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2012 Meas. Sci. Technol. 23 105105

(http://iopscience.iop.org/0957-0233/23/10/105105)

Meas. Sci. Technol. **23** (2012) 105105 (6pp) doi:10.1088/0957-0233/23/10/105105

A new calibration method for tri-axial field sensors in strap-down navigation systems

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Received 26 March 2012, in final form 10 July 2012
Published 3 September 2012
Online at stacks.iop.org/MST/23/105105

Abstract
This paper presents a novel calibration method for tri-axial field sensors, such as magnetometers and accelerometers, in strap-down navigation systems. Strap-down tri-axial sensors have been widely used as they have the advantages of small size and low cost, but they need to be calibrated in order to ensure their accuracy. The most commonly used calibration method for a tri-axial field sensor is based on ellipsoid fitting, which has no requirement for external references. However, the self-calibration based on ellipsoid fitting is unable to determine and compensate the mutual misalignment between different sensors in a multi-sensor system. Therefore, a novel calibration method that employs the invariance of the dot product of two constant vectors is introduced in this paper. The proposed method, which is named dot product invariance method, brings a complete solution for the error model of tri-axial field sensors, and can solve the problem of alignment in a multi-sensor system. Its effectiveness and superiority over the ellipsoid fitting method are illustrated by numerical simulations, and its application on a digital magnetic compass shows significant enhancement of the heading accuracy.

Keywords: calibration method, self-calibration, ellipsoid fitting, magnetometer, accelerometer, strap-down navigation system

(Some figures may appear in colour only in the online journal)

1. Introduction

Tri-axial field sensors, such as AMR (anisotropic magneto-resistive) magnetometers and MEMS (micro-electronics mechanical system) accelerometers, are widely used in strap-down navigation systems to provide compact, low cost and low power consumption solutions to determine the orientation and/or attitude. But these sensors usually have different kinds of errors, e.g. bias, scale factors and misalignment [1, 2]. Moreover, in the case of tri-axial magnetometers, the impact of the adjacent magnetic materials, which are commonly referred to as the soft and hard iron interferences, should also be taken into account [3–5]. Therefore, calibration of the strap-down tri-axial field sensors must be performed before they are used.

The error model of a tri-axial field sensor can be described as

$$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}. \qquad (1)$$

In (1), $\mathbf{u} = (u_1 \quad u_2 \quad u_3)^{\mathrm{T}}$ indicates the input of the sensor, i.e. the coordinate components of the 3D vector field, and $\mathbf{v} = (v_1 \quad v_2 \quad v_3)^{\mathrm{T}}$ is the sensor output. The vector $\mathbf{b} = (b_1 \quad b_2 \quad b_3)^{\mathrm{T}}$ and the matrix $\mathbf{K} = (k_{ij})_{3\times 3}$ include all the linear and time-invariant sensor errors. The sensor bias, hard iron interferences and other attitude-independent errors are included in **b**. All the errors proportional to the sensor inputs, e.g. errors caused by the scale factors, soft iron interferences and misalignments, are included in **K**.

In some previous studies, the matrix **K** was divided into several 3 × 3 matrices that account for different kinds of sensor errors. For instance, in the case of a tri-axial magnetometer, the errors included in **K** can be classified into the following types [3, 4].

I. Scale factors, which can be described as a diagonal matrix:

$$\mathbf{C}_{\mathrm{sf}} = \begin{pmatrix} 1+sf_x & 0 & 0 \\ 0 & 1+sf_y & 0 \\ 0 & 0 & 1+sf_z \end{pmatrix}.$$

II. Misalignment, or non-orthogonal errors, which can be described as a skew symmetric matrix plus a unit matrix:

$$\mathbf{C}_m = \begin{pmatrix} 1 & -\varepsilon_z & \varepsilon_y \\ \varepsilon_z & 1 & -\varepsilon_x \\ -\varepsilon_y & \varepsilon_x & 1 \end{pmatrix}.$$

III. Soft iron errors (for magnetometers):

$$\mathbf{C}_{\mathrm{si}} = \begin{pmatrix} \alpha_{xx} & \alpha_{xy} & \alpha_{xz} \\ \alpha_{yx} & \alpha_{yy} & \alpha_{yz} \\ \alpha_{zx} & \alpha_{zy} & \alpha_{zz} \end{pmatrix}.$$

The matrix **K** is the combination of all the above errors, i.e. $\mathbf{K} = \mathbf{C}_{\mathrm{sf}}\mathbf{C}_m\mathbf{C}_{\mathrm{si}}$. However, it is unnecessary and usually impractical to distinguish the error sources, as they have mathematically equivalent impacts on the sensor outputs. Therefore, a uniform expression such as (1) is adopted in the following discussion. This error model can also be applied to other tri-axial field sensors, e.g. the accelerometer [6, 7].

To compensate the sensor errors, the parameters in **b** and **K** must be determined. Let $\mathbf{L} = \mathbf{K}^{-1}$ and $\mathbf{d} = \mathbf{K}^{-1}\mathbf{b}$, the real value of **u** can be recovered from **v** as $\mathbf{u} = \mathbf{Lv} - \mathbf{d}$ once **b** and **K** have been acquired.

Traditional calibration methods, such as the swinging method for magnetometers [3] and the multi-position method for accelerometers [8], require precise external references and may be costly. To reduce the cost of the calibration procedures, many previous studies on the calibration of tri-axial field sensors focus on methods that require no external equipment or references, i.e. self-calibration or auto-calibration [6, 7, 9–11]. Without a precision reference, the input of the sensor is unknown, thus the self-calibration methods usually make use of a time-invariant vector field, such as the Earth's magnetic field for magnetometers and the gravity field for the accelerometers. The strength of the vector field should be a constant scalar, which is used as a constraint in the calibration, so this kind of calibration method is called 'scalar calibration' [1]. Furthermore, these methods are also known as 'ellipsoid fitting' [12, 13], as the acquisition data that contain sensor errors and interferences are assumed to be distributed on an ellipsoid. This method has been commonly used to calibrate magnetometers and accelerometers [14–19].

Previous studies have already proved the effectiveness of the ellipsoid fitting method in the calibration of a single triad of strap-down field sensors, especially magnetometers. However, a navigation system usually contains multiple tri-axial sensors in order to determine the complete attitude of its host platform, e.g. an electronically gimbaled AMR compass [1] or a 9-DOF (degrees of freedom) navigation system [17]. The mutual misalignment between different sensor triads in such a system can become a major source of error [1, 14, 17]. It will be pointed out in the following section that the ellipsoid fitting method is unable to eliminate this kind of error. Therefore, an additional procedure is needed to solve this problem [12, 14, 17].

In this paper, an improved calibration scheme that utilizes the dot product of two vectors is presented, which can include the mutual misalignment as well as other error sources in its calibration. The rest of this paper is organized as follows. The conventional calibration method using ellipsoid fitting is reviewed and discussed in section 2. Then the dot product invariance (DPI) method is introduced in section 3. The DPI method is evaluated by a numerical simulation in section 4, and its application on a digital magnetic compass is presented in section 5. Finally, concluding remarks are given in section 6.

2. Theory of the ellipsoid fitting method

As mentioned in section 1, the ellipsoid fitting method makes use of the field strength, which is a constant scalar

$$\|\mathbf{u}\|^2 = \mathbf{v}^{\mathrm{T}}\mathbf{L}^{\mathrm{T}}\mathbf{L}\mathbf{v} - 2\mathbf{d}^{\mathrm{T}}\mathbf{L}\mathbf{v} + \mathbf{d}^{\mathrm{T}}\mathbf{d} = \mathrm{const}. \tag{2}$$

In most cases, $\mathbf{L}^{\mathrm{T}}\mathbf{L}$ is positive definite, thus (2) stands for an ellipsoid. This ellipsoid can be fitted by the classical least-squares (LS) algorithm, which requires a set of sampling data $\mathbf{v}_i$ and minimizes $\sum_i \left(\mathbf{v}_i^T\mathbf{L}^T\mathbf{L}\mathbf{v}_i - 2\mathbf{d}^T\mathbf{L}\mathbf{v}_i + \mathbf{d}^T\mathbf{d} - \mathrm{const}\right)^2$. However, it can be noticed that (2) contains only 9 independent parameters, while there is a total of 12 elements in **L** and **d**. In other words, the ellipsoid fitting method cannot determine all the parameters that are needed for error compensation.

From another point of view, the ellipsoid fitting method can give an optimal estimation of $\mathbf{L}^{\mathrm{T}}\mathbf{L}$, but there are countless solutions of the decomposition of $\mathbf{L}^{\mathrm{T}}\mathbf{L}$ [9, 13]. For a certain solution of **L**, **RL** will be another solution if **R** is an orthogonal matrix. Its geometrical meaning is that any 3D rotation (which is defined by **R**) does not change the sphere that is recovered from the ellipsoid.

To implement the calibration process, a certain form of the matrix **L** must be chosen among the countless solutions. A lower triangular form of **L** has been applied in [9], which is calculated by the Cholesky decomposition and implies that the *x*-axis and the *x*–*y* plane of the sensor triad is unchanged during the calibration. A symmetrical form of **L** has been adopted in [13, 18, 19], which can be solved by the singular value decomposition (SVD). A more complex decomposition of **L** can be found in [12].

Although the above solutions can achieve the calibration, problems still exist. According to the theory of matrix analysis, any two solutions of **L** can be related by a certain orthogonal matrix. In other words, the ellipsoid fitting method may cause a 3D rotation of the sensor triad with respect to the host platform's coordinate system (the body frame). This 3D rotation cannot be eliminated by the ellipsoid method itself.

In the case of a single sensor triad, the above problem can be eliminated by additional procedures, as will be stated in section 3. But in a multi-sensor system, the situation may be even more complex. Consider the digital compass that is introduced in [1], which contains a tri-axial magnetometer and a tri-axial accelerometer. Actually there are three different coordinate systems in the compass [1]:

I. the coordinate system of the host platform, i.e. the body frame;
II. the coordinate system of the accelerometer;
III. the coordinate system of the magnetometer.

The ellipsoid fitting method can be used to calibrate each sensor triad, but it cannot ensure that the two sensor triads are aligned together since it only takes the errors of a single sensor triad into account. Moreover, it may even cause an extra rotation of the sensor triad when it chooses a certain form of the matrix **L**, and thus increase the misalignment. This is an inevitable limitation of the ellipsoid fitting method. Since the compass is electronically gimbaled and the accelerometer is used to determine the tilt angles, this mutual misalignment will lead to an incomplete or incorrect tilt compensation, and a heading error will arise [1, 14]. Similar situations may appear in a 9-DOF navigation system [17].

There is another drawback of the ellipsoid fitting method. As the sensors have certain noise, a noise item ε should be added to the right side of (1). Since (2) has a quadratic form, a least-squares estimation that is based on (2) may not give an unbiased result, even though ε is assumed to be zero-mean [9, 18]. Numerical simulations and analysis of the impact of this noise were presented in [9]. Since the impact is small when the noise goes to zero, this noise was not included in the calculation of the ellipsoid fitting in some previous studies, such as [13]. But in the case of a low grade sensor that has significant noise, the noise should be treated as an additional parameter in the LS estimation [18], in order to get more reliable results.

3. The dot product invariance method

Several approaches to determine and compensate the mutual misalignment can be found in the existing literature. A two-step calibration procedure has been proposed in [1], while a theoretical solution based on the result of the orthogonal Procrustes problem has been introduced in [12]. Moreover, there is a more feasible method named 'elementary in-plane rotation calibration' presented in [14]. The elementary in-plane rotation calibration is carried out by a rotation around a certain axis. As the vector field projection onto the rotation axis should be a constant, the mutual misalignment can be recognized during this procedure.

The projection of one vector onto another can be naturally associated with, and calculated by, the dot product of these two vectors. If these two vectors are both constant in the reference frame, such as the Earth's magnetic field **h** and gravity **g**, their dot product $\mathbf{h}\cdot\mathbf{g}$ will also be a constant. The DPI method is derived from the above discussion.

For the error model $\mathbf{u} = \mathbf{L}\mathbf{v} - \mathbf{d}$ introduced in section 2, if the real value of the input **u** is known, **L** and **d** can be solved directly. Otherwise, if **u** remains unknown but constant, and there is another constant vector **w**, **L** and **d** can be calculated according to the expression of the two vectors' dot product

$$\mathbf{w}\cdot\mathbf{u} = \mathbf{w}^{\mathrm{T}}\mathbf{L}\mathbf{v} - \mathbf{w}^{\mathrm{T}}\mathbf{d} = \text{const}. \tag{3}$$

Using the classical LS method, all 12 elements in **L** and **d** can be determined by (3), and the mutual misalignment can be removed as well. This process requires a set of sampling data of **w** and **v** (denoted as $\mathbf{w}_i$ and $\mathbf{v}_i$), and then minimizing $\sum_i \left(\mathbf{w}_i^T\mathbf{L}\mathbf{v}_i - \mathbf{w}_i^T\mathbf{d} - \text{const}\right)^2$. Since (3) is a linear equation, the noise can be naturally removed by the LS estimation.

It is worth mentioning that the dot product $\mathbf{h}\cdot\mathbf{g}$ has appeared in the identification and compensation of the mutual misalignment between the accelerometer and magnetometer in [17]. But this calibration is only for the mutual misalignment. It should be emphasized that the DPI method proposed here is a complete solution for the error model described by (1), rather than a simple approach to compensate the mutual misalignment.

The key point of the DPI method is that an auxiliary vector **w** should be available. If an external attitude reference is available, this auxiliary vector can be generated by the attitude reference data. Furthermore, in a multi-sensor system, this vector can be provided by a pre-calibrated sensor triad. For instance, in a digital magnetic compass that consists of a tri-axial magnetometer and a tri-axial accelerometer, if the accelerometer has been pre-calibrated, the gravity vector can be used in the DPI method for the calibration of the magnetometer. Experiments of this case will be introduced in section 5.

4. Numerical simulation of the DPI method

In this section, the calibration of a tri-axial magnetometer using the DPI method will be simulated.

Assuming that the magnetic field vector is $\mathbf{h} = (500\quad 0\quad 0)^{\mathrm{T}}$ mG (1 mG $= 10^{-7}$ T) in the reference frame. The magnetometer has a resolution of 0.1 mG (approximately equal to the level of commercial grade AMR sensors, e.g. the Honeywell HMC1021/1022/1043), and its errors are set to be

$$\mathbf{K} = \begin{pmatrix} 31\,000 & 150 & -250 \\ -100 & 32\,000 & 350 \\ 200 & -300 & 32\,767 \end{pmatrix},$$

and

$$\mathbf{b} = \begin{pmatrix} 500 \\ 600 \\ 700 \end{pmatrix}.$$

The x, y and z axes of the reference frame point to the north, east and down directions respectively, i.e. the NED coordinate system. The elements of **K** and **b** are expressed in a 16-bit fixed point data format, which is adopted in the micro-processors. Moreover, the matrix **K** has been normalized so that its largest element is adjusted to the upper bound of the data format.

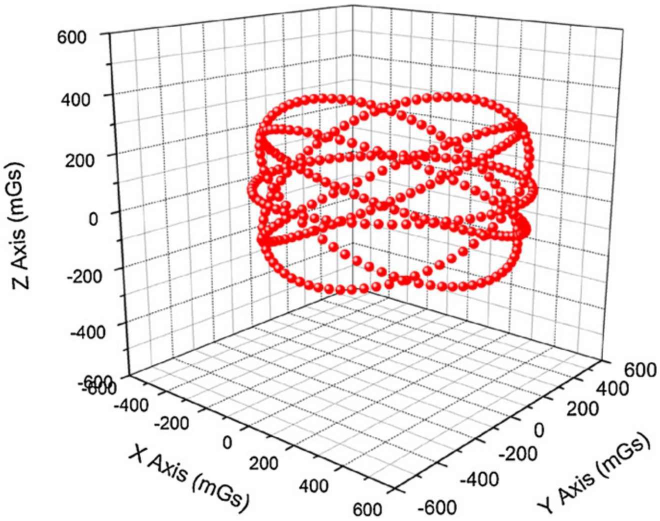

Figure 1. 3D representation of the simulation data.

All the data points are generated in such a way that they imitate five rotations around the vertical axis and the 'host platform' keeps level or 30° inclined to four different directions (front, back, left and right) during these rotations. The simulation data are shown in figure 1.

The exact solution of $\mathbf{L}$ can be calculated by the inverse matrix of $\mathbf{K}$, that is (in 16-bit fixed point format, normalized)

$$\mathbf{L}_{\text{true}} = \begin{pmatrix} 32\,767 & -151 & 252 \\ 105 & 31\,741 & -338 \\ -199 & 292 & 30\,996 \end{pmatrix}.$$

Next, the ellipsoid fitting method and the DPI method are applied to resolve $\mathbf{L}$ from the simulation data. Both methods use the classical LS algorithm. The constant in (3), which equals the dot product, depends on the choice of the auxiliary vector. The constant in (2) is equal to $\|\mathbf{h}\|^2$. Brief introductions of the LS algorithm for ellipsoid fitting can be found in [4, 12, 13]. Besides, more details of this algorithm were presented in [20].

The ellipsoid fitting method gives

$$\mathbf{L}_{\text{EF}} = \begin{pmatrix} 32\,767 & 0 & 0 \\ -51 & 31\,744 & 0 \\ 61 & -57 & 31\,007 \end{pmatrix}.$$

The above solution is a lower triangular matrix, which is calculated by the Cholesky decomposition. Details of the Cholesky decomposition can be found in [21].

Meanwhile, several calibration results of the DPI method with different auxiliary vectors are presented in table 1.

It can be seen that the results of $\mathbf{b}$ are robust, but the matrix $\mathbf{L}_{\text{DPI}}$ may contain large errors when the auxiliary vector is parallel or perpendicular to the magnetic field. To make a quantitative evaluation of the calibration results of $\mathbf{L}$, an indicator $I(\mathbf{L})$ defined by the Frobenius norm is adopted:

Figure 2. Variation of $I(\mathbf{L})$.

$$I(\mathbf{L}) = \|\mathbf{KL} - \sqrt[3]{|\mathbf{K}||\mathbf{L}|}\mathbf{I}_{3\times 3}\|_F. \tag{4}$$

The indicator $I(\mathbf{L})$ describes the deviation of $\mathbf{KL}$ from a scalar matrix (a unit matrix $\mathbf{I}_{3\times 3}$ with an incidental scale factor $\sqrt[3]{|\mathbf{K}||\mathbf{L}|}$). It is similar to, but not the same as, the evaluation factor that has been used in [9]. The evaluation factor in [9] replaced $\mathbf{KL}$ with its SVD result. This replacement implies that the 3D rotation caused by the ellipsoid fitting method has been neglected, thus the evaluation factor in [9] cannot assess the mutual misalignment. However, the indicator $I(\mathbf{L})$ can overcome this deficiency.

Figure 2 shows the variation of $I(\mathbf{L})$ with the angle between the auxiliary vector $\mathbf{w}$ and the magnetic field $\mathbf{h}$.

According to the simulation results in table 1 and figure 2, the following conclusions can be drawn.

I. The matrix $\mathbf{L}$ cannot be solved exactly when the auxiliary vector is parallel to the magnetic field. That is because (3) will degenerate to an equivalent form of (2) under this situation. As discussed in section 2, (2) contains only nine independent parameters, and it is insufficient to determine the matrix $\mathbf{L}$.

II. On the other hand, if the auxiliary vector is perpendicular to the magnetic field, the dot product of them is close to zero, and that may lead to significant truncation errors. It is illustrated by the sharp increase of $I(\mathbf{L})$ in figure 2.

III. Despite the above two cases, the indicator $I(\mathbf{L})$ of the DPI method is much lower than that of the ellipsoid method in most situations. This suggests that the DPI method will have better calibration results.

Table 1. Calibration results.

Auxiliary vector	$(\cos 0.1^\circ \quad \sin 0.1^\circ \quad 0)^{\mathrm{T}}$	$(\cos 30^\circ \quad \sin 30^\circ \quad 0)^{\mathrm{T}}$	$(\cos 60^\circ \quad \sin 60^\circ \quad 0)^{\mathrm{T}}$	$(0 \quad 1 \quad 0)^{\mathrm{T}}$
$\mathbf{L}_{\text{DPI}}$	$\begin{pmatrix} 32\,767 & -632 & -2070 \\ 569 & 31\,743 & 1236 \\ 2250 & -1335 & 31\,029 \end{pmatrix}$	$\begin{pmatrix} 32\,767 & -135 & 249 \\ 88 & 31\,742 & -341 \\ -195 & 290 & 31\,005 \end{pmatrix}$	$\begin{pmatrix} 32\,767 & -140 & 253 \\ 93 & 31\,741 & -339 \\ -197 & 291 & 31\,004 \end{pmatrix}$	$\begin{pmatrix} 32\,767 & 3965 & 209 \\ -4143 & 31\,747 & -369 \\ -199 & 291 & 31\,004 \end{pmatrix}$
$\mathbf{b}$	$(500 \quad 600 \quad 700)^{\mathrm{T}}$	$(500 \quad 600 \quad 700)^{\mathrm{T}}$	$(500 \quad 600 \quad 700)^{\mathrm{T}}$	$(500 \quad 600 \quad 700)^{\mathrm{T}}$

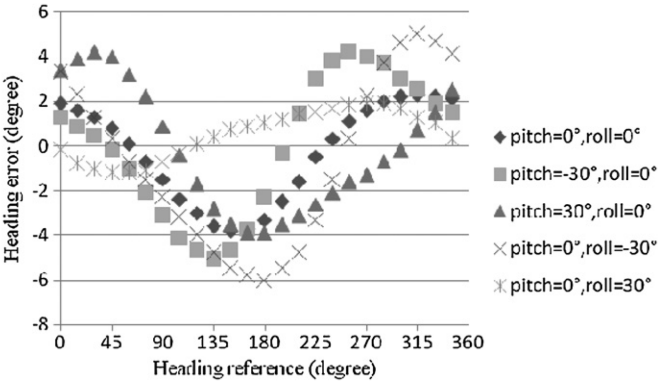

Figure 3. Heading errors before calibration.

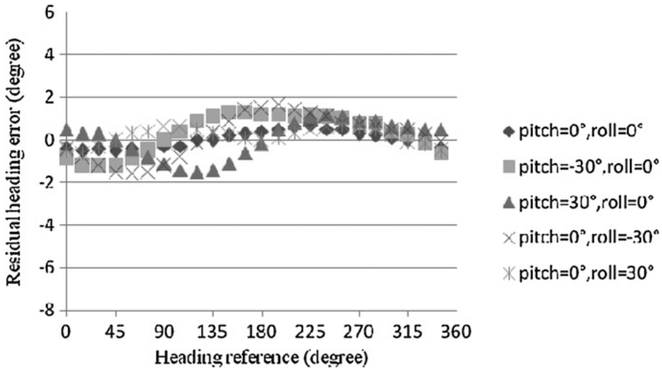

Figure 4. Calibration result of the ellipsoid fitting method.

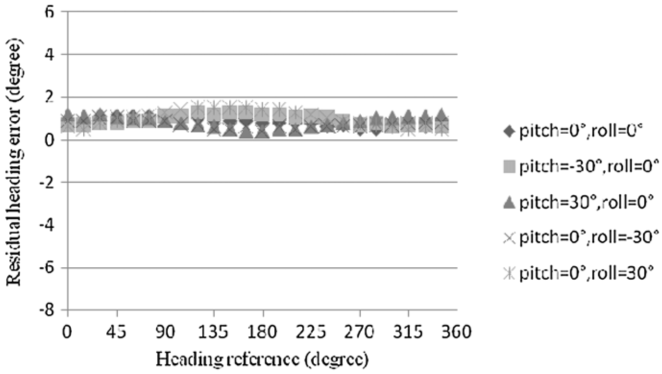

Figure 5. Calibration result of the DPI method.

5. An application of the DPI method

In this section, the DPI method will be applied to a digital magnetic compass module, which consists of a tri-axial magnetometer HMC5843 and a tri-axial accelerometer ADXL345. A non-magnetic turntable that has three degrees of freedom (3-DOF) and a resolution of 0.1° is utilized to provide the attitude reference.

As mentioned in section 3, the DPI method needs an auxiliary vector during the calibration. Although this vector can be generated according to the attitude reference data, the gravity vector measured by the accelerometer in the compass is utilized as the auxiliary vector in the experiment. Therefore, the accelerometer in the compass has been pre-calibrated in order to provide precise measurements of the gravity vector. The residual errors of the pitch and roll angles are limited within $0.1^\circ(1\sigma)$. Since the accelerometer is less sensitive to environment changes than the magnetometer, it may be used to provide the auxiliary vector for the DPI method when performing an in-use calibration of the compass.

Figure 3 shows the heading error of the compass with different pitch and roll angles before the calibration of its magnetometer, which is up to $2.73^\circ(1\sigma)$.

The calibration of the magnetometer is carried out by rotating the compass in various directions, and the readings of the magnetometer are collected during this procedure. After that, the ellipsoid fitting method and the DPI method are used to compensate the errors of the magnetometer.

After the calibration, the residual heading error of the ellipsoid fitting method is $0.77^\circ(1\sigma)$, while that of the DPI method is 0.27°, as shown in figures 4 and 5. It can be seen that the DPI method brings a significant enhancement of the compass heading accuracy, due to the elimination of the mutual misalignment between the magnetometer and accelerometer triads. From another point of view, since the pre-calibration has already aligned the accelerometer triad to the body frame, the effects of the DPI method can also be interpreted as correcting the absolute orientation (with respect to the body frame) of the magnetometer.

6. Conclusion

A novel calibration method for tri-axial field sensors, which is based on the invariance of the dot product of two constant vectors, is introduced in this paper. It is suitable for, but not limited to, the calibration of 3D field sensors in strap-down navigation systems.

The proposed method, which can be referred to as the DPI method, utilizes an auxiliary vector that has a constant dot product with the vector field measured by the sensor triad. The auxiliary vector can be generated according to the external attitude reference, and can also be provided by a pre-calibrated sensor in the system, e.g. gravity.

Numerical simulations have been performed to verify the effectiveness of the DPI method among different choices of the auxiliary vector. The simulation results suggest that the auxiliary vector should not be parallel or perpendicular to the vector field, to avoid affecting the calibration adversely. Despite these two cases, the DPI method can effectively eliminate the sensor errors, especially the mutual misalignment in a multi-sensor system. Meanwhile, since the equation of the DPI method is linear, its calculation is more efficient and concise than that of the conventional ellipsoid fitting method. Both the simulation and experiment results have illustrated that the DPI method can effectively enhance the accuracy of a tri-axial field sensor in a strap-down navigation system.

Acknowledgment

This work was supported by the Guangxi Scientific Research and Technology Development Programs (11107006-19).

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