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On parameter identification of the Furuta pendulum

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Abstract

The Furuta pendulum is a well–known underactuated mechanical system used by many control researchers to test new control techniques. In this paper, the parameter identification of a Furuta pendulum prototype designed at IPN–CITEDI is presented. The procedure used to achieve the parameter identification of the experimental system consisted in using the filtered dynamic model and the standard least–squares algorithm. Comparisons between numerical simulation and experiment show a manner of validating the accuracy of the obtained parameter estimation.

Keywords: Filtered dynamic model, Furuta pendulum, Least–squares, Parameter identification, Real–time experiment

1. Introduction

The Furuta pendulum is a very popular experiment used for educational purposes. This is an academic benchmark in modern control theory and underactuated mechanical systems. It consist of an arm which rotates in the horizontal plane and attached to the arm is a pendulum which is free to rotate in the vertical plane [1]. It was firstly developed by K. Furuta at Tokyo Institute of Technology and was called the TITECH pendulum . Since the angular acceleration of the pendulum cannot be controlled directly, the Furuta pendulum is an underactuated mechanical system [2]. A lot of papers have used the system to demonstrate linear and nonlinear control laws. The reason is that the Furuta pendulum belong to the class of underactuated systems, which have more degrees–of–freedom that control inputs. Therefore, underactuated systems are difficult to control.

On the other hand, the parameter identification of a mechanical system is useful to design model–based controllers. In order to predict the behavior of the system and a control strategy for a set of operation conditions (initial conditions, disturbances, control gains, model nonlinearities, etc.) numerical simulations are necessary. Thus, the simulated system can provide useful information to design a new control law. See for instance [3], [4],[5], [6], were advantages, limitations and applications of the parameter identification techniques are discussed.

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In this paper, a method to identify the parameters of a Furuta pendulum is tested experimentally. An original prototype design is presented. See Figure 1, where a CAD draw and picture of the system are depicted. A numerical simulation carried out with estimated parameters is compared with respect to the real–experiment from which the parameters were estimated. Good results are observed.

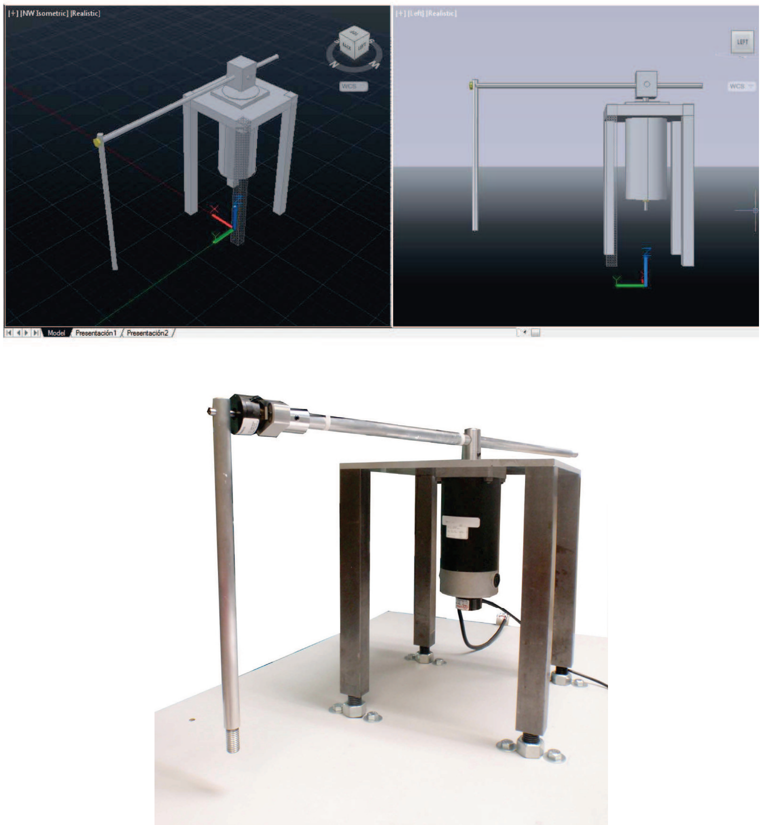

Fig. 1. CAD design of Furuta pendulum and real prototype build at IPN–CITEDI.

The identification procedure used on this paper consists in computing the filtered dynamic model of the Furuta pendulum to obtain a regression input–output system and then using the least–squares method. The Furura pendulum modeling takes into account the friction torque, assumed to be captured by viscous and Coulomb friction models [7].

The organization of this paper is as follows: In Section 2, the parameter identification procedure is reviewed. The results are given in Section 3. Finally, some concluding remarks are drawn in Section 4.

2. Parameter identification of the Furuta pendulum dynamic model

2.1. Least-squares algorithm

The least–squares method is a technique for parameter estimation and has the characteristic of easy application if the model has the property of being linear in the parameters. Let us consider regression model [8]

$$u(k) = Y(k)\theta, \tag{1}$$

where $u(k)$ represents the output vector, $Y(k)$ is the regressor matrix of known functions, θ is the vector of unknown parameters and $k = 0, 1, 2, \ldots$, is the integer time index. Let $\hat{\theta}(k)$ be an estimation of the vector of unknown parameters θ. Thus, $\hat{\theta}(k)$ can be obtained through the least–square algorithm [8]

$$\hat{\theta}(k) = \left\{ \sum_{i=0}^{k} Y(i)^T Y(i) \right\}^{-1} \left\{ \sum_{i=0}^{k} Y(i)^T u(i) \right\}, \tag{2}$$

where $\hat{\theta}(k)$ represents the vector of estimated parameters as a function of the time index k.

2.2. Joint velocity calculation

In first instance, a low pass filter $f_p(z)$ should be designed in order to eliminate the high frequency components of the joint position of the pendulum. These high frequency components are due to the optical encoders of the motors [6], [3].

Once the filter $f_p(z)$ has been designed, off–line implementation can be done through a zero–phase digital filtering with aim of avoiding distortion in the samples of the joint position. See for instance [9]. This digital filtering can be implemented easily by using the Matlab function `filtfilt`. The filtered joint position $q_f(k)$ is obtained by applying the `filtfilt` function in the joint position $q(k)$. The parameters of the `filtfilt` filter are those of the designed low pass filter $f_p(z)$. In other words, `filtfilt` computes the filtered position

$$q_f(k) = f_p(z)q(k), \tag{3}$$

where $q(k)$ is the vector of joint position, which is obtained from encoder measurements, and $f_p(z)$ is the designed low pass filter, with zero–phase distortion.

We used the Matlab function `fdatool` to obtain the coefficients of the the filter $f_p(z)$, whose characteristics are given in Table 1; see [3] for further details.

Table 1. Filter characteristics.

Response Type	Lowpass
Design Method	FIR - Windows
Window	Nutall
Specify Order	30
Frequency Specifications	Normalized
wc	0.07

Finally, the joint velocity $\dot{q}(k)$ can be computed off–line by applying the central differentiation algorithm [6], [3],

$$\dot{q}(k) = \frac{q_f(k+1) - q_f(k-1)}{2T}, \tag{4}$$

where k is integer time index, T is the sampling period and $q_f(k)$ denotes the filtered joint position vector.

2.3. Dynamic filtered model

Taking into account the presence of viscous and Coulomb friction at the joints, the Furuta pendulum dynamic model can be written as [2], [10], [11],

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + F_v\dot{q} + F_c \tanh(r\dot{q}) + g(q) = u, \tag{5}$$

where

$$q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}, \tag{6}$$

$$M(q) = \begin{bmatrix} \theta_1 + \theta_2 \sin^2(q_2) & \theta_3 \cos(q_2) \\ \theta_3 \cos(q_2) & \theta_4 \end{bmatrix}, \tag{7}$$

$$C(q,\dot{q}) = \begin{bmatrix} \frac{1}{2}\theta_2\dot{q}_1\sin(2q_2) & -\theta_3\dot{q}_2\sin(q_2) + \frac{1}{2}\theta_2\dot{q}_1\sin(2q_2) \\ -\frac{1}{2}\theta_2\dot{q}_1\sin(2q_2) & 0 \end{bmatrix}, \tag{8}$$

$$g(q) = \begin{bmatrix} 0 \\ -\theta_5\sin(q_2) \end{bmatrix}, \tag{9}$$

$$F_v = \begin{bmatrix} \theta_6 & 0 \\ 0 & \theta_7 \end{bmatrix}, \tag{10}$$

$$F_c = \begin{bmatrix} \theta_8 & 0 \\ 0 & \theta_9 \end{bmatrix}, \tag{11}$$

$$\tanh(r\dot{q}) = \begin{bmatrix} \tanh(r\dot{q}_1) \\ \tanh(r\dot{q}_2) \end{bmatrix}, \tag{12}$$

and

$$u = \begin{bmatrix} \tau \\ 0 \end{bmatrix}. \tag{13}$$

See the textbook [2] for a detailed discussion on the obtention of the dynamic model (5). The physical meaning of the input vector (13) is that the system is equipped with one actuator only, which deliver the torque input τ. In other words, there is only one control input and it is denoted as τ.

The physical description of the parameters θ_i is given in Table 2. Notice that J_1 is the inertia constant

Table 2. Parameter definitions.

Symbol	Definition	Symbol	Definition
θ_1	$m_2L_1^2 + J_1$	θ_6	F_{v1}
θ_2	$m_2l_2^2$	θ_7	F_{v2}
θ_3	$L_1l_2m_2$	θ_8	F_{c1}
θ_4	$m_2l_2^2 + J_2$	θ_9	F_{c2}
θ_5	l_2m_2g		

of the arm, m_2 is the mass of the pendulum, L_1 is the total length of the arm, l_2 is the distance to the center of gravity of the pendulum, J_2 is Inertia of the Pendulum around its center of gravity, F_{v1} is viscous friction coefficient of the motor, F_{v2} is viscous friction coefficient of the pendulum, F_{c1} is Coulomb friction coefficient of motor, F_{c2} is Coulomb friction coefficient of pendulum, g is the gravity acceleration, r is a constant to approach the "sign" function, q_1 is the position of the arm, and q_2 is the position of the pendulum.

The model (5) can be written as

$$\frac{d}{dt}\Omega_a(q,\dot{q})\theta + \Omega_b(q,\dot{q})\theta = u, \tag{14}$$

where Ω_a, Ω_b and θ are given by

$$\Omega_a = \begin{bmatrix} \dot{q}_1 & \dot{q}_1\sin^2(q_2) & \dot{q}_2\cos(q_2) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \dot{q}_1\cos(q_2) & \dot{q}_2 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Omega_b = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \dot{q}_1 & 0 & \tanh(r\dot{q}_1) & 0 \\ 0 & -\frac{1}{2}\sin(2q_2)\dot{q}_1{}^2 & \dot{q}_1\dot{q}_2\sin(q_2) & 0 & -\sin(q_2) & 0 & \dot{q}_2 & 0 & \tanh(r\dot{q}_2) \end{bmatrix},$$

and

$$\theta = \begin{bmatrix} \theta_1 & \theta_2 & \theta_3 & \theta_4 & \theta_5 & \theta_6 & \theta_7 & \theta_8 & \theta_9 \end{bmatrix}^T.$$

Now, let us define a low–pass filter

$$f(s) = \frac{\lambda}{s+\lambda}, \tag{15}$$

By multiplying both sides of equation (14), the filtered Furuta pendulum model is obtained, that is,

$$[sf(s)\Omega_a(\dot{q}, q) + sf(s)\Omega_b(\dot{q}, q)]\theta = f(s)u, \tag{16}$$

which at the same time can be expressed as

$$[\Omega_{af}(\dot{q}, q) + \Omega_{bf}(\dot{q}, q)]\theta = u_f, \tag{17}$$

where

$$\Omega_{af}(\dot{q}, q) = sf(s)\Omega_a(\dot{q}, q), \tag{18}$$

$$\Omega_{bf}(\dot{q}, q) = f(s)\Omega_b(\dot{q}, q), \tag{19}$$

and

$$u_f = f(s)u. \tag{20}$$

Finally, the following compact notation is used

$$u_f = \Omega_f(q, \dot{q})\theta, \tag{21}$$

$$\Omega_f(q, \dot{q}) = \Omega_{af}(\dot{q}, q) + \Omega_{bf}(\dot{q}, q).$$

Notice that to compute the filtered model (21), the on–line velocity and acceleration measurements are not required [3], [6],[4].

The joint velocity $\dot{q}(k)$ should be first computed as explained in Section 2.2. Then, the matrices $\Omega_{af}(k)$ and $\Omega_{bf}(k)$ should be obtained by using the discrete version of the filters $f(s)$ and $sf(s)$.

The joint acceleration is not necessary since it is obviated through the application of the filter $f(s)$ in the calculation of the matrices Ω_{af} and Ω_{bf}

The least–squares identification method in equation (2) can by applied by using

$$u(k) = u_f(k), \tag{22}$$

$$Y(k) = \Omega_f(k), \tag{23}$$

while $\hat{\theta}(k)$ becomes an estimation of the Furuta pendulum parameters described in Table 2.

3. Results

The described identification process has been implemented in the Furuta pendulum built at IPN–CITEDI Research Center. The experimental system has a direct current motor connected to the servo amplifier in voltage mode. The position of the arm and the pendulum is measured through optical encoders. A data acquisition board is used to read the encoder signals and to transfer voltage to the servo amplifier input. For simplicity, we have assumed to that the voltage transferred to the servo amplifier input is equal to the torque delivered by the motor.

It is noteworthy that the dynamics of the servo amplifier and the direct current motor is neglected.

3.1. Identification

An important constituent of the identification problem is the selection of the input signal for exciting the pendulum dynamics. The selected input signal was selected as

$$\tau(t) = Kx(t) + d(t), \tag{24}$$

where

$$x(t) = \begin{bmatrix} q_1 \\ \dot{q}_1 \\ q_2 \\ \dot{q}_2 \end{bmatrix},$$

$$K = \delta[-9.1265 \ -5.2953 \ -41.5431 \ -5.6176], \quad \delta = 0.8,$$

$$d(t) = d_1(t) + d_2(t) + d_3(t),$$

and $d_1(t)$, $d_2(t)$ and $d_3(t)$ are periodic signals with the following characteristics

$$d_1(t) = \begin{cases} -1 & for \quad 0 \leq t \leq 1, \\ 1 & for \quad 1 < t \leq 2, \end{cases}$$

$$d_2(t) = \begin{cases} -0.5 & for \quad 0 \leq t \leq 0.25, \\ 0.5 & for \quad 0.25 < t \leq 0.5, \end{cases}$$

$$d_3(t) = \begin{cases} -0.25 & for \quad 0 \leq t \leq 0.1667, \\ 0.25 & for \quad 0.1667 < t \leq 0.3333. \end{cases}$$

Notice that $d_1(t) = d_1(t+2)$, $d_2(t) = d_2(t+0.5)$ and $d_3(t) = d_3(t+0.3333)$ for all $t \geq 0$.

In fact the control input (24) with $d(t) = 0$, for all $t \geq 0$, is a controller able to stabilize at the system the equilibrium point $x = [0\ 0\ 0\ 0]^T$. The signal $d(t)$ is used to excite the Furuta pendulum dynamics and then to obtain acceptable estimated parameters $\hat{\theta}$ that capture the system dynamics at low and fast velocity.

Figure 2 shows the time evolution of the estimated parameters by using the identification procedure, which consists in the application of the least–square algorithm in the filtered dynamic model. The last value of $\hat{\theta}_i$ is shown in Table 3. The last value of $\hat{\theta}$ has been considered a good enough approximation of the true parameter vector θ.

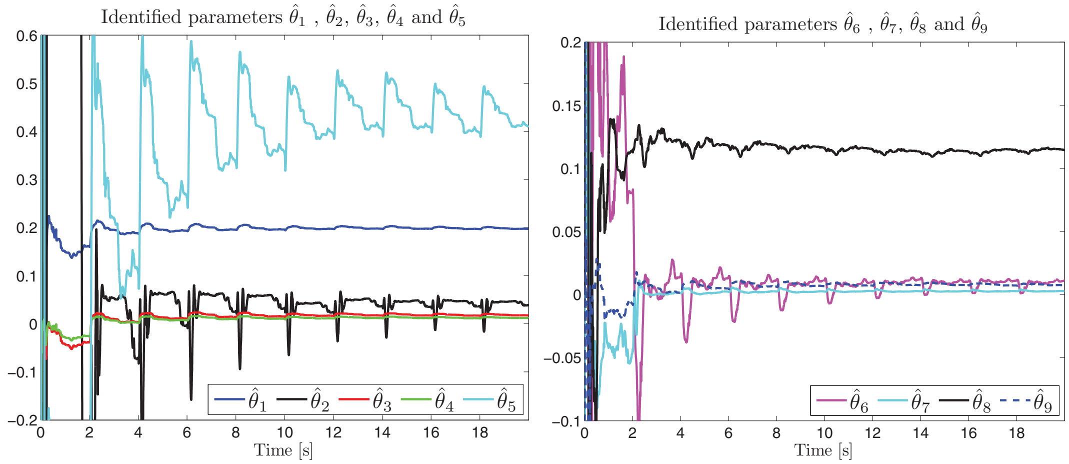

Fig. 2. Time evolution of the estimated parameters $\hat{\theta}_i(t)$.

Table 3. Numerical values of the obtained parameter estimation. We have assume that $\hat{\theta} = \theta$.

Symbol	Definition	Symbol	Definition
θ_1	0.19717	θ_6	0.0108
θ_2	0.03909	θ_7	0.00267
θ_3	0.01746	θ_8	0.11456
θ_4	0.01187	θ_9	0.00732
θ_5	0.40808		

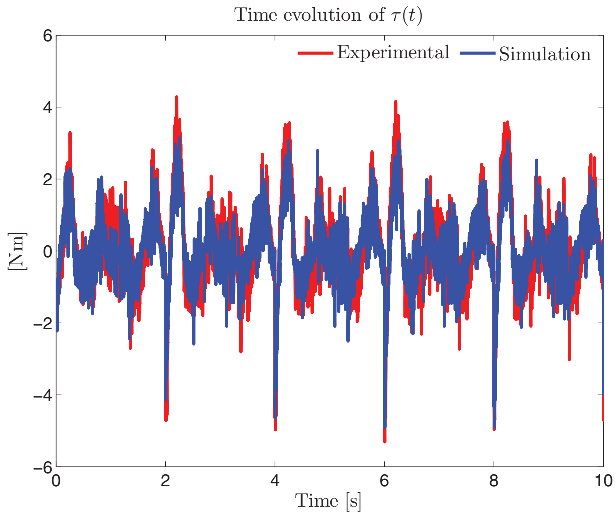

Fig. 3. Time evolution of the excitation signal $\tau(t)$ for both numerical simulation and real–time experiment. Let us notice that we have assumed that the servo amplifier input voltage is equal to the delivered torque.

The real–time experiment is compared with respect to a numerical simulation, which was achieved by using the model (5) with estimated parameters $\hat{\theta}$ shown in Table 3 and the input signal (24). Numerical simulation and experiment started at the initial condition $x(0) = [0\ 0\ 0\ 0]^T$. The numerical simulation assumes that the controller is implemented in discrete time. Figure 3 shows the time evolution of the control input τ for both the numerical simulation and experimental result. Similarly, Figure 4 depicts the arm position $q_1(t)$ and pendulum position $q_2(t)$ obtained from numerical simulation and real–time implementation.

It is observed in Figures 3 and 4 that, although simulation and experiment present a good matching in their time evolution, there are some time interval in that the numerical simulation deviates from the experimental results. This is attributed to the simple friction model adopted and to the dynamics of the servo amplifier and direct current motor.

4. Conclusions

In this paper the experimental parameter identification of a Furuta pendulum has been presented. The used procedure involves on–line and off–line steps, which can be summarized as follows:

- On–line: Select a proper input τ to excite the dynamics of the Furuta pendulum.
- On–line: Capture the joint position $q(k)$ and the control input $\tau(k)$.
- Off–line: Use a zero–phase filtering on $q(k)$ to eliminate high frequency components. For example, implement equation (3) using the `filtfilt` Matlab function.

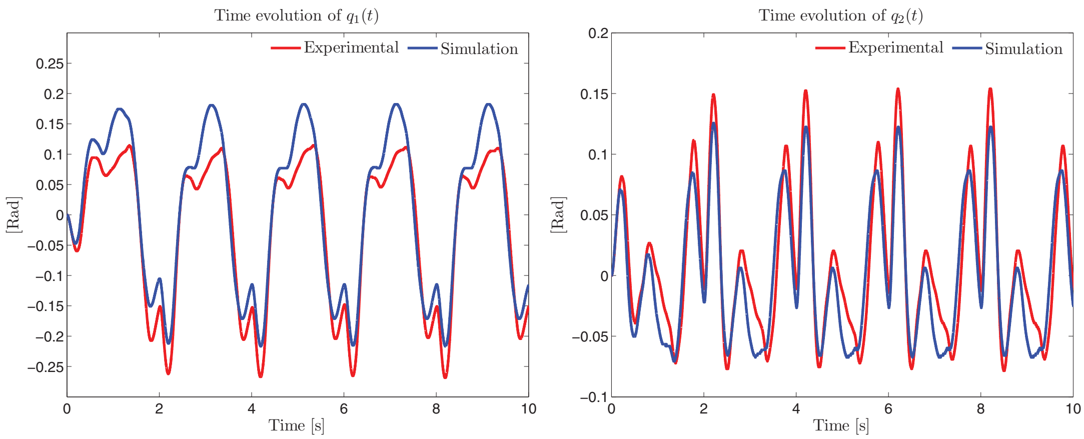

Fig. 4. Time evolution of the arm position $q_1(t)$ and of the pendulum position $q_2(t)$ for both numerical simulation and real–time experiment.

- Off–line: Use central differentiation algorithm (4) to estimate the joint velocity.
- Off–line: With the captured input $u(k)$, joint position $q(k)$ the estimated velocity $\dot{q}(t)$, implement in discrete form the equations (18), (19) and (20).
- Use the classical least–square algorithm in (2) by using $u(k)$ in (22) and $Y(k)$ in (23).

The used identification procedure has the advantage that joint velocity and acceleration measurements are not required since the identification procedure is achieved off–line.

An identification experiment was successfully carried out, which was compared with respect to numerical simulations. The comparison between numerical simulation and experiment is a form of validating the estimated parameters.

Further research consists in performing a comparison with respect to some on–line identification method.

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