Swing-Up Control

Topics Covered

- Energy control.
- Nonlinear control.
- Control switching logic.

Prerequisites

- Filtering laboratory experiment.
- Rotary Pendulum Modeling laboratory experiment.
- Rotary pendulum module is attached to the QUBE-Servo 2.

1 Background

1.1 Energy Control

In theory, if the arm angle is kept constant and the pendulum is given an initial perturbation, the pendulum will keep on swinging with constant amplitude. The idea of energy control is based on the preservation of energy in ideal systems: The sum of kinetic and potential energy is constant. However, friction will be damping the oscillation in practice and the overall system energy will not be constant. It is possible to capture the loss of energy with respect to the pivot acceleration, which in turn can be used to find a controller to swing up the pendulum.

The dynamics of the pendulum can be redefined in terms of the pivot acceleration u as

$$J_p\ddot{\alpha} + \frac{1}{2}M_p g L_p \sin\alpha = \frac{1}{2}M_p g u \cos\alpha. \quad (1.1)$$

Here, u is the linear acceleration of the pendulum.

The potential energy of the pendulum is

$$E_p = \frac{1}{2}M_p g L_p \left(1 - \cos\alpha\right),$$

and the kinetic energy is

$$E_k = \frac{1}{2}J_p\dot{\alpha}^2.$$

The pendulum angle α and the lengths of the pendulum are illustrated in the free body diagram in Figure 1.1.

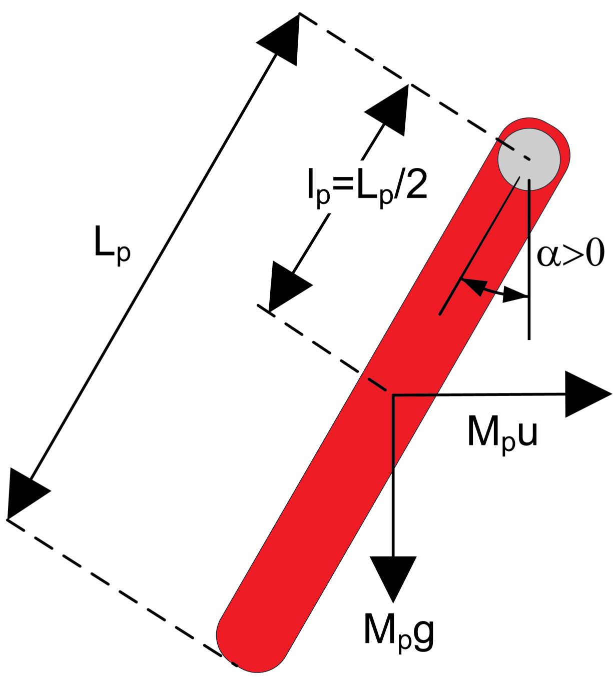


Figure 1.1: Free-body diagram of pendulum

The potential energy is zero when the pendulum is at rest at $\alpha = 0$ and equals $M_p g L_p$ when the pendulum is upright at $\alpha = \pm\pi$. The sum of the potential and kinetic energy of the pendulum is

$$E = \frac{1}{2}J_p\dot{\alpha}^2 + \frac{1}{2}M_p g L_p \left(1 - \cos\alpha\right). \quad (1.2)$$

Differentiating Equation Equation 1.2 yields

$$\dot{E} = \dot{\alpha}\left(J_p\ddot{\alpha} + \frac{1}{2}M_p g L_p \sin\alpha\right). \quad (1.3)$$

Recalling Equation (Equation 1.1) and rearranging terms as

$$J_p\ddot{\alpha} = -M_p g l_p \sin\alpha + M_p u l_p \cos\alpha$$

eventually yields

$$\dot{E} = M_p u l_p \dot{\alpha} \cos\alpha.$$

Since the acceleration of the pivot is proportional to current driving the arm motor and thus also proportional to the drive voltage, it is possible to control the energy of the pendulum with the proportional control law

$$u = (E_r - E)\dot{\alpha}\cos\alpha. \tag{1.4}$$

By setting the reference energy to the pendulum potential energy ($E_r = E_p$), the control law will swing the link to its upright position. Notice that the control law is nonlinear because the proportional gain depends on the cosine of the pendulum angle α. Further, the control changes sign when $\dot{\alpha}$ changes sign and when the angle is ± 90 degrees.

For the system energy to change quickly, the magnitude of the control signal must be large. As a result the following swing-up controller is implemented in the controller as

$$u = \text{sat}_{u_{max}}\left(\mu(E_r - E)\text{sign}(\dot{\alpha}\cos\alpha)\right) \tag{1.5}$$

where μ is a tunable control gain and the $\text{sat}_{u_{max}}$ function saturates the control signal at the maximum acceleration of the pendulum pivot, u_{max}. The expression $\text{sign}(\dot{\alpha}\cos\alpha)$ is used to enable faster control switching.

1.2 Hybrid Swing-Up Control

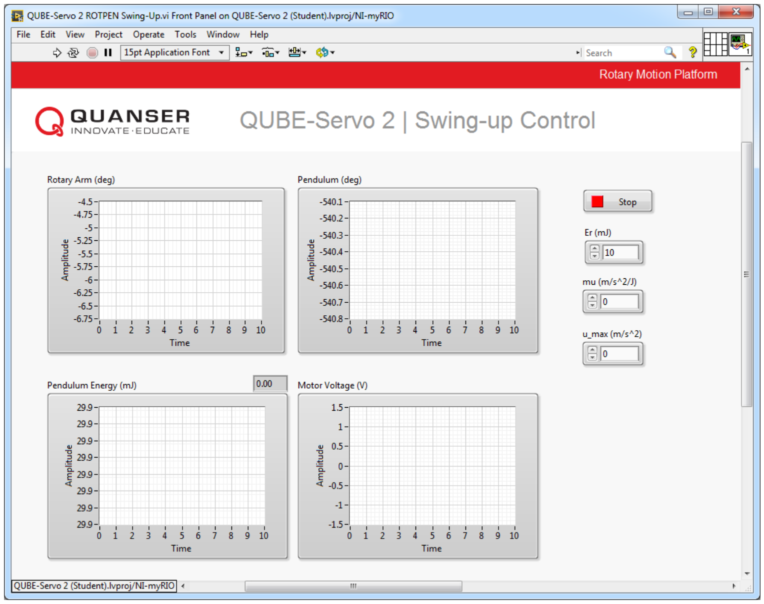
The energy swing-up control in Equation 1.4 (or Equation 1.5) can be combined with the balancing control law from the Balance Control Lab to obtain a control law that swings up the pendulum and then balances it.

Similarly as described in the Balance Control Lab, the balance control is to be enabled when the pendulum is within ± 20 degrees. When it is not enabled, the swing-up control is engaged. Thus the switching can be described mathematically by:

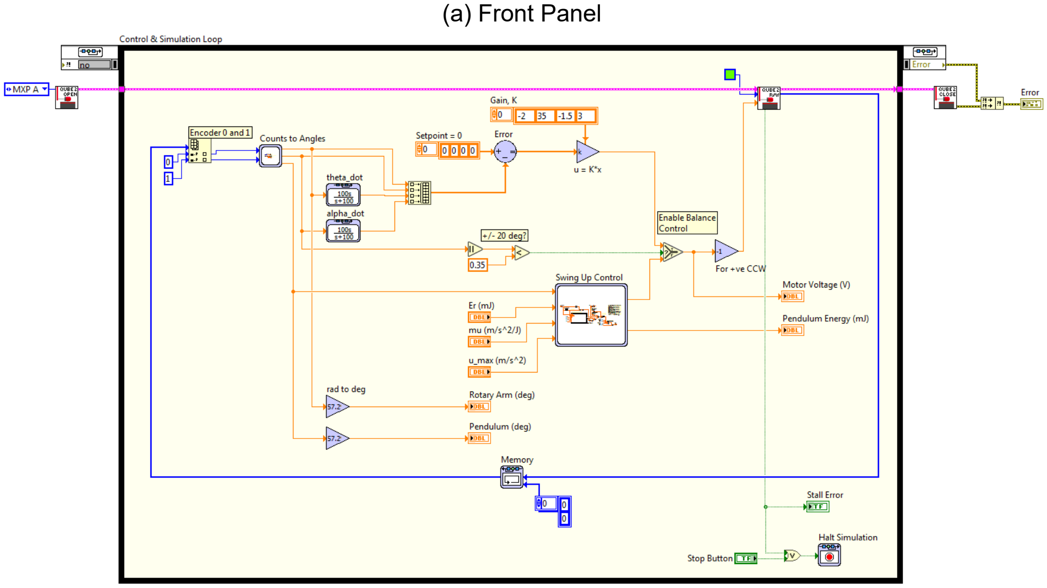
$$u = \begin{cases} u_{bal} & \text{if } |\alpha| - \pi \leq 20^\circ \\ u_{swing\_up} & \text{otherwise} \end{cases} \tag{1.6}$$

2 In-Lab Exercises

The VI shown in Figure 2.1 swings-up and balances the pendulum on the QUBE-Servo 2 Rotary Pendulum system. The *Swing-Up Control* subsystem implements the energy control described in Background section of this lab.



(a) Front Panel



(b) Block Diagram

Figure 2.1: VI that implements the swing-up controller

2.1 Energy Control

1. Open `QUBE-Servo 2 ROTPEN Swing-Up.vi`.
2. To turn the swing-up control off, set `mu` to 0.
3. Run the VI.

4. Manually rotate the pendulum at different levels and examine the pendulum angle and energy in the Pendulum (deg) and Pendulum Energy (mJ) charts.

5. What do you notice about the energy when the pendulum is moved at different positions? Record the energy when the pendulum is being balanced upright. Does this reading make sense in terms of the equations developed in the Background section of this lab?

6. Click on the Stop button to bring the pendulum down to the initial, downward position.

7. Set the swing-up control parameters (Constant and Gain blocks connected to the inputs of the Swing-Up Control subsystem) to the following values:

   - mu (m/s^2/J) = 50
   - Er (mJ) = 10.0
   - u_max (m/s^2) = 6

8. If the pendulum is not moving, gently perturb the pendulum with your hand from the downward position.

9. Vary the reference energy, Er, between 10.0 mJ and 20.0 mJ. As it is changed, examine the pendulum angle and energy response in Pendulum (deg) and the Pendulum Energy (mJ) charts and the control signal in the Motor Voltage (V) chart. Attach the responses showing how changing the reference energy affects the system.

10. Fix Er to 20.0 mJ and vary the swing-up control gain *mu* between 20 and 60 $\text{m/s}^2/\text{J}$. Describe how this changes the performance of the energy control.

11. Click on the Stop button to stop the VI.

2.2 Hybrid Swing-Up Control

1. Open `QUBE-Servo 2 ROTPEN Swing-Up (Student).vi`.

2. Set the swing-up control parameters to the following:

   - mu (m/s^2/J) = 20
   - u_max (m/s^2) = 6

3. Based on your observations in the previous lab, 2.1, what should the reference energy be set to?

4. Make sure the pendulum is hanging down motionless and the encoder cable is not interfering with the pendulum.

5. Run the VI.

6. The pendulum should begin going back and forth. If not, perturb the pendulum lightly with your hand. **Click on the Stop button if the pendulum goes unstable.**

7. Gradually increase the swing-up gain μ until the pendulum swings up to the vertical position. Capture a response of the swing-up and record the swing-up gain that was required. Show the pendulum angle, pendulum energy, and motor voltage.

8. Click on the Stop button to stop the VI.

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