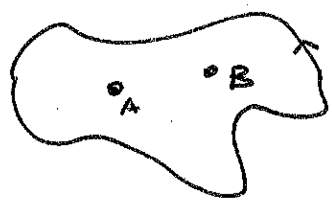
Notes:

FRIDAY: we review $\mathbb{C}$ analysis

- POSTING SOME RESOURCES on ilearn

2. PREP: "ANALYTIC" / HOLOMORPHIC ← "nice"
calculus on complex plane
↳ line integrals

GOAL: RESIDUE THEOREM



$\propto$ Res A + Res B

why? $\mathcal{O}_x G(x,x') = \delta(x-x')$

$$\cancel{\int \frac{dk}{2\pi} \tilde{G}(k,x') e^{ikx} = \int \frac{dk}{2\pi}}$$

$$\mathcal{O}_x \int đk\, e^{-ikx} \tilde{G}(k,x') = \int đk\, e^{-ikx} \underline{e^{+ikx'}}$$

$\left(đ = \frac{d}{2\pi}\right)$

if $\mathcal{O}_x = \sum_{n=0}^{\infty} p^n(x) \left(\frac{d}{dx}\right)^n = P\left(x, \frac{d}{dx}\right)$

then: LHS:

$$\int đk\, e^{-ikx} \underline{P(x,-ik)\tilde{G}(k,x')}$$

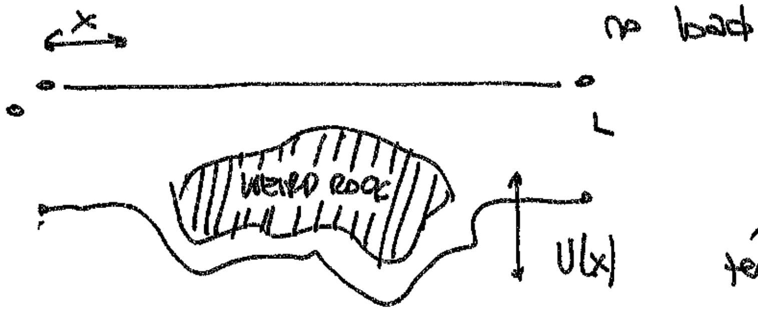
IDENTIFY FOURIER COEFF:

Do this $\mathbb{C}$ int

$$\tilde{G}(k,x') = \frac{e^{ikx'}}{P(x,-ik)} \Rightarrow \boxed{G(x,x') = \int đk \frac{e^{-ik(x-x')}}{P(x,-ik)}}$$

example of PATCHING (BUTKOV CH. 12.1)

string under load



$$T\left(\frac{d}{dx}\right)^2 u = F(x)$$

tension

force/len

$$\Rightarrow \boxed{\left(\frac{d}{dx}\right)^2 u(x) = f(x)}$$

solve by Green's func:

$$u(x) = \int_0^L \underline{G(x,x')}\, f(x')\, dx'$$

find by patching.

$$\left(\frac{d}{dx}\right)^2 G(x,x') = \delta(x-x')$$

$$G(x,x') = \begin{cases} G^<(x) & x < x' : \left(\frac{d}{dx}\right)^2 G^< = 0 \\ G^>(x) & x > x' : \left(\frac{d}{dx}\right)^2 G^> = 0 \end{cases}$$

$$G^< = ax + b \qquad G^> = Ax + B$$

BC: $G(0,x') = 0 \longrightarrow b = 0$

$G(L,x') = 0 \longrightarrow B = -AL$

Key step:

$$\int_{x'-\varepsilon}^{x'+\varepsilon} \frac{d}{dx}\left(\frac{d}{dx}G\right) dx = \int_{x'-\varepsilon}^{x'+\varepsilon} dx\; \delta(x-x')$$

$$\boxed{\frac{d}{dx}G\Big|_{x'-\varepsilon}^{x'+\varepsilon} = 1}$$ JUMP CONDITION

1st derivative changes suddenly by a unit impulse @ x'

BUT G is <u>continuous</u>.

$$\boxed{G(x'+\varepsilon) = G(x'-\varepsilon)}$$

do: $\partial_x G^>(x') - \partial_x G^<(x') = 1$

$$\boxed{A - a = 1}$$

$$G^>(x') = G^<(x')$$

$$\boxed{Ax' - AL = ax'}$$

$A - a = 1$

$A(x'-L) = ax'$

then you're done: $A(x'-L) = Ax' - x'$

$$\Rightarrow A = x'/L$$

$$a = \frac{x'-L}{L}$$

$$G(x,x') = \begin{cases} \left(\frac{x'-L}{L}\right)x = & x<x' \\ \frac{x'}{L}x - x' = \left(\frac{x-L}{L}\right)x' & x>x' \end{cases}$$

note: $G(x,x') = G(x',x)$