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Monotone empirical bayes test for the parameter of pareto distribution under random censorship

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Abstract

This article considers the monotone empirical Bayes (MEB) test problem for the parameter of Pareto distribution with censored data. We construct the MEB test rule for the parameter of Pareto distribution under random censorship. It is shown that the proposed MEB test have asymptotically optimal property. The convergence rate of the MEB test is obtained and the rate can approach to $O(n^{-1/2})$ arbitrarily under suitable conditions.

Keywords: *Empirical bayes test, Random censorship, Convergence rate, Pareto distribution*

2010 Mathematics Subject Classification: *62C12*

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1. Introduction

In recent years there has been a great deal of interest in the analysis of empirical Bayes test. Liang (1999) has studied monotone empirical Bayes tests for a discrete normal distribution. Xu (2004) has investigated empirical Bayes test for truncation parameters using linex loss. Shi (2005) has studied two-sided empirical Bayes test for truncation parameter using NA samples. Li (2003) has studied empirical Bayes tests for lower truncation parameters. For random censored data, Blum (1997) and Breslow (1974) have given the estimator of the distribution function, Mielniczuk (1986) and Wang (2000) have studied asympotic properties of kernel density estimator function. Kaplan (1958) has given the nonparametric estimator for censored data. Wang (2005, 2007) study monotone empirical Bayes test for scale parameter and convergence rates of empirical Bayes estimation for the parameter of the uniform distribution $U(0, \theta)$ under random censored data. However, for Pareto distribution with censored data, there are no papers related to empirical Bayes test. It is well known that Pareto probability laws are formulated by Pareto and initially dealt with the distribution of income over a population. Pareto distribution is nowadays popular in describing a variety of other phenomena such as city population sizes, occurrence of natural resources and stock price fluctuations. In this article, we shall study Empirical Bayes test for the parameter of Pareto distribution under random censorship.

Let X denote a random variable arising from the Pareto distribution, having the following conditional density:

$$f(x \mid \theta) = \frac{\theta\alpha^{\theta}}{x^{\theta+1}}, \tag{1.1}$$

where $\alpha > 0$ is a constant and $\theta \in (0, +\infty)$.

The hypothesis to be tested in the paper is

$$H_0 : \theta \le \theta_0 \text{ versus } H_1 : \theta > \theta_0,$$

where θ_0 is a known positive constant. We adopt the following weighted linear loss function:

$$L(\theta, d_0) = a(\theta - \theta_0) I(\theta > \theta_0)$$

and

$$L(\theta,\ d_1) = a(\theta_0 - \theta) I(\theta \geq \theta_0),$$

where $D = \{d_0, d_1\}$ is the action space with d_i accepting $H_i\,(i = 0,1)$, $L(\theta, d_i)$ indicates the loss when the decision is in favor of H_i.

Assume that the parameter θ has an unknown non-degenerate prior distribution

$G(\theta)$ with support on $\Omega = \{\theta \mid \theta > 0\}$. Hence the marginal p.d.f. of $r.v.X$ is

$$\mathrm{f(x)} = \int_0^{+\infty} \mathrm{f(x}|\theta\mathrm{)dG(}\theta\mathrm{)}.$$

Let $\delta(x) = P(\text{accept}H_0 \mid X = x)$ be a randomized decision rule. Then the Bayes risk of $\delta(x)$ can be written as,

$$R(\delta(x),\ G(\theta)) = \int_\Omega \int_\Delta \left[L_0(\theta, d_0) f(x|\theta)\delta(x) + L_1(\theta, d_1) f(x|\theta)(1 - \delta(x)) \right] dx dG(\theta)$$

$$= a\int_\Delta Q(x)\delta(x)\,dx + C_G, \tag{1.2}$$

where $\Delta = [x/m,\ x]$, $C_G = \int_\Omega \int_\Delta [L(\theta,\ d_1) f(x \mid \theta)] dx dG(\theta)$,

$$\mathrm{Q(x)} = \int_\Delta (\theta - \theta_0) f(x|\theta)\,dG(\theta) = -(\theta_0 - 1) f(x) - x f^{(1)}(x), \tag{1.3}$$

and $f^{(1)}(x)$ denotes the first derivative of $f(x)$.

From (1.2), the best Bayes test rule is minimzing $R(\delta_G(x), G(\theta))$ would have the form

$$\delta_G(x) = \begin{cases} 1 & Q(x) \leq 0, \\ 0 & Q(x) > 0. \end{cases} \tag{1.4}$$

The minimum Bayes risk is

$$R_G = R(\delta_G, G(\theta)) = a\int_\Theta Q(x)\delta_G(x)\,dx + C_G. \tag{1.5}$$

However, Bayes test $\delta_G(x)$ in (1.4) is unavailable to use since the prior $G(\theta)$ is unknown. As an alternative we can use the MEB approach to estimate $G(\theta)$ so as to obtain an MEB test.

The paper is organized as follows. Section 2 proposes an MEB test under random censorship. In Section 3, we present some useful lemmas. Section 4 is devoted to obtaining the main result.

2. Monotone Empirical Bayes Test

In the MEB framework, we make the following assumptions: let $(X_1,\theta_1),\cdots,(X_n,\theta_n)$ and (X,θ) be independent random vectors, the $\theta_i\,(i=1,\cdots,n)$ and θ are independently identically distributed (i.i.d.) and have the common prior distribution $G(\theta)$; $X_1,\cdots,X_n$ and X are i.i.d. and have the common marginal density $f(x)$.

We suppose that the sequence $X_1,\cdots,X_n$ is censored from the right by $Y_1,\cdots,Y_n$. Let the sequence $X_1,\cdots,X_n$ and $Y_1,\cdots,Y_n$ be independent of each other. Let $Z_i=\min\{X_i,Y_i\}$ and $\delta_i=I(X_i\le Y_i)$, $i=1,\cdots,n$. If we consider the situation when only the censored sample (Z_i,δ_i), $\mathrm{i}=1,\cdots,\mathrm{n}$. is observed.

Define $\bar{F}=1-F=1-\int_{-\infty}^{x} f(x)dx$. Then Z_i are i.i.d. with the distribution Function H. An optimal nonparametric estimator of F is the well-known Kaplan-Meier (see Kaplan(1958)) estimate given by

$$1-\hat{F}_n(x)=\prod_{i=1}^{n}\left[\frac{\bar{H}_n(Z_i)}{\bar{H}_n(Z_i)+\frac{1}{n}}\right]^{\delta_i(x)}, \tag{2.1}$$

where $H(x)=P(Z_i\le x)$, $H_n(x)$ is the empirical distribution function $H(x)$ and $\bar{H}_n=1-H_n$.

Estimation of the underlying density function $f(x)$ has been discussed by numerous authors (see Wang (2000), Mielniczuk (1986) and Lemdani(2002) among others). The usual procedure is to replace the empirical distribution function $F_n(x)$ in $f_n(x)=h_n^{-1}\int k(x-y/h_n)dF_n(y)$ by the PL estimator $\hat{F}(y)$. Then we can obtain the estimator of $f(x)$ given by

$$f_n(x)=\frac{1}{h_n}\int k_0\left(\frac{x-y}{h_n}\right)d\hat{F}_n(y), \tag{2.2}$$

where $0<h_n\to 0$ as $n\to\infty$.

Similarly, we can define a kernel estimator for $f^{(1)}(x)$ as

$$f_n^{(1)}(x) = \frac{1}{h_n^2} \int k_1\left(\frac{x-y}{h_n}\right) d\hat{F}_n(y). \tag{2.3}$$

We make the following assumptions about the kernel function $k_i(x)(i=0,1)$:

(A1) $k_i(x)(i=0,1)$ are continuously differentiable with compact support $[0,1]$;

(A2) $\int_0^1 x^j k_i(x)dx = \begin{cases} (-1)^j & j=i \\ 0 & j \neq i \end{cases}$, j = 0,1, , s-1;

(A3) $\int_0^1 x^s k_i(x)dx \neq 0, k_i(0) = k_i(1) = 0$, where $s > 2$ is an arbitrary but fixed integer.

Let $Q_n(x)$ be the estimator of $Q(x)$.

$$Q_n(x) = -(\theta_0+1) f_n(x) - x f_n^{(1)}(x). \tag{2.4}$$

We define the EBT of $\delta_n(x)$ by

$$\delta_n(x) = \begin{cases} 1 & Q_n(x) \leq 0, \\ 0 & Q_n(x) > 0. \end{cases} \tag{2.5}$$

The Bayes risk of $\delta_n(x)$ is

$$R_n = R_n(\delta_n, G(\theta)) = a\int_\Theta Q_n(x)\delta_n(x)dx + C_G. \tag{2.6}$$

If $\lim_{n\to\infty} R(\delta_n, G) = R(\delta_G, G)$, the EB test $\delta_n(x)$ is said to be asymptotically optimal. Moreover if for $q>0$, $R_n - R_G = O(n^{-q})$, then the EB test $\delta_n(x)$ is said to be asymptotically optimal with convergence rate of $O(n^{-q})$.

3. Several Lemmas

Lemma 3.1: *Let T be such that* $1 - H(T) > \delta$ *with some* $\delta > 0$. *Then the Process* $\hat{F}(t) - F(t), -\infty < t < \infty, 1 - H(t) > 0$ *can be represented as*

$$\hat{F}(t) - F(t) = \frac{1}{n}\sum_{j=1}^{n}\left[1 - F(t)\right] M_j(t) + \frac{1}{n} R_n(t),$$

in such a way that

$$P\left(\sup_{t\le T}\left|R_n(t)\right| > \frac{2c}{\delta}\log^2 n + x\log n\right) \le 2K\exp\left(-\lambda\delta^2 x\right), \qquad x > 0.$$

where $M_1(t),\ M_2(t),\cdots,M_n(t)$ are Gaussian processes with $EM_i(t)=0$ and covariance function

$$EM(\mathrm{s})M(t) = EM(s)^2 = \int_{-\infty}^{s}\frac{dF(t)}{\left(1-W(t)\right)\left[1-F(t)\right]^2},$$

here $C>0,\ K>0,\ \lambda>0$ are some universal constants.

Proof: See Major (1998) and Lemdani (2002).

Lemma 3.2: *Let $f_n(x)$ and $f_n^{(1)}(x)$ be defined in (2.2) and (2.3),respectively. If $f(x)$ is the s-th ($s \ge 2$) continuously differentiable and the kernel function $k_i(x)$ ($i = 0$, 1) satisfy the assumptions(A1)-(A3). Then for $x < T$, taking $h_n = n^{-1/(2s+1)}$, we have*

$$E_n\left[f_n^i(x)-f^i(x)\right]^2 \le \left\{c_{1i}\left[f^s(x)\right]^2 + c_{2i}f(x)\left[1-W(x)\right]^{-1}\right\}n^{-2(s-i)/(2s+1)}$$

Proof: Integrating by parts, we have

$$\begin{aligned}
&\left[f_n^{(i)}(x)-f^{(i)}(x)\right]^2\\
&=\left[\frac{1}{h_n^{1+i}}\int k_i\left(\frac{x-t}{h_n}\right)d\hat{F}(t)-\frac{1}{h_n^{1+i}}\int k_i\left(\frac{x-t}{h_n}\right)dF(t)+\frac{1}{h_n^{1+i}}\int k_i\left(\frac{x-t}{h_n}\right)dF(t)-f^{(i)}(x)\right]^2\\
&\le 2\left[\frac{1}{h_n^{1+i}}\int k_i\left(\frac{x-t}{h_n}\right)d\hat{F}(t)-\frac{1}{h_n^{1+i}}\int k_i\left(\frac{x-t}{h_n}\right)dF(t)\right]^2\\
&+2\left[\frac{1}{h_n^{1+i}}\int k_i\left(\frac{x-t}{h_n}\right)dF(t)-f^{(i)}(x)\right]^2\\
&=2\left\{\frac{1}{h_n^{1+i}}\int k_i\left(\frac{x-t}{h_n}\right)d\left(\hat{F}(t)-F(t)\right)^2\right\}+2\left[\frac{1}{h_n^{1+i}}\int k_i\left(\frac{x-t}{h_n}\right)dF(t)-f^{(i)}(x)\right]^2\\
&\le 2\left\{\frac{1}{h_n^{2+i}}\int\left[\hat{F}(t)-F(t)\right]k_i^{(1)}\left(\frac{x-t}{h_n}\right)dt^2\right\}+2\left[\frac{1}{h_n^{1+i}}\int k_i(u)f\left(x-uh_n\right)du-f^{(i)}(x)\right]^2
\end{aligned}$$

$$\triangleq 2I_1^2 + 2I_2^2. \tag{3.1}$$

From lemma 3.1

$$I_1 = \frac{1}{h_n^{2+i}} \int \left[\hat{F}(t) - F(t) \right] k_i^{(1)} \left(\frac{x-t}{h_n} \right) dt = \frac{1}{nh_n^{1+i}} \int_0^1 \sum_{j=1}^{n} \left[1 - F\left(x - uh_n\right) \right]$$
$$M_j\left(x-uh_n\right) k_i^{(1)}\left(u\right) du \quad + \frac{1}{nh_n^{1+i}} \int_0^1 R_n\left(x-uh_n\right)$$
$$k_i^{(1)}(u)du \triangleq I_{11} + I_{12} \tag{3.2}$$

Note that

$$I_{11}^2 = \frac{1}{n^2 h_n^{2+2i}} \int_0^1 \int_0^1 \overline{F}\left(x-uh_n\right) \overline{F}\left(x-vh_n\right) \sum_{j=1}^{n} M_j\left(x-uh_n\right)$$
$$\sum_{j=1}^{n} M_l\left(x-uh_n\right) k_i^{(1)}\left(u\right) du k_i^{(1)}\left(v\right) dv \tag{3.3}$$

Hence

$$E_n I_{11}^2 = \frac{1}{nh_n^{2+2i}} \int_0^1 \int_0^1 E\left[M_j\left(x-uh_n\right) M_l\left(x-uh_n\right) \right] \overline{F}\left(x-uh_n\right)$$
$$\overline{F}\left(x-vh_n\right) k_i^{(1)}\left(u\right) k_i^{(1)}\left(v\right) dudv$$
$$= \frac{1}{nh_n^{2+2i}} \int_0^1 \int_0^u d\left(x-uh_n\right) \overline{F}\left(x-uh_n\right) \overline{F}\left(x-vh_n\right) k_i^{(1)}\left(u\right)$$
$$k_i^{(1)}\left(v\right) dudv + \frac{1}{nh_n^{2+2i}} \int_0^1 \int_u^1 d\left(x-uh_n\right) \overline{F}\left(x-uh_n\right)$$
$$\overline{F}\left(x-vh_n\right) k_i^{(1)}\left(u\right) k_i^{(1)}\left(v\right) dudv$$
$$\triangleq \frac{1}{nh_n^{2+2i}} \int_0^1 \left(\tau_1\left(u\right) + \tau_2\left(u\right) \right) k_i^{(1)}\left(u\right) du \tag{3.4}$$

$$\tau_1(u) = d\left(x\right) \overline{F}^2(x) \left\{ \int_0^u 1 + h_n \left[u \frac{f(x)}{\overline{F}(x)} + v \frac{f(x)}{\overline{F}(x)} - u \frac{d^{(1)}(x)}{d(x)} \right] + o\left(h_n\right) \right\} k_i^{(1)}(v) dv$$

$$\tau_2(u) = d(x)\overline{F}^2(x)\left\{\int_u^1 1 + h_n\left[u\frac{f(x)}{\overline{F}(x)} + v\frac{f(x)}{\overline{F}(x)} - u\frac{d^{(1)}(x)}{d(x)}\right] + o(h_n)\right\}k_i^{(1)}(v)dv.$$

here $d^{(1)}(x)$ denotes the derivative of : $d(x) = \int_{-\infty}^{s}\frac{dF(t)}{(1-W(t))(1-F(t))^2}$.

If the assumptions of $k_i(x)$ (A1)-(A3) hold, it generates

$$E_n I_{11}^2 = \frac{f(x)}{[1-W(x)]nh_n^{2i+1}}\int_0^1 k_i^2(u)\,du + o\left(\frac{1}{nh_n^{2i+1}}\right) \tag{3.5}$$

Also, using Lemma 3.1, it is easily seen that

$$E_n I_{12}^2 = c_{1i}\left(\frac{1}{nh_n^{i+1}}\right)^2 E\left[\sup_{t\le T}|R_n(t)|\right]^2 \le c_{1i}\left(\frac{1}{nh_n^{i+1}}\right)^2$$

$$\int_0^{+\infty} xP\left(\sup_{t\le T}|R_n(t)| \ge x\right)dx = O\left(\frac{\log^4 n}{n^2 h_n^{2i+2}}\right). \tag{3.6}$$

On the other hand, expanding $f(x-uh_n)$ and using the assumption (A1)-(A3), we know:

$$I_2^2 \le 2\left\{\frac{1}{h_n^i}\int_0^1 K_i(u)\left[f(x) + \sum_{k=1}^{s-1}\frac{f^k(x)}{k!}(-uh_n)^k + \frac{f^s(x)}{s!}(-uh_n)^s\right]du - f^{(i)}(x)\right\}^2$$

$$= 2\left\{\frac{1}{h_n^i}\int_0^1 K_i(u)\frac{f^i(x)}{i!}(-uh_n)^i\,du + \frac{1}{h_n^i}\int_0^1 K_i(u)\frac{f^s(x^*)}{s!}(-uh_n)^s du - f^{(i)}(x)\right\}^2$$

$$= 2\left\{\frac{1}{h_n^i}\int_0^1 K_i(u)\frac{f^s(x^*)}{s!}(-uh_n)^s du\right\}^2 = 2\left\{h_n^{s-i}\frac{f^s(x^*)}{s!}\int_0^1 K_i(u)(-u)^s du\right\}^2$$

$$= 2h_n^{2(s-i)}\left[\frac{f^s(x^*)}{s!}\right]^2 + o\left(h_n^{2(s-i)}\right), \qquad x^* \in (x-uh_n, x). \tag{3.7}$$

Following from (3.1)-(3.7) and taking $h_n = n^{-1/(2s+1)}$, we conclude that the conclusion of the Lemma 3.2 is true.

Lemma 3.3: *If the conditions of Lemma3.2 hold, and the following condition are satisfied(A4)* $\sup\left|f^{(s)}(x)\right| < 1$, *and* $\sup\left|f(x)(1-W(x))\right| < \infty$. *Then*

(1) let $f^{(i)}(x)$ is continuously, taking $h_n = n^{-1/(2s+1)}$, we have

$$\lim_{n\to\infty} E\left|f_n^{(i)}(x) - f^{(i)}(x)\right|^2 = 0$$

(2) let $f(x) \in C_{s,\alpha}$, taking $h_n = n^{-1/(2s+1)}$, then for any $0 < \lambda \le 1$, we have

$$\lim_{n\to\infty} E\left|f_n^{(i)}(x) - f^{(i)}(x)\right|^{2\lambda} \le cn^{-\frac{2\lambda(s-i)}{2s+1}}.$$

Proof: From Lemma 3.2 and (A4), let $h_n \to 0$ we can easily obtain this Lemma.

Lemma 3.4: *If R_G denotes the Bayes risk of δ_G, and R_n is defined by (2.6). Then*

$$0 \le R_n - R_G \le \int |Q(x)| P\left[\left|Q_n(x) - Q(x)\right| \ge Q(x)\right] dx \tag{3.8}$$

Proof: See Lemma 3.1 in Xu (2004).

4. Rate of Convergence

Theorem 4.1: *Let δ_G and δ_n be defined by (1.4) and (2.5) respectively. Suppose the conditions (A1)-(A4) hold and $\int_\Omega \theta dG(\theta) \le \infty$. Then as $h_n = n^{-1/(2s+1)}$, we have*

$$\lim_{n\to\infty} R(\delta_n, G) = R(\delta_G, G).$$

Proof of Theorem 4.1. According to Lemma 3.4.

$$0 \le R_n - R_G \le a\int |Q(x)|\, P\left[\left|Q_n(x) - Q(x)\right| \ge Q(x)\right] dx$$

Let $\Psi_n(x) = |Q(x)| P\left[\left|Q_n(x) - Q(x)\right| \ge Q(x)\right]$, hence $\Psi_n(x) \le |Q(x)|$.

$$\int_\Omega |Q(x)|\, dx \le |\theta_0| \int_\Omega f(x)\, dx + \int_\Omega \int_\Theta \theta f(x|\theta)\, dG(\theta) dx = |\theta_0| + \int_\Omega \theta dG(\theta) \le \infty. \tag{4.1}$$

$$0 \le \lim_{n\to\infty} \left|R(\delta_n, G) - R(\delta_G, G)\right| \le \int_\Omega \left[\lim_{n\to\infty} \Psi_n(x)\right] dx,$$

$$\Psi_n(x) \le \left|1+\theta_0\right| \left[E\left| f_n(x) - f(x) \right|^2 \right]^{\frac{1}{2}} + |x| \left[E\left| f_n^{(1)}(x) - f^{(1)}(x) \right|^2 \right]^{\frac{1}{2}}$$

From Lemma 3.3

$$0 \le \lim_{n\to\infty} \Psi_n(x) \le \left|1+\theta_0\right| \left[\lim_{n\to\infty} E\left| f_n(x) - f(x) \right|^2 \right]^{\frac{1}{2}}$$
$$+ |x| \left[\lim_{n\to\infty} E\left| f_n^{(1)}(x) - f^{(1)}(x) \right|^2 \right]^{\frac{1}{2}} = 0 \tag{4.2}$$

we can obtain this Theorem.

Theorem 4.2: *Let δ_G and δ_n be defined by (1.4) and (2.5) respectively. Suppose the conditions (A1)-(A4) hold and $\int_\Omega x^{m\lambda} \left|Q(x)\right|^{(1-\lambda)} dx \le \infty$. Then as $h_n = n^{-1/(2s+1)}$, we have*

$$R(\delta_n, G) - R(\delta_G, G) = O\left(n^{-\frac{\lambda(s-1)}{2s+1}} \right).$$

Proof of Theorem 4.2.

$$\begin{aligned} 0 &\le R(\delta_n, G) - R(\delta_G, G) \le c\int_\Omega \left|Q(x)\right|^{(1-\lambda)} E_n \left|Q_n(x) - Q(x)\right|^\lambda dx \\ &\le c_1 \int_\Omega \left|Q(x)\right|^{(1-\lambda)} E_n \left| f_n(x) - f(x)\right|^\lambda dx + c_2 \int_\Omega x^\lambda \left|Q(x)\right|^{(1-\lambda)} \\ &E_n \left| f_n^{(1)}(x) - f^{(1)}(x)\right|^\lambda dx \\ &= A_n + B_n. \end{aligned} \tag{4.3}$$

From Lemma 3.3

$$A_n \le c_1 n^{-\frac{\lambda s}{2s+1}} \int_\Omega \left|Q(x)\right|^{(1-\lambda)} dx \le c_3 n^{-\frac{\lambda s}{2s+1}}, \tag{4.4}$$

$$B_n \le c_2 n^{-\frac{\lambda(s-1)}{2s+1}} \int_\Omega x^\lambda \left|Q(x)\right|^{(1-\lambda)} dx \le c_4 n^{-\frac{\lambda(s-1)}{2s+1}}. \tag{4.5}$$

From (4.4)-(4.5), we have

$$R\left(\delta_n, G\right) - R\left(\delta_G, G\right) = O\left(n^{-\frac{\lambda(s-1)}{2s+1}}\right),$$

which completes the proof.

Remark: In Theorem 4.2, if $\lambda \to 1$ and $s \to \infty$, $O\left(n^{-\frac{\lambda(s-1)}{2s+1}}\right)$ can close to $O(n^{-½})$ arbitrary.

5. An Example

Let $f(x \mid \theta) = \dfrac{\theta\alpha^{\theta}}{x^{\theta+1}}$, if we take the prior density $g(\theta) = \theta^{-1}\alpha^{-\theta}e^{-\theta}$. Then $f(x) = \frac{1}{x^2}$, $Q(x) = -(\theta_0 + 1)\frac{1}{x^2}$. We can easily prove the Theoerm 4.2 conditions are satisfied and accordingly the rate of convergence can be arbitrarily close to $O(n^{-½})$ under the condition that $\lambda \to 1$ and s $\to \infty$.

6. Conclusion

We study MEB test for testing the hypothesis $H_0 : \theta > \theta_0$ *versus* $H_1 : \theta > \theta_0$ for the parameter of Pareto distribution with censored data. By using a kernel estimation of the density function, we construct the MEB test rule for the parameter of Pareto distribution under random censorship. We prove the proposed MEB test has asymptotically optimal property. Moreover, we obtain the convergence rate of the MEB test. Given suitable conditions, we are able to show that the convergence rate can be approach to $O(n^{-½})$.

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