Peskin: ch. 12

$$Z[J] = \int \mathcal{D}\phi \; e^{i\int d^dx \, \mathcal{L} + J\phi}$$

this is an infinite product of H.O. $\rightarrow \prod_x d\phi(x) = \underbrace{\prod_k d\phi(k)}$

change of basis

WILSONIAN RG:

well, a scale at least

A THEORY IS DEFINED W/RT A CUTOFF, Λ
TREAT THIS AS THE SCALE AT WHICH THEORY IS NO LONGER VALID.

RG: what happens as we change Λ?

SPECIFICALLY: LET'S STAY IN THIS THEORY, $Z[J]$ but perform some of the $d\phi(k)$ integral.

$$Z[J] = \int \mathcal{D}_\Lambda\phi \; e^{i\int d^dx \cdots}$$

$\prod_{|k|<\Lambda} d\phi(k)$ } what we mean by cutoff

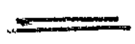
LET US perform an integral over momenta close to the cutoff: $b\Lambda < |k| < \Lambda$

$\boxed{b \lesssim 1}$

will end up with

$$Z[J] = \int \mathcal{D}_{b\Lambda}\phi \; e^{i\int \cdots}$$

thy w/ lower cutoff.

WRITE  to be a $\hat{\phi}$ LINE (hi momentum only) $\leftarrow b\Lambda < k < \Lambda$

 to be ϕ LINE ($k < b\Lambda$)

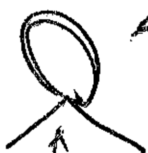
WE HAVE "GENERATED" MIXED INTERACTIONS:

 as well as 

with Feynman rules $\sim \lambda$ up to combinatorics
(these "new" rules were always there, we're just making a big deal about identifying the kinds of momentum modes)

nb: $\hat{\phi}$ PROPAGATOR HAS A MOMENTUM RESTRICTION.

then we can calculate

 $\leftarrow$ integrate over a shell of momenta near Λ

$\frac{\lambda}{4}$

$$= \frac{\lambda}{2}\int_{b\Lambda}^{\Lambda} d^d k \, \frac{1}{k^2} = \frac{\cancel{\lambda}}{(4\pi)^{d/2}\,\Gamma(d/2)} \; \frac{1-b^{d-2}}{d-2}\,\Lambda^{d-2}$$

$$\equiv \Delta m^2$$

$$Z = \int \mathcal{D}_{b\Lambda}\phi \; e^{iS[\phi]} \int \mathcal{D}\hat{\phi} \; e^{iS[\hat{\phi}] + iS_{int}[\phi,\hat{\phi}]}$$

$\uparrow$ $J=0$ for simplicity $\qquad$ $b\Lambda < k < \Lambda$

doing the  loop is integrating out the $\hat{\phi}$ in $iS_{int}[\phi,\hat{\phi}]$ and leaving a correction to $S[\phi]$

SIMILARLY:  $= \dfrac{-3\lambda^2}{16\pi^2}\,\underbrace{\dfrac{1-b^{d-4}}{d-4}}\,\Lambda^{d-4}$ $\to 1-\epsilon\Lambda$

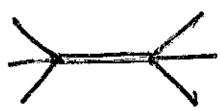
$d = 4-\epsilon$

$b^{d-4} = e^{\ln b^{-\epsilon}} \approx 1-\epsilon\ln b$

$= \dfrac{-3\lambda^2}{16\pi^2}\ln\dfrac{1}{b}$

$= \delta\lambda$ (& similar diagrams)

BUT WE ALSO GET <u>NEW</u> INTERACTIONS:

 $\Leftarrow$ gives ϕ^6 interaction!

(& more @ higher order)

$$Z = \int \mathcal{D}_{b\Lambda}\phi\, e^{i\int d^4x\, \mathcal{L}_{eff}} = \int \mathcal{D}_{\Lambda}\phi\, e^{i\int d^4x\, \mathcal{L}_{ORIG}}$$

$\uparrow$

$= \mathcal{L}_{ORIG}$ + CORRECTIONS & new terms

how are $\mathcal{L}_{orig}$ & $\mathcal{L}_{eff}$ RELATED?
LET'S TRY TO COMPARE THEM

Rescale dummy variables of $\mathcal{L}_{eff}$ thy to give same appearance of $\mathcal{L}_{orig}$ thy

$K' = \dfrac{K}{b} \longrightarrow x' = xb$ (st FOURIER TRANSF UNCHANGED)

s.t. this goes up to Λ
in the $K < b\Lambda$
regime

$$S_{eff} = \int d^d x \quad \tfrac{1}{2}(1+\Delta Z)(\partial\phi)^2 + \tfrac{1}{2}(m^2+\Delta m^2)\phi^2$$

$$+ \tfrac{1}{4!}(\lambda+\Delta\lambda)\phi^4 + \Delta C(\partial\phi)^4 + \Delta D\,\phi^6$$

$$= \int d^d x' \left(b^{-d}\right)\Big[\ \tfrac{1}{2}(1+\Delta Z)b^2(\partial\phi)^2 + \tfrac{1}{2}(m^2+\Delta m^2)\phi^2$$

$$+\tfrac{1}{4!}(\lambda+\Delta\lambda)\phi^4 + \Delta C b^4(\partial\phi)^4 + \Delta D\,\phi^6 + \cdots\Big]$$

canonical normaliz-

$$\phi' = \sqrt{b^{2-d}(1+\Delta Z)}\ \phi$$

$$= \int d^d x'\Big[\ \tfrac{1}{2}(\partial\phi')^2 + \tfrac{1}{2}(m')^2(\phi')^2 + \tfrac{1}{4!}\lambda'(\phi')^4 + \cdots\Big]$$

$$(m')^2 = \frac{(m^2+\Delta m^2)}{1+\Delta Z}\, b^{-2}$$

$$\lambda' = \frac{\lambda+\Delta\lambda}{(1+\Delta Z)^2}\, b^{d-4}$$

$$C' = \frac{C+\Delta C}{(1+\Delta Z)^2}\, b^{d}$$

$$D' = \frac{D+\Delta D}{(1+\Delta Z)^3}\, b^{2d-6}$$

now we are transforming in space of theories.

SUPPOSE WE SCATTER PARTICLES @ $p_i \ll \Lambda$

CAN USE $\mathcal{L}_{ORIG}$ OR $\mathcal{L}_{EFF}$, w/ momentum shells integrated out until p_i

PREDICTIONS ARE THE SAME.

(BUT): FOR $\mathcal{L}_{ORIG}$, HI-K FLUCTUATIONS APPEAR @ LOOP ORDER

FOR EFF. $\mathcal{L}$: HI-K FLUCTUATIONS ALREADY ACCOUNTED FOR IN $\mathcal{L}_{eff}$ PARAMS.

INCLUDING "NONRENORMALIZ." TERMS.

What does this do?

STUDY THY NEAR "GAUSSIAN FIXED POINT" (free)

↳ ALL $\Delta m^2, \Delta\lambda, \ldots$ ARE HIGHER ORDER IN (SMALL)

THEN WILSONIAN RG: $(b<1)$

$(m')^2 = m^2 b^{-2}$ → GROWS IN IR! (RELEVANT)

$\lambda' = \lambda b^{d-4}$ → MARGINAL (ANOM. DIM MATTERS)

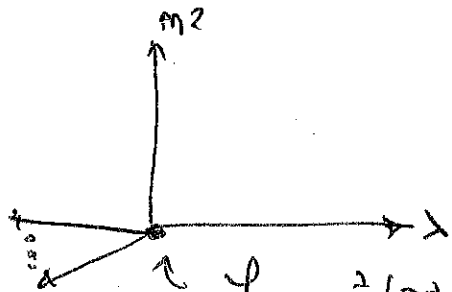
$C' = C b^{d}$ → SHRINKS IN IR

$D' = D b^{2d-6}$ ↗ (IRRELEVANT)

→ next page

RG FLOW : changing Λ (or μ in REN. COND.) changes your theory.

IMAGINE A **THEORY SPACE** :



other params in $\mathcal{L}$

$\mathcal{L}_0 = \frac{1}{2}(\partial\phi)^2$, free $\mathcal{L}$.

RG is a flow through this space.
↳ transformation is a **RESCALING**
so endpoints are scale invariant theories (CFT).

start in **neighborhood** of $\mathcal{L}_0$ in theory space.
ASSUME ALL PARAMS ARE "SMALL" (w/ approp def of small for dimensionful things)

then $\underbrace{\Delta m^2, \Delta\lambda}$, etc are smaller

all loop induced, H.O. in coupling

RG transf:

$$m'^2 = m^2 \, b^{-2} \quad \leftarrow \text{GROWS: RELEVANT}$$

$$\lambda' = \lambda \, b^{d-4}$$

$$\left.\begin{aligned} c' &= c \, b^{d} \\ D' &= D \, b^{2d-6} \end{aligned}\right\} \text{SHRINK: IRREL}$$

$b < 1$

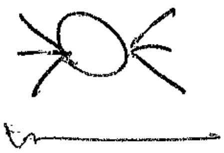
marginal in $d=4$ (dep on loop correction)
(OBVIOUS FROM DIM ANALYSIS)

this ties to another point:

RENORMALIZABILITY

IF YOU ~~HAVE~~ *need* A counter-term ... the ORIGINAL TERM HAD to BE IN $\mathcal{L}$.

eg. $g_5 \phi^5$ int generates

 $\sim \int d^4K \frac{1}{K^4} \sim \log \Lambda$

this needs a $\Delta g_6 \phi^6$ term

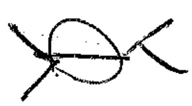
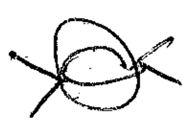
... so orig $\mathcal{L}$ HAD A $g_6 \phi^6$ term.

but if you have $g_6 \phi^6$...

 $\sim \log \Lambda \Rightarrow \Delta g_8 \phi^8$

and ad infinitum!

CRISIS: PREDICTIVITY. If you want to truly measure λ_4, need to also measure all the others b/c they contribute:

 +  + ...

$\sim g_6^2$ $\quad\quad$ $\sim g_8^2$

need ∞ renorm. conditions

NB: if we stick to RENORMALIZABLE THY (relevant + marginal terms only), no such problem.

OLD STYLE: only Renormalizable theories make sense

↳ else: "non-predictive, non-theory."

RG flow: IN IR, NONREN terms stop mattering.

↳ eTheory projects onto a subspace of RELEVANT / MARGINAL params.

(ie onto a slice of REN theories)

What about predictivity?

↳ didn't we have to absorb ∞'s into counter terms ad infinitum?

↳ BECAUSE IRRELEVANT TERMS GET SMALL, WE CAN DESCRIBE THAT BY REN. INT. and least-non-ren terms.

↳ effective theory.

↳ Jose will discuss on Tue.

to do: Polchinski RG flow.