LAST TIME

$$\langle q_1 \cdots q_M \rangle = \frac{1}{\mathcal{N}} \int d^N q \, (q_1 \cdots q_M) \, e^{-\frac{1}{2} q A q - V(q)}$$

correlation function

LINEAR NONLIN

$$Z_V[J] = \int d^N q \, e^{-\frac{1}{2} q A q - V(q) + J \cdot q}$$

$$\mathcal{N} = Z_{V=0}[J=0] = \sqrt{\frac{(2\pi)^N}{\det A}}$$

SOURCE. (new variable to make Z a generating function)

WRITE CORR. FUNC WRT $\delta/\delta J$ of $Z_V[J]$:

$$\langle q_1 \cdots \rangle = \frac{1}{\mathcal{N}} \left(\frac{\delta}{\delta J_1} \cdots \right) Z_V[J] \Big|_{J=0}$$

WE HAVE A NICE FORM OF $Z[J]$ BUT NOT $Z_V[J]$

SO DO PERT. EXPANSION

$$Z_V[J] = \left(1 + V\left(\frac{\delta}{\delta J}\right) + \frac{1}{2} V\left(\frac{\delta}{\delta J}\right)^2 + \cdots \right) Z[J]$$

$$Z[J] = \mathcal{N} e^{-\frac{1}{2} J A^{-1} J}$$

$\frac{\delta}{\delta J}$ of this is doable!

WHAT WE DERIVE:

CORRELATION BETWEEN M POINTS:
SUM OVER GRAPHS CONNECTING M POINTS USING LINES $(A^{-1})_{ij}$ AND VERTICES GIVEN BY $V(\delta/\delta J)$

↳ LOCALITY: $V(\delta/\delta J)$ HAS POWERS OF $(\delta/\delta J)$ @ SAME POINT.

→ (A^{-1}) IS PROPAGATION OF "INFO"
→ $V(\delta/\delta J)$ IS VERTEX OF "INTERACTION"

TRICK: (notes 1.4)

$$Z_V = e^{-V(\delta/\delta J)} e^{\frac{1}{2} J A^{-1} J}$$

claim: $$g\left(\frac{\delta}{\delta J}\right) f(J) = f\left(\frac{\delta}{\delta q}\right) g(q) e^{J\cdot q}\Big|_{q=0}$$

if: $g(q) = e^{\alpha\cdot q}$ $\quad f(J) = e^{\beta\cdot J}$ ⊛

then:

$$\text{LHS} = e^{\alpha\cdot \delta/\delta J} e^{\beta\cdot J} = e^{\alpha\cdot\beta} e^{\beta\cdot J} = e^{\beta\cdot(J+\alpha)}$$

$$\text{RHS}: e^{\beta\cdot\frac{\delta}{\delta q}} \underbrace{e^{\alpha q} e^{J\cdot q}}_{e^{(\alpha+J)q}}\Big|_{q=0} = e^{\beta(\alpha+J)} e^{\alpha\cdot q} e^{J\cdot q}\Big|_{q=0} = e^{\beta(J+\alpha)}$$

generalize: WE MAY WRITE "ANY" f & g AS FOURIER SERIES, SO EA TERM REDUCES TO ⊛

so: $$Z_V[J] = e^{\frac{1}{2}\frac{\delta}{\delta q} A^{-1} \frac{\delta}{\delta q}} e^{-V(q) + J\cdot q}\Big|_{q=0}$$ (*)

expand each exp.

$$= \left(1 + \frac{\delta}{\delta q^i}(A^{-1})_{ij}\frac{\delta}{\delta q^j} + \cdots\right)\left(1 - V(q) + \frac{1}{2}V(q)^2 + \cdots\right)\cdots$$

notation: $$V_i = \frac{\delta}{\delta q^i} V$$
$$V_{ij} = \frac{\delta}{\delta q^i}\frac{\delta}{\delta q^j} V$$
$$\vdots$$

$$Z_V[J]\Big|_{J=0} = 1 - \tfrac{1}{2} A^{-1}_{ij} V_{ij}$$

if $M^2 q_i^2$ in V

1

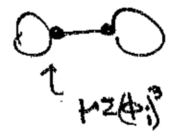
$$+ \tfrac{1}{8}\Big(-A^{-1}_{ij} A^{-1}_{kl} V_{ijkl}$$

if $\lambda \xi (q_i)^4$ in V

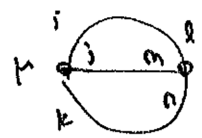
$$+ \big(A^{-1}_{ij} V_{ij}\big)\big(A^{-1}_{kl} V_{kl}\big)$$

$$+ 2\, V_{ij} A^{-1}_{jk} A^{-1}_{kl} V_{kl}$$

$$+ V_{ijk} A^{-1}_{ij} A^{-1}_{kl} A^{-1}_{mn} V_{lmn} \Big)$$



$$+ \tfrac{1}{12} V_{ijk} A^{-1}_{il} A^{-1}_{jm} A^{-1}_{kn} V_{lmn} + \ldots$$



if $\mu\, Z(\phi_i)^3$

$Z_V[J]\big|_{J=0}$ IS A SUM OVER <u>VACUUM POLARIZATIONS</u> (BUBBLE DIAGRAMS)

[when we normalize our correlation functions, we <u>actually</u> normalize w/rt $Z_V[0]$

EXTERNAL LINES: When $\exists$ <u>EXTERNAL SOURCES</u> of ϕ

$$Z_V[J] = e^{\frac{1}{2}\frac{\delta}{\delta q} A^{-1} \frac{\delta}{\delta q}} \Big[e^{-V(q)}\, e^{Jq} \Big]$$

SOURCES

When $\delta/\delta q$'s HIT e^{Jq}, WE GET A^{-1} MOTIONS THAT END AT THE SOURCE.

<u>QM</u> $H = \frac{1}{2m} P^2 + V(q)$

$$i \frac{d}{dt} |\psi\rangle = \hat{H} |\psi\rangle \Rightarrow |\psi(t)\rangle = e^{-i\hat{H}t} |\psi_0\rangle$$

POSITION STATE: $\hat{q}(t)\, |q,t\rangle = q\, |q,t\rangle$

s.t. WAVEFUNCTION IS $\psi(q,t) = \langle q | \psi(t)\rangle$

& $\hat{H} \to -\frac{1}{2m} \frac{d^2}{dq^2} + V(q)$ ACTING ON $\psi(q,t)$

PATH INTEGRAL:

$$\psi(q,t) = \langle q| \; e^{-i\hat{H}t} \; |\psi_0\rangle$$

insert $\int dq_0 \, |q_0\rangle\langle q_0| = \mathbb{1}$

$$= \boxed{\int dq_0 \; K(q, q_0; t) \; \psi(q_0, 0)}$$

GREEN'S FUNCTION

$K = \langle q | e^{-i\hat{H}t} | q_0\rangle$

IN FACT, BREAK UP INTO LITTLE TIME SLICES:

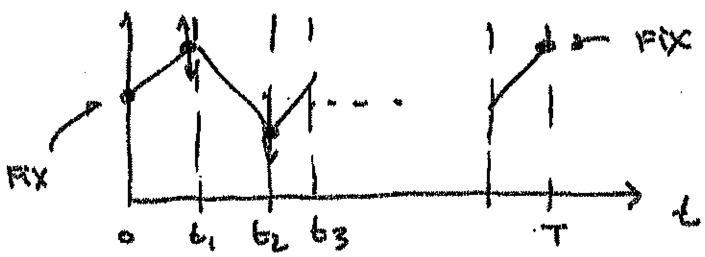
$$e^{-i\hat{H}T} = e^{-i\hat{H}(t_N - t_{N-1})} \, e^{-i\hat{H}(t_{N-1} - t_{N-2})} \cdots e^{-i\hat{H}(t_1 - 0)}$$

INSERT COMPLETE SETS OF STATES

$$K(q,q_0;T) = \int d^N q \prod_r \langle q_{r+1}| e^{-i\hat{H}\underbrace{(t_{r+1}-t_r)}_{\delta t}} |q_r\rangle$$

T ← finite time

{ what are we integrating?
ALL POSSIBLE INTERMEDIATE STATES



$\underbrace{t_1 \; t_2 \; t_3 \; \cdots}$ integrate over "abacus positions"

EVALUATE FOR SMALL δt IN $\boxed{V=0 \text{ LIMIT}}$ ← free prop.

$$K_0(q,q';t) = \langle q| e^{-i\hat{p}^2/2m\, t} |q'\rangle$$

USE $\langle q|p\rangle = e^{ipq}$
$\langle p|q'\rangle = e^{-ipq'}$

{ will insert $\mathbb{1} = \int dp\, |p\rangle\langle p|$

$$K_0(q,q';t) = \int dp\, \langle q| e^{-i\frac{\hat{p}^2}{2m}t} |p\rangle\langle p|q'\rangle$$

$$= \int dp\; e^{-i\frac{p^2}{2m}t}\; \underline{e^{ip(q-q')}}$$

Gaussian

$$= \sqrt{\frac{m}{2\pi i t}}\; e^{i\frac{m(q-q')^2}{2t}}$$

{ in $t \to 0$ limit we recover δ

Restoring $V(q)$

↳ note that $\hat{H} = \frac{\hat{p}^2}{2m} + V(\hat{q})$ $[\hat{q},\hat{p}] \neq 0$

BUT: $e^{\epsilon(\hat{A}+\hat{B})} = e^{\epsilon\hat{A}} e^{\epsilon\hat{B}} (1 + O(\epsilon^2))$

this is why we slice up into small δt

$$\langle q_{r+1} | e^{-i\hat{H}\delta t} | q_r \rangle = \langle q_{r+1} | e^{-i\frac{\hat{p}^2}{2m}\delta t} e^{-iV(\hat{q})\delta t} | q_r \rangle$$

$$= \sqrt{\frac{m}{2\pi i\,\delta t}}\; e^{\frac{1}{2}m\left(\frac{q_{r+1}-q_r}{\delta t}\right)^2 - iV(q_r)\delta t}$$

minus!

or N+1?

$$K(q,q_0;T) = \left(\frac{m}{2\pi i\delta t}\right)^{N/2} \int \prod_r e^{i\,\text{—}}$$

$$\equiv \int \mathcal{D}q\; e^{iS[q]}$$

QM TELLS US TO USE $\mathcal{L}$ VS. $\cancel{H}$

(QFT VS. STAT. MECH)

→ see OSBORN §1.

$$\langle q | e^{i\hat{H}T} | q_0 \rangle = \int \mathcal{D}q(t)\; e^{i\int_0^T dt\, L(\dot{q},q)}$$

See p.12

UNITS: $e^{iS} = e^{iS/\hbar}$

of FEYNMAN PHASOR

CLASSICAL LIMIT: $\hbar \to 0$

steepest descent (sum over very fast osc.)

the BIGGEST CONTRIBUTION is CLASSICAL PATH

↳ EULER-$\mathcal{L}$

$\delta S = 0 \Rightarrow \frac{d}{dt}\frac{\delta L}{\delta \dot{q}} - \frac{\delta L}{\delta q} = 0$

sum of e^{iS} unit vectors