0.1. NCERT 12.10.5.9

Find the position vector of a point $\mathbf{R}$ which divides the line joining two points $\mathbf{P}$ and $\mathbf{Q}$ whose Position Vectors are $2\mathbf{a} + \mathbf{b}$ and $\mathbf{a} - 3\mathbf{b}$ externally in the ratio $1 : 2$.

Solution: let us assume $\mathbf{a}$ and $\mathbf{b}$ and the given ratio is

Symbol	Value	Description
$\mathbf{a}$	$\begin{pmatrix} 1 \\ -3 \end{pmatrix}$	vector $\mathbf{a}$
$\mathbf{b}$	$\begin{pmatrix} 0 \\ 2 \end{pmatrix}$	vector $\mathbf{b}$
k	2	ratio

Table 1: Vectors $\mathbf{a}$ and $\mathbf{b}$, ratio k

using section formula

$$\mathbf{R} = \frac{\mathbf{Q} - k.\mathbf{P}}{1 - k} \tag{1}$$

where $\mathbf{P}$ and $\mathbf{Q}$ depends on $\mathbf{a}$ and $\mathbf{b}$ then,

$$\mathbf{P} = (2\mathbf{a} + \mathbf{b}) = 2\begin{pmatrix} 1 \\ -3 \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \end{pmatrix} \tag{2}$$

$$\mathbf{Q} = (\mathbf{a} - 3\mathbf{b}) = \begin{pmatrix} 1 \\ -3 \end{pmatrix} - 3\begin{pmatrix} 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ -9 \end{pmatrix} \tag{3}$$

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where $\mathbf{R}$ can be calculated as

$$\mathbf{R} = \frac{(\mathbf{a} - 3\mathbf{b}) - k.(2\mathbf{a} + \mathbf{b})}{1 - k} \tag{4}$$

by substituting $\mathbf{a}$ and $\mathbf{b}$ values we get $\mathbf{R}$ as

$$\mathbf{R} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \tag{5}$$

Symbol	Value	Description
$\mathbf{P}$	$(2\mathbf{a} + \mathbf{b})$	position vector $\mathbf{P}$
$\mathbf{Q}$	$(\mathbf{a} - 3\mathbf{b})$	position vector $\mathbf{Q}$
$\mathbf{R}$	$\frac{\mathbf{Q} - k.(\mathbf{P})}{1 - k}$	position vector $\mathbf{R}$

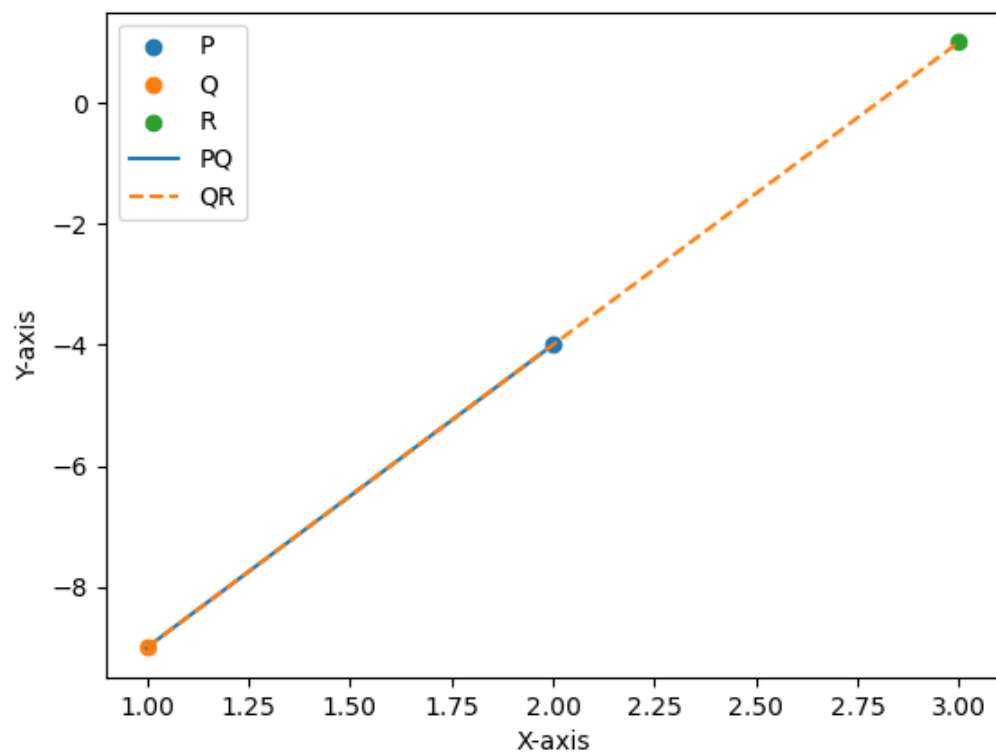Table 2: Vectors $\mathbf{P}$, $\mathbf{Q}$, $\mathbf{R}$

Figure 1: Point vectors **P**, **Q**, **R**