TANEDO 24 Jan 2020

Polchinski : Renormalization & Effective $\mathcal{L}$ (1983)

toy example in §2

let $t = \ln\Lambda$ s.t. $\Lambda \frac{d}{d\Lambda} = \frac{d}{dt}$ ← .

GIVEN 4-point & 6-point couplings :

$$\dot{g}_4 = \beta_4(\underbrace{g_4}_{\lambda_4}, \underbrace{\Lambda^2 g_6}_{\lambda_6}) \qquad \dot{g}_6 = \Lambda^{-2}\beta_6(g_4, \Lambda^2 g_6)$$

scale in the theory.

THE DIMLESS FLOW EQS ARE :

$$\boxed{\begin{aligned}\dot{\lambda}_4 &= \beta_4(\lambda_4, \lambda_6)\\ \dot{\lambda}_6 &= 2\lambda_6 + \beta_6(\lambda_4,\lambda_6)\end{aligned}}$$

flow only depends on couplings, not $t = \ln\Lambda$.

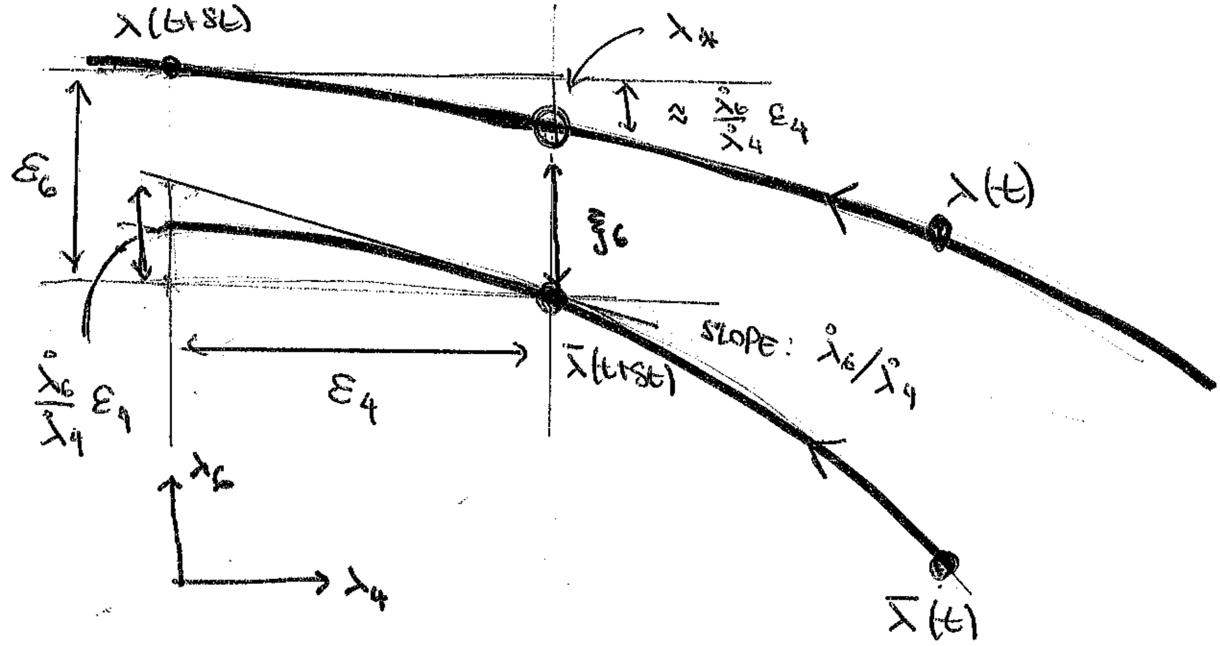
consider deviation btwn nearby trajectories

$\varepsilon_i \equiv \lambda_i - \bar{\lambda}_i$

these deviations obey a flow equation

$$\boxed{\begin{aligned}\dot{\varepsilon}_4 &= \partial_4\beta_4\varepsilon_4 + \partial_6\beta_4\varepsilon_6\\ \dot{\varepsilon}_6 &= 2\varepsilon_6 + \partial_4\beta_6\varepsilon_4 + \partial_6\beta_6\varepsilon_6\end{aligned}}$$

← $\partial_i = \frac{\partial}{\partial\lambda_i}$



two theories with $\bar{\lambda}_4(t) = \lambda_4(t)$ flow differently in the IR.

the λ trajectory may, for example, flow faster in the λ_4 direction; thus $\lambda(t+\delta t)$ may appear quite far from $\bar{\lambda}(t+\delta t)$.

the key point is that $\exists$ a point λ_* on the $\lambda(t)$ trajectory that <u>IS</u> close to $\bar{\lambda}$.

We show this by arguing that the separation in the irrelevant direction shrinks w/ RG flow.

$$\xi_6 \equiv \varepsilon_6 - \frac{\mathring{\lambda}_6}{\mathring{\lambda}_4}\varepsilon_4$$

$$\mathring{\xi}_6 = \mathring{\varepsilon}_6 - \frac{d}{dt}\left[\frac{\dot\lambda_6}{\dot\lambda_4}\varepsilon_4\right]$$

$$= 2\varepsilon_6 + \partial_4\beta_6\varepsilon_4 + \partial_6\beta_6\varepsilon_6$$

$$- \frac{\ddot\lambda_6}{\dot\lambda_4}\varepsilon_4 + \underbrace{\frac{\dot\lambda_6}{\dot\lambda_4}\frac{\ddot\lambda_4}{\dot\lambda_4}}\varepsilon_4 - \frac{\dot\lambda_6}{\dot\lambda_4}\mathring{\varepsilon}_4$$

$$\frac{\ddot\lambda_4}{\dot\lambda_4} = \frac{d\ln\beta_4}{dt} \qquad \mathring{\varepsilon}_4 = \partial_4\beta_4\varepsilon_4 + \partial_6\beta_6\varepsilon_6$$

$$\mathring{\beta}_6 = \partial_4\beta_6\dot\lambda_4 + \partial_6\beta_6\dot\lambda_6$$

$$\dot\lambda_6 = 2\lambda_6 + \beta_6$$

$$\boxed{-\frac{\varepsilon_4}{\dot\lambda_4}\ddot\lambda_6 = -2\frac{\dot\lambda_6}{\dot\lambda_4}\varepsilon_4 - \partial_4\beta_6\varepsilon_4 - \frac{\dot\lambda_6}{\dot\lambda_4}\partial_6\beta_6\varepsilon_4}$$

$$\mathring{\xi}_6 = \underbrace{2\varepsilon_6}_{2\xi_6} + \cancel{\partial_4\beta_6\varepsilon_4} + \underbrace{\partial_6\beta_6\varepsilon_6}_{\partial_6\beta_6\xi_6}$$

$$-2\frac{\dot\lambda_6}{\dot\lambda_4}\varepsilon_4 - \cancel{\partial_4\beta_6\varepsilon_4} - \frac{\dot\lambda_6}{\dot\lambda_4}\partial_6\beta_6\varepsilon_4$$

$$+ \frac{\dot\lambda_6}{\dot\lambda_4}\frac{d\ln\beta_4}{dt}\varepsilon_4 - \frac{\dot\lambda_6}{\dot\lambda_4}\partial_4\beta_4\varepsilon_4 - \frac{\dot\lambda_6}{\dot\lambda_4}\partial_6\beta_6\dot\lambda_6$$

$$\mathring{\xi}_6 - 2\xi_6 = \partial_6\beta_6\xi_6 + \frac{\dot\lambda_6}{\dot\lambda_4}\frac{d\ln\beta_4}{dt}\varepsilon_4 - \frac{\dot\lambda_6}{\dot\lambda_4}\partial_4\beta_4\varepsilon_4$$

$$- \frac{\dot\lambda_6}{\dot\lambda_4}\underline{\partial_6\beta_4\varepsilon_6}$$

$$\frac{d\beta_4}{dt} = \partial_6\beta_4\dot\lambda_6 + \partial_4\beta_4\dot\lambda_4$$

$$\Rightarrow \partial_6\beta_4 = \frac{1}{\dot\lambda_6}\left(\mathring{\beta}_4 - \partial_4\beta_4\dot\lambda_4\right)$$

$$\dot{\xi}_6 - 2\xi_6 = \partial_6\beta_6\xi_6 + \frac{\dot{\lambda}_6}{\dot{\lambda}_4}\frac{d\ln\beta_4}{dt}\varepsilon_4 - \frac{\dot{\lambda}_6}{\dot{\lambda}_4}\partial_4\beta_4\varepsilon_4$$

$$-\frac{\dot{\lambda}_6}{\dot{\lambda}_4}\cdot\frac{1}{\dot{\lambda}_6}\left(\dot{\beta}_4 - \partial_4\beta_4\dot{\lambda}_4\right)\varepsilon_6$$

$$= -\underbrace{\frac{\dot{\beta}_4}{\dot{\lambda}_4}}_{=\frac{d}{dt}\ln\beta_4}\varepsilon_6 + \partial_4\beta_4\varepsilon_6$$

$$= \partial_6\beta_6\xi_6 - \frac{d}{dt}\ln\beta_4\Big(\underbrace{\varepsilon_6 - \frac{\dot{\lambda}_6}{\dot{\lambda}_4}\varepsilon_4}_{\xi_6}\Big) + \partial_4\beta_4\Big(\underbrace{\varepsilon_6 - \frac{\dot{\lambda}_6}{\dot{\lambda}_4}\varepsilon_4}_{\xi_6}\Big)$$

$$= \left[\partial_6\beta_6 + \partial_4\beta_4 - \frac{d}{dt}\ln\beta_4\right]\xi_6 \quad \checkmark$$