1 NCERT 12.10.5.9

Find the position vector of a point R which divides the line joining two points P and Q whose Position Vectors are $2\vec{a}+\vec{b}$ and $\vec{a}-3\vec{b}$ externally in the ration $1:2$.Also, Show that P is the mid point of the line segment RQ

Construction steps:

1. Let us consider vector of point R which divides the line joining two points P and Q externally in the ratio $m:n$ then for finding the external point R using section formulae.

$$\vec{OR}=\frac{n.\vec{OP}-m.\vec{OQ}}{n-m} \tag{1}$$

2. After finding point vector R we also know the point vectors P and Q,As given R is external point to find the mid point of line segment RQ we need to apply mid point formulae

$$midpoint=\frac{\vec{OR}+\vec{OQ}}{2} \tag{2}$$

   if mid point vector is equal to the point vector P, then we can say that Point vector P is the mid point of the line segment RQ

3. Let us, Assume Point vectors

$$\vec{OP}=2\vec{a}+\vec{b} \tag{3}$$
$$\vec{OQ}=\vec{a}-3\vec{b} \tag{4}$$

   and the ratio is

$$m:n=2:1 \tag{5}$$

   For finding the position vector R,

$$\vec{OR}=\frac{2(2\vec{a}+\vec{b})-1(\vec{a}-3\vec{b})}{2-1} \tag{6}$$
$$\vec{OR}=\frac{4\vec{a}+2\vec{b}-\vec{a}+3\vec{b}}{1} \tag{7}$$
$$\vec{OR}=3\vec{a}+5\vec{b} \tag{8}$$

4. Now The point vectors P,Q and R are

$$\vec{OP}=2\vec{a}+\vec{b} \tag{9}$$
$$\vec{OQ}=\vec{a}-3\vec{b} \tag{10}$$
$$\vec{OR}=3\vec{a}+5\vec{b} \tag{11}$$

For showing P is the midpoint of the line segment RQ

$$\vec{OP} = \frac{\vec{OR} + \vec{OQ}}{2} \tag{12}$$

$$\vec{OP} = \frac{3\vec{a} + 5\vec{b} + \vec{a} - 3\vec{b}}{2} \tag{13}$$

$$\vec{OP} = 2\vec{a} + \vec{b} \tag{14}$$

So,it is proved that P is the midpoint of RQ.

5. The Plot showing the Vector representation of the point vectors P,Q and R is

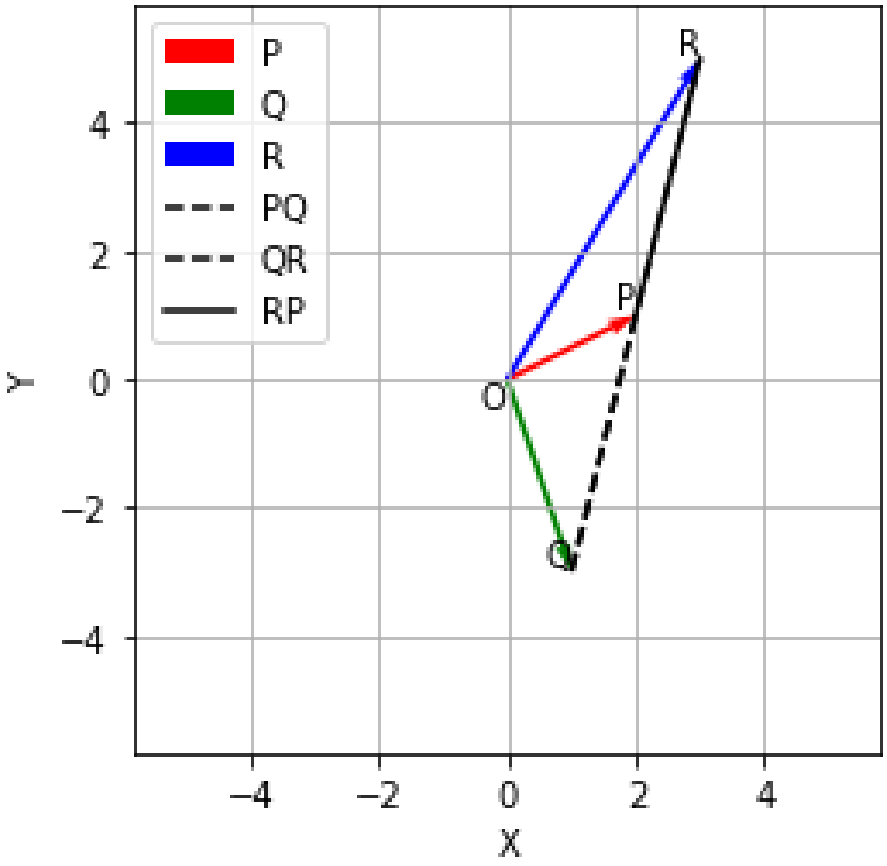

Figure 1: point vectors P,Q,R