Analytical Modeling For Determination Of Pull-In Voltage For An Electrostatic Actuated MEMS Cantilever Beam

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***Abstract*—A new analytical model has been developed for determining the pull-in voltage of an electrostatic actuated microelectromechanical system (MEMS) cantilever beam. The model takes into account the beam curvature caused by residual stress or stress gradient in the beam material. It can be used to model straight, curled and beam with mixture of straight and curled sections. Modeling result has been compared with published work of other researchers as well as with experimental measurement of this work and is found to be accurate.**

***Index Terms*—Analytical model, MEMS, pull-in voltage, cantilever, stress-induced curling, stress gradient, residual stress.**

I. Introduction

CANTILEVER BEAM based electrostatic actuation is a very common design used in MEMS devices. Pull-in voltage is one of the most important criteria in an electrostatic actuated MEMS cantilever design. Accurate modeling of pull-in voltage is very challenging due to high nonlinearity and instability. Effects such as fringing field, residual stress and stress gradient induced curling have further complicated the modeling of such device. Numerical method based modeling, such as the finite element method is often used for the modeling of such phenomena, and has been implemented in various commercial MEMS simulation softwares such as Coventor, Intellisuite, Ansys and Femlab. However, the finite element method provides little insight into the relationship among the device design parameters. An analytical model can provide a better insight into the system behaviour, but derivation of a model is extremely complex. There has been numerous work on analytical modeling of the pull-in voltage for simple MEMS structure such as cantilever beam and fixed-fixed beam [1]-[4]. However, most of the models have been developed through data fit with experimentally measured data. Among those models, only that in [2] is added with a correction factor to cater for curled cantilever beam, while the rest are meant for straight cantilever beam. The new model presented in this paper has been derived fully theoretically to calculate the pull-in voltage for an electrostatic actuated MEMS cantilever beam, both straight and curled It provides relatively accurate result compared with work conducted elsewhere [2], as well as with measured result of this work. The model has been based on cantilever fabricated using the Multi User MEMS Processes (MUMPS) but is general enough to be extended into cantilever fabricated using other MEMS fabrication process.

II. The MEMS Cantilever Structure

In MUMPS fabrication process, a MEMS device is fabricated by surface micromachining method. Layers of silicon nitride, polysilicon and sacrificial phosphosilicate glass (PSG) are deposited in a Integrated Circuit fabrication alike process to create a multilayer structure. The PSG is etched away in the final stage, a process named releasing, to create a movable structure such as cantilever beam. Fig. 1 shows the cantilevers which are fabricated in this work. A SMF fiber put on the substrate helps in demonstrating the small size of the cantilevers.

In MUMPS cantilever based electrostatic actuation design presented in this work, a voltage difference is applied between the actuated beam and the substrate, with an insulating layer of silicon nitride, as shown in fig.2 through fig.4 The electrostatic pressure will pull the cantilever beam towards the substrate. At a certain threshold voltage known as pull-in voltage, the cantilever beam snaps down touching the nitride layer.

Fig. 1. Fabricated MEMS cantilever

A. Straight Cantilever Beam

Ideally, a fabricated beam without residual stress and stress gradient will be straight. Straight cantilever beam model has been derived in [1]. Fig.2 shows the schematic of a straight cantilever beam fabricated using MUMPS process.

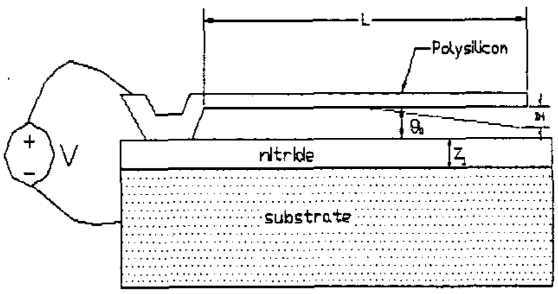

Fig. 2. A straight MEMS cantilever beam.

B. Curled Cantilever Beam

In practice, residual stress and stress gradient exist in beam material. This will cause the beam to curl, as reported in [2], [5],[6],[7]. Fig.3 shows curled up cantilever beam after the release process due to stress gradient in the polysilicon beam.

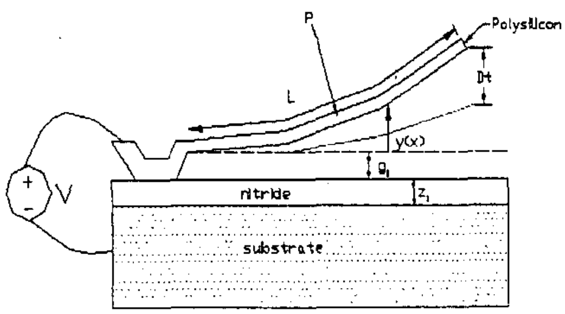

Fig. 3. A MUMPS cantilever beam curls upward due to stress gradient.

C. Partially Curled Cantilever Beam

In Multi User MEMS Processes (MUMPS) fabrication process, it is possible to design the the curling of a cantilever beam through the deposition of metal, a technique known as stress induced curling[5]. Through the deposition of metal at desired location, a cantilever beam with non-continuous curling can be achieved. In Fig. 4, a cantilever beam with a straight-curl-straight region is shown.

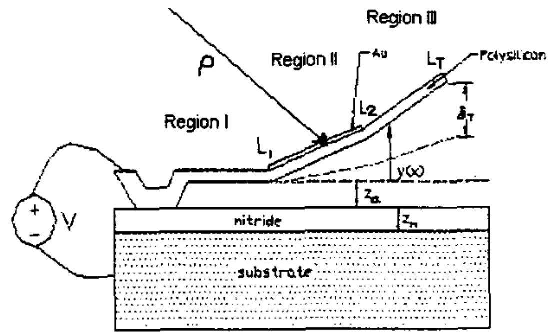

Fig. 4. A partially curled cantilever beam.

In this work, a model has been developed for both curled and partially curled cantilever beam.

III. THE CURLED CANTILEVER BEAM MODEL

There has been work published on analytical modeling for electrostatic pull-in voltage of a straight MEMS cantilever beam [1],[8],[9] as in fig.1. However, no analytical model has been published on curled cantilever model as in fig.3 or fig.4 The model presented in this work is aimed at including the curling effect in the analytical model developed so as to provide a more accurate result on pull-in voltage analysis.

From[10], a concentrated load at position x, q(x), causes a tip deflection $\delta(x)$ given by

$$\delta(x) = \frac{x^2}{6EI}(3L - x)wq(x)dx \tag{1}$$

where

E =Young modulus of the beam
I =moment of inertia of the beam
w =width of the beam

For electrostatic loading of a straight beam, assuming square law curvature for the loaded beam, q(x) is given in [1] as

$$q(x)=\frac{k\varepsilon_0}{2}\left(\frac{V}{g_0-(x/L)^2\delta_T}\right)^2 \qquad (2)$$

where

- g_0 =gap separating the fixed-end of the beam with the substrate
- k =effective dielectric constant for the gap
- ε_0 =permittivity of free space
- V =applied voltage across the beam and the substrate
- δ_T =total tip deflection

In this work, the straight beam model in (2) is modified to include curvature of the beam. A new model is derived as

$$q(x)=\frac{k\varepsilon_0}{2}\left(\frac{V}{g_0+y(x)}\right)^2 \qquad (3)$$

where y(x) is the extra gap at position x due to beam curvature, which can be calculated from geometry (detail is given in Appendix A) as

$$y(x)=\rho\left(1-\cos\left(x/\rho\right)\right) \qquad (4)$$

where

ρ =radius of curvature for the curled beam

Substituting (3) and (4) into (1), the total tip deflection δ_T can be found by integrating over the length of the beam for (1)

$$\delta_T=\frac{Lwk\varepsilon_0V^2}{4EI}\int_0^L\left(\frac{x}{g_0+\rho(1-\cos(x/\rho))}\right)^2dx \qquad (5)$$

The pull-in voltage V_{pi} can be approximated by solving V in (5) when $\delta_T=y(L)/2$ (i.e. pull-in voltage occurs when the beam has deflected half way through the entire gap). This gives

$$V_{pi}=\sqrt{\frac{2EI\rho(1-\cos(L/\rho))}{Lwk\varepsilon_0\int_0^L\left(\frac{x}{g_0+\rho(1-\cos(x/\rho))}\right)^2dx}} \qquad (6)$$

In deriving (6), a number of assumptions have been made. Fringing field at the edges of the beam is ignored, higher order terms (i.e. ρ^n and x^n, n>=3) is discarded, and k, the effective dielectric constant for air (K_A=1) and silicon nitride (K_N=7.46)[2] is an approximated value. A value of 1.2046 for k has been calculated as in Appendix B and is found to give good result for the model. (6) can then be solved using Taylor series expansion.

Despite all these assumptions and simplifications (6) gives relatively high accuracy consistency with published measured result. In Fig.4, comparison is made with measured result obtained from MTEST test structure [2]. From fig. 5, it can be observed that the straight cantilever beam model in (1) is only accurate at short beam length, where curling of the beam is less significant. It significantly deviates from the measured result as the beam length increase.

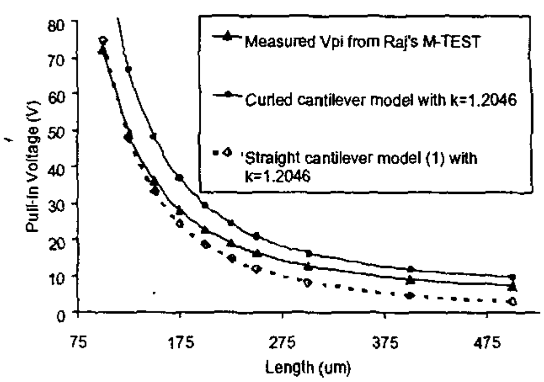

Fig. 5. Comparison of modeling result with measured V_{pi} from Raj's M-TEST structure [2] where h=2.1µm, E=153GPa, g_0=2.4µm, ?=40mm.

IV. Partially Curled Model

For the partially curled model, modeling of the partially curled cantilever beam is done by performing three separate modeling on each section of the straight-curled-straight beam, as

$$\delta_T = \frac{L_T w k \varepsilon_0 V^2}{4EI}\left[\int_0^{L_1}\left(\frac{x}{Z_m-(x/L_1)^2-Z_a}\right)^2 dx+\int_{L_1}^{L_2}\left(\frac{x}{Z_n+\rho\left(1-\cos\left(\frac{L_2-L_1}{\rho}\right)\right)}\right) + \int_{L_2}^{L_T}\left(\frac{x}{Z_n+\rho\left(1-\cos\left(\frac{L_2-L_1}{\rho}\right)\right)+(x-L_2)\sin\left(\frac{L_2-L_1}{\rho}\right)}\right)dx\right] \tag{7}$$

where

$Z_m = Z_n + Z_a$

From (7), the deflection versus applied voltage graph can be plotted as in Fig. 6 and Fig. 7. As in the curled cantilever model, pull-in occurs when the deflection equals half of the entire separating gap.

Comparison of the partially curled model with the reference design [5] is shown in Fig. 6 whereas comparison with measured result of this work is shown in Fig. 7. The partially curled model is able to give a good approximation to the actual pull-in voltage value.

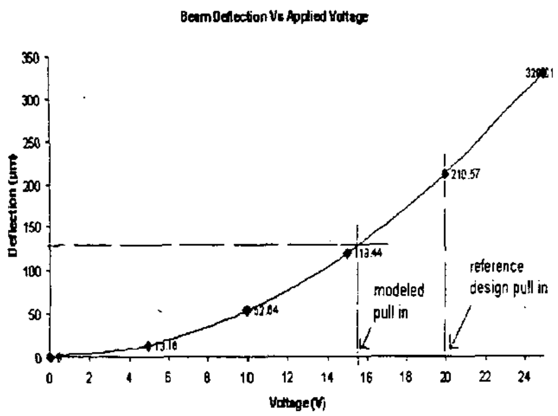

Fig. 6. Comparison with reference design [5].

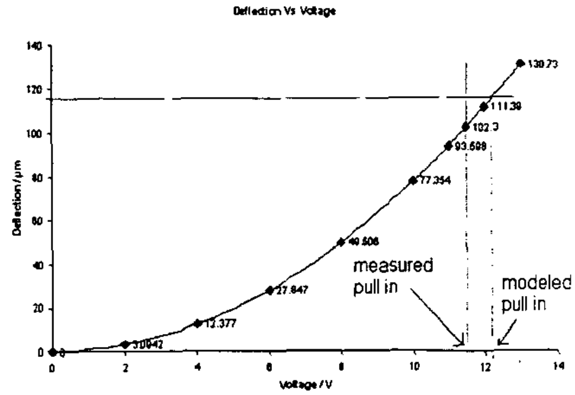

Fig. 7. Comparison with measured pull-in voltage of this work.

V. Conclusion

A closed form analytical model has been developed for MEMS cantilever beam structure. Despite all the assumptions and simplifications made, the model manages to approximate selected experimentally verified measured system to a more accurate order compared with previous work. The model can thus be used to accurately predict the performance of a cantilever based actuation MEMS design. More measured data is needed in order to fully characterize the model. Work is currently being undertaken to fabricate a new set of partially curled test structure to further characterize the model.

Appendix

A. Derivation of (4)

Based on geometry as shown in fig. 8.

y(L) = ?-h
= ?-? cos?
= ?-? cos(L / ?)
= ?(1- cos(L / ?)) (8)

For small ?, we can assume that the increment in x is equal to the increment along L, we get (4), that is

y(x)= ?(1- cos(x / ?)) (9)

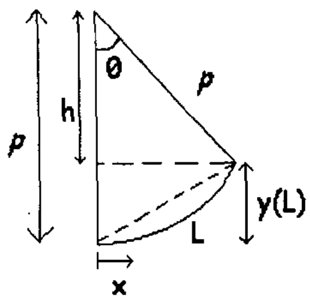

Fig. 8. Deriving y(L) and y(x) from geometry.

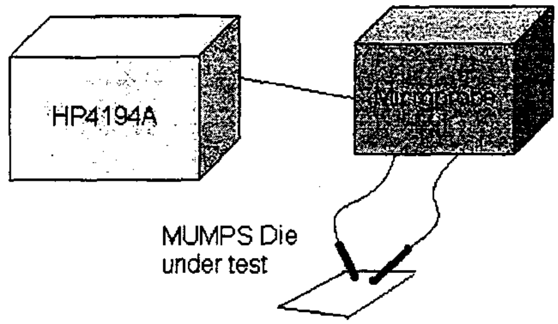

Fig. 9. Experimental setup for measurement

B. Calculation of effective dielectric constant, k

For a curled cantilever as depicted in fig.2, the effective dielectric constant across the electrostatic gap (nitride layer with thickness Z_1 and dielectric constant K_N and free space air gap g_0 with dielectric constant equals 1) is calculated based on the average height of the beam from the substrate.

The capacitance across the electrostatic gap is given by

$$C = K_N \varepsilon_0 w \int_0^L \frac{1}{z_1 + K_N\left(g_0 + \rho\left(1 - \cos(x/\rho)\right)\right)} dx \quad (8)$$

k is calculated based on dividing the capacitance with nitride layer by capacitance without nitride layer (i.e setting K_N =1). For the MUMPS cantilever structure in this work K_N =7.46 and this gives k=1.2046.

C. Experimental Setup For The Measurement Of Pull-In Voltage

Measurement has been done using HP4194A impedance meter. The change of capacitance across the cantilever and the silicon substrate is measured as the cantilever gets deflected through a gradual increase in applied voltage. A block diagram of the setup is shown in Fig. 9.

A typical graph obtained from the measurement is shown in Fig. 10. A Sudden increase in capacitance at 11.5V signifies the pull-in phenomena.

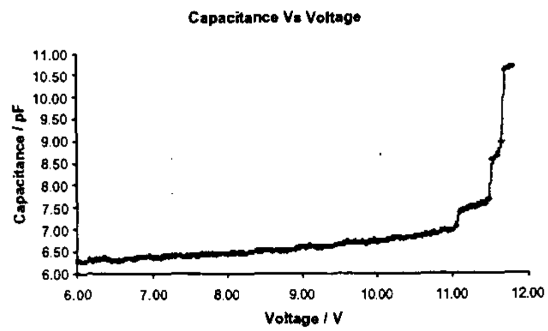

Fig. 10. Typical measurement result

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