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Summary Sheet

Dragons and Ecosystem: $1 + n$ Meta-population Model

Abstract

Dragons are the virtual creatures in the fictional television series *Game of Thrones*. It is interesting and meaningful to consider what would happen if the three dragons were living today. The purpose of this problem is to simulate the competition game when new species are introduced into the ecosystem. And the result of the simulation can be used for reference in the realistic situations.

Our paper propose the $1 + n$ Meta-population Model, where 1 refers to the Growth Model Based on Energy Flow focus on the dragon's growth and n refers to the Improved n-population Meta-populaiton Model focus on the ecosystem.

We first form the **Growth Model Based on Energy Flow** to represent the growth process of dragons in the food chain. We calculate the energy expenditure using the Kleiber's Law and find the relationship between the energy intake and dragon's mass based on the energy flow in the food chain. Referencing the animals similar to dragons, we set the upper bound of dragon's mass as $1.17 \times 10^3 kg$ and time-varying growth rates.

Then **Improved n-population Meta-populaiton Model** is generated to quantatively estimate the dragons' influence on the populations in the ecosystem. Based on the Tilman n-population Meta-population Model, we form the initial ecosystem including n kinds of populations and then introduce dragons into it. We regard dragons as the interference in the ecosystem. According to the amount and the abundance of living beings in the ecosystem, we can analyze the influence of dragons. Besides, we further take human beings' reconstruction of environment into consideration, offering guidance for human beings.

After that, we apply our models to the different kinds of environments including temperate deciduous forest, desert and ice sheet. By calculating, we know that an adult dragon per should take in $2.4 \times 10^{11} J$ energy per month and the region must be larger than $1.8 \times 10^7 m^2$ to support three dragons.

We then analyze the ecological response when dragons are introduced in the ecosystem, which conforms to strong population extinction mechanism and Odd-even Symmetry Law.

In addition, we conduct sensitivity analysis about the area of the habitat and the time-lag.

Finally, we discuss strengths and weaknesses of our models.

Keywords: Energy Flow; n-population Meta-population Model; Growth Model; Habitat-occupancy Proportion; Ecosystem influence

Dragons and Ecosystem: $1 + n$ Meta-population Model

January 29, 2019

Contents

1 Introduction

In the fictional television series *Game of Thrones*, based on the epic fantasy novel series *A Song of Ice and Fire*, three dragons are raised by Daenerys Targaryen, the "Mother of Dragons." As warm-blooded animals, the dragons' behaviors should follow the basic law of biology. Besides, the dragons also have special characteristics such as the ability to breathe fire, longevity and so on, making them capable to greatly influence the ecosystem.

It is interesting and meaningful to consider what would happen if the three dragons were living today. Based on the reasonable physical and biological assumptions of dragons' characteristics, we do research on the requirements and ecological impacts of the dragons. Not only can it offer guidance about how to maintain the ecological settings of the story, but it also provides insights into the game in realistic ecology.

The game of ecology, as one of the most important problems in ecology, has been drawing researchers' attention for a long time. Verhulst et. al [1] proposed Logistic Growth Model to simulate the population growth in an asset controlling system. Lotka and Volterra [2] presented the predator–prey equations to study the predator–prey competition. Hanski et. al [3] proposed Two-population Meta-population Model. However, none of the above works take environmental factors and other populations into account at the same time. Moreover, no prior works have ever introduced dragons, or any kind of virtual creature, into the ecosystem.

In our work, we propose the $1 + n$ Meta-population Model combined by two models to simulate the growth of dragons and their influences on the ecosystem.

The remainder of this article is organized as follows. In Section 2, we put forward the assumptions and symbols used in this paper. Section 3 shows the Growth Model Based on Energy Flow. Section 4 presents Improved n-population Meta-population Model. In Section 5, we apply the models to the different kinds of environment. In Section 6, we present the results of the simulation and make analysis on them. In Section 7, we discuss the strengths and weaknesses of our models. And finally, we conclude the paper in Section 8.

2 Assumptions and Symbols

2.1 Assumptions

First and foremost, we make some basic assumptions and explain their rationales.

Assumption. 1 *The dragon's biological structure and behaviours are similar to the warm-blooded animals.*

This assumption is the prerequisite of our model. With this assumption, we can analyze dragons' characteristics based on the existing knowledge.

Assumption. 2 *During the time scale we discuss in this paper, the earth's ecological environment keeps stable.*

In our model, we don't consider the incidents that may cause great climate changes such as meteorite impacts or frequent volcanoes, since their probabilities are small.

Assumption. 3 *The existing communities are climax communities and have reached the homeostasis.*

The existing communities on the earth, going through a long-time succession, have adapted to the local climate and reached a stable state.

Assumption. 4 *During the time scale we discuss in this paper, a dragon's body function won't change as the dragon grows old.*

Dragons live a long life.

Assumption. 5 *Dragons are at the top of the food chain.*

Considering the power of dragons, the dragons can have influence on all living beings in the ecosystem while few living beings can compete with the dragon.

Assumption. 6 *A dragon's body function won't change when it is hurt.*

Dragons can resist tremendous trauma.

Assumption. 7 *There is a upper bound for dragon's body size.*

With the limitation of gravity and biological structure, the body size of a dragon cannot infinitely increase.

2.2 Symbols

The symbols we define in this paper are shown in the Table. 1.

Table 1: Symbols mentioned in this paper

Symbol	Definition	Unit
E_{expend}	Energy expenditure	J
E_{in}	Energy intake	J
E_{fire}	Energy used in breathing fire	J
$E_{assimilate}$	Energy the animal assimilates	J
AR	Assimilation rate	−
NAR	Net assimilation rate	−
M	Mass of a dragon	kg
q	Metabolic rate	$J \cdot kg^{-1} \cdot h^{-1}$
σ	Bone strength	$N \cdot m^{-2}$
NPP	Net primary productivity	$J \cdot m^{-2}$
s	Minimal area to support a dragon's living	m^2
S	A dragon's living scope	m^2
p_i	Habitat-occupancy proportion of the population i	−
c_i	Migration rate of the population i	−
d_i	Mean death rate of the population i	$month^{-1}$
η	Reduction ratio of the habitat-occupancy	−
ξ	Reduction ratio of the population types	−
ϵ	Time-lag	$month$
μ	Human beings' impact factor	−

3 Growth Model Based on Energy Flow

In this section, we present the Growth Model Based on Energy Flow to simulate the growing process of a dragon, covering both energy expenditure and energy intake tasks. The framework of the model is shown in Figure. 1. At first, we calculate the dragon's energy expenditure using Kleiber's Law, which considers two motion states and the fire-breathing process. Then, we analyze the assimilation process in the food chain where energy is translated from primary producers to senior consumers. Finally, we establish the relationship between the amount of the absorbed energy and the dragon's mass. Also, this model further discusses the upper bound of the dragon's body size with some physical limitations.

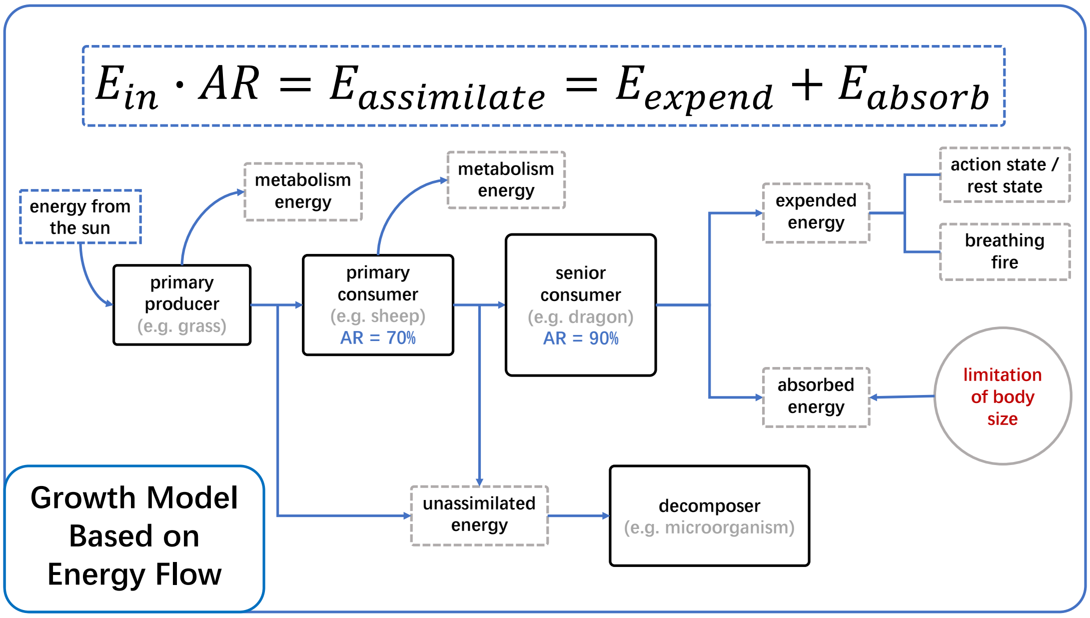

Figure 1: Framework of the Growth Model Based on Energy Flow

3.1 Energy Expenditure

A dragon's energy expenditure E_{expend} is mainly affected by two factors: its growth condition and its motion state. The former can be characterized by the dragon's mass M, while the latter is classified into rest state and action state. Besides, the ability to breathe fire is one of the biggest differences between dragons and ordinary animals. Without any fire-breathing creatures for reference, we use a physical model to calculate the energy use E_{fire} in fire puffing.

3.1.1 Kleiber's Law

Because the metabolic rate q is changing as the dragon grows, the energy expenditure varies during the growth processes. In this paper, we firstly calculate the metabolic rate q in order to get the energy expenditure.

In nature, for very small organisms, such as algae, bacteria, and protozoa, their metabolic rate per gram of biomass is much higher than that of large organisms, such as trees and vertebrates. This phenomenon shows that the metabolic rate negatively relates with the animal's mass, which quantitatively presents the growth degree.

As the pioneer of researchers on biology work, Max Kleiber (1947) [4] observed and summarized that, for the vast majority of animals, an animal's metabolic rate scales to the $\frac{3}{4}$ power of the animal's mass. This is named as Kleiber's law and can be written as

$$q = q_0 M^b \tag{1}$$

where q is the metabolic rate, q_0 is the characteristic constant, $b = \frac{3}{4}$ is the proportional coefficient.

3.1.2 Action and Rest

Animals will adapt their metabolic rates q to different motion states, so will the dragons do. In Equation (1), the adaption is reflected in the characteristic constant q_0. The motion state of a dragon can be divided into two types: rest state and action state. The rest state includes sleeping and relaxing while the action state includes flying, preying and so on. Dragons consume much more energies in the action state than in the rest state.

By this classification, we calculate the characteristic constant q_{0r} for the rest state and q_{0a} for the action state. According to the Assumption. 1, dragons' motion mechanism is similar to birds and bats, so we use the metabolic rate of different kinds of birds and bats to fit the q_0. Parts of the data are present in the Table. 2, whose third and forth columns are the metabolic rate per gram $\hat{q}$ [5], and the complete data are present in the Appendix A:

$$\hat{q} = \frac{q}{M} = q_0 M^{b-1}. \tag{2}$$

$\hat{q}$ represents the metabolic rate of a unit mass of the dragon. Equation (2) can be derived to:

$$\ln \hat{q} = (b-1)\ln M + \ln q_0, \tag{3}$$

revealing the linear relation between $\ln \hat{q}$ and $\ln M$. We use linear fitting to calculate the q_0.

Table 2: Metabolic rate in rest state and action state

Flying animals	Body weight (g)	Metabolic rate ($\mathrm{cal}\cdot g^{-1}\cdot hr^{-1}$)	
		Rest	Action
Budgerigar	35	15.8	105
Gull	300	7.3	54
pigeon	448	7.2	58
Gyrfalcon	2057	4.9	36

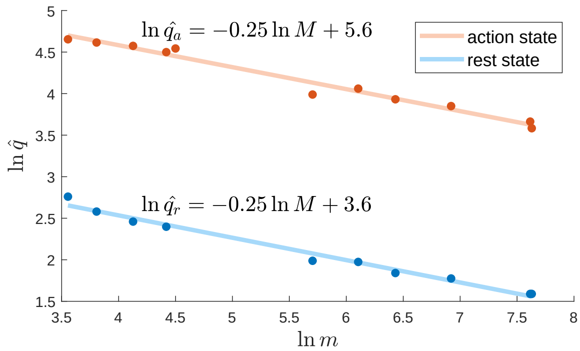

Figure 2: Linear fitting to calculate q_{0r} and q_{0a}

As shown in the Figure. 2, the relationships between q_r, q_a and $\ln M$ are:

$$\ln \hat{q}_r = -0.25 \ln M + 3.6,$$

$$\ln \hat{q}_a = -0.25 \ln M + 5.6,$$

so the characteristic constant or the rest state $q_{0r} = 36.6 cal \cdot hr^{-1} \cdot g^{-\frac{3}{4}}$ and the characteristic constant for the action state $q_{0a} = 270.4 cal \cdot hr^{-1} \cdot g^{-\frac{3}{4}}$, so using the Equation (1), the energy expenditure E_{expend} can be expressed as:

$$\frac{\mathrm{d}E_{expend}}{\mathrm{d}t} = \alpha_r q_{0r} M^{\frac{3}{4}} + \alpha_a q_{0a} M^{\frac{3}{4}}, \tag{4}$$

where α_r and α_a are the coeffcient of proportionality indicating the proportion of the rest state and action state in the total time. It's easy to get that:

$$\alpha_r + \alpha_a = 1.$$

3.1.3 Mechanism and Energy Consumption of Fire-breathing

There has never been a creature that can breathe fire like the dragon. In order to simulate the process of dragon's fire-breathing, we have made a reasonable model based on reality rather than merely imagination. The dragon has a gland in its mouth, which can produce flammable mixed liquid and spray it out. Like electric eels, the dragon has muscle tissue providing a voltage, which is high enough to ignite the flammable mixed liquid. And through this way, the dragon can breathe fire as it likes.

The fire-breathing process is too complicated to calculate energy consumption in biology. So in the later calculation, we simplify the process and give a estimation of the energy it costs. From *A Song of Ice and Fire*, we learn that the dragon prefers grilled meat. In our model, the dragon breathes fire mostly to roast food. By estimation and calculation, we derived the energy to roast food E_{fire} is:

$$E_{fire} = 6.69 \times 10^6 \times m,$$

where m is the mass of the food the dargon eats.

3.2 Energy Intake

We take the dragon's caloric intake into consideration of the food chain and the food web. All living things are in the food chain, where most organisms consume or are consumed by more than one type of organisms. Based on the relationship among each nutrient class in the food web, there are four nutrient classes: primary producers (chlorophytes), primary consumers (herbivores), senior consumers (carnivores) and decomposers (microorganisms). Dragons, as carnivores, are classified into senior consumer [6].

There are great energy losses during the transfer in the food webs. As the energy is transferred to the higher nutrient classes, the amount decreases. Fortunately, the quality of the energy increases at the same time, leading to the higher conversion efficiency.

We define the assimilation rate AR to quantitatively describe the conversion efficiency. AR can be expressed as

$$AR = \frac{E_{assimilate}}{E_{in}},$$

where E_{in} is the energy gained by ingestion, $E_{assimilate}$ is the energy that the animal assimilates. Table.3 shows the approximate percentage values of the biological production, respiration and the assimilation of different nutrient classes. We can use respiration to calculate net assimilation rate.

Table 3: Biological production, respiration and assimilation rate of each nutrient class

Nutrient class	Biological production($\%$)	Respiration($\%$)	Assimilation rate($\%$)
Primary producer	$60 \sim 70$	$30 \sim 40$	$60 \sim 70$
Primary consumer	$40 \sim 50$	$50 \sim 60$	$70 \sim 80$
Senior consumer	$5 \sim 10$	$90 \sim 95$	$90 \sim 95$

In this paper, we take the AR of dragons as 90% to calculate the caloric intake of a dragon.

3.3 Absorption

The energy a dragon assimilates is used in three aspects: absorption, motion expenditure and breathing fire. This process can be expressed as:

$$E_{assimilate} = E_{absorb} + E_{expend} + E_{fire}. \tag{5}$$

By estimation and calculation, E_{in} is:

$$E_{in} = 2 \times 10^7 \times m$$

$E_{assimilate}$ is:

$$\begin{aligned} E_{assimilate} - E_{fire} &= E_{in} \times AR - E_{fire} \\ &= E_{in} \times (AR - \frac{E_{fire}}{E_{in}}) \\ &\overset{def}{=} E_{in} \times K \end{aligned} \tag{6}$$

Using Equations. (4), (5) and (6), we can get Equation (7):

$$\frac{\mathrm{d}E_{in}}{\mathrm{d}t} \times K = \frac{\mathrm{d}E_{absorb}}{\mathrm{d}t} + (\alpha_r q_{0r} + \alpha_a q_{0a}) \cdot M_t^b \tag{7}$$

The dragon's absorption energy E_{absorb} is invested into growth, which is measured by the mass of the dragon in this paper. We define the dragon's mass at time point t by M_t, so the mass addition can be expressed as:

$$\lambda \frac{\mathrm{d}M}{\mathrm{d}t} = \frac{\mathrm{d}E_{absorb}}{\mathrm{d}t} = \frac{\mathrm{d}E_{in}}{\mathrm{d}t} \times K - (\alpha_r q_{0r} + \alpha_a q_{0a}) \cdot M_t^b \tag{8}$$

3.4 Body Size

3.4.1 Limit on Body Size

Due to the current global environment, the dragon's body size has certain limitations. In other words, the dragon cannot grow indefinitely.

Creatures living on Earth are affected by gravity. Bones play a supporting role in the biological structure to fight against gravity. Table. 4 shows compressive strength of several kinds of bones. By these data, we can estimate the bone strength of the dragon.

Table 4: Compressive strength of several kinds of bones

Kind of bones	Thigh bone	Shin bone	Shoulder bone
Compressive strength (N$\cdot m^{-2}$)	8.8×10^7	7.6×10^7	7.1×10^7

We think that the dragon is a powerful creature, and its bone strength is slightly higher than the existing creature. In this paper, we define the bone strength of dragon σ as:

$$\sigma = 1.2 \times 10^8 N \cdot m^{-2}$$

The normal stress that the dragon's bones can withstand F can be expressed as:

$$F = \sigma \cdot A. \tag{9}$$

The gravity of the dragon G can be expressed as:

$$G = \rho V g = Mg, \tag{10}$$

where $g = 9.8N \cdot kg^{-1}$, the density of the dragon $\rho = 1 \times 10^3 kg \cdot m^{-2}$.

In order to simplify the model, we consider the cross-sectional area of the bone and the biological volume have the following relationship:

$$\sqrt[3]{V} = 10\sqrt[2]{A} \tag{11}$$

The bones should be strong enough for the dragon to do daily activities:

$$F \geq 25G \tag{12}$$

Using Equations. (9), (10), (11), (12),we can get Equation. (13):

$$M \leq (\frac{\sigma}{2500\rho g})^3 \cdot \rho = 1.17 \times 10^5 kg \tag{13}$$

3.4.2 Mass Growth Function

The growth rates of living beings in nature aren't constant in different states of life. Von Bertalanffy [7] proposed the VB Equation to analyze the change of growth rate. It is reasonable to assume that dragons' growth also satisfies the VB Equation, so the relationship between the dragon's mass M and time t can be expressed as

$$\frac{\mathrm{d}M}{\mathrm{d}t} = 3kM^{2/3}(M_\infty^{1/3} - M^{1/3}), \tag{14}$$

where the unit of the time is $month$.

Integrating the Equation (14), we can get the equation 15:

$$M(t) = [(1 - e^{-kt})M_\infty^{1/3} + e^{-kt}M_0^{1/3}]^3. \tag{15}$$

The dragons are $10kg$ when hatched and are $30 \sim 40kg$ after a year grow. Using Equation (13), we can get

$$M(0) = 10kg, M(12) = 35kg, M(\infty) = 1.2 \times 10^5 kg.$$

So the k can be calculated:

$$k = 0.0020.$$

The growth process of the dragon is shown in the Figure. 3. The line means the relationship between the dragon's mass M and time t; the deeper is the color, the larger is the growth rate. So we can know that the dragon grows most fast aged between 30 and 80 years old.

4 Improved n-population Meta-population Model

In this section, the dragons are introduced into the ecosystem. We propose an Improved n-population Meta-population Model to quantitively estimate the dragon's influence on other populations in a given region.

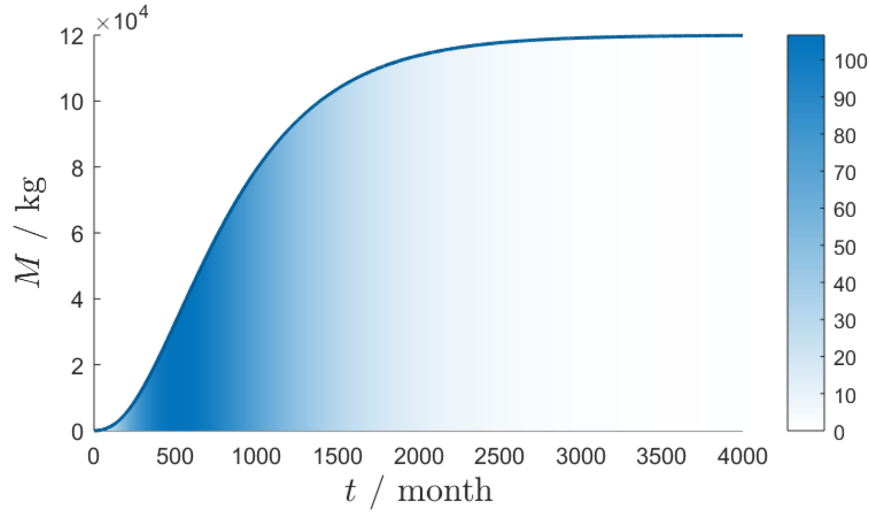

Figure 3: The relationship between the dragon's mass M and time t

4.1 Minimal Area of the Habitat

The area required to support the three dragons depends on the energy it can provide. The energy comes from solar energy which is transformed to chemical energy by the green plants during the photosynthesis. The primary productivity PP of a ecosystem denotes the rate of the solar energy and the chemical energy that a producer transforms, which is proportional to the assimilation rate AR of the producer. And net primary productivity NPP is the primary productivity taking off the energy consumption in respiratory. We use net primary productivity to estimate the sum of the primary productivity, which is proportional to the net assimilation rate NAR of the producer. Table. 5 shows the net primary productivity per unit area $\widehat{NPP}$ in different kinds of ecosystem [8].

Table 5: Net primary productivity per unit area in ecosystems

Type of ecosystem	Net primary productivity per unit area ($g \cdot m^{-2} \cdot a^{-1}$)		
	Area($10^6 km^2$)	Range	Average
Temperate deciduous forest	7.0	600∼2500	1200
Desert	18.0	10∼250	90
Ice sheet	24.0	0∼10	3

Note that in Table. 5, the primary productivity is measured by the mass of consumer. For the primary consumers, there is a linear relationship between the mass and the energy. The scale coefficient $= 1.8 \times 10^7 J/kg$.

Table. 3 presents the respiration and assimilation rate of each nutrient class, so we can calculate the net assimilation rate NAR:

$$NAR = (1 - Respiration) \cdot AR,$$

so the net assimilation rate of primary consumer $NAR_{pc} \approx 20\%$.

The ingestion energy E_{in} in the Equation (8) comes from the energy from the primary consumers, which can be expressed as:

$$E_{in} = \widehat{NPP} \cdot s \cdot NAR_{pc}, \tag{16}$$

where s is the minimal area to support the three dragons' living.

4.2 Tilman n-population Meta-population Model

We form a model to simulate ecosystem where the dragon will live. We characterize the initial ecosystem.

The population denotes all the individuals in a given area. However, because of the limitation of the geographical conditions, most of populations live in the fragmented habitats. As the habitats are divided into fragments, the distribution of populations will change from continuous to dispersive. Taking spatial structure and dynamics into consideration, we define a group of spatially separated populations of the same species as meta-population, which interacts at some level [9].

The loss and division of the habitat are the major reasons of the population's extinction. To investigate the ecosystem with n kinds of population, Tilman et al. [10] proposed the n-population Meta-population Model:

$$\frac{\mathrm{d}p_i}{\mathrm{d}t} = c_i p_i (1 - D - \Sigma_{j=1}^{i} p_j) - d_i p_i - \Sigma_{j=1}^{i-1} p_i c_i p_j \tag{17}$$

where $i = 1, 2, \cdots, n$, is the sort according to the population's competitive ability in the community; p_i is the habitat-occupancy proportion of population i; c_i is the migration rate of the population i; d_i is the mean death rate of the population i; D is the ratio of destroyed habitat to total habitat.

Equation (17) reveals that coexistence of n kinds of populations is the result of the homeostasis in competitive ability, migration ability and mortality of different kinds of populations.

The model sorts the population based on the initial habitat-occupancy proportion. In other words, the population occupying the biggest area of the habitat is defined as the strongest term, which can compete with any weak population. When the area of habitat decreases, the strong population will invade the weak population. So the weak population is supposed to have a higher migration rate to survive, which is shown in the last part of Equation (17) $\Sigma_{j=1}^{i-1} p_i c_i p_j$.

Besides, this model assumes the death rate of each population is equal and both of the initial habitat-occupancy proportion p_i^0 and the migration rate c_i are given by the geometric distribution:

$$d_i = d$$
$$p_i|_{D=0} = q(1-q)^{i-1}$$
$$c_i = d_i/(1-q)^{2i-1}$$

4.3 Dragon's Influence on the Ecosystem

Finally the dragon is brought into a given region as introduced species, and we will estimated its influence on the ecosystem.

Once the introduced species appear in the ecosystem, they will decrease the indigenous species' habitat through competition in moisture, nutrients and so on. From mathematical point of view, the introduced species can be deemed as the interference in the ecosystem which has reached the homeostasis. The effect of the introduced species depends on whether the interference can be rapidly magnified.

According to the Assumption. 5, dragons don't have any natural enemies, but have the strongest competitive ability among the community, so the dragon can be regarded as a destructive factor in the ecosystem and the dragon's living scope can be regarded as the destroyed habitat.

The minimal area to support a dragon's living is changing with time, so we set s as $s(t)$. It is reasonable that the scope of a dragon's activity is much larger than $s(t)$. In this paper, we set the dragon's living scope $S(t)$ as:

$$S(t) = 5s(t). \tag{18}$$

So the ratio D of destroyed habitat to total habitat can be expressed as:

$$D_1(t) = \frac{S(t)}{S_0}, \tag{19}$$

where S_0 is the area of the given region. So the Equation (17) can be written as:

$$\frac{\mathrm{d}p_i}{\mathrm{d}t} = c_i p_i(1 - \frac{S(t)}{S_0} - \Sigma_{j=1}^{i} p_j) - d_i p_i - \Sigma_{j=1}^{i-1} p_i c_i p_j \tag{20}$$

4.4 Human being' Restoration to the Ecosystem

The introduction of dragons may cause the destruction to the existing ecosystem, so it is necessary for human beings to help reconstruct the environment. In this part, we quantatively calculate the impact factory μ of human's conservation.

Human beings reconstruct the environment by repairing the habitat that dragons have destroyed. However, human beings cannot completely repair the destruction the dragons make. We set human beings' impact factor as μ. Besides, considering the observation, analysis and action of human beings have certain hysteresis, we set the time difference between dragons' destruction and human beings' reconstruction as ϵ. So the ratio of destroyed habitat to the total habitat D can be updated to Equation (21).

$$D_2(t) = \mu D_1(t - \epsilon) = \mu \frac{S(t - \epsilon)}{S_0} \tag{21}$$

Note that $D_2(t) = 0, t \leq \epsilon$. So Equation (22) can be expressed as:

$$\frac{\mathrm{d}p_i}{\mathrm{d}t} = c_i p_i(1 - \frac{S(t)}{S_0} + \mu \frac{S(t - \epsilon)}{S_0} - \Sigma_{j=1}^{i} p_j) - d_i p_i - \Sigma_{j=1}^{i-1} p_i c_i p_j \tag{22}$$

5 Implementation

Considering the evolution of population and the succession of the community is a long-term process, numerical simulation is one of the most effective methods to simulate this process. In this section, we simulate the model we proposed above in three regions with different kinds of climates.

5.1 Solution Method

The Equation (22) is a first order linear differential equation system, which is hard to work out analytical solutions. To solve this problem, we use Matlab to get numerical solutions. With the help of ode functions in Matlab, we simulated the dragon's influence to the ecosystem under different conditions. The code is present in the Appendix B, C, D, E.

5.2 Standard of Measurement

In this paper, we put forward two measure indices to quantatively estimate dragons' influence: the reduction ratio of the habitat-occupancy and the reduction ratio of the population types.

The reduction ratio of the habitat-occupancy η is calculated by:

$$\eta(t) = \frac{\Sigma_{j=1}^{n} p_j(t)}{\Sigma_{j=1}^{n} p_j(0)},$$

where p_0 is the habitat-occupancy of all populations in the ecosystem initially. This index shows the amount of the survival living beings when the dragons are introduced into the ecosystem, since the habitat-occupancy of all populations is proportional to the amount of living beings.

The reduction ratio of the population types ξ is calculated by:

$$\xi(t) = \frac{n(t)}{n},$$

where n_0 is the number of the population types initially. This index shows the abundance of the survival living beings when the dragons are introduced into the ecosystem.

5.3 Basic Parameter Setting

As with other animals that migrate, dragons might travel to different regions of the world with very different climates. In this paper, we simulate the process of the dragon's growth in the temperate deciduous forest (a warm temperate region), desert (an arid region) and ice sheet (an arctic region). The features of these regions can be expressed as:

1. Temperate deciduous forest

   Temperate deciduous forests are a variety of temperate forest dominated by trees that lose their leaves each year. They are found in areas with warm moist summers and cool winters [11].

2. Desert

   A desert is a barren area of landscape where little precipitation occurs and, consequently, living conditions are hostile for plant and animal life. The lack of vegetation exposes the unprotected surface of the ground to the processes of denudation [12].

3. Ice sheet

   An ice sheet, also known as a continental glacier, is a mass of glacial ice that covers surrounding terrain [13].

Given the features above, we set the parameters in different regions as Table. 6.

Table 6: Parameter settings in different ecosystems

Kind of ecosystems	$\widehat{NPP}$	α_r	i	q	d
Temperate deciduous forest	1.200	0.75	60	0.1	0.02
Desert	0.090	0.6	30	0.18	0.02
Ice sheet	0.003	0.3	15	0.3	0.02

6 Results and Sensitivity Analyses

6.1 Calculation on the Energy and the Area Requirement

By the numerical calculation of the computer, we get the Figure. 4. The left one describes the area required to support dragons expanding with the weight gain of the dragons. The middle one describes the energy intake increasing with the weight gain of the dragons. The left one describes the energy expenditure increasing with the weight gain of the dragons. From the left one, we can see that in different ecosystems the area required has the same trend but different the order of magnitude. The energy intake is slightly larger than the energy expenditure, which means the energy of dragon assimilation is mainly used for metabolic consumption.

By calculating, a young dragon weighting $12.9t$ needs $6.16 \times 10^{10} J$ energy per month and requires at least $1.5 \times 10^6 m^2$ region to support itself while an adult dragon whose mass is $79.2t$ needs $2.4 \times 10^{11} J$ energy per month and requires at least $5.9 \times 10^6 m^2$. To support three dragons, the area of the region must be larger than $1.8 \times 10^7 m^2$, which equals to 2500 football fields.

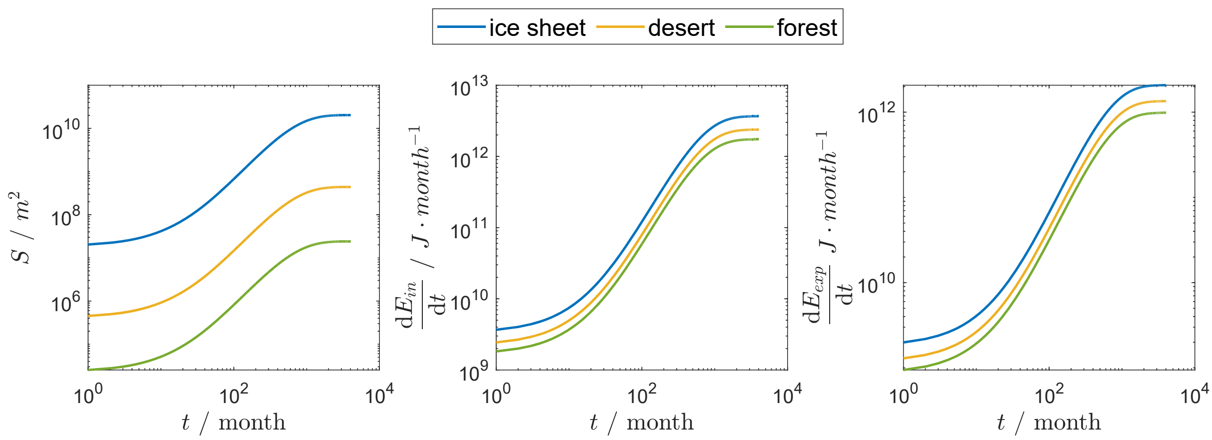

Figure 4: Calculation on the energy and the area required to support dragons

6.2 The Dragons' Influence on the Ecosystem

6.2.1 Influence in Different Ecosystems

We use the indices we set in Section 5 to quantatively estimate the influence on the ecosystem. We present the result in the figures below.

The left part of each figure shows the habitat-occupancy proportion of the population which is changing over time. Each line in the left figure represents a population in the ecosystem. The p_i at $t = 0$ can show the competition ability of the population i initially. In other words, the population with maximum p is the strongest population at first. Once $p_i = 0$, it indicates that the population i has extincted. So using the amount of types of survival populations, we can predict the abundance of the survival living beings at any time.

The right part of each figure shows the reduction ratio of the habitat-occupancy which is changing over time. This index indicts the amount of the survival living beings when dragons are introduced into the ecosystem.

Then we analyze the influence of dragons in different environments with different areas S_0 of the whole habitat, probing into the sensitivity of it.

Temperate deciduous forests

Figures 5, 6, 7 show the influence in the temperate deciduous forests. In this region, dragons need small space to survive. Because of the high abundance of the populations, the ecosystem is stable and hardly influenced by dragons. But it takes a long time for the ecosystem to recover once destroyed. Dominant populations are much more likely to be affected by the dragons than the vulnerable populations. Comparing the three figures, we can know that, as the S_0 gets larger, the populations are more difficult to be extincted by dragons.

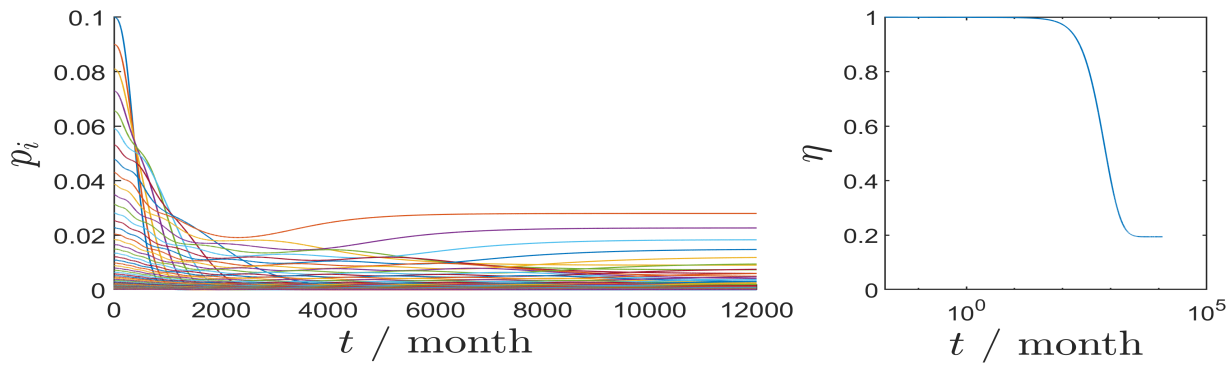

Figure 5: Simulation on the habitat-occupancy proportion of the population and the reduction ratio of the habitat-occupancy in temperate deciduous forest ($S_0 = 3 \times 10^7$). The reduction ratio of the population types $\xi(12000) = 83.33\%$.

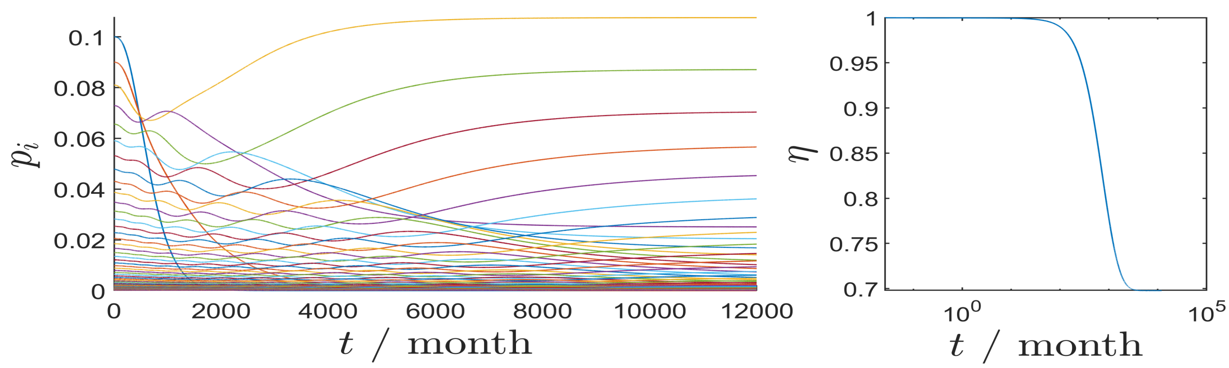

Figure 6: Simulation on the habitat-occupancy proportion of the population and the reduction ratio of the habitat-occupancy in temperate deciduous forest ($S_0 = 8 \times 10^7$). The reduction ratio of the population types $\xi(12000) = 96.67\%$.

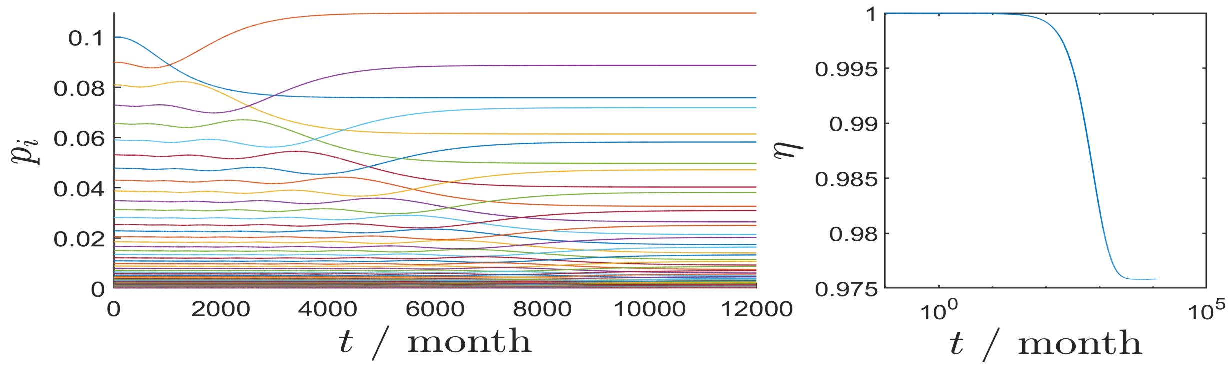

Figure 7: Simulation on the habitat-occupancy proportion of the population and the reduction ratio of the habitat-occupancy in temperate deciduous forest ($S_0 = 1 \times 10^9$). The reduction ratio of the population types $\xi(12000) = 100\%$.

Desert

Figures. 8, 9, 10 show the influence in desert. In this region, the area required is 10 times bigger than that in the temperate deciduous forests. In Figures.8 and 10 , we find that only the top populations are hugely affected by the dragons while the weak population are not seriously affected. In Figure. 9, we find that in the early 2000 years, the status of adjacent populations are interchanged. And as the temperate deciduous forest, the extinction probability of populations is smaller in larger area.

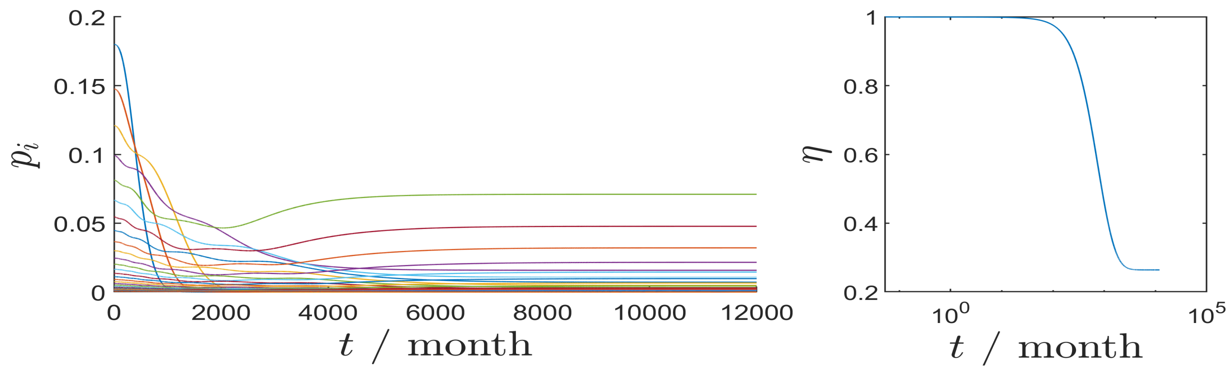

Figure 8: Simulation on the habitat-occupancy proportion of the population and the reduction ratio of the habitat-occupancy in desert ($S_0 = 6 \times 10^8$). The reduction ratio of the population types $\xi(12000) = 86.67\%$.

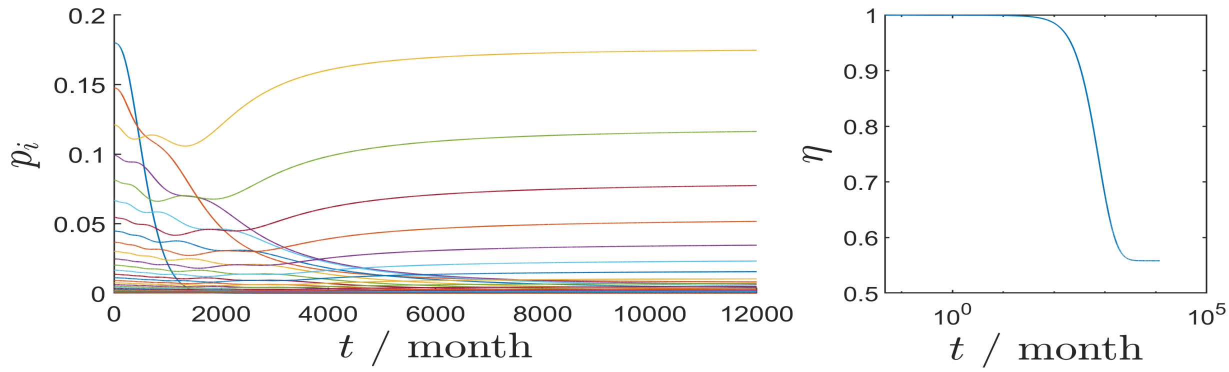

Figure 9: Simulation on the habitat-occupancy proportion of the population and the reduction ratio of the habitat-occupancy in desert ($S_0 = 1 \times 10^9$). The reduction ratio of the population types $\xi(12000) = 93.33\%$.

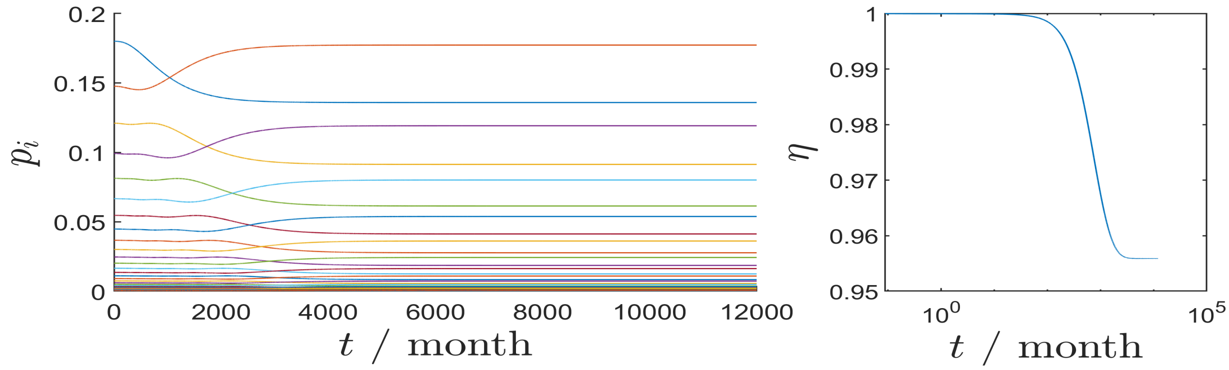

Figure 10: Simulation on the habitat-occupancy proportion of the population and the reduction ratio of the habitat-occupancy in desert ($S_0 = 1 \times 10^{10}$). The reduction ratio of the population types $\xi(12000) = 100\%$.

Ice sheet

Figures. 11, 12, 13 show the influence in ice sheet. In this region, the dragons need largest space to survive among all kinds of regions. The ecosystem is easily to collapse, for example, as shown in Figure. 11, the reduction ratio η of the population types at the $t = 12000 month$ is 0, which means all populations extinct because of the dragons. However, it takes little time for the ecosystem to reach stable state.

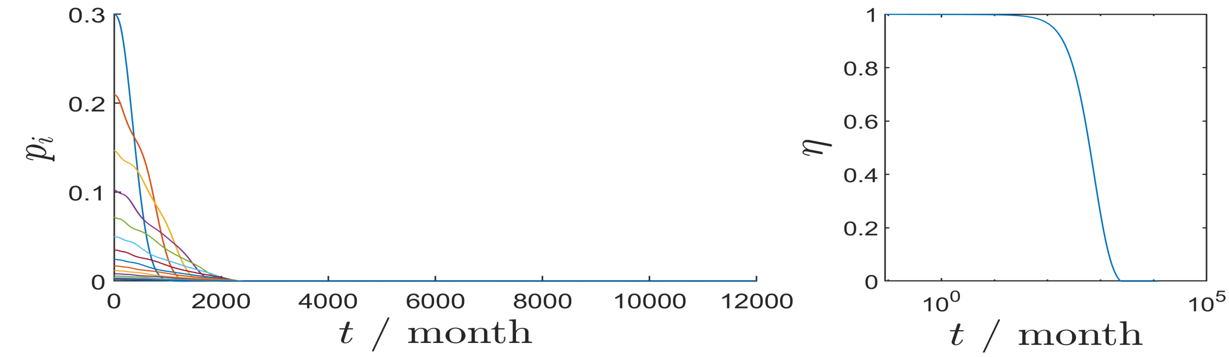

Figure 11: Simulation on the habitat-occupancy proportion of the population and the reduction ratio of the habitat-occupancy in ice sheet ($S_0 = 2 \times 10^{10}$). The reduction ratio of the population types $\xi(12000) = 0\%$.

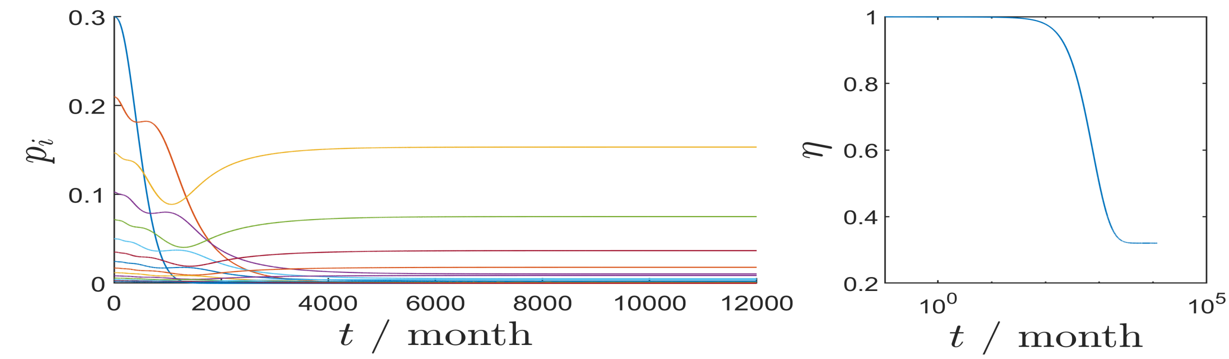

Figure 12: Simulation on the habitat-occupancy proportion of the population and the reduction ratio of the habitat-occupancy in ice sheet ($S_0 = 3 \times 10^{10}$). The reduction ratio of the population types $\xi(12000) = 86.67\%$.

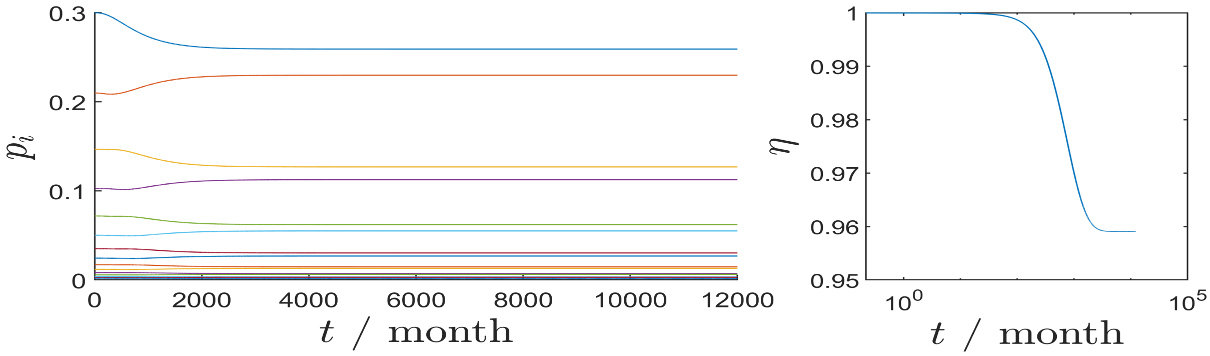

Figure 13: Simulation on the habitat-occupancy proportion of the population and the reduction ratio of the habitat-occupancy in ice sheet ($S_0 = 5 \times 10^{11}$). The reduction ratio of the population types $\xi(12000) = 100\%$.

6.2.2 The Law of Ecological Response

In the n-population meta-population model, different populations have different responses to the dragon. Although the dynamic model is very complicated, there are laws we can summarize.

By further analyzing the phenomenon we observe above, we explore two laws:

1. **Strong population extinction**

   The relaxation time of the ecosystem is very long; the time scale is $10^3 a$. When n-population meta-population model reachs equilibrium, habitat-occupancy proportion of the populations p_i^e satisfies this condition $dp_i/dt = 0$. It means the steady state solution is:

   $$p_i^e = \begin{cases} \hat{p}_i & if \quad \hat{p}_i > 0 \quad [\hat{p}_i = 1 - D_1 - \frac{m_i}{c_i} - \Sigma_{j=1}^{i} p_j^e (1 + \frac{c_j}{c_i})] \\ 0 & if \quad \hat{p}_i \leq 0. \end{cases} \tag{23}$$

   When

   $$D_1 \geq 1 - m_i/c_i, \tag{24}$$

   the top i populations will go extinction. The Equation (24) is the threshold condition of the first kind of extinction.

2. **Odd-even Symmetry Law**

   We analyze the dynamics characteristics in the ecosystem when the dragons are introduced. As shown in the Figure. , after a long-time evolving and developing, the sort of the populations follow the Odd-even Symmetry Law: if the number of the populations extincting is odd, the populations with even number will eventually evolve into strong populations; If the number of the populations extincting is odd, the populations with even number will eventually evolve into strong populations.

6.3 Human Beings' Efforts on Environment Reconstruction

After considering the human beings' effort on environment reconstruction used Equation (22), the populations' changes are shown in the Figures. 14, 15, 16 and 17. Our simulations are made in the temperate deciduous forest, and we will probe into the sensitivity of the parameter ϵ to analyze the impact of the hysteretic time.

Figure. 15 shows the ecosystem without human beings' reconstruction. We set incremental μ in Figures. 15 and 16. As human beings enhance the reconstruction in the environment, the probability for populations grows. We set incremental ϵ in Figures. 16 and 17, which shows the timely reconstruction is more efficient.

We summarize the influence of the human beings' reconstruction:

1. A small amount of human beings' help can save most of the populations, so it is worth to reconstruct the environment .
2. The more the human beings repair the habitat, the better the ecosystem will recover.
3. The more timely the human beings repair the habitat, the faster the ecosystem will recover.
4. Though the ecosystem will be unstable, human beings can still save a number of endangered populations long time after the destruction done by the dragon.

Through the analysis of human beings' reconstruction, there are the specific measures to reconstruct the environment which is influenced by dragons.

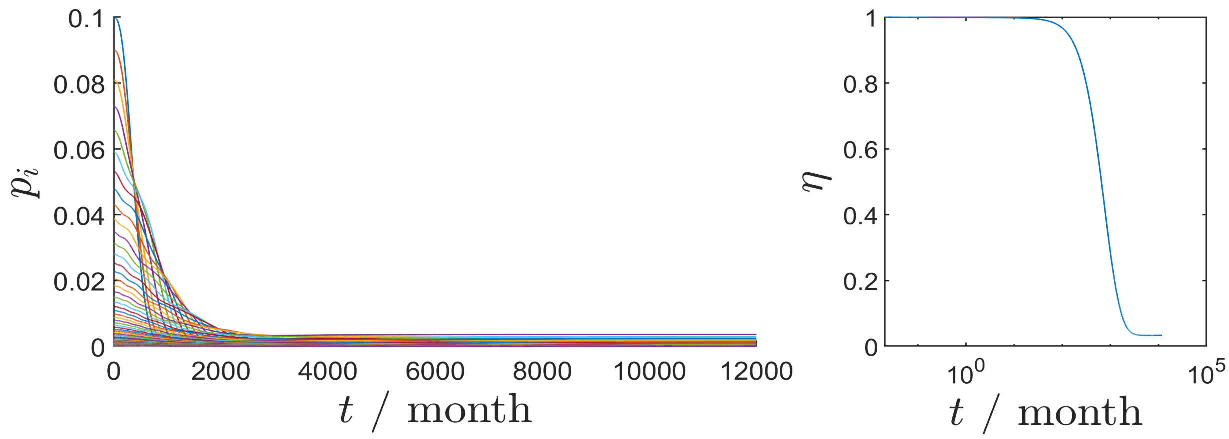

Figure 14: Simulation on the habitat-occupancy proportion of the population and the reduction ratio of the habitat-occupancy in temperate deciduous forest without human beings' effort ($S_0 = 2.5 \times 10^7$). The reduction ratio of the population types $\xi(12000) = 56.67\%$.

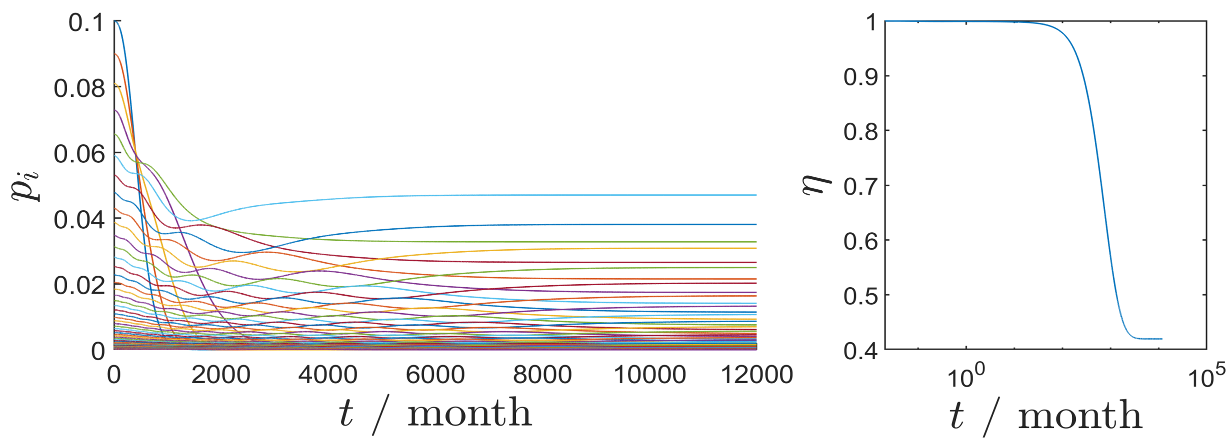

Figure 15: Simulation on the habitat-occupancy proportion of the population and the reduction ratio of the habitat-occupancy in temperate deciduous forest within human beings' effort ($S_0 = 2.5 \times 10^7$, $\mu = 0.4$, $\epsilon = 12$). The reduction ratio of the population types $\xi(12000) = 93.33\%$.

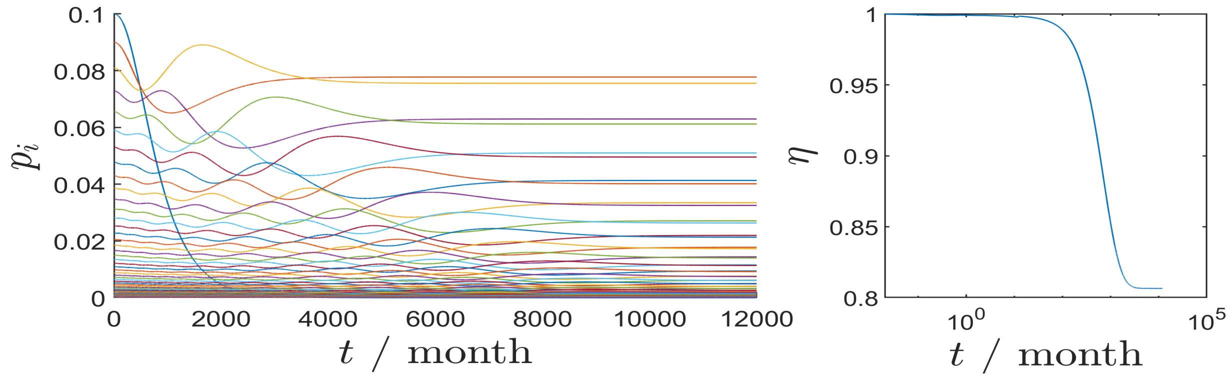

Figure 16: Simulation on the habitat-occupancy proportion of the population and the reduction ratio of the habitat-occupancy in temperate deciduous forest within human beings' effort ($S_0 = 2.5 \times 10^7$, $\mu = 0.8$, $\epsilon = 12$). The reduction ratio of the population types $\xi(12000) = 98.33\%$.

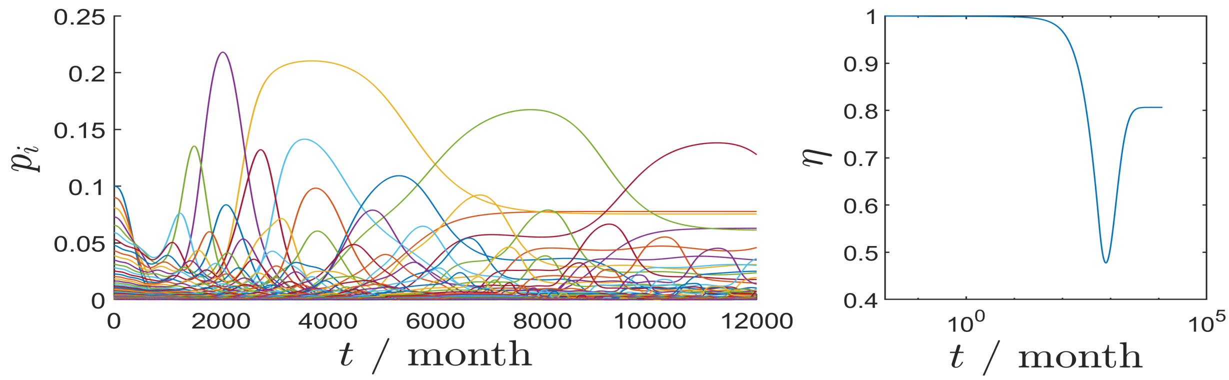

Figure 17: Simulation on the habitat-occupancy proportion of the population and the reduction ratio of the habitat-occupancy in temperate deciduous forest within human beings' effort ($S_0 = 2.5 \times 10^7$, $\mu = 0.8$, $\epsilon = 600$). The reduction ratio of the population types $\xi(12000) = 91.67\%$.

1. Human beings should establish the monitoring system for biodiversity in the given area. The basic monitoring methods include general investigation, taking photographs at the fixed points and so on [14].

2. Human beings should introduce native species which extinct because of the dragons' invasion to keep the biodiversity in the ecosystem.

3. For the area that has been seriously damaged by dragons, human beings should change the physical environment for the ecosystem. For example, human beings can add soil, nutrients and moisture to the land burned by dragons.

4. Human beings can properly provide for the dragons, offering livestock as dragons' food sources, which can reduce the influence of dragons' predation.

7 Strengths and Weaknesses

7.1 Strengths

1. We reference both data of the real animals and descriptions in the fictions to assume the dragon's characteristics, making them reasonable and scientific.

2. Our model adequately considers the influence between different populations in the ecosystem, which can be a reference for the game of ecology in the real world.

3. Our model can simulate a long-time developing process of the ecosystem, predicting the population types and the amount of living beings at any time point.

4. The parameters in our model can be adjusted according to the environment, so the model can be applied to different kinds of ecosystem.

7.2 Weaknesses

1. To simulate the ecosystem in nature with more than 10^6 kinds of populations, the model's computation will be large.

2. Although our model considers the differences in the different regions, we ignore the seasonal variation in the same locality.

8 Conclusion

Our paper provides the $1 + n$ Meta-population Model combined by two models to efficiently simulate the growth of dragons and their influence on the ecosystem. The Growth Model Based on Energy Flow is generated to simulate the growing process of a dragon in the food chain. Then we proposed Improved n-population Meta-population Model to quantitively estimate the dragons' influence on the ecosystem. We apply the models to the different kinds of environment including temperate deciduous forest, desert and ice sheet. calculating the energy expenditure and intake of dragons and area required to support the dragons. We also simulate the dragons' influence on other populations in the ecosystem and discuss the ecological response. Finally, we analyze the human beings' effect on environment reconstruction and provide some specific measures.

Dear George Martin,

As loyal readers of *A Song of Ice and Fire* who have deep interest in the dragons appearing in the fiction, we would like to consider these three fictional dragons are living today. We form two models dubbed as Growth Model and Improved n-population Meta-population Model to simulate the dragons' lives in the real world. And we get interesting and meaningful results, which may help to set more scientific assumptions about dragons' characteristics in your fictions.

At first, we discuss the growth process of the dragon, which determines the body size and image characteristics of the dragon. The growth rate of dragons doesn't keep constant during different states of lives. According our calculation, the dragon grows most fast aged between 30 and 80 years old, which is the puberty of the dragon; then the dragon reaches an adult age and its growth rate will gradually slow down.

Although dragons are virtual organisms, the characteristics of the dragons conform to the physical and biological conditions. Due to the current global environment, the dragon's body size has certain upper bound. In other words, the dragon cannot grow indefinitely. Referencing the data of the real animals, we build a physical model considering the gravity affection and calculate the upper bound of dragon's mass is $1.17 \times 10^5 kg$, equalling 30 Asian elephants.

Then we calculate the energy intake and energy expenditure of the dragon, by which we can analyze the scope of a dragon's activity. We propose a Growth Model based energy flow to quantatively estimate the amount of energy. In our calculation, an adult dragon must eat 480 sheep weighting $25kg$.

Besides, it is necessary to reasonably plan the scope of a dragon's activity so that the size of cities can be set more scientifically. In our model, a young dragon weighting $12.9t$ requires at least $1.5 \times 10^6 m^2$ region to support itself while an adult dragon whose mass is $79.2t$ requires at least $5.9 \times 10^6 m^2$. So to support three dragons, the area of the region must be larger than $1.8 \times 10^7 m^2$, which is equal to 2500 football field.

What's more, we propose an improved n-population meta-population model to simulation the dragons' influence on the ecosystem during the given time. Once the introduced species appear in the ecosystem, they will decrease the indigenous species' habitat through competition in moisture, nutrients and so on. Dragons don't have any natural enemies, but have the strongest competitive ability among the community. Since there is no competitor, dragons may cause great damage to the local ecological. They may lead to the extinction of the local populations.

We analyze the influence of the dragons in temperate deciduous forests, desert and ice sheet. These are the typical places that the dragons have lived in the fiction. Our simulation result shows that the dragons struggle to live in the ice sheet. So it is reasonable to properly weaken the dragons' power in the ice sheet. Besides, in our model, the larger habitat the dragons have, the less damage they will do to ecological environment. So in the novel, the dragons are supposed to have a wider range of activities.

We also estimate the human beings' effort in our model. In fact, if people can offer food to dragons, the dragons can live longer. It is of strategical importance to Daenerys Targaryen.

Thanks for taking your time out of your busy schedule to read my letter. I hope my advice can help and your new season of television series will make a hit.

Yours sincerely

Team 1919022

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Appendices

Appendix A Metabolic Rate

The metabolic rates of different kinds of birds are shown in the Table. 7.

Table 7: Metabolic rate in rest state and action state

Flying animals	Body weight (g)	Metabolic rate (cal$\cdot g^{-1} \cdot hr^{-1}$)	
		Rest	Action
Hummingbird	3	14.3	204
Budgerigar	35	15.8	105
Sparrow	45	13.2	101
sparrow	62	11.7	97
Coturnix	83	11	90
Gyrfalcon	90	3.8	94
Gull	300	7.3	54
Pigeon	448	7.2	58
Pigeon	621	6.3	51
Eagle	1013	5.9	47
Gyrfalcon	2029	4.9	39
Gyrfalcon	2057	4.9	36

Appendix B Energy-weight Regression Code

energyWeightRegression.m

```
clear,clc;
f1 = figure;
hold all;
body_weight_1 = [35,45,62,83,90,300,448,621,1013,2029,2057];
action_q = [105,101,97,90,94,54,58,51,47,39,36];
log_action_q = log(action_q);
log_body_weight_1=log(body_weight_1);
p1 = polyfit(log_body_weight_1,log_action_q,1);
rest_line = plot(log_body_weight_1,
   polyval(p1,log_body_weight_1),
   'LineWidth',3,'Color',[0.98,0.80,0.71]);
rest_dot = plot(log_body_weight_1,log_action_q,'.',
   'MarkerSize',18,'Color',[0.85,0.33,0.10]);
xlabel('$\ln{m}$','Interpreter','latex','FontSize',15);
ylabel('$\ln{\hat{q}}$','Interpreter','latex','FontSize',15);
text(4.20,4.75,
   {'$\ln{\hat{q_a}}=-0.25\ln{M}+5.6$'},
   'Interpreter','latex','FontSize',15);

body_weight_2 = [35,45,62,83,300,448,621,1013,2029,2057];
rest_q=[15.8,13.2,11.7,11,7.3,7.2,6.3,5.9,4.9,4.9];
log_rest_q = log(rest_q);
log_body_weight_2=log(body_weight_2);
p2 = polyfit(log_body_weight_2,log_rest_q,1);
action_line =
   plot(log_body_weight_2,polyval(p2,log_body_weight_2),
```

```
   'LineWidth',3,'Color',[0.65,0.85,0.98]);
action_dot = plot(log_body_weight_2,log_rest_q,'.',
   'MarkerSize',18,'Color',[0.00,0.45,0.74]);
xlabel('$\ln{m}$','Interpreter','latex','FontSize',15);
ylabel('$\ln{\hat{q}}$','Interpreter','latex','FontSize',15);
text(4.20,2.65,{'$\ln{\hat{q_r}}=-0.25\ln{M}+3.6$'},
   'Interpreter','latex','FontSize',15);
legend([rest_line, action_line],'action state','rest
   state','FontSize',12)
set(gcf,'position',[0,0,600,337.5])

display(p1)
display(p2)

% get(gcf,'position')
```

Appendix C Weight Calculation Code

weight.m

```
function dw_dt=weight(t,w,k)
   dw_dt = 3*k*w^(2/3)*((120*10^3)^(1/3)-w^(1/3));
```

calcWeightFunc.m

```
clear,clc;
max_t = 4000;
init_w = 10;
min_diff = 100;
min_diff_k = 0;
max_growth_rate = 0;
for i = 0.0001:0.00001:0.008
   k = i;
   [~,w]=ode45(@(t,w) weight(t,w,k),[0,12],init_w);

   % [~,ind]=min(abs(t-12));
   tmp = abs(w(end)-35);
   if tmp < min_diff
      min_diff = tmp;
      min_diff_k = k;
   end
end

[t,w]=ode45(@(t,w) weight(t,w,min_diff_k),[0,max_t],init_w);

display(min_diff_k)
display(min_diff)
```

weightFigure.m

```
clear,clc;
mass_init = 10;
mass_inf = 120*10^3;
max_t = 4000;
k = 0.002;
% syms t;
% m_t = ((1-exp(1)^(-k*t))*mass_inf^(1/3)+
   exp(1)^(-k*t)*mass_init^(1/3))^3;
% dm_dt = diff(m_t,t);
% m_t = @(t) ...
% dm_dt = @(t) ...
```

```
t = 0:1:max_t;
m = arrayfun(m_t,t);
max_dm_dt = max(arrayfun(dm_dt,t));

f1 = figure;
hold on;
blue_color = [0,0.45,0.74];
white_color = [1,1,1];
[~,arr_size] = size(t);
for i = 1:arr_size
   ly = [0,m_t(t(i))];
   lx = [t(i),t(i)];
   alpha = dm_dt(t(i))/max_dm_dt;
   tmp_color = blue_color.*alpha + white_color.*(1-alpha);
   plot(lx,ly,'-','Color',tmp_color);
end
plot(t,m,'-','LineWidth',1.5,'Color',blue_color.*0.8);
map = zeros(101,3);
for i = 1:101
   alpha = 0.01*(i - 1);
   map(i,:) = blue_color.*alpha + white_color.*(1-alpha);
end
colormap(map);
tks = 0:10:max_dm_dt;
[~,tick_size] = size(tks);
tl = cell(1,tick_size);
for i = 1:tick_size
   tl(1,i) = {num2str(tks(i))};
end
clb = colorbar('Ticks',0:(10/max_dm_dt):1, 'TickLabels',tl);
xlabel('$t$ / month','Interpreter','latex','FontSize',15);
ylabel('$M$ / kg','Interpreter','latex','FontSize',15);
set(gcf,'position',[329,202,560,300])
```

Appendix D Population Calculation Code

population.m

```
function dp_dt=population(t,p,d,z,e,c,m,max_i)
   dp_dt = zeros(max_i,1);
   for i = 1:max_i
      sum_a = 0;
      sum_b = 0;
      for j = 1:i
         sum_a = sum_a + p(j);
      end
      for j = 1:(i - 1)
         sum_b = sum_b + p(i)*c(j)*p(j);
      end
      if t-e<0
         dp_dt(i,1) = c(i)*p(i)*(1-d(t)-sum_a) - m(i)*p(i) -
            sum_b;
      else
         dp_dt(i,1) = c(i)*p(i)*(1-d(t)+z*d(t-e)-sum_a) -
            m(i)*p(i) - sum_b;
      end
   end
```

evalPopulationChange.m

```
clear,clc;
max_i = 15;
max_t = 12000;
```

```
lbd = 20000000;
mass_init = 10;
mass_inf = 120*10^3;
k = 0.002;
pp = 0.003;
ar = 0.3;
z = 0;
e = 12;
s = 2e10;
syms t;
m_t = ((1-exp(1)^(-k*t))*mass_inf^(1/3)+
   exp(1)^(-k*t)*mass_init^(1/3))^3;
dm_dt = diff(m_t,t);
% Change: func_d = @(t) blah_blah_blah(copied text)
% func_d = 3*(lbd*dm_dt +
   m_t^(3/4)*(36.6*ar+(1-ar)*270.4)*5.35e5)/(pp*33930)/s
func_d = @(t) ...;
input_d = zeros(max_t,1);
for ti = 1:max_t
   input_d(ti) = func_d(ti);
end
q = 0.3;
m = ones(max_i,1);
m = m.*0.02;
c = zeros(max_i,1);
init_p = zeros(max_i,1);
for i = 1:max_i
   init_p(i) = q*(1-q)^(i-1);
   c(i) = m(i) / (1-q)^(2*i-1);
end

[t,p]=ode45(@(t,p)
   population(t,p,func_d,z,e,c,m,max_i),[0,max_t],init_p);

f = figure(2);
subplot('Position',[0.1 0.2 0.5 0.7]);
% axis([0,max_t,0,max(max(p))])
hold on;
for j = 1:max_i
   plot(t,p(:,j),'-');
end
xlabel('$t$ / month','Interpreter','latex','FontSize',15);
ylabel('$p_{i}$','Interpreter','latex','FontSize',15);

subplot('Position',[0.7 0.2 0.25 0.7]);
% axis([0,max_t,0,1.1])
[t_size,~] = size(t);
sum_pi_t0 = sum(p(1,:));
y = zeros(t_size,1);
for j = 1:t_size
   y(j) = sum(p(j,:))/sum_pi_t0;
end
semilogx(t,y,'-');
xlabel('$t$ / month','Interpreter','latex','FontSize',15);
ylabel('$\eta$','Interpreter','latex','FontSize',15);

set(gcf,'position',[360 265 553 353])

count = 0;
for i = 1:max_i
   if p(end,i) < 10e-5
      count = count + 1;
   end
end
count/max_i

% [t,p]=ode45(@(t,p)
   population(t,p,func_d,z,e,c,m,max_i),[0,max_t],init_p);
%
% subplot(2,2,1);
% hold on;
```

```
% for j = 1:max_i
%    plot(t,p(:,j),'-');
% end
%
% subplot(2,2,2);
% [t_size,~] = size(t);
% sum_pi_t0 = sum(p(1,:));
% y = zeros(t_size,1);
% for j = 1:t_size
%    y(j) = sum(p(j,:))/sum_pi_t0;
% end
% semilogx(t,y,'-');
```

Appendix E Space Energy Analysis Code

functionDE.m

```
clear,clc;

lbd = 20000000;
mass_init = 10;
mass_inf = 120*10^3;
k = 0.002;
syms t;
m_t = ((1-exp(1)^(-k*t))*mass_inf^(1/3)+
exp(1)^(-k*t)*mass_init^(1/3))^3;
dm_dt = diff(m_t,t);

pp = 1.2;
ar = 0.75;
func_d1 = 3*(lbd*dm_dt +
   m_t^(3/4)*(36.6*ar+(1-ar)*270.4)*5.35e5)/(pp*33930)

pp = 0.09;
ar = 0.6;
func_d2 = 3*(lbd*dm_dt +
   m_t^(3/4)*(36.6*ar+(1-ar)*270.4)*5.35e5)/(pp*33930)

pp = 0.003;
ar = 0.3;
func_d3 = 3*(lbd*dm_dt +
   m_t^(3/4)*(36.6*ar+(1-ar)*270.4)*5.35e5)/(pp*33930)

assimi_rate = 0.5655;
lbd = 20000000;
mass_init = 10;
mass_inf = 120*10^3;
k = 0.002;
syms t;
m_t = ((1-exp(1)^(-k*t))*mass_inf^(1/3)+
exp(1)^(-k*t)*mass_init^(1/3))^3;
dm_dt = diff(m_t,t);

ar = 0.75;
e_exp1 = (ar*36.6+(1-ar)*270.4)*5.35e5*m_t^(3/4)*3
e_in1 = (e_exp1 + 3*lbd*dm_dt)/assimi_rate

ar = 0.6;
e_exp2 = (ar*36.6+(1-ar)*270.4)*5.35e5*m_t^(3/4)*3
e_in2 = (e_exp2 + 3*lbd*dm_dt)/assimi_rate

ar = 0.3;
e_exp3 = (ar*36.6+(1-ar)*270.4)*5.35e5*m_t^(3/4)*3
e_in3 = (e_exp3 + 3*lbd*dm_dt)/assimi_rate
```

calcDragonSpaceAndEnergy.m

```
clear,clc;
max_t = 4000;

blue = [0.00,0.45,0.74];
green = [0.47,0.67,0.19];
yellow = [0.93,0.69,0.13];

func_d1 = @(t) ...
func_d2 = @(t) ...
func_d3 = @(t) ...
d = zeros(max_t,1);

e_exp1 = @(t) ...
e_in1 = @(t) ...
e_exp2 = @(t) ...
e_in2 = @(t) ...
e_exp3 = @(t) ...
e_in3 = @(t) ...
e_in = zeros(max_t,1);
e_exp = zeros(max_t,1);

f = figure(3);
subplot('Position',[0.07 0.2 0.2 0.5]);
for ti = 1:max_t
   d(ti) = func_d3(ti);
end
loglog(1:max_t,d,'Color',blue,'LineWidth',1.5);
hold on;
for ti = 1:max_t
   d(ti) = func_d2(ti);
end
loglog(1:max_t,d,'Color',yellow,'LineWidth',1.5);
for ti = 1:max_t
   d(ti) = func_d1(ti);
end
loglog(1:max_t,d,'Color',green,'LineWidth',1.5);
axis([0,10e3,0,10e10])
set(gca,'FontSize',12);
xlabel('$t$ / month','Interpreter','latex','FontSize',15);
ylabel('$S$ / $m^{2}$','Interpreter','latex','FontSize',15);
legend('ice sheet','desert','forest','Location','northoutside');

subplot('Position',[0.35 0.2 0.2 0.5]);
for ti = 1:max_t
   e_in(ti) = e_in3(ti);
end
loglog(1:max_t,e_in,'Color',blue,'LineWidth',1.5);
hold on;
for ti = 1:max_t
   e_in(ti) = e_in2(ti);
end
loglog(1:max_t,e_in,'Color',yellow,'LineWidth',1.5);
for ti = 1:max_t
   e_in(ti) = e_in1(ti);
end
loglog(1:max_t,e_in,'Color',green,'LineWidth',1.5);
set(gca,'FontSize',12);
xlabel('$t$ / month','Interpreter','latex','FontSize',15);
ylabel('$$\frac{\mathrm{d}E_{in}}{\mathrm{d}t}$$ / $J \cdot
   month^{-1}$','Interpreter','latex','FontSize',15);

subplot('Position',[0.63 0.2 0.2 0.5]);
for ti = 1:max_t
   e_exp(ti) = e_exp3(ti);
end
loglog(1:max_t,e_exp,'Color',blue,'LineWidth',1.5);
hold on;
for ti = 1:max_t
   e_exp(ti) = e_exp2(ti);
```

```
end
loglog(1:max_t,e_exp,'Color',yellow,'LineWidth',1.5);
for ti = 1:max_t
   e_exp(ti) = e_exp1(ti);
end
loglog(1:max_t,e_exp,'Color',green,'LineWidth',1.5);
set(gca,'FontSize',12);
xlabel('$t$ / month','Interpreter','latex','FontSize',15);
ylabel('$$\frac{\mathrm{d}E_{exp}}{\mathrm{d}t}$$ $J \cdot
   month^{-1}$','Interpreter','latex','FontSize',15);

set(gcf,'position',[15.7,209.7,1258,333.3])
```