

Contents

1	Triangle	1
1.1	Vectors	1
1.2	Median	1
1.3	Altitude	1

Chapter 1

Triangle

Consider a triangle with vertices

$$\mathbf{A} = \begin{pmatrix} -3 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}, \mathbf{c} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}, \quad (1.1)$$

1.1. Vectors

1.2. Median

1.3. Altitude

1.3.1. \mathbf{D}_1 is a point on BC such that

$$AD_1 \perp BC \quad (1.3.1.1)$$

and AD_1 is defined to be the altitude. Find the normal vector of AD_1 .

Solution: Given

$$\mathbf{A} = \begin{pmatrix} -3 \\ 0 \end{pmatrix} \quad (1.3.1.2)$$

$$\mathbf{B} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} \quad (1.3.1.3)$$

$$\mathbf{C} = \begin{pmatrix} 1 \\ 4 \end{pmatrix} \quad (1.3.1.4)$$

The normal vector of AD_1 is orthogonal to AD_1 and hence parallel to BC

Direction vector \mathbf{m}_{BC}

$$= \mathbf{C} - \mathbf{B} \quad (1.3.1.5)$$

$$= \begin{pmatrix} 1 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 2 \end{pmatrix} \quad (1.3.1.6)$$

$$m_{BC} = \begin{pmatrix} -3 \\ 2 \end{pmatrix} \quad (1.3.1.7)$$

Normal vector of AD_1 is

$$\mathbf{n} = \begin{pmatrix} -3 \\ 2 \end{pmatrix} \quad (1.3.1.8)$$

1.3.2. Find the equation of AD_1 .

Solution: from (1.3.1.8)

$$\mathbf{n} = \begin{pmatrix} -3 \\ 2 \end{pmatrix} \quad (1.3.2.1)$$

The equation of AD_1 is

$$\mathbf{n}^\top (\mathbf{x} - \mathbf{A}) = 0 \quad (1.3.2.2)$$

$$\Rightarrow \begin{pmatrix} -3 & 2 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -3 & 2 \end{pmatrix} \begin{pmatrix} -3 \\ 0 \end{pmatrix} \quad (1.3.2.3)$$

$$\begin{pmatrix} -3 & 2 \end{pmatrix} \mathbf{x} = 9 \quad (1.3.2.4)$$

see Fig. 1.1

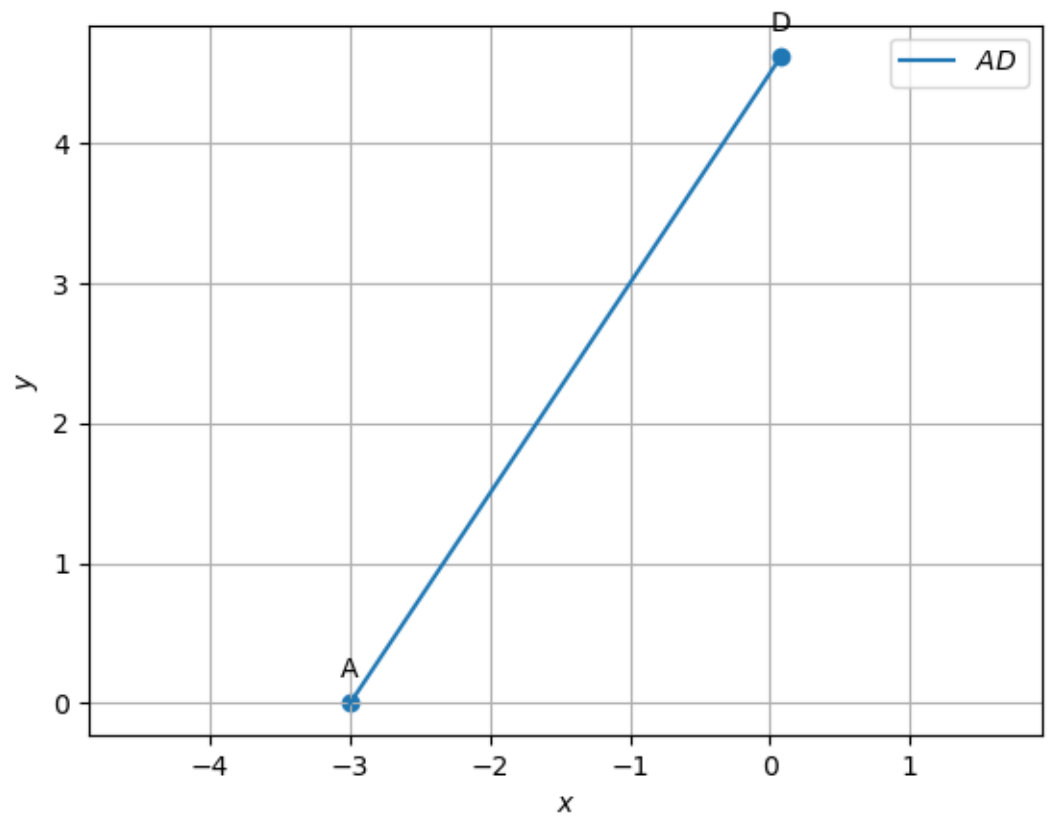


Figure 1.1: Line AD

1.3.3. Find the equations of the altitudes BE_1 and CF_1 to the sides AC and AB respectively.

Solution: Given

$$\mathbf{A} = \begin{pmatrix} -3 \\ 0 \end{pmatrix} \quad (1.3.3.1)$$

$$\mathbf{B} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} \quad (1.3.3.2)$$

$$\mathbf{C} = \begin{pmatrix} 1 \\ 4 \end{pmatrix} \quad (1.3.3.3)$$

Direction vector

$$\mathbf{m}_{AB} = \mathbf{B} - \mathbf{A} \quad (1.3.3.4)$$

$$= \begin{pmatrix} 4 \\ 2 \end{pmatrix} - \begin{pmatrix} -3 \\ 0 \end{pmatrix} \quad (1.3.3.5)$$

$$= \begin{pmatrix} 7 \\ 2 \end{pmatrix} \quad (1.3.3.6)$$

$$\mathbf{m}_{AC} = \mathbf{C} - \mathbf{A} \quad (1.3.3.7)$$

$$= \begin{pmatrix} 1 \\ 4 \end{pmatrix} - \begin{pmatrix} -3 \\ 0 \end{pmatrix} \quad (1.3.3.8)$$

$$= \begin{pmatrix} 4 \\ 4 \end{pmatrix} \quad (1.3.3.9)$$

$$(1.3.3.10)$$

Normal vector of BE_1 is orthogonal to BE_1 and hence parallel to AC

and normal vector of CF_1 is orthogonal to CF_1 and hence parallel to AB

$$\mathbf{n}_{BE_1} = \mathbf{m}_{AC} \quad (1.3.3.11)$$

$$= \begin{pmatrix} 4 \\ 4 \end{pmatrix} \quad (1.3.3.12)$$

$$\mathbf{n}_{CF_1} = \mathbf{m}_{AB} \quad (1.3.3.13)$$

$$= \begin{pmatrix} 7 \\ 2 \end{pmatrix} \quad (1.3.3.14)$$

$$(1.3.3.15)$$

Equation of line is represented by :

$$\mathbf{n}^\top (\mathbf{x} - \mathbf{p}) = 0 \quad (1.3.3.16)$$

(a) The equation of line CF_1

$$\mathbf{n}_{CF_1}^\top (\mathbf{x} - \mathbf{C}) = 0 \quad (1.3.3.17)$$

$$\mathbf{n}^\top CF_1 \mathbf{x} = \mathbf{n}^\top CF_1 \mathbf{C} \quad (1.3.3.18)$$

$$\begin{pmatrix} 7 \\ 2 \end{pmatrix}^\top \mathbf{x} = \begin{pmatrix} 7 \\ 2 \end{pmatrix}^\top \begin{pmatrix} 1 \\ 4 \end{pmatrix} \quad (1.3.3.19)$$

$$\begin{pmatrix} 7 & 2 \end{pmatrix} \mathbf{x} = 15 \quad (1.3.3.20)$$

(b) The equation of line BE_1

$$\mathbf{n}_{BE_1}^\top (\mathbf{x} - \mathbf{B}) = 0 \quad (1.3.3.21)$$

$$\mathbf{n}^\top CF_1 \mathbf{x} = \mathbf{n}^\top BE_1 \mathbf{B} \quad (1.3.3.22)$$

$$\begin{pmatrix} 4 \\ 4 \end{pmatrix}^\top \mathbf{x} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}^\top \begin{pmatrix} 4 \\ 2 \end{pmatrix} \quad (1.3.3.23)$$

$$\begin{pmatrix} 4 & 4 \end{pmatrix} \mathbf{x} = 2 \quad (1.3.3.24)$$

,

1.3.4. Find the intersection \mathbf{H} of BE_1 and CF_1 .

Solution: Equation of \mathbf{BE}_1

$$\begin{pmatrix} 7 & 2 \end{pmatrix} \mathbf{x} = 15 \quad (1.3.4.1)$$

Equation of \mathbf{CF}_1

$$\begin{pmatrix} 4 & 4 \end{pmatrix} \mathbf{x} = 24 \quad (1.3.4.2)$$

Therefore ,we need to solve the following equation to get \mathbf{H} :

$$\begin{pmatrix} 7 & 2 \\ 4 & 4 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 15 \\ 24 \end{pmatrix} \quad (1.3.4.3)$$

which can be solved as yielding

$$\mathbf{H} = \begin{pmatrix} \frac{3}{5} \\ \frac{27}{5} \end{pmatrix}, \quad (1.3.4.4)$$

See Fig. 1.3

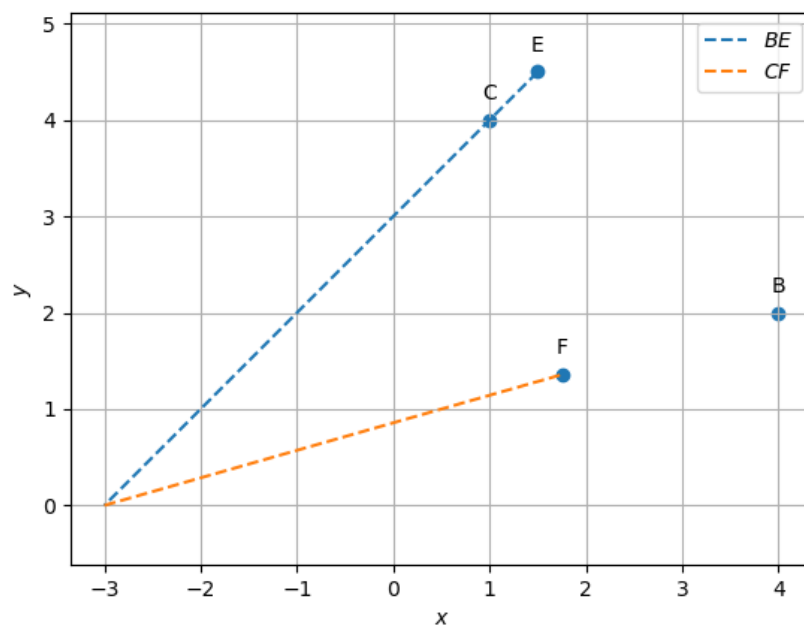


Figure 1.2: Intersection point \mathbf{H} of altitudes BE_1 and CF_1 plotted using python

1.3.5. Verify that

$$(\mathbf{A} - \mathbf{H})^\top (\mathbf{B} - \mathbf{C}) = 0 \quad (1.3.5.1)$$

Solution:

$$\mathbf{A} = \begin{pmatrix} -3 \\ 0 \end{pmatrix} \quad (1.3.5.2)$$

$$\mathbf{B} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} \quad (1.3.5.3)$$

$$\mathbf{C} = \begin{pmatrix} 1 \\ 4 \end{pmatrix} \quad (1.3.5.4)$$

$$\mathbf{H} = \begin{pmatrix} \frac{3}{5} \\ \frac{27}{5} \end{pmatrix} \quad (1.3.5.5)$$

$$\mathbf{A} - \mathbf{H} = \begin{pmatrix} \frac{-18}{5} \\ \frac{-27}{5} \end{pmatrix}, \mathbf{B} - \mathbf{C} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} \quad (1.3.5.6)$$

$$\Rightarrow (\mathbf{A} - \mathbf{H})^\top (\mathbf{B} - \mathbf{C}) = \begin{pmatrix} \frac{-18}{5} & \frac{-27}{5} \end{pmatrix} \begin{pmatrix} 3 \\ -2 \end{pmatrix} = 0 \quad (1.3.5.7)$$

see Fig. 1.4

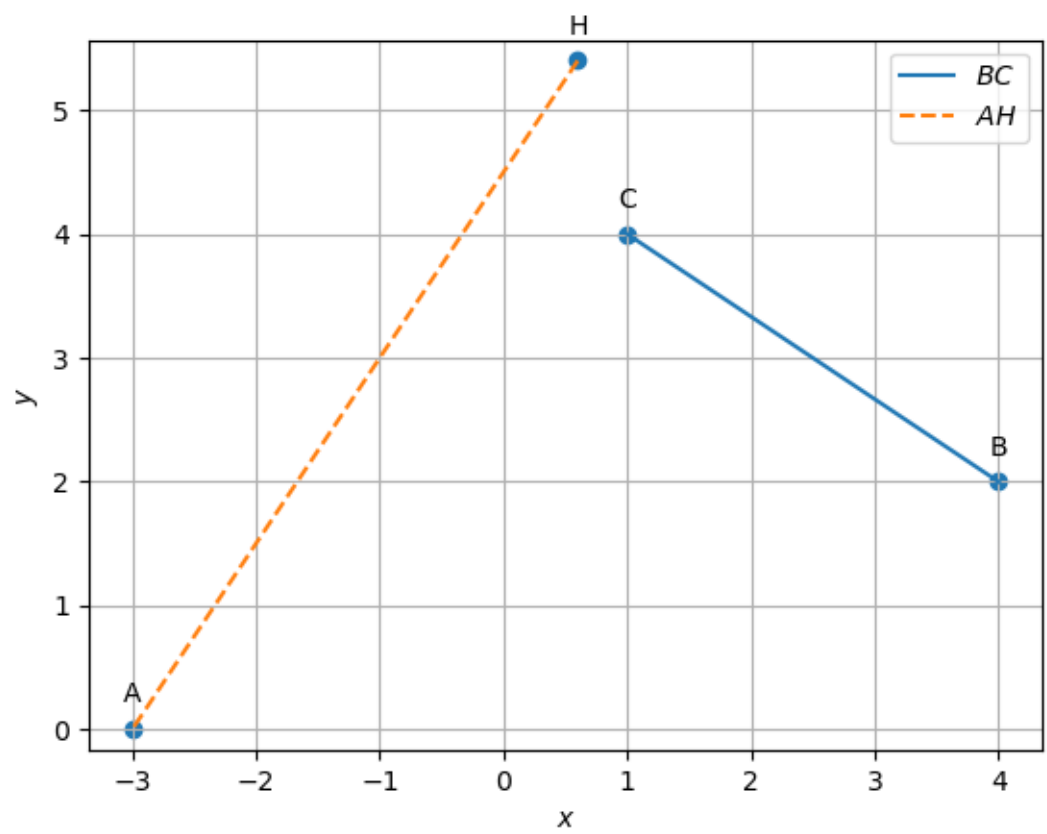


Figure 1.3: Plot of points A,B,C and H