$$Z[J] = \int \mathcal{D}\phi \; e^{iS[\phi]} \; \underbrace{e^{i\int d^dx\, J\phi}}_{\text{kind of like power } \tau.}$$

in HW:

$$Z[J] = \langle 0 | \hat{T}\, e^{i\int d^4x\, J\phi} | 0 \rangle$$

the vacuum persistence amplitude in the presence of source

so: $\phi \to \frac{\delta}{i\delta J}$ analog of $x \to \frac{\partial}{i\partial p}$

so that $Z[J] = e^{-i\int d^dx\, V[\phi]} \; \underbrace{Z_0[J]}_{\text{free, } Z[0]=1 \text{ by normaliz.}}$

$\mathcal{N}^{-1}$

$$\boxed{\text{w/ } \mathcal{N} = Z[0]}$$

for HW ex 12.3

(See GREINER ex 12.4 for vacuum bubble cancel.)

LET'S DIG IN MORE EXPLICITLY ⟶ GREINER

$$\cancel{\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4}$$

TAYLOR EXP:

H.O. in COUPLING

$$Z[J] = Z_0[J]\,(1 + Z_1[J] + Z_2[J] + \dots)$$

BUT: norm is a little tricky

MORE CAREFUL: $\delta_J = \frac{\delta}{\delta J}$

$$\star \quad \underbrace{e^{-i\int V(\delta_J)} Z_0[J]}_{Z[J] \text{ w/o } \frac{1}{N}} \equiv Z_0[J]\left(1 + u_1[J] + u_2[J] + \cdots\right)$$

taylor exp of $e^{-i\int V(\delta_J)}$:

$$u_1[J] = Z_0[J]^{-1}\left(-i\int V(\delta_J)\right) Z_0[J]$$

$$u_2[J] = Z_0[J]^{-1}\ \tfrac{1}{2}\left(-i\int V(\delta_J)\right)\left(-i\int V(\delta_J)\right) Z_0[J]$$

so: $Z[J] = \dfrac{\star}{Z[0]} = \dfrac{1 + u_1[J] + u_2[J] + \cdots}{1 + u_1[0] + u_2[0] + \cdots}$

ö now compare to $\ddot{\smile}$:

$$Z_1[J] = u_1[J] - u_1[0]$$

$$Z_2[J] = u_2[J] - u_2[0] + (u_1[J] - u_1[0])(-u_1[0])$$

Greiner ex 12.4 consider: $V[\phi] = \frac{\lambda}{4!}\phi^4$ let's see this @ work

$$u_1[J] = Z_0^{-1}\left(\frac{-i\lambda}{4!}\int d^4x\left(\frac{\delta}{i\delta J}\right)^4\right) Z_0$$

free

$$Z_0 = e^{-i\frac{1}{2}\int dx\,dy\, J_y \Delta_{xy} J_x}$$

$$\frac{\delta}{i\delta J(x)} Z_0 = -\int d^4y\, \Delta_F(x-y)\, J(y)\, Z_0$$

nb $\dfrac{\delta J(z)}{\delta J(x)} = \delta(z-x)$

$$\left(\frac{\delta}{i\delta J(x)}\right)^2 Z_0 = \left[\frac{-1}{i}\Delta_F(0) + \left(\int d^4y\, \Delta_F(x-y)\, J(y)\right)^2\right] Z_0$$

$$\left(\frac{\delta}{i\delta J(x)}\right)^3 Z_0 = 2\,\frac{1}{i}\Delta_F(0)\left(\int d^4y\, \Delta_F(x-y)\, J(y)\right) Z_0$$

$$+ \left[\cdots\right]\left(-\int d^4y\, \Delta_F(x-y)\, J(y)\right) Z_0$$

$$= \left[\frac{3}{i}\Delta_F(0)\left(\int d^4y\, \Delta_F(x-y)\, J(y)\right) - \left(\int d^4y\, \Delta_F(x-y)\, J(y)\right)^3\right] Z_0$$

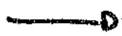
$$\left(\frac{\delta}{i\delta J(x)}\right)^4 Z_0 = \left[3\left(\frac{\Delta_F(0)}{i}\right)^2 - 6\,\frac{1}{i}\Delta_F(0)\left(\int d^4y\, \Delta_F(x-y)\, J(y)\right)^2 + \left(\int d^4y\, \Delta_F(x-y)\, J_y\right)^4\right] Z_0$$

each $\times$ } each $-\!\bullet$
has an integral over positions

<u>RULES</u> :

 $= i\Delta_F(x-y)$

 $= i\Delta_F(0)$

 $= \int d^4x\; J(x)$

 $= -i\lambda \int d^4x$

<u>δ°: $U_1[J]$ from P 2</u>

$$U_1[J] = Z_0^{-1}\, \frac{-i\lambda}{4!} \int d^4x \left(\frac{\delta}{i\delta J}\right)^4 Z_0$$

$= \frac{1}{4!}\Big(3\,\infty + 6$  $+$  $\Big)$

$$Z[J] = \frac{1 + U_1[J] + O(\lambda^2)}{1 + U_1[0] + O(\lambda^2)}\, Z_0[J]$$

$J=0$ kills line endings ← only $\frac{3\,\infty}{4!}$

$$= \big(1 + (U_1[J] - U_1[0]) + \cdots\big)\, Z_0[J]$$

** $= \Big[1 + \frac{1}{4!}\Big(6$  $+$  $\Big) + \cdots \Big]\, e^{\frac{1}{2}}$ 

$\int d^4y_1 d^4y_2\, \Delta_{xy_1} \Delta_{xy_2} J_1 J_2\, \Delta(0)$

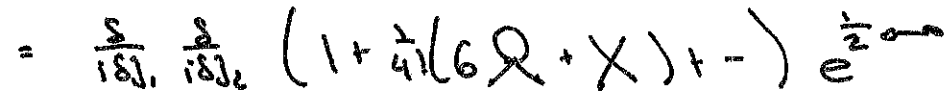
$\int d^4y_1 \cdots d^4y_4\, \Delta_{xy_1} \cdots \Delta_{xy_4} J_1 J_2 J_3 J_4$

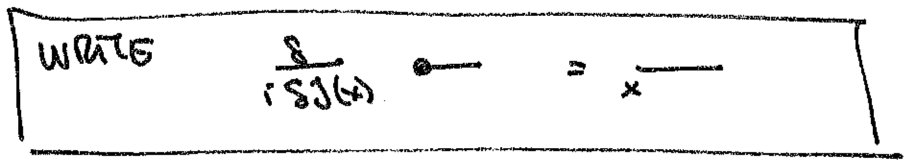
POINT: VACUUM BUBBLES GO AWAY.

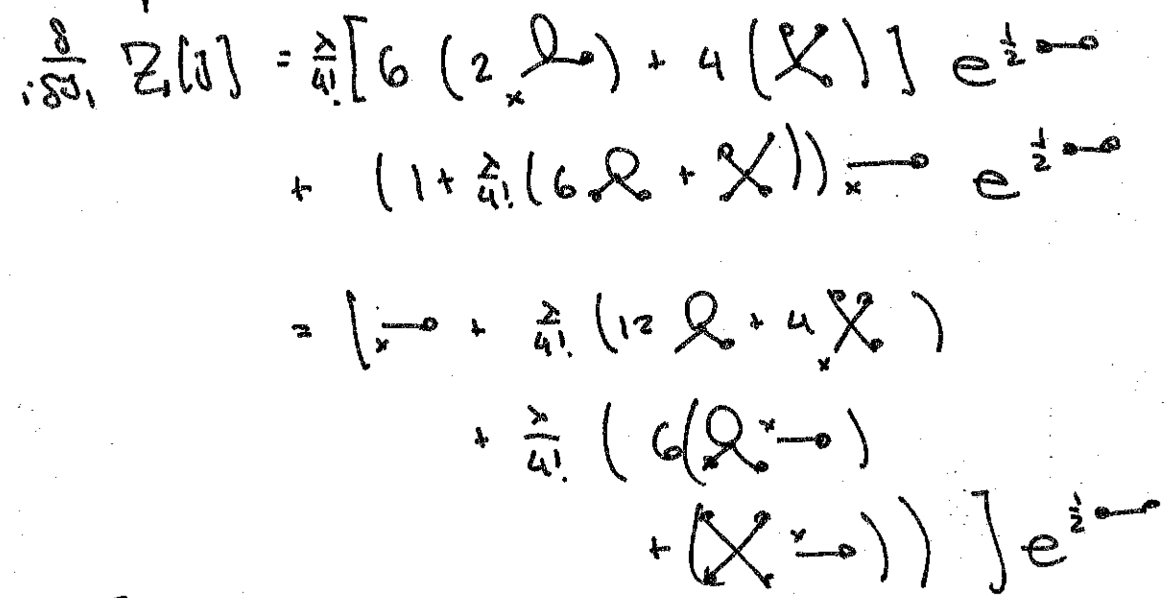
GREEN'S FUNC / CORR FUNC of INTERACTING THY

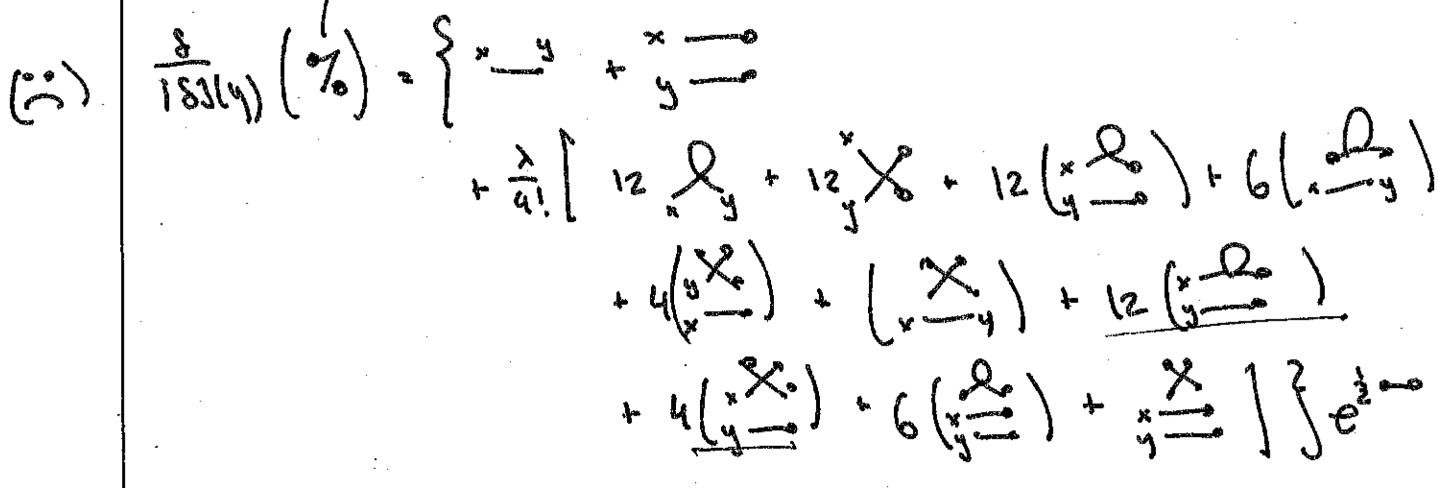
$$\langle \phi_1 \phi_2 \rangle = \frac{\delta^2 Z[J]}{i\delta J_1 \, i\delta J_2}\Big|_{J=0}$$

$\uparrow$ $\phi(x_1)\phi(x_2)$









then we take $J=0$: kill any

$$\langle \phi_1 \phi_2 \rangle = {}_x\,{}_y + \frac{\lambda}{2}\ {}_x\ {}_y + \mathcal{O}(\lambda^2)$$

$$= i\Delta_F(x-y) - \frac{\lambda}{2}\Delta_F(0)\int d^4z\, \Delta_F(x-z)\,\Delta_F(y-z) + \cdots$$

this is formally infinite

$\leadsto$ need renormaliz.

<u>what about 4-point?</u>

start from (;;) ... ignore anything $\mathcal{O}(J^3)$ ~~$\mathcal{O}[J^1]$~~

... dies from $J|_0$ ~~or $(\delta/\delta J)^2$~~

~~$(;;) = \{ \ldots + \frac{\lambda}{4!}[12 \ldots + 12 (\ldots) + 6(\ldots)$~~

~~$+ 12(\ldots)]\}\, e^{\frac{i}{2}}$~~

PIECES : terms w/ 1 line ending after $\delta/\delta J(z)$

~~$\prod_{x,y,z}\left(\frac{\delta}{i\delta J}\right) = \frac{\lambda}{4!}\,12$~~

$$Z[J] = \left(1 + \frac{\lambda}{4!}\left(\underline{6} + \ \right) + \cdots\right) e^{\frac{i}{2}}$$

$$\frac{\delta}{i\,\delta J_x}\frac{\delta}{i\,\delta J_y}\frac{\delta}{i\,\delta J_z} Z[J] = \frac{\lambda}{4!}\,12\Big[\ {}^{x}\ {}_{y}\ {}_{z} + \text{perm.}$$

$$+\ {}_x\ {}_{y}\ {}_{z} + \text{perm}\ \Big]\, e^{\frac{i}{2}}$$

$$+\ \mathcal{O}[J^2]$$

$\llcorner$ dies on 4th der.

↓ PLUS EXPR.

$$\frac{\delta}{i\delta J_w}\left(\%\right)\Big|_{J=0} = \frac{\lambda}{4!}\,12\left[\; + \text{PERM}\;\right]$$

6 total

$$\frac{\delta}{i\delta J_x}\frac{\delta}{i\delta J_y}\frac{\delta}{i\delta J_z}Z[J]_X = \frac{\lambda}{4!}\,24\; e^{\frac{1}{2}} + \cdots$$

DIES on 4th deriv.

$$\frac{\delta}{i\delta J_w}\left(\%\right)\Big|_{J=0} = \frac{\lambda}{4!}\,24$$

RESULT :

$$\langle \phi_1\phi_2\phi_3\phi_4\rangle = \; + \; + $$

$$+\frac{1}{2}\lambda\left(+ + + + + \right)$$

symmetry factor!

$$+\lambda$$

normalized!

Connected GREEN's fn's / CORRELATORS:

$$Z[J] = e^{iW[J]}$$

claim: generates ~~connected~~ correlation functions

(nb why would we care?)

$$iW[J] = \ln Z[J] = \ln\left[\left(1 + \tfrac{\lambda}{4!}(6 \quad + \quad) + \cdots\right) e^{\frac{1}{2}}\right]$$

$$= \ln e^{\frac{1}{2}} + \ln\left[1 + \tfrac{\lambda}{4!}(6 \quad + \quad)\right] + \cdots$$

$$= \tfrac{1}{2} \quad + \tfrac{\lambda}{4!}(6 \quad + \quad) + \cdots$$

$$\underbrace{\frac{\delta}{i\delta J_x} \cdots \frac{\delta}{i\delta J_w}}_{\text{4 POINT}} W[J] = \lambda \quad + O(\lambda^2)$$

no "mixed terms" b/c $e^{\frac{1}{2}}$ is treated additively.

so: W generates connected diagrams

GREINER: ~~Quantum theory of Fields~~ Field Quantiz ch. 12