following TONG - GAUGE THEORY

LAST TIME: triangle diagrams $\gamma_5 \triangleleft \gamma \gamma$

TODAY: ANOMALY CANCELLATION for MODEL BUILDING

NO ANOMALIES IN GAUGE SYM!

WHEN DO BAD THINGS HAPPEN? CHIRAL GAUGE THEORIES

← Weyl FERMIONS

not in VECTOR-LIKE THEORIES w/ all fermions DIRAL
(& no γ^5 interactions)

↓

if you can write a bare mass term
$M\bar{\Psi}\Psi \longleftrightarrow M(\psi_L\psi_R + \psi_L^\dagger\psi_R^\dagger)$

have opposite charge
& cancel contribution in triangle

ABELIAN CHIRAL GAUGE THY

WRITE EVERYTHING AS LH FERMION

↳ (RH FERMION)† = LH FERMION
SO THIS IS JUST A CHOICE OF WHICH PARTICLE IS "ANTI"

eg N_L LH FERMIONS w/ CHARGE Q^L_a for a^{th} type

N_R ——— " ——— Q^R_b ———

triangles cancel if $\underbrace{\sum_a^{N_L}(Q^L_a)^3 = \sum_b^{N_R}(Q^R_b)^3}_{\text{eg if VECTORLIKE (but more GENERAL)}}$

in terms of LH-only: $Q_i = \begin{cases} Q^L_a & \text{for } i=a, \\ -Q^R_b & \text{for } i=b \end{cases}$

$$\Rightarrow \boxed{\sum_{i=1}^{N_L+N_R}(Q_i)^3 = 0}$$

eg (if) CHARGE IS QUANTIZED & $Q_i \in \mathbb{Z}$, then for 3 TYPES OF FERMION w/ $Q = \{x, y, -z\}$

↑ @ least one is neg.

$$\Rightarrow \underbrace{x^2 + y^3 - z^3 = 0}$$

no solution (FERMAT)

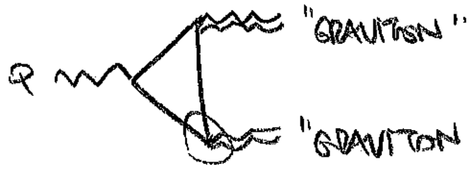
SIMPLE NONTRIVIAL CASES: 4 types:

$$3^3 + 4^3 + 5^3 = 6^3 \qquad (=216)$$

$$1^3 + 12^3 = 9^3 + 10^3$$ ← HARDY'S STORY ABOUT TAXI # 1729 @ RAMANUJAN'S HOSPITAL BED.

MIXED GRAVITATIONAL ANOMALY

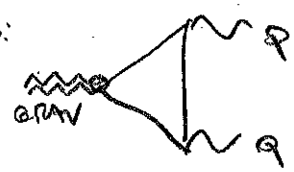
COUPLE TO GRAVITY ← not "quantum grav" just allowing for CURVED SPACE



+ CROSSED $\Rightarrow$ $\boxed{\sum_q^N Q_q = 0}$

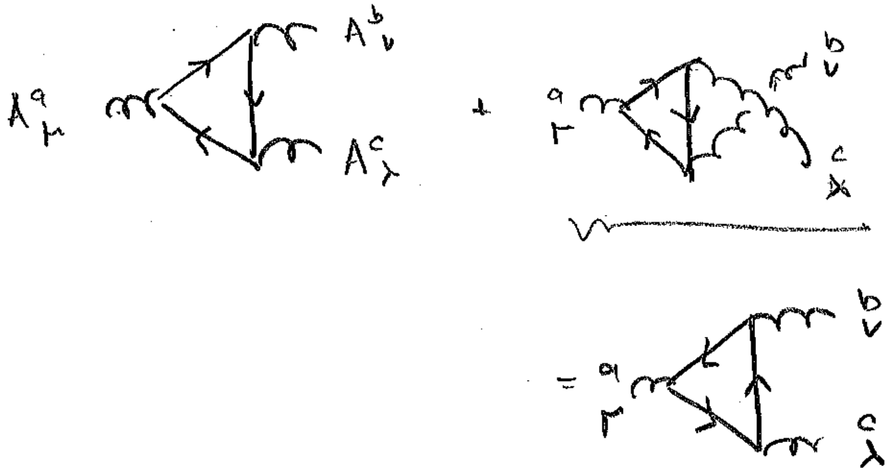
↑ REALLY CURRENT OF DIFFEOMORPHISM INVARIANCE

nb:



$= 0$ b/c DIFFEO IS NON ABELIAN s.t. tr (DIFFEO) $= 0$.

NON ABELIAN (one group, simple)



obs : symmetric w/ $\{\nu, \lambda\}$ $\rightarrow$ imposed on GROUP INDICES

but nothing special about ν & λ, really just orient. of fermion loop. $\rightarrow$ actually totally sym.

CHIRAL (ψ_L) or DIRAC w/ $\left(\frac{1\pm\gamma^5}{2}\right)$ COUPLINGS

RESULT: for a $\boxed{\text{WEYL}}$ fermion in Rep R $\rightarrow$ generator T^a_R,

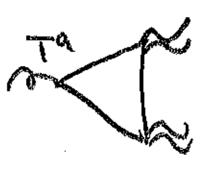
the contribution to the ANOMALY is

$$\propto d^{abc}(\underline{R}) = \mathrm{tr}\, T^a_R \{T^b_R, T^c_R\}$$

$$\Rightarrow \boxed{\sum_{i=1}^{N_L} d^{abc}(\underline{R}_{Li}) = \sum_{j=1}^{N_R} d^{abc}(\underline{R}_{Rj})}$$

$\rightarrow$ so: EQUAL # OF LH & RH FERMIONS IN SAME REP IS SUFFICIENT (vectorlike)

(MIXED NON-ABELIAN – GRAVITY



$$\sim \mathrm{tr}\, T^a_R = 0$$
)

ONLY CERTAIN GAUGE GROUPS CAN BE ANOMALOUS :

- $\boxed{d^{abc} = 0 \text{ for any } \mathbb{R} \text{ rep (eg ADJOINT)}}$

$d^{abc}(T_R)$

so gauginos are safe

or PSEUDO-REAL (SU(2) FUNDAMENTAL)

$H^a \longrightarrow H^\dagger_a$

$\searrow \varepsilon_{ab} H^b$

given $T^a \longrightarrow e^{i\alpha^a T^a} = g$

conjugate is $e^{-i\alpha^a T^{*a}}$

$$\bar{T}^a = -T^{*a} = -(T^a)^T \quad \text{(HERMITIAN)}$$

REAL: $T = \bar{T}$

$$\Rightarrow d^{abc} = \mathrm{tr}\, T^a\{T^b, T^c\} = -\mathrm{tr}\, \bar{T}^a\{\bar{T}^b, \bar{T}^c\}$$

$$= -\mathrm{tr}\, T^a\{T^b, T^c\}$$

$\boxed{\equiv 0}$

for $\mathbb{R}$ rep.

for PSEUDO$\mathbb{R}$: $\bar{T}^a = U T^a U^{-1}$ for some UNITARY U

so this follows trivially from tr.

REMARK : consistent w/ "massless fermion" property of ANOMALY

b/c MAJORANA ($\mathbb{R}$) fermions can have bare mass the only NON-ABELIAN GROUPS w/ $\mathbb{C}$ REPS:

$$\begin{cases} SU(N) & N \geq 3 \\ SO(4N+2) & \longrightarrow d^{abc} = 0 \text{ for } N \geq 2 \\ E_6 & \longrightarrow d^{abc} = 0 \end{cases}$$

$SO(6) \cong SU(4)$ @ ALGEBRA

($[U(1)]$ ← ABELIAN)

So: only $\boxed{SU(N) \quad N \geq 3}$ can give anomalies

↑ turns out to be IMPORTANT for SM ∵

So NOW WHAT? do we have to solve for $d^{abc}(R)$ for EACH REP?

EVEN EASIER: these factors ARE ALL RELATED (in the same way CASIMIR ops RELATED) so connect to FUNDAMENTAL REP

$$d^{abc}(R) = \underbrace{A(R)}_{\text{ANOMALY coefficient}} d^{abc}(\square)$$

← standard

↑ so just check that sum of ANOMALY COEFFICIENTS VANISH

eg: $A(\bar{\square}) = -A(\square)$

$$A(R_1 \times R_2) = \dim(R_1)\, A(R_2) + A(R_1)\, \dim(R_2)$$

"flavor" w/rt GAUGE

eg: $3 \otimes \bar{3} = 8 \oplus 1 \rightarrow A(8) = 3A(\square) + 3A(\bar{\square}) - A(1) = 3 + (-3) + 0 = 0$

↑ IR.

<u>REMARK</u>: SU(2) ANOMALY IS SPECIAL → nonperturbative

$$d^{abc}(\square) = \mathrm{tr}\, \sigma^a \{\sigma^b, \sigma^c\} = 0$$

<u>CLAIM</u>: SU(2) w/ ODD # WEYL FERMIONS IS ANOMALOUS

MOTIVATION:

DIRAC FERMION:

$$Z = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A \, e^{iS[A,\psi]}$$

$$= \int \mathcal{D}A \underbrace{\det i\not{D}}_{\text{REGULATED}} \, e^{iS[A]}$$

WEYL FERMION:

$$\det i\not{D} \to \det i\sigma^\mu D_\mu$$

BUT this is NOT a GOOD OPERATOR
MAPS LH FERMION to RH!

BETTER: $\det\left(i\not{D}\, \frac{1+\gamma^5}{2}\right) \stackrel{?}{=} \sqrt{\det i\not{D}}$

PROJECT OUT RH

SIGN AMBIGUITY

RELATED TO FERMI SEA UNDER CHANGE IN A!

↓

related to LARGE GAUGE TRANSFORMS
VACUUM STRUCTURE: $U(x) \to 1$ as $x \to \infty$
s.t. $\mathbb{R}^4 \to S^4$

$$U: S^4 \to SU(2) \longleftrightarrow \Pi_4(SU(2)) = \mathbb{Z}_2$$

can give either sign
by LARGE GAUGE TRANS.

(measure transforms!)

eg. Standard MODEL: $SU(3) \times SU(2) \times U(1)$ ← ×6

		SU(3)	SU(2)	U(1)
$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	→ Q	□	□	1
	u_R	□		4
	d_R	□		−2
$\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	→ L		□	−3
	e_R			−6

1 GEN SUFFICIENT

CHECK ANOMALY CANCELLATION

$\underline{SU(3)^3}$: vectorlike

$\underline{SU(2)^3}$: vectorlike

$$\underline{U(1)^3}: \quad \underbrace{\left[2\times(-3)^3 + 6\times(1)^3\right]}_{LH} - \underbrace{\left[(-6)^3 + 3\times(4)^3 + 3\times(-2)^3\right]}_{RH} = 0 \quad !!$$

$$\underline{U(1)\ GRAV^2}: \quad \left[2\times(-3) + 6\right] - \left[-6 + 3\times 4 + 3\times(-2)\right] = 0$$

MIXED ANOMALY: PAIRS OF NON-ABELIAN w/ U(1)

$$SU(2)^2 \times U(1) \quad : \quad \underbrace{(-3)}_{L} + \underbrace{3\times(1)}_{Q} = 0$$

$$SU(3)^2 \times U(1) \quad : \quad 2\times(1) - 4 - 2 = 0$$

HOW NONTRIVIAL IS THIS? LET q, u, d, l, e BE Y CHARGES

$$\begin{aligned} SU(2)^2 \times U(1) &: \quad 3q + l = 0 \\ SU(3)^2 \times U(1) &: \quad 2q - u - d = 0 \\ U(1)^3 &: \quad 6q^3 + 2l^3 - 3u^3 - 3d^3 - e^3 = 0 \end{aligned}$$

2 sols:

① trivial: $u = -d$; all else $= 0$

② $e = 2l = -3(u+d) = -6q$

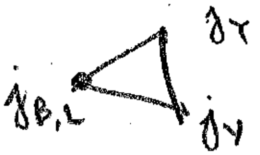
$u - d = \pm 6q$

ANOMALY CANCEL ⇒ QUANTIZE Y!

↳ not nec. evidence of GUT!

	Q	u_R	d_R	L	e_R	ν_R	
B:	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$				← BARYON #
L:				1	1	1	← LEPTON #

eg:



or other GAUGE

$$\sum_L BY^2 - \sum_R BY^2 = \frac{1}{3}\Big(6 - 3\times(4)^2 - 3\times(-2)^2\Big) = -18$$

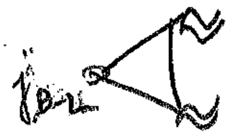
$\qquad Q \qquad u_R \qquad d_R$

$$\sum_L LY^2 - \sum_R LY^2 = 2(-3)^2 = -18$$

$\qquad L$

same anomaly!
⇒ (B-L) nonanom

GRAV.



$= (\frac{1}{3})\times 6$	$= 2$	←	Q
$-(\frac{1}{3})\times 3$	-1	←	u_R
$-(\frac{1}{3})\times 3$	-1	←	d_R
$(1)\times 2$	$+2$	←	L
$-(1)$	-1	←	e_R
$\boxed{-(1)}$	$\boxed{-1}$	←	$\boxed{\nu_R}$

RH ν REQ. to CANCEL MIXED (B-L)GRAV ANOMALY!