The Soft Iron and Hard Iron Calibration Method using Extended Kalman Filter for Attitude and Heading Reference System

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Abstract —**The calibration method of the soft iron and hard iron distortion based on attitude and heading reference system (AHRS) can boil down to the estimation of 12 parameters of magnetic deviation, normally using 12-state kalman filter (KF) algorithm. The performance of compensation is limited by the accuracy of local inclination angle of magnetic field and initial heading. A 14-state extended kalman filter (EKF) algorithm is developed to calibrate magnetic deviation, local magnetic inclination angle error and initial heading error all together. The calibration procedure is to change the attitude of AHRS and rotate it two cycles. As the strapdown matrix can hold high precision after initial alignment of AHRS in short time for the gyro's short-term precision, the magnetic field vector can be projected onto the body frame of AHRS. The experiment results demonstrate that 14-state EKF outperforms 12-state KF, with measurement errors exist in the initial heading and local inclination angle. The heading accuracy (variance) after compensation is 0.4 degree for tilt angle ranging between 0 and 60 degree.**

I. INTRODUCTION

Advances in Micro-Electro-Mechanical System (MEMS) technology have made magnetoresistive (MR) sensors play an important role in the low cost attitude and heading reference system (AHRS) which is the linchpin of heading accuracy of AHRS. Among the many factors affecting the system errors of 3-axis magnetoresistive sensors (i.e. bias, scale factor error and install alignment error, etc.), one large contributor comes from the soft iron and hard iron distortion in installation environment. The compensation methods of the soft iron and hard iron distortion can be categorized into two groups: with or without information outside. With the aid of outside calibrating information, like rotation table, hexahedron, speed sensor or GPS, the methods in the former group can reach high calibration accuracy[1, 2]. But the application condition of the methods is limit. In the second group, we can use least-squares method to recognize the model of compass errors or we can use ellipse-specific fitting error compensation method based on extended kalman filter (EKF) [3-5]. The least-squares method can not compensate the soft iron and hard iron distortion on magnetic field of up direction. The performance of the ellipse-specific fitting error compensation method highly depends on the initial value of filter and the algorithm is complicated.

This paper develops the compensation method of the soft iron and hard iron distortion based on AHRS. We discussed the application condition in two cases. With local inclination angle of magnetic field and initial heading angle as known, when AHRS changes attitude angle randomly and rotates two cycles after initial alignment, we can project the magnetic field vector onto reference frame of AHRS. The 3-axis bias, scale factor error and install alignment error, which are equivalent to the influence of soft iron and hard iron distortions, can be estimated by 12-state kalman filter (KF). With local inclination angle of magnetic field and initial heading angle as unknown, we employ 14-state EKF to calibrate the initial heading angle and local inclination angle

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of magnetic field, in addition to 12 parameters of soft iron and hard iron effects. The algorithm is deduced in the paper in detail and the experiment process is also introduced which is easy to follow up.

The experiment results demonstrate that 14-state EKF can estimate precisely the initial heading angle and local inclination angle of magnetic field, when initial heading angle error is 10 degree and local inclination angle error of magnetic field is 5 degree. In calibrating 12 parameters of soft iron and hard iron effects, 14-state EKF is outperforming 12-state KF also. The heading accuracy (variance) after compensation is 0.4 degree for tilt angle from 0 to 60 degree.

II. THE THEORY OF MAGNETIC DEVIATION COMPENSATION

The AHRS consists of a 3-axis sensor assembly with MEMS accelerometer, MEMS gyro and magnetoresistive sensor on each axis. The device also includes temperature sensor, 16-bit A/D converter and DSP board. Analog sensor output is converted to 3-axis measurement of angle rate, acceleration and magnetic field via anti-alias filtering, A/D convert and temperature compensation. After the information is transferred to the data processor, it is further processed with strapdown algorithm and data fusion to provide roll, pitch and heading. The result angles of strapdown algorithm can hold high precision in a short period for the gyro's short-term precision, and the error variance of the angles increases with time due to gyro noise. In case of static state, the gravity field and the earth's magnetic field observations can aid the attitude angle estimation without unbounded growth in the accumulation error. With the incorporation of accelerometer, gyro and magnetometer measurement, EKF can give the optimum solution of roll, pitch and heading [6]. Therefore, the veracity of earth magnetic field observation direct influences the heading accuracy of AHRS.

In application environment, the magnetoresistive sensor shows sensitivities to the soft iron and hard iron distortion in addition to the earth's magnetic field. The hard iron distortion is equivalent to a constant magnetic field vector observed by the 3-axis magnetic sensors. Compensating for soft iron effects is a bit more difficult than for hard iron effects. The effects produced by the soft iron distortion which is correlating with the hard iron can be attributed to the hard iron distortion. As a result, the magnetic field caused by soft iron distortion is in proportion to the earth's magnetic field. Poisson has introduced the mathematics model to illustrate the relationship between the earth's magnetic field and the magnetic field sensing by the magnetic sensors. The latter one is the magnetic field combining the former one and the magnetic field caused by the soft iron and hard iron distortion [7]. The model can be represented as the following 12-parameter matrix formula:

$$\mathbf{m} = \mathbf{k}\mathbf{m}^b + \mathbf{b} \tag{1}$$

Where

$$\mathbf{m} = \begin{bmatrix} m_x \\ m_y \\ m_z \end{bmatrix}, \quad \mathbf{m}^b = \begin{bmatrix} m^b{}_x \\ m^b{}_y \\ m^b{}_z \end{bmatrix},$$

$$\mathbf{k} = \begin{bmatrix} 1+a & b & c \\ d & 1+e & f \\ g & h & 1+k \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

$\mathbf{m}^b$ is the earth's magnetic field vector represented in vehicle-frame coordinates.

$\mathbf{m}$ is the combination magnetic field represented in vehicle-frame coordinates sensed by 3-axis magnetic sensors.

In the above, p, q, r are determined by hard iron materials. $a, b, c, d, e, f, g, h, k$ are determined by soft iron materials. They are generally called coefficients of magnetic deviation.

Rearranging (1) yields

$$\mathbf{m}^b = \mathbf{s}\begin{bmatrix} 1 \\ m_x \\ m_y \\ m_z \end{bmatrix} \tag{2}$$

Here we introduce the matrix $\mathbf{s}$ which is convenient for compensation. In the subsequent section, we will also name the twelve coefficients of matrix $\mathbf{s}$ as the twelve coefficients of magnetic deviation without confusion of original meaning.

III. 12-STATE KF ALGORITHM

For the convenience of derivation, we choose the following coordinate frame throughout the remains of this paper. The z axis points from the earth's center toward the system location. The y axis points toward the magnetic north in the plane orthogonal to the z axis. The x axis points east to complete the orthogonal, right-handed rectangular coordinate system. We assume that the projection of the earth's magnetic field vector on y axis is m_0. The local magnetic inclination angle, the angle of the local magnetic field to the surface of the earth, is θ_{incl}. The earth's magnetic field vector can be completely described by $\mathbf{m}^t = \begin{bmatrix} 0 & m_0 & -m_0 \tan\theta_{incl} \end{bmatrix}^T$. We can aim the AHRS's heading direction at the magnetic north and rotate the AHRS continuously after initial alignment. The rotation matrix $\mathbf{R}_{b2t}$ exhibits high accuracy during rotation for the gyro's short-term precision, therefore

$$\mathbf{m}^b = \mathbf{R}_{t2b}\mathbf{m}^t \tag{3}$$

Where $\mathbf{R}_{b2t} = \mathbf{R}_{t2b}^T$.

The 12-state KF states are defined to be

$$\begin{aligned}\mathbf{X} = [&s_{11} \quad s_{12} \quad s_{13} \quad s_{14} \quad s_{21} \quad s_{22} \\ &s_{23} \quad s_{24} \quad s_{31} \quad s_{32} \quad s_{33} \quad s_{34}]^T\end{aligned} \tag{4}$$

These twelve states are best represented as constant (but unknown) random variable so that they are modeled as $\dot{\mathbf{X}}(t) = 0$. There are measurement errors in the output of the magnetoresistive sensor and the rotation matrix $\mathbf{R}_{b2t}$. The measurement equation can be derived from (2) by adding measurement errors.

$$\mathbf{Z} = \mathbf{m}^b = \mathbf{H}\mathbf{X} + \mathbf{V} \tag{5}$$

where

$$\mathbf{m}^T = [m_x \quad m_y \quad m_z], \mathbf{V} = [v_x \quad v_y \quad v_z]^T,$$

$$\mathbf{H} = \begin{bmatrix} 1 & \mathbf{m}^T & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \mathbf{m}^T & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \mathbf{m}^T \end{bmatrix}$$

$\mathbf{V}$ is the earth's magnetic field error on the east-north(magnetic north)-up coordinate frame which is equivalent to the measurement error of magnetic sensor and the rotation matrix.

The twelve coefficients of magnetic deviation can be estimated step by step using KF as described below

$$\widehat{\mathbf{X}}_{k,k-1} = \widehat{\mathbf{X}}_{k-1} \tag{6}$$

$$\hat{\mathbf{X}}_k = \widehat{\mathbf{X}}_{k,k-1} + \mathbf{K}_k(\mathbf{Z}_k - \mathbf{H}_k\widehat{\mathbf{X}}_{k,k-1}) \tag{7}$$

$$\mathbf{K}_k = \mathbf{P}_{k,k-1}\mathbf{H}_k^T(\mathbf{H}_k\mathbf{P}_{k,k-1}\mathbf{H}_k^T + \mathbf{R}_k)^{-1} \tag{8}$$

$$\mathbf{P}_{k,k-1} = \mathbf{P}_{k-1} \tag{9}$$

$$\mathbf{P}_k = (\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)\mathbf{P}_{k,k-1} \tag{10}$$

IV. 14-STATE EKF ALGORITHM

As far as the true magnetic north and local inclination angle of magnetic field are concerned, they are difficult to acquire in normal application situation without the aid of outside information. And the self-acquisition of the magnetic north and local magnetic inclination angle will probably introduce large measurement error which does harm to the calibration of magnetic deviation. As a result of rotating the AHRS continuously, the measurement is redundant. We can take initial heading angle ψ_0 and local magnetic inclination angle θ_{incl} into account as the states of 14-state EKF in addition to the twelve coefficients of magnetic deviation.

Equation (3) can be transformed into

$$\mathbf{m}^b = \mathbf{R}_{t2b}\mathbf{R}_{\psi_0}\mathbf{m}^t = \mathbf{R}_{t2b}\mathbf{R}_{\psi_0}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \tan\theta_{incl} \end{bmatrix}\begin{bmatrix} 0 \\ m_0 \\ -m_0 \end{bmatrix} \tag{11}$$

Substituting (2) into (11) yields

$$\begin{bmatrix} 0 \\ m_0 \\ -m_0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \dfrac{1}{\tan\theta_{incl}} \end{bmatrix} \mathbf{R}_{\psi_0}^T \mathbf{R}_{b2t} \mathbf{s} \begin{bmatrix} 1 \\ m_x \\ m_y \\ m_z \end{bmatrix} \quad (12)$$

where

$$\mathbf{R}_{b2t} = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \text{ and}$$

$$\mathbf{R}_{\psi_0} = \begin{bmatrix} \cos\psi_0 & \sin\psi_0 & 0 \\ -\sin\psi_0 & \cos\psi_0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The 14-state EKF states are defined to be

$$\begin{aligned} \mathbf{X} = [&s_{11} \quad s_{12} \quad s_{13} \quad s_{14} \quad s_{21} \quad s_{22} \quad s_{23} \quad s_{24} \\ &s_{31} \quad s_{32} \quad s_{33} \quad s_{34} \quad \psi_0 \quad \tan\theta_{incl}]^T \end{aligned} \quad (13)$$

These fourteen states are best represented as constant (but unknown) random variable so that they are modeled as $\dot{\mathbf{X}}(t) = 0$. The measurement equation can be derived from (12) by adding measurement errors.

$$\mathbf{Z} = \begin{bmatrix} 0 \\ m_0 \\ -m_0 \end{bmatrix} = \mathbf{h}(\mathbf{X}) + \mathbf{V} \quad (14)$$

where

$$\mathbf{h}(\mathbf{X}) = \begin{bmatrix} h_1(\mathbf{X}) \\ h_2(\mathbf{X}) \\ h_3(\mathbf{X}) \end{bmatrix},$$

$$\begin{aligned} h_1(\mathbf{X}) = &(R_{11}\cos\psi_0 - R_{21}\sin\psi_0)(s_{11} + m_x s_{12} + m_y s_{13} + m_z s_{14}) \\ &+ (R_{12}\cos\psi_0 - R_{22}\sin\psi_0)(s_{21} + m_x s_{22} + m_y s_{23} + m_z s_{24}) \\ &+ (R_{13}\cos\psi_0 - R_{23}\sin\psi_0)(s_{31} + m_x s_{32} + m_y s_{33} + m_z s_{34}) \end{aligned} \quad (15)$$

$$\begin{aligned} h_2(\mathbf{X}) = &(R_{11}\sin\psi_0 + R_{21}\cos\psi_0)(s_{11} + m_x s_{12} + m_y s_{13} + m_z s_{14}) \\ &+ (R_{12}\sin\psi_0 + R_{22}\cos\psi_0)(s_{21} + m_x s_{22} + m_y s_{23} + m_z s_{24}) \\ &+ (R_{13}\sin\psi_0 + R_{23}\cos\psi_0)(s_{31} + m_x s_{32} + m_y s_{33} + m_z s_{34}) \end{aligned} \quad (16)$$

$$\begin{aligned} h_3(\mathbf{X}) = &R_{31}(s_{11} + m_x s_{12} + m_y s_{13} + m_z s_{14}) / \tan\theta_{incl} \\ &+ R_{32}(s_{21} + m_x s_{22} + m_y s_{23} + m_z s_{24}) / \tan\theta_{incl} \\ &+ R_{33}(s_{31} + m_x s_{32} + m_y s_{33} + m_z s_{34}) / \tan\theta_{incl} \end{aligned} \quad (17)$$

In (15) – (17), the measurement matrix $\mathbf{H}$ is dependent on the states of EKF in a nonlinear fashion. We need to linearize (15) – (17) and calculate the partial differential of $\mathbf{H}_k$ to obtain the measurement matrix of EKF. We are not going into detail about the process here due to the length of the paper.

According to EKF implementation (18) – (22), the optimum estimation of the fourteen states will be produced. Then the solving of local magnetic inclination angle θ_{incl} will proceed to the next step.

$$\hat{\mathbf{X}}_{k,k-1} = \hat{\mathbf{X}}_{k-1} \quad (18)$$

$$\hat{\mathbf{X}}_k = \hat{\mathbf{X}}_{k,k-1} + \mathbf{K}_k(\mathbf{Z}_k - \mathbf{h}(\hat{\mathbf{X}}_{k,k-1}, k)) \quad (19)$$

$$\mathbf{K}_k = \mathbf{P}_{k,k-1}\mathbf{H}_k^T(\mathbf{H}_k\mathbf{P}_{k,k-1}\mathbf{H}_k^T + \mathbf{R}_k)^{-1} \quad (20)$$

$$\mathbf{P}_{k,k-1} = \mathbf{P}_{k-1} \quad (21)$$

$$\mathbf{P}_k = (\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)\mathbf{P}_{k,k-1} \quad (22)$$

V. TEST RESULTS

A. *Experiment with calibration*

The AHRS is mounted on a hexahedron which is made of aluminum shown in Fig. 1. A sheet iron is attached immovably to the hexahedron, right on the shell of AHRS (just above the magnetoresistive sensors). The goal is to induce artificially the soft iron and hard iron distortion to AHRS. A simple calibration procedure is designed to emulate the down-to-earth calibration case of unmanned aerial vehicles (UAV):

- Level the AHRS on a horizontal surface
- Rotate the AHRS one cycle with axis of rotation which is perpendicular to the horizontal plane

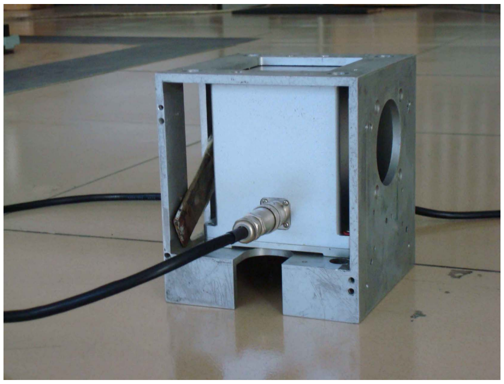

Fig. 1. AHRS mounted on a hexahedron

- Incline the AHRS about 30 degree
- Rotate the AHRS one cycle with the same axis of rotation

The roll, pitch and heading during calibration procedure are solved by the strapdown algorithm after initial alignment. The curve is shown in Fig. 2.

The true local magnetic inclination angle approximates to 56.8 degree and the true initial heading angle is about 0 degree.

On the assumption that inaccuracy of measurement exists, we simulate the accurate level of the magnetic inclination angle and the initial heading angle into three cases which are described as following:

- Case 1: measurement with no errors,

  We set $\psi_0 = 0^\circ, \theta_{incl} = 56.8^\circ$.

- Case 2: measurement with small errors,

  We set $\psi_0 = 3^\circ, \theta_{incl} = 55.8^\circ$.

- Case 3: measurement with large errors,

  We set $\psi_0 = 10^\circ, \theta_{incl} = 51.8^\circ$.

12-state KF and 14-state EKF are employed respectively to estimate the twelve coefficients of magnetic deviation with all three cases.

Following initial state value is set if the 12-state KF approach is chosen:

$$\mathbf{X}_0 = [0 \;\; 1 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 1 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 1]^T$$

Following initial state value is set if the 14-state EKF approach is chosen:

$$\mathbf{X}_0 = [0 \;\; 1 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 1 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 1 \;\; \psi_0 \;\; \tan(\theta_{incl})]^T$$

$\sigma = 1mgauss$, where σ is the error variance.

As Fig. 3, 4 shows, the 14-state EKF can estimate effectively the inclination angle, the initial heading angle and the twelve coefficients of magnetic deviation in case 3. When we rotate the AHRS in the second cycle, the tilt angle is changing from 0 to 30 degree. The observability of the system state on z axis increases. The estimation of the inclination angle, the initial heading angle and the twelve coefficients of magnetic deviation is updated sharply. The system states converge finally.

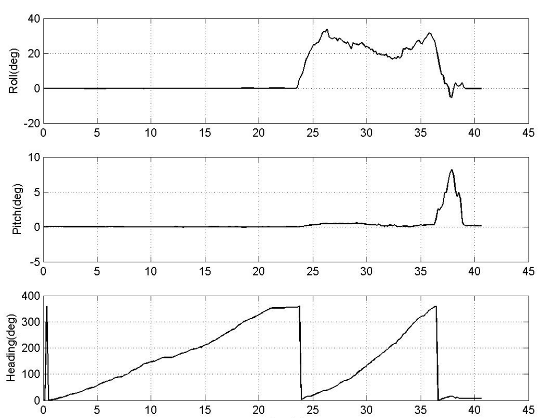

Fig. 2. The curve of roll, pitch and heading

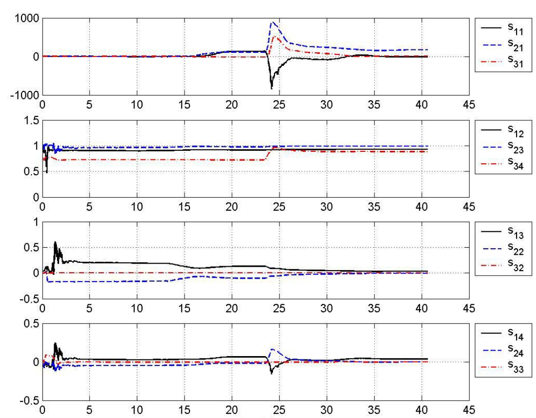

Fig. 3. Estimation curve of the twelve coefficients of magnetic deviation

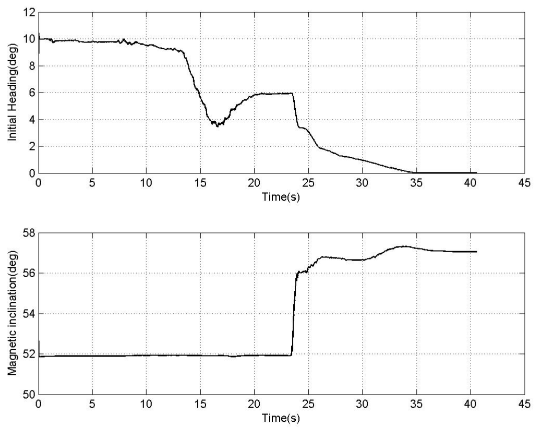

Fig. 4. Estimation curve of the initial heading angle and the inclination angle

B. Experiment with rotation

In this experiment, the AHRS is placed with x axis, y axis and z axis direct upward separately. When the AHRS is turning around in a circle in a horizontal plane, the magnetic readings of the horizontal plane before compensation produce the curves shown in Figure 5. Note here that the plot is three ellipses. The ideal effect after compensation of the soft iron and hard iron distortion is to change three ellipses to three circles, with circle centers are on the 0, 0 origin and the radiuses are the same.

Fig. 5 shows the results of compensation adopting 12-state KF algorithm. Fig. 6 shows the results of compensation adopting 14-state EKF algorithm. When the magnetic inclination angle and the initial heading angle are deviating from the true value, the outcomes of 14-state EKF are three cycles which fits one another. The curves of 12-state KF method present three ellipses instead of three cycles and they are not superposed. When the magnetic inclination angle and the initial heading angle are accurate, the outcomes of 12-state KF are three cycles which fits one another approximately. The test data indicates that the estimation validity of 12-state KF largely depends on the accuracy of the magnetic inclination angle and the initial heading angle. And these factors don't affect the estimation result of 14-state EKF method.

C. Experiment with heading

In this test, the AHRS is mounted on one end of an aluminum stick shown in Figure 7. The other end is fixed on

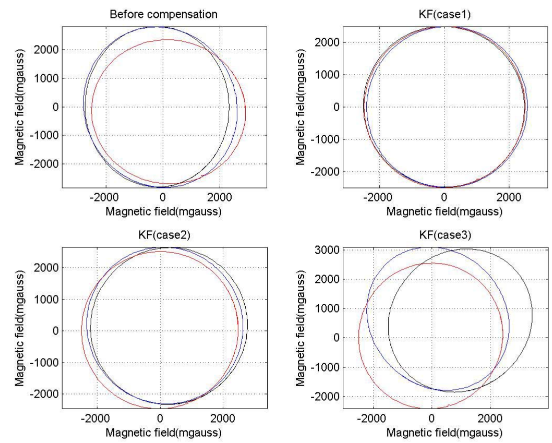

Fig. 5. The magnetic readings of the horizontal plane using KF

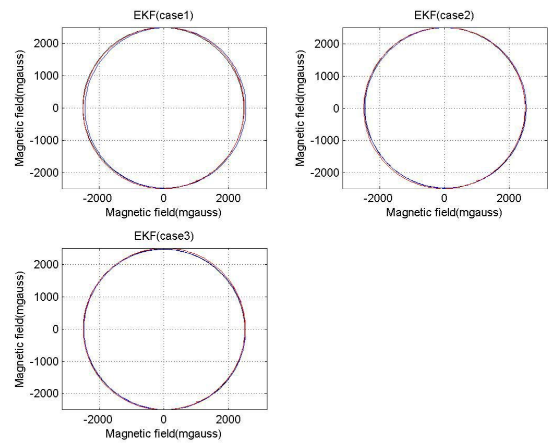

Fig. 6. The magnetic readings of the horizontal plane using EKF

the rotation table which also has position function. The sheet iron is still securely attached to the AHRS with the effect of soft iron and hard iron distortion. Tilt the AHRS with 0, 30 and 60 degrees separately and circumrotate the aluminum stick with one cycle. The data is collected every 30 degrees and the AHRS is sampled on twelve points every cycle.

Comparison of the heading angle errors curves before and after compensation is shown in Fig. 8, 9 and 10, which are relative to the tilt angle of 0, 30 and 60 degrees. The maximum error values before compensation are 7.9, 10.7 and 9.9 degrees respectively. In case 2 that the magnetic inclination angle and the initial heading angle include small errors, the maximum error values after compensation are 0.8, 5.8 and 8.8 degrees using 12-state KF algorithm. On the other

Fig. 7. AHRS on the rotation table

hand, if the magnetic inclination angle and the initial heading angle include no errors in case 1, the maximum error values after compensation are 0.6, 1.0 and 1.6 degrees using 12-state KF algorithm. With regard to 14-state EKF method, the maximum error values after compensation are 0.7, 0.8 and 1.0 degrees in case 3 that the magnetic inclination angle and the initial heading angle include large errors. The test data illustrates that errors of the magnetic inclination angle and the initial heading angle are large influences on 12-state KF's compensation effect. As is listed on Table I, the heading errors variances of 14-state EKF are less than 0.4 degree in all three cases. Concerning to 14-state EKF's compensation effect, it is not related to the errors and the performance is the best.

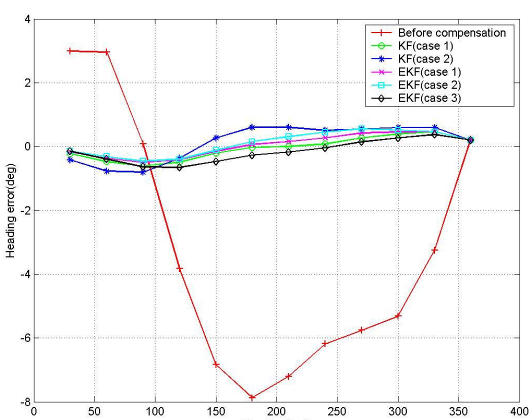

Fig. 8. Comparison of heading angle error before and after compensation, tilt angle is 0 degree

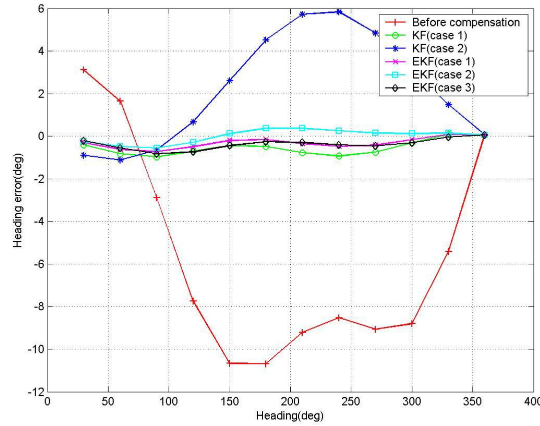

Fig. 9. Comparison of heading angle error before and after compensation, tilt angle is 30 degree

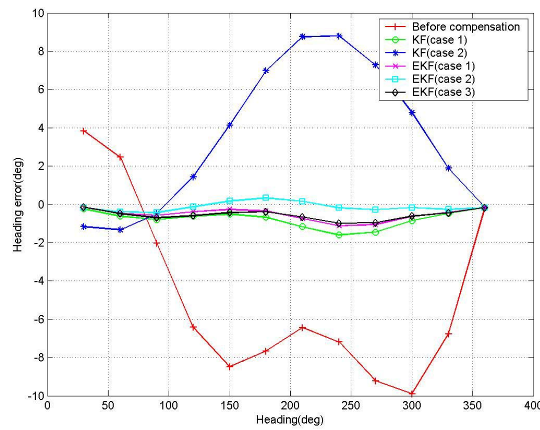

Fig. 10. Comparison of heading angle error before and after compensation, tilt angle is 60 degree

TABLE I

COMPARISON OF HEADING ANGLE ERROR VARIANCE AFTER COMPENSATION

Tilt angle (deg)	KF case 1	KF case 2	EKF case 1	EKF case 2	EKF case 3
0	0.4	0.6	0.3	0.4	0.3
30	0.4	2.6	0.3	0.3	0.3
60	0.4	3.8	0.3	0.2	0.3

VI. CONCLUSION

In real application environment, the effects of the soft iron and hard iron distortion will distort, or bend the earth's field.

The magnetic deviation should be accounted for and removed from the magnetic readings. With local inclination angle of magnetic field and initial heading angle as unknown, we employ 14-state EKF to calibrate the initial heading angle and local inclination angle of magnetic field, in addition to 12 parameters of soft iron and hard iron effects. The algorithm outperforms 12-state KF methods when there are large errors in the measurement of local magnetic inclination angle and initial heading angle. The experiment results demonstrate that the heading accuracy (variance) after 14-state EKF estimation is 0.4 degree for tilt angle of 60 degree giving that the measurement error of initial heading angle and local magnetic inclination angle are 10 degree and 5 degree respectively. The whole calibration procedure is as simple as rotating the AHRS two cycles with attitude angle changing. This paper has alluded to improving the performance of the AHRS on applications of UAV.

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