FROM LAST TIME: Where did integration rules for θ come from?

see: Catto, Choun, Gürcan, Khalfan, Kurt } technical
"Grassmann #s & ..."

or eg: *field quantization*, GREINER (12.8)

Given rules for differentiation
↳ RIEMANN SUM FAILS

eg. $\left(\frac{d}{d\theta}\right)^2 = 0$; no inverse → no antideriv.

WANT: $\int d\theta \, (a f(\theta) + b g(\theta)) = a \int d\theta \, f(\theta) + b \int d\theta \, g(\theta)$

NEED: $\underset{\equiv 0}{\int d\theta}$ & $\underset{\equiv 1}{\int d\theta \, \theta}$ defined

Some intuition:

- *for function space that is L_2* (dies @ $\pm\infty$)

$$\int dx \frac{df}{dx} = 0$$

similarly: $\int d\theta = \int d\theta \frac{d}{d\theta}\theta = 0$

- but θ is not a total derivative
so let's define $\int d\theta \, \theta \equiv 1$.

- $\int d\theta \, \underbrace{f(\theta + \overset{\swarrow c\#}{\eta})}_{f(\theta) = a + b\theta} = \int d\theta \, a + b(\theta + \eta) = \int d\theta \, f(\theta) + \underbrace{\int d\theta \, \eta}_{\text{want} = 0}$

SHARE TEX

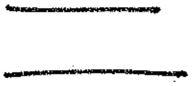
Diagrammatica

last thing we did: $Z = e^{iW}$

generating function of connected diagrams
↑
care about correlations that are dynamically generated

(eg in a "HOT" system, can get lots of correlations from RANDOM FLUCTUATIONS)

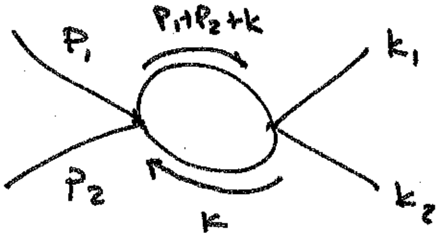
discon.



vs.

conn:

loops: intuition/experience thus far



momentum conservation does not constrain d^dk integral

↳ get contributions from ARBITRARILY HIGH momentum. A MOMENT'S THOUGHT: this is insane.

... maybe it's suppressed?

$$\int d^dk \frac{i}{k^2-m^2+i\epsilon} \frac{i}{(k+p_1+p_2)^2-m^2+i\epsilon}$$

$$\sim \int d^dk \frac{i}{k^4} \longrightarrow \log \Lambda$$

for $d=4$

⇒ not suppressed.. ARBIT. LARGE!

(regulariz + renorm ...)

BUT SUPPOSING these have some finite value...

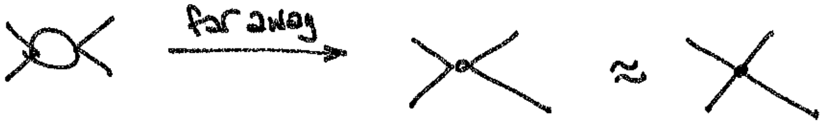
these loops represent "corrections" to vertices

↑ @ least Hi-Momentum Modes

↑ LOCAL terms in S

(relative to what? p_i & k_i ... nb if $M^2 \gg p_i^2, k_i^2$, then loop is local)

for "low" ext momenta, the loop is very local in position space! from "far away", looks like point interaction.



will make precise

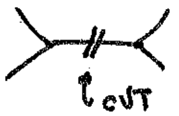
Another obs: loops are very "quantum" internal virtual particles w/ unconstrained 4-momentum.

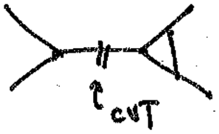
So it is useful to separate out "loopy things" that naturally lump together.

DEFINE 1PI: one particle irreducible

graph that cannot be separated by cutting one internal line

1P REDUCIBLE:



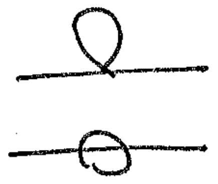


1PI:

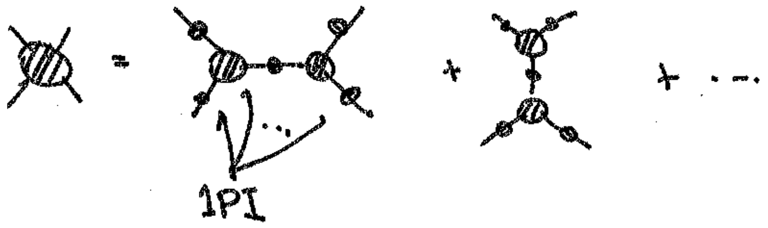
→ 1PI diagrams "naturally" sum together as "local" interactions w/ quantum effects built in; then we can connect these local blobs by [nonlocal] propagators

↑ which themselves get 1PI corrections

eg



eventually want "skeleton diagram" decomposition:



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in fact, we can write a generating functional for the 1PI graphs: $\Gamma[\varphi]$.

define: classical field, φ, in bg of source J

$$\varphi(x) = \langle \phi(x) \rangle_J = \frac{\delta W[J]}{\delta J(x)}$$

↑ ASSUME: WE CAN WRITE $\varphi = \varphi[J]$
& assume that $J=0 \Leftrightarrow \varphi = 0$
(not the case w/ Higgs)

$$\text{nb: } \langle 0 | \varphi(x) | 0 \rangle_J = \frac{\delta}{i\delta J} Z = \Big(\underbrace{\frac{\delta}{i\delta J} W}_{\text{gives "vev" in presence of source}} \Big) Z$$

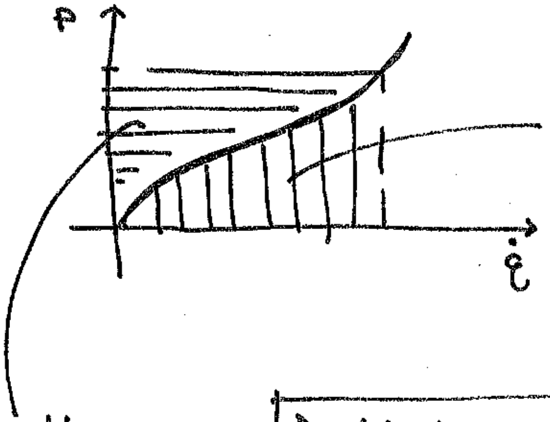
from: physics stack ex. 4384

<u>define</u> : $\boxed{W[J] + \Gamma[\varphi] = \int d^d x \; \varphi(x) J(x)}$

↑

Reminiscent of Legendre Transform in MECHANICS.

REMINDER: Legendre : $\dot{q} \longleftrightarrow p$



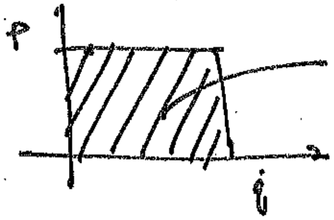
this area: $\int^{\dot{q}} p(\dot{q}) \equiv L$

$\Rightarrow \boxed{\frac{\partial L}{\partial \dot{q}} = p}$

this area: $\boxed{\int \dot{q}(p) \; dp \equiv H}$

$\Rightarrow \boxed{\frac{\partial H}{\partial p} = \dot{q}}$

total AREA :



$p\dot{q}$

so: $L + H = p\dot{q}$

the definition of Γ w/r.t W + $\int d^d x \; \varphi J$ is a FUNCTIONAL extension of the LEGENDRE TRANSFORM

$$\frac{\delta}{\delta\varphi(x)} W[J] + \frac{\delta}{\delta\varphi(x)} \Gamma[\varphi] = \int d^d x' \left(\underbrace{\frac{\delta\varphi(x')}{\delta\varphi(x)}}_{\delta^{(d)}(x-x')} J(x') + \varphi(x') \frac{\delta J(x')}{\delta(x)} \right)$$

dep on φ through
$\varphi = \delta W[J]/\delta J$

$$\frac{\delta}{\delta\varphi(x)} W[J] = \int d^d x' \underbrace{\frac{\delta W[J]}{\delta J(x')}}_{= \varphi(x')} \frac{\delta J(x')}{\delta\varphi(x)}$$

Analog of:

$$\frac{\partial}{\partial r_i} f(q(r)) = \sum_j \frac{\partial f}{\partial q_j} \frac{\partial q_j}{\partial r_i}$$

(CHAIN RULE)

these two terms cancel

$$\Rightarrow \boxed{\frac{\delta}{\delta\varphi(x)} \Gamma[\varphi] = J(x)}$$

analog of $\frac{\partial H}{\partial p} = \dot{q}$

$\dot{q} \longrightarrow J$

$p \longrightarrow \varphi$

$L \longrightarrow W$

$H \longrightarrow \Gamma$

we still don't see why Γ generates connected graphs... let's get there

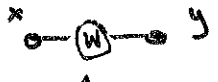
<u>OBSERVE</u>: let's relate $\delta/\delta J$ to $\delta/\delta\varphi$

$$\frac{\delta}{\delta J(x)} = \int d^d y \, \frac{\delta\varphi(y)}{\delta J(x)} \frac{\delta}{\delta\varphi(y)}$$

$$= \int d^d y \, \underbrace{\frac{\delta^2 W[J]}{\delta J(x)\delta J(y)}} \frac{\delta}{\delta\varphi(y)}$$

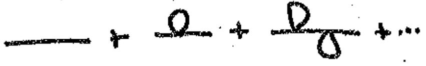
$$\varphi(y) = \frac{\delta W[J]}{\delta J(y)}$$

nb: $\frac{\delta}{\delta J(x)} \frac{\delta}{\delta J(y)} W[J] =$



connected

eg



$$= i \int d^d y \, G_2(x,y) \frac{\delta}{\delta\varphi(y)}$$

$\delta/\delta\varphi(y) = -i \, \delta/\delta J(x)$

ADDS LINE TO A GRAPH

---

define shorthand: $\Gamma_n(x_1 \cdots x_n) = -i \frac{\delta}{\delta\varphi(x_1)} \cdots \frac{\delta}{\delta\varphi(x_n)} \Gamma[\varphi]$

<u>our main claim</u>: $\Gamma_n(\cdots)\big|_{\varphi=0} = \langle\phi(x_1) \cdots \phi(x_n)\rangle_{\text{conn \& 1PI}}$

interesting result:

$$\frac{\delta J(z)}{\delta J(x)} = \Big(i\int d^dy\, G_2(x,y)\,\frac{\delta}{\delta\varphi(y)}\Big)\,\overbrace{J(z)}^{\frac{\delta}{\delta\varphi(z)}\Gamma[\varphi]}$$

$$\delta^{(d)}(x-z) = i\int d^dy\, G_2(x,y)\,\underbrace{\frac{\delta}{\delta\varphi(y)}\frac{\delta}{\delta\varphi(z)}\Gamma[\varphi]}_{\text{2-POINT of }\Gamma}$$

$$= -\int d^dy\, G_2(x,y)\,\Gamma_2(y,z)$$

$$\Rightarrow \mathbb{1} = -G_2\Gamma_2$$

$$\Rightarrow \boxed{G_2 = -\Gamma_2^{-1}}$$

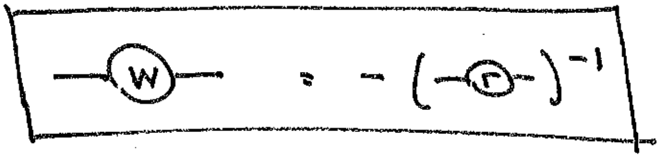
connected 2-point func

hm!

analog of

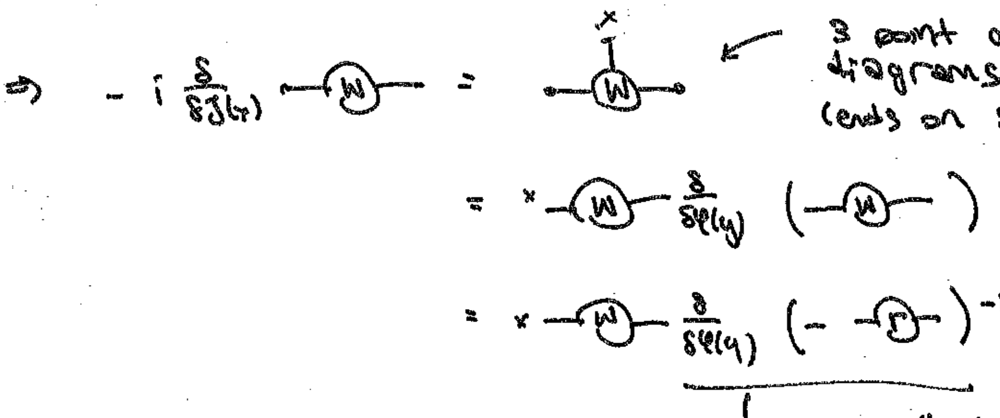
$$1 + x + x^2 + \cdots = \frac{1}{1-x}$$

$$- + \ell + \ell\ell + \cdots = \frac{1}{1-\ell} -$$



$-i\frac{\delta}{\delta J}$ ADDS EXT LINE TO G_N

$\frac{\delta}{\delta\varphi}$ ADDS EXT LINE TO Γ_N



3 point connected diagrams (ends on sources)

need to write wrt Γ b/c δ/δφ(y)

now what?

matrix analogy : ← func analogy: $d(\frac{1}{x}) = -\frac{dx}{x^2}$

$$\underset{\shortparallel\;0}{d(\underbrace{A A^{-1}}_{\mathbb{1}})} = (dA)A^{-1} + A\,d(A^{-1}) \Rightarrow \frac{d}{d\lambda}(A^{-1}) = -A^{-1}\left(\frac{d}{d\lambda}A\right)A^{-1}$$

continuum limit :

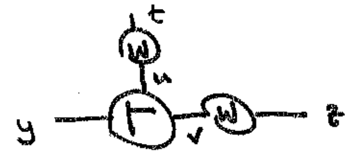
$$\frac{\delta}{\delta\varphi(y)}\left[-\Gamma_2(t,z)\right]^{-1} = \int d^du\, d^dv\; \Gamma_2(t,u)^{-1} \boxed{\frac{\delta\Gamma_2(u,v)}{\delta\varphi(y)}}\; \Gamma_2(v,z)^{-1}$$

$\{$ 3 point! Γ_3

2 point W!

$$= \int d^du\, d^dv\;\; G_2(t,u)\,\Gamma_3(y,u,v)\,G_2(v,z)$$

=



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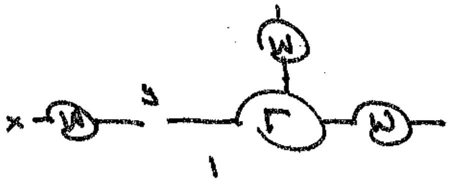
return to:

$-i\,\frac{\delta}{\delta J(x)}$

=



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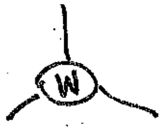

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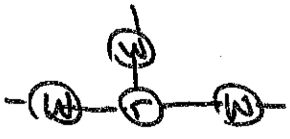
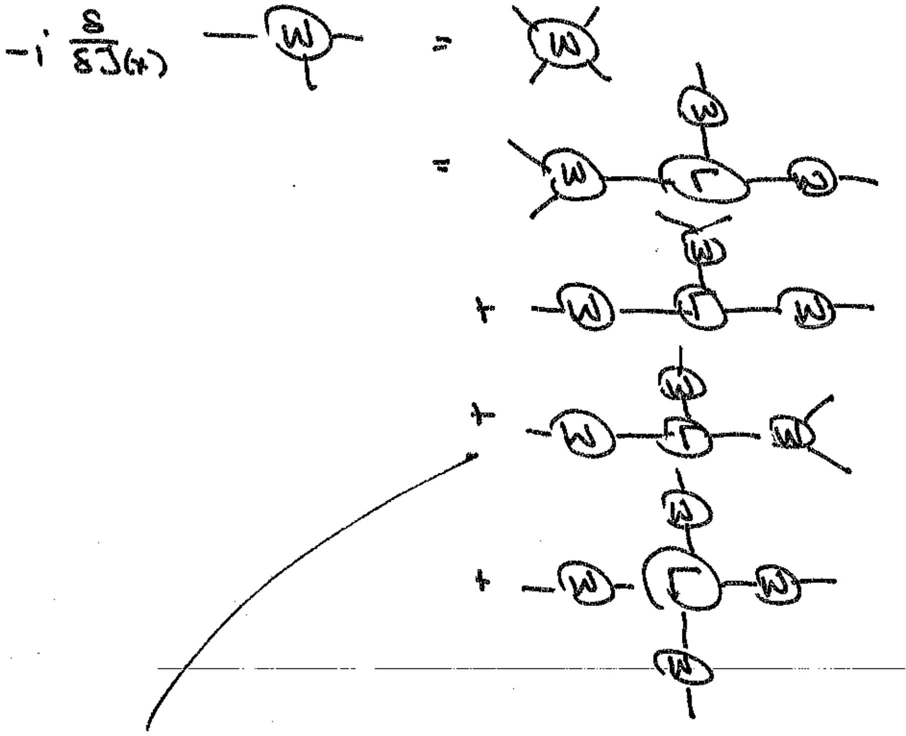


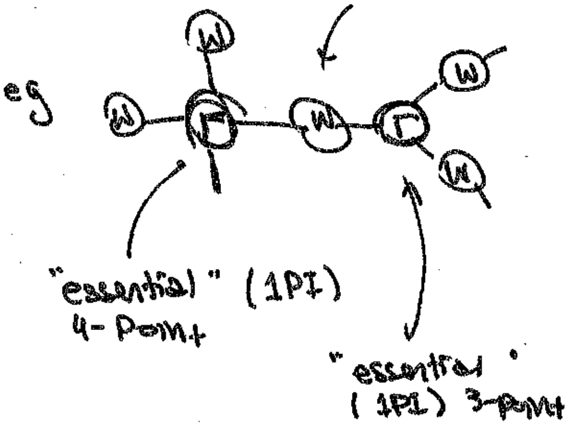
$G_3(x,y,z)$ $\qquad$ $\int d^du\, d^dv\, d^dw\; G(x,w)\, G(y,u)\, G(z,v)\, \Gamma_3(w,u,v)$

if you meditate on this : clear that Γ is 1PI

EASY to BUILD UP SKELETON:



all connected 2-point :

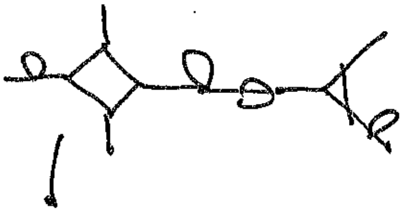

eg


the W's account for all nonlocal prop, including all corrections

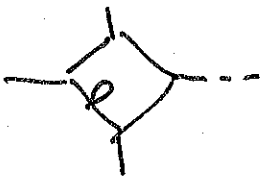
so the Γ_i's are all "local" ... all the 1PI

the point:

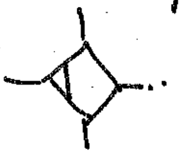
Γ is quantum effective action.

eg:


eg what about ?



still 1PI.



still 1PI . . .

~~① tree = classical~~

~~ignore all loops~~.

Srednicki § 21

so: Γ generates 1PI graphs.

drop this 2 12 BANKS eq. 3.4 p. 28

claim: Γ is also the QUANTUM (EFFECTIVE) ACTION

meaning: tree level graphs wrt Γ (instead of W)

gives complete amplitude including loops.

"W" for Γ

$$\tilde{Z}[J] \equiv \int \mathcal{D}\varphi \, e^{i\Gamma[\varphi] + i\int d^4x \, J\varphi} \equiv e^{i\tilde{W}[J]}$$

instead of usual (CLASSICAL!) ACTION.

how do we identify tree graphs from the functional perspective.

ħ-ology (counting of ħ's) — unit of quantum-ness

ACTION HAS UNITS OF ħ. LET'S MAKE THIS FACTOR EXPLICIT → it will be a trick to isolate tree graphs

$$e^{\frac{i}{\hbar}(S + \cdots)}$$

$$\uparrow$$

$$\frac{1}{\hbar}(\partial\phi)^2 - \frac{1}{\hbar}V(\phi) + iJ\phi/\hbar$$

$\boxed{\sim\hbar}$

EA internal line, I

$\sim 1/\hbar$ each vertex, V

$\sim \frac{1}{\hbar}$ each EXTERNAL LINE, E

So in a graph:

$$\hbar^{I-E-V} = \hbar^{L-1}$$

↖ # loops! (obvious in iterative steps)

$$\text{So: } e^{i\tilde{W}} \xrightarrow{\hbar\to 0} \boxed{e^{\frac{i}{\hbar}\tilde{W}_{L=0}}}$$

— ħ → 0 limit: isolates tree terms in W!

$$\uparrow$$

$$= \Sigma \text{ DIAGRAMS} = \sum_{L=0} \hbar^{L-1}\tilde{W}_L$$

OUR CLAIM is $W = \tilde{W}_{L=0}$.

Remarks

$$\Gamma[\varphi_c] \sim -\int d^4x \, V_{eff}(\varphi_c)$$

"CLASSICAL", constant

potential of field
DETERMINES SYM BREAKING
↳ not the classical pot (!!)
+ affected by Renormaliz (!!)

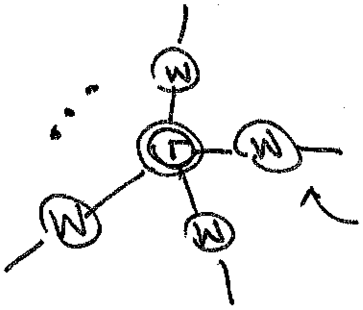
- *What's all this good for?*

if we can say things about Γ_n
then we know it's <u>true</u>

(vs. only true @ tree level)

↳ important for, eg. renormalizability
of gauge thy.

- <u>AMPUTATION</u> of DIAGRAMS (cf Peskin p. 113–114)



S-matrix element?
<u>LSZ REDUCTION</u>: RELATE
POLE OF CORRELATION FUNC. to
S-matrix element.

some propagator

$$\frac{iZ}{p^2-M^2}$$