P208 F LEC 3: INTELLIGENT FALLING

17 Sen'17

BIG PIC: 1. GEOMETRY OF SPACETIME DEFINES
HOW STUFF FALLS

2. STUFF SOURCES CURVATURE (distorts geometry)

WE FOCUS (next few weeks) on ①, analogous to "test charge" analysis of electrostatics

THEN @, EINSTEIN EQ: analog of FULLY COVARIANT ELECTRODYNAMICS

1 recall: no issue of causality

THEN Q+Q, GRAVITATIONAL WAVES

Wis of GOUNALENCE PRINCIPLE
LOCALLY INERTIAL FRAMES
UGH FRAMES

CHRISTOFFELS

## THE EQUIVALENCE PRINCIPLE

EXPT: grav. accelerates all stuff proportionally to its inertial mass, no matter what the stuff is

2 ie: indep of additional quantum #'s

HYPOTHESIS: if everything falls the same in a grav. field, maybe gravity is really a modification to "falling"

## EQUIVALENCE

what gravity?

What gravity?

What gravity?

Cin space

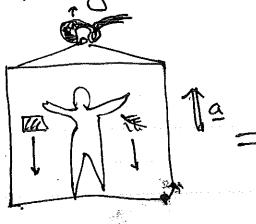
LOCAL EXPERIMENTS CANNOT DISTINGUISH

BETWEEN FREE FAIL "IN GRAV.

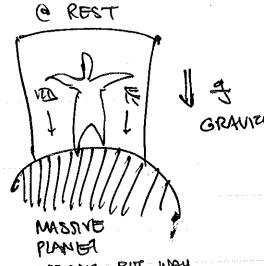
THOUS" (along spacetime geometry)

vs. in a nice, so inertial frame

if they had a window, they'd see how bad things are alternatively



SPACE ELEVATOR NO grav, but mechanical acc.



more impressible when it's both replied!

Jen blediction

SP: maybe gravity is just falling

AS PHYSICISTS, WE USE COORDINATE SYSTEMS

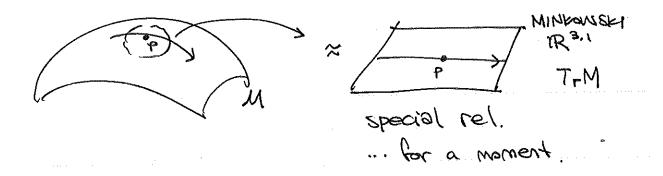
AS A CRUTCH -> WANT TO FIGURE OUT

HOW COORDINATE SYSTEMS "FALL" THROUGH

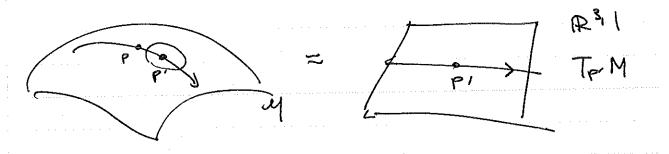
SPACETIME.

GEOMETRIC PICTURE
I HAVE ONE PICTURE THAT I WILL DRAW
OVER 1 OVER AGAIN
TeM = targent space @ P
M=SPREAME
(3) Soon) whatever that means (will figure
80 3
A "TEST PARTICLE" (test observer) WILL FREE FALL ALONG A PATH X(T)
THROUGH OPACETIME. trajectory
some goal of ours is to develop the
and where we can take
"curry geometry" and figure out what the free fall trajectory is
LOCALLY (in space i time — ie, near on event) THO TEST PARTICLE SEES TEM.
Tom is FLAT = MINKOWSKi

## in that sapathet little region around p



## SIMILARLY FOR NEIGHBORING POINTS

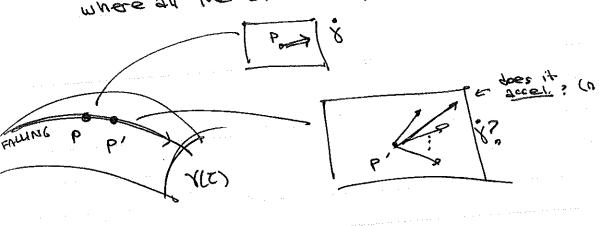


BUT THESE TANGENT PLANES ARE DIFFERENT EACH IS SEPARATELY ISOMORPHIC, BUT THEY ARE PATCHED TOGETHER BY THE GEOMETRY OF THE CURVED SPACE.

BY THE WAY "obvious" that tangent space is Minkowski, some way that you can use sheets of paper to engres a 8D object. VECTORS ( ) DUAL VEC ... i tensors live on tangent spaces generalizations; eg cotongent space for dual vectors) gle e Trum eg: GEOMETRICS g(t) TM 7 if (t) does not live on M lives in a different space, still. btu: in this sense "position rector" 15 a very weight (MRONG) notion! position is a point 9 (4) EM M FOR TON FOR TON DIFF OBJECTS:

ETRY IS ALL ABOUT PATCHING THESE TANGENT OPACES TO GETHER BY

where all the calculus appears to live



R GAAL IS TO DEVELOP MALHINERY OF THIS HPING, THEN USE IT TO ASSESS CURVATURES

S YOU CAN IMABINE, COORDINATES LAY A CENTRAL ROLE.

NEED A WAY TO DISTINGUISH n fact: "COORD WRYATURE" FROM ACTUAL CURVATURES

> werd metric \* curvature

yy)(jy)	
Wenb. 3-2	PRINCIPLE OF EQUIVALENCE @ WORK
	Eg near black hale, in expanding univ,
	Equiv. >> I freely falling coord system  s.t. observes "ATTACHEO"  To THIS SYS (co-falling)
	don't realize they're m
or TARDIS	p nb 1 think this is why the starship enterprise doesn't jerk around when
	It goes to warp prive:  CEOMETRY CHANGES, IT JUST KEEPS FAUNG
	LET Y & BE THESE ODORDINATES
	INSIDE TARDIS:  POSITION OF TEST PARTICLE  d <sup>2</sup> yd
	4 7 2
	PROPER TIME: time in the y system

- 1. When dx = 0, ie no spatial displacement,  $d\tau = dt$ ; so this is indeed the time meas. by obs @ rest in the frame.
- 2. dt2 is invariant... I CAN MEASURE
  IT IN ANY FRAME BY GX') GW')
  DOESN'T MATTER IF IN X' FRAME,
  THE TEST PARTICLE is MOVING.

So: in free folling frame, test particle observes

$$\frac{d^2y^2}{dt^2} = 0$$
The courteries courts

ie: ANOTHER COORD SYSTEM X THAT MAY BE CURVILLE AR?

	of Courses: no need to be in TARDIS"! LIKETIDAL LO CAN HAVE ACCEL FRAME ("Sees GRAN FORCE")
	SO X' COULD BE COORD OF SOMBOLE NOT ON ELEVATION
	HOW DO WE CHANGE COORDINATES?
	-> calculus
	, , ,
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	$Q = \frac{\partial^2 x^{\mu}}{\partial \mathcal{L}^2} + \frac{\partial x^{\rho}}{\partial y^{\alpha}} \frac{\partial^2 y^{\alpha}}{\partial x^{\mu} \partial x^{\nu}} \frac{\partial x^{\mu}}{\partial x^{\nu}} \frac{\partial x^{\nu}}{\partial x^{\nu}}$
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and the second s	quantities)
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WHAT ABOUT METRIC? IN Y' OFORDS,

- 482 = 122 = (44)2

TAKE INVARIANT I "do calculus" to change vors:

 $dT^{2} = \int V dy^{+} dy^{-} \qquad \qquad \leftarrow - 816N \text{ if EAST WAST}$   $= \int d\beta \left( \frac{\partial y^{-}}{\partial x^{+}} dx^{+} \right) \left( \frac{\partial y^{B}}{\partial x^{-}} dx^{-} \right) \left( \frac{\partial y^{B}}{\partial x^{-}} dx^{-} \right)$   $= \int d\beta \left( \frac{\partial y^{-}}{\partial x^{+}} dx^{-} \right) \left( \frac{\partial y^{B}}{\partial x^{-}} dx^{-} \right)$   $= \int d\beta \left( \frac{\partial y^{-}}{\partial x^{+}} dx^{-} \right) \left( \frac{\partial y^{B}}{\partial x^{-}} dx^{-} \right)$   $= \int d\beta \left( \frac{\partial y^{-}}{\partial x^{+}} dx^{-} \right) \left( \frac{\partial y^{B}}{\partial x^{-}} dx^{-} \right)$   $= \int d\beta \left( \frac{\partial y^{-}}{\partial x^{+}} dx^{-} \right) \left( \frac{\partial y^{B}}{\partial x^{-}} dx^{-} \right)$   $= \int d\beta \left( \frac{\partial y^{-}}{\partial x^{+}} dx^{-} \right) \left( \frac{\partial y^{B}}{\partial x^{-}} dx^{-} \right)$   $= \int d\beta \left( \frac{\partial y^{-}}{\partial x^{+}} dx^{-} \right) \left( \frac{\partial y^{B}}{\partial x^{-}} dx^{-} \right)$   $= \int d\beta \left( \frac{\partial y^{-}}{\partial x^{-}} dx^{-} \right) \left( \frac{\partial y^{B}}{\partial x^{-}} dx^{-} \right)$   $= \int d\beta \left( \frac{\partial y^{-}}{\partial x^{-}} dx^{-} \right) \left( \frac{\partial y^{B}}{\partial x^{-}} dx^{-} \right)$   $= \int d\beta \left( \frac{\partial y^{-}}{\partial x^{-}} dx^{-} \right) \left( \frac{\partial y^{B}}{\partial x^{-}} dx^{-} \right)$   $= \int d\beta \left( \frac{\partial y^{-}}{\partial x^{-}} dx^{-} \right) \left( \frac{\partial y^{B}}{\partial x^{-}} dx^{-} \right)$   $= \int d\beta \left( \frac{\partial y^{-}}{\partial x^{-}} dx^{-} \right) \left( \frac{\partial y^{B}}{\partial x^{-}} dx^{-} \right)$   $= \int d\beta \left( \frac{\partial y^{-}}{\partial x^{-}} dx^{-} \right) \left( \frac{\partial y^{B}}{\partial x^{-}} dx^{-} \right)$   $= \int d\beta \left( \frac{\partial y^{-}}{\partial x^{-}} dx^{-} \right) \left( \frac{\partial y^{B}}{\partial x^{-}} dx^{-} \right)$   $= \int d\beta \left( \frac{\partial y^{-}}{\partial x^{-}} dx^{-} \right) \left( \frac{\partial y^{B}}{\partial x^{-}} dx^{-} \right)$   $= \int d\beta \left( \frac{\partial y^{-}}{\partial x^{-}} dx^{-} \right) \left( \frac{\partial y^{B}}{\partial x^{-}} dx^{-} \right)$   $= \int d\beta \left( \frac{\partial y^{-}}{\partial x^{-}} dx^{-} \right) \left( \frac{\partial y^{B}}{\partial x^{-}} dx^{-} \right)$   $= \int d\beta \left( \frac{\partial y^{-}}{\partial x^{-}} dx^{-} \right) \left( \frac{\partial y^{B}}{\partial x^{-}} dx^{-} \right)$   $= \int d\beta \left( \frac{\partial y^{-}}{\partial x^{-}} dx^{-} \right) \left( \frac{\partial y^{B}}{\partial x^{-}} dx^{-} \right)$   $= \int d\beta \left( \frac{\partial y^{-}}{\partial x^{-}} dx^{-} \right) \left( \frac{\partial y^{B}}{\partial x^{-}} dx^{-} \right)$   $= \int d\beta \left( \frac{\partial y^{-}}{\partial x^{-}} dx^{-} \right) \left( \frac{\partial y^{B}}{\partial x^{-}} dx^{-} \right)$   $= \int d\beta \left( \frac{\partial y^{-}}{\partial x^{-}} dx^{-} \right)$   $= \int d\beta \left( \frac{\partial y^{-}}{\partial x^{-}} dx^{-} \right)$   $= \int d\beta \left( \frac{\partial y^{-}}{\partial x^{-}} dx^{-} \right)$   $= \int d\beta \left( \frac{\partial y^{-}}{\partial x^{-}} dx^{-} \right)$   $= \int d\beta \left( \frac{\partial y^{-}}{\partial x^{-}} dx^{-} \right)$   $= \int d\beta \left( \frac{\partial y^{-}}{\partial x^{-}} dx^{-} \right)$   $= \int d\beta \left( \frac{\partial y^{-}}{\partial x^{-}} dx^{-} \right)$   $= \int d\beta \left( \frac{\partial y^{-}}{\partial x^{-}} dx^{-} \right)$   $= \int d\beta \left( \frac{\partial y^{-}}{\partial x^{-}} dx^{-} \right)$ 

= Nab 3xx 3xx dx dx

= gru(x) netre in x coords

X-DOP IN GON!

SPACE STILL PLAT (inside TARDIS)

BUT GOORDS CURVED.

	What about PHOTONS?
	t photon was ectory
: - પુરુ કરા દુવારા દુવારા પ્રાપ્ત ક્લાના કરવા કરવા કરવા કરવા કરવા કરવા કરવા કરવ	4c2 = 0
	so dy/dz means nothing to a photon
	PATOLONS NEWS AGE
	CAN PICK DIFFERENT PARAMETUSE.
	A convoluent one is yo (time in y coords)
e . - Nasakananan karti Shiri Almanin yi Sir 1884 ili Papininyi yakani yi maninin kunin ani ani ani ani ani ani ani	" 2 the one w/ minus sign
	then: FREE FAIL OF PHOTOSI:
* Society of the September of the Septem	
arii kardissiin da kalkiilis kardis a kilkii kalkiikiin ariista kardissiin oo gissiin artii aa aan kandisaastad	$\frac{d^2 y^2}{d(y^2)^2}$
	dz2=0= MAB dy dy8
- mark halanda an kumin kala pilikakan 20 min halanda 1950 cika kan singin menghangkan kebangan dan kemunan ka	$d\sigma^2$
—— (1) (1) о оби 19 година од дв. 189 година обишт градина об об объект на години почетива обиштова од обиштов	
and the second s	$\Rightarrow \frac{d^2x^{\prime\prime}}{d\sigma^2} + \frac{1}{1} \frac{\nu}{\rho} \frac{dx^{\prime\prime}}{d\sigma} \frac{dx^{\prime\prime}}{d\sigma} = 0$
	> do2 'VP do do
i viran kasilasi ilikula ana karaka karaka kasila iliman kasilasi kasilasi ilin ka manamanan kara ana karaka	0 = 9m/(x) do do
а, <sub>т</sub> ана <sub>в</sub> е најумена и того в е чавојује изголога на правоса на ветом на веће војуме на <sup>ве</sup> в до јуме на ве	
er mengan sama ana na kisandi ni kalibi dikendi Kinamilan pelakun kisandi ni kalibi salah salah salah salah sa Salah salah sa	

PHOTON, CONTS: but  $\sigma = y^{\circ}$ ... WHAT IF I ONLY CARLE ABOUT X COORDS?

eg: in x more inates, how long does
IT TAKE A PHOTON TO TRAVEL A
DISTANCE dx?

0 = + gru de de de (- lar EAST COAST)

= 900 dt2 + 2giodxidt + gijdxidxj

1,3: SPATIAL INDICES

X': SPATIAL DIR, BUT NOT NEC CARTESIAN
SANITU CHECK: MINUS SIGNS?

fowe THIS FOR dt (t=x0)

dt = - giodxi - Ngio goo - gis goo) dxidxi

integrate this to get finite time it takes for light to traverse some diajectory

(M) => 9m (X(m) = N 4B b + b B 2 = 88 60 65

80: GRVEN TECONSTRUCT y to O((x-x(0))2)
"locally"

Q is APAIT. I (\*) HAS A REDUMBANCY:

If b" SOURS (\*),

SO DOES 19 6 b 8 m

if your is an mestial frame, so is  $X^{a}BY^{B}+C^{a}$ 

· WE HAVEN'T SAID ANYTHING AROUT INTEGRABILITY OF FIRE You AT ALL X.

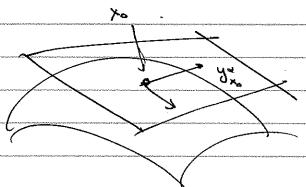
	80: somersion pathon g, e > x,
	"PREE FALLING" stationary" LIF, MINKOW. I FEEL GRAV.
	is encoded in (Ffu = gravity lives were? Asso coops curu)
	THESE ARE RELATED (st GRAVITY "LIVES" IN gru)
	Shr = 3x yx NxB Lu = 2x 3x
	3 Xb = 3xh9xb 3xh NaB
e u	WARUT!  S MARUT!  S MARUT!  S MARUTSIS  T DU A DUB  T
Like Indian	tensors) = The sx sx r r de
	+ trp axe axe NdB
	= [troger + troger]

WANT TO SOWE FOR T.

BUT FIRST: IMPORTANT ASIDE - we're taking spacetime derivatives of Sm

3m(x0) = 24x.(x) 2yx.(x) Nap /x=x0

( the Goods we set up & Xo



 $y_{x_0}(x) = a^{\circ} + b^{\circ}_{r}(x^{r} - x_{\circ}) + --$ 

SO WHEN WE DIFFERENTIATE

THERE ARE 2 KINDS OF "CHANGES IN 9"

- 1. change in orgument of functions

  you (x)
- 2. change in where we set up drop" locally

80 m 38m(xm), worry about

Section of the Party of the Par	
and the sale of the sale of	WETTPYING OUR =x 8 900 EXPRESSION REQUIRES
William or don't change	SOME INPUT;
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	UNUISTIRED ANSETZ: LOCALLY INERTIAL COORS 4.
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-	WILL SHARE PHYS. CHOSEN S.A. THE FIRST
de la companya de la	motive what. O DERIVATIVES OF THE
	clocks METRIC VANISH Q X.
- Contraction	ASSOCIATED S
An American Series	meaning: x'=x+sx.
- Company	Jy Syx, DyB,
The state of the s	Sm(X?) = DXL DXL (Nb /x=X?
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Andrew particular despensable	24x 248 1/00/11/
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Villean operation of the same	Uxo PLAT HERE
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-	, '
-	EXCEND THIS GOOD WERE SYSTEM TO XS', AWAY

FROM WHERE IT IS FURT

PECAL: WANT TO "SOWE" DQ = Tg + Tg

BUT CONTRACTIONS

MRE HARD

From P- 16 quick
(JUSTLANING OUR RESULT)

1
6 oost of UNINGIFIED ANSAGE

a = Ledon + Le don

 $\frac{3 \times 6}{3000} = \sqrt{88} \frac{3 \times 6}{3} \left[ \frac{3 \times 6}{3000} \frac{3 \times 6}{3000} \right]$ 

91B

DROP

 $= \frac{3(x^{\circ})_{6}}{3} \left[ \sqrt{\frac{3x}{9}} \frac{3x}{9} \frac{3x}{3} \frac{3x}{3} \right] = \frac{3(x^{\circ})_{6}}{3} \left[ \frac{3x}{9} \frac{3x}{3} \frac{3x}{3} \frac{3x}{3} \right] = \frac{3x}{9}$ 

Olivina de va	
	BACK TO TARK: + (8)
	playing w/ Indices
	3292 7 = SKUT* + 9KHT*
	- 8 v 9 + > J - 9 K > T = 9 K F T = 3 K F T =
	2n = 2gkv + 3m
	Using symm. With these
	HIT WITH INVERSE METRIC: 3 200
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