A Technique for Fast Magnetometer Calibration with Little Space Coverage

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BIOGRAPHY

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ABSTRACT

One of the most challenging aspects in pedestrian navigation solutions is heading determination, especially when GNSS is not available. Different sensors are used to determine heading in different ways, such as gyroscopes and magnetometers. Gyroscopes can provide the angular rate from which the heading can be calculated, they can be available in low-cost and light weight. Nevertheless, the problem of using gyroscopes as a sole source of heading is that gyroscopes readings are drifting with time in addition to the accumulated errors due to mathematical integration operation. Magnetometer is available in low-cost as well, it does not suffer from mathematical integration errors, and it can provide an absolute heading from the magnetic north. However, magnetometer readings are usually affected by magnetic fields, other than the earth's field, and by other error sources. Therefore calibration is required to use magnetometer as a reliable source of heading. In this paper, a technique is proposed for fast automatic 3D-space magnetometer calibration requiring small space coverage. There is no user involvement during calibration and there are no required specific movements. The proposed technique performs 3D-space magnetometer calibration by calibrating the three magnetometer readings in the device frame which makes the magnetometer useful for determining heading in untethered devices, especially in pedestrian navigation. In these applications, portable navigation devices (such as smartphones) are always moving freely between the user's hands, belt, pocket, or placed on the ear during phone calls. The proposed technique is also capable of calibrating magnetometers in

tethered devices, given that the platform to which the device is tethered is capable of performing 3D motion.

1. INTRODUCTION

Magnetometers are sensors that can sense the magnetic field of earth. They are used long ago in navigation for determining the heading from north. The earth magnetic field can be expressed as a dipole magnet where the magnetic north and south poles lie in an axis that does not coincide with the earth's true north and south poles. The difference between the true north and the magnetic north defines an angle called "declination angle" that should be accounted for when determining heading using magnetometer. Heading can be obtained from the two levelled horizontal magnetometer signals as follows:

$$\psi = -\,arctan\left(\frac{h_y}{h_x}\right) \pm\ D \qquad (1)$$

where $\mathrm{h_x}$ and $\mathrm{h_y}$ represent the two levelled measurements of the earth's magnetic field vector in a coordinate system attached to the compass body (or the device comprising the magnetometer).The angle ψ is the angle the device makes with the earth true north, it represents the device heading, and D represents the declination angle [1].

The presence of electronic magnetometers and the availability of microcontrollers in low-cost that can be programmed to perform a real-time calibration enabled researchers to implement real-time magnetometer calibration. However, the calibration always suffers from one of two major problems. The first is that the user is involved in the calibration process; he should perform specific movements. The second is that the calibration requires large space coverage which makes the calibration process slow and not efficient in navigation applications if the user is not involved in the calibration process.

Different approaches were used for calibrating magnetometers; the main goal of magnetometer calibration is to use magnetometer as a heading source. One of the old and well known methods for calibrating compass is "Compass Swinging". Compass swinging was used for compass calibration to be used for heading determination in marine [1] and aviation [2]. The procedure involves leveling and rotating the vehicle or the aircraft containing the compass through a series of known headings[3]. The main drawback of using traditional compass swinging is that the method is limited to use with two-axis systems as it cannot be used to calibrate a 3D compass [3, 4]. It also requires the user to be instructed to rotate the compass in certain predefined directions [3] which involve the user in the calibration process.

Another approach for compass calibration that does not require an external heading source is known as Ellipse/Ellipsoid fitting. In this approach, calibration is done in the magnetic field domain depending on the fact that the error-free locus made by the compass is a circle when it makes full rotation in 2D or a sphere when rotated in 3D covering all possible orientations [5-8]. In some implementations it is assumed that the locus the magnetometer readings forms a translated hyperbolic shape in 2D (ex. ellipse) or translated hyperboloid shape in 3D (ex. ellipsoid) when rotated in normal operation conditions (in the presence of ferrous interference) [3, 5, 6, 9-11]. Either geometric [4] or algebraic [3, 12-14] methods can be used to best fit the magnetometer measurements to the assumed manifold such fitting the measurements to an ellipse in 2D or to an ellipsoid in 3D.

The main drawback of the methods depending on the Ellipse/Ellipsoid fitting approach is that they require the device having the magnetometers to rotate at least 360 degrees in horizontal plane in case of 2D. In case of 3D calibration the device should cover a big portion of an ellipsoid in 3D to define the ellipsoid eccentricity and rotation [15]. Consequently, in 3D calibration, the calibration process nearly requires rotating the device having the magnetometers in all directions to cover all possible orientations [4]. This drawback either makes the calibration process slow, or it may require the user to perform certain movements (such as for example moving the device in "figure eight" if the device is portable) or rotating the vehicle for one complete loop to cover 360 degrees. The other drawback in the latter scenarios is that the user becomes involved in the calibration process which is not efficient in daily life scenarios [4, 15]. For example, when the user requires an accurate heading from magnetometer without getting involved in a calibration process; either from his portable device (smart phone or personal navigator), or from his vehicle navigation device.

In this paper, a technique is proposed that performs full 3D-space magnetometer calibration. The user is not required to get involved in the calibration process; therefore the technique is fully automatic. Moreover, the technique is fast as it requires very small space coverage.

The proposed technique performs 3D-space calibration when external heading information is available without requiring large space coverage, i.e. very small pitch and roll changes together with one turn in heading (for example a 70 degree turn). To assure the quality of the calibration, two quality check routines are developed. The first routine is called directly after the calibration to decide whether the calculated calibration parameters are correct or not. The other quality check is called periodically after calibration to consider the case if the magnetic environment is changed.

2. AUTOMATIC 3D MAGNETOMETER CALIBRATION WITH LITTLE SPACE COVERAGE

Magnetometers readings are usually affected by magnetic fields other than the earth magnetic field, these fields in addition to some other error sources perturbs the magnetometer readings [7, 8]. Some examples of error sources and their effects on magnetometer readings are among others: (i) Hard iron distortion that is considered a constant bias added to each axis of sensor output. (ii) Sensitivity errors that arise due to different sensitivities of magnetometer sensors in different axis.(iii) Soft iron effect arises from the interaction of earth's magnetic field and any magnetically soft iron material such as nickel or iron. In most cases, hard iron distortion has a much larger contribution to the total error than soft iron. (iv) Temperature can affect the magnetic sensors causing inaccurate heading determination [9]. (v) Other error sources may be due to different factors such as sensor material or sensor fabrication that can cause errors in the magnetometer readings. Also the magnetometer readings can be affected by sensor noise that also adds other source of error.

The above-mentioned error sources are the most effective sources that make the magnetometer readings perturbed, therefore they require calibration.

In this paper, a technique is proposed for 3D-space magnetometer calibration. This work is patent pending. The proposed technique has several key advantages. First, the technique is fully automatic; the user is not involved in the calibration process. Consequently there are no instructions or certain process that the user has to perform to calibrate the magnetometer, for example moving the portable device in "figure eight". Moreover, the calibration technique requires little space coverage compared to other calibration techniques in literature [3-5]. The technique is able to make use of different available navigational information that is available in different navigation scenarios. In some navigation scenarios heading information can be available from another source such as GNSS or an integrated navigation solution (INS/ GNSS) that does not use magnetometer updates till calibration is done.

In addition to the proposed calibration technique, two methods are developed to assure the quality of the calibration. Two quality check routines are developed where the first routine is called directly after the calibration to decide whether the calculated calibration parameters are correct or not. The other quality check routine is called periodically after calibration to consider the case if the magnetic environment is changed for example entering an elevator after walking in a corridor.

The major application of the proposed technique is heading determination in untethered devices such as personal navigators and smart phones which can work in walking and/or driving. However, the technique can be used in tethered devices when the platform is capable of changing the pitch and roll orientations sufficiently to be able to perform the 3D-space calibration for example in airborne applications with high dynamics.

To perform magnetometer calibration using the proposed technique, reference earth magnetic field information is required for the region where the calibration is performed. Earth magnetic field information can be obtained according to any model that describes the earth magnetic field (In this paper, International Geomagnetic Reference Field (IGRF) model is used). Information that may be required from the earth magnetic field model are (i) the value of the components of the magnetic field vector (from which the 3D magnitude of the magnetic field can be calculated); and (ii) the declination angle.

In order to acquire the earth's magnetic field information the position of the device on earth in terms of latitude, longitude and altitude is required. The position can be obtained using any absolute navigation information updates (such as GNSS or WiFi), or an integrated navigation system.

The technique in this paper requires an external heading information source to be used during calibration to supply the calibration method by different readings of the device heading. Afterward the calibrated magnetometer readings can provide heading to update the navigation solution when GNSS is not available. During calibration, the device heading can be calculated using:

(i) Any absolute navigational information (for example, GNSS or WiFi).
(ii) Any integrated navigation solution using different sensors and/or systems such as accelerometers, gyroscopes, magnetometers, barometer, odometer, or any navigational information (for example, GNSS or WiFi) [16, 17].

The magnetometer heading always represents the device heading. Therefore in cases when the device and the platform have a heading misalignment (i.e. the heading of the device is not the same as the heading of the platform), and when an absolute navigational information source is used to obtain platform heading; heading compensation for the misalignment between device and platform is required to obtain the device heading. Solving heading misalignment problem is out of the scope of this work and it is assumed compensated using another module. The external heading information mentioned above is the device heading to be able to perform the calibration.

The proposed technique requires the pitch and roll angles of the device comprising the magnetometer to perform calibration. The pitch and roll angles can be calculated from any one of the following:

(i) Accelerometers readings or averaged accelerometer readings [18].
(ii) Gyroscopes to maintain a known inertial reference orientation at all times using quaternions [6], [18].
(iii) Integrated navigation solution using different sensors and/or systems such as: accelerometers, gyroscopes, magnetometers, barometer, odometer, or

any navigational information updates (for example, GNSS or WiFi) [18].

The developed technique in this paper is capable of calculating different calibration parameters to correct for all different error sources that affect magnetometers; however, biases and scale factors in magnetometers readings are the only calibration parameters that are calculated and calibrated for in this research work. This provides less space coverage during calibration and simpler calibration equations without affecting the heading accuracy.

This technique can be used when the device's pitch and roll angles are changing with the regular usage; such as holding the mobile in hand while hand dangling during walking, putting the mobile device in pocket or using the mobile for voice calls and placing it close to the ear. The method does not require the user to perform certain specific movements with the mobile to perform the calibration; therefore the method is completely automatic and online. Figure 1 shows a hand dangling use-case for a smart phone. The user is walking while hand dangling the device where the device's pitch and roll angles changes with values that can range from 15 to 30 degrees.

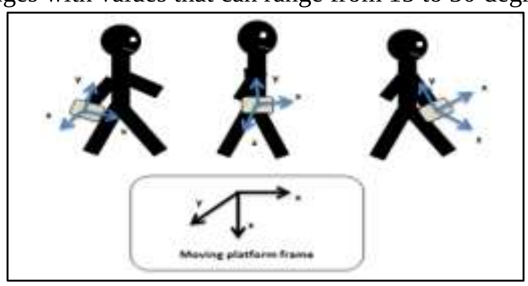

Figure 1 Hand dangling while walking

The following equations are given to describe how the present technique makes use of external heading information to calibrate magnetometer by calculating 3D biases and scale factors in magnetometer readings.

The technique starts with equation (2) describing the unperturbed reference magnetic field vector $\boldsymbol{H_b}$ in device frame. The subscript b stands for body which is considered the device in this technique.

$$\boldsymbol{H_b} = \begin{bmatrix} h_x \\ h_y \\ h_z \end{bmatrix} = \boldsymbol{R_l^b} \cdot \begin{bmatrix} h_N \\ h_E \\ h_D \end{bmatrix} \quad (2)$$

where $\boldsymbol{h_N}, \boldsymbol{h_E}, \boldsymbol{h_D}$ are the three magnetic field components taken from IGRF model in the local level frame, where N represents North, E for East and D for Down, while $\boldsymbol{h_x}, \boldsymbol{h_y}, \boldsymbol{h_z}$ represent the three components of the earth's magnetic field in the device frame (X-Y-Z) forming the vector $\boldsymbol{H_b}$. The device frame is the frame of the device comprising the magnetometer

$\boldsymbol{R_l^b}$ is a 3x3 rotation matrix. It converts the magnetic field vector from the local level frame to the device frame [18].

$$R_l^b = \begin{pmatrix} \cos(p)\cos(h) & \cos(p)\sin(h) & -\sin(p) \\ -\cos(r)\sin(h)+\sin(r)\sin(p)\cos(h) & \cos(r)\cos(h)+\sin(r)\sin(p)\sin(h) & \sin(r)\cos(p) \\ \sin(r)\sin(h)+\cos(r)\sin(p)\cos(h) & -\sin(r)\cos(h)+\cos(r)\sin(p)\sin(h) & \cos(r)\cos(p) \end{pmatrix} \quad (3)$$

In the $\boldsymbol{R_l^b}$ matrix, the letter ($\boldsymbol{h}$) refers to an external heading. The declination angle can be acquired from the IGRF model. The letter ($\boldsymbol{p}$) refers to the pitch angle while the letter ($\boldsymbol{r}$) refers to the roll angle of the device.

Assuming the magnetic field components are modeled as follows:

$$h_x = \frac{h'_x - b_x}{\gamma_x} \quad (4)$$

$$h_y = \frac{h'_y - b_y}{\gamma_y} \quad (5)$$

$$h_z = \frac{h'_z - b_z}{\gamma_z} \quad (6)$$

In equations (4), (5) and (6) $\boldsymbol{b_x}, \boldsymbol{b_y}$ and $\boldsymbol{b_z}$ represent the reading biases in the device frame. $\boldsymbol{\gamma_x}$, $\boldsymbol{\gamma_y}$ and $\boldsymbol{\gamma_z}$ represent the reading scale factor, while $\boldsymbol{h'_x}, \boldsymbol{h'_y}$ **and** $\boldsymbol{h'_z}$ represent the raw 3D magnetometer readings. The previous equations can be written as follows:

$$h_x = h'_x \cdot \left(\frac{1}{\gamma_x}\right) - \left(\frac{b_x}{\gamma_x}\right), h_y = h'_y \cdot \left(\frac{1}{\gamma_y}\right) - \left(\frac{b_y}{\gamma_y}\right), \; h_z = h'_z \cdot \left(\frac{1}{\gamma_z}\right) - \left(\frac{b_z}{\gamma_z}\right) \quad (7)$$

The unknowns are represented as follows:

$$A = \left(\frac{1}{\gamma_x}\right), \; B = \left(\frac{b_x}{\gamma_x}\right), \; C = \left(\frac{1}{\gamma_y}\right), \; D = \left(\frac{b_y}{\gamma_y}\right), \; E = \left(\frac{b_z}{\gamma_z}\right) \quad (8)$$

Using least square (LS) approach for solving the previous equations using N readings in which the heading, pitch and roll values are unique enough to solve the system of equations.

The unknown values representing biases and scale factors can be calculated for 3D magnetometer using LS approach, as follows,

$$\gamma_x = \frac{1}{A}, b_x = \frac{B}{A}, \gamma_y = \frac{1}{C}, b_y = \frac{D}{C}, \gamma_z = \frac{1}{C}, b_z = \frac{D}{C} \quad (9)$$

The device comprising the magnetometer is assumed to be aligned to the platform (ex. the human body) having zero heading misalignment during calibration. This technique can be used to calibrate magnetometer for heading determination in a Pedestrian Dead Reckoning (PDR) solution where the device used is untethered and free to move by a pedestrian (portable device) after being calibrated.

The minimum required data is exactly two heading readings from two different directions, exactly two different pitch values and two roll values. By experimental trials, the minimum absolute difference between the two heading values required is set empirically to 70 to 80 degrees. Also by experimental trials the minimum absolute difference between the two pitch values required is set empirically to 20 degrees and the same for roll. To collect sufficient data for this technique to perform 3D calibration for biases and scale factors, the device is required to be moved covering one heading turn, which is very common and easy to achieve during walking, and to change its pitch and roll only by 20 degrees each. This gives the chance to perform 3D magnetometer calibration while hand dangling or when the mobile is in the trousers pocket while the pedestrian turns by 70 to 90 degrees (heading turn).

To assure the quality of the calibration, two quality check routines are developed. The first routine is used directly after the calibration, to decide whether the calculated calibration parameters are correct or not. The other quality check is used periodically after calibration to consider the case if the magnetic environment is changed. Some examples of the parameters that can assess the calibration quality are such as (i) checking the scale factor is in the suitable range according to experimental results; (ii) comparing the magnetometer heading after calibration with a reference heading when external heading information is available; (iii) comparing the calibrated magnetometer readings with the magnetic field components acquired according to the earth magnetic field model.

4. EXPERIMENTAL RESULTS

The performance of the developed magnetometer calibration technique is examined by different pedestrian trajectories using a portable device that is free to move with different pitch, roll and angles.

In the following experiments, a low-cost prototype unit consisting of a six degrees of freedom inertial unit from Invensense (i.e. tri-axial gyroscopes and tri-axial accelerometer) (MPU-6050), tri-axial magnetometers from Honeywell (HMC5883L), barometer from Measurement Specialties (MS5803), and a GPS receiver from u-blox (LEA-5T) was used.

The calibrated magnetometer heading is compared to a reference heading obtained from an integrated navigation solution that integrates accelerometers, gyroscopes, barometer with GPS in open sky, assuming zero heading misalignment between the device and the platform (the human body). This reference heading is used with the pitch and roll obtained from the integrated navigation solution to perform the 3D. The same integrated solution is used to acquire the magnetic field information from the IGRF model.

The Mean error is used as a measure to assess the calibration quality in the following experiments. The Mean error after calibration is compared to the Mean error before calibration. Mean error is only calculated when magnetometer heading is used, i.e. after data collection is done.

Three pedestrian trajectories are conducted to test the proposed magnetometer calibration. The first trajectory is a PDR walking trajectory in which heading is calculated from magnetometer after 3D calibration is performed. The pedestrian moves in rectangular loops holding the device in his hand, this device usage scenario is called “hand held” use-case, in each two consecutive rectangular loops he holds the device with a different pitch and/or roll, and then changes the devices pitch and/or roll in the next two rectangular loops. The figures show the heading after data collection is made and calibration is applied. Data is collected using the same trajectory by changing the device heading by 90 degrees, changing the pitch by 20 degrees and roll by 20 degrees. In this experiment, for demonstration the calibration parameters are calculated offline and then applied to calculate the heading for the whole trajectory. However, the implementation is capable to run in real-time.

Figure 2 shows the heading calculated using magnetometer in handheld use-case after calibration is performed compared to the reference heading.

Another two trajectories are conducted for more demonstration. These trajectories are Hand-dangling trajectory and Trousers-pocket trajectory. In the Hand-dangling use-case trajectory, the device is held in the users hand while walking where he dangles his hands normally. In the trousers pocket use-case trajectory the user puts the device in his pocket and walks normally.

In order to prove the robustness of the proposed 3D calibration technique, the calibration parameters (biases and scale factors in magnetometer readings) calculated in one trajectory will be used to calibrate magnetometer readings in the other two trajectories. The mean error of the heading compared to the reference in all different combinations using parameters of one trajectory in a different one did not exceed 7 degrees on average.

Figure 3 shows the heading calculated using magnetometer after calibration is performed in Hand-dangling trajectory. In this testing trajectory, for demonstration, data is collected and calibration parameters are calculated from the same hand-dangling trajectory data then the calibration parameters are applied offline to calculate the heading for the whole trajectory. Figure 4 shows the heading calculated using magnetometer after calibration is performed in Trousers-pocket trajectory. In this testing trajectory, for demonstration, data is also collected and calibration parameters are calculated from the same Trousers-pocket trajectory data then the calibration parameters are applied offline to calculate the heading for the whole trajectory. The heading is shown in the figures after calibration is performed. The calibrated magnetometer heading is compared to a reference heading obtained from an integrated navigation solution that integrates accelerometers, gyroscopes, barometer with GPS in open sky, assuming zero heading misalignment between the device and the platform.

It should be noted that the discontinuity of some of the drawn curves in the figures is due to the cyclicity of the heading and roll values where their range is from -180 degrees to 180 degrees.

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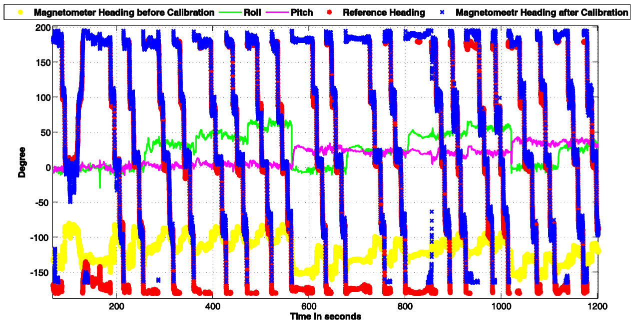

Figure 2 Handheld trajectory magnetometer heading

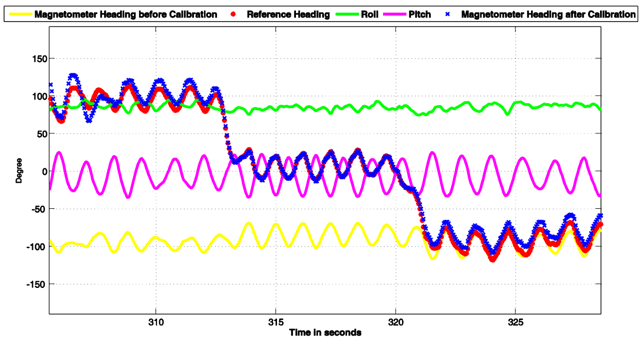

Figure 3 Zoomed section of Hand-dangling trajectory magnetometer heading

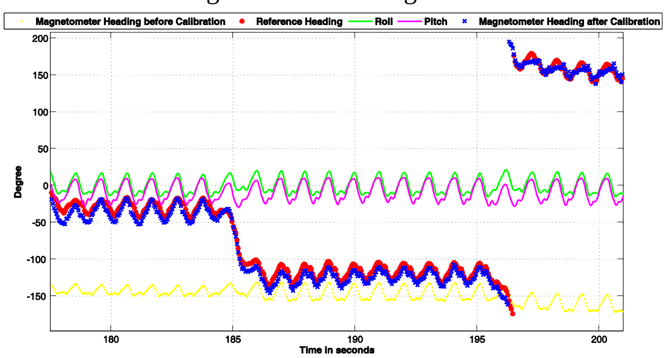

Figure 4 Zoomed section of Trousers-pocket trajectory magnetometer heading

5. CONCLUSION

In this paper, 3D-space magnetometer calibration is performed in an automatic and fast approach. The proposed technique can perform magnetometer calibration with small space coverage and does not require any user involvement. No specific movements are required from the user to perform the calibration like some other calibration techniques from literature. The proposed magnetometer calibration technique depends on different available navigation information from different navigation sensors and/or systems included in most modern navigation devices. Thus, they can perform automatic calibration in real-time. Moreover, quality check routines are developed to check the calibration performance, to make sure that the magnetometer can be used as a reliable source for heading calculation in navigation applications. Experimental results proved that the calibrated magnetometer using the proposed technique can be used as a reliable heading source in portable navigation applications.

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