

Pendulum Moment of Inertia

Topics Covered

- Finding moment of inertia analytically and experimentally.

Prerequisites

- Integration laboratory experiment.
- Rotary pendulum module is attached to the QUBE-Servo 2.

1 Background

The free-body diagram of the QUBE-Servo 2 pendulum system is shown in Figure 1.1.

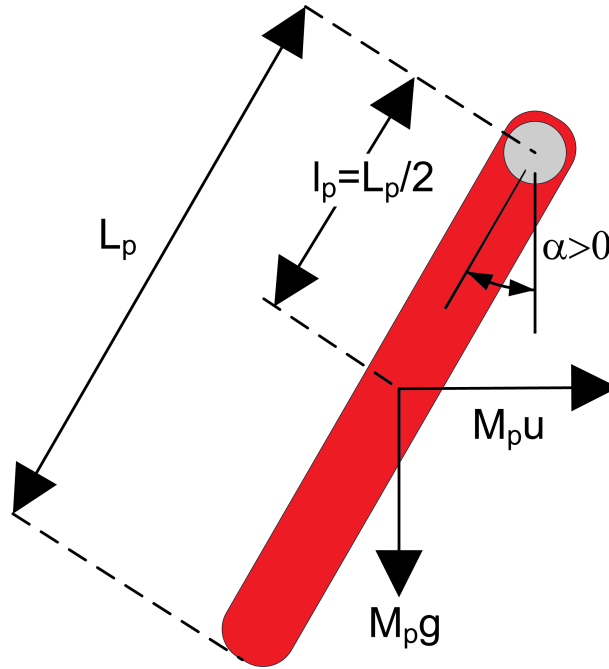


Figure 1.1: Free-body diagram of pendulum

From the free-body diagram in Figure 1.1, the resulting nonlinear equation of motion of the pendulum is

$$J_p \ddot{\alpha}(t) = M_p g \frac{L_p}{2} \sin(\alpha(t)), \quad (1.1)$$

where J_p is the moment of inertia of the pendulum at the pivot axis, M_p is the total mass of the pendulum assembly, and L_p is the length of the pendulum (from pivot to end). The center of mass position is at $L_p/2$, as depicted in Figure 1.1.

The moment of inertia of the pendulum can be found experimentally. Assuming the pendulum is not actuated, linearizing Equation 1.1 and solving for the differential equation yields

$$J_p = \frac{M_p g l_p}{(2\pi f)^2}, \quad (1.2)$$

where f is the measured frequency of the pendulum as the arm remains rigid. The frequency is calculated using

$$f = \frac{n_{cyc}}{\Delta t} \quad (1.3)$$

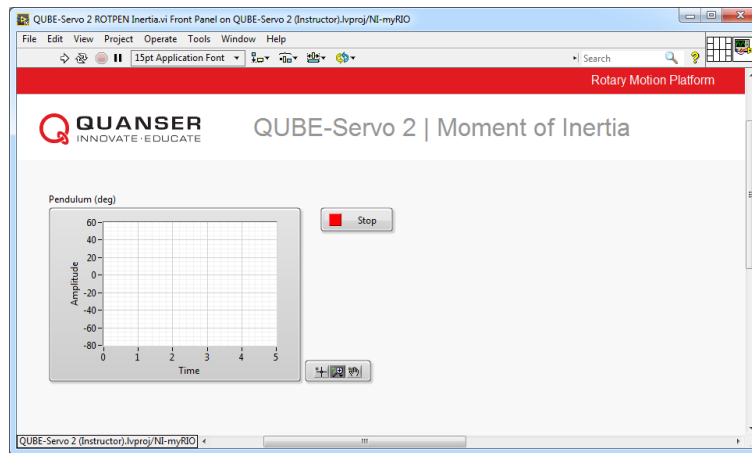
where n_{cyc} is the number of cycles and Δt is the duration of these cycles. Alternatively, J_p can be calculated using the moment of inertia expression

$$J = \int r^2 dm, \quad (1.4)$$

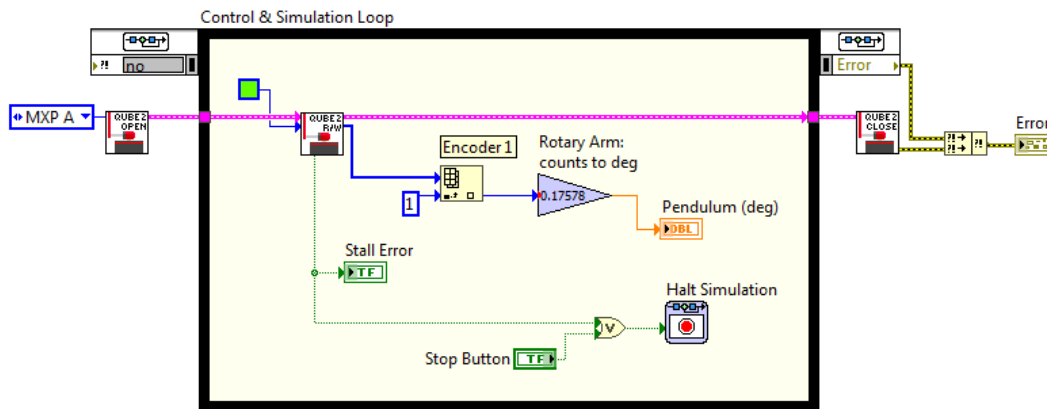
where r is the perpendicular distance between the element mass dm and the axis of rotation.

2 In-Lab Exercises

Based on the model already designed in Integration laboratory experiment, design a model that measures the pendulum angle using the encoder as shown in Figure 2.1.



(a) Front Panel



(b) Block Diagram

Figure 2.1: Displays measured pendulum angle

1. Find the moment of inertia acting about the pendulum pivot using the free-body diagram. Make sure you evaluate it numerically using the parameters defined in the QUBE-Servo 2 User Manual.
Hint: For solid objects with a uniform density, you can express the differential mass in terms of differential length.
2. Build the VI shown in Figure 2.1. Enter the value of 0.17578 (equivalent to $360/(4 \times 512)$) in the encoder sensor gain to measure the angular displacement of the pendulum in degrees.
3. Run the VI. With the controller running, manually perturb the pendulum while holding the rotary arm in place. The scope response should be similar to Figure 2.2.
4. Find the frequency and moment of inertia of the pendulum using the observed results. If needed, use the *Graph* palette to zoom on the response.
5. Compare the moment of inertia calculated analytically in Exercise 1 and the moment of inertia found experimentally. Is there a large discrepancy between them?
6. Click on the Stop button to stop the VI.

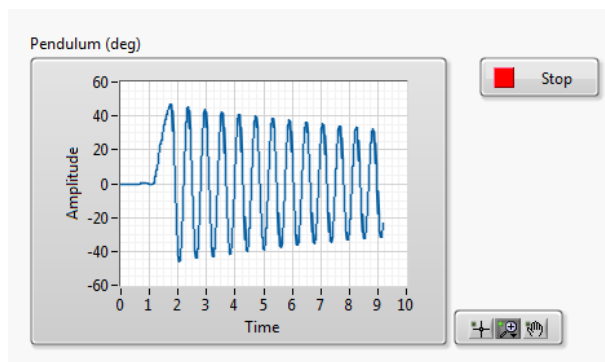


Figure 2.2: Free-oscillation response of pendulum

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