Lead Compensator

Topics Covered

- Lead Compensator Design.
- Bode plots.

Prerequisites

- QUBE-Servo 2 Integration Lab.
- PD Control Lab.

1 Background

PID control uses integration and derivative operations to design a compensator such that the overall system response follows a desired trajectory. More generally, the design of a controller for a system can be regarded as a filter design problem. In this context, a PI controller is a *low-pass filter*, and because it introduces a negative phase over some frequency range, therefore it is also called a **phase lag controller**. Contrary, a PD controller is a *high-pass filter* that introduces a positive phase and is also referred to as a **phase lead controller**.

The generic transfer function of a simple lead or lag compensator can be expressed as

$$C_{lead,lag}(s) = K_1 \frac{s+z}{s+p} = K_c \frac{1+\alpha T s}{1+Ts} \tag{1.1}$$

which is a low-pass or phase lag controller for $\alpha < 1$ (or $p < z$), and a high-pass or phase lead controller for $\alpha > 1$ (or $p > z$).

The proportional gain of the lead compensator is used to attain a certain crossover frequency. In general, increasing the gain, and respectively the crossover frequency, essentially increases the bandwidth of the system, thus decreasing the system's peak time (speeding up the response). A gain of $K_c > 1$ decreases the system's phase margin and, if K_c is chosen too large, will lead to large overshoots in the system response. For design purposes, K_c is often chosen such that it increases the bandwidth of the system to about half the desired bandwidth. The lead compensator will add additional gain such that the combination of K_c and lead compensator result in the desired system bandwidth.

Even though lag compensators work well in theory, they often struggle with the saturation limits of actual hardware, and may not be able to achieve a zero steady-state error specification. In this lab, we will design a lead compensator in series with an integrator as in Figure 1.1 to achieve zero steady-state error. The resulting controller has the form

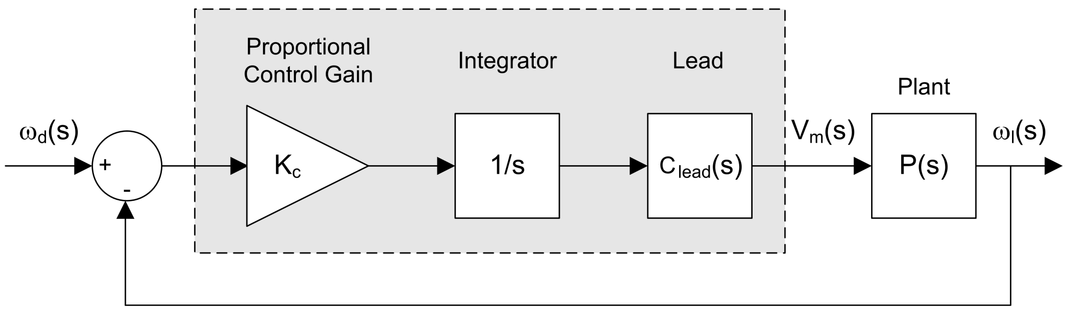

Figure 1.1: Closed-loop speed control with lead compensator

$$C(s) = K_c \frac{1+\alpha T s}{(1+Ts)s}. \tag{1.2}$$

1.1 Lead compensator design procedure

The two main design parameters for a lead compensator are the desired phase margin and the desired crossover frequency. The phase margin mainly affects the shape of the response, and a higher phase margin implies a more stable response with less overshoot. As a rule of thumb, the overshoot PO will not go beyond 5 % for a phase margin of at least 75 deg.

The crossover frequency is defined as the frequency where the gain of the system is 1 (or 0 dB in a Bode plot). This parameter mainly affects the speed of the response and a larger ω_m implies a decrease in the peak time. As a rule of thumb, the peak time t_p will not be more than 0.05 s with a crossover frequency of at least 75 rad.

The design process for a lead compensator can be summarized as follows:

1. Generate the Bode plot of the open-loop uncompensated system.
2. The lead compensator itself will add some gain to the closed-loop system response. To make sure that the bandwidth requirement of the design can be met, a proportional gain K_c needs to be added such that the open-loop crossover frequency is about a factor of two below the desired system bandwidth.
3. Determine the necessary additional phase lead ϕ_m for the plant with open-loop gain K_c. To do so, compute

$$\phi_m = PM_{des} - PM_{meas} + 5, \tag{1.3}$$

i.e. add 5 degrees to the desired phase margin and subtract the open-loop measured phase margin.

4. Compute α. To attain the maximum phase ϕ_m at the frequency ω_m as shown in Figure 1.2, the compensator is required to add $20\log_{10}(\alpha)$ of gain. Here, ω_m is the geometric mean of the two corner frequencies from the zero and pole of the lead compensator, respectively, i.e.

$$\log_{10}(\omega_m) = \frac{1}{2}\left(\log_{10}\left(\frac{1}{\alpha T}\right) - \log_{10}\left(\frac{1}{T}\right)\right). \tag{1.4}$$

Solving for ω_m reveals

$$\omega_m = \frac{1}{\sqrt{\alpha}T}. \tag{1.5}$$

The proportional gain of the lead compensator is used to attain a certain crossover frequency. In general, increasing the gain, and respectively the crossover frequency, essentially increases the bandwidth of the system, thus decreasing the system's peak time (speeding up the response). A gain of $K_c > 1$ decreases the system's phase margin and, if K_c is chosen too large, will lead to large overshoots in the system response.

The lead compensator is used to dampen the overshoot and increase the overall stability of the system by increasing the phase margin. The frequency response of the lead compensator in (Equation 1.1) is given by substituting $s = j\omega$ as

$$C_{lead}(j\omega) = \frac{1 + j\omega\alpha T}{1 + j\omega T}, \tag{1.6}$$

with the corresponding magnitude and phase

$$\begin{aligned} |C_{lead}(j\omega)| &= \sqrt{\frac{1+\omega^2\alpha^2T^2}{1+\omega^2T^2}}, \\ \phi_m &= \arctan(\omega\alpha T) - \arctan(\omega T). \end{aligned} \tag{1.7}$$

Using the trigonometric identity

$$\tan(\alpha - \beta) = \frac{\tan(\alpha) + \tan(\beta)}{1 + \tan(\alpha)\tan(\beta)}$$

on Equation 1.7 yields

$$\tan(\phi_m(j\omega)) = \frac{\omega\alpha T - \omega T}{1 + (\omega\alpha T)(\omega T)}. \tag{1.8}$$

Noting

$$\tan(\alpha) = \pm\frac{\sin(\alpha)}{\sqrt{1 - \sin^2(\alpha)}},$$

and using Equation 1.5, one can find

$$\sin(\phi_m) = \frac{\alpha - 1}{\alpha + 1}. \tag{1.9}$$

Thus, if ϕ_m is known, α can be determined by solving

$$\alpha = \frac{1 + \sin(\phi_m)}{1 - \sin(\phi_m)}. \tag{1.10}$$

5. Determine the value of T using Equation 1.5. To do so, place the corner frequencies of the lead compensator such that ϕ_m is located at ω_m, i.e. the new gain crossover frequency (the geometric mean of $1/\alpha T$ and $1/T$) where the compensator has a gain of 10 dB. By design, ω_m is the frequency at which the system with compensator has 0 dB gain. Therefore, ω_m has to be placed at the frequency where the magnitude of the uncompensated system is $G(j\omega) = -10\log_{10}\alpha$ dB. ω_m is then obtained by finding the corresponding frequency in the uncompensated Bode plot.

6. Determine the pole and zero of the lead compensator.

7. Check whether or not the compensator fulfills the design requirements. To do so, draw the Bode plot of the compensated system and check the resulting phase margin and check whether or not the system response meet the desired characteristics. Repeat the design steps for a different ϕ_m if necessary.

A typical Bode plot of a lead compensator is shown in Figure 1.2.

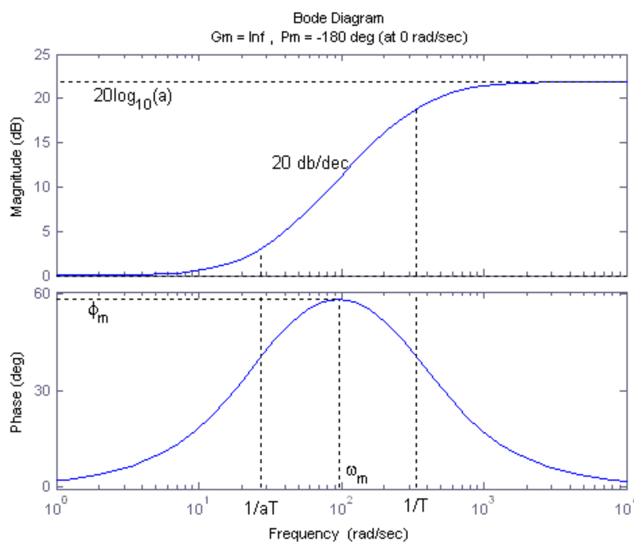

Figure 1.2: Bode plot of a typical lead compensator

2 In-Lab Exercises

In this lab, you will design a lead compensator for the speed control of the QUBE-Servo 2. Recall that the input voltage to output speed transfer function for the QUBE-Servo 2 is given by

$$P(s) = \frac{K}{\tau s + 1}. \tag{2.1}$$

As stated in the background section, we want to design a controller that is in series with an integrator to guarantee zero steady-state. For the design purpose of the lead compensator, we assume that the integrator is part of the plant model, i.e.

$$P_i(s) = P(s)\frac{1}{s}. \tag{2.2}$$

The control design should fulfill the following design requirements for steady-state error (e_{ss}), peak time (t_p), percentage overshoot (PO), phase margin (PM) and system bandwidth (ω_m):

$$\begin{aligned} e_{ss} &= 0, \\ t_p &= 0.05 \text{ s}, \\ PO &\leq 5 \text{ \%}, \\ PM &\geq 75 \text{ deg}, \\ \omega_m &\geq 75.0 \text{ rad/s}. \end{aligned} \tag{2.3}$$

1. Find the magnitude of the frequency response of the system transfer function (Equation 2.2) that is in series with an integrator ($|P_i(s)|$) in terms of the frequency ω.
2. The system has a gain of 1 (or 0 dB) at the crossover frequency ω_g. Find an expression for the crossover frequency in terms of the model parameters K and τ for $P_i(s)$. Use this expression to determine the crossover frequency for the QUBE-Servo 2 using $K = 21.9$ and $\tau = 0.15$.

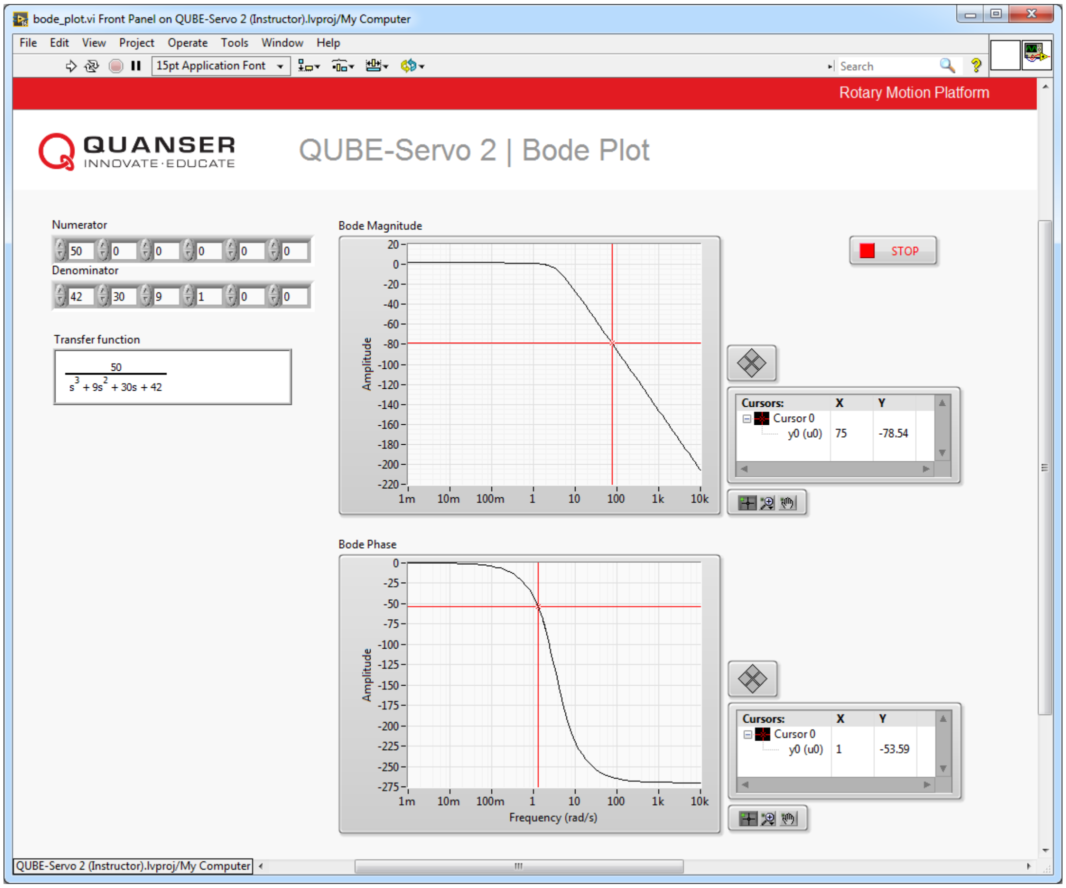

Figure 2.1: LabVIEW VI to generate a Bode plot.

3. Generate the Bode plot of $P_i(s)$ using `bode_plot.vi` shown in Figure 2.1. To open the VI, expand the Windows target as well as the *Bode Plot* virtual folder. Using $K = 21.9$ and $\tau = 0.15$ (or using the values you determined after performing the Bump Test Modeling laboratory experiment), enter appropriate values in the `Numerator` and `Denominator` numeric controls to generate the desired transfer function. Compare your derived frequency from the previous step to that obtained using the cursor palette in the `Bode Magnitude` and `Bode Phase` XY Graphs.

4. Find the proportional gain K_c that is necessary such that $K_c P_i(s)$ has a crossover frequency of 35 rad/s (about half the desired closed-loop bandwidth).

5. Determine the necessary phase lead ϕ_m that the lead compensator needs to add for the system $K_c P_i(s)$.

6. Compute α.

7. Determine ω_m.

8. Does ω_m meet the design requirement of $\omega_m \geq 75$ rad/s? Comment on what you could do to ensure you meet this requirement.

9. Determine the transfer function of the lead compensator. Start by evaluating T.

10. Determine the pole and zero location of the lead compensator. Generate the Bode plot of your lead compensator and verify that you have the desired phase margin at the desired frequency.

11. Validate your result by obtaining the closed-loop bode plot with proportional gain K_c and lead compensator $C_L(s)$. Do you have the desired phase margin at the desired frequency?

12. Open `QUBE-Servo 2 Lead.vi` and implement your lead compensator $C_L(s)$ with proportional gain $K_c = 8.21$. Run your controller. Does the system response match the desired characteristics? Try varying the value of K_c and see if you can improve the overall system response.

Quanser Inc.
119 Spy Court
Markham, Ontario
L3R 5H6
Canada
info@quanser.com
Phone: 1-905-940-3575
Fax: 1-905-940-3576

Printed in Markham, Ontario.

For more information on the solutions Quanser Inc. offers, please visit the web site at:
http://www.quanser.com