$$\underbrace{\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x}_{\mathcal{O}_t x(t)} = \underbrace{F(t)}$$

new time scale

choose $\boxed{F(t) = F_0 e^{-\alpha t}}$
EXP falling BUP.

b/c $t_2 > t$ ⟷ causal

$$x(t) = A\,x_1(t) + B\,x_2(t) + \int_{t_1}^{t} F(t')\,G(t-t')\,dt'$$

$\mathcal{O}_t x_{1,2} = 0$

$$G(t-t') = \frac{e^{-\gamma(t-t')}\sin\left(\sqrt{\omega_0^2-\gamma^2}\,(t-t')\right)}{\sqrt{\omega_0^2-\gamma^2}}$$

define : $\boxed{\begin{aligned}\Delta t &= t - t' \\ \bar{\omega} &= \sqrt{\omega_0^2-\gamma^2}\end{aligned}}$ physical signific!

NOW DO THE dt' integral.

USE: $\sin\theta = \frac{1}{2i}\left(e^{i\theta} - e^{-i\theta}\right)$

$$G(\Delta t) = \frac{1}{\bar{\omega}} e^{-\gamma t} e^{\gamma t'} \cdot \frac{1}{2i}\left(e^{i\bar{\omega}t} e^{-i\bar{\omega}t'} - e^{-i\bar{\omega}t} e^{i\bar{\omega}t'}\right)$$

for convenience ⟶ $e^{i\bar{\omega}t} \equiv C$ $\quad e^{-i\bar{\omega}t} \equiv C^{-1}$

$$G(\Delta t)F(t') = \frac{F_0}{\bar{\omega}}\frac{e^{-\gamma t}}{2i}\, e^{(\gamma-\alpha)t'}\left(C e^{-i\bar{\omega}t'} - C^{-1} e^{i\bar{\omega}t'}\right)$$

$\boxed{\bar{\gamma} \equiv \gamma - \alpha}$ DAMPING VS. BUP SIZE

$$GF(t') = \frac{F_0}{\bar\omega}\frac{e^{-\gamma t}}{2i}\left[C e^{\beta_- t'} - \frac{1}{C} e^{\beta_+ t'} \right]$$

$$\boxed{\beta_\pm = \bar\gamma \pm i\bar\omega} = \gamma - \alpha \pm i\bar\omega$$

ALL t' DEPENDENCE IS IN $e^{\beta_\pm t'}$

$$\int_0^t dt'\, GF(t') = \frac{F_0}{\bar\omega}\frac{e^{-\gamma t}}{2i}\left[\frac{C}{\beta_-} e^{\beta_- t} - \frac{C}{\beta_-} - \frac{1}{C\beta_+} e^{\beta_+ t} + \frac{1}{C\beta_+} \right]$$

$e^{i\bar\omega t}$

Note: $e^{-\gamma t}\, C e^{\beta_- t} = e^{-\alpha t}$

$e^{-\gamma t}\, \frac{1}{C} e^{\beta_+ t} = e^{-\alpha t}$

$$= \frac{F_0}{\bar\omega}\frac{1}{2i}\left[\frac{1}{\beta_-} e^{-\alpha t} - \frac{1}{\beta_-} e^{-\gamma t + i\bar\omega t} - \frac{1}{\beta_+} e^{-\alpha t} + \frac{1}{\beta_+} e^{-\gamma t - i\bar\omega t} \right]$$

$$\frac{\beta_+ - \beta_-}{\beta_+\beta_-} e^{-\alpha t} - e^{-\gamma t}\left(\frac{\beta_+ e^{i\bar\omega t} - \beta_- e^{-i\bar\omega t}}{\beta_+\beta_-} \right)$$

$2i\bar\omega$

$$\beta_+\beta_- = (\gamma - \alpha)^2 + \bar\omega^2$$

$$= \cancel{\gamma^2} - 2\gamma\alpha + \alpha^2 + \omega_0^2 - \cancel{\gamma^2}$$

$$= \omega_0^2 - 2\gamma\alpha + \alpha^2$$

$$= \frac{F_0}{\bar{\omega}} \frac{1}{2i} \left[\frac{2i\bar{\omega}}{\omega_0^2 - 2\gamma\alpha + \alpha^2} e^{-\alpha t} - \frac{\beta_+ e^{i\bar{\omega}t} - \beta_- e^{-i\bar{\omega}t}}{\beta_+ \beta_-} e^{-\gamma t} \right]$$

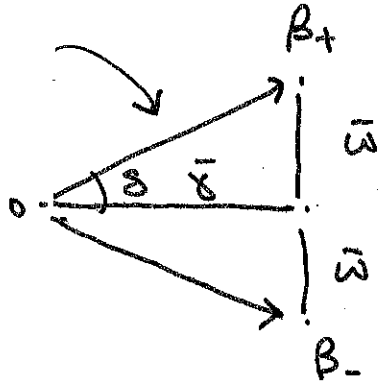
want to write $\beta_+ e^{i\bar{\omega}t} - \beta_- e^{-i\bar{\omega}t}$

$$= r(\underbrace{e^{i\theta} - e^{-i\theta}}_{2i\sin\theta})$$

HOW TO DO THIS? $\beta_\pm = \bar{\gamma} \pm i\bar{\omega}$

real

$r = \sqrt{\bar{\gamma}^2 + \bar{\omega}^2}$



$\boxed{\tan\delta = \frac{\bar{\omega}}{\bar{\gamma}}}$

$= \sqrt{(\gamma-\alpha)^2 + \omega_0^2 - \gamma^2}$

$= \sqrt{\omega_0^2 - 2\gamma\alpha + \alpha^2} \qquad = \sqrt{\beta_+\beta_-}$

$\beta_\pm = r e^{\pm i\delta}$

$$= \frac{F_0}{\bar{\omega}} \frac{1}{2i} \left[\frac{2i\bar{\omega}}{\omega_0^2 - 2\gamma\alpha + \alpha^2} e^{-\alpha t} - \frac{r e^{i(\bar{\omega} t + \delta)} - r e^{-i(\bar{\omega} t + \delta)}}{\omega_0^2 - 2\gamma\alpha + \alpha^2} e^{-\gamma t} \right]$$

$$= \frac{F_0}{r^2} e^{-\alpha t} - \frac{F_0}{r} \frac{\sin(\bar{\omega} t + \delta)}{\bar{\omega}} e^{-\gamma t}$$

$$\left[\begin{array}{l} r^2 = \omega_0^2 - 2\gamma\alpha + \alpha^2 \\ \bar{\omega}^2 = \omega_0^2 - \gamma^2 \\ \delta = \tan^{-1}(\bar{\omega}/\bar{\gamma}) \\ \bar{\gamma} = \gamma - \alpha \end{array} \right.$$

t' was set to 0.

for $\boxed{t \gg 1/\alpha}$ 1st term vanishes.

RECOVER OSCILLATORY MOTION

for $\boxed{\gamma \to 0}$ no damping

$$x(t) = \underbrace{\frac{F_0}{\omega_0} \frac{\sin(\omega_0 t - \delta)}{\sqrt{\omega_0^2 + \alpha^2}}}_{\text{no damping, long time limit}} + \underbrace{\frac{F_0}{\omega_0^2 + \alpha^2} e^{-\alpha t}}_{\alpha F(t)}$$

ENERGY : init: $E=0$, how much transferred in?
($\gamma \to 0$ limit)

$$E = \frac{1}{2}\dot{x}^2 + \frac{1}{2}\omega_0^2 x^2$$

$$= \frac{1}{2}\left(\frac{F_0}{\omega_0}\,\frac{\omega_0 \cos(\omega_0 t - \delta)}{\sqrt{\omega_0^2+\alpha^2}}\right)^2$$

$$+ \frac{1}{2}\omega_0^2\left(\frac{F_0}{\omega_0}\,\frac{\sin(\omega_0 t - \delta)}{\sqrt{\omega_0^2+\alpha^2}}\right)^2$$

$$= \boxed{\frac{1}{2}\,\frac{F_0^2}{\omega_0^2+\alpha^2}}$$