LEC 6: CUPVATURE ... 7 DIGRESSIONS

26 PEB

- · PARAUEL TRANSPORT
- · Reorv, finally... but briefly
- · GEOMETRIC INTERMOET: INTHINSIE VS extonois
- · GEOMETRIC INTERMISE Z: lie derivative

GEODESIC MOTION
$$\frac{d^2x^m}{dz^2} + \int_{\rho\sigma}^{\mu} \frac{dx^{\rho}}{dz} \frac{dx^{\sigma}}{dz} = 0$$

- Geodesic: path in spacetime of maximal proper length, s
- PARAUEL TRANSPORTS ITS VELOCITY VECTOR

Carroll 3.3

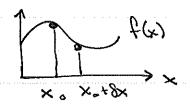
Parallel Transport

Why this is important:

HOW DO WE COMPARE VECTORS (tensors)

AT DIFFERENT POSITIONS?

important question - generalizes derivative m calculus

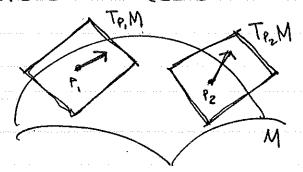


compare 2 numbers

f(xo) 1 f(xo+8x)

as 8x =0.

BUT HOW DO YOU COMPARE <u>VECTORS</u> will different bases?



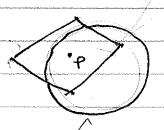
HOW DO THE COORDINATES OF TP, M COMPARED TO TP2 M? HOW DO WE KNOW IF THE VECTOR @ P, is THE SAME AS THE ONE @ P2?

Simple answer: just take the vector, AND WALK IT OVER! flat space: "let me just and keep and keep COOKED WHE OR PATH angle btun vector and my path" N BUT ON 82: transport from AN VS. ABN QNES DIFFERENT VECTORS @ N

80: PARAUEL TRANSPORT & COMPARE" E- MONTOCA! is a way to blackose curvature

COMPARE TO EQUIVALENCE PRINCIPLE

T.M - R311



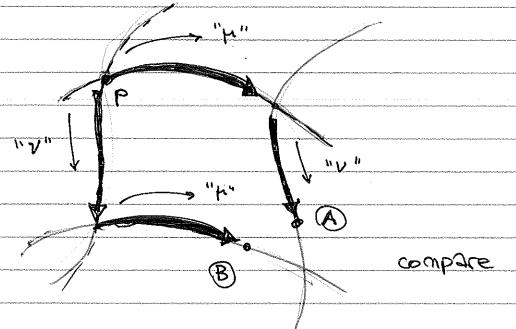
rochry Winkowsky

By the way: this strategy of COMPARING
VECTORS after // transport
is BASIS-INDEPENDENT.

recall: how do / Know if space is curved, or if i'm just using curvilinear coordinates?

"funny metric" is not a robust diagnosis for physical curvature

HERE'S THE STRATERY: flow infinitesimally along different geodesics



So TAKE A VECTOR V @ xot 1 PUSH IT IN at Provided for (xo+a)

THEN PUSH IT IN AMOTHER DIRECTION (+b")

a,b + a" Dr VP

-> VP + b" Dr VP + a"b" Dr Dr VP

IF WE DID THIS IN THE OTHER ORDER, WE'D HAVE GOTTEN

VP big VP + ard, VP + arb DrDv VP + br Dv VP

THE DIFFERENCE OF PUSHING 9,6 VS. b, a
18:

about [Dr. Dv] VP

T DrbvVP-DrDrVP

NOW USE DaVB = DaVB + TBVX

IN ORDER TO DO THIS, NEED ONE GENERALIZATION:

you are "fixing" each transformation independently

TYB -> (2x') / (2x) o Tro

BAB TRANSF COMES FROM
DERIVATIVE HITING THESE TRANSF MATRICES
BUT DERIVATIVE HITS THEM "ONE OF A TIME"
BY LEIBNIZ RULE

0+7/(\$)6)\$ + T\$ [(\$)6) = 'T6

SO WE CORPORT THEM ONE Q A TIME!

and a cook now report a policy page to the least of the conference and	USING THIS:
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and around restrictly model of memory should	

WE CALL THIS THE RIEMANN TENSOR it's actually a tensor

[Dr. Du]VP = RPomuVe (- Thu D, VP)

Torrand: Terry

can be set to a

- · PART OF CDr. Dull P
- · observe R is ontisym in MAN V

on more general tensor:

[DH. D.]QdB=R@xHVQB-RBHVQdx

zee 17 / Geometric Interlude

I. EXTRINGIC US INTRINGIC CURVATURE

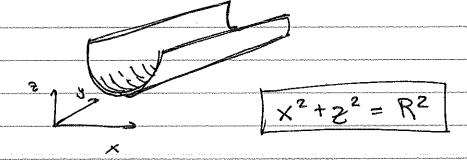
C SPACETIME

EMBEDDINGS: When you think

of cheveo space as A

SUBSPACE IN A FLAT, LARBERZ SPACE (SUFFECE)

HEURISTIC EXAMPLE



CURVATURE HAS SOMETHING TO DO WITH SECOND DERNATIVES; es near x=0

HOSSIAN

HESSPAN HAS 2 INVARIANTS det H = 0 = intrinsic curvature tr H = - YR = extrinsic curvature this is a flat piece of paper

just curted up in 300 pm.

LET'S GO TO CYLINDRICAL COORDINATES

$$x' = \emptyset$$

$$x^2 = Z$$

$$z = Z$$

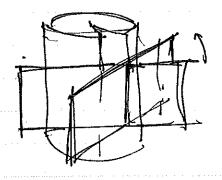
THE CYLINDER IS EMBEDDED IN IR3 (flat) WHICH HAS A NATURAL AMBIGNT BASIS



THE CYLINDER IS GIVEN BY POINTS ER3 S.A. $X = \begin{pmatrix} R \cos \varphi \\ R \sin \varphi \end{pmatrix}$ $\forall \psi, \Xi$

BASIS OF THE TANGENT VECTORS & A GIVEN POINT ARE e; = 3:X = 2x/2xi $\begin{array}{ccc}
- R \sin \varphi \\
R \cos \varphi \\
0
\end{array}
\qquad
\begin{array}{c}
e_{2} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1
\end{array}$ DEPENDS ON 4 BIC TANGENT PLANES ROTATE AS YOU MOVE ALONG CYLINDER! DISTANCE BYON NEARBY POPUTS: dX = 2: X dx' porting R3 in mores 985 = 9x.9x = (9:X.92X) 9x, 9x, = e: .e: dx dx [recoll: gm = (2-yd) (2-yd)) HOW DO C: CHANGE AS WE MOVE?

2; e; = 2; 2; X



CHANGE 17 UMICED. to tangent DIRECTIONS

2, e; = Ti, e, + Ksin FUL BASIS FOR R3

AFFINE ONNECTION tells us about how tongent plane pass @1 point changes as you go to neighboring point; but projected along TPM

complying to to WI CUPUTAURE from embredon

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	tangent space
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$v_{2}(m_{2},m_{2}) \leq m_{2}(m_{2},m_{2}) + m_{2}(m$	V'(x)
	X(z) SOME TRAJECTORY
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a yang penagan ayan ngapaga an ayan _{kan} ang an ayan yan ayan ayan ayan ayan ayan	W \(\(\times \) = W \(\times \) + d\(\times \) \\

THEN DEFINE

= W'(x) 3x' = W+WyyV(xL)

THEN DEFINE

"change of coords"

LIE: DOMPARE W'(X') - W'(X')

e same positions

= 9c (N,D^M - M,D^N/L)

sous onest sous des suce "pad"

= dt [V, W] or dt ZvW

C vooks vike soo in ribumin tenser per!

2) EMPHASIZES THE UTILITY OF THE PICTURE THAT VECTORS ARE DIFFERENTIAL OPERATIONS, $V = V^M \partial_V$.

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militaria mara a amin'ima i massama saa i kamiliano i inaa ilibah T	· ALSO IMPORTANT WHEN DEPINING GOODDINATE GRIDS.
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	GIVE INTEGRAL OURNES. THESE CAN BE CAPORDINATEDS
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