LECTURES 2 & 3 | 1 OCT 2018

REMINDER: MTG 3 today @ 2:10 pm in READING RM.

from last week's note cards:

- this will not be a thm-pf math course, nor will it be an advanced topics course (maybe last 2-3 wks, but only for fun)

  We will try to be insightful, but will focus on the basics. [YOU ARE FREE TO DROP CLASS – just let me know]

- FOOD / COOKING: I SUGGEST THAT YOU LEARN HOW TO MAKE AN EASY PASTA. AFFORDABLE, CUSTOMIZABLE, INGREDIENTS KEEP WELL.

- re: *how I learn* – some great responses

  ① find a study group that suits your style ... be willing to adopt!

  ② find a reference that you like. APPEL is nice, BOAS is fine (more basic)

  └ I LIKE THE IDEA OF YOUTUBE EXPLAINERS let me know if you find good ones!

  ③ I will not give solutions. CONFER W/ COLLEAGUES, ASK IAN... BUT THERE IS NO ONE RIGHT WAY.

- "I've never done a contour integral"

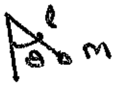
  └ that's okay! make sure you have a book that you like.

$\boxed{\text{Qs}}$?

HW1 is due Wed!

LAST TIME: DIM. ANALYSIS. $[g] \sim L^a M^b T^c \dots$

why? 1. check work
2. understand ratios } LAST TIME



$\tau \sim \sqrt{l/g}\, f(\theta_0)$

THIS IS PHYSICS!

related

3. SCALING

$\underline{r}_0(t)$ is a trajectory through a potential $U(\underline{r})$

SUPPOSE WE KNOW $r_0(t)$
eg. DATA POINTS, fit to an ellipse, ...

can generate other solutions
w/o "doing math."

SCALE TIME: $t = \alpha t'$ ← new var

α ← some number LIKE PICKING UNIT

IF $U(\underline{r})$ is STATIC, only LHS changes

STATIC → DOESN'T CARE IF WE'RE COUNTING in seconds or minutes ... only what is the POSITION of the SATELLITE

BUT RHS HAS
$[\partial U/\partial r] = ML\boxed{T^{-2}}$
... from G_N

ALL WE KNOW $$m\left(\frac{d}{dt}\right)^2 \underline{r}_0(t) = -\frac{\partial U(\underline{r}_0)}{\partial \underline{r}}$$

PLUG IN CHANGE of VARS:

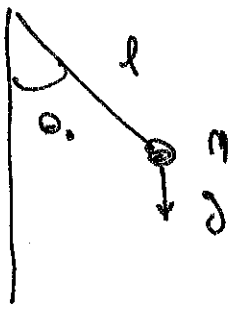
$$\underbrace{m\,\alpha^{-2}}\left(\frac{d}{dt'}\right)^2 \underline{r}_0(\alpha t') = -\frac{\partial U(\underline{r}_0(\alpha t'))}{\partial \underline{r}}$$

$m' \equiv m/\alpha^2$

so: $\alpha = 2$: same trajectory, twice as fast
... valid if 1/4th the mass.

4. ERROR ESTIMATE

$\int$ BTW: LAST TIME: no explicit $(\underline{x}, t)$ dependence!



$$\Rightarrow \quad \tau \sim \sqrt{l/g}\; f(\theta_0)$$

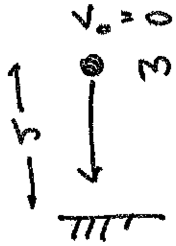
why not $l/R_\oplus$?
why not a/g ?

they shouldn't matter ... $\boxed{\text{to leading order}}$

mantra of every Taylor expansion in physics.

"L.O. IS GOOD ENOUGH" ... how good?

eg in HW :



$\boxed{?\ t_0}$ time to hit floor

$$\text{HS: } \ddot{x}\,g \rightarrow x = \tfrac{1}{2} g t^2 + \overset{v_0 = 0}{\cancel{v_0 t}} + \overset{\text{COORDS}}{\cancel{d}}$$

$$\boxed{t_0 = \sqrt{2h/g}}$$

$$\text{ERROR} = \boxed{\frac{t_r - t_0}{t_0}}$$

L.O.

(maybe should be t_r ?

MORE REALISTIC
(NLO ... OR ESTIMATE OF NLO)

$f(\dots) = \frac{t_r - t_0}{t_0}$ } DIMless

↑ what does the ERROR DEPEND ON?

ξ : dim parameter.

★ • as $\boxed{\xi \to 0, \quad f \to 0}$ — when a param goes to zero, our approx. becomes exact

• if t_0 is "good" approx, f is "small"

↑ means something b/c f is dimless

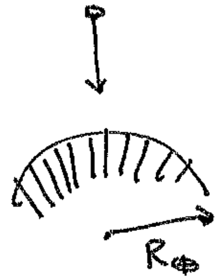
$$f(\xi) = \cancel{f(0)} + \xi \underbrace{f'(0)}_{\text{DIMLESS, ASSUME } O(1)} + O(\xi^2)$$

0 from ★

$$\boxed{\text{ERROR} \sim O(\xi)}$$

← kind of an obvious result.

eg.



We approx ROUND EARTH BY FLAT EARTH

PARAMETER: $\boxed{\xi = \frac{h}{R_\oplus}}$ or $\frac{R_\oplus}{h}$

✓

↑ BAD BEHAVIOR $\xi \to 0$

so ERROR is $\boxed{h/R_\oplus}$

closer to ground, better the approx

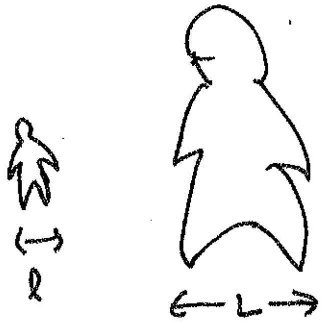
↳ APPROX is the $V \sim 1/r$ SCALING

(why didn't we use G_N?)

if there's time:

CRUELTY DIET:
<u>NO</u> FOOD.
(DO <u>NOT</u> try @ home!!)

WEIGHT LOSS allometry.



which loses more weight in a given time period?

how does it scale w/ ℓ/L ??

<u>HINT</u>: assume weight loss is from "BURNING FAT" & that fat has many <u>CARBON</u>.

(where does it go? NOT IN POOP.)

<u>LINEAR ALGEBRA</u>

goal: $\underbrace{\text{function space}}_{\text{"collection of all functions"}} = \infty \text{ dim } \underbrace{\text{[linear] vector space}}_{\text{"something you 'do' LINEAR ALGEBRA on"}}$

Reminder $\boxed{\text{LINEAR}}$

given vector $\underline{v}$, a <u>linear transform</u> A acts as $\underbrace{A\underline{v}}_{\text{matrix multiplication}}$

$$\underline{V} = \begin{pmatrix} v^1 \\ v^2 \\ \vdots \\ v^N \end{pmatrix} \Bigg\}$$ dim of vector space is

dim $\mathbb{V} = N$, # components

$\forall (\underline{v} \in \mathbb{V})$

call a generic component v^i

MATRIX MULTIPLICATION: in 2 dim

$$A = \begin{pmatrix} A^1{}_1 & A^1{}_2 \\ A^2{}_1 & A^2{}_2 \end{pmatrix} \leftarrow \text{or} \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

s.t. $A^i{}_j$ IS A GENERIC ELEMENT OF THE MATRIX A

UPPER/LOWER INDICES IS A CONVENTION. COULD DO EVERYTHING LOWER.

(notation is neither math nor physics ... but it can help...)

$$A\vec{V} = \begin{pmatrix} A^1{}_1 & A^1{}_2 \\ A^2{}_1 & A^2{}_2 \end{pmatrix} \begin{pmatrix} V^1 \\ V^2 \end{pmatrix} = \begin{pmatrix} A^1{}_1 V^1 + A^1{}_2 V^2 \\ A^2{}_1 V^1 + A^2{}_2 V^2 \end{pmatrix}$$

generic element of $(\overrightarrow{Av})$, a vector:

$$\boxed{(Av)^i = \sum_j A^i{}_j V^j}$$

I WILL OFTEN USE EINSTEIN NOTATION: REPEATED UP/DOWN INDICES ARE SUMMED OVER

$$= \boxed{A^i{}_j V^j}$$ object w/ one index part of a vector.

$\boxed{\text{D2}}$: CHECK THIS w/ EXPLICIT MATRICES!

LINEAR TRANSFORM : satisfies

$$(A+B)\vec{v} = A\vec{v} + B\vec{v}$$

$$A(\underset{\uparrow \#}{\lambda}\vec{v}) = \lambda(A\vec{v})$$

CHECK THE FIRST ONE IN OUR NOTATION

A w/ generic element $A^i{}_j$

B ——"—— $B^i{}_j$

↑ I AM OFTEN SLOPPY & WILL SAY $A^i{}_j$ TO REFER TO BOTH the individual element & the matrix

$$A^i{}_j + B^i{}_j = (\underbrace{A+B}_{\text{new transform; i-j element}})^i{}_j$$

↑ true element by element

LET'S BE PEDANTIC: $\underline{V} = \begin{pmatrix} V^1 \\ \vdots \\ V^N \end{pmatrix}$ is more than an array/list of N numbers: it means

$$\underset{\substack{\uparrow \\ \text{ABSTRACT OBJECT}}}{\underline{V}} = V^1 \underset{\substack{\uparrow \\ \text{maybe} \\ \begin{pmatrix} 1 \\ 0 \\ \vdots \end{pmatrix}}}{\underline{e}_1} + V^2 \underset{\substack{\uparrow \\ \text{maybe} \\ \begin{pmatrix} 0 \\ 1 \\ 0 \\ \vdots \end{pmatrix}}}{\underline{e}_2} + \cdots$$

$\underline{e}_i$ ← BASIS of vector space

... also an abstract object!

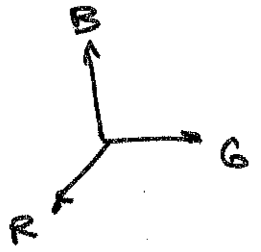
eg. color space is a vector space, 3 DIM

A color $\underline{c} = \begin{pmatrix} c^1 \\ c^2 \\ c^3 \end{pmatrix} \rightleftarrows c^i \in (0, 255)$

is SPECIFIED BY:

$$c^1 \times \underset{\substack{\uparrow \\ R}}{\left(\text{unit of RED light}\right)} + c^2 \underset{\substack{\uparrow \\ G}}{\left(\text{unit of GREEN light}\right)} + c^3 \underset{\substack{\uparrow \\ B}}{\left(\text{unit of BLUE LIGHT}\right)}$$

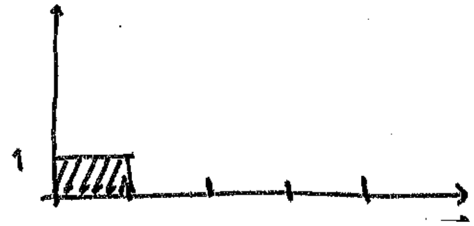
can still plot $\underline{c}$ on



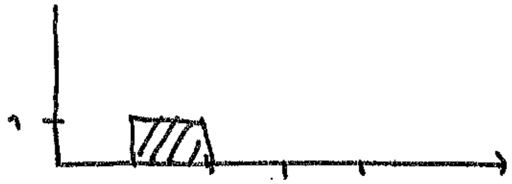
but this 3d space is an abstract representation of a physical phenomenon: COLOR

in general: $\underline{V} = \overset{\text{just a \#}}{v^i}\ \underbrace{\underline{e}_{(i)}}_{\text{carries vector-ness}}$ ← SUM over i

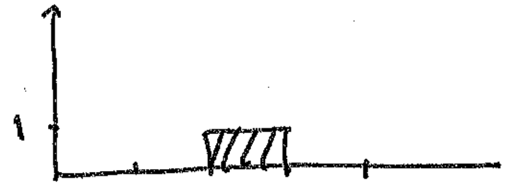
Here's another funny linear space:



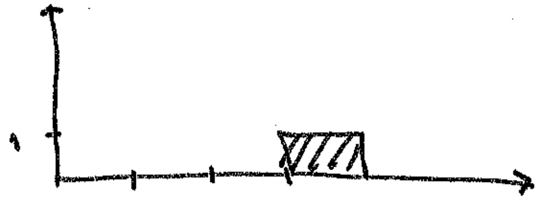
$= \underline{e}_1$



$= \underline{e}_2$

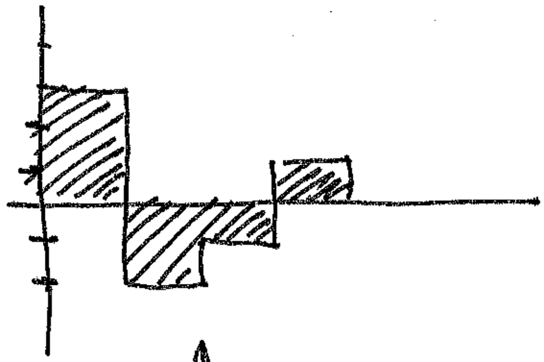


$= \underline{e}_3$



$= \underline{e}_4$

so that

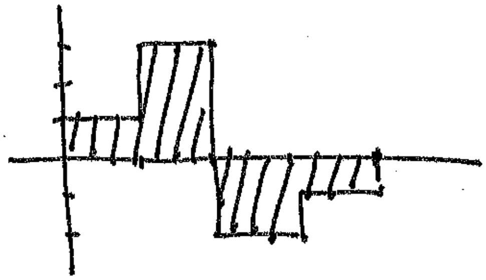


$$= \underline{V} = \begin{pmatrix} 3 \\ -2 \\ -1 \\ 1 \end{pmatrix}$$

$$= 3\underline{e}_1 - 2\underline{e}_2 - \underline{e}_3 + e_4$$

↑ looks like a histogram!

now suppose I have a linear transformation, A, that acts on the system & does this:



$$A\vec{V} = \begin{pmatrix} 1 \\ 3 \\ -2 \\ -1 \end{pmatrix}$$

q. do you know what A is?

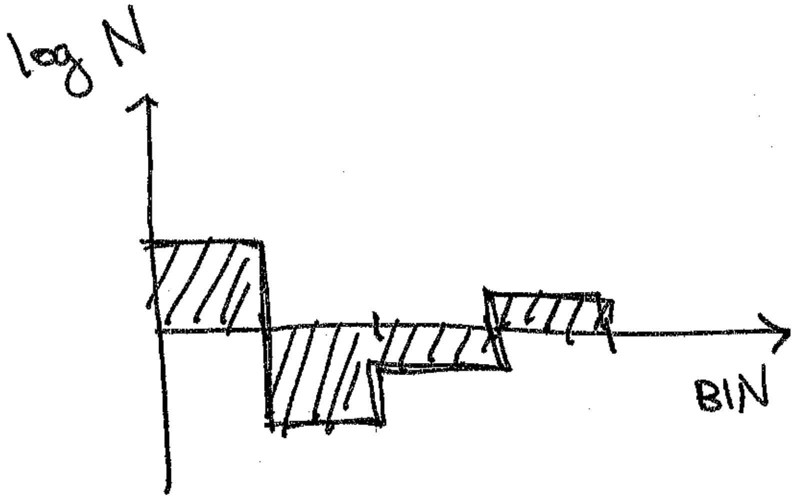
q. what could this have to do w/ physics?

A GOOD REFERENCE :

Linear Algebra Done Right

S. Axler → free on SpringerLink

Q. do you know how?



HOW TO TALK ABOUT PLOTS

1. explain the axes
2. explain the context
3. explain the trend
4. tell us what to think