from this morning: | DISCUSS LEGENDRE

$$A\,A^{-1} = \mathbb{1}$$

$$A^i{}_j\, |i\rangle\langle j|\ (A^{-1})^k{}_\ell\, |k\rangle\langle \ell|$$

$$= A^i{}_j\,(A^{-1})^k{}_\ell\, |i\rangle\underbrace{\langle j|k\rangle}_{\delta^j{}_k}\langle \ell|$$

$$= A^i{}_j\,(A^{-1})^j{}_\ell\, |i\rangle\langle \ell| = \delta^i{}_\ell\, |i\rangle\langle \ell|$$

i.e.: $\boxed{\sum_j A^i{}_j\,(A^{-1})^j{}_\ell = \delta^i{}_\ell}$

compare to:

$$\int dx\ \mathcal{O}_x\ G(x,x') = \int \delta(x-x')\ dx$$

eg $\left(\frac{d}{dx}\right)^2$

MEANING: $\underline{\underline{A}}\,\underline{\psi} = \underline{s} \longrightarrow \underline{\psi} = \underline{\underline{A}}^{-1}\,\underline{s}$

$$\underline{\underline{A}}\,\underline{\psi} = \underline{\underline{A}}\,\underline{\underline{A}}^{-1}\,\underline{s} = \underline{s} \checkmark$$

function: $\psi(x) = \int dx'\ G(x,x')\, s(x')$

$$\mathcal{O}_x\,\psi(x) = \int dx' \left(\mathcal{O}_x\, G(x,x')\right) s(x')$$

$$= \int dx'\ \delta(x-x')\ s(x') = s(x) \checkmark$$

What is δ function?

$$\begin{cases} \delta(x) = 0 \quad \text{for } x \neq 0 \\ \int_{-\infty}^{\infty} dx\, \delta(x - x_0) = 1 \end{cases}$$

that's a good definition

any "deep" interpretation of "what happens @ $\delta(0)$ is not relevant.

by comparison:

$$\sum_i |i\rangle\langle i| = \underline{\underline{\mathbb{1}}}$$

$|1\rangle\langle 1| + |2\rangle\langle 2| + \dots$

one term

$\delta(x - x_0)$ is like $\delta_{x_0}^{x} |x\rangle\langle x_0|$

$f(x)$ as a vector in "histogram basis"

$$\underline{f} = \underbrace{f(x_1)\,|x_1\rangle + f(x_2)\,|x_2\rangle + \dots}$$

$$\int \delta(x - y_i) f(x)\, dx \rightarrow \sum_i \delta_{x_i}^{y} |y\rangle\langle x_i| \left(\underline{f} \right)$$

Q. is this like a determinant?

$$\int e_n(x)^* e_m(x)\, dx = \delta_{nm}$$

$$\sum_n e_n(x)^* e_m(y) = \delta(x-y)$$

NO: DETERMINANTS ARE SPECIAL.
eg come from change of basis

$$d^3x \rightarrow d^3y \, J$$

JACOBIAN IS A DET.

ANTISYMMETRY IS IMPORTANT.
HAS TO DO W/ TAKING N-DIM VOLUME ELEMENT.

(comes from generalization of dual vector!)

ex. ~~MATH~~ MATHEWS & WALKER 9-4

unit string w/ frequency k $(= \omega/c)$
& DIRICHLET BC @ $x = 0, 1$

note: chose units.

$$\frac{d^2 f}{dx^2} + k^2 f = 0$$

interested in RHS $= F(x)$ (driving force)

$$\Theta_x = \left[\left(\frac{d}{dx}\right)^2 + k^2 \right]$$

EIGENFUNCTIONS?

still $\sqrt{2} \sin(n\pi x)$

CHK: RESTORE UNITS OF LEN.

$$G(x, x') = \sum_n \frac{f^*(x) f(x')}{\lambda_n}$$

$$\boxed{\lambda_n = -n^2\pi^2 + k^2}$$

$$= \boxed{2 \sum_n \frac{\sin(n\pi x) \sin(n\pi x')}{k^2 - n^2\pi^2}}$$

SO SOLUTION TO $\left[\frac{d^2}{dx^2} + k^2 \right] f = F(x)$

is $f(x) = \int_0^1 dx' \, G(x, x') \, F(x')$

$$\underbrace{\left[\left(\frac{d}{dx}\right)^2 + k^2\right] G(x,x')} = \delta(x-x')$$

FIRST SOLVE $[\cdots]_x\, G(x) = 0$

in the two regions its true.

$$G^< \qquad\qquad G^>$$

$$x'$$

x = 0 gives $G^<(0) = 0$

$$G(x,x') = \begin{cases} G^< = a \sin(kx) & x < x' \\ G^> = b \sin k(x-1) & x > x' \end{cases}$$

2 coefficients to be fixed

why? set $G^>(x=1) = 0$ w/o setting coeff = 0.

$$\lim_{\varepsilon \to 0} \int_{x'-\varepsilon}^{x'+\varepsilon} \left[\left(\frac{d}{dx}\right)^2 + k^2\right] G(x,x')\, dx = \int_{x'-\varepsilon}^{x'+\varepsilon} \delta(x-x')\, dx$$

only makes sense as an integral!

$$\underbrace{\int \frac{d}{dx}\left(\frac{d}{dx} G\right) dx} + \underbrace{\int k^2 G \, dx}_{\to 0 \text{ AS } \varepsilon \to 0} = 1$$

$$\boxed{\left.\frac{d}{dx} G \right|_{x-\varepsilon}^{x+\varepsilon} = 1}$$ JUMP CONDITION

continuity: $\boxed{G(x+\varepsilon, x') = G(x-\varepsilon, x')}$

RESULT:

$$k a \cos k x' + 1 = k b \cos k(x' - 1)$$
$$a \sin k x' = b \sin k(x' - 1)$$

$$\Rightarrow \quad a = \frac{\sin k(x' - 1)}{k \sin k}$$

$$b = \frac{\sin k x'}{k \sin k}$$

$$\boxed{G = \frac{1}{k \sin k} \begin{cases} \sin k x \sin k(x' - 1) & x < x' \\ \sin k x' \sin k(x - 1) & x > x' \end{cases}}$$