P230B LEC 8 | WIN 2020 | 30 Jan

Regularization & Renormalization

WORK w/ FINITE FIELD THEORY

make sense of scale dependence

INTRODUCE SCALE

go from bare Lagrangian to "renormalized" Lagrangian

WILL EXPLAIN

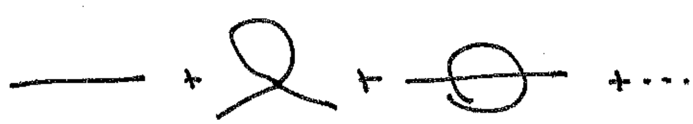
... w/ counterterms

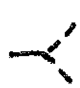
REALLY JUST SHIFTING PARAMS s.t. the MASSES, COUPLINGS WE WRITE MEAN WHAT WE THOUGHT THEY MEANT @ TREE LEVEL.

the lesson: interactions make theory weird.

nonlinearities in S

QFT vs. QM : even what we mean by "particle" is subtle!



if $\exists$  + 

is  possible?

then ϕ unstable

... picks up a width

$m^2 \to m^2 + i\Gamma m$

[from BLUMENTHAL & LANCASTER] ← nice qualitative discussion

↳ *if you really want to be a practitioner, then* DO the "DEEP" READING, DO THE PROBLEMS!

(dressed/ renormalized/ quasi-) PARTICLE = (on shell) PARTICLE + MULTIPARTICLE STATES (from int.)

in operator language:

if we had a full solution to S, then some creation operator $a^{F\,+}_{p}$ creates excitations.

↳ no interactions – it's just a complete state that propagates. only trivial diagrams.

BUT WE ONLY HAVE FULL SOLUTIONS TO THE QUADRATIC PART OF S (kinetic term + mass) SO WE DO PERT THY W/RT a_p^+.

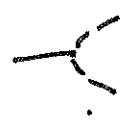
a_p^+ DOES NOT CREATE single states of "full" thy.

$$a_p^+ |0\rangle = |p\rangle \underbrace{\langle p| a_p^\dagger |\Omega\rangle}_{\sqrt{Z_p}} + \left(\begin{array}{c}\text{multiparticle}\\ \text{continuum}\end{array}\right)$$

$|0\rangle$: QUANTUM VAC ($|\Omega\rangle$)

$|p\rangle$: 1 PARTICLE STATE : $p^2 = m^2$

$\sqrt{Z_p}$: AMPLITUDE TO CREATE $|p\rangle$ state from free creation operator.

multiparticle continuum: eg 

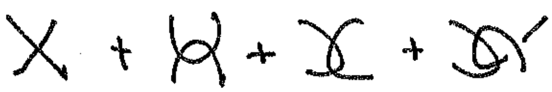
$p^2 = \underline{(k_1+k_2)^2}$

continuum above $(2M)^2$

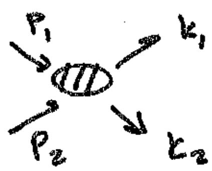
Significance:

Field strength renormaliz: $\boxed{\text{See prev Lec p. 12 - 14}}$

BLUNDELL ex 82.1

 $= -i\lambda + ia\lambda^2 \sum_I V_I(P_I^2)$

$V_I = \ln\Lambda^2 - \ln P_I^2$



$s = (p_1+p_2)^2 = P_I^2$

$t = (p_1-k_1)^2 = P_{II}^2$

$u = (p_1-k_2)^2 = P_{III}^2$

a is some #

from HARD CUT: $\int_0^\Lambda d^4 k \frac{i}{k^2-m^2+i\epsilon}\frac{i}{(p_1-k)^2-m^2+i\epsilon} = -2ia\ln\frac{\Lambda^2}{p^2}$

killed by sym fac.

s.t. $i\mathcal{M} = -i\lambda + ia\lambda^2(\underbrace{3\ln\Lambda^2} - \ln s - \ln t - \ln u)$

this is the "bad" term (well... the worst one)

IN TERMS OF RENORMALIZED FIELD $\phi = \sqrt{Z}\,\phi_R$
$= (1+\delta_Z)\phi_R$

$\mathcal{L} = \frac{1}{2}Z(\partial\phi_R)^2 - \frac{1}{2}m_0^2 Z\phi_R^2 - \frac{\lambda_0}{4!}Z^2\phi_R^4$

BARE PARAMS - not "physical"

$\boxed{Z m_0^2 = m^2 + \delta_m}$ $\boxed{Z^2\lambda_0^2 = \lambda + \delta_\lambda}$

where m^2 & λ are correct @ some kinematic point

renorm. condition comes w/ a scale !

@ 1 loop LEVEL, $Z = 1$

did not give p^2-dependence

& that would correct $(\partial\phi)^2$

REN COND :

physical

$= -i\lambda_p$ @ some kinematic config (choice of s, t, u)

1PI 4-POINT (AMPUTATE the 4-POINT)

"@ this specific kinematic configuration,

Γ_4 is exactly λ

(tree level result; loop cancels against counter term by choice of counter term.)

CALL THIS CHOICE OF CONFIG: s_0, t_0, u_0

eg $(4m^2, 0, 0)$

then : $i\mathcal{M}|_{\dots} = -i\lambda \underbrace{+ia\lambda^2(3\ln\Lambda^2 - \ln s - \ln t - \ln u) - i\delta_\lambda}$

$= -i\lambda_p$ @ $s = s_0, \dots$ $\qquad i\delta_\lambda = -ia\lambda^2(3\ln\Lambda^2 - \ln s_0 - \dots)$

$-i\lambda = -i\lambda_p - ia\lambda_p^2(3\ln\Lambda^2 - \ln s_0 - \dots)$

nlo corr.

$i\mathcal{M} = -i\lambda_p - ia\lambda_p^2\left(\ln\frac{s}{s_0}, \ln\frac{t}{t_0}, \ln\frac{u}{u_0}\right) + \dots$

OBSERVE: Λ does not show up in $i\mathcal{M}$.
no logs of UV to IR scale.

BUT: DOES introduce dependence on s_0, t_0, u_0
for simplicity (to match BLUNDELL + LANCASTER)
↳ set $s_0 = t_0 = u_0 = \mu$, some scale.

(nb: here I think the Lancaster book misses the main points — let's discuss;
BETTER: POLCHINSKI - renormaliz of eff $\mathcal{L}$)

$$i\mathcal{M} = -i\lambda_p - ia\lambda_p^2\left(\ln\left(\frac{s}{\mu^2}\right) + \cdots\right)$$

PERTURBATION THY:
second term smaller than first.

... uh: BUT WHAT IF this term is BIG!

↳ this is a problem

DEF of theory (Wilsonian)

params in $\mathcal{L}$ and scale (μ) @ which we define them in some way.

$\boxed{(m, \lambda, \mu)}$

λ_p
(DEF P relation)

call this $\lambda(\mu)$

(m', λ', μ') — treat μ as a PARAMETER OVER THY SPACE

↳ different theory in same class.

$\boxed{\text{DIFFERENT}}$

@ this point there is no reason to connect these two completely different theories.

Renormalization (group):

by changing theory, you can avoid nonperturbative regime

"flow" in theory space

change the theory a little bit (so parameters of the are DIFFERENT)

in such a way that the OBS result is the same, but logs are under control.

Same for both theories

$$iM = -i\lambda(\mu) + ia\lambda(\mu)^2\left[\ln\frac{\mu^2}{s} + \cdots\right]$$

$$iM(\mu' \text{ thy}) - iM(\mu \text{ thy}) = 0$$

$$\Rightarrow 0 = -i\lambda(\mu') + ia\,\lambda(\mu')^2\left[\ln\frac{(\mu')^2}{s} + \cdots\right]$$
$$+ i\lambda(\mu) - ia\,\lambda(\mu)^2\left(\ln\left(\frac{\mu^2}{s}\right) + \cdots\right)$$

this is A PERT EXP.
for nearby μ, μ', these should also be nearby

so $\lambda(\mu') = \lambda(\mu) + O(\lambda(\mu)^2)$

$$\Rightarrow \lambda(\mu') = \lambda(\mu) + a\lambda(\mu)^2\left(3\ln\left(\frac{\mu'}{\mu}\right)^2\right)$$

this gives a flow eqn :

$$\underbrace{\lambda(\mu') - \lambda(\mu)}_{d\lambda(\mu)} = 6a\,\lambda(\mu)^2 \underbrace{\ln\left(\frac{\mu'}{\mu}\right)}$$

$$\ln\left(1 + \frac{\mu'-\mu}{\mu}\right) = \ln\left(1 + \frac{d\mu}{\mu}\right) \approx \frac{d\mu}{\mu}$$

$$\Rightarrow \boxed{\underbrace{\mu \frac{d\lambda}{d\mu}}_{\frac{d\lambda}{d\ln\mu}} = 6a\,\lambda(\mu)^2} \equiv \beta(\lambda)$$

"BETA FUNCTION"

this is a differential eqn that we can solve to RE-SUM the LOGARITHMS & always stay close to PERTURBATIVE REGIME

Analog : solving ODE vs. using ODE as input to 1st O term in Taylor expansion.

Key : we are changing theories

OFTEN WE TALK ABOUT "RUNNING COUPLINGS" that change w/ scale. IT'S not that a theory has some exotic behavior.

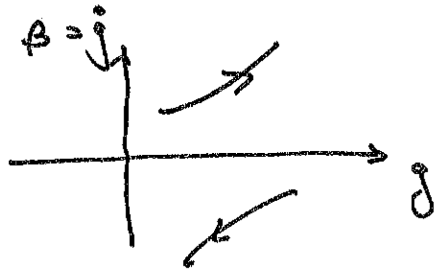
I'm changing to a different theory that describes the SAME physics but is slightly MORE well behaved w.r.t. PERT. EXPANSION.

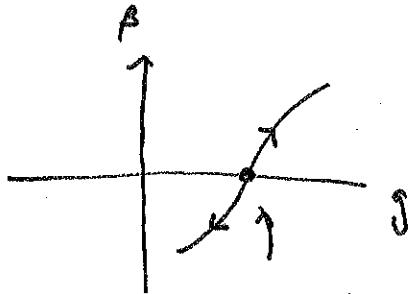
then I'm doing this continuously to reach a different scale.

Kinds of behavior for 1 coupling.

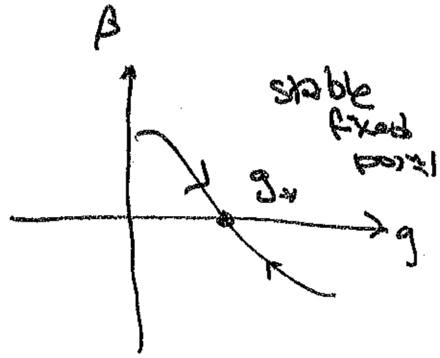
"PHASE SPACE": arrow: DECREASING $t = \ln \mu$

$\beta = \dot{g}$





g_* unstable fixed point



nb: endpoints of flow are theories w/ $\beta(g_*) = 0$

↳ scale change leaves thy invariant

$\Rightarrow$ $\boxed{\text{CONFORMAL THEORY}}$

th/0809149
strassler

trivial version: free theory

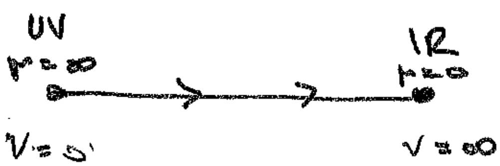
$$\mathcal{L} = \tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{2} m^2\phi^2$$

"scale we study thy @"

scale of thy.

$m^2 = \nu^2 \mu^2 \rightarrow \nu^2 = \frac{m^2}{\mu^2}$

dimless PARAM.

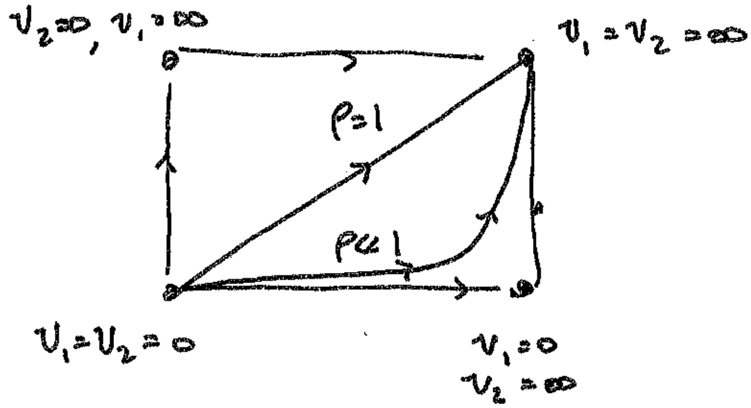


@ $\mu^2 = \infty$, m^2 is negligible!
FREE, MASSLESS THY

@ $\mu^2 = 0$, m^2 IS HUGE!
no PROPAGATING DOF.

theory w/ two free fields :

$$v_1^2 = m_1^2/\mu^2 \qquad v_2^2 = m_2^2/\mu^2 \qquad \rho = v_1/v_2$$



MASS TERM IS A RELEVANT OPERATOR
IF IT IS NONZERO @ SOME SCALE, IT GETS BIG IN IR

dimensionful statement!

$$\beta_v = \mu \frac{dv}{d\mu} = -v \quad \longleftarrow \text{CLASSICAL SCALING}$$

grows in IR

For comparison : $g_6 \phi^6$ is IRRELEVANT.

DIMLESS COUPLING IS $\lambda_6 = \mu^2 g_6$

$$\beta_{\lambda_6} = \lambda_6 \frac{d\lambda_6}{d\mu} = 2\mu g_6 + \underbrace{\mu \frac{dg_6}{d\mu}}$$

CLASSICAL SCALING

QUANTUM SCALING PROPORTIONAL TO COUPLINGS