PRACTICAL GOAL: $\mu \overset{k}{\sim\sim\sim} \nu = \dfrac{-i\eta_{\mu\nu}}{k^2+i\varepsilon}$ what happens for DIFF GAUGE CHOICE

REVIEW

GAUGE REDUNDANCY

↳ physically equivalent / indistinguishable

mathematically equivalent

BECOMES PROBLEMATIC FOR QUANTIZING THEORY

$$Z = \int \mathcal{D}A\, e^{iS[A]} \qquad S[A] = S[A^g]$$

$$\sim \underbrace{\mathcal{D}(\text{gauges})}_{\text{gives infinite, identical configs}}\ \underbrace{\mathcal{D}(A/G)}_{\text{physically distinct configurations}}\ e^{iS[A]}$$

$$\longrightarrow S[A] \sim A_\mu \boxed{\mathcal{O}^{\mu\nu}} A^\nu$$

not invertible

$$-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} \rightsquigarrow A_\mu\left[-k^2\eta_{\mu\nu} + k_\mu k_\nu\right]A_\nu$$

PROPAGATOR: $\boxed{\mathcal{O}^{\mu\nu} G_{\nu\rho} = i\delta^{(4)}(x-x')}$

when $\mathcal{O}^{\mu\nu}$ vanishes, there is no inverse.

SOLUTION: fix the damn gauge

EASIEST: $Z_{phys} = \int \mathcal{D}A\, e^{iS[A]}\, \delta(G[A])$

$G[A] = 0$ IS GAUGE FIXING

PROBLEMS / DESIRES :

1. NORM of GAUGE-FIXING δ

↳ $G[A]$ is a function of A
so need to normalize

analog: $\delta(f(x)) = \frac{\delta(x-x_0)}{|\partial f/\partial x|_{x_0}}$

$$\delta(x-x_0) = \delta(f(x)) \, |\partial f/\partial x|_{x_0}$$

(both sides integrate to 1)

FUNCTIONAL VERSION:

$$\delta(A - \underset{\sim}{A}) = \delta[G[A]] \boxed{\left|\det \frac{\delta G}{\delta A}\right|}$$

physical (A/G)

$G[\underset{\sim}{A}] = 0$ DETERMINES WHAT IS PHYS.

δA IS CHANGING A ALONG A GAUGE TRANSFORM

$A \to A^\alpha = A + \partial\alpha$ ← i'm suppressing μ & (x); setting $e = 1$

so $\frac{\delta}{\delta A}$ is $\frac{\delta}{\delta\alpha}$

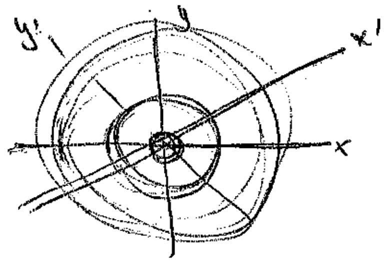
BOTH SIDES INTEGRATE TO 1 WHEN INTEGRATING OVER GAUGE CHOICES, α

2. I LIKE GAUGE INVARIANCE, IT MAKES IT EASY TO WRITE TERMS IN S.

↳ I don't want to simply fix GAUGE,
I want to factor out the redundant part of the integral over $\mathcal{D}A$

these are somewhat contradictory goals.

ANALOG LAST TIME:



ROTATIONALLY INVT INTEGRAND
can do integral (1D) along any axis.

WANT TO WRITE AS A "OVER SOME AXIS" INTEGRAL TIMES A "VOLUME OF ROTATION SPACE" FACTOR.

$$\int dx\,dy\; I = \int d\theta \int d^2x\; \delta(f(x))\left[\frac{\partial f}{\partial x}\Big|_{x_0}\right] \; I(x,y)$$

PROJECTS TO ONE AXIS DEFINED BY f

VOLUME OF REDUNDANCY

ORIG. INTEGRAND

3. I WANT THIS TO LOOK LIKE A NORMAL QFT PARTITION FUNCTION

$$Z = \int \mathcal{D}(\text{field}) \cdots \; e^{i\int d^4x \;\cdots}$$

(no other terms that are not in exp!)

... even if that means more fields & terms in action

WITH FUTURE SIMPLIFICATIONS IN MIND

LET'S GENERALIZE LORENTZ GAUGE

$G[A] = \partial \cdot A - \omega$ ← some arbitrary function $\omega(x)$ that is part of gauge fixing

(NOT the gauge transformation parameter, $\alpha(x)$)

ABELIAN (E&M-LIKE)

$\omega = 0 \rightarrow$ LORENTZ.

RECALL: $A \rightarrow A_\mu^\alpha = A_\mu + \partial_\mu \alpha$

$G[A^\alpha] = \partial \cdot A + \partial^2 \alpha - \omega$

$\boxed{\frac{\delta G[A^\alpha]}{\delta \alpha} = \partial^2} \rightarrow \det \frac{\delta G}{\delta \alpha} = \boxed{\det \partial^2}$

RECALL: nothing weird about $\partial^2 = -k^2$

$\det \partial^2 = -\prod k^2$ over all modes of the system.

this is some big, but constant factor in a discrete lattice

& some ∞ but constant factor in a continuum field.

(this becomes more nuanced for non-Abelian fields!)

HOW to CHOOSE $\omega(x)$? AVERAGE OVER IT.

$$\underset{\uparrow\ \text{norm}}{N_\xi} \int \underbrace{\mathcal{D}\omega(x)}_{\text{sample } \underline{\text{all}}\ \omega(x)} \ \underbrace{e^{-i\int d^4x \frac{\omega^2}{2\xi}}}_{\text{GAUSSIAN WEIGHT (FUNCTIONAL)}} \quad (20)$$

$\omega^2 \leftarrow \omega(x)^2$

$\underline{\text{const}}\ \xi$

(this is called R_ξ GAUGE.)

I DO NOT KNOW WHAT R MEANS

why? the $\mathcal{D}\omega$ can collapse the Fadeev-Popov δ-function

$$Z = \underbrace{(\det \partial^2)}_{\det(\delta G/\delta\alpha)} \underbrace{\int \mathcal{D}\alpha}_{\substack{\text{Redundancy} \\ \text{("Vol of } G\text{")}}} \underbrace{\int \mathcal{D}A\, e^{iS[A]}}_{\text{"ORIGINAL" } Z} \delta(\underbrace{\partial\cdot A - \omega}_{G[A]})$$

$\leftarrow$ & integ over

$$= \underbrace{\boxed{N_\xi \det \partial^2}}_{\text{const.}} \left[\int \mathcal{D}\alpha\right] \underbrace{\left[\mathcal{D}A\, e^{iS[A]} \boxed{e^{-i\int d^4x \frac{(\partial A)^2}{2\xi}}}\right]}_{\text{"PHYSICAL" } Z}$$

the infinity that bugged us. JUST PULL IT OUT.

this is $\mathcal{O}(\partial^2)\,\mathcal{O}(A^2)$
→ KINETIC TERM PIECE!

⇒ will solve the non-invert. problem w/

$\mathcal{O}^{\mu\nu} G_{\nu\rho} = i\delta$!

WITH THIS NEW TERM, $\Theta^{\mu\nu}$ IS NOW:

$$\Theta^{\mu\nu} = -k^2\eta^{\mu\nu} + \left(1-\frac{1}{\xi}\right)k^\mu k^\nu$$

then $\Theta^{\mu\nu} G_{\nu\rho} = i\delta^\mu{}_\rho$

$$\Rightarrow \boxed{G_{\nu\rho} = \frac{-i}{k^2+i\varepsilon}\left(\eta^{\mu\nu} - (1-\xi)\frac{k^\mu k^\nu}{k^2}\right)}$$

Photon propagator

MORE GENERAL THAN WHAT WE USED IN P230A

Feynman Gauge : $\xi = 1$

BUT STILL NOT THE MOST GENERAL
WE CHOSE GAUSSIAN WEIGHT
(ξ GENERALIZED LORENZ GAUGE)

OTHER CHOICES: $\xi = 0$ LANDAU GAUGE

$\xi = \infty$ UNITARITY GAUGE

PROJECTS OUT LONG. MODE

NOW CONSIDER NON-ABELIAN CASE

$$Z = \left(\int \mathcal{D}\alpha^a\right) \int \mathcal{D}A \, e^{iS[A]} \, \delta(G(A)) \, \det\left(\frac{\delta G[A^\alpha]}{\delta\alpha}\right)$$

$$(A^\alpha)^a_\mu = A^a_\mu + \underbrace{D_\mu\alpha^a}$$

$$= A^a_\mu + \partial_\mu\alpha^a + f^{abc}A^b_\mu\alpha^c$$

COV. DER ACTING ON ADJOINT INDEX

cannot pull this out of $\mathcal{D}A$

A APPEARS INSIDE VIA D_μ !!

$$G(A) = \partial^\mu A^a_\mu(x) - \omega^a(x)$$

$$\frac{\delta G(A^\alpha)}{\delta\alpha} = \frac{\delta}{\delta\alpha}\left(\partial(A + D\alpha) - \omega\right)$$

$$= \partial^\mu D_\mu \longleftarrow \text{cf } \partial^\mu \partial_\mu \text{ for } U(1)$$

B/C U(1) GAUGE FIELD IS NEUTRAL UNDER U(1) WHEREAS NONABELIAN GAUGE FIELDS SELF-INTERACT

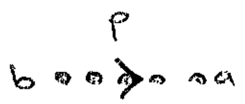
now we have to deal with $\det(\partial^\mu D_\mu)$

<u>trick</u>: GRASSMANN VARIABLES!

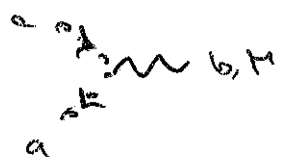
$$\det(\partial^\mu D_\mu) = \int \mathcal{D}c\,\mathcal{D}\bar{c}\; e^{i\int d^4x\; \underbrace{\bar{c}(-\partial D)c}_{\text{KINETIC TERM!}}}$$

ANTICOMMUTING SCALARS *that just became "semi-dynamical"*

nb c & $\bar{c}$ are IN ADJOINT REP: they have SAME INDEX AS A^a_μ.



$= \dfrac{i\delta^{ab}}{p^2}$



$= -g f^{abc} p^\mu$

you can read this off the "$\mathcal{L}$"

$$\mathcal{L} = \underbrace{-\frac{1}{4}(F^a_{\mu\nu})^2}_{\text{KIN.}} \underbrace{- \frac{1}{\xi}(\partial A^a)^2}_{\text{GAUGE FIX}} + \underbrace{\bar{\psi}(i\not{\partial} - m)\psi}_{\text{MATTER KIN}} + \underbrace{\bar{c}^a(-\partial D)^{ac} c^c}_{\text{GHOST (NON AB. ONLY)}}$$

What are they good for?

↳ cancel unphysical DOF.

<u>OBSERVE</u>: you can only produce ghosts from gauge fields.

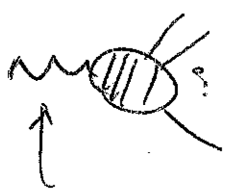
↳ no MATTER-GHOST INTERACTION (no ectoplasm!)

<u>WHERE IT SHOWS UP</u>:

never in external states

↳ we're just saying this for now
RIGOROUS ARGUMENT: BRST sym.

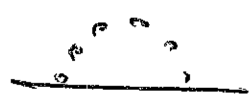
... never needed them in ext. states!



this is <u>physical</u>, no redundancy.

~~INSTEAD~~ SO WORRY ABOUT LOOPS?

↳ there unphysical polarizations might propagate!

 ← but no  analog!

↑ this also has no GAUGE redundant internal gauge polz. ∴ the spin structure acts as a projection.

⇒ only in PURE GAUGE