MIT CSAIL

6.819/6.869 Advances in Computer Vision

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Problem Set 8

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Problem 1 *:Markov Network*

```
import numpy as np

def markov_chain(alpha):
    phi_bc = phi_ef = np.array([[0.9, 0.1], [0.1, 0.9]])
    psi_ab = psi_bd = psi_de = np.array([[alpha, 1-alpha], [1-alpha, alpha]])

    # marginal of e. Message propagates from f to e. f observed to be 0.
    m_ef_e = phi_ef.dot(np.array([[1.], [0.]]))
    # marginal of d. Message propagates from e to d.
    m_de_d = psi_de.dot(m_ef_e)
    # marginal of b from d.
    m_bd_b = psi_bd.dot(m_de_d)
    # marginal of b from c. Message propagates from c to b. c observed to be 1.
    m_bc_b = phi_bc.dot(np.array([[0.], [1.]]))
    # marginal of a. Messages from 2 ways.
    m_ab_a = psi_ab.dot(m_bd_b * m_bc_b)

    m_ab_a = m_ab_a / np.sum(m_ab_a)
    return m_ab_a

alpha = 0.9

marginal_a = markov_chain(alpha)
print(marginal_a)
```

Output:

```
[3] alpha = 0.99

    marginal_a = markov_chain(alpha)
    print(marginal_a)

    [[0.45971599]
     [0.54028401]]
```

```
[4] alpha = 0.9

    marginal_a = markov_chain(alpha)
    print(marginal_a)

    [[0.30487805]
     [0.69512195]]
```

```
[5] alpha = 0.6

    marginal_a = markov_chain(alpha)
    print(marginal_a)

    [[0.42118227]
     [0.57881773]]
```

Figure 1: Output with different alpha value

(a) When $\alpha = 0.9$,

$$P(a) = \begin{bmatrix} 0.3049 \\ 0.6951 \end{bmatrix}$$

(b) When $\alpha = 0.6$,

$$P(a) = \begin{bmatrix} 0.4212 \\ 0.5788 \end{bmatrix}$$

Discussion:

When alpha changes, the compatibility matrices changes accordingly. The marginal distributions are thus affected. From the numerical values, D tends to be in state 0 when alpha=0.9, but are more equally distributed when alpha=0.6. Since marginal probability of B from C is fixed by the observation of C, B's probability from F is affected after the chain of events with changing alpha.

When $\alpha = 0.6$, the marginal probability of A is rather close to equal probability of being in state 0 or 1. The transition between pair AB, BD, and DE are weak, given the distribution of C and F won't help much in determining marginal of A, resulting in equal probability;

While $\alpha = 0.9$, the marginal probability of A has a higher chance (0.7) of staying in 1, and lower chance of being 0. With alpha getting larger, the information propagated from a closer neighbour C is also more certain comparing to a further away node F. Thus, A has a higher chance to be in the same state as C.