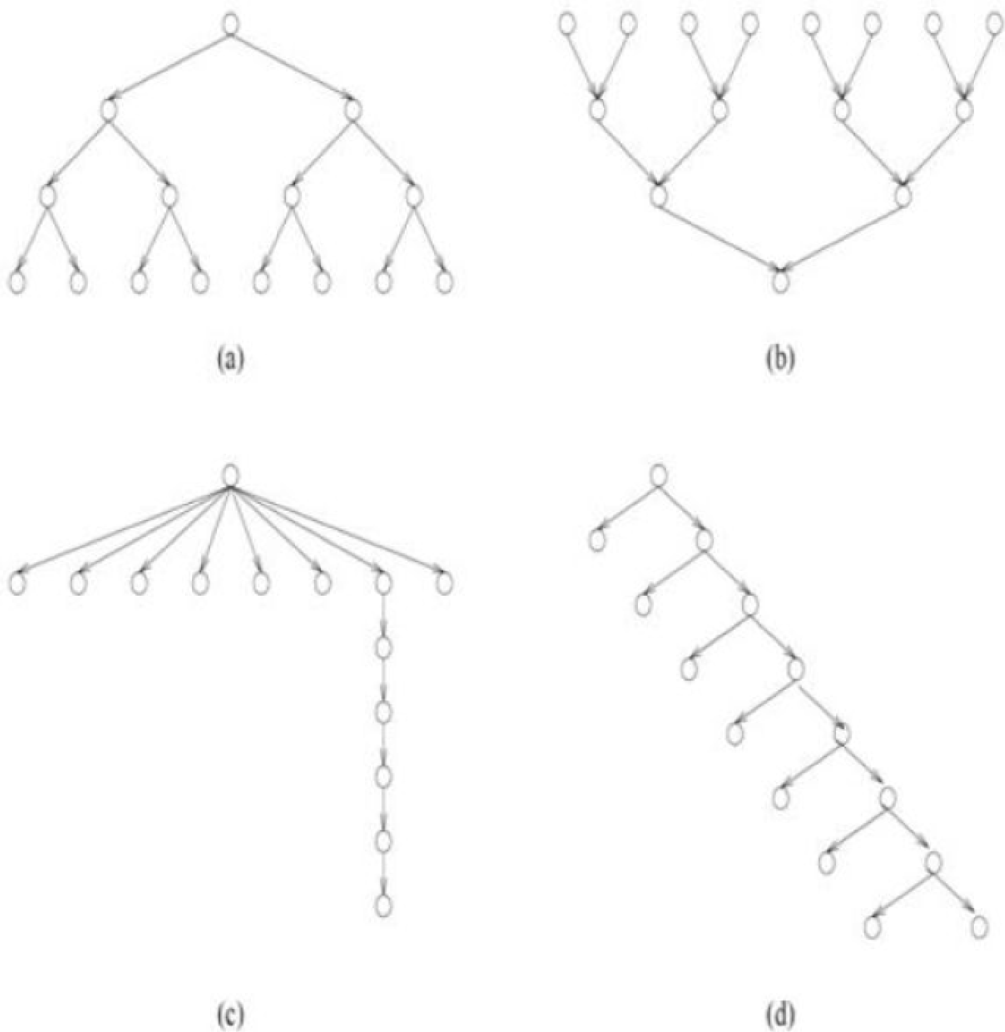
1.2. Graph Parallelization (10 points)

For the computation graphs a), b), c) and d) shown in the figure, determine the following assuming unit time for computation:

1. Maximum degree of concurrency
2. Critical path length
3. The minimum number of processors needed to obtain the maximum speed-up
4. The maximum achievable speed-up if the number of processors is limited to (i) 2 and (ii) 5



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> **Graph (a):**
>
> - Maximum degree of concurrency: 8

- Critical path length: 4 (any branch from grandparent to leave node)
- The minimum number of processors needed to obtain the maximum speed-up: 8

- The maximum achievable speed-up if the number of processors is limited to (i) 2 and (ii) 5:
  - (i) if limited to 2: It takes 8 computation unit time to finish the graph computation. The maximum achievable speed-up is

$$15/8 = 1.875$$

  - (ii) if limited to 5: As critical path is 4, it is the lower bound. By traversing through, I found it takes at minimum 5 computation unit to finish the graph computation. The maximum achievable speed-up is

$$15/5 = 3$$

> **Graph (b):** - essentially similar to graph (a)

- Maximum degree of concurrency: 8
- Critical path length: 4 (any branch from grandparent to leave node)
- The minimum number of processors needed to obtain the maximum speed-up: 8
- The maximum achievable speed-up if the number of processors is limited to (i) 2 and (ii) 5:
  - (i) if limited to 2: It takes 8 computation unit time to finish the graph computation. The maximum achievable speed-up is

$$15/8 = 1.875$$

  - (ii) if limited to 5: By traversing through from lower bottom to top, I found it still takes at minimum 5 computation unit to finish the graph computation. The maximum achievable speed-up is

$$15/5 = 3$$

> **Graph (c):**

- Maximum degree of concurrency: 8
- Critical path length: 7 (from grandparent to the deepest leave node)
- The minimum number of processors needed to obtain the maximum speed-up: 3
- The maximum achievable speed-up if the number of processors is limited to (i) 2 and (ii) 5:
  - (i) if limited to 2: It takes 8 computation unit time (horizontal 6 units + hanging 6 units at the same time; plus 2 more units for the right most one, and top one) to finish the graph computation. The maximum achievable speed-up is

$$14/8 = 1.75$$

  - (ii) if limited to 5: It exceeds minimum number of processor needed (which is 3) to get maximum speed up, it takes at minimum 7 computation unit to finish the graph computation, same as critical path length. The maximum achievable speed-up is

$$14/7 = 2$$

> **Graph (d):**

- Maximum degree of concurrency: 8 (all the leave nodes can be computed together)
- Critical path length: 8 (from top parent node to the deepest leave node)
- The minimum number of processors needed to obtain the maximum speed-up: 2 (calculate two nodes together at each depth, acheiving min time of critical path length - 8)
- The maximum achievable speed-up if the number of processors is limited to (i) 2 and (ii) 5:
    - (i) if limited to 2: It is the minimum number of processor needed to get maximum speed up, thus it takes at minimum 8 computation unit to finish the graph computation, same as critical path length. The maximum achievable speed-up is

      $$15/8 = 1.875$$

    - (ii) if limited to 5: It exceeds the minimum number of processor needed to get maximum speed up, thus it takes at minimum 8 computation unit to finish the graph computation, same as critical path length. The maximum achievable speed-up is

      $$15/8 = 1.875$$

In [ ]: