TODAY: Return assignments
Review + Big picture

one last note card: CANDID, DIRECT FEEDBACK
NO name!

this class in a nutshell:

- DIMENSIONAL ANALYSIS
- VECTOR SPACES ⟶ ∞ DIM
  - $\mathcal{O}$ AS "MATRICES"
  - DIFF. EQ AS LIN. ALG

$$\mathcal{O} f(x) = s(x) \Longrightarrow f(x) = \underbrace{\mathcal{O}^{-1}}_{\text{indices in a basis}} s(x)$$

$$\equiv \int dx' \, G(x,x') s(x') \quad \left(\equiv \sum_j G^i{}_j s^j\right)$$

DEFINITION: $\boxed{\mathcal{O}_x G(x,x') = \delta(x-x')}$ (def)

So WHAT? Fourier trick: $G(x,x') = \int dk \, \underbrace{e^{-ikx}}_{\text{ALL x-DEP}} \tilde{G}(k,x')$

$$\mathcal{O}_x G = \int dk \, (\mathcal{O}_x e^{-ikx}) \tilde{G} = \int dk \, e^{-ik(x-x')}$$

$\mathcal{O}_x e^{-ikx} = P(k) e^{-ikx}$, $P(k)$: POLYNOMIAL IN k — match coeff.

$$\Rightarrow \boxed{\tilde{G}(k,x') = \frac{e^{+ik\cdot x'}}{P(k)}} \leftarrow \text{eg } -k^2 + \omega_0^2$$

$$G(x,x') = \int dk \; \frac{e^{-ik(x-x')}}{P(k)}$$

↑ tough integral

① TRICK: Residue thm.

② POLE PUSHING

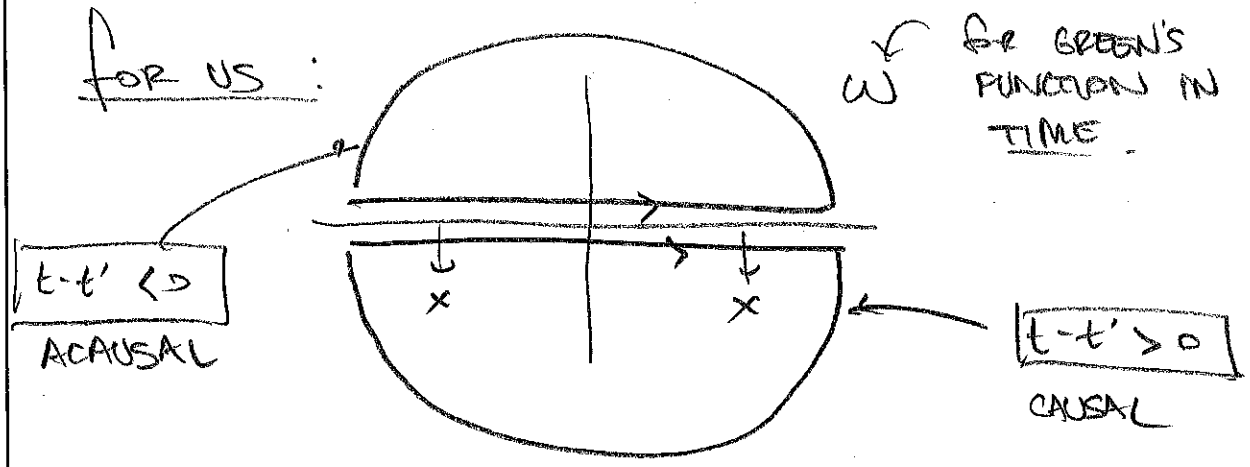
move poles off integ. contour.

how to pick?

USUALLY CAUSALITY.

FOR US:

ω ← for GREEN'S FUNCTION IN TIME.



then: $\boxed{G(x,x') = 2\pi i \sum_j \text{Res}(z_j)}$

so WHAT? $Of = s$

$$f(x) = \int dx' \, G(x,x') \, s(x')$$

Well, what if i don't like that?

OTHER OPTIONS:

① Series solution

Go to BASIS of EIGENFUNCTIONS

↳ vs. POSITION SPACE
vs. MOMENTUM SPACE

$$\mathcal{O}\left(\underbrace{\sum f_i e^i(x)}_{f(x)}\right) = \underbrace{\sum s_i e^i(x)}_{s(x)}$$

$$\sum_i f_i \lambda_i e^i(x) = \sum_i s_i e^i(x)$$

match. (hit w/ BRA on both sides)

② Piece-wise solution to HOMOG.

(*) $\mathcal{O}G(x,x') = \delta(x-x')$

$\Downarrow$

$\mathcal{O}G_>(x,x') = 0$ — $G_>$ only for $x > x'$

$\mathcal{O}G_<(x,x') = 0$ — $G_<$ ——— $x < x'$

HOMOG. eq.; SOLVE USING BAG o TRICKS.

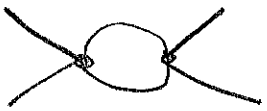
2 UNKNOWN functions → 2x the PARAMETERS

↳ USE (*) + integrals of (*) to match two functions over $\delta(x-x')$

what if my problem isn't fmg LINEAR?!

↑ MOST PROBLEMS ARE NOT "fully" LINEAR

1. simulate / numerical
2. PERTURBATION THEORY IN NON-LINEAR PART
   ↳ eg. Feynman Diagrams



combo of LINEAR + NON LINEAR

each line: linear propagation of a fluctuation
↳ like SHO or EM wave

VERTEX: nonlinear stuff happens.

PROBABILITY for THIS TO HAPPEN SHOULD BE "SMALL"

3. NATURE is KIND
   ↳ other handle
   → TOPOLOGICAL PROTECTION
   → EFFECTIVE PERTURBATIVE THEORY
   → symmetries, mathematical structure

So now what?

so much of what we do is linear algebra.
↑ DIM. ANALYSIS

more math: DIFF. GEOM. ↔ LINEAR ALGEBRA + CALCULUS