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Blockwise Empirical Likelihood Method for Spatial Dependent Data

TANG Jie(唐洁), ZOU Yunlong(邹云龙), QIN Yongsong(秦永松), LI Yufang(黎玉芳)*
(Department of Statistics, Guangxi Normal University, Guilin, Guangxi 541006, China)

Abstract: Existing blockwise empirical likelihood (BEL) method blocks the observations or their analogues, which is proven useful under some dependent data settings. In this paper, we introduce a new BEL (NBEL) method by blocking the scoring functions under high dimensional cases. We study the construction of confidence regions for the parameters in spatial autoregressive models with spatial autoregressive disturbances (SARAR models) with high dimension of parameters by using the NBEL method. It is shown that the NBEL ratio statistics are asymptotically χ^2-type distributed, which are used to obtain the NBEL based confidence regions for the parameters in SARAR models. A simulation study is conducted to compare the performances of the NBEL and the usual EL methods.

Key words: SARAR model; Empirical likelihood; Confidence region; High-dimensional statistical inference
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1. Introduction

We firstly outline the developments of the empirical likelihood (EL) method under **independent samples**. The likelihood method is undoubtedly one of the most popular methods in statistics. Owen[1, 2] introduced the EL as a general nonparametric methodology for creating likelihood-type inference. Many researchers have contributed to the EL method for various models under independent samples. For instance, Owen[3] used the EL method to construct confidence regions for a linear model with independence errors, Kolaczyk[4] popularized the EL method for generalized linear models, and Chen[5] obtained the EL confidence intervals for a probability density function, among others. A prerequisite for the existence of the EL ratio statistics is that 0 must be inside the convex hull of the scoring function set. To address this problem, by adding an additional pseudo-observation, Chen et al.[6] proposed an adjusted EL (AEL) method to guarantee that the convex hull of the scoring function set contains 0. A similar AEL method can also be found in [7]. Owen[8] provided a good review on the developments of the EL method under independent data as well as some developments for dependent data. Chen and Van Keilegom[9] reviewed EL methods for regression problems.

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Biography: Li Yufang, female, Han, Guangxi, associate professor, major in empirical likelihood.

Secondly, we briefly review the developments of the EL method for **dependent data** including spatial data. The EL methodology has also been studied under dependent data by several scholars. Kitamura[10] first proposed blockwise EL (BEL) method to construct confidence intervals for parameters with mixing samples. Zhang[11] further studied the BEL method under associated samples. Chen and Wong[12] investigated the BEL method for the quantiles of a population under mixing samples. Qin and Li[13] proposed a new BEL method to construct confidence intervals for regression vectors in linear models with associated model errors. Nordman and Lahiri[14] provided a comprehensive review on the EL method for dependent data. Spatial data are also dependent, which appear in many fields such as econometrics, epidemiology, environmental science, image analysis, oceanography and many others. The EL method applied to spatial data is developed by Jin and Lee[15] and Qin[16]. However, the usual EL method fails in the cases when dimensions is large in comparison with the sample size. The extension of EL to high dimensional problems, especially for dependent data, is itself a challenging task.

Our study is motivated by the interest in dimension reduction based on the EL method for spatial models since there is no EL method used for dimension reduction in spatial models. We propose to use a new blockwise EL (NBEL) method to treat this issue in this article. Instead of the penalized EL method in Tang and Leng[17] and Leng and Tang[18] based on sparse models to reduce the dimension of the parameter vector and unlike the usual BEL method in Chang et al.[19] that blocks the data to reduce the dependence of data, the NBEL method blocks the scoring functions used in the usual EL approach to produce lower dimensional scoring functions. Instead of existing EL methods applicable only to the case that the sample size is larger than the dimension of parameters, the EL method presented in this article also works when the sample size is lower than the dimension of parameters. This similar EL method has also been successfully applied to test whether the regression coefficients are equivalent to given values in high dimensional linear models in [20, 21]. For more relevant literatures, one can be referred to Peng et al[20, 22].

The research of the EL method based on dimension reduction for non-sparse models is still in its infancy, and there is no research work on the NBEL method for spatial autoregressive models with spatial autoregressive disturbances (SARAR models). The results in this article show that the NBEL ratio statistics are asymptotically χ^2-type distributed unrelated to the dimension of parameters, which are used to obtain a NBEL-based confidence region for the parameters in SARAR models. A simulation study is conducted to compare the performances of the NBEL and the usual EL method. Simulation results show that the NBEL confidence regions perform better than the usual EL method, especially when the data dimension is larger than the sample size. However, the power of the NBEL test performs not as good as the usual EL test when the distance between the parameters and the true parameters is small (See Remark 2.1 for explanation). Nevertheless, the performance of the NBEL test is comparable with the usual EL test when the distance between the parameters and the true parameters is large.

In Section 2, the NBEL method is presented. Section 3 uses the NBEL method to construct confidence regions for SARAR models. Simulation results are stated in Section 4. All technical proofs are in Section 5.

2. NBEL method and estimating equations

In this section, we present one basic concept and two conclusions on existing usual EL and the recommended NBEL method.

2.1 Estimation equations

Suppose that $X \in R^d$ is a population and $X_1, X_2, \cdots, X_n$ are the i.i.d. observations of X. We further assume that there are r known functions $g_j(x,\theta), 1 \le j \le r$, such that

$$Eg_j(X,\theta) = 0, 1 \le j \le r,$$

where $\theta \in \Theta \subseteq R^p$.

2.2 Usual EL for estimation equations

Suppose that 0 is inside the convex hull of the $g(X_i,\theta), 1 \le i \le n$. Define the scoring function

$$g(x,\theta) = (g_1(x,\theta), g_2(x,\theta), \cdots, g_r(x,\theta))^\tau, x \in R^d, \theta \in \Theta,$$

and the EL statistic (e.g., Qin and Lawless[23]):

$$\ell_E(\theta) = \sum_{i=1}^{n} \log\{1 + t^\tau(\theta) g(X_i,\theta)\},$$

where $t(\theta) \in R^r$ is the solution of the following equations:

$$\sum_{i=1}^{n} \frac{g(X_i,\theta)}{1 + t^\tau(\theta) g(X_i,\theta)} = 0.$$

Following Qin and Lawless[23], one can obtain the following result which states the limiting distribution of $\ell_E(\theta)$.

Theorem 2.1 Suppose that $E\|g(X,\theta_0)\|^3 < \infty$ and $Cov(g(X,\theta_0))$ is positive definite, where $\|a\|$ is the L_2 norm of the vector $a \in R^r$ and θ_0 is the true value of θ. Then for fixed p and r, as $n \to \infty$,

$$2\ell_E(\theta_0) \xrightarrow{d} \chi_r^2,$$

where χ_r^2 is a chi-squared distributed random variable with r degrees of freedom.

2.3 NBEL method for estimation equations

Instead of the usual block method which blocks the data, we block the scoring function as follows. Let l is a positive number and $l \le r$ and let $s = [r/l]$, where $[a]$ stands for the integer part of a. Let

$$\tilde{g}_i(X,\theta) = \frac{1}{s} \sum_{j=(i-1)s+1}^{is} g_j(X,\theta), 1 \le i \le l,$$

and if $r > sl$

$$\tilde{g}_{l+1}(X,\theta) = \frac{1}{r-sl} \sum_{j=sl+1}^{r} g_j(X,\theta),$$

otherwise, $\tilde{g}_{l+1}(X,\theta)$ vanishes.

Let $l_0 = l + 1$ if $r > sl$, otherwise $l_0 = l$. Define the block scoring function

$$\tilde{g}(x,\theta) = (\tilde{g}_1(x,\theta), \tilde{g}_2(x,\theta), \cdots, \tilde{g}_{l_0}(x,\theta))^\tau, x \in R^d, \theta \in \Theta,$$

and the NBEL statistic:

$$\tilde{\ell}_E(\theta) = \sum_{i=1}^{n} \log\{1 + \tilde{t}^\tau(\theta)\tilde{g}(X_i,\theta)\},$$

where $\tilde{t}(\theta) \in R^{l_0}$ is the solution of the following equations:

$$\sum_{i=1}^{n} \frac{\tilde{g}(X_i,\theta)}{1 + \tilde{t}^\tau(\theta)\tilde{g}(X_i,\theta)} = 0.$$

In fact, let $\mathbf{1_s}$ present the s-dimensional vector with 1 as its components and if $r = sl$ define

$$H = \begin{pmatrix} s^{-1}\mathbf{1}_\mathbf{s}^\tau & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & s^{-1}\mathbf{1}_\mathbf{s}^\tau & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & s^{-1}\mathbf{1}_\mathbf{s}^\tau \end{pmatrix}_{l\times r},$$

and if $r > sl$ one can similarly define the matrix H as

$$H = \begin{pmatrix} s^{-1}\mathbf{1}_\mathbf{s}^\tau & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & s^{-1}\mathbf{1}_\mathbf{s}^\tau & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & s^{-1}\mathbf{1}_\mathbf{s}^\tau & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & (r-sl)^{-1}\mathbf{1}_\mathbf{r-sl}^\tau \end{pmatrix}_{(l+1)\times r}.$$

Then

$$\tilde{g}(x,\theta) = Hg(x,\theta).$$

Based on Theorem 2.1, it is ready to obtain the limiting distribution of $\tilde{\ell}_E(\theta)$ presented in Theorem 2.2.

Theorem 2.2 Suppose that the conditions of Theorem 2.1 hold true. Then for fixed p and r, as $n \to \infty$,

$$2\tilde{\ell}_E(\theta_0) \xrightarrow{d} \chi^2_{l_0},$$

where $\chi^2_{l_0}$ is a chi-squared distributed random variable with l_0 degrees of freedom.

Remark 2.1 It is clear by the results in [23], that the asymptotic efficiency of the NBEL estimator is less than or equal to that of the usual EL estimator, which provides an explanation that in the simulations the NBEL rejection rates under an alternative hypothesis perform not as good as the usual EL method when the distance between the parameters and the true parameters is relatively small.

3. NBEL method for SARAR models

The previous section introduced the NBEL method, and then the process that the NBEL method is applied to construct confidence regions for SARAR models are showed. Meanwhile, the estimation equations used in NBEL method rely on the estimation equations in EL framework, so we introduce the existing EL conclusion on spatial models before giving the asymptotic distribution based on NBEL method for SARAR models.

3.1 Usual EL for SARAR models

In this article, the following spatial autoregressive model with spatial autoregressive disturbances (SARAR model) is investigated:

$$Y_n = \rho_1 W_n Y_n + X_n\beta + u_{(n)}, u_{(n)} = \rho_2 M_n u_{(n)} + \epsilon_{(n)}, \tag{3.1}$$

where n is spatial sample size, $Y_n = (y_1, y_2, ..., y_n)^\tau$ is an $n \times 1$ vector of observations on the dependent variable, the matrix $X_n = (x_1, x_2, \cdots, x_n)^\tau$ with $x_i, i = 1, 2, \cdots, n$ is a $p \times 1$ exogenous vector of observations on the independent variable, β is the $p\times 1$ vector of regression parameters, the scalar parameters $\rho_j, j = 1, 2,$ are spatial autoregressive coefficients with $|\rho_j| < 1, j = 1, 2,$ W_n and M_n are known $n\times n$ spatial weight matrices whose diagonal elements are zero, the disturbance vector $u_{(n)} = (u_1, u_2, \cdots, u_n)^\tau$ contains a spatially autocorrelated setting, the vector $\epsilon_{(n)} = (\epsilon_1, \epsilon_2, \cdots, \epsilon_n)^\tau$ represents the random innovations which satisfies

$$E\epsilon_{(n)} = 0, Var(\epsilon_{(n)}) = \sigma^2 I_n.$$

The $\rho_1 W_n$ term is a spacial lag in the dependent variable and its coefficient represents the spatial influence due to neighbors realized dependent variable. The $\rho_2 M_n$ term is a spacial lag in the disturbances and its coefficient represents the spacial effect of unobservables on neighboring units. This model is introduced by Cliff and Ord[24] and Anselin[25]. The usual EL method to construct the confidence region for the parameters in this model is proposed in [16]. For the sake of conceptual integrity, we introduce the usual EL method in the following.

Let $A_n(\rho_1) = I_n - \rho_1 W_n$, $B_n(\rho_2) = I_n - \rho_2 M_n$ and suppose that $A_n(\rho_1)$ and $B_n(\rho_2)$ are nonsingular. Then (3.1) can be written as

$$Y_n = A_n^{-1}(\rho_1)X_n\beta + A_n^{-1}(\rho_1)B_n^{-1}(\rho_2)\epsilon_{(n)}.$$

At this moment, suppose that $\epsilon_{(n)}$ is normally distributed, which is firstly used to derive the EL statistic only and not employed in our main results. Then the log-likelihood function based on the response vector Y_n is

$$L = -\frac{n}{2}\log(2\pi) - \frac{n}{2}\log\sigma^2 + \log|A_n(\rho_1)| + \log|B_n(\rho_2)| - \frac{1}{2\sigma^2}\epsilon_{(n)}^\tau\epsilon_{(n)},$$

where $\epsilon_{(n)} = B_n(\rho_2)\{A_n(\rho_1)Y_n - X_n\beta\}$. Let $G_n = B_n(\rho_2)W_nA_n^{-1}(\rho_1)B_n^{-1}(\rho_2), H_n = M_nB_n^{-1}(\rho_2), \tilde{G}_n = \frac{1}{2}(G_n + G_n^\tau)$ and $\tilde{H}_n = \frac{1}{2}(H_n + H_n^\tau)$. It can be shown that (e.g. [25], p. 74-75)

$$\partial L/\partial\beta = \frac{1}{\sigma^2}X_n^\tau B_n^\tau(\rho_2)\epsilon_{(n)},$$

$$\partial L/\partial\rho_1 = \frac{1}{\sigma^2}\{B_n(\rho_2)W_nA_n^{-1}(\rho_1)X_n\beta\}^\tau\epsilon_{(n)} + \frac{1}{\sigma^2}\{\epsilon_{(n)}^\tau\tilde{G}_n\epsilon_{(n)} - \sigma^2 tr(\tilde{G}_n)\},$$

$$\partial L/\partial\rho_2 = \frac{1}{\sigma^2}\{\epsilon_{(n)}^\tau\tilde{H}_n\epsilon_{(n)} - \sigma^2 tr(\tilde{H}_n)\},$$

$$\partial L/\partial\sigma^2 = \frac{1}{2\sigma^4}\{\epsilon_{(n)}^\tau\epsilon_{(n)} - n\sigma^2\}.$$

Letting above derivatives be 0, we obtain the following estimating equations:

$$X_n^\tau B_n^\tau(\rho_2)\epsilon_{(n)} = 0, \tag{3.2}$$

$$\{B_n(\rho_2)W_nA_n^{-1}(\rho_1)X_n\beta\}^\tau\epsilon_{(n)} + \{\epsilon_{(n)}^\tau\tilde{G}_n\epsilon_{(n)} - \sigma^2 tr(\tilde{G}_n)\} = 0, \tag{3.3}$$

$$\epsilon_{(n)}^\tau\tilde{H}_n\epsilon_{(n)} - \sigma^2 tr(\tilde{H}_n) = 0, \tag{3.4}$$

$$\epsilon_{(n)}^\tau\epsilon_{(n)} - n\sigma^2 = 0. \tag{3.5}$$

Let $\tilde{g}_{ij}$, $\tilde{h}_{ij}$, b_i and s_i denote the (i, j) element of the matrix $\tilde{G}_n$, the (i, j) element of the matrix $\tilde{H}_n$, the i-th column of the matrix $X_n^\tau B_n^\tau(\rho_2)$ and i-th component of the vector $B_n(\rho_2)W_nA_n^{-1}(\rho_1)X_n\beta$, respectively, and adapt the convention that any sum with an upper index of less than one is zero. To deal with the quadratic forms in (3.2) and (3.3), define the σ-fields: $\mathcal{F}_0 = \{\emptyset, \Omega\}, \mathcal{F}_i = \sigma(\epsilon_1, \epsilon_2, \cdots, \epsilon_i), 1 \le i \le n$. Let

$$\tilde{Y}_{in} = \tilde{g}_{ii}(\epsilon_i^2 - \sigma^2) + 2\epsilon_i \sum_{j=1}^{i-1} \tilde{g}_{ij}\epsilon_j, \quad \tilde{Z}_{in} = \tilde{h}_{ii}(\epsilon_i^2 - \sigma^2) + 2\epsilon_i \sum_{j=1}^{i-1} \tilde{h}_{ij}\epsilon_j. \tag{3.6}$$

Then $\mathcal{F}_{i-1} \subseteq \mathcal{F}_i$, $\tilde{Y}_{in}$ is $\mathcal{F}_i$-measurable and $E(\tilde{Y}_{in}|\mathcal{F}_{i-1}) = 0$. Thus $\{\tilde{Y}_{in}, \mathcal{F}_i, 1 \le i \le n\}$ and $\{\tilde{Z}_{in}, \mathcal{F}_i, 1 \le i \le n\}$ form two martingale difference arrays and

$$\epsilon_{(n)}^{\tau}\tilde{G}_n\epsilon_{(n)} - \sigma^2 tr(\tilde{G}_n) = \sum_{i=1}^{n} \tilde{Y}_{in}, \quad \epsilon_{(n)}^{\tau}\tilde{H}_n\epsilon_{(n)} - \sigma^2 tr(\tilde{H}_n) = \sum_{i=1}^{n} \tilde{Z}_{in}. \tag{3.7}$$

Based on (3.2) to (3.7), we can get the scoring function:

$$\omega_i(\theta) = \begin{pmatrix} b_i\epsilon_i \\ \tilde{g}_{ii}(\epsilon_i^2 - \sigma^2) + 2\epsilon_i \sum_{j=1}^{i-1} \tilde{g}_{ij}\epsilon_j + s_i\epsilon_i \\ \tilde{h}_{ii}(\epsilon_i^2 - \sigma^2) + 2\epsilon_i \sum_{j=1}^{i-1} \tilde{h}_{ij}\epsilon_j \\ \epsilon_i^2 - \sigma^2 \end{pmatrix}_{(p+3)\times 1},$$

where ϵ_i is the i-th component of $\epsilon_{(n)} = B_n(\rho_2)\{A_n(\rho_1)Y_n - X_n\beta\}$.

Qin[16] defined the following EL ration statistic for $\theta = (\beta^{\tau}, \rho_1, \rho_2, \sigma^2)^{\tau} \in R^{p+3}$:

$$L_n(\theta) = \sup\left\{\prod_{i=1}^{n}(n\hat{p}_i) : \hat{p}_i \ge 0, \sum_{i=1}^{n}\hat{p}_i = 1, \sum_{i=1}^{n}\hat{p}_i\omega_i(\theta) = 0\right\}.$$

Following Owen[2], we have

$$\ell_n(\theta) \hat{=} -2\log L_n(\theta) = 2\sum_{i=1}^{n}\log\{1 + \hat{\lambda}^{\tau}(\theta)\omega_i(\theta)\},$$

where $\hat{\lambda}(\theta) \in R^{p+3}$ is the solution of the following equation:

$$\frac{1}{n}\sum_{i=1}^{n}\frac{\omega_i(\theta)}{1 + \hat{\lambda}^{\tau}(\theta)\omega_i(\theta)} = 0.$$

Let $\mu_j = E(\epsilon_1^j), j = 3, 4$, Use $Vec(diagA)$ to denote the vector formed by the diagonal elements of a matrix A and use $\|a\|$ to denote the L_2-norm of a vector a. To obtain the asymptotical distribution of $\tilde{\ell}_n(\theta)$, we need following assumptions.

A1. $\{\epsilon_i, 1 \le i \le n\}$ are independent and identically distributed random variables with mean 0, variance σ^2 and $E|\epsilon_1|^{4+\eta_1} < \infty$ for some $\eta_1 > 0$.

A2. $W_n, M_n, A_n^{-1}(\rho_1), B_n^{-1}(\rho_2)$ and $\{x_i\}$ be as described above. They satisfy the following conditions:

(i) The row and column sums of $W_n, M_n, A_n^{-1}(\rho_1)$ and $B_n^{-1}(\rho_2)$ are uniformly bounded in absolute value,

(ii) $\{x_i\}, i = 1, 2, ..., n$ are uniformly bounded.

A3. There is a constants $c_j > 0, j = 1, 2$, such that $0 < c_1 \le \lambda_{min}(n^{-1}\Sigma_{p+3}) \le \lambda_{max}(n^{-1}\Sigma_{p+3}) \le c_2 < \infty$, where $\lambda_{min}(A)$ and $\lambda_{max}(A)$ denote the minimum and maximum eigenvalues of a matrix A, respectively,

$$\Sigma_{p+3} = \Sigma_{p+3}^{\tau} = Cov\left\{\sum_{i=1}^{n}\omega_i(\theta)\right\} = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} & \Sigma_{14} \\ \Sigma_{21} & \Sigma_{22} & \Sigma_{23} & \Sigma_{24} \\ \Sigma_{31} & \Sigma_{32} & \Sigma_{33} & \Sigma_{34} \\ \Sigma_{41} & \Sigma_{42} & \Sigma_{43} & \Sigma_{44} \end{pmatrix}, \tag{3.8}$$

where

$$\begin{aligned}
\Sigma_{11} &= \sigma^2\{B_n(\rho_2)X_n\}'B_n(\rho_2)X_n,\\
\Sigma_{12} &= \sigma^2\{B_n(\rho_2)X_n\}'B_n(\rho_2)W_nA_n^{-1}(\rho_1)X_n\beta+\mu_3\{B_n(\rho_2)X_n\}'Vec(diag\tilde{G}_n),\\
\Sigma_{13} &= \mu_3\{B_n(\rho_2)X_n\}'Vec(diag\tilde{H}_n),\quad \Sigma_{14}=\mu_3\{B_n(\rho_2)X_n\}'\mathbf{1}_n,\\
\Sigma_{22} &= 2\sigma^4tr(\tilde{G}_n{}^2)+\sigma^2\{B_n(\rho_2)W_nA_n^{-1}(\rho_1)X_n\beta\}'B_n(\rho_2)W_nA_n^{-1}(\rho_1)X_n\beta\\
&\quad+(\mu_4-3\sigma^4)\|Vec(diag\tilde{G}_n)\|^2+2\mu_3\{B_n(\rho_2)W_nA_n^{-1}(\rho_1)X_n\beta\ \}'Vec(diag\tilde{G}_n),\\
\Sigma_{23} &= 2\sigma^4tr(\tilde{G}_n\tilde{H}_n)+(\mu_4-3\sigma^4)Vec'(diag\tilde{G}_n)Vec(diag\tilde{H}_n)\\
&\quad+\mu_3\{B_n(\rho_2)W_nA_n^{-1}(\rho_1)X_n\beta\ \}'Vec(diag\tilde{H}_n),\\
\Sigma_{24} &= (\mu_4-\sigma^4)tr(\tilde{G}_n)+\mu_3\{B_n(\rho_2)W_nA_n^{-1}(\rho_1)X_n\beta\ \}'\mathbf{1}_n,\\
\Sigma_{33} &= 2\sigma^4tr(\tilde{H}_n{}^2)(\mu_4-3\sigma^4)\|Vec(diag\tilde{H}_n)\|^2,\\
\Sigma_{34} &= (\mu_4-\sigma^4)tr(\tilde{H}_n),\quad \Sigma_{44}=n(\mu_4-\sigma^4).
\end{aligned}$$

We now state the main results in [16].

Theorem 3.1 Suppose that conditions A1-A3 hold. Let θ_0 be the true value of θ, as $n\to\infty$,

$$\ell_n(\theta_0)\xrightarrow{d}\chi^2_{p+3},$$

where χ^2_{p+3} is a chi-squared distributed random variable with $p+3$ degrees of freedom.

3.2 NBEL method for SARAR models

Following the NBEL method proposed in Section 2.3, we define the NBEL scoring functions for the SARSAR model as follows. Let l is a positive number and $l\le p+3$ and let $s=[(p+3)/l]$, where $[a]$ stands for the integer part of a. If $p+3=sl$, define

$$H=\begin{pmatrix} s^{-1}\mathbf{1}_{\mathrm{s}}^\tau & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0}\\ \mathbf{0} & s^{-1}\mathbf{1}_{\mathrm{s}}^\tau & \mathbf{0} & \cdots & \mathbf{0}\\ \vdots & \vdots & \vdots & \cdots & \vdots\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & s^{-1}\mathbf{1}_{\mathrm{s}}^\tau \end{pmatrix}_{l\times(p+3)},$$

and if $p+3>sl$, define

$$H=\begin{pmatrix} s^{-1}\mathbf{1}_{\mathrm{s}}^\tau & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & s^{-1}\mathbf{1}_{\mathrm{s}}^\tau & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0}\\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & s^{-1}\mathbf{1}_{\mathrm{s}}^\tau & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & (p+3-sl)^{-1}\mathbf{1}_{\mathbf{p+3-sl}}^\tau \end{pmatrix}_{(l+1)\times(p+3)}.$$

Let

$$\widetilde{\omega}_i(\theta)=H\omega_i(\theta),1\le i\le n.$$

Then define the NBEL statistic for the SARSAR model:

$$\tilde{\ell}_n(\theta)=\sum_{i=1}^n\log\{1+\tilde{t}^\tau(\theta)\widetilde{\omega}_i(\theta)\},$$

where $\tilde{t}(\theta)$ is the solution of the following equations:

$$\sum_{i=1}^n\frac{\widetilde{\omega}_i(\theta)}{1+\tilde{t}^\tau(\theta)\widetilde{\omega}_i(\theta)}=0.$$

Let $l_0 = l + 1$ if $p + 3 > sl$, otherwise $l_0 = l$. We now state the main results.

Theorem 3.2 Suppose that conditions A1-A3 hold. Let θ_0 be the true value of θ, as $n \to \infty$,

$$\tilde{\ell}_n(\theta_0) \xrightarrow{d} \chi^2_{l_0},$$

where $\chi^2_{l_0}$ is a chi-squared distributed random variable with l_0 degrees of freedom.

Let $z_\alpha(l_0)$ satisfy $P(\chi^2_{l_0} \geq z_\alpha(l_0)) = \alpha$ for $0 < \alpha < 1$. It follows from Theorem 3.2 that an NBEL-based confidence intervals (CIs) for θ with asymptotically correct coverage probability (CP) $1 - \alpha$ can be constructed as

$$\{\theta : \tilde{\ell}_n(\theta) \leq z_\alpha(l_0)\}. \tag{3.9}$$

4. Simulations

According to Qin[16], let $z_\alpha(p+3)$ satisfy $P(\chi^2_{p+3} \geq z_\alpha(p+3)) = \alpha$ for $0 < \alpha < 1$, and the usual EL-based CI for θ is given by equation (4.1)

$$\{\theta : \ell_n(\theta) \leq z_\alpha(p+3)\}. \tag{4.1}$$

We conducted a small simulation study to compare the finite sample performances of the confidence regions based on the usual EL and the NBEL methods with confidence level $1-\alpha = 0.95$. In the simulations, we take $l = 1$, i.e. there is only one block for the scoring functions. One can download R codes related to this article at `https://github.com/Tang-Jay/NBEL`.

In the simulations, we used the model: $Y_n = \rho_1 W_n Y_n + X_n \beta_0 + u_{(n)}, u_{(n)} = \rho_2 M_n u_{(n)} + \epsilon_{(n)}$, with $(\rho_1, \rho_2) = (0.85, 0.15)$, $\beta_0 = \mathbf{1}_{p+3}$, and $\{X_i\} \sim N(\mathbf{0}, I_p)$, $i = 1, 2, \cdots, n$. To make p and n increase simultaneously, we took $p = [3n^{index}]$, where $[x]$ is the integer part of x.

For the contiguity weight matrix $W_n = (w_{ij})$, we took $w_{ij} = 1$ if spatial units i and j are neighbours by queen contiguity rule (namely, they share common border or vertex), $w_{ij} = 0$ otherwise ([25], p. 18). We considered some ideal cases of spatial units: $n = m \times m$ and $M_n = W_n$.

Experiment 1

Denote the true value of θ as $\theta_0' = (\beta_0', \rho_1, \rho_2, \sigma_0^2)$, and then let $m = 10, 15, 20, 30$ and $index = 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9$. In addition, $\epsilon_i's$ are taken from $N(0,1)$, $N(0, 0.75)$, $t(5)$ and $\chi^2_4 - 4$, respectively, and σ_0^2 are chosen as 1, 0.75, 2.5, 8, respectively. We report the proportion of $\tilde{\ell}_n(\theta_0) \leq z_{0.05}(1)$ and $\ell_n(\theta_0) \leq z_{0.05}(p+3)$ respectively in our $2,000$ simulations. We use the mark '-' to denote the invalid result. The results of simulations are reported in Tab. 4.1.

Experiment 2

Denote $\theta' = (\beta', \rho_1, \rho_2, \sigma^2)$, and let $m = 10, 15, 17, 20, 25, 30$, $index = 0, 0.1, 0.2$,$0.3, 0.4, 0.5$, $(\beta, \sigma^2)' = (\beta_0, \sigma_0^2)' + \Delta \mathbf{1}_{p+1}'$, $\Delta = 0, 0.1, 1, 2$, $\epsilon_i \sim N(0,1)$. We report the proportion of $\tilde{\ell}_n(\theta) \geq z_{0.05}(1)$ and $\ell_n(\theta) \geq z_{0.05}(p+3)$, respectively in our 2,000 simulations. The results of simulations are reported in Tab. 4.2.

Experimental Analysis

In terms of the type one error, we designed Experiment 1 whose results are shown in Tab. 4.1 to compare the coverage probabilities of the EL method and the NBEL method in the confidence regions under the true value parameters. If the coverage probabilities of one method in the confidence regions under the Null Hypothesis are closer to 0.95, it indicates that the statistical inference is more better, because the probability of the method making the type one error is closer to the given nominal level of 0.05.

Tab. 4.1 shows that the confidence regions based on EL behave well with coverage probabilities very close to the nominal level 0.95, as n increases, only when $index \leq 0.2$. When $0.2 < index \leq 0.5$, the coverage probabilities of the confidence regions based on EL fall to the range [0.022, 0.905] for $N(0,1)$ distribution , [0.020, 0.909] for $N(0,0.75)$, [0.011, 0.840] for $t(5)$ and [0.010, 0.849] for χ^2 distribution, which are far from the nominal level 0.95, as p increases, and when $index > 0.5$, the EL method is even invalid. Tab. 4.1 also shows that the confidence regions based on NBEL behave well with coverage probabilities very close to the nominal level 0.95, as n increases, whether $index \leq 0.2$ or not. When $0.2 < index \leq 0.5$, the coverage probabilities of the confidence regions based on NBEL fall to the range [0.938, 0.952] for $N(0,1)$ distribution, [0.937, 0.956] for $N(0,0.75)$, [0.928, 0.948] for $t(5)$ and [0.919, 0.952] for χ^2 distribution, which are close to the nominal level 0.95, as p increases, and even if $index > 0.5$, the NBEL method is still valid. In a word, the probability of EL making the type one error is very high or the statistical inference of the EL method become ineffective in high-dimensional situations, and under the same conditions the probability of NBEL making the type one error remains stable near the nominal level, and NBEL is even suitable for high-dimensional situations.

In terms of the type two error, we designed Experiment 2 whose results are shown in Tab. 4.2 to compare the frequencies of the EL method and the NBEL method in the rejection regions under the false value parameters. If the frequencies of one method in the rejection regions under the Alternative Hypothesis are closer to 1, it indicates that the statistical inference is more better, because the probability of the method making the type two error is closer to zero.

For the Hypothesis testing problem $H_0 : \theta = \theta_0$ and $H_1 : \theta \neq \theta_0$, the model parameters satisfy $\|\theta - \theta_0\| = \sqrt{p+1}\Delta$, $\Delta = 0, 0.1, 1, 2$, which represents the different distances between the experimental parameters and the true value parameters from near to far, where it is best to get closer to 0.05 for the frequencies of the rejection regions only when $\Delta = 0$. Tab. 4.2 shows that the rejection rates based on usual EL and NBEL tests both behave well with coverage probabilities very close to 1, as $\|\theta_0 - \theta\|$ increases. Although the NBEL rejection regions perform not as good as EL method when the distance between the θ and θ_0 is small, the probability of the NBEL method making the type two error gradually decreases to zero with the improvement of dimension or distance from true value. In other words, the frequencies of the rejection regions based on the EL method is superior to the NBEL methods.

In order to visually compare sample quantiles and theoretic quantiles, we drew some Q-Q plots. When $\epsilon_i \sim N(0,1)$, we pictured the sample quantiles of $\ell_n(\theta_0)$ with the quantiles of $\chi^2(p+3)$ shown in Fig. 4.1 and the sample quantiles of $\tilde{\ell}_n(\theta_0)$ with the quantiles of $\chi^2(1)$ in Fig. 4.2 - Fig. 4.3 . Based on the simulation results, in low dimension case as n increases, all EL sample distributions and the theoretic distributions agree well. When the dimension is high, all the EL sample distributions fit the theoretic distributions poor, but all the NBEL sample distributions agree the theoretic distributions well even when the dimension is far greater than the number of samples. Therefore, the simulation experiment verified the theoretical results.

Tab. 4.1 Coverage probabilities of the EL and NBEL confidence regions

n	p	$\epsilon_i \sim N(0,1)$		$\epsilon_i \sim N(0,0.75)$		$\epsilon_i \sim t(5)$		$\epsilon_i \sim \chi^2(4)-4$	
		EL	NBEL	EL	NBEL	EL	NBEL	EL	NBEL
100	3	0.861	0.948	0.872	0.940	0.778	0.904	0.775	0.910
	5	0.824	0.944	0.824	0.941	0.720	0.931	0.730	0.920
	8	0.740	0.950	0.718	0.957	0.591	0.918	0.624	0.919
	12	0.548	0.943	0.545	0.937	0.392	0.928	0.452	0.919
	19	0.250	0.940	0.266	0.938	0.132	0.930	0.182	0.920
	30	0.022	0.941	0.020	0.940	0.011	0.928	0.010	0.929
	48	-	0.946	-	0.943	-	0.931	-	0.928
	75	-	0.949	-	0.945	-	0.928	-	0.944
	119	-	0.948	-	0.945	-	0.933	-	0.933
	189	-	0.945	-	0.940	-	0.936	-	0.943
225	3	0.920	0.946	0.917	0.948	0.866	0.919	0.877	0.921
	5	0.910	0.945	0.907	0.952	0.857	0.928	0.861	0.927
	9	0.863	0.941	0.878	0.949	0.786	0.926	0.788	0.921
	15	0.774	0.949	0.783	0.953	0.614	0.942	0.665	0.936
	26	0.508	0.944	0.508	0.944	0.317	0.938	0.384	0.932
	45	0.096	0.950	0.084	0.954	0.025	0.941	0.047	0.943
	77	-	0.948	-	0.942	-	0.943	-	0.938
	133	-	0.946	-	0.947	-	0.945	-	0.931
	228	-	0.958	-	0.937	-	0.942	-	0.933
	393	-	0.948	-	0.952	-	0.947	-	0.946
400	3	0.931	0.950	0.929	0.950	0.895	0.929	0.907	0.941
	5	0.929	0.949	0.925	0.936	0.891	0.937	0.897	0.940
	10	0.905	0.952	0.911	0.956	0.856	0.936	0.850	0.945
	18	0.856	0.942	0.850	0.949	0.723	0.945	0.740	0.941
	33	0.655	0.938	0.655	0.952	0.459	0.948	0.476	0.950
	60	0.163	0.944	0.186	0.946	0.061	0.943	0.088	0.939
	109	-	0.951	-	0.950	-	0.948	-	0.959
	199	-	0.951	-	0.947	-	0.941	-	0.940
	362	-	0.957	-	0.947	-	0.946	-	0.938
	659	-	0.941	-	0.950	-	0.946	-	0.948
900	3	0.951	0.948	0.935	0.948	0.921	0.936	0.931	0.945
	6	0.942	0.946	0.941	0.955	0.915	0.946	0.922	0.949
	12	0.928	0.945	0.936	0.940	0.906	0.945	0.902	0.946
	23	0.905	0.948	0.909	0.948	0.840	0.937	0.849	0.942
	46	0.792	0.952	0.798	0.956	0.624	0.945	0.625	0.952
	90	0.353	0.946	0.337	0.951	0.118	0.942	0.172	0.949
	178	-	0.938	-	0.951	-	0.948	-	0.948
	351	-	0.950	-	0.948	-	0.954	-	0.955
	693	-	0.955	-	0.946	-	0.942	-	0.957
	1368	-	0.945	-	0.947	-	0.953	-	0.953

In sum, our simulation results recommend NBEL method when the dimension is high given the following reasons that the probability of NBEL making the type one error is close to the nominal level, the frequencies of the rejection regions based on the NBEL method are slightly inferior to the EL methods, and all the NBEL sample distributions agree the theoretic distributions well.

Tab. 4.2 Frequencies of rejection of the EL and NBEL tests

n	p	$\Delta = 0$		$\Delta = 0.1$		$\Delta = 1$		$\Delta = 2$	
		EL	NBEL	EL	NBEL	EL	NBEL	EL	NBEL
100	3	0.146	0.058	0.205	0.091	0.994	0.083	1.000	0.976
	5	0.180	0.062	0.256	0.091	1.000	0.427	1.000	0.999
	8	0.282	0.066	0.325	0.082	1.000	0.732	1.000	1.000
	12	0.453	0.053	0.475	0.082	1.000	0.851	1.000	1.000
	19	0.755	0.058	0.726	0.065	1.000	0.891	1.000	1.000
	30	0.979	0.046	0.968	0.091	1.000	0.945	1.000	1.000
225	3	0.079	0.050	0.140	0.094	1.000	0.358	1.000	1.000
	5	0.097	0.044	0.129	0.089	1.000	0.907	1.000	1.000
	9	0.114	0.063	0.181	0.099	1.000	0.994	1.000	1.000
	15	0.220	0.060	0.263	0.100	1.000	1.000	1.000	1.000
	26	0.513	0.057	0.525	0.100	1.000	1.000	1.000	1.000
	45	0.925	0.045	0.911	0.108	1.000	1.000	1.000	1.000
324	3	0.066	0.041	0.132	0.128	1.000	0.577	1.000	1.000
	5	0.078	0.058	0.129	0.113	1.000	0.989	1.000	1.000
	10	0.114	0.048	0.168	0.116	1.000	1.000	1.000	1.000
	17	0.183	0.052	0.240	0.132	1.000	1.000	1.000	1.000
	30	0.392	0.052	0.526	0.136	1.000	1.000	1.000	1.000
	54	0.857	0.052	0.953	0.136	1.000	1.000	1.000	1.000
400	3	0.072	0.061	0.146	0.141	1.000	0.735	1.000	1.000
	5	0.062	0.061	0.160	0.166	1.000	0.999	1.000	1.000
	10	0.092	0.047	0.143	0.117	1.000	1.000	1.000	1.000
	18	0.137	0.046	0.271	0.141	1.000	1.000	1.000	1.000
	33	0.342	0.048	0.586	0.142	1.000	1.000	1.000	1.000
	60	0.807	0.050	0.984	0.177	1.000	1.000	1.000	1.000
625	3	0.059	0.056	0.152	0.191	1.000	0.926	1.000	1.000
	6	0.060	0.058	0.134	0.184	1.000	1.000	1.000	1.000
	11	0.076	0.052	0.151	0.168	1.000	1.000	1.000	1.000
	21	0.120	0.054	0.351	0.170	1.000	1.000	1.000	1.000
	39	0.253	0.050	0.857	0.194	1.000	1.000	1.000	1.000
	75	0.740	0.056	1.000	0.234	1.000	1.000	1.000	1.000
900	3	0.049	0.058	0.212	0.226	1.000	0.989	1.000	1.000
	6	0.050	0.055	0.168	0.221	1.000	1.000	1.000	1.000
	12	0.061	0.055	0.216	0.224	1.000	1.000	1.000	1.000
	23	0.101	0.046	0.552	0.235	1.000	1.000	1.000	1.000
	46	0.221	0.059	0.994	0.272	1.000	1.000	1.000	1.000
	90	0.654	0.049	1.000	0.334	1.000	1.000	1.000	1.000

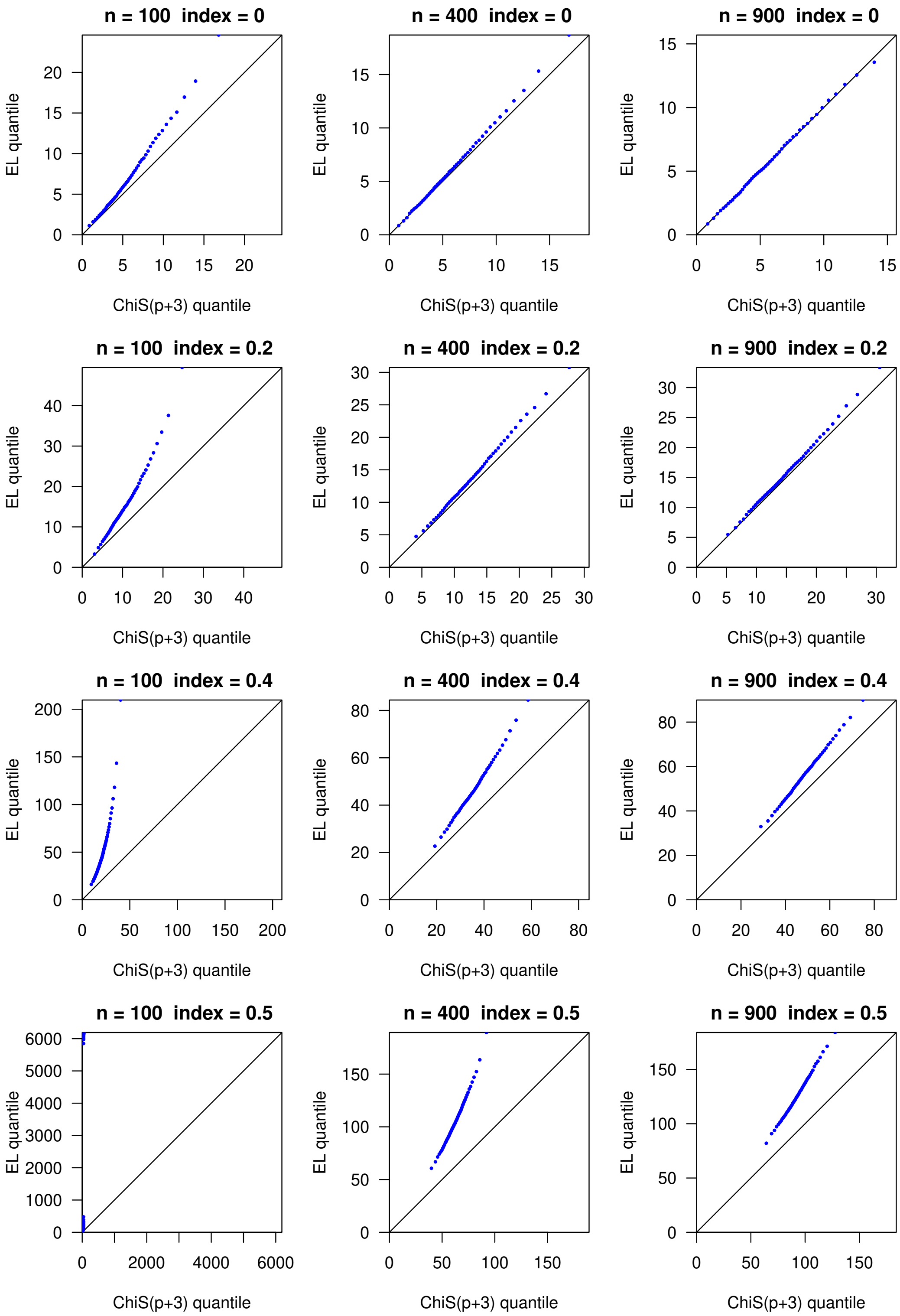

Fig. 4.1 Q-Q plots of $\ell_n(\theta_0)$ and χ^2_{p+3} with $\epsilon_i \sim N(0,1)$

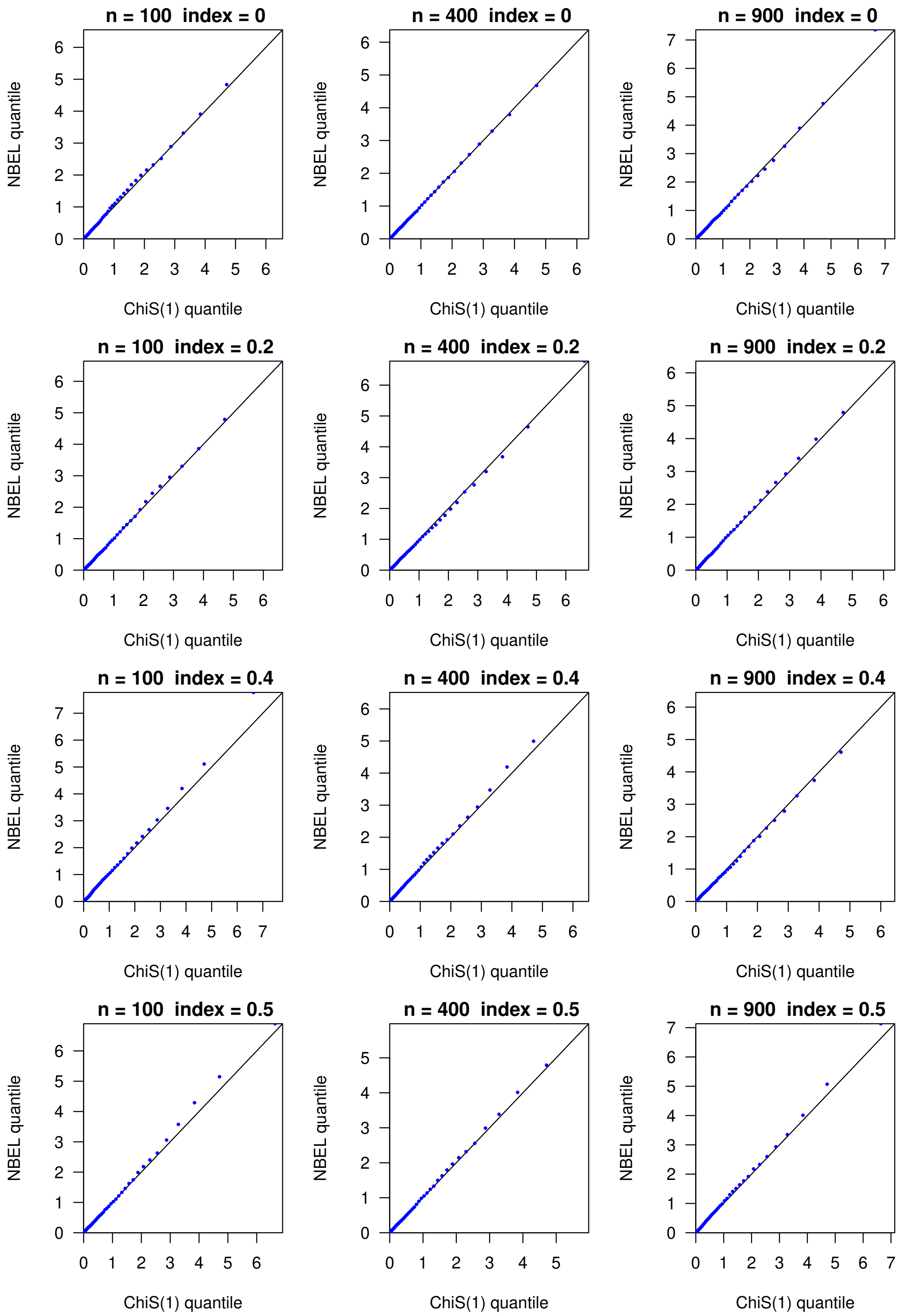

Fig. 4.2 Q-Q plots of $\tilde{\ell}_n(\theta_0)$ and χ_1^2 with $\epsilon_i \sim N(0,1)$

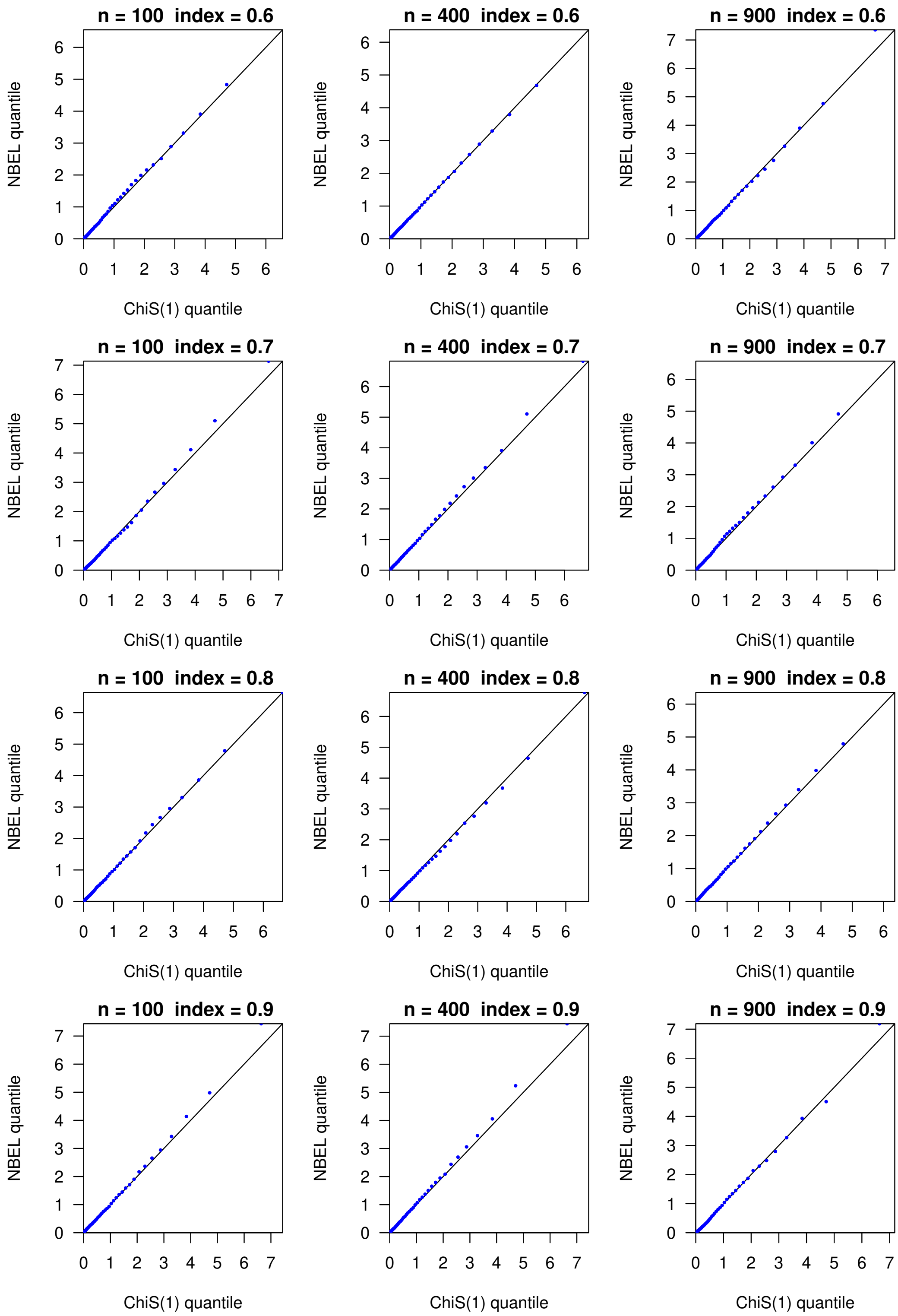

Fig. 4.3 Q-Q plots of $\tilde{\ell}_n(\theta_0)$ and χ_1^2 with $\epsilon_i \sim N(0,1)$

5. Proofs

Lemma 5.1 Suppose that Assumptions A1-A3 are satisfied, then as $n \to \infty$,

$$\max_{1\le i\le n} \|\omega_i(\theta)\| = o_p(n^{1/2}), \tag{5.1}$$

$$\Sigma_{p+3}^{-1/2} \sum_{i=1}^{n} \omega_i(\theta) \xrightarrow{d} N(0,1), \tag{5.2}$$

$$n^{-1} \sum_{i=1}^{n} \omega_i(\theta)\omega_i^{\tau}(\theta) = n^{-1}\Sigma_{p+3} + o_p(1), \tag{5.3}$$

$$\sum_{i=1}^{n} \|\omega_i(\theta)\|^3 = O_p(n), \tag{5.4}$$

where Σ_{p+3} is given in (3.8).

Proof See Lemma 3 in [16].

Lemma 5.2 Suppose that Assumptions A1-A3 are satisfied, then as $n \to \infty$,

$$Z_n = \max_{1\le i\le n} \|\tilde{\omega}_i(\theta)\| = o_p(n^{1/2}), \tag{5.5}$$

$$\sum_{i=1}^{n} \tilde{\omega}_i(\theta) \xrightarrow{d} N(\mathbf{0}_{l_0}, H\Sigma_{p+3}H^{\tau}), \tag{5.6}$$

$$n^{-1} \sum_{i=1}^{n} \tilde{\omega}_i(\theta)\tilde{\omega}_i^{\tau}(\theta) = n^{-1}H\Sigma_{p+3}H^{\tau} + o_p(1), \tag{5.7}$$

$$\sum_{i=1}^{n} \|\tilde{\omega}_i(\theta)\|^3 = O_p(n), \tag{5.8}$$

where Σ_{p+3} is given in (3.8) and $\mathbf{0}_{l_0}$ presents the l_0-dimensional vector with 0 as its components.

Proof Use ω_{ij} to denote the j-th element of the vector $\omega_i(\theta)$. If $p+3 = sl$, by $\tilde{\omega}_i(\theta) = H\omega_i(\theta)$, we have

$$\|\tilde{\omega}_i(\theta)\| = \|H\omega_i(\theta)\| = (\omega_i^{\tau}(\theta)H^{\tau}H\omega_i(\theta))^{1/2} \le \left(\lambda_{max}(H^{\tau}H)\|\omega_i(\theta)\|^2\right)^{1/2} \le C\|\omega_i(\theta)\|,$$

where $\lambda_{max}(H^{\tau}H)$ denotes the maximum eigenvalues of a matrix $H^{\tau}H$. Further,

$$\sum_{i=1}^{n} \|\tilde{\omega}_i(\theta)\|^3 = \sum_{i=1}^{n} \|H\omega_i(\theta)\|^3 \le C\sum_{i=1}^{n} \|\omega_i(\theta)\|^3.$$

If $p+3 > sl$, the above equation still holds, and the proof is analogous. Combining with (5.1) and (5.4), we have $\max_{1\le i\le n} \|\tilde{\omega}_i(\theta)\| = o_p(n^{1/2})$ and $\sum_{i=1}^{n} \|\tilde{\omega}_i(\theta)\|^3 = O_p(n)$. (5.5) and (5.8) is proved.

Obviously,

$$\sum_{i=1}^{n} \tilde{\omega}_i(\theta) = H\sum_{i=1}^{n} \omega_i(\theta),$$

$$n^{-1}\sum_{i=1}^{n} \tilde{\omega}_i(\theta)\tilde{\omega}_i^{\tau}(\theta) = Hn^{-1}\sum_{i=1}^{n} \omega_i(\theta)\omega_i^{\tau}(\theta)H^{\tau}.$$

Using (5.2) and (5.3), we obtain (5.6) and (5.7).

Proof of Theorem 3.2 The proof is analogous to the proof of Theorem 1 in [16].

6. Conclusions

In this article we consider a specification for spatial autoregressive models with spatial autoregressive disturbances with high dimension of parameters, which is studied in detail under the situation that the parameters dimension p is diverging, and even the parameters dimension is lager than the sample size n.

Our simulation shows that the NBEL method is recommended. Firstly, The confidence regions based on NBEL method closer to the nominal level 0.95 than those based on EL method as the number of spatial units n is large enough, whether the error term ϵ_i is normally distributed or not. Secondly, the calculation based on NBEL method simpler and faster than EL method. In other words, the NBEL method performs much better than the EL method when the parameters dimension is high, especially in high-dimensional situations where usual empirical likelihood methods may become invalid, overcoming the drawbacks of the traditional methods.

More research on the performs of the NBEL methods in different and high-dimensional models with a spatial lag or a space-specific effect is left for future discussion.

References:

[1] OWEN A B. Empirical likelihood ratio confidence intervals for a single functional. Biometrika, 1988, 75, 237-249.

[2] OWEN A B. Empirical likelihood ratio confidence regions. Annals of Statistics, 1990, 18, 90-120.

[3] OWEN A B. Empirical likelihood for linear models. Annals of Statistics, 1991, 19, 1725-1747.

[4] KOLACZYK E D. Emprical likelihood for generalized linear models. Statistica Sinica, 1994, 4, 199-218.

[5] CHEN S X. Empirical likelihood confidence intervals for nonparametric density estimation[J]. Biometrika, 1996, 83, 329-341.

[6] CHEN J, VARIYATH A M, ABRAHAM B. Adjusted empirical likelihood and its properties[J]. J. Comput. Graph. Statist., 2008, 17, 426-443.

[7] EMERSON S C, OWEN A B. Calibration of the empirical likelihood method for a vector mean. Electronic Journal of Statistics, 2009, 3, 1161-1192.

[8] OWEN A B. Empirical Likelihood. New York: Chapman & Hall, 2001.

[9] CHEN S X, VAN KEILEGO I. A review on empirical likelihood methods for regression. Test, 2009, 18, 415-447.

[10] KITAMURA Y. Empirical likelihood methods with weakly dependent processes. Ann. Statist., 1997, 25, 2084-2102.

[11] ZHANG J. Empirical likelihood for NA series. Statist. Probab. Lett., 2006, 76, 153-160.

[12] CHEN S X, WONG C M. Smoothed block empirical likelihood for quantiles of weakly dependent processes. Statist. Sinica, 2009, 19, 71-81.

[13] QIN Y S, LI Y H. Empirical likelihood for linear models under negatively associated errors. J. Multivariate Anal., 2011, 102, 153-163.

[14] NORDMAN D J, LAHIRI S N. A review of empirical likelihood methods for time series. J. Statist. Plann. Inference, 2014, 155, 1-18.

[15] JIN F, LEE L F. GEL estimation and tests of spatial autoregressive models. Journal of Econometrics, 2019, 208, 585-612.

[16] QIN Y S. Empirical likelihood for spatial autoregressive models with spatial autoregressive disturbances. Sankhyā A: Indian J. Statist., 2021, 83, 1-25.

[17] TANG C Y, LENG C L. Penalized high dimensional empirical likelihood. Biometrika, 2010, 97, 905-920.

[18] LENG C L, TANG C Y. Penalized empirical likelihood and growing dimensional general estimating equations. Biometrika, 2012, 99, 703-716.

[19] CHANG J Y, CHEN S X, CHEN X H. High dimensional generalized empirical likelihood for moment restrictions with dependent data[J]. Journal of Econometrics, 2015, 185, 283-304.
[20] PENG L, QI Y C, WANG R D. Empirical likelihood test for high dimensional linear models. Statistics and Probability Letters, 2014, 86, 85-90.
[21] ZENG L L. Empirical likelihood methods for high dimensional linear regression models[D] (in Chinese). Nanjing Normal University, 2016.
[22] PENG L, QI Y C, WANG F. Test for a mean vector with fixed or divergent dimension. Statistical Science, 2014, 29, 113-127.
[23] QIN J, LAWLESS J. Empirical likelihood and general estimating equations. Ann. Statist., 1994, 22, 300-325.
[24] CLIFF A D, ORD J K. Spatial autocorrelation[M]. London: Pion Ltd, 1973.
[25] ANSELIN L. Spatial econometrics: methods and models[M]. Amsterdam: Kluwer Academic Publishers, 1988.

空间相依数据的分块经验似然推断

唐洁, 邹云龙, 秦永松, 黎玉芳 *
(广西师范大学数学与统计学院, 广西 桂林 541004)

摘要: 对数据进行分块处理的分块经验似然方法被证实是一种对相依数据行之有效的手段. 本文通过对得分函数进行分块得到新分块经验似然方法，并应用到高维相依数据情形. 我们研究高维下含空间自相关误差的空间自回归模型的新分块经验似然方法, 证明该经验似然统计量的极限分布为卡方分布, 并由此构造高维参数置信区间. 模拟对比了普通经验似然方法与新分块经验似然方法在各自置信区间的表现.

关键词: SARAR 模型; 经验似然; 置信区域; 高维统计推断