EFT in 90 minutes

Why EFT's ?

i) We don't know physics beyond a scale Λ, but it can affect observables below Λ $\Rightarrow$ we need a consistent, coherent & reliable way of parameterizing this ignorance (these effects)

ii) Sometimes can't do ab-initio calculations (eg QCD @ low energies) $\Rightarrow$ use EFT's to do calculations in those "difficult" regimes.

What do we know?

i) The particles we are going to observe

ii) The symmetries of the corresponding fields

e.g. if looking @ BSM physics.

particles $\rightarrow$ all SM particles

symmetries $\rightarrow$ $U(1) \times SU(2) \times SU(3)$

[global symmetries like B & L are accidental]
actually, B-L

Another example, low energy QCD

particles $\rightarrow$ pions

symmetries $\rightarrow$ $SU(2) \times SU(2)$ chiral

Will use the first example below (low energy QCD requires more background)

The general recipe

Construct all local operators using the fields of interest & obeying the symmetries of interest

The effective Lagrangian is then

$$\mathcal{L}_{eff} = \sum c_{\mathcal{O}} \mathcal{O}$$

local operator

which is renormalizable:

- All divergences correspond to local operators

$\Rightarrow$ renormalize one of the $\mathcal{O}$ (absorbed in one of the $c_{\mathcal{O}}$)

The problem is: ∞ coefficients $\Rightarrow$ ∞ data to fix the $c_{\mathcal{O}}$!

Hierarchies

<u>However</u>: experimental precision is not zero

Successful EFT's have an ordering

$\{\mathcal{O}\} = \{\mathcal{O}\}_{\text{lead}} \cup \{\mathcal{O}\}_{\text{sub-lead}} \cup \{\mathcal{O}\}_{\text{sub-sub-lead}} \cdots$

such that the effects of each group get smaller $\Rightarrow$ eventually can be ignored

Example: for BSM physics (assumed weakly coupled):

if $\dim \mathcal{O} = n \Rightarrow c_{\mathcal{O}} \propto \Lambda^{4-n}$

scale of NP (example below)

$\Rightarrow$ large $n \Rightarrow$ small $c_{\mathcal{O}}$

$\Rightarrow \{\mathcal{O}\}_{\text{lead}} \rightarrow \dim \leq 4$

$\{\mathcal{O}\}_{\text{sub-lead}} \rightarrow \dim = 5$

$\{\mathcal{O}\}_{\text{sub-sub-lead}} \rightarrow \dim 6$

Example

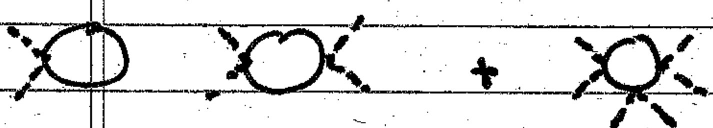
"SM": ϕ Symmetry: $\phi \leftrightarrow -\phi$

"NP": Φ

If we knew the full theory

$$\mathcal{L} = \tfrac{1}{2}(\partial\phi)^2 - V_\phi(\phi) + \tfrac{1}{2}(\partial\Phi)^2 - \tfrac{\lambda}{4}\phi^2\Phi^2 - V_\Phi(\Phi)$$

We see Φ only virtually



+ many others

Assume $V_\Phi = \frac{1}{2}M^2\Phi^2 + \cdots$

Need: $-\frac{1}{2i}\mathrm{Tr}\ln(\Box + j + M^2)$; $j = \frac{1}{2}\lambda\phi^2$

$$= -\frac{1}{2i}\mathrm{Tr}\left(\frac{1}{\Box+M^2}\, j - \frac{1}{\Box+M^2}\, j\, \frac{1}{\Box+M^2}\, j + \cdots\right)$$

1st term $\propto \int d^4x\, \phi^2(x) \rightarrow$ unobservable

2nd term $\frac{1}{2i}\int d^4x\, d^4y\, d^4p\, d^4q\; e^{-i(p-q)(x-y)}\, j(x)\, j(y)$

$$\frac{1}{p^2-M^2}\,\frac{1}{q^2-M^2}$$

$\Rightarrow$ need $\int d^4q \, \frac{1}{q^2+M^2} \frac{1}{(q+p)^2+M^2}$

$$= \int_0^1 dx \int d^4q \, \frac{1}{(q^2-\xi)^2} \qquad \xi = M^2 - x(1-x)p^2$$

$$= \frac{2\pi^2 i}{(2\pi)^4} \int_0^1 dx \int_0^\Lambda dq \, \frac{q^3}{(q^2+\xi)^2}$$

$$= \frac{i}{8\pi^2} \int_0^1 dx \left\{ \tfrac{1}{2} \ln \frac{\Lambda^2}{\xi} - 1 \right\}$$

$$= \frac{i}{8\pi^2} \left\{ \tfrac{1}{2} \ln \frac{\Lambda^2}{M^2} - 1 - \tfrac{1}{2} \int_0^1 dx \ln \left[1 - x(1-x) \frac{p^2}{M^2} \right] \right\}$$

$$= \frac{i}{8\pi^2} \Big\{ \underbrace{\ln \frac{\Lambda}{M} - 1}_{\text{renormalizes } \phi^4(x)} + \frac{1}{12} \frac{p^2}{M^2} + \frac{1}{120} \left(\frac{p^2}{M^2} \right)^2 + \cdots \Big\}$$

Observable terms

$$\frac{1}{16\pi^2} \left\{ -\frac{1}{12} \int d^4x \, \phi^2(x) \frac{\Box}{M^2} \phi^2(x) \right.$$

$$\left. + \frac{1}{120} \int d^4x \, \phi^2(x) \frac{\Box^2}{M^4} \phi^2(x) \; \cdots \right\}$$

To note: · divergences are unobservable

· finite terms vanish as $M \to \infty$

· there's a "loop factor" of $\frac{1}{16\pi^2}$

If we don't know the NP

— Light fields: ϕ

— Light symmetries: $\phi \leftrightarrow -\phi$

$\Rightarrow$ $\mathcal{O}$: n ϕ's & l ∂'s $\rightarrow$ dim $n+l$

n: even (symmetry) $\quad$ l: even (Lor. inv.)

dim ≤ 4 : unobservable (renormalize $\frac{1}{2}(\partial\phi)^2 - V_\phi$

dim $= 6$: $\phi\,\Box^2\phi$, $\phi^2\,\Box\,\phi^2$, ϕ^6

$$\Rightarrow \mathcal{L}_{eff} = \frac{1}{\Lambda^2}\{\underbrace{c_1\phi\,\Box^2\phi}_{(\Box\phi)^2} + \underbrace{c_2\,\phi^2\Box\phi^2}_{\frac{4}{3}\phi^3\Box\phi} + c_3\,\phi^6\} + \cdots$$

Can show that the S matrix only depends on

$$\frac{\lambda_\phi^2}{36}c_1 - \frac{2}{9}\lambda_\phi c_2 + c_3$$

(when $V_\phi = \frac{1}{2}m^2\phi^2 + \frac{1}{24}\lambda_\phi\phi^4$)

not on $c_{1,2,3}$ independently. All leading!! NP effects are

parametrized by 1 coupling

A cautionary tale

$$\mathcal{L}_{\text{eff}} = \tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{2}m^2\phi^2 - \tfrac{1}{24}\lambda\phi^4$$

$$+ \frac{c_1}{\Lambda^2}\phi\,\Box^2\phi + \frac{c_2}{\Lambda^2}\phi^2\Box\phi^2 + c_3\phi^6\frac{1}{\Lambda^2}$$

$\Rightarrow$ the propagator is $\left(p^2 - m^2 + \frac{2c_1}{\Lambda^2}p^4\right)^{-1}$

with poles @ $p^2 = -\frac{1}{4c_1}\left[\Lambda^2 \pm \sqrt{1+8c_1\frac{m^2}{\Lambda^2}}\right]$

$$= \begin{cases} -\dfrac{\Lambda^2}{2c_1} - m^2 + \cdots \\ m^2 - 2c_1\dfrac{m^4}{\Lambda^2} + \cdots \end{cases}$$

but (even if $c_1 < 0$) the pole @ $\sim\Lambda^2$ is not physical: the theory is valid only at scales $\ll \Lambda$.

Renormalization

Generic operator has

$$\left.\begin{array}{l} \cdot\ d \text{ derivatives} \\ \cdot\ b \text{ Boson fields} \\ \cdot\ f \text{ Fermion fields} \end{array}\right\} \Rightarrow \mathcal{O} \sim D^d B^b F^f$$

the coefficient is

$$c_{\mathcal{O}} \sim \lambda(b,f)\, \Lambda^{-\Delta_{\mathcal{O}}}$$

$$\Delta_{\mathcal{O}} = \dim \mathcal{O} - 4 = b + \tfrac{3}{2} f + d - 4$$

A graph with I_b boson lines

I_f fermion lines

L loops

and vertices with d_v derivatives

has a naïve deg of divergence

$N_{div} = 4L - 2I_b - I_f + \sum d_v - d$

(d from an overall mom. conserv. δ-funct.)

If the graph renormalizes $\mathcal{O}$ & has

vertices generated by $\{\mathcal{O}_v\}$ then

$N_{div} = \sum \Delta_{\mathcal{O}_v} - \Delta_{\mathcal{O}}$

leading!

Check: power of Λ (using Λ as cutoff)

- each $\mathcal{O}_v$: $\Lambda^{-\Delta_{\mathcal{O}_v}}$

- divergence : $\Lambda^{N_{div}}$

total : $\prod \Lambda^{-\Delta_{\mathcal{O}_v}} \cdot \Lambda^{N_{div}} = \Lambda^{-\Delta_{\mathcal{O}}}$

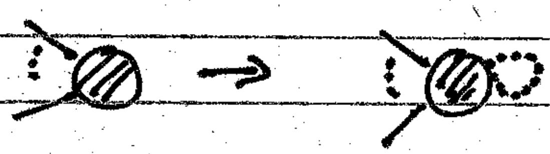
same as $\mathcal{O}$ ✓

Natural estimate of λ:

$$\delta\lambda = \frac{1}{(16\pi^2)^L} \prod_v \lambda(b_v, f_v)$$

$\Rightarrow \quad \lambda \sim \delta\lambda$ for any graph

i) Replace $\mathcal{O}_v \to B^2\, \mathcal{O}_v$ for one $\mathcal{O}_v$



$$\Rightarrow \quad \lambda(b+2, f) \cdot \frac{1}{16\pi^2} = \lambda(b, f)$$

$$\Rightarrow \quad \lambda(b, f) = (4\pi)^{b-1}\, \lambda(1, f)$$

ii) For fermions:

$$\lambda(b, f) = (4\pi)^{f-2}\, \lambda(b, 2)$$

$\Rightarrow \quad \lambda(1, f) = (4\pi)^{N_0} \qquad N_0 = \#\text{ fields} - 2$

$\qquad\qquad\qquad\qquad\qquad\quad = b + f - 2$

Types of divergences

- If $N_{div} = 0$ $\quad \delta c_{\mathcal{O}} \sim \frac{(4\pi)^{N_{\mathcal{O}}}}{\Lambda^{\Delta_{\mathcal{O}}}} \times (\text{power of } \ln\Lambda)$

- If $N_{div} > 0$ $\quad \delta c_{\mathcal{O}} \sim \frac{(4\pi)^{N_{\mathcal{O}}}}{\Lambda^{\Delta_{\mathcal{O}}}} \left\{ 1 + \text{const} \frac{m}{\Lambda} + \cdots + \text{const} \left(\frac{m}{\Lambda}\right)^{N_{div}} \times (\text{pow. } \ln\Lambda) \right\}$

m = light mass

RG from $\ln\Lambda$ terms. $\Rightarrow$ leading RG

from $N_{div} = 0$ graphs : $\sum \Delta_{\mathcal{O}_v} = \Delta_{\mathcal{O}}$

If $\Delta_{\mathcal{O}} \geq 0$ $\Rightarrow$ $\Delta_{\mathcal{O}} \geq \Delta_{\mathcal{O}_v}$

$\Rightarrow$ the leading RG flow for $\mathcal{O}$ is generated by lower dim operators

Subleading RG effects are $\propto$ to a light mass

$A_0 \geq 0$ <u>except</u> for SR vertices $\sim B^3$

& $B =$ scalar.

If $\mathcal{L} \supset \mu B^3$ & $\mu \sim O(\Lambda)$

$\Rightarrow m_B \sim \Lambda \Rightarrow \mu \sim$ light mass and

SR effects are also subleading.

PTG operators

Weakly coupled N.P.

- $\mathcal{O}$ generated @ tree level

$$c_{\mathcal{O}} \sim \frac{1}{\Lambda^n} \prod(\text{couplings})$$

- $\mathcal{O}$ generated @ L-loops

$$c_{\mathcal{O}} \sim \frac{1}{(16\pi^2)^L} \frac{1}{\Lambda^n} \prod(\text{couplings})$$

Tree generated $\longrightarrow$ more sensitive to NP.

Given (mild) assumptions about N.P. :

$$\{\mathcal{O}\} \to \{\mathcal{O}\}_{\text{loop}}, \quad \{\mathcal{O}\} \to \{\mathcal{O}\}_{\text{PTG}}$$

$\{\mathcal{O}\}_{\text{loop}}$: $\mathcal{O}$'s <u>always</u> generated by loop.

$\{\mathcal{O}\}_{\text{PTG}}$: $\mathcal{O}$ that <u>may</u> be generated @ tree level (Potentially Tree Generated)

Example (w/o proof)

If the NP (BSM) is a gauge theory with scalars & fermions

$$\mathcal{O} \simeq \epsilon_{IJK} W^{I}{}_{\mu}{}^{\nu} \cdot W^{J}{}_{\nu}{}^{\alpha} W^{K}{}_{\alpha}{}^{\mu}$$

is <u>always</u> loop generated. But

$$\mathcal{O} = (\bar{\ell}\gamma_\mu \ell)(\bar{\ell}\gamma^\mu \ell)$$

is PTG

Given a PTG $\mathcal{O}$: can easily find the NP that does generate it @ tree level.

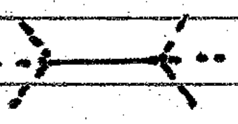
Example: "SH" : $\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - V(\phi)$

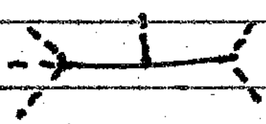
effective: $\mathcal{L}_{eff} = \frac{c}{\Lambda^2}\phi^6$

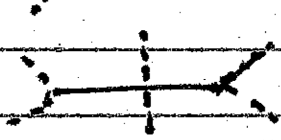
$\mathcal{O} \sim$  graph with I internal lines, V vertices & zero loops. 6 ext. legs

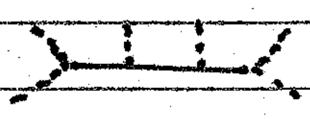
$\Rightarrow 0 = I - V + 1 \qquad 3V_3 + 4V_4 = 2I + E$

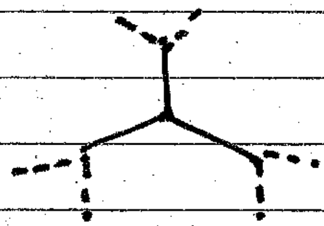
(assumed renorm. NP $\Rightarrow$ only vertices w/3 or 4 legs ; $V = V_3 + V_4$) $\Rightarrow$ graphs:

 heavy scalar

 ✓ ✓

 ✓ ✓ or vector

 ✓ ✓



heavy scalar

Equivalence thm

A QM example:

Classical Lagrangian $L = \frac{1}{2} m \dot{x}^2 - V$

classical e.o.m. $m\ddot{x} + V' = 0$

Modified Lagrangian

$$L \to L + \underbrace{\epsilon A(x)(m\ddot{x} + V')}_{-\epsilon(mA'\dot{x}^2 - AV') + \text{tot time der.}} + O(\epsilon^2)$$

$$\Rightarrow p = \frac{\partial L}{\partial \dot{x}} = m(1 - 2\epsilon A')\dot{x}$$

$$H = p\dot{x} - L = \underbrace{\frac{p^2}{2m} + V}_{H_0} + \epsilon \underbrace{\left(-\frac{A'}{m}p^2 + AV'\right)}_{H'} + \cdots$$

Quantize with

$$A'p^2 \to \frac{1}{4}\{\{p, A'\}p\}$$

then

$$H = UH_0U^\dagger + \cdots \quad U = \exp\left(-\frac{i}{2}\epsilon\{p, A\}\right)$$

The point: if $L \to L + \delta L = L'$

$\uparrow$ vanishes "on shell" (when the eom from L are applied)

$\Rightarrow$ the theories defined by L & L' are unitarily equivalent $\Rightarrow$ the same theory

Similarly if $\mathcal{L}_{tot} = \mathcal{L} + \Sigma c_{\mathcal{O}} \mathcal{O}$

and $\mathcal{O}$ & $\mathcal{O}'$ are such that $\mathcal{O}' - a\mathcal{O} = 0$

on shell

$$\mathcal{L}_{tot} = \mathcal{L} + \underbrace{c_{\mathcal{O}} \mathcal{O} + c_{\mathcal{O}'} \mathcal{O}'}_{(c_{\mathcal{O}} + c_{\mathcal{O}'} a)\mathcal{O} + c_{\mathcal{O}'}(\mathcal{O}' - a\mathcal{O})} + \cdots$$

$\Rightarrow \mathcal{L}_{tot}$ is unit. equivalent to

$$\mathcal{L}_{tot}' = \mathcal{L} + (c_{\mathcal{O}} + c_{\mathcal{O}'} a)\mathcal{O} + \cdots$$

the S-matrix depends on $c_{\mathcal{O}} + a c_{\mathcal{O}'}$ not $c_{\mathcal{O}}$

& Co' separately

Application: $\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - V$

$$\mathcal{L}_{eff} = \frac{1}{\Lambda^2}\{c_1 \phi \Box^2 \phi + c_2 \phi^2 \Box \phi^2$$

$$+ c_3 \phi^6\} + \cdots$$

write $\phi \Box^2 \phi = (\Box\phi)^2 + \text{tot. der.}$

use eom: $\mathcal{E} = \Box\phi + V' = 0$

write $\Box\phi = \mathcal{E} - V' \Rightarrow (\Box\phi)^2 = \mathcal{E}^2 - 2\mathcal{E}V' + V'^2$

similarly:

$$\phi^2 \Box \phi^2 = \frac{4}{3}\phi^3 \Box \phi + \text{tot der}$$

$$= \frac{4}{3}\phi^3 \mathcal{E} - \frac{4}{3}\phi^3 V'$$

So $\mathcal{L} + \mathcal{L}_{eff}$ is equiv. to $\mathcal{L} + \mathcal{L}_{eff}'$ with

$$\mathcal{L}_{eff}' = \frac{1}{\Lambda^2}\{c_1 V'^2 - \frac{4}{3} c_2 \phi^3 V' + c_3 \phi^6\}$$

with $V = \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{24}\phi^4$, $\quad V' = m^2\phi + \frac{\lambda}{6}\phi^3$

$$\mathcal{L}_{\text{eff}}' = (\text{terms} \propto \phi^2 \text{ \& } \phi^4) \leftarrow \text{renormalize } m^2 \text{ \& } \lambda$$

$$+ \frac{1}{\Lambda^2}\left\{ c_1 \cdot \frac{\lambda^2}{36} - \frac{2}{9} c_2 \lambda + c_3 \right\} \phi^6$$

If the NP generates c_1 or c_3, the low-energy theory cannot distinguish