Zee 1.3

FIELD THEORY:

$$Z[J] = \int \mathcal{D}\phi \; e^{iS[\phi] + i\int d^d x \, J(x)\,\phi(x)}$$

Zee calls this "from mattress to field"

↳ Zee §1.3

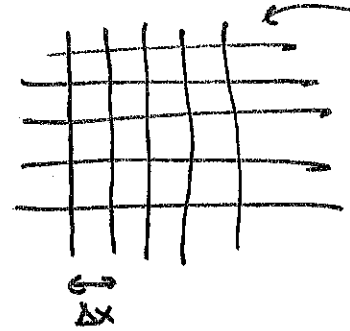
DISCRETE: $q_1, q_2, \cdots$ lattice of H.O. ← why?

$$S = \int_0^T dt \sum_i \frac{1}{2} M \dot{q}_i^2 - V[q]$$

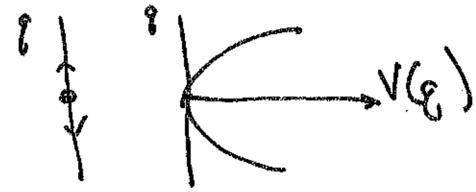
$$\uparrow$$

$$\sum_{\langle ij \rangle} \frac{1}{2} k_{ij} (q_i - q_j)^2$$

$\langle$n.n.$\rangle$



@ EACH SITE



$$\left[\begin{array}{lcl} q_i & \longrightarrow & q(x) \\ (q_i - q_j)^2 & \longrightarrow & \Delta x^2 \dfrac{\partial^2 q(x)}{\partial x^2} + \cdots \\ \sum_i & \longrightarrow & \displaystyle\int \frac{d^D x}{(\Delta x)^D} \\ M & \longrightarrow & \rho\, (\Delta x)^D \quad \leftarrow \text{density} \times \text{vol} \end{array}\right.$$

↳ not the "mass" of field.

$$S = \int_0^T dt \int \frac{d^D x}{(\Delta x)^D} \left[\frac{1}{2} \rho (\Delta x)^D \frac{\partial^2 q}{\partial t^2} - \frac{1}{2} (\Delta x)^2 k \frac{\partial^2 q}{\partial x^2} \right]$$

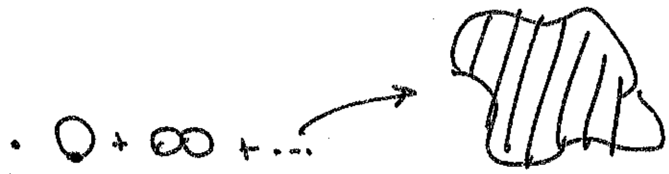
$$= \int dt \, d^D x \; \frac{1}{2} \left[\rho \frac{\partial^2 q}{\partial t^2} - \frac{k}{(\Delta x)^{D-2}} \frac{\partial^2 q}{\partial x^2} \right]$$

RESCALE FIELD (OR LET IT BE AN OVERALL NORM

$$\longrightarrow \int d^d x \; \frac{1}{2} \Big(\underbrace{\partial_t^2 - c^2 \partial_x^2}_{\partial^\mu \partial_\mu} \Big) \varphi^2 \leftarrow \varphi = q/\sqrt{\rho}$$

$d = D + 1$

INTUIT:



$\leftarrow Z[J]\big|_{J=0}$

PARTITION FUNC

HIT IT W/ $\delta/\delta J$'s TO PULL OUT EXT LEGS

$\left(\frac{\delta}{\delta J_1}\cdots\right)$  = 

= 

not interesting {

↳ in fact, these exponentiate ~~…~~

& factor out in $\frac{1}{N}\left(\frac{\delta}{\delta J}\right)^n Z\big|_{J=0}$

$N = Z[0]$

THINGS TO READ

HO: as much of §1 as you want

↳ eg. 1.22 "few comments"

① WICK ROTATION & CONVERGENCE

② INSTANTONS

eg. 1.2.1: harmonic osc.
SOLVE FOR GREEN'S FUNC.
EXPLICITLY.

other treatments:

Zee: All of ch.1 → more insight

Blundell (GIFTED AM.) ch. 21 - stat mech
22 - Gen func.
23 - QM

next wk: PATH INT for FIELDS

↳ 2ND half of Zee ch.1

HO: §2: BOSONS only

What you need: where FEYNMAN GRAPH-OLOGY come from

next step: tricks for
GEN FUNC OF connected DIAGRAMS
then GEN func of 1PI DIAGRAMS

<u>scalar field theory:</u>

$$Z[J] = \int \underbrace{\mathcal{D}\varphi}_{\text{or } d[\varphi]} \; e^{iS + i\int d^{d}x \; J(x)\varphi(x)}$$

$d = 1+D$

$S = \int d^{d}x \; \frac{1}{2}\left(\varphi \Theta \varphi - V(\varphi)\right)$

$$= e^{-i\int d^{d}x \, V(-i\,\delta/\delta J)} \; e^{-\frac{i}{2}\int d^{d}x \, d^{d}y \; J(x)\Delta(x-y)J(y)}$$

$\sum_{ij}$ $\qquad (A^{-1})_{ij}$

$$= e^{\frac{i}{2}\int d^{d}x \, d^{d}y \; \frac{\delta}{\delta\phi(x)}\Delta(x-y)\frac{\delta}{\delta\phi(y)}}$$

$$\times \; e^{i\int d^{d}x \left[-V[\phi(x)] + J(x)\phi(x)\right]}\Big|_{\phi=0}$$

using:

$$G\left(i\frac{\delta}{\delta J}\right)F[iJ] = F\left[\frac{\delta}{\delta\phi}\right]G[\phi]\, e^{i\int d^{d}x \, \phi(x)J(x)}$$