$\boxed{\text{THIS CLASS}}$

LINEAR DYNAMICS: $\mathcal{O}\,\psi = s$

↳ IDENTIFY GREEN'S FUNC OF $\mathcal{O}$
USE FOURIER, CONTOUR INTEGRAL

↳ (OR COMPLETENESS [series exp], PATCHING [piecewise])

CAUSALITY ↔ ANALYTICITY

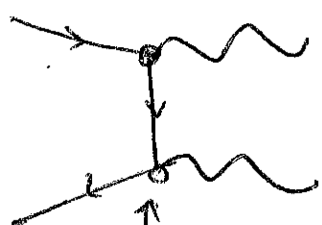
$$\psi = \int d^n x'\; G(x,x')\, s(x')$$

what about non-linear?

TWO STANDARD APPROACHES

1. SIMULATE → computational
2. PERTURBATION THEORY

↳ LINEAR DYNAMICS W/ PERTURBATIONS OF NON-LINEAR INSERTIONS.



EA LINE IS LINEAR EVOLUTION (GREEN'S FUNCTION)

VERTICES REPRESENT NONLINEARITY

must be <u>small</u> w/rt G

" & now for something completely different "

Probability

let $P(A)$ be the probability of A

IF A & B ARE MUTUALLY EXCLUSIVE.

$$P(A \& B) = P(A)P(B)$$

eg as opposed to entangled

Conditional Probability

$$P(A|B) = \frac{P(A \& B)}{P(B)}$$

"assuming B is true"
⇓
divide by $P(B)$

prob of A given B is true

Bayes' Theorem

$$P(A \& B) = P(B \& A)$$

$$\parallel \qquad\qquad \parallel$$

$$P(A|B)P(B) \qquad P(B|A)P(A)$$

$$\boxed{P(A|B) = \frac{P(B|A)P(A)}{P(B)}}$$

So what? often A = proposed theory
B = observed data

want to test

what we have

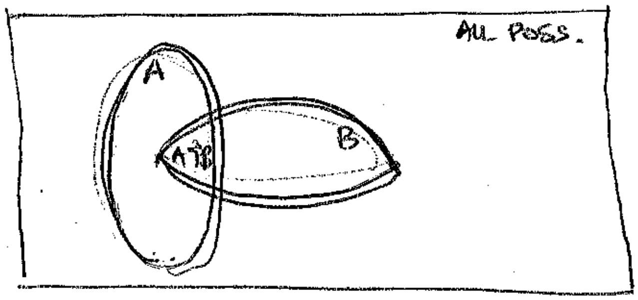
WE CAN CALCULATE: P(data | theory)
↳ given that theory is <u>true</u>,
what is the chance to have
measured the observed data. } p-value

WHAT WE USUALLY <u>WANT</u> IS
how likely is a hypothesis (THEORY)
given the observed data?

BAYES' THM RELATES P(thy | data)
TO P(data | thy) ... need to know
P(data) & P(thy)

?!

VISUAL (via Bob cousins)



$$P(A) = \frac{\bigcirc}{\square} \qquad P(B) = \frac{\bigcirc}{\square}$$

$$P(A|B) = \frac{\triangle}{\bigcirc} \qquad P(B|A) = \frac{\triangle}{\bigcirc}$$

$$P(A+B) = \frac{\triangle}{\square}$$

So: $$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

$$\frac{\triangle}{\bigcirc} = \frac{\triangle}{\bigcirc}\ \frac{\bigcirc}{\square}\ \frac{\square}{\bigcirc}$$

$$P(B|A) \quad P(A) \quad P(B)^{-1}$$

concrete eg:

$$P\left(\begin{smallmatrix}\text{USES}\\ \text{ROOT}\end{smallmatrix} \middle| \text{PARTICLE PHYS}\right) \sim 50\%.$$

theorists don't use root

$$P\left(\begin{smallmatrix}\text{PARTICLE}\\ \text{PHYS.}\end{smallmatrix} \middle| \begin{smallmatrix}\text{USES}\\ \text{ROOT}\end{smallmatrix}\right) \sim 100\%.$$

only Hep-ex people use ROOT.

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continuous variables

$$p(x)\,dx \leftarrow P(x' \in (x, x+dx))$$

PROB DISTRIB

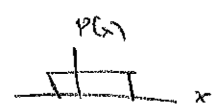
BORN TO BE INTEGRATED

$$\langle x \rangle = \int dx \; x \; p(x)$$

$$\langle x^2 \rangle = \int dx \; x^2 \; p(x)$$

$$\langle f(x) \rangle = \int dx \; f(x) \; p(x) \qquad \text{etc.}$$

common distributions.

- BOX $\quad p(x) =$ 

- BINOMIAL (discrete)

$$p(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

P(positive)

given BINARY EVENT, PROB. OF k POSITIVE RESULTS GIVEN n TRIALS

WHAT ABOUT WHEN NO FIXED "# TRIALS"? eg # OF LIGHTNING BOLTS IN A STORM?

limit: unknown # trials $\rightarrow$ poisson.

$$P(k) = \frac{1}{k!}\underbrace{\boxed{\frac{n!}{(n-k)!}}}_{\rightarrow n^k \text{ as } n\rightarrow\infty}\left(\frac{\lambda}{n}\right)^k \underbrace{\boxed{\left(1-\frac{\lambda}{n}\right)^{n-k}}}_{e^{-\lambda} \text{ AS } n\rightarrow\infty}$$

λ is # of expected events in some big interval.

then split into n tiny intervals w/ only one event per interval

$$\boxed{P(k) = \frac{1}{k!}e^{-\lambda}\lambda^k} \quad \text{poisson}$$

<u>GAUSSIAN</u> : $p(x)\,dx = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu^2)/2\sigma^2}$

$\hookrightarrow$ central limit thm :

IN LIMIT OF LARGE # OF MEASUREMENTS THE DISTRIB. OF A MEASURED PARAM IS GAUSSIAN.

Pf/ exercise. ONE WAY ends up using Fourier transf.

$$\tilde{P}(k) \sim e^{-\# k^2 - \underbrace{\cdots}_{O(1/\sqrt{N}) \text{ SUPPRESSED}}}$$

$\downarrow$

$p(x) \sim e^{-\# x^2}$ (+negligible)

YOU SHOULD DO THIS. WHY? REMINDER THAT CLT FAILS. (memento mori)

INFERENCE

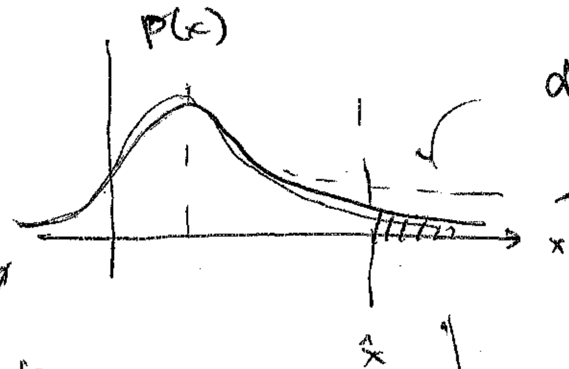
given data $\hat{x}$ & $\underbrace{\text{theory}}_{\text{PARAMS.}}$ H

how compatible?

$$\underbrace{L(H)}_{\text{likelihood}} = p(\hat{x}|H)$$

← the only thing that's easy to calculate

$\boxed{\text{P-values}}$: how crazy is $\hat{x}$ if H is true?



$$\alpha = \int_{\hat{x}}^{\infty} dx \; p(x|H)$$

← TAILS MAKE A DIFFERENCE! (where CLT can fail!!)

can also do two sided...

if this is small then unlikely that the H would produce observation $\hat{x}$

for GAUSSIAN $p(x)$: $p = 16\% \Rightarrow 1\sigma$

$p = 2.3\% \rightarrow 2\sigma$

$p = 0.14\% \rightarrow 3\sigma$

⋮

Better & harder question:
prob of H given $\hat{x}$?

$$P(H|\hat{x}) = \frac{P(\hat{x}|H)\,P(H)}{P(\hat{x})}$$

ḿ ?!

= 1, we definitely measured $\hat{x}$

P(H): PRIOR PROBABILITY

subjective-as-hell probability prior to any measurement.

POSTERIOR PROBABILITY
(euphemism for butt.)

USUALLY the HYPOTHESIS HAS A PARAMETER THAT WE WANT TO ESTIMATE, a.

$$P(H(a)|\hat{x}) = P(\hat{x}|H(a)) \cdot \underbrace{P(H(a)) \cdot \frac{1}{P(\hat{x})}}$$

= 1

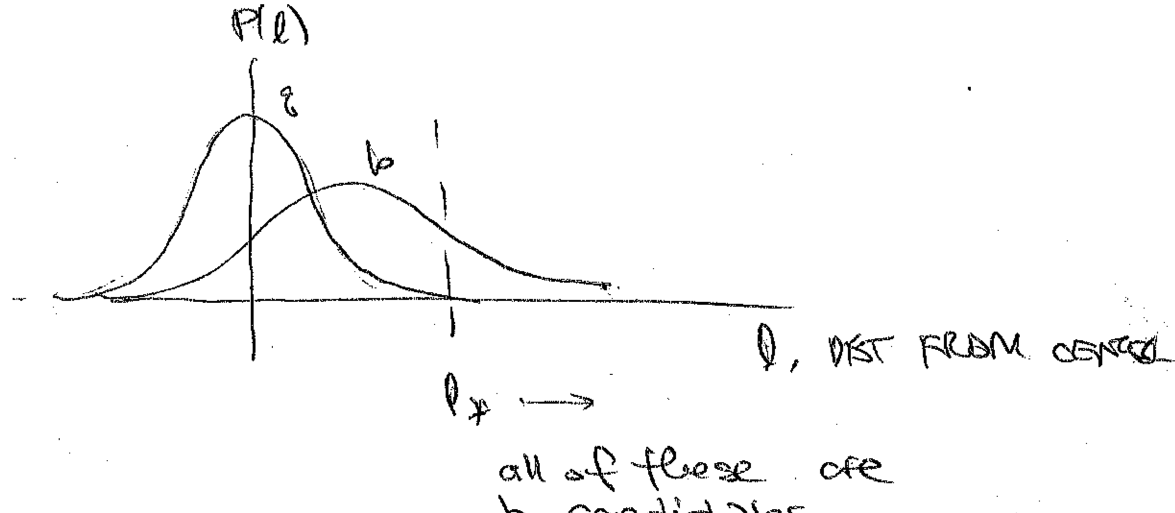
eg flat prior on a

$$P(H(a)) = 1/\int da'\, P(\hat{x}|a')$$

eg. @ LHC: observe sprays of particles coming from

light quark → q: EXPLODE EARLY

or b quarks → b: EXPLODE LATE



~~FREQUENTIST~~

want: measure the decays of b-jets.

TWO EFFECTS:

EFFICIENCY: given actual b, how likely to catch?

$$\varepsilon_b = \int_{l_*}^{\infty} dl \; p(l \mid b)$$

PURITY: given something we think is a b, HOW LIKELY THAT IT IS?

$$\varepsilon_l = \int_{l_*}^{\infty} dl \; p(l \mid q)$$

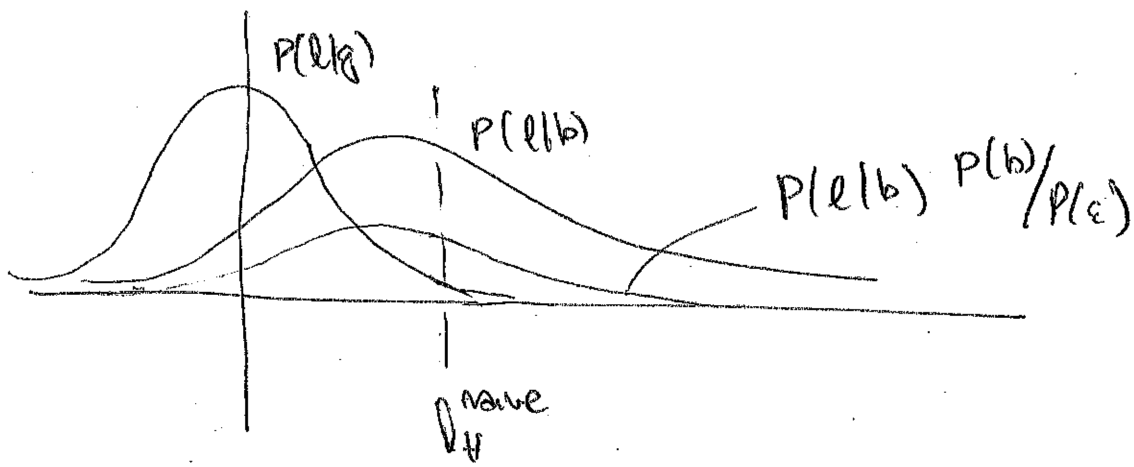
FREQUENTIST: l_* CHOSEN TO BALANCE high ε_b & low ε_l

BAYESIAN: ACTUALLY -- LHC PRODUCES WAY MORE qj's than bj's

Relative probabilities

$$\frac{P(l \mid b)\; P(b)}{P(l \mid g)\; P(g)}$$

this is small.



not as much separation

2x AS LIKELY TO BE CHANGELING

10 K CHANGELINGS

	on WATCH LIST	not on WATCH LIST
CHANGELING	10	9,990
HUMANOID	99,990	199,890,010

100 K FLAGGED

total: 200M