Something different, broadly useful

Symmetry breaking & Goldstone bosons

SPONTANEOUS SYM BREAKING: $G \to G' \subset G$

↑ VACUUM of theory DOES NOT RESPECT SYMMETRY G, even if S DOES. } some field gets a vev

Result: DEGREES of freedom SEPARATE INTO HEAVY & MASSLESS MODES

SYMMETRY G is NONLINEARLY REALIZED

when G is a gauge symmetry:

↳ GAUGE BOSON EATS MASSLESS MODE & TOGETHER BECOME MASSIVE GAUGE BOSON.

↑ eg Higgs mechanism

when G is an approximate symmetry

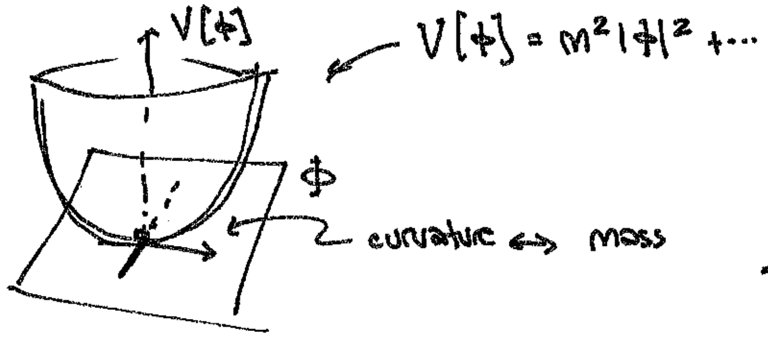
↳ GOLDSTONE is approximately massless eg PIONS

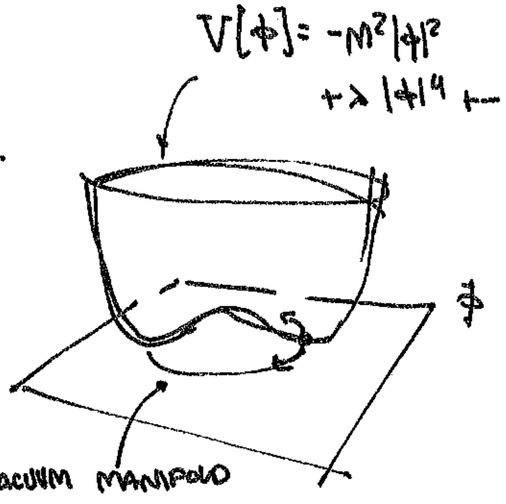
when G is an anomalous symmetry

↳ funny things can happen

eg. Goldstone can get vev (eg: AXION)

the PICTURE:





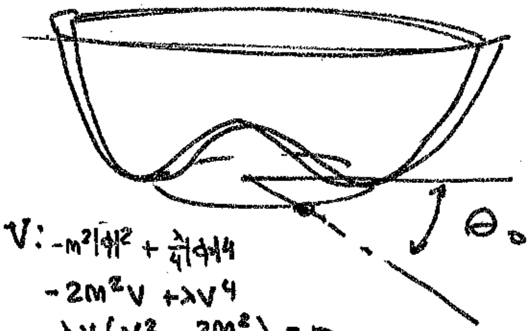
above example: $U(1) \longrightarrow \phi$

$U(1): \phi \rightarrow e^{i\theta} \phi$

some parameter

for the "wine bottle" potential $\langle \phi \rangle = e^{i\theta_0} v$

vev (minimum of V)



min V: $-m^2|\phi|^2 + \frac{\lambda}{4}|\phi|^4$

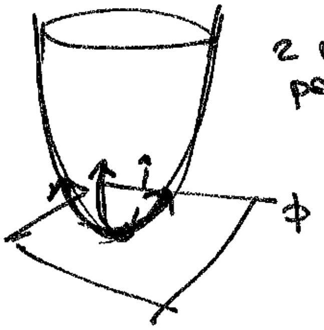
$-2m^2 v + \lambda v^4$

$\lambda v (v^2 - \frac{2m^2}{\lambda}) = 0$

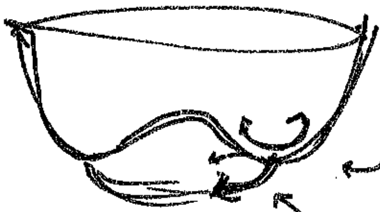
θ_0 PARAMETERIZES THE VEV

vacua are identical $\forall\ \theta_0$

PARTICLES ARE EXCITATIONS IN ϕ DIRECTIONS



2 massive particles



MASSIVE (RADIAL) MODE

GOLDSTONE MODE ALONG VACUUM MANIFOLD

nb: ACTUALLY A MANIFOLD w/ A METRIC

WHAT DOES THEORY LOOK LIKE @ LOW E?

EASY WAY: PICK $\theta_0 = 0$ + quantize w/rt $\phi_1 + i\phi_2 + v$

~~$\partial\phi = \partial\phi_1 + i\partial\phi_2 \leftrightarrow$ unchanged~~

~~$|\phi|^2 = v^2 + 2v\phi_1 + \phi_1^2 + \phi_2^2$~~

~~$|\phi|^4 = v^4 + 4v^2\phi_1^2 + \phi_1^4 + \phi_2^4$~~

... but that's not actually easy

$$\phi = \frac{1}{\sqrt{2}}\left(a(x) + v \quad + i b(x) \right) \qquad v^2 = \frac{2m^2}{\lambda}$$

(the $1/\sqrt{2}$ that accompanies HIGGS VEV)

$$|\phi|^2 = \frac{1}{2}\left[(a+v)^2 + b^2 \right]$$

$$|\phi|^4 = \frac{1}{4}\left[(a+v)^4 + 2(a+v)^2 b^2 + b^4 \right]$$

mass terms for a & b

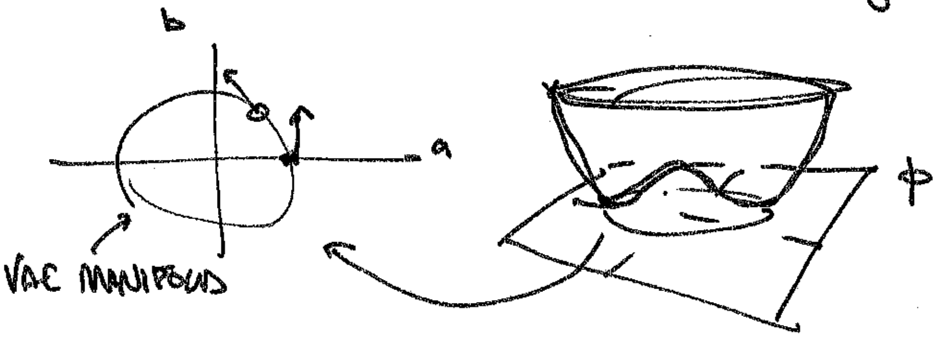
<u>look @ b masses:</u>

$$V \supset -\frac{1}{2}m^2 b^2 + \frac{\lambda}{4}(2v^2)b^2$$

$$= -\frac{m^2}{2}\left(\underbrace{1 - \frac{\lambda v^2}{2m^2}}_{=0} \right) b^2$$

so imaginary part is <u>massless.</u>

Depends on picking V on R line



BUT @ SOME OTHER POINT ON VACUUM, A DIFFERENT LIN COMB IS MASSLESS

BETTER PARAMETERIZATION ("NLSM, CCWZ, ...")

$$\phi = (r(x) + v)\, e^{i\theta(x)}$$

not the right DIMS

SHOULD HAVE FACTORS OF $\sqrt{2}$ FOR R.DOF. BUT WILL NORMALIZE LATER

$$\phi = \underline{(r+v)}\, e^{i\varphi/v}$$

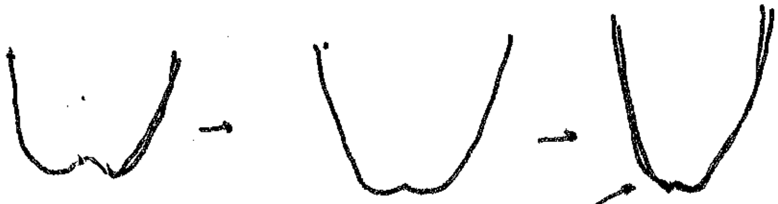
↳ only scale available

CLEARLY φ ALWAYS POINTS IN THE GOLDSTONE DIRECTION WHEN EXPANDING ABOUT A MINIMUM

WE KNOW THE RADIAL MODE IS HEAVY COMPARED TO φ

USUALLY WE CARE ABOUT <u>IR</u> PHYSICS

↳ eg IN DEEP UV, DON'T EVEN NOTICE THE BUMP :



Using $(r+v)e^{i\varphi/v}$ fails when $v \to 0$

(or: $v/p^2 \to 0$)

<u>Cost</u>: φ is a <u>nonlinear</u> REALIZATION (not a linear transformation of a+ib)

<u>REMARK</u>: thm: FIELD REDEFINITIONS DO NOT CHANGE CORRELATION FUNCTIONS AS LONG AS LEADING ORDER TERM IS LINEAR

$$\phi \to \Phi(\phi) \sim \underset{\substack{\uparrow \\ \text{eg for LSZ}}}{a\phi} + \cdots$$

<u>PROPERTIES</u>

Higgs Mechanism

SUPPOSE the U(1) is GAUGED.

↳ Gauge sym is A REDUNDANCY

MUST NOT BE BROKEN ← eg no ANOMALIES

BUT MAY BE REALIZED NONLINEARLY

↑ SPONTANEOUSLY BROKEN

Do you still have Goldstones?

↳ "isn't THE TRANSF A REDUNDANCY?"

yes... but the parameterization of the field ~~is actu~~ describes actual DoF. ← $\phi = r(x)\, e^{i\varphi(x)/v}$

the 2 IR DoF of ϕ away from $\phi = 0$

nb: an analogous (poetic?) way of saying this is that the gauge symm. is local, but there is still a global piece.

$$\mathcal{L} = |\partial\phi| \to |D\phi|^2 + \cdots$$

LET'S DO THIS IN THE LINEAR BASIS

$$\phi = \frac{1}{\sqrt{2}}\left(v + a(x) + ib(x)\right)$$

$$\underset{\partial + ieA}{D}\phi = \frac{1}{\sqrt{2}}\left(\underset{\sim}{ie}Av + \partial a + \underset{\sim}{ieA}a + \underset{\sim}{i\partial b} - eAb\right)$$

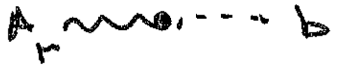
$$|D\phi|^2 = \frac{1}{2}\left[\underset{\text{kin term}}{(\partial a - eAb)^2} + (\underset{\text{mass}}{eAv} + eaA + \underset{\text{kin term}}{\partial b})^2\right]$$

mixing

MOST INTERESTING PIECES :

$$|D\phi|^2 \supset \underbrace{\tfrac{1}{2}e^2v^2}_{\tfrac{1}{2}M_A^2} A^2 + evA_\mu\partial^\mu b$$

Mass term!
(Meißner effect



$ie v(-ik^\mu) = M_A k^\mu$

from FS

"*Gauge boson "eats" Goldstone*"
↳ ξ becomes longitudinal polarization

nb : VACUUM POLE REMAINS TRANSVERSE

$$= iM_A^2\eta^{\mu\nu} + M_A k^\mu \frac{i}{k^2}(-M_A k^\nu)$$

outgoing

$$= iM_A^2 \underbrace{\left(\eta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right)}$$

PROJECTION onto transverse directions
(eg dot into K_ν & perp. vector)

What about GAUGE INVARIANCE?

still have it. CAN GAUGE AWAY the GOLDSTONE
↳ no longer part of theory as an independent particle!

DO U(1) to make $\sqrt{2}\phi = a(x) + v. \in \mathbb{R}$

("every excitation of ϕ in b(x) dir is COMPENSATED by a GAUGE "COUNTER EXCITATION" $\alpha(x)$ "

from $|D\phi|^2$ on p.5:

$$\mathcal{L} = -\frac{1}{4}F^2 + \frac{1}{2}(\partial a)^2 + \frac{1}{2}\underbrace{e^2 v^2}_{\nearrow\ M_A^2} A^2 + \underbrace{\frac{1}{2}e^2 a^2 A^2}_{\substack{\nearrow \\ \text{still have} \\ \text{coupling} \\ \text{to remnant}}} + \underset{\substack{\uparrow \\ \text{check ...} \\ \text{I may} \\ \text{be wrong ?}}}{e^2 v a A^2}$$

So far we've been semi-classical.
WHAT IF WE QUANTIZE THIS THEORY?

AGAIN: $\phi = \frac{1}{\sqrt{2}}\Big(\underbrace{(v+h(x))}_{a} + i\,\underbrace{\varphi(x)}_{b}\Big)$

GAUGE: $a \to a - \alpha b \Rightarrow \delta h = -\alpha\varphi$
$b \to b + \alpha a \Rightarrow \delta\varphi = \alpha(v+h)$
$A \to A - \frac{1}{e}\partial\alpha$

$$\mathcal{L}\Big|_{VAC} = -\frac{1}{4}F^2 + \frac{1}{2}(\partial h - eA\varphi)^2 + \frac{1}{2}(\partial\varphi + eA(v+h))^2 - V$$

RECALL: GAUGE THEORY QUANTIZ $\to$ Fadeev-Popov

$$Z \propto \int \mathcal{D}A\,\mathcal{D}h\,\mathcal{D}\varphi\; e^{i\int d^4x\, \mathcal{L} - \frac{1}{2\xi}G^2}\;\underline{\det\frac{\delta G}{\delta\alpha}}$$

$\nearrow$ from R_ξ GAUGE FIXING
$\delta(G(x) - \omega(x))\, e^{\frac{i}{2\xi}\int d^4x\, \omega^2}$

$\searrow$ GHOSTS (not for ABELIAN) $\vdots$ USUALLY!!

convenient choice for GAUGE FIXING:

$$G = \partial^\mu A_\mu - \xi e v \varphi$$

$$Z \propto \int \cdots\; e^{i\int d^4x\, \mathcal{L}'}$$

MASS

$$\mathcal{L}'\big|_{\text{QUAD}} = -\frac{1}{2} A_\mu \Big(\overbrace{-\eta^{\mu\nu}\partial^2 - \left(1-\tfrac{1}{\xi}\right)\partial^\mu\partial^\nu}^{\text{old}} - (ev)^2\eta^{\mu\nu} \Big) A_\nu$$

$$+\frac{1}{2}(\partial h)^2 - \frac{1}{2}M_h^2 h^2 + \frac{1}{2}(\partial\varphi)^2 - \underbrace{\frac{\xi}{2}(ev)^2\varphi^2}$$

from V

ξ-DEP MASS for GOLDSTONE?!

$M_\varphi^2 = \xi M_A^2$

what about FADEEV-POPOV?

$$\frac{\delta G}{\delta \alpha} = -\frac{1}{e}\partial^2 - \xi e v(v+h)$$

→ convert into term in $\mathcal{L}$ w/ GHOST FIELDS

$$\mathcal{L}_{\text{ghost}} = \bar{c}\left[-\partial^2 - \xi M_A^2\left(1+\frac{h}{v}\right)\right]c$$

NO COUPLING to GAUGE FIELD

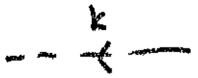
so we never needed this for $U(1)$... just said "∞ constant"

COUPLES TO HIGGS! not just a constant

QUADRATIC TERMS GIVE PROPAGATORS :

A: 

$$= \frac{-i}{k^2-M_A^2}\left(\eta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2-\xi M_A^2}(1-\xi)\right)$$

φ 

$$= \frac{i}{k^2-\xi M_A^2}$$

c 

$$= \frac{i}{k^2-\xi M_A^2}$$

ξ-DEP MASS

$\xi = 0$ LANDAU
$\xi = \infty$ UNITARITY

h 

$$= \frac{i}{k^2-M_h^2}$$

ξ finite: PROPAGATORS ~ $1/k^2$
+ POWER COUNTING ARGUMENTS HOLD
(re: divergences + renormalizability)

(R_ξ sometimes called Renormaliz. gauges.

$\xi = \infty$: UNPHYSICAL DOF DISAPPEAR (DECOUPLE)

$$\sim\sim\sim \; = \frac{-i}{k^2 - M_A^2}\left(\eta^{\mu\nu} - \frac{k^\mu k^\nu}{M_A^2}\right)$$

GOLDSTONE: - - - = 0 (decouple)

⇝ only physical intermediate states
(CUTKOSKY RULES GIVE UNITARITY of S)

⇝ unitarity of S matrix is visible
in different ways.

Non Abelian : sketch of χRAL P.T.

↳ NLΣM for PIONS

universe : $\boxed{q + \bar{q} \text{ confine}}$

↑ ASYMP FREEDOM ↔ IR SLAVERY
BROKEN (ANOMALOUS) SCALE SYM → ∃ SCALE, Λ_{QCD}

vacuum : $\langle \bar{q} q \rangle \neq 0$ ← nb $\langle q_L q_R \rangle$
s.t. this BREAKS EW

Global sym of QCD : $SU(3)_L \times SU(3)_R \times U(1)_B \; (\times \overset{\text{anomalous}}{U(1)_A})$

↑ anomalous in full SM

nb : EW is a subgroup
$Y = T^3_R + \frac{1}{2} B$

χRAL P.T. (easy way)

1. IDENTIFY ORDER PARAM (vev)
2. ACT ON vev w/ BROKEN GENERATORS
3. PROMOTE TRANSF PARAMETERS TO FIELDS.
4. READ OFF THE THEORY.

$\langle q_L q_R \rangle \sim U_0$ ← BIFUNDAMENTAL UNDER $SU(3)_L \times SU(3)_R$

↑

$U_0 \sim \mathbb{1}$ BREAKS TO $SU(3)_V$

$\boxed{U \to U_L U U_R^\dagger}$ ← $U_L = U_R$ still a sym.

$\underbrace{U_L = U_R^\dagger}_{SU(3)_A}$ broken transform BY broken GENERATORS :

$$U \underset{\mathbb{1}}{U_0} U = U^2 = e^{2i\varepsilon^a T^a}$$

generators of SU(3)

TRANSF PARAM OF $SU(3)_A$

identify param w/ GOLDSTONE: $\varepsilon \to \pi/f$

$\langle U \rangle = f\,\mathbb{1}$

nb $U(x) = e^{2i\frac{\pi(x)^a}{f}T^a}$

$\approx 1 + 2i\frac{\pi(x)^a}{f}T^a + \cdots$

under transf by <u>UNBROKEN SYM</u>:

$$U(x) \to U_V U(x) U_V^\dagger = U_V \left(1 + 2i\frac{\pi}{f}T + \cdots\right) U_V^\dagger$$

<u>linear</u> transf w/r/t UNBROKEN SYM ✓

(true for all other TERMS w/ $\mathbb{1} = U_V^\dagger U$)

under <u>BROKEN SYM</u> :

$$U(x) \to U_A U(x) U_A^\dagger \equiv e^{2i\frac{(\pi')^a}{f}T^a} = 1 + 2i\frac{\pi'^a}{f}T^a + \cdots$$

$$\shortparallel$$

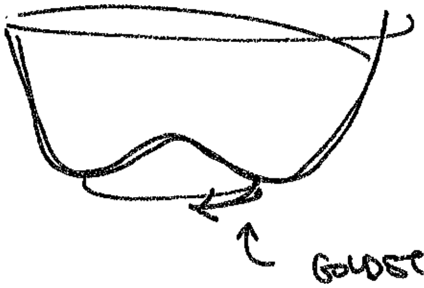
$$(1 + ic^aT^a)\left(1 + 2i\frac{\pi^a}{f}T^a\right)(1 + ic^aT^a)$$

$$\to \pi'^a T^a = \pi^a T^a + f c^a T^a$$

$$\boxed{\pi \to \pi + c}$$ <u>shift symm</u>

shift sym of Goldstone

① "OBVIOUS"



GOLDSTONE SHIFT <u>IS</u> sym. transf in BROKEN DIRECTION!

② IMPLIES ALL GOLDSTONE INTERACTIONS ARE SHIFT-SYMMETRIC.

↳ eg no MASS TERM. ✓

What's the $\mathcal{L}$?

GUESS the KIN TERM (based on sym)

$$\mathcal{L} = \frac{f^2}{4} \mathrm{Tr}\left[(\partial U^\dagger)(\partial U) \right]$$

$$U = 1 + 2i\frac{\pi}{f}T + \cdots$$

USE: $\mathrm{Tr}\, T^a T^b = \frac{1}{2}\delta^{ab}$ in fundamental

nb: what if we gauge weak SU(2)? $q = \left(\begin{array}{c} u \\ d \\ \hline s \end{array}\right)$

$$\partial U \to DU = \partial U - igW^a\left(\tfrac{1}{2}\sigma^a\right)U$$

$$\frac{1}{2}\left(\begin{array}{c|c} \sigma^a & \\ \hline & 0 \end{array}\right)$$

note: ∞ # of terms! (GET interactions)

↳ all 1/f SUPPRESSED

PS: CHECK that YOU BREAK WEAK SYM.

from Goldstone to Pseudo Goldstone

$SU(3)_L \times SU(3)_R$ was never a perfect flavor symmetry.

① u,d,s have different masses!

② u,d,s have different charges!

PARAMETERIZE these EFFECTS

↳ SPURION ANALYSIS.

PRETEND those SMALL EXPLICIT BREAKINGS ARE SPONTANEOUS BREAKINGS.

$$M = \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix}$$

how could this affect π^a ?

$$\Delta\mathcal{L} \sim \mathrm{Tr}\left[M\,U(x)\right] \supset \tfrac{1}{2}\mathrm{Tr}\left[M\left(\frac{\pi^a T^a}{f}\right)^2\right]$$

$+h.c.$

↑ kills linear

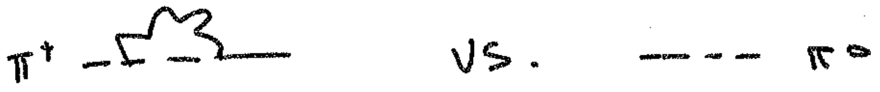
GIVES A MASS TO GOLDSTONES.

↳ HIERARCHY OF QUARK MASSES → RELATIONS ON PSEUDO-GOLDSTONES

eg in $m_u = m_d$ LIMIT,

$$m_\eta^2 + m_\pi^2 = 4m_K^2 \quad \text{(Gell-Mann–Okubo)}$$

EM SPLITTING



$$\hookrightarrow \Delta\mathcal{L} \sim e^2\,\mathrm{Tr}[\underset{\uparrow}{Q}\,U^\dagger Q U]$$

$$\begin{pmatrix} 2/3 & & \\ & -1/3 & \\ & & -1/3 \end{pmatrix}$$

nb: QUAD sensitivity to f

<u>remark</u>: Gauging a subgroup of a global symmetry explicitly breaks that symmetry.

NDA : when a theory breaks down

shift sym

 $\sim \int d^4k \left(\frac{p^2 k^2}{f^4}\right)\frac{1}{k^4}$

$\sim \frac{\Lambda^2 p^2}{16\pi^2 f^4}$

$\sim$  $\times \boxed{\frac{\Lambda^2}{16\pi^2 f^2}}$

$\uparrow$

$< 1 \Rightarrow \boxed{\Lambda \sim 4\pi f}$

CUTOFF ~ GeV

XRAL SYM BR ~ 100 MeV

ESTIMATING COUPLINGS :

1. $1/f$ for EACH GOLDSTONE ← f IS XRAL SYM BR (SSB SCALE) (LIKE A COUPLING)
2. Λ for REMAINING DIMS ← CUTOFF of thy (~ M_ρ IN QCD) NON-GOLDSTONE STATES

$\overset{\shortparallel}{M}$

~~strongly coupled sector field:~~

~~$\chi = \frac{\phi}{f} = \delta \frac{\phi}{M}$~~

$\uparrow$

~~$\boxed{g = \frac{M}{f}}$ eff coupling~~

IDENTIFY CHARACTERISTIC COUPLING $\boxed{g = \frac{1}{f} \cdot M}$

one scale

1S1C

one coupling

WRITE $\mathcal{L}$ WRT DIMLESS $\hat{\mathcal{L}}$

$$\mathcal{L} \sim \underbrace{M^4}_{[\mathcal{L}]=4} \hat{\mathcal{L}}\left(\frac{\partial}{M},\ \underbrace{g\frac{\phi}{M}}_{\text{"to create }\phi\text{"}},\ \frac{g\psi}{M^{3/2}},\ \dots\right)$$

CAN DO FURTHER DIM ANALYSIS: $e^{iS/\hbar}$ ← expansion in $\hbar$ loops

$$\partial\phi\,\partial\phi \sim \hbar \quad \Rightarrow \quad \phi \sim \sqrt{\hbar} \quad \forall \text{ field}$$

COUPLING: $g \sim 1/\sqrt{\hbar}$

$$\Rightarrow \mathcal{L} = \frac{M^4}{g^2}\hat{\mathcal{L}}\left(\frac{\partial}{M},\ g\frac{\phi}{M},\ \dots\right)$$

eg: YUKAWA COUPLING:

$$\Delta\mathcal{L} = \frac{M^4}{g^2}\left(\frac{g\phi}{M}\right)\left(\frac{g\psi}{M^{3/2}}\right)\left(\frac{g\psi}{M^{3/2}}\right) = \underset{\substack{\uparrow \\ \text{Yukawa}}}{g}\,\phi\psi\psi$$