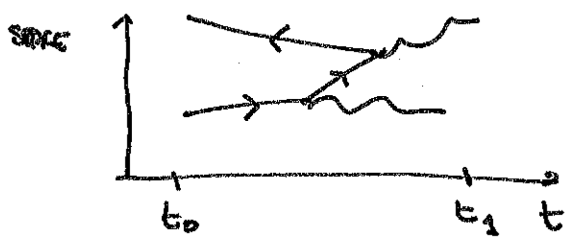
Dynamics & QM

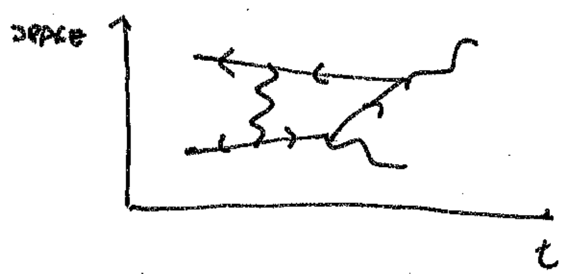
want amplitude

$$\langle \text{out} | \underbrace{\text{time evolution}}_{e^{iHt}} | \text{IN} \rangle$$

Taylor EXP

"evidently": this can be expanded as a sum over HISTORIES





More vertices are higher order terms in the expansion

Kinematics

$\to p^\mu$ conservation

$\to p^2 = m^2$

$$p^\mu = (E_p, p_x, p_y, p_z)$$

EACH IS SEPARATELY CONSERVED
in fact, if you know nothing about P41, then these are just 4 indep. quantities that a particle has.

$\to$ nb: ROTATING YOUR FRAME CAN MIX UP p_x, p_y, p_z !

BUT E ACTUALLY DEPENDS ON MOMENTUM!

$$\Rightarrow \boxed{E^2 - \vec{p}^2 = m^2}$$

encodes $E = mc^2$ ... but also kinetic E
(on your hw)

EXPECT: $E = m + \frac{1}{2}mv^2 + \cdots$

POWERFUL NOTATION (just like DIAGRAMS are powerful notation)

Use height of index as a code

$$p_\mu \equiv (E_p, \underbrace{-p_x, -p_y, -p_z}_{\text{just put minus sign on spatial components}})$$

WEIRD THING TO DEFINE, BUT VERY USEFUL.

A physical 4-momentum OF A PARTICLE satisfies

$$\underbrace{E^2 - \vec{p}^2}_{} = m^2 \quad \leftarrow \text{name: } \underline{\text{on-shell}}$$

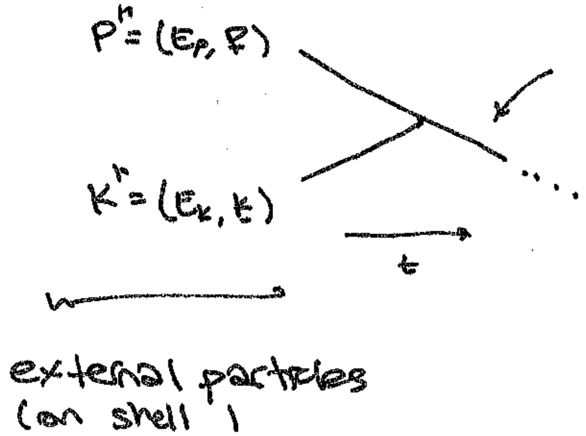
$$\sum_\mu p^\mu p_\mu \equiv p^\mu p_\mu \equiv p^2 \quad \leftarrow \text{notation}$$

"unphysical" 4-momentum: $p^2 \neq m^2$

↳ then E & $\underline{p}$ are not related
... this is "nonsensical" momentum, classically.

KINEMATICS: 4-momenta add as you'd expect

consider this part of a graph



this particle has 4-momentum
$q^\mu = (p+k)^\mu$
$= (E_p + E_k, \underline{p} + \underline{k})$

→ *in other words*, EACH VERTEX CONSERVES 4-MOMENTUM.

<u>Q</u>: if p & k are on-shell, is q on shell?

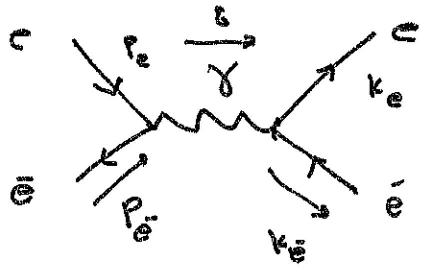
$E_p^2 - \underline{p}^2 = m^2$

$E_k^2 - \underline{k}^2 = m^2$

$(E_p + E_k)^2 - (\underline{p} + \underline{k})^2 \stackrel{?}{=} m^2$

(simplify: assumed all have same mass)

NO! ... that's weird.



$M_e = 0.5$ MeV

$M_\gamma = 0$

LET'S START W/ C of MASS FRAME :

$P_e = (E, \underline{p})$

$P_{\bar{e}} = (E, -\underline{p})$

on shell (physical)

then: $q = P_e + P_{\bar{e}} = (2E, 0)$

$q = K_e + K_{\bar{e}}$

on shell

$k = (E_e, \underline{k}_e)$ $\quad \bar{k} = (E_{\bar{e}}, \underline{k}_{\bar{e}})$

$E_e + E_{\bar{e}} = 2E$

$\boxed{\underline{K}_e + \underline{K}_{\bar{e}} = 0}$

$\underline{k}_e = -\underline{k}_{\bar{e}}$ (opp momenta)

$\Rightarrow E_e = E_{\bar{e}}$

$\Rightarrow E_e = E$ & $|\underline{k}| = |\underline{p}|$

so that: $K = (E, |\underline{p}|\,\hat{n})$

$\bar{K} = (E, -|\underline{p}|\,\hat{n})$

clearly total 4-momentum is conserved in a graph. ↳ each vertex conserves momentum lines dont change momentum.

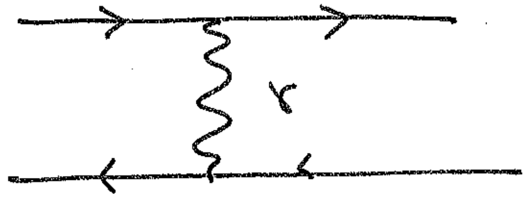
BUT REQUIRING on-shell external states

$\Rightarrow$ internal states have unphysical 4-momenta

$\Rightarrow$ <u>VIRTUAL PARTICLES</u>

INSIDE A DIAGRAM, 4-mom. conserved
but on-shellness is out the window
... that's fine! we never see those particles. they're "UNPHYSICAL PATHS" that we sum over.

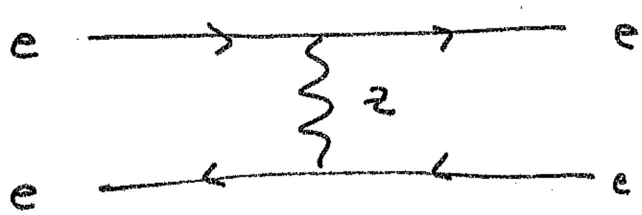
Poetic analogy:



$e\bar{e} \to e\bar{e}$

classically: Coulomb potential (eg, imagine state)

NEW THEORY: QED + Z



$M_Z = 90$ GeV
$\uparrow$
$\gg M_e$

is the Z or the γ closer to being on-shell?

$\hookrightarrow \Delta E \, \Delta t \gtrsim \hbar$
$\uparrow$
IF WE HAVE TO BORROW A LOT ENERGY FROM THE VACUUM, then WE HAVE TO PAY IT BACK SOONER.

so "VIRTUAL Z" CAN'T PROPAGATE AS LONG AS SLIGHTLY-VIRTUAL γ.

<u>incidentally</u>

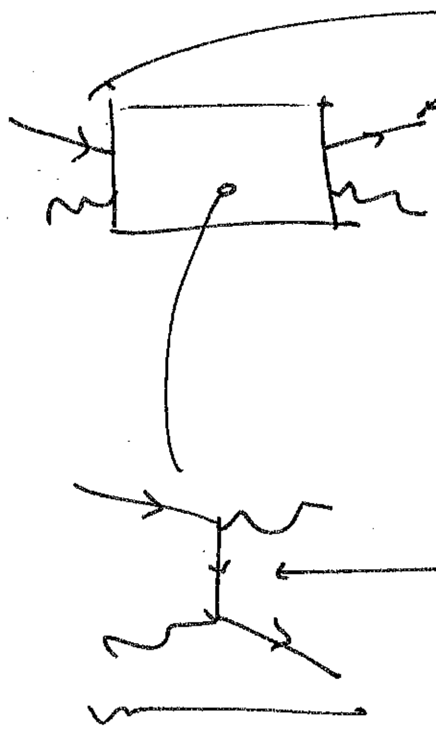
$$V_{\text{coulomb}} = \frac{e^2}{4\pi}\,\frac{-1}{r}$$

$$V_{\text{yukawa}} = \frac{g^2}{4\pi}\,\frac{-1}{r}\,e^{-Mr}$$

mass of

up to $O(1)$ factors
that I'm being sloppy about

<u>SD</u> :



in & out states
<u>OBSERVED</u>
↳ *physical*

momenta are
<u>ON-SHELL</u>

$(p^2 = m^2)$

INTERNAL STATES
ARE <u>NOT</u> OBSERVED

momenta are (in gen)
<u>OFF-SHELL</u>

$p^2 \neq m^2$

& momentum is
conserved

(like pipes, or current)

<u>New Rules for the GAME</u>: KINEMATICS

- in states & out states must conserve total 4-mom / be on-shell
- all intermediate vertices: 4-mom. conserved

INDICES

$p^\mu \longleftrightarrow$ why is this useful?

INDEX IS A CLUE ABOUT HOW IT TRANSFORMS.

simpler version: $v^i \leftarrow \begin{pmatrix} v^x \\ v^y \end{pmatrix}$ 2-vector

symmetry: 2D ROTATION $R = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

under a rotation: $\underline{v} \longrightarrow \underline{\underline{R}}\,\underline{v} = \underline{v}'$

INDEXOLOGY: (column) vectors: upper index

$$R^i{}_j = \begin{pmatrix} R^1{}_1 & R^1{}_2 \\ R^2{}_1 & R^2{}_2 \end{pmatrix}$$

SECOND INDEX LOWER

FIRST INDEX UPPER

SAME CONVENTION:

any repeated upper & lower index are summed ← "contracted"

so: $\underline{W} = \underline{\underline{R}}\,\underline{v} \Leftrightarrow W^i = \sum_j R^i{}_j v^j \equiv R^i{}_j v^j$

Watch carefully:

$$W^{1} = R^{1}{}_{1} V^{1} + R^{1}{}_{2} V^{2}$$

$$W^{2} = R^{2}{}_{1} V^{1} + R^{2}{}_{2} V^{2}$$

compare to "matrix mult":

$$\begin{pmatrix} W^1 \\ W^2 \end{pmatrix} = \begin{pmatrix} R^1{}_1 & R^1{}_2 \\ R^2{}_1 & R^2{}_2 \end{pmatrix} \begin{pmatrix} V^1 \\ V^2 \end{pmatrix}$$

$$= \begin{pmatrix} R^1{}_1 V^1 + R^1{}_2 V^2 \\ R^2{}_1 V^1 + R^2{}_2 V^2 \end{pmatrix}$$

so what? we can also define <u>ROW</u> vectors.

$$(s_1 \quad s_2) \begin{pmatrix} W^1 \\ W^2 \end{pmatrix} = s_1 W^1 + s_2 W^2 = \underbrace{\underline{S} \cdot \underline{W}}_{\text{INVARIANT.}}$$

convention: <u>LOWER</u> index

evidently, ROW VECTORS transform "OPPOSITELY" to COLUMN VECTORS!

related to this projection

so rot. mat.

Rule: lower index object transforms w/ $R^\dagger$ ← here $\dagger = T$

$\underline{\underline{R}}^\dagger = \underline{\underline{R}}^{-1}$ for ROTATIONS

$$= \begin{pmatrix} c & -s \\ s & c \end{pmatrix}$$

s.t. $R^\dagger R = R R^\dagger = \mathbb{1}$

$\underline{\underline{R}}^\dagger = (R^\dagger)^i{}_j =$

$$= \begin{pmatrix} \bar{R}^1{}_1 & \bar{R}^1{}_2 \\ \bar{R}^2{}_1 & \bar{R}^2{}_2 \end{pmatrix}$$ ← writing $\bar{R}$ for simplicity $\bar{R}^i{}_j = R^j{}_i$

just #'s!

$$s_i \longmapsto s'_i = s_j \bar{R}^j{}_i = \bar{R}^j{}_i s_j$$

$$w^i \longrightarrow w'^i = R^i{}_k w^k$$

$$s_i w^i \longrightarrow s'_i w'^i = \bar{R}^j{}_i R^i{}_k s_j w^k$$

$$= (\bar{R}R)^j{}_k s_j w^k$$

$\mathbb{1}^j{}_k = \delta^j{}_k$ $$= (\mathbb{1})^j{}_k s_j w^k$$

$$= s_k w^k$$ ✓

invariant.

Rule: objects w/ many indices transform as product of R & R^{-1} matrices, each acting on a different index

$$I^{ij} \rightarrow R^i{}_k R^j{}_l I^{kl}$$

eg moment of inertia

$$T^{ij}{}_k \rightarrow R^i{}_l R^j{}_m \underbrace{\bar{R}^n{}_k}_{= \underbrace{R^k{}_n}} T^{ij}{}_k$$

BUT NOW SUMMATION CONVENTION IS WEIRD.

can you ever raise/lower indices?
yes → use metric; in 3 space, metric is just the identity: δ_{ij}

why are we doing this?
LAWS of PHYSICS ARE WRITTEN W/RT. ROTATIONAL INVARIANTS

$$V \sim \frac{1}{r} \quad \text{not} \quad V(x,y,z)$$

NEXT WK: GENERALIZE TO SPACETIME: lorentz symmetry
then GENERALIZE TO INTERNAL SYM. (DYNAMICS!)