Measurement of the 3-D Shape of Specular Polyhedrons Using an M-Array Coded Light Source

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***Abstract*—We reconstruct the shape of a specular object from the image of an M-array coded light source observed after reflection by the object. Using the "window property" of the M-array, a correspondence is found between a point on the image of the reflected light source and a point on the light source. By assuming that the local surface of the object is a plane, surface normal vectors are determined from the correspondence, and the three-dimensional shape of the object is reconstructed. This method enables us to measure the shape of a specular polyhedron using simple equipment. The method was tested for a polyhedron with side length of 10 cm, set 50 cm from a camera. The shape of the polyhedron was reconstructed correctly. The measurement precision was about 1 mm.**

I. Introduction

Determination of the three-dimensional shape of objects is one of the most important problems in measurement. Measurement of the shape of specular objects such as metallic parts has many applications in industry. However, their shape could not be measured with ease by the methods which are applied to diffusing objects [1], [2], because surfaces of specular objects hardly diffuse incident light. A number of methods have been proposed to measure the shape of specular objects [3], [4]. However, many of them require special equipment to illuminate the surface, and need to measure the amount of reflected light accurately in order to determine the orientation of the surfaces of an object.

In this paper, we propose a method for measuring the three-dimensional shape of a specular polyhedron by using an M-array coded light source. The shape of a specular object is reconstructed from the image of the coded light source observed after being reflected by the object. This method requires only simple equipment, a coded light source, and a CCD camera.

An M-array [5] is a two-dimensional pseudo-random pattern constructed from the M-sequence (maximum-length sequence). Structured light coded by an M-sequence or an M-array has been used to measure the shape of objects with diffuse surfaces [6]–[8]. Here, we use an M-array coded light source for illuminating a specular object.

The method was tested for a polyhedron with side length of 10 cm, set 50 cm from a camera. The principle of the method and the results of the experiment together with the estimation of precision are described.

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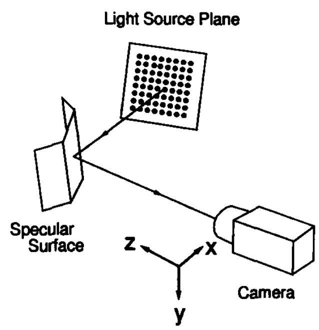

Fig. 1. System configuration of measurement. An image of an M-array coded light source reflected on a specular surface is acquired by a CCD camera.

II. Principle

Fig. 1 illustrates the system configuration of measurement [9]. An image of a light source reflected on a specular surface is acquired by a CCD camera. We assume that the positions of the light-source plane and the camera are exactly known. The light-source plane is coded according to the M-array.

As the first step of measurement, a point of the light source corresponding to each point on the image of the reflected light source is determined using the property of the M-array. As the next step, surface-normal vectors are determined from the obtained correspondence. The three-dimensional shape of the object is then reconstructed using the normal vectors.

A. M-Array Coded Light Source

An M-sequence a_i of degree n is a binary sequence of length $N = 2^n - 1$ specified by a primitive polynomial.

We take a number $N = 2^{k_1 k_2} - 1$ such that $N_1 = 2^{k_1} - 1$ and $N_2 = N/N_1$ are relatively prime and greater than 1. Then the M-array $b(j, k)$ of size $N_1 \times N_2$ is obtained by

$$b(i \bmod N_1, i \bmod N_2) = a_i, \quad (i = 1, \cdots, N) \qquad (1)$$

where a_i is an M-array sequence. This means that the M-sequence fills an $N_1 \times N_2$ array by writing a_i down the main diagonal and continuing from the opposite side whenever an edge is reached (Fig. 2). We encode the light source plane with circles assigned by gray and black according to the M-array. Fig. 3 is the light source, M-array of 15 $\times$ 17 used, which is constructed from an M-sequence of degree 8.

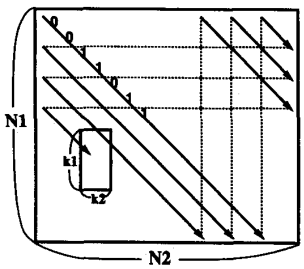

Fig. 2. Construction of an M-array from an M-sequence.

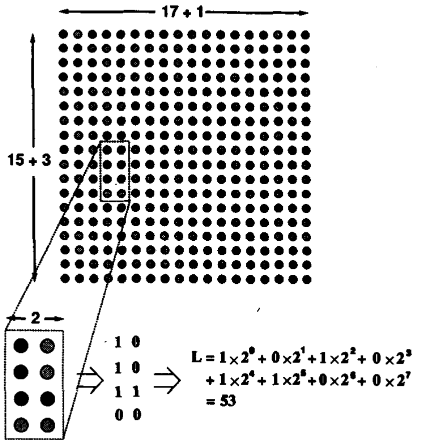

Fig. 3. An M-array coded light source and the window property. Gray and black correspond to "0" and "1" of M-sequence, respectively.

In order to reconstruct the shape of a specular object, we use the "window property" of an M-array: that is, if a prescribed size $(k_1 \times k_2)$ of window is slid over the array, each of the $2^{k_1 k_2} - 1$ possible nonzero arrays is seen through the window once and only once, when the end of the array is considered to be connected to the other side of the array. We construct a light source of size $(N_1 + k_1 - 1) \times (N_2 + k_2 - 1)$ from the M-array of size $N_1 \times N_2$. In the 18 × 18 binary pattern of Fig. 3, every 4 × 2 pattern appears once.

B. Determination of Correspondence Using M-Array

Specular objects in an acquired image reflect the patterns on the light source plane. If a prescribed size $(k_1 \times k_2)$ of pattern is seen in the image, we can determine the position on the light-source plane where the detected pattern comes from. Using the window property we can determine all the correspondences over the image using the following procedures.

1) Brightness of an image varies with the orientation of the reflecting surface. The acquired image must be normalized to correctly recognize the light source pattern. We take two images for one object: one is the image $I_1(x,y)$ of coded light source, and the other is the image $I_1'(x,y)$ of uniform light source. A normalized image $I_2(x,y)$ is obtained by
$$I_2(x,y) = I_1(x,y)/I_1'(x,y). \tag{2}$$

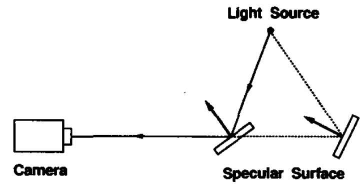

Fig. 4. Uncertainty of the position of a specular surface. The surface could not be determined by the direction of the reflected ray and the corresponding point on the light source.

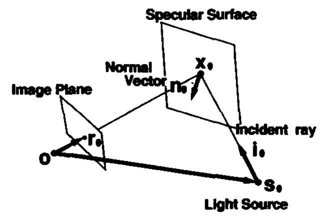

Fig. 5. Geometry of vectors.

2) The normalized image is quantized into three levels using two thresholds. The black level means "1," and the gray level "0," respectively. The white level shows the background. We determine the center of a pattern as the "center of gravity" of the points belonging to the pattern.
3) The point on the light-source plane is found where the pattern matches the "window" pattern. The matching is attained efficiently by using a table-lookup procedure.

C. Reconstruction of 3-D Shape

As described above, we have determined the direction of the reflected ray and the corresponding point of the light source. However, they are not enough to determine the three-dimensional shape of the object, because, as shown in Fig. 4, the surface may take any point on the reflected ray. Thus we introduce another constraint to determine the normal vectors.

We assume that the object is a polyhedron so that the local surface of the object is a plane. Fig. 5 illustrates the geometry of vectors we define, where $\boldsymbol{o}$ is the center of the camera, $\boldsymbol{x}_0$ the reflecting point on the specular surface, $\boldsymbol{r}_0$ and $\boldsymbol{s}_0$ the corresponding point on the image plane and the light-source plane, respectively, and $\boldsymbol{i}_0$ a unit vector of the incident ray. The vector $\boldsymbol{r}_0$ indicates the direction of the reflected ray. As the normal vector $\boldsymbol{n}_0$ at point $\boldsymbol{x}_0$ lies on the plane spanned by $\boldsymbol{r}_0$ and $\boldsymbol{s}_0$, an equation
$$(\boldsymbol{r}_0 \times \boldsymbol{s}_0, \boldsymbol{n}_0) = 0 \tag{3}$$
holds, where (,) and × mean inner and outer products of vectors, respectively. If we take another point $\boldsymbol{x}_1$ on the surface close to $\boldsymbol{x}_0$, we obtain another equation
$$(\boldsymbol{r}_1 \times \boldsymbol{s}_1, \boldsymbol{n}_1) = 0 \tag{4}$$
where $\boldsymbol{r}_1$ and $\boldsymbol{s}_1$ are the point on the image and the point on the light source, respectively. If the points $\boldsymbol{x}_0$ and $\boldsymbol{x}_1$ lie on

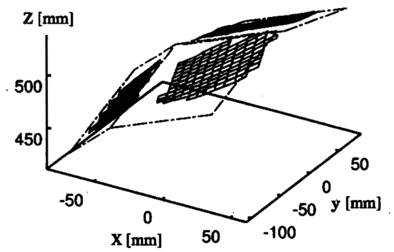

Fig. 6. Result of 3-D shape reconstruction of a polyhedron. Calculated points on the surface are connected by lines. The dashed lines show the regression planes.

the same plane, the normal vectors $\boldsymbol{n}_0$ and $\boldsymbol{n}_1$ have the same direction: that is

$$\boldsymbol{n}_0 = \boldsymbol{n}_1. \tag{5}$$

Thus $\boldsymbol{n}_0$ is given by

$$\boldsymbol{n}_0 = \frac{(\boldsymbol{r}_0 \times \boldsymbol{s}_0) \times (\boldsymbol{r}_1 \times \boldsymbol{s}_1)}{|(\boldsymbol{r}_0 \times \boldsymbol{s}_0) \times (\boldsymbol{r}_1 \times \boldsymbol{s}_1)|}. \tag{6}$$

In practice we determine the normal vector which minimizes the value of

$$\sum_{i=0}^{8} (\boldsymbol{r}_i \times \boldsymbol{s}_i, \boldsymbol{n}_0)^2 \tag{7}$$

to reduce the effect of measurement error, where $\boldsymbol{s}_i$ $(i = 1, \cdots, 8)$ are the eight nearest points around $\boldsymbol{s}_0$ on the light-source plane and $\boldsymbol{r}_i$ $(i = 1, \cdots, 8)$ the corresponding points on the image plane.

After the direction of $\boldsymbol{n}_0$ is determined, the position on the specular surface $\boldsymbol{x}_0$ can be determined by using the condition that the normal vector $\boldsymbol{n}_0$ bisect the angle between the incident and the reflected rays. The vector of incident ray $\boldsymbol{i}_0$ is expressed by

$$\boldsymbol{i}_0 = \frac{2(\boldsymbol{r}_0, \boldsymbol{n}_0)\boldsymbol{n}_0 - \boldsymbol{r}_0}{|\boldsymbol{r}_0|}. \tag{8}$$

The point on the specular surface is written in two ways

$$\boldsymbol{x}_0 = t\,\boldsymbol{r}_0 \tag{9}$$

and

$$\boldsymbol{x}_0 = u\,\boldsymbol{i}_0 + \boldsymbol{s}_0 \tag{10}$$

where t and u are real-valued parameters. From (9) and (10), we obtain the equation

$$t\,\boldsymbol{r}_0 \times \boldsymbol{i}_0 = \boldsymbol{s}_0 \times \boldsymbol{i}_0. \tag{11}$$

Thus t is written as

$$t = \frac{(\boldsymbol{s}_0 \times \boldsymbol{i}_0, \boldsymbol{r}_0 \times \boldsymbol{i}_0)}{(\boldsymbol{r}_0 \times \boldsymbol{i}_0, \boldsymbol{r}_0 \times \boldsymbol{i}_0)}. \tag{12}$$

From (8), (9), and (12), we can determine point $\boldsymbol{x}_0$ on the specular surface.

TABLE I
RESIDUAL FROM A REGRESSION PLANE

	Mean Residual (mm)	Maximum Residual (mm)
1	0.65	1.72
2	0.62	1.68
3	0.43	1.03
4	0.65	1.68
5	0.64	1.34
6	0.27	0.57

III. Experimental Results and Discussion

A specular polyhedron composed of three specular planes was used for the experiment. Each side of the object is about 10 cm long, and the distance from the object to the camera was about 50 cm, and that to the light-source plane about 15 cm. Images were acquired by a high-resolution CCD camera which provides a digitized image of 1024 × 1024 pixels.

The result of reconstruction is shown in Fig. 6. Calculated points on a surface are connected by lines. The dashed lines show the regression planes obtained from the calculated points. It is seen that each plane of the polyhedron is reconstructed correctly.

In order to estimate the precision of this measurement, one specular plane was measured six times with varying orientation. Table I shows the residual from the regression plane. The mean residual is less than 1 mm.

IV. Conclusions

We have described a method for measuring the three-dimensional shape of a specular polyhedron by using an M-array coded light source. The shape of objects which consist of three specular planes with side lengths of about 10 cm and set at about 50 cm apart from a CCD camera was reconstructed correctly. The measurement precision was estimated to be about 1 mm. The reconstruction algorithm is based on the assumption that the surface of an object is locally a plane. This algorithm, therefore, may cause error if it is applied to a surface with a large curvature. Extension of the algorithm to reconstruction of specular objects with curved surfaces is the subject of a future study.

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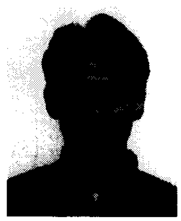

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Sadao Fujimura (M'80), for a photograph and biography, please see p. 627 of this issue of this TRANSACTIONS.