

HOMEWORK 4: Damped Harmonic Oscillator

COURSE: Physics 231, *Methods of Theoretical Physics* (2018)

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DUE BY: Mon, November 26 (or before the last lecture of the term)

1 The 1D Damped Harmonic Oscillator: application

This problem is based on Byron and Fuller, *Mathematics of Classical and Quantum Physics*, section 7.3. You can also find the procedure outlined¹ in the 2017 lecture notes from this class.

1.1 Find the Green's Function

The damped harmonic oscillator with driving force (per unit mass) $F(t)$.

$$\ddot{x}(t) + 2\gamma\dot{x}(t) + \omega_0^2 x(t) = F(t) . \quad (1.1)$$

Following the same procedure that we did for the un-damped harmonic oscillator, show that the Green's function is:

$$G(t - t') = \frac{e^{-\gamma(t-t')} \sin \left[\sqrt{\omega_0^2 - \gamma^2} (t - t') \right]}{\sqrt{\omega_0^2 - \gamma^2}} . \quad (1.2)$$

HINT: Do a Fourier transform to determine $\tilde{G}(k)$, then perform the integral to obtain $G(t)$. In the *undamped* case, we had to pick a $\pm i\epsilon$ prescription to push the poles off the real axis. The choice of prescription determined whether we were calculating the advanced, retarded, or Feynman Green's function. In this case, there is no such prescription to choose—do you see why?

1.2 A full problem

Let's go back to the complete problem. We would like to find the solution to the forced, damped spring with some force function $F(t)$, that is, (1.1). The solutions of this system are $x(t)$, given by

$$x(t) = Ax_1(t) + Bx_2(t) + \int_{t_1}^{t_2} \frac{e^{-\gamma(t-t')} \sin \left[\sqrt{\omega_0^2 - \gamma^2} (t - t') \right] F(t')}{\sqrt{\omega_0^2 - \gamma^2}} dt' , \quad (1.3)$$

where $x_{1,2}(t)$ are solutions to the *homogeneous* differential equation, that is (1.1) with $F(t) = 0$. Recall that we may take the upper limit of integration to be $t_2 = t$. The lower limit, t_1 can be taken to be the time at which the driving force is applied.

¹With possible egregious sign errors!

Extra Credit

These problems are not graded and are for your edification. You are strongly encouraged to explore and discuss these topics, especially if they are in a field of interest to you.

Curriculum Vitae

I strongly encourage you to do this. Write a curriculum vitae. I suggest using a L^AT_EX template, e.g. from <https://www.overleaf.com/gallery/tagged/cv> . For more information about what a CV is versus a resume, see <https://gradschool.cornell.edu/academic-progress/pathways-to-success/prepare-for-your-career/take-action/resumes-and-cvs/>

A future CV

Take your CV from the previous problem. Remove your name, this is no longer a CV about you. Imagine a hypothetical graduate student nearing completion. The hypothetical graduate student has the same initial condition as you—so their CV looked just like yours in their first year at UCR. Now fill in the CV with the kinds of accomplishments (feel free to make up paper titles and collaborators) that make this hypothetical graduate student an excellent candidate for the types of positions you envision after graduate school. Feel free to ask for advice from more senior people in your field.