Non-Linear Optimization Assignment 2021

SC42056 Optimization for Systems and Control

E_1, E_2, and E_3 are parameters ranging from 0 to 18 for each group according to a specific sum of the last three numbers of the student or employee IDs:

$E_1 = D_{a,1} + D_{b,1}$, $E_2 = D_{a,2} + D_{b,2}$, $E_3 = D_{a,3} + D_{b,3}$

with $D_{a,3}$ the right-most digit of the ID of the first group member, $D_{b,3}$ the right-most digit of the ID of the other group member, $D_{a,2}$ the one but last digit of the ID of the first group member, etc.

Important: Please note that all questions regarding this assignment should be asked via the Brightspace Discussion forum.

Our indoor habits significantly increase the energy demand for Heating, Ventilation, and Air-Conditioning (HVAC) in buildings. In addition, the recent COVID-19 pandemic has increased the requirements for ventilation, which implies even more power consumption to ensure the indoor air quality requirements. This problem has motivated the development of new HVAC technologies, in order to ensure the thermal comfort in indoor spaces while saving energy. For obvious reasons, HVAC technologies driven by renewable energy sources are the most popular. However, such energy sources cannot always provide a sufficiently high level of energy output, since the supply they provide mostly relies on the actual weather conditions. As a consequence, the HVAC devices must include a reliable backup system and an effective operational control strategy. Among the options available, optimal control methods are very well suited to operate the non-linear HVAC system with satisfactory performance. In this assignment, we will model and control the behavior of the HVAC system shown in Figure 1.

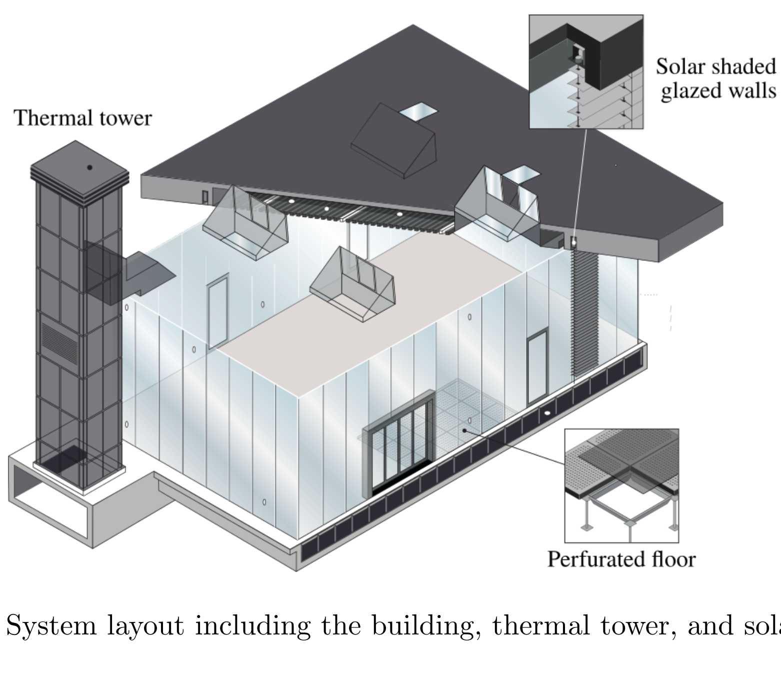

Figure 1: System layout including the building, thermal tower, and solar shades.

The system considered consists of an innovative building with vertical glass facades, where external steerable shades determine when the solar radiation is allowed to come into the building. Additionally, the attached thermal tower provides natural ventilation by the temperature difference between indoor and outdoor air, where the air comes into the building through the bottom of the tower and goes out by its top. The goal of this assignment is to determine the dynamic apertures of the shades and the tower (i.e., the fraction of heat and air flow that is allowed to enter) that minimize the use of the auxiliary backup system.

The principle of energy conservation will be used to model the behavior of the system, in which five main equations describe the system dynamics. The first one expresses the air mass flow rate produced by the thermal tower:

$$\dot{m}(k) = \rho_\mathrm{a} u(k) \phi \sqrt{2gH \max\left[0, \frac{T_\mathrm{a}(k) - T_\mathrm{o}(k)}{T_\mathrm{a}(k)}\right]} \tag{1}$$

where ρ_a is the specific mass of the air, $u(k)$ the ventilation fraction, ϕ the outlet cross-sectional area, g the gravitational constant, H the tower height, $T_\mathrm{a}(k)$ the indoor air temperature, $T_\mathrm{o}(k)$ the outdoor air temperature. Moreover, the index k is the time step corresponding to the time instant $t = k\Delta t$, where Δt is the simulation time step. Note that the temperatures in (1) must be in kelvin (K).

The second equation expresses the indoor air temperature as a heat balance between the internal heat generation, ventilation, and convection:

$$T_\mathrm{a}(\overset{K}{k+1}) = T_\mathrm{a}(k) + [\overset{K-1}{\dot{q}_\mathrm{p}(k)} + \dot{m}(k) C_\mathrm{a}[T_\mathrm{o}(k) - T_\mathrm{a}(k)] + \sum_{i=1}^{3} h_\mathrm{a} A_i [\overset{K-1}{T_{\mathrm{w},i}(k)} - T_\mathrm{a}(k)]] \frac{\Delta t}{\rho_\mathrm{a} V_\mathrm{a} C_\mathrm{a}} \tag{2}$$

where $\dot{q}_\mathrm{p}(k)$ is the heat rate from people occupancy, C_a the specific heat of air, h_a the internal convection coefficient, A_i the heat transfer area, V_a the volume of air, and $T_{\mathrm{w},i}(k)$ the temperature of the vertical glass facades ($T_{\mathrm{w},1}$), ceiling ($T_{\mathrm{w},2}$), and floor ($T_{\mathrm{w},3}$). The temperature of the glass facades is modeled as follows:

$$\begin{aligned} \overset{K-1}{T_{\mathrm{w},1}(k+1)} = T_{\mathrm{w},1}(k) + [v(k) I(k) \alpha_{\mathrm{w},1} + \lambda[T_\mathrm{sky}^4(k) - T_{\mathrm{w},1}^4(k)] + h_\mathrm{o}[T_\mathrm{o}(k) - T_{\mathrm{w},1}(k)] \\ + h_\mathrm{a}[T_{\mathrm{w},1}(k) - T_\mathrm{a}(k)]] \frac{A_{\mathrm{w},1} \Delta t}{\rho_{\mathrm{w},1} V_{\mathrm{w},1} C_{\mathrm{w},1}} \end{aligned} \tag{3}$$

where $v(k)$ is the shading fraction, $I(k)$ the solar incidence, $\alpha_{\mathrm{w},1}$ the wall's absorptance, λ the radiation coefficient, h_o the external convection coefficient, and $T_\mathrm{sky}(k)$ the sky temperature. Similarly, for determining the temperature of the ceiling we have:

$$\begin{aligned} T_{\mathrm{w},2}(k+1) = T_{\mathrm{w},2}(k) + [I(k) \alpha_{\mathrm{w},2} + \lambda[T_\mathrm{sky}^4(k) - T_{\mathrm{w},2}^4(k)] + h_\mathrm{o}[T_\mathrm{o}(k) - T_{\mathrm{w},2}(k)] \\ + h_\mathrm{a}[T_{\mathrm{w},2}(k) - T_\mathrm{a}(k)]] \frac{A_{\mathrm{w},2} \Delta t}{\rho_{\mathrm{w},2} V_{\mathrm{w},2} C_{\mathrm{w},2}} \end{aligned} \tag{4}$$

The temperatures in the third term in (3) and (4) should be expressed in kelvin (K) before they are raised to the power 4.

At last, for determining the temperature of the floor the following expression should be considered:

$$T_{\mathrm{w},3}(k+1) = T_{\mathrm{w},3}(k) + [0.5v(k)I(k)\tau_{\mathrm{w},1}\alpha_{\mathrm{w},3} + h_{\mathrm{a}}[T_{\mathrm{w},3}(k) - T_{\mathrm{a}}(k)]]\frac{A_{\mathrm{w},3}\Delta t}{\rho_{\mathrm{w},3}V_{\mathrm{w},3}C_{\mathrm{w},3}} \tag{5}$$

where $\tau_{\mathrm{w},1}$ refers to the glass transmittance.

The total energy required by the auxiliary backup system to reach thermal comfort during each time step is taken as the output of the system:

$$q_{\mathrm{backup}}(k) = \dot{m}(k)C_{\mathrm{a}}|T_{\mathrm{a}}(k) - T_{\mathrm{ref}}|\Delta t + \beta[T_{\mathrm{a}}(k) - T_{\mathrm{ref}}]^2 \tag{6}$$

where T_{ref} is the reference indoor temperature and β is the penalization factor for thermal comfort (Assume that $\beta = 10^6$ K^{-2}).

For the system considered, the dimensions and properties of each segment are shown in the Table 1. In addition, we consider the tower height $H = 5$ m and the cross-flow area $\phi = 3$ m^2, while the heat transfer coefficients are $h_{\mathrm{a}} = 5\frac{\mathrm{W}}{\mathrm{m}^2\mathrm{K}}$, $h_{\mathrm{o}} = 25\frac{\mathrm{W}}{\mathrm{m}^2\mathrm{K}}$, and $\lambda = 4.5 \cdot 10^{-8}\frac{\mathrm{W}}{\mathrm{m}^2\mathrm{K}^4}$.

Table 1: System properties

Segment	Subscript	$V[\mathrm{m}^3]$	$A[\mathrm{m}^2]$	$C[\frac{\mathrm{J}}{\mathrm{kgK}}]$	$\rho[\frac{\mathrm{kg}}{\mathrm{m}^3}]$	$\alpha[-]$	$\tau[-]$
Air	a	1430	–	1005	1.2	–	–
Facade	w, 1	17.5	350	900	2500	0.085	0.9
Ceiling	w, 2	15	286	840	2000	0.2	–
Floor	w, 3	80	286	840	2000	0.6	–

Moreover, we have $\Delta t = 5$ min, $T_{\mathrm{ref}} = 21$°C, and $T_{\mathrm{sky}}(k) = T_{\mathrm{o}}(k) - 8$ °C. Assume that at $k = 0$, the temperature of all the segments is equal to 16°C, while the solar incidence, outside temperature, and heat rate from occupancy is defined as follows:

$$I(k) = \begin{cases} 300 + E_1 \text{ W/m}^2 & \text{if } k < 48 \\ 700 + E_1 \text{ W/m}^2 & \text{if } 48 \leq k \leq 96 \\ 300 + E_1 \text{ W/m}^2 & \text{if } k > 96 \end{cases}$$

$$T_{\mathrm{o}}(k) = \begin{cases} 12 + 0.1E_2 \text{ °C} & \text{if } k < 48 \\ 18 + 0.1E_2 \text{ °C} & \text{if } 48 \leq k \leq 96 \\ 17 + 0.1E_2 \text{ °C} & \text{if } k > 96 \end{cases}$$

$$\dot{q}_{\mathrm{p}}(k) = \begin{cases} 5000 + 10E_3 \text{ W} & \text{if } k < 48 \\ 25000 + 10E_3 \text{ W} & \text{if } 48 \leq k \leq 96 \\ 20000 + 10E_3 \text{ W} & \text{if } k > 96 \end{cases}$$

Tasks:

1. Formulate the discrete-time state-space model that determines the ~~tower flow rate and~~ temperature of each building segment for the following time step $k+1$:

$$\begin{cases} x(k+1) = f(x(k), u(k), v(k)) \\ y(k) = g(x(k), u(k)) \end{cases} \tag{7}$$

Note: The state $x(k)$ includes ~~5 variables:~~ 4 temperatures (one for each segment) ~~and 1 flow rate~~. The output $y(k)$ includes one variable: the backup demand for the time step k. The values for $u(k)$ and $v(k)$ are fractions ranging between 0 and 1 and, therefore, they have no unit. **All of the system variables have to be considered in their corresponding SI unit: energy in joule (J), heat power in watt (W), mass flow in kg/s, and temperature in kelvin (K).**

2. Formulate the optimization problem in order to find the values of $u(k)$ and $v(k)$ that minimize the total energy demanded by auxiliary backup system $q_{\text{backup}}(k)$ for the following 12 hours, i.e., for the period $[0, 144\Delta t]$.

3. Select an appropriate optimization algorithm (explaining your choice) and write the corresponding MATLAB code.

4. Run the optimization algorithm using two different starting points. Is there a substantial difference between the solutions? Explain the results obtained.

5. Can you prove that the solution obtained is the global optimum? How can the solution be improved (if possible)?

6. Find the heating and ventilation signals $u(k)$ and $v(k)$ that minimize the energy demanded by the auxiliary backup system for the following 12 hours (i.e., for the period $[0, 144\Delta t]$) while limiting the minimum air flow rates to 0.5 kg/s.

7. Plot the backup energy rates and the dynamic values of $u(k)$ and $v(k)$ obtained in Tasks 2-5 and 6, and then compare them with the values obtained for the no control case (i.e., the case where $u(k) = 1$ and $v(k) = 1 \quad \forall\ k$). Moreover, find the minimal backup demand for the three different cases and compare them. Explain the results obtained.

The solutions of the assignment should be uploaded to Brightspace before Monday, October 25, 2021 at 17:00 as two <u>separate</u> files:

1. A written report on the practical exercise as a single .pdf file (no other formats allowed).

2. The .m files with the Matlab code you used as a single zip file; please make sure that the code is error free.

After uploading, please verify the uploaded files so as make sure that you have uploaded the correct files and that they are not broken.

Please also note that you will lose 0.5 point from your grade for this assignment for each (started) day of delay in case you exceed the deadline.