# **Computational Methods For The Black Hole Imaging**

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### **Abstract**

This project aims to study some of the image reconstruction methods developed for the Very Long Baseline Interferometry (VLBI) data. Observation of distant, non-visual cosmic objects is often challenged by the diffraction limit of telescopes. Interferometry essentially overcomes this by observing in the frequency domain, but due to various limits (number of telescope dishes, atmospheric inconsistency, etc.) the data is very sparse and noisy. Different image reconstructions methods are proposed which stand behind different assumptions of the data. We aim to give a comprehensive comparison of them and discuss their potentials for the future imaging of the milky way galaxy black hole SgrA\*. To be specific, we will apply one of such algorithms on the VLBI Challenge data, discuss performance and propose improvements if possible.

#### 1. Introduction

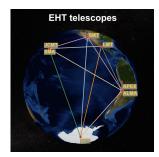
#### 1.1. Very Long Baseline Interferometry

Very long baseline interferometry (VLBI) refers to using multiple telescopes to emulate samples from a single large telescope with diameter equal to the maximum distances between telescopes in the array.

Classic optics states that the angular resolution of a telescope is limited by its diameter and the incident wavelength, that is,  $\theta \sim \frac{\lambda}{D}$ . To achieve higher resolving power at radio frequency, one must employ an extremely large telescope dish, which is nearly impossible in engineering terms. Interferometry, however, gets around this problem since it is the baseline length between 2 telescopes that dictates its resolving power. By forming an array of telescopes globally, we are able to jointly use results from different baselines to acquire images of distant objects. More details about baseline interferometry can be found at [6].

#### 1.2. Event Horizon Telescope

One domain for the application of VLBI measures is to image the immediate environment around a blackhole's



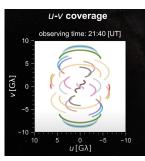


Figure 1. EHT telescopes and Earth Rotation Synthesis
The left image shows EHT telescopes around the world. Each
pair of telescopes form a baseline. As the Earth rotates, each
baseline forms a pair of symmetrical elliptical shape, shown on
the right. Images from [12].

with angular resolution comparable to its event horizon. The Event Horizon Telescope [3] is an international initiative to collaboratively capture images of black holes using a virtual earth-sized telescope, by linking together existing telescopes using novel systems. Its goal is to leverage global collaboration to create new instructions with angular resolving power at short wavelengths that are highest possible from the surface of the earth. The first image of a blackhole is captured using VLBI reconstruction algorithms in the galaxy Messier 87 in the center of the Virgo A galaxy.

In this project, we will evaluate the performance of various reconstruction algorithms on EHT data based on training and testing data from EHT Imaging Challenge [5]. This could better help researchers understand bias of each imaging algorithm, and develop better and robust algorithms to reconstruct an image for SgrA\*, our Milky Way Galaxy's blackhole, which is said to be more challenging than M87.

#### 1.3. Earth Rotation Synthesis

Given K telescopes, we could produce  $\binom{K}{2}$  baselines, each with a unique spatial frequency components that can be visualized in a 2D plot with x axis representing East-West frequency (**u**), and y axis representing North-South frequency (**v**). Due to Earth's self-rotation, the angle between a baseline and the direction of the source  $(\hat{s})$  changes, so a single baseline could produce an elliptical-shaped track

of u-v pairs. This is called earth rotation synthesis [10], depicted in Fig.1, and allows us to cover more spatial frequencies in the u-v frequency plane.

#### 1.4. Image Reconstruction

The VLBI operates in the spatial frequency domain, so the actual image needs to be reconstructed from its raw data. The reconstruction algorithm for VLBI measurements is an on-going research field. An ideal algorithm could find an explanation that respects prior assumptions while still satisfying the observed data, among an infinite number of possible image assumptions that explain the data. Besides, the algorithm should also be able to reconstruct images in fine angular resolution at the mm/sub-mm wavelengths, while at the same time be robust to the additional measurement errors due to rapidly changing inhomogeneities in various telescopes' atmosphere environments. The section 2 of this report contains an overview of these algorithms.

#### 2. Related Work

The challenge of image reconstruction is that reconstructed images are not unique. Reconstructions require prior assumptions, information and constraints. Some strong constraints are image positivity, restricted field of view (FOV). While prior assumptions could favor some physically motivated properties such as image smoothness, entropy, sparsity, or maximum resolution of reconstruction features [9].

Such algorithms could be broadly classified into two methodologies, which are inverse modeling and forward modeling. The former approach includes deconvolution methods such as CLEAN; the latter includes regularized maximum likelihood (RML) algorithms such as maximum entropy method (MEM) [9].

## 2.1. Inverse Modeling

Inverse modeling begins with an inverse Fourier transform of sampled image, after which the algorithm deconvolves the effects with the limited baseline coverage.

CLEAN: The CLEAN algorithm dates back to 1974 [6]. The essence of it revolves around an iterative procedure that locates the point with highest reconstructed intensity and removes the point-source response at that location. This is due to the fact that the "dirty beam" (i.e. the inverse Fourier Transform of sampling function in the spatial frequency domain) contains sidelobes caused by sparse sampling. Several improvements have been proposed based on CLEAN [7].

## 2.2. Forward Modeling

Forward modeling represents an image as an array of pixels, and apply Fourier transform of this array to evalu-

ate consistency between data and image. Forward modeling approaches have been intensively developed for EHT.

MEM: The Maximum Entropy Method defines a target entropy function that is determined solely by the image intensity [6]. In reconstructing the image, it also aims to maximize the entropy with the restraint that the Fourier Transform of the reconstructed image must fit the observed visibility values. The selection of entropy function largely dictates the outcome of the algorithm and is intended to introduce as little details in the unmeasured area as possible.

**BSMEM**: The BiSpectrum Maximum Entropy Method uses a fully Bayesian approach to find the best possible image by using the entropy as a prior [8]. In searching for the image, gradient descent method is used to iteratively optimize the image and narrow down the search range.

**SQUEEZE**: This algorithm takes a Markov chain Monte Carlo method to sample images from posterior distribution. It is not limited in its choice of regularizers or constraints, but comes with a large number of parameters to finetune [11].

CHIRP: The Continuous High-resolution Image Reconstruction using Patch Priors (CHIRP) algorithm takes a Bayesian approach with improved forward model approximation to better model spatial frequency measurement. Meanwhile, it could model atmospheric noise with simpler problem formulation and optimization strategy [11].

#### 2.3. Open Source Software Packages

Three open source software packages are used with scripted imaging pipelines in the second stage creation of the first M87 EHT result. Each pipeline has some fixed choices (e.g., the convergence criterion, the pixel size, etc.) but takes additional parameters (e.g., the regularizer weights, the total compact flux density) as arguments. [11]

**DIFMAP**: a scripted version of the CLEAN algorithm to carry out the parameter search. [1]

**EHT-imaging**: a RML method that uses a template imaging script that use MEM, L1 norm, total variation (TV) and total squared variation (TSV) as regularizers. [2]

**SMILI**: a RML method that reconstruct image using low-band EHT data that use L1, TV, TSV regularizers. [4]

#### 3. Proposed Method

## 3.1. Bias Evaluation Approach

To evaluate different algorithms, we would test image reconstruction algorithms on a suite of synthetic dataset, so that we could optimize the algorithm with objective performance assessment. To be specific, using the EHT challenge dataset, we will reconstruct synthetic data from a large survey of imaging parameters and then conduct a fair quantitative comparison of the results with the corresponding

ground truth images. This allows us to select and evaluate parameters objectively to use in EHT tasks.

It could be uninformative in highly degraded VLBI reconstruction to use traditional pixel-wise error metrics such as MSE and PSNR. This will first use MSE metrics with the perceptually motivated structural similarity (SSIM) index proposed by Katherine et al.[11], and explore possible alternative better metrics of evaluation.

## 4. Progress

Conducted some literature review on

- · Background
- Existing reconstruction work on M87
- · Different reconstruction algorithms
- · EHT challenge

#### 5. Future Plan

- Work with EHT challenge dataset of synthetic test sets, scoping down problem into a reasonable size
- Start with the open-source implementations, and experiment with one or two classic algorithms. A comparison of the results and potential improvements will be conducted if time permits.
- Look into alternative error metrics for SgrA\*, if time permits

## References

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